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MODAL IDENTIFICATION OF STRUCTURES FROM THE RESPONSES AND RANDOM DECREMENT SIGNATURES

By

Sam R. Ibrahim

G.L. Goglia, Principal Investigator

Final Report

Prepared for the
National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia

Under
Research Grant NSG 1325
B. R. Hanks, Technical Monitor
Structures and Dynamics Division

October 1977

Old Dominion University Research Foundation
Norfolk, Virginia 23508
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December 5, 1977

Mr. B.R. Hanks
National Aeronautics and Space Administration
Langley Research Center - Mail Code 230
Hampton, VA 23665

Dear Mr. Hanks:

Reference research grant NSG 1325, G.L. Goglia principal investigator.

Enclosed is the final technical report on work performed under the referenced grant.

If there are any question on the report, please contact this office.

Sincerely,

Maxine Lippman
Publications Coordinator

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ABSTRACT

Part I of this report describes the theory and application of a method which utilizes the free response of a structure to determine its vibration parameters. The time domain free response is digitized and used in a digital computer program to determine the number of modes excited, the natural frequencies, the damping factors, and the modal vectors. The underlying theory is developed, including the basis of the computational procedures required, as well as the requirements regarding the sampling rate in the digitizing procedure. Consideration is given to the practical application of the theory. The technique is applied to a complex "generalized payload" model previously tested using sine sweep method and analyzed by NASTRAN. Ten modes of the payload model are identified.

In case free decay response is not readily available, an algorithm is developed in part II to obtain the free responses of a structure from its random responses, due to some unknown or known random input or inputs, using the random decrement technique without changing time correlation between signals. The algorithm is tested using ramdom responses from a "generalized payload" model and from the "space shuttle" model. The resulting free responses are then used to identify the modal characteristics of the two systems.

Key Words:

Vibration Testing
Random Decrement Technique
Modal Testing
Identification

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MODAL IDENTIFICATION OF STRUCTURES FROM THE RESPONSES AND RANDOM DECREMENT SIGNATURES

By

Sam R. Ibrahim[1]

PART I

TIME DOMAIN MODAL IDENTIFICATION TECHNIQUE

Introduction

The experimental determination of the natural frequencies and modes of structures is usually pursued through the application of frequency response or other frequency domain methods. Many test procedures have been proposed (refs. 1 to 10), differing in the manner in which the structure is excited, the quantities which are measured, and the manner in which the experimental data are analyzed. Analysis of frequency domain methods have shown that there are limits on the degree of damping and the closeness of natural frequencies beyond which frequency response methods cannot yield accurate information about the vibration parameters (refs. 11 to 14). These limitations are essentially due to modal interference which can obscure the separate observation of individual modes and natural frequencies. Most modal vibration test methods are, then, based on assumptions of negligible mode coupling, although some methods have been introduced to deal with modal interference (refs. 15 to 17). Special methods, though, have the main disadvantage that it must somehow be determined in advance that special attention is, in fact, required.

These particular problems encountered using frequency response methods have led to the consideration of time domain based methods

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in vibration testing. The direct use of time response information, without transformation to the frequency domain, should not necessarily require assumptions about the interference of modes due to heavy damping or closely spaced natural frequencies, and would thus eliminate the necessity for special procedures in these cases.

The theory and application of such a time domain method were presented in references 18 and 19. The method involves the use of transient response data in determining the differential equations of a lumped parameter model of the system under test, followed by analysis of the mathematical model to determine the vibration parameters. An alternative time domain technique is described in this paper. This method differs from the one noted previously in that here a mathematical model or differential equation of the structure is not developed; rather, the structure's free response is used directly in a computational procedure which yields the vibration parameters. In both methods, either the acceleration, velocity or displacement response may be used, but in the earlier method it was required that the recorded response be numerically integrated twice in the computational procedure, while in the present case no such integration is necessary.

Theory

The test procedure described utilizes the free response of the structure under test. As for the previously referenced time domain method (refs. 18, 19), the free response is generated after the sudden termination of excitation of the structure by a forcing function containing contributions in the frequency range of interest. It is necessary that a mode contribute to the response in order that it can be detected. Narrow band random excitation and rapid frequency sweeps have been used with success in laboratory experiments. In this way, a considerable amount of energy may be built up in the structure during excitation, to be dissipated during the free response. This contrasts with transient procedures which depend upon, and can be seriously limited by, the amount of energy which can be imparted to the structure by a pulse or a step.

Dependent upon the form of excitation, the response may, in theory, contain an infinite number of modes, although it is not physically possible in practice. For the theoretical development described here, it is assumed that a finite, but for the present known, number of modes is excited. While this assumption is necessary and fundamental to the technique described, it also allows the testing of a complex structure to be done in a series of tests, each of which covers a frequency spectrum only as wide as desired. This makes the procedure analogous to a series of tests on simple systems rather than a single test of a large system, keeping both instrumentation and computational complexities to a lower level.

It is convenient to imagine the response that is thus monitored as being equivalent to that ot a hypothetical n-degree of freedom lumped parameter system with all modes excited and in which the masses undergo the same motions as the measuring stations on the structure under test. The natural frequencies of the hypothetical lumped system are the same as those of the structure, and the mode shapes of the lumped system are equivalent to the modal displacements of the structure measured at the measuring stations. The identification of the vibration parameters of the structure is now pursued through the determination of the vibration parameters of the lumped system.

The lumped system is assumed to be described by the following equation during its free response:

$$\underline{\underline{M}}\ddot{\underline{x}} + \underline{\underline{C}}\dot{\underline{x}} + \underline{\underline{K}}\underline{x} = \underline{0} \qquad (1)$$

$\underline{\underline{M}}$, $\underline{\underline{C}}$, and $\underline{\underline{K}}$ are nxn matrices, while $\underline{x}$, $\dot{\underline{x}}$, and $\ddot{x}$ are n-dimensional vectors. The solution of this equation is assumed to be

$$\underline{x} = \underline{p}e^{\lambda t} \qquad (2)$$

whence

$$[\lambda^2\underline{M} + \lambda\underline{C} + \underline{K}]\underline{p} = 0 \tag{3}$$

Equation (3) defines 2n values for λ, which are also the roots of the characteristic equation of the system. For each root, there is a corresponding vector, $\underline{p}$, of relative displacements of the coordinates of the system; λ and the elements of $\underline{p}$ are real for overdamped modes, but for underdamped modes they are complex, and occur in conjugate pairs. Each conjugate pair combines to produce a real mode shape corresponding to a single natural frequency and damping factor. For a complex conjugate pair of modal vectors having the first element of each normalized to unity and having the kth elements $c \pm id$, the displacement of the kth coordinate is $\sqrt{c^2 + d^2}$, and its motion lags that of the first coordinate by $\tan^{-1}(d/c)$. For a complex conjugate pair of roots $a \pm ib$, the damped natural frequency is $\omega_{nd} = b$, the undamped natural frequency is $\omega_n = \sqrt{a^2 + b^2}$, and the damping ratio is

$$\eta = \frac{a}{\sqrt{a^2 + b^2}}$$

The problem of modal vibration testing is to determine, from the test data, the values of λ and $\underline{p}$ which satisfy equation (3). The test data required may be any one of the displacement, velocity or acceleration responses assumed for the present to be measured at the n stations. The response, be it displacement, velocity or acceleration, consists of a sum of contributions made by all the modes, and can be written as

$$\underline{x} = \sum_{j=1}^{2n} \underline{p}_j \, e^{\lambda_j t} \tag{4}$$

The response at time t_i is

$$\underline{x}(t_i) = \underline{x}_i = \sum_{j=1}^{2n} \underline{p}_j e^{\lambda_j t_i} \tag{5}$$

and the response vectors measured at 2n different instances of time can be written in matrix form as

$$[\underline{x}_1 \underline{x}_2 \cdots \underline{x}_{2n}] = [\underline{p}_1\ \underline{p}_2 \cdots \underline{p}_{2n}] \begin{bmatrix} e^{\lambda_1 t_1} & e^{\lambda_1 t_2} & e^{\lambda_1 t_{2n}} \\ e^{\lambda_2 t_1} & \cdots\cdots & e^{\lambda_2 t_{2n}} \\ \vdots & & \vdots \\ e^{\lambda_{2n} t_1} & & e^{\lambda_{2n} t_{2n}} \end{bmatrix} \tag{6}$$

or

$$\underline{X} = \underline{P}\underline{\Lambda} \tag{7}$$

Responses that occur at time Δt later with respect to those of equation (7) are

$$[\underline{y}_1\ \underline{y}_2 \cdots \underline{y}_{2n}] = [\underline{p}_1\ \underline{p}_2 \cdots \underline{p}_{2n}] \begin{bmatrix} e^{\lambda_1 (t_1+\Delta t)} & \cdots\cdots & e^{\lambda_1 (t_{2n}+\Delta t)} \\ \vdots & & \vdots \\ e^{\lambda_{2n} (t_1+\Delta t)} & \cdots\cdots & e^{\lambda_{2n} (t_{2n}+\Delta t)} \end{bmatrix} \tag{8}$$

where

$$\underline{y}_i = \underline{x}(t_i + \Delta t), \quad i = 1,2,\ldots,2n$$

Equation (8) can be rewritten as

$$[\underline{y}_1 \ \underline{y}_2 \cdots \underline{y}_{2n}] = [\underline{q}_1 \ \underline{q}_2 \cdots \underline{q}_{2n}]\underline{\Lambda} \qquad (9)$$

or

$$\underline{Y} = \underline{Q}\underline{\Lambda} \qquad (10)$$

where

$$\underline{q}_i = \underline{p}_i e^{\lambda_i \Delta t}, \ i = 1,2,\ldots,2n \qquad (11)$$

In a similar manner, responses that occur at time Δt later with respect to those of $\underline{Y}$ are

$$[\underline{z}_1 \ \underline{z}_2 \cdots \underline{z}_{2n}] = [\underline{r}_1 \ \underline{r}_2 \cdots \underline{r}_{2n}] \qquad (12)$$

or

$$\underline{Z} = \underline{R}\underline{\Lambda} \qquad (13)$$

where

$$\underline{z}_i = \underline{x}(t_i + 2\Delta t), \ i = 1,2,\ldots,2n \qquad (14)$$

and

$$\underline{r}_i = \underline{q}_i e^{\lambda_i \Delta t} = \underline{p}_i e^{2\lambda_i \Delta}, i = 1,2,\ldots,2n \qquad (15)$$

The responses given by equations (7), (10) and (13) can be manipulated to solve for the eigenvalues and modal vectors. First, equations (7) and (10) are grouped to be written as

$$\begin{bmatrix} \underline{X} \\ \underline{Y} \end{bmatrix} = \begin{bmatrix} \underline{P} \\ \underline{Q} \end{bmatrix} \underline{\Lambda} \tag{16}$$

or

$$\underline{\Phi} = \underline{\Psi}\underline{\Lambda} \tag{17}$$

and equations (10) and (13) are written as

$$\begin{bmatrix} \underline{Y} \\ \underline{Z} \end{bmatrix} = \begin{bmatrix} \underline{Q} \\ \underline{R} \end{bmatrix} \underline{\Lambda} \tag{18}$$

or

$$\hat{\underline{\Phi}} = \hat{\underline{\psi}}\underline{\Lambda} \tag{19}$$

It is shown in the appendix that the inverses of the matrices $\underline{\Phi}$ and $\hat{\underline{\Phi}}$ exist. Also, the inverses of $\underline{\psi}$ and $\hat{\underline{\Psi}}$ exist, because their columns are proportional to the modal vectors, which must be linearly independent. Hence, equations (17) and (19) can be manipulated to eliminate $\underline{\Lambda}$, giving the result

$$\hat{\underline{\Phi}}\, \hat{\underline{\Phi}}^{-1}\, \underline{\Psi} = \hat{\underline{\Psi}} \tag{20}$$

This equation relates each column, $\underline{\Psi}_i$, of $\underline{\Psi}$ to the corresponding column, $\hat{\underline{\Psi}}_i$, of $\hat{\underline{\Psi}}^{-i}$ through

$$\hat{\underline{\Phi}}\, \underline{\Phi}^{-1}\, \underline{\Psi}_i = \hat{\underline{\Psi}}_i,\ i = 1,2,\ldots,2n \tag{21}$$

The column vectors $\underline{\Psi}_i$ and $\hat{\underline{\Psi}}_i$ are also related through equations (11) and (15) by

$$\hat{\underline{\Psi}}_i \doteq e^{\lambda_i \Delta t} \underline{\Psi}_i \tag{22}$$

Equations (21) and (22) can be combined to give

$$\hat{\underline{\underline{\Phi}}}\underline{\underline{\Phi}}^{-1} \underline{\Psi}_i = e^{\lambda_i \Delta t} \underline{\Psi}_i \tag{23}$$

This equation is an eigenvalue problem which enables the measured response to be used in the calculation of eigenvectors and eigenvalues which are related to the vibration parameters of the structure. The modal vectors are merely the first n elements of the eigenvectors of $\hat{\underline{\underline{\Phi}}}\underline{\underline{\Phi}}^{-1}$, but the relationship between the eigenvalues and λ_1, the characteristic roots of equation (1), requires further analysis. This relationship is discussed below with reference to an eigenvalue $\beta + i\gamma$, of $\hat{\underline{\underline{\Phi}}}\underline{\underline{\Phi}}^{-1}$, which is related to a characteristic root $a + ib$. It is to be noted that the eigenvalue $\beta - i\gamma$ and the corresponding characteristic root $a - ib$ would also exist, but for simplicity it will henceforth be assumed that $b \geq 0$ with the understanding that for every $b > 0$ the conjugate root $a - ib$ also exists. The eigenvalue $\beta + i\gamma$ is related to $a + ib$ through

$$\beta + i\gamma = e^{(a+ib)\Delta t} \tag{24}$$

Thus,

$$\beta = e^{a\Delta t} \cos b\Delta t$$

and

$$\gamma = e^{a\Delta t} \sin b\Delta t$$

from which

$$a = \frac{1}{2\Delta t} \ln \ (\gamma^2 + \beta^2) \tag{25}$$

and

$$b = \frac{i}{\Delta t} \tan^{-1} \ (\frac{\gamma}{\beta}) \tag{26}$$

Hence, real eigenvalues of $\underline{\hat{\Phi}}\underline{\Phi}^{-1}$ correspond to critically or over-damped modes because they represent real characteristic roots. Natural frequencies of underdamped modes are represented by complex conjugate eigenvalues and must be determined using equations (25) and (26).

Sampling Rate

Equation (26) does not allow the natural frequencies to be determined uniquely, because it can be written as

$$b = \frac{1}{\Delta t} [\tan^{-1} \ (\frac{\gamma}{\beta}) \ + k\pi] \ldots \begin{cases} 0<\tan^{-1}(\frac{\gamma}{\beta})<\pi \\ \quad k = 0,1,2,\ldots \end{cases} \tag{27}$$

Thus for each damped natural frequency b, there is a relationship between Δt and the value of k which should be used in equation (27); Δt is the time delay used to generate the delayed response matrices $\underline{Y}$ and $\underline{Z}$ from $\underline{X}$, and thus $\frac{1}{\Delta t}$ represents the sampling rate required to obtain these delayed responses, although consecutively sampled response vectors need not be used as the columns of $\underline{X}$.

To avoid ambiguity in the use of equation (27), it is necessary to specify that all the modes which contribute to the response correspond to frequencies which can be calculated from the equation by using only a single value for k. It is required then that for all values of b in the frequency range of interest,

$$\frac{k\pi}{\Delta t}<b<\frac{(k+1)\pi}{\Delta t} \tag{28}$$

for some value of k. Writing the minimum and maximum values of damped natural frequencies as related to the minimum and maximum values of the frequency range of interest as

$$b_{min}>2\pi f_{min}$$

$$b_{max}<2\pi f_{max}$$

and the sampling frequency f_s as

$$f_s = \frac{1}{\Delta t},$$

equation (28) requires that

$$\left.\begin{aligned} f_s k &< 2f_{min} \\ f_s(k+1) &> 2f_{max} \end{aligned}\right\} k = 0,1,2,\ldots$$

or

$$\frac{2f_{max}}{k+1}<f_s<\frac{2f_{min}}{k}, \; k = 0,1,2,\ldots \tag{29}$$

This equation can be used to relate the frequency range of interest to the acceptable value or values of k for use in equation (27) through

$$\frac{2f_{max}}{k+1}<\frac{2f_{min}}{k}$$

or

$$\frac{f_{max}}{f_{min}} < \frac{k+1}{k}, \quad k = 0,1,2,... \tag{30}$$

The determination of frequencies in a range from f_{min} equal to zero, to some upper limit f_{max}, requires, from equation (30), that $k = 0$. In addition, equation (29) requires that

$$f_s > 2f_{max} \tag{31}$$

This requirement on f_s may present practical difficulties if f_{max} is excessively high, but it is seen below that the requirement for determining high frequencies can be made less demanding if testing is done for a frequency range having a lower limit greater than zero and thus allowing values other than zero to be used for k. Equation (30) defines the maximum width of the frequency range which can be used with the various values of k or, conversely, it can be thought of as defining the possible values of k which can be used for a desired frequency band. This information can then be used in equation (29) to determine the allowable sampling frequencies. The maximum width of the frequency range is f_{max}/f_{min} equal to two, for which the lowest sampling frequency is f_{max} and k equals unity. As the frequency range decreases, the sampling rate may be chosen from an allowable range, and with frequency ranges that are sufficiently narrow it becomes possible to choose sampling frequencies from several allowable ranges, each range corresponding to a different value of k. This is demonstrated in an example below.

It is supposed that all the natural frequencies up to 1000 Hz are to be determined for a structure. Two approaches can be taken: either the entire range can be covered at once, or it may be subdivided into several narrower ranges, each to be covered separately.

A single test covering the range from 0 to 1000 Hz would require from equation (31), that the sampling frequency be any value above

2000 Hz, and k = 0 should be used in equation (27).

Division into narrower frequency ranges must keep in mind that, for any range having a nonzero lower limit, the upper frequency limit must be not more than twice the lower frequency limit. (If the upper frequency limit is more than twice the lower frequency limit, k should be 0). Suitable ranges for the case being considered would be 0 to 400 Hz, 400 to 700 Hz, and 700 to 1000 Hz. Each frequency range must be studied using response information which contains (through filtering or other means) only frequency components in the range of interest. The frequency range 0 to 400 Hz must be studied using a sampling frequency of at least 800 Hz, with k = 0 in equation (27). The 400 to 700 Hz range requires, from equation (30), that

$$\frac{700}{400} < \frac{k+1}{k}$$

hence k = 0, 1 are acceptable, with corresponding sampling rates as determined from equation (29). These rates are $f_s > 1400$ Hz for k = 0, and 700 Hz $< f_s <$ 800 Hz for k = 1 in equation (29). For the 700 to 1000 Hz range, k = 0, 1, 2 are acceptable with sampling rates $f_s >$ 2000 Hz for K = 0, 1000 Hz $< f_s <$ 1400 Hz for k = 1 and 667 Hz $< f_s <$ 700 Hz for k = 2.

In comparison of sampling rates for the two approaches, the first requires a rate of at least 2000 Hz, while the second requires a rate of at least 800 Hz. The requirements for the second approach can be reduced still further if narrower frequency ranges are chosen.

Another solution to the problem of requiring high sampling rates for a certain frequency range of interest is the use of a tape recorder at a high recording speed, with the recorded response then played back at a lower speed during the digitization process. This can reduce the required sampling rate by a factor of the ratio of the two recorder speeds that have been used. This ratio is to be used later as a correction factor to obtain the actual structural frequencies.

Practical Application

An important assumption made in deriving the theory is that the number of measuring stations on the structure equals the number of degrees of freedom of the hypothetical lumped parameter system with all modes excited. Thus in performing a test it would be necessary to know in advance the number of equivalent degrees of freedom to be excited, so that the correct number of measuring stations could be employed. In practice, this information is not usually available, and even if it were it might not be possible or desirable to use the same number of measuring stations as there are equivalent degrees of freedom. This section describes a procedure whereby the theory developed thus far may be used in conjunction with any convenient number of measuring stations. As few as a single station may be used for determining any number of frequencies in a single test, and modal displacements at any number of points may be determined using as few as two stations at a time in a series of tests using one station as a reference.

Three possibilities exist in the relationship between the equivalent number of degrees of freedom excited and the number of stations at which measurements are made. The number of stations used may be greater than, equal to, or fewer than the number of degrees of freedom. Each situation requires a different computational approach, so the determination of which of the three possibilities is actually present is the first goal in the analysis of experimental data. These same possibilities arise and are dealt with in reference 19 and parallel procedures are used in dealing with them here. Verification is dealt with fully in reference 19, so emphasis here is placed only on describing the procedures employed.

The first step in the analysis of experimental data is the determination of the number of degrees of freedom of the associated hypothetical lumped parameter system. This is done by determining the number of independent modal vectors which contribute to the response, on the basis that the rank of the matrix $\underline{\Phi}$ is equal to the number of independent modal vectors, 2k, used to make up its columns. It is kept in mind that a pair of complex conjugate modal vectors corresponds to a single real underdamped mode, and thus for a structure in which all modes are underdamped, the number of real modes is half the number of modal vectors. Also, overdamping results in two real modal vectors corresponding to a single degree of freedom. Hence, the existance of 2k independent modal vectors corresponds to a corresponding lumped parameter system having k degrees of freedom.

First, the matrix $\underline{\Phi}$ is formed using the responses obtained from all the measuring stations which have been used. Then, in theory, it should only be necessary to successively calculate the determinants of submatrices of $\underline{\Phi}$, using the first m elements of the responses of m stations at 2m instants of time, where m takes on values from unity up to a value at which the determinant becomes zero. This value of m equals k + 1 whence the response of only k stations need to be used to identify the vibration parameters of the structure. In practice the computed determinant is never zero due to measurement noise and computer round-off; hence, instead of using the value of the determinant itself, the ratio of two successively calculated determinants should be used (ref. 20). The responses of the excess measurement stations may be used in a modification of the computational procedure as described in (ref. 19).

If the determinant check described above does not reveal the number of modes excited, then the number of modes is either equal to or greater than the number of measuring stations. In either case, it is required to increase the apparent number of stations through the generation of response vectors of higher order, with a corresponding increase in the number of time instances considered, thus increasing the order of $\underline{\Phi}$ and allowing the determinant check to be continued. The response vectors of higher order take the form

$$\underline{x}' = \begin{bmatrix} \underline{x} \\ \underline{x}_a \end{bmatrix} \tag{32}$$

where

$$\underline{x}_a = \underline{x}\ (t + \Delta\tau) \tag{33}$$

in which $\Delta\tau$ is any convenient value which must be different from Δt. The procedure can be repeated to triple, quadruple, etc. the order of the apparent response vector using other values for $\Delta\tau$, using the determinant check after each increase until the number of modes is determined. The frequencies are then computed for the appropriately enlarged $\underline{\Phi}$, and the mode shapes are give by the eigenvector elements which correspond to the original measuring stations.

Another problem of practical importance which arises is in the introduction of errors due to measurements and recording noise. In the next section, two methods are suggested to reduce the effect of measurement errors on the identified parameters. These methods were used in the second (payload model) experiment.

Measurements And Recording Noise

Two methods are used to minimize the effect of different kinds of noise on the identified parameters. These two methods are

1. Least Square Error Minimization: This can be accomplished by using more data than needed and finding the parameters with least error that satisfy the data. In such a case $\hat{\underline{\Phi}}$ and $\underline{\Phi}$ will be rectangular matrices and equation (23) will be:

$$[\hat{\underline{\Phi}}\underline{\Phi}^T]\ [\underline{\Phi}\underline{\Phi}^T]^{-1}\ \underline{\Psi}_i = e^{\lambda_i \Delta t}\ \underline{\Psi}_i \tag{34}$$

2. <u>The Use of Overspecified Math Model</u>: In this case the number of degrees of freedom of the mathematical model is larger than the number of modes to be identified. This will give an escape for some of the noise and thus improve the accuracy of the identified modes.

Experimental Results

The payload model is shown in figure 1. Sixteen accelerometers were fixed to the eight bulkheads, eight accelerometers on each side (fig. 2). Two data groups were used. Data group one had accelerometers 1 to 8. Data group two had accelerometers 9 to 16, and accelerometer 8 was a common accelerometer for the two data groups. A random input was applied at station 8. The input was cut off, and free responses from data group one were recorded on a tape recorder. The procedure was repeated for data group two. A two-way switch was used to cut off the random input and at the same time generate a D.C. signal, recorded on a separate channel of the tape recorder, was used to determine the start of the free response.

The free responses were filtered to eliminate frequency components higher than 350 Hz and then digitized at a sampling rate of 2000 samples/second. Only 500 points for each channel were stored to be used as data for the identification program. This corresponds to a record length of 0.25 second.

The noise/signal ratio for the resulting data was estimated at about 22 percent. This estimate was based on comparing two responses from station 8 that were recorded simultaneously on two channels of the tape recorder. The root mean square of the two records, rms and RMS, were calculated and the noise/signal ratio was estimated using the following formula:

$$N/S = \sqrt{\frac{(RMS - rms)^2}{RMS \times rms}}$$

Higher order response vectors were generated using equations (32) and (33). Results of the determinant check are shown in table 1, from which it is indicated that the system had 10 modes in its response. The determinant check indication is not as strong as in the cantilever beam case because of the higher level of noise in the payload model response.

Although it was known from the determinant check that the system's responses contained about 10 modes, a mathematical model with 20 degrees of freedom was used to identify the system. Also, a math model of 40 D.O.F. was used. The structures mode can be differentiated from noise modes by observing that the system's modes occur consistently in different computer runs. Table 2 shows the frequencies of the 10 modes obtained by this technique using a math model of 20 and 40 D.O.F. Also listed are frequencies obtained by sine sweep test and NASTRAN. Figures 3a to 3f show some of the identified mode shapes.

Conclusions

The theory and application of a time domain modal vibration testing technique are presented. The results of the two experiments reported in this work are very encouraging. The second experiment (payload model) proved that the technique is insensitive to measurement noise. While the data used for this experiment had about 22 percent noise, the identified frequencies compared extremely well with the analytical (NASTRAN) and the other experimental (sine sweep) frequencies. Maximum error in the identified frequencies was in the range of 2.5 percent.

Another important feature of this technique is the ability to use an overspecified math mode to identify a number of modes much less than the number of degrees of freedom of the math model. This is very useful when the number of modes in the structure's response is not exactly known because the determinant check, due to high noise levels in the data, might be inconclusive.

Simplicity and economy of the experimental procedure were main factors in designing this technique. Any structure, however complex, can be identified in stages using only two stations at a time. Also, the data needed for the identification program was minimized. The free response needed can be either displacement, velocity acceleration or strain response. The length of record needed to identify a certain structure was noted to be relatively small. Only 0.25 second of data was used to identify the 10 modes of the payload model.

PART II

RANDOM DECREMENT TECHNIQUE FOR MODAL IDENTIFICATION OF STRUCTURES

Introduction

In general, the experimental identification of structural modes of vibration is carried out by measuring the input (or inputs) to the structure under test and the resulting responses due to this input. Some vibration testing techniques, in order to simplify the identification procedure, use the free responses of structures. In such cases, although the input excitation need not be measured, some initial excitation is applied to the structure, and free responses are measured immediately after the initial exciting force is removed.

There are situations where controlled excitation or initial excitation cannot be used. For example, if the structure to be tested is in operation, applying any kind of external force may cause undesirable interruption. Another example is the case of in-flight response measurements where a complete knowledge of the excitation is not usually available. In such cases, the use of the "random decrement signature" technique (a special averaging procedure which is used to determine the step and/or impulse response from the random response) to obtain the free responses is promising.

The random decrement signature technique (ref. 21) has been successfully used for failure detection and damping measurement of structures in single station, single mode response cases. Application of the random decrement signature technique to a multiple of signals changes the time correlation between the individual signals. If the resulting responses are to be used to identify several modes of a structure, the random decrement signature technique must be modified to keep the time correlation between signals unchanged.

In this paper, an algorithm is developed to obtain the free responses of a linear structure from its random responses, due to some unknown or known random input or inputs, using the random decrement technique without changing time correlation between signals.

The algorithm is tested by applying it to random responses obtained from two real structures. The first structure is a generalized payload model previously tested using sine sweep method and analyzed by NASA Structural Analysis (NASTRAN). The second structure is the 1/8 scale space shuttle model with modal parameters previously determined using sine sweep method and Fast Fourier Transform (FFT). Only responses from four stations on the Solid Rocket Boosters (SRB's) were considered in the case of the space shuttle model. The filtered random responses from these two structures were recorded and digitized. The free responses were then obtained from the digitized random responses using the modified random decrement technique. The resulting free responses were used as data for a time domain identification technique, described in Part I, to identify the modal parameters of these structures.

The basic concept of the "random decrement signature" (ref. 21) is based on the fact that a random response of a structure due to a random input is composed of two parts:

(1) Deterministic part (impulse and/or step), and

(2) Random part (assumed to have a zero average).

By averaging enough samples of the same random response, the random part of the response will average out leaving the deterministic part of the response. To avoid averaging out the deterministic part of the signal, the samples can be taken starting always with:

(a) a constant level: this will give the free decay step response;
(b) positive slope and zero level: this will give the free decay positive impulse response;

(c) negative slope and zero level: this will give the free decay negative impulse response.

In figure 4, if $y(t)$ is the random response, the free decay response will be

$$x(\tau) = \frac{1}{N} \sum_{n=1}^{N} y\ (t_n + \tau) \tag{35}$$

with the condition $t_n = t$

$t_n = t$ when $y = y_x$ for case a,

$tn = t$ when $y = 0$ and $dy/dt > 0$ for case b,

or $t_n = t$ when $y = 0$ and $dy/dt < 0$ for case c.

Random Decrement Signatures and Modal Identification of Structures

The response resulting from applying the random decrement signature technique to a random response output of a structure under test is the free decay response. This response can be used in any identification technique (with force input equal to zero) to identify the natural frequencies and damping factors of the structure under test. This procedure has been widely used in flutter testing (refs. 22,23). In flutter testing, determining natural frequencies and damping factors is the primary goal, and one response signal from the structure is sufficient to obtain such information.

For complete modal identification of a structure, mode shapes, frequencies and damping factors of a structure under test are to be determined. This can be done by using a random input and the random responses of the structure at the stations of interest due to that input in some identification technique. If complete knowledge of the random input to the structure is not available, the use of random decrement signature technique seems to be the answer.

Applying the random decrement signature technique to each individual signal of the multiple of signals simultaneously recorded will change the time correlation between these individual signals. Using the resulting signatures to identify the structure will give the correct frequencies and damping factors, but will give erroneous mode shapes because of the change in the time correlation between the individual signals.

Multiple-Signal Random Decrement Technique

In this section, an algorithm will be developed to obtain the free decay responses of a multiple of random response signals simultaneously recorded from a structure using the random decrement technique without changing the time correlation between signals.

To derive and prove this algorithm, a two-station response will be used and then the algorithm can be generalized to any number of stations.

In figure 5, if y_1, (t) and y_2 (t) are the random responses of a structure at stations one and two due to some known or unknown random input, a free decay response of these two stations can be written as:

$$\begin{bmatrix} x_1 & (\tau) \\ x_2 & (\tau) \end{bmatrix} = \frac{1}{N} \sum_{i=1}^{N} \begin{bmatrix} y_1 & (t_i + \tau) \\ y_2 & (t_i + \tau) \end{bmatrix} \tag{36}$$

with condition:

$$t_i = t \text{ when } y_1 \ (t) = y_s \tag{37}$$

The condition imposed by equation (37) implies that the free decay response of station one will have initial conditions of x_1 $(0) = y_s$ and $(dx_1/d\tau)_{\tau=0} = 0$. This means that the free decay response of station one resulting from the random decrement averaging process cannot diminish by the possibility of being averaged out.

Unlike station one, the random response of station two has no condition on the start of the samples. Averaging of station two samples exactly follows that of station one.

The question now is whether it is possible for the deterministic response of station two to be averaged out since no condition was applied to the start of its samples. The answer to this question is readily available by examining equations (36) and (37). Since the free decay of station one exists, and since the two stations' random responses were recorded simultaneously on the same structure and these two stations are dynamically coupled, it is impossible to have response from one station and no response from the other.

To state the algorithm for a multiple of signals: if $\underset{\sim}{y}(t)$ is the random response vector of n stations on a structure due to some known or unknown random input or inputs with zero mean; the free decay response vector for these n stations can be written as:

$$\underset{\sim}{x}(\tau) = \frac{1}{N}\sum_{i=1}^{N} \underset{\sim}{y}(t_i + \tau) \qquad (38)$$

with one of the following conditions

$$t = t_i \text{ when } y_\ell(t) = y_s = \text{(constant level)}$$

$$\text{or} \quad y_\ell(t) = 0 \text{ and } y_\ell(t) > 0$$

$$\text{or} \quad y_\ell(t) = 0 \text{ and } y_\ell(t) < 0$$

where t is any arbitrary leading station of the n stations and N is the number of averages.

Multiple-Signal Randomdec Computation

Assume $y_{i,j}$ ($i = 1,2,..n$ and $j = 1,2,...m$) is the random response recorded simultaneously from n stations on a structure and m is the number of data points for every station. Any of these stations can be assigned as the leading station. This is completely arbitrary as long as all the stations are recorded simultaneously. The randomdec response $x_{i,k}$ according to equation (38) will be:

$$x_{i,k} = \frac{1}{N} \sum_{r=1}^{N} y_{i,(r+k)} \quad ,$$

where $r = 1,2.. N$ designates zero crossings with positive slopes of the leading station response and $k = 1,...M$; N will be the total number of averages, and M will be the number of randomdec data points for each station.

Experimental Results

Payload Model.— Figure 1 shows the payload model structure. The model was previously analyzed by "NASTRAN" and tested using the sine sweep test.

The model was tested using the time domain technique both with and without the random decrement analysis. Sixteen accelerometers were fixed to the eight bulkheads, and their responses were recorded on magnetic tape in two data groups. The first data group contained accelerometers 1 through 8 on one side of the model, and the second data group contained accelerometers 9 through 16 on the other side, as well as accelerometer 8 to relate the 2 data groups.

A random input was applied at station 8. The experiment was designed so that a double switch can cut off the random input and at the same time generates a step signal to be recorded on a separate channel; indicating the start of the free decay. The random part and the free decay part of the responses of data group 1 were recorded. The same procedure was repeated for data group 2.

The free decay part of the response was used as data for the time domain technique. Technique and results are reported in Part I of this report.

To test the algorithm developed in this paper, the random responses (filtered above 500 Hz) of the 17 stations were digitized and analyzed by a computer program to obtain free decay responses using the multiple signal random decrement technique described by equation (38).

It is to be noted here that since the responses from the 17 stations were not recorded simultaneously, a leading station must be used for each data group. In this experiment, station 1 was used as a leading station for stations 1 through 8, and station 9 was the leading station for stations 9 through 16 and station 8. Figures 6a and 6b show the random response of stations 2 and 10, while figures 6c and 6d show their calculated free decay.

To illustrate the importance of using a leading station from the same run, the free decay response of station 10 (group 2) calculated with station 1 (from group 1) as leading station is shown in figure 6e. Comparing figures 6d and 6e, it is clear that the level of the free decay response in figure 6e is much lower than that of figure 6d.

The number of samples averaged to obtain the free decay response from the random response was 753. All samples were chosen such that the response of the leading station starts with zero level and positive slope.

Table 3 shows the natural frequencies and damping factors obtained from the time domain technique using recorded free decay responses and using calculated randomdec responses. Also shown are frequencies from NASTRAN analysis (ref. 24).

Table 4 shows a comparison of the identified modal shapes of the 74.15-Hz mode (first bending), one obtained from recorded free decay data and the other from calculated randomdec data. Comparison is excellent considering that the recorded free decay data had 22 percent noise in it and that the random responses had about 10 percent saturated point. Another factor that contributed to the discrepancy between the two modes in comparison took place during the analog to digital conversion of the random responses of the two data groups. Signals simultaneously recorded should be simultaneously digitized. In this experiment, responses were digitized in four groups (1,3,5,7), (2,4,6,8), (9,11,13,15,17), and (10,12,14,16). If this regrouping must be used, one station must be common between each two subgroups. In this experiment the regrouping should have been (1,3,5,7), (1,2,4,6,8), (9,11,13,15,17), and (9,10,12,14,16).

The 1/8 scale Space Shuttle Model. - The 1/8 scale "space Shuttle" model is shown in figure 7. Modal survey testing of the model by FFT (ref. 25) has been carried out at NASA's Langley Research Center as part of the Space Shuttle Program.

To test the algorithm developed in this paper, the random responses of 4 stations on the 2 "Solid Rocket Boosters," figure 7, in the Y-direction already recorded on analog tape were digitized at a rate of 500 samples/second. These four random responses were then used to calculate the randomdec free decay responses using the multiple signal random decrement technique (eq. 38). The resulting free responses were then used as data for the time domain identification technique to identify the vibration modal characteristics of these four stations.

Figures 8a and 8b show the random responses of stations 1 and 4. The free decay responses resulting from sample averages are shown in figures 8c and 8d. All samples of station 1, as a leading station, were chosen to start with zero levels and positive slopes.

Frequencies identified by the random decrement/time domain technique are listed in table 5 together with the FFT frequencies. It is to be noted that all the four stations considered in this experiment were on the two SRB's and in the Y-direction.

Comparison of modal shapes of the 76.3 Hz, FFT, and the 7623 Hz, "time domain," modes (assuming that they are the same mode) is shown in table 6. It is to be noted that time domain technique identified modes with frequencies very close to the 76.23-Hz mode (76.81 and 76.85), while the FFT did not show these modes. If the difference in the two modal vectors in comparison is not because of different gains or different data processing, the FFT modal vector might have contained contributions from the two very close modes.

Conclusions

The multiple signal random decrement technique described in this paper makes it possible to calculate the free decay response of a multiple of random response signals simultaneously recorded from a structure under test. The resulting free decay response can be used to identify the vibration modal characteristics of structures without altering phase relations between individual stations.

This approach is extremely useful when complete knowledge of the random input to the structure under test is not available. It is also useful for identification techniques that use free decay data.

APPENDIX

According to equations (17) and (19), the matrices $\underline{\Phi}$ and $\hat{\underline{\Phi}}$ are the results of multiplying the two matrices $\hat{\underline{\Psi}}$ and $\underline{\Lambda}$, and $\hat{\underline{\Psi}}$ and $\underline{\Lambda}$. For $\underline{\Phi}$ and $\hat{\underline{\Phi}}$ to have inverses, $\underline{\Psi}$, $\hat{\underline{\Psi}}$ and $\underline{\Lambda}$ should be nonsingular. The columns of $\underline{\Psi}$ and $\hat{\underline{\Psi}}$ are the modal vectors, which are known to be linearly independent, hence neither $\underline{\Psi}$ or $\hat{\underline{\Psi}}$ can be singular. Thus, for $\underline{\Phi}$ and $\hat{\underline{\Phi}}$ to be nonsingular the requirement is that $\underline{\Lambda}$ be nonsingular.

$\underline{\Lambda}$ is a 2n X 2n matrix having elements of the form

$$\lambda_{ij} = e^{\lambda_i t_j} \tag{A-1}$$

If t_1 is arbitrarily chosen to be zero and the time between samples is always the same, then

$$t_j = (j-1)\ \delta t \tag{A-2}$$

$$\underline{\Lambda} = \begin{bmatrix} 1 & e^{\lambda_1 \delta t} & e^{2\lambda_1 \delta 2} & \cdots & e^{(2n-1)\lambda_1 \delta t} \\ \\ 1 & e^{\lambda_2 \delta t} & e^{2\lambda_2 \delta t} & \cdots & e^{(2n-1)\lambda_2 \delta t} \\ \\ 1 & e^{\lambda_{2n} \delta t} & e^{2\lambda_{2n} \delta t} & \cdots & e^{(2n-1)\lambda_{2n} \delta t} \end{bmatrix} \tag{A-3}$$

Substituting $e^{\lambda_i \delta t} = \alpha_1$,

$$\underline{\Lambda} = \begin{bmatrix} 1 & \alpha_1 & \alpha_1^2 & \cdots & \alpha_1^{2n-1} \\ 1 & \alpha_2 & \alpha_2^2 & \cdots & \alpha_2^{2n-1} \\ 1 & \alpha_{2n} & \alpha_{2n}^2 & \cdots & \alpha_{2n}^{2n-1} \end{bmatrix} \qquad \text{(A-4)}$$

$\underline{\Lambda}$ is written as the Vandermonde matrix (21), for which the determinant is

$$|\underline{\Lambda}| = \prod_{1 \leq i \leq j \leq 2n} (\alpha_i - \alpha_j)$$

$$= \prod_{1 \leq i \leq j \leq 2n} \left(e^{\lambda_i \delta t} - e^{\lambda_j \delta t}\right) \qquad \text{(A-5)}$$

Equation (A-5) shows that $\underline{\Lambda}$ is singular only if there are equal λ's, which is unlikely. Thus in the usual case the inverse of $\underline{\Lambda}$ exists, and $\underline{\Phi}$ and $\hat{\underline{\Phi}}$ are nonsingular.

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Table 1. Determinant check.

No. of Stations (N)	$\|Det\|_N$	$\|Det\|_N / \|Det\|_{N+1}$
1	0.50×10^{0}	0.35×10^{1}
2	0.14×10^{0}	0.32×10^{2}
3	0.46×10^{-2}	0.10×10^{3}
4	0.43×10^{-4}	0.10×10^{3}
5	0.45×10^{-6}	0.11×10^{3}
6	0.40×10^{-8}	0.13×10^{5}
7	0.31×10^{-12}	0.88×10^{4}
8	0.35×10^{-16}	0.66×10^{4}
9	0.52×10^{-20}	0.12×10^{6}
10	0.44×10^{-25}	0.20×10^{10}
11	0.25×10^{-34}	0.10×10^{10}
12	0.25×10^{-43}	0.24×10^{11}
13	0.10×10^{-53}	0.34×10^{10}
14	0.30×10^{-63}	0.77×10^{10}
15	0.39×10^{-73}	0.13×10^{11}
16	0.30×10^{-83}	0.24×10^{11}
17	0.13×10^{-93}	0.13×10^{11}
18	0.97×10^{-106}	0.46×10^{11}
19	0.21×10^{-116}	0.16×10^{12}
20	0.13×10^{-127}	

Table 2. Frequencies for the payload model.

Mode No.	Motion	Time Domain 20 D.O.F.	Time Domain 40 D.O.F.	Frequency Sweep	NASTRAN
1	1st Bending	74.20	74.15	74.6	73.4
2	1st Torsion	78.75	78.75	79.7	80.1
3	1st Bending (Yaw)	119.88	119.83	120.7	117.3
4	2nd Torsion	156.63	156.63	158.5	158.9
5	2nd Bending	161.95	161.93	163.1	159.9
6	3rd Torsion	216.51	216.44	219.2	218.6
7	3rd Bending	245.00	245.18	246.7	244.6
8	2nd Bending (Yaw)	259.55	261.04	263.7	253.1
9	4th Torsion	280.95	280.94	283.7	283.0
10	5th Torsion	325.31	325.31	328.0	—

Table 3. Identified frequencies and damping factors for the payload model.

	Time Domain With Recorded Free Decay Responses		Domain With Calculated Randomdec Responses		NASTRAN
Mode No.	Frequency (Hz)	Damping Factor	Frequency (Hz)	Damping Factor	Frequency (Hz)
1	74.15	0.0019	74.15	0.0029	73.4
2	78.75	0.0017	78.75	0.0010	80.1
3	119.83	0.0013	120.27	0.0010	117.3
4	156.63	0.0007	156.61	0.0013	158.9
5	161.93	0.0007	161.77	0.0007	159.9
6	216.44	0.0013	216.47	0.0010	218.6
7	245.18	0.0034	245.00	0.0036	244.6
8	261.04	0.0007	260.70	0.0004	253.1
9	280.94	0.0018	280.75	0.0017	283.0
10	325.31	0.0005	325.30	0.0002	---

Table 4. An identified modal vector for the payload model (frequency = 74.15 Hz).

Station	With Recorded Free Decay Responses		With Calculated Randomdec Responses	
	Amplitude	Phase (degrees)	Amplitude	Phase (degrees)
1	83.37	-6.3	86.79	-2.6
2	18.02	-13.2	20.10	-2.2
3	39.86	179.2	41.43	177.8
4	72.58	178.7	75.7	179.1
5	71.58	174.4	74.46	177.6
6	30.77	176.0	35.13	180.5
7	31.43	1.2	31.63	-2.4
8	100.00	0.0	100.00	0.0
9	99.34	-1.4	96.02	-14.0
10	34.69	7.1	28.26	-1.7
11	28.82	164.3	35.05	160.0
12	68.58	176.4	71.62	175.6
13	71.63	175.8	71.15	164.1
14	39.92	182.12	38.54	178.2
15	15.43	-27.00	23.44	-21.9
16	73.00	-9.8	84.35	-5.1

Table 5. Shuttle model identified frequencies.

FFT	Randomdec/Time Domain	Remarks
13.8	-----	Gear-Train Rotation
16.5	16.57	
17.6	18.17	
20.5	20.71	
21.6	-----	ORB. Roll Mode
24.3	-----	ET 1st Torsion
---	25.50	
26.0	26.29	
---	26.69	
28.2	-----	ORB. Longitudinal
30.0	-----	
32.2	32.10	
34.5	-----	
---	36.36	
28.0	38.25	
43.2	43.01	
---	45.00	
47.5	-----	
48.5	-----	ET Z-Direction Bending
51.0	50.2	
57.5	58.05	
58.5	59.2	
---	60.07	
---	61.68	
69.0	68.65	
---	71.55	
76.3	76.23	
---	76.81	
---	76.85	

Table 6. An identified shuttle model mode.

	FFT		Time Domain	
Sta.	Relative Amplitude	Phase (degrees)	Relative Amplitude	Phase (degrees)
1	100.00	0.0	100.00	0.0
2	55.00	0.4	61.65	0.98
3	86.46	19.1	98.77	22.3
4	54.07	18.7	33.96	15.4

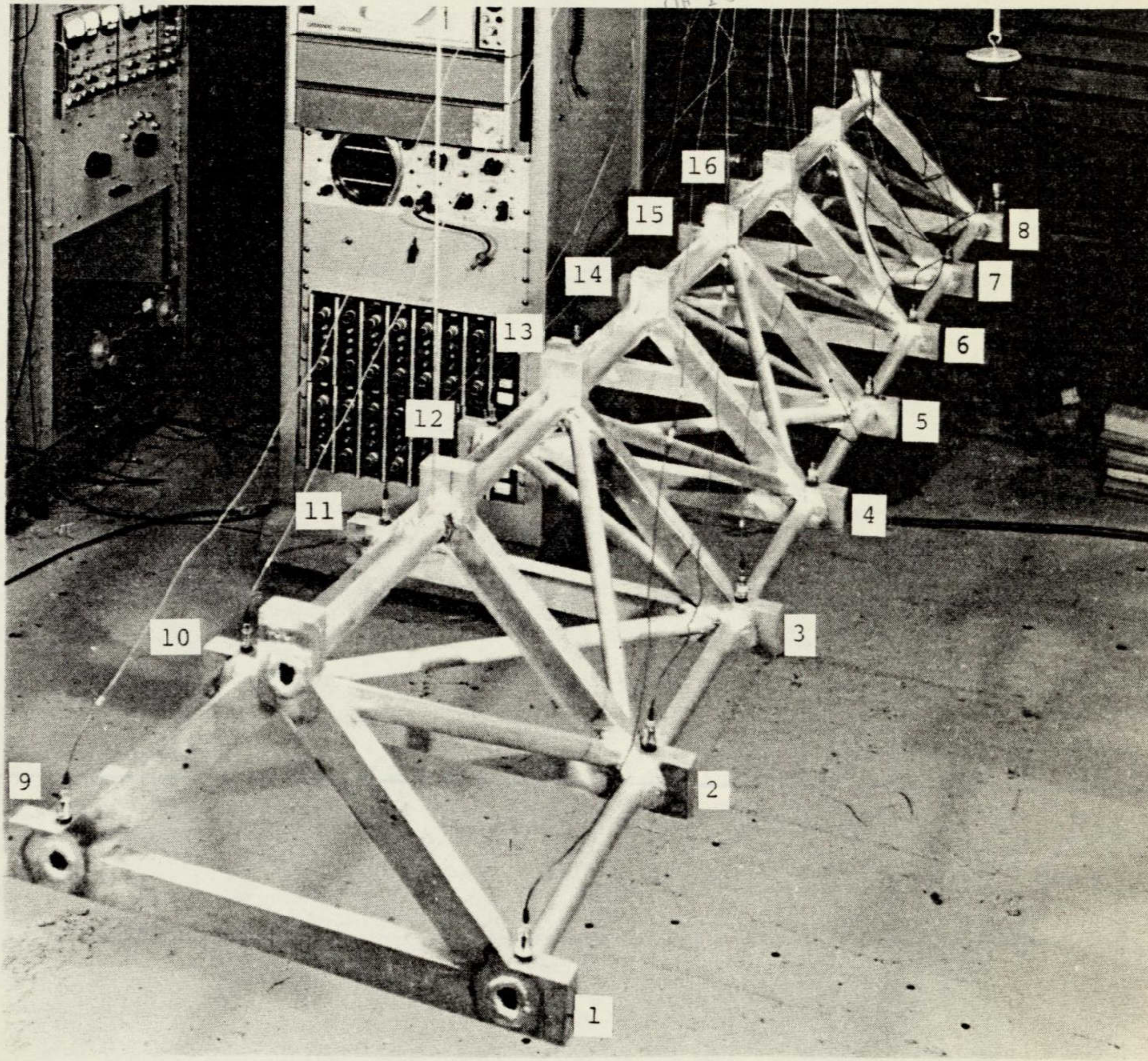

Figure 1. Payload model.

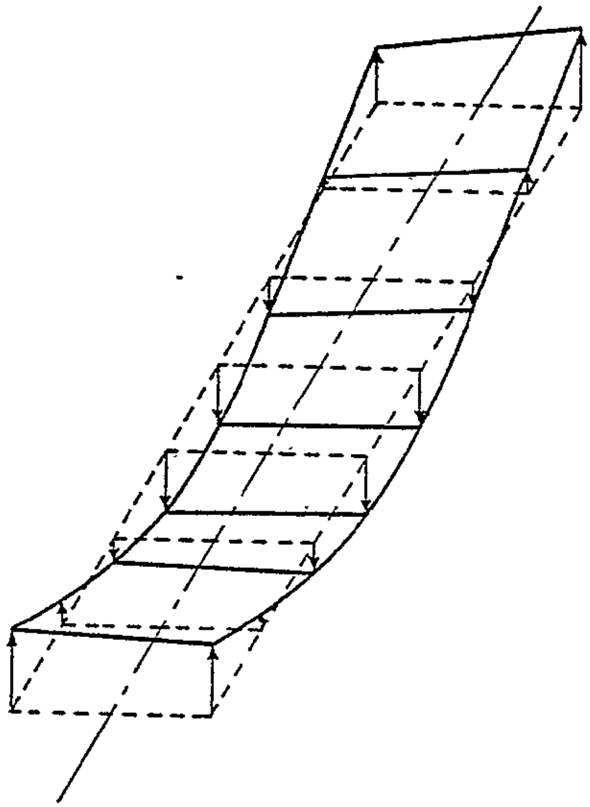

Figure 3a. Mode No. 1, first bending.

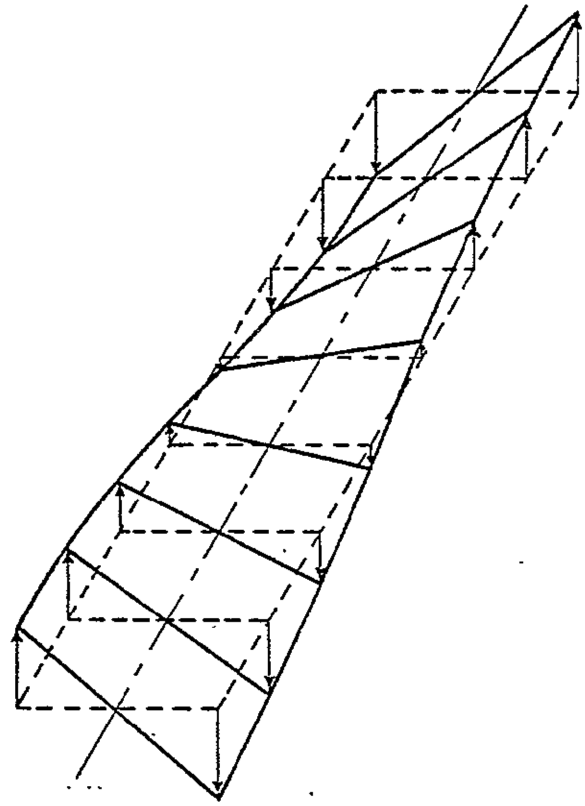

Figure 3b. Mode No. 2, first torsion.

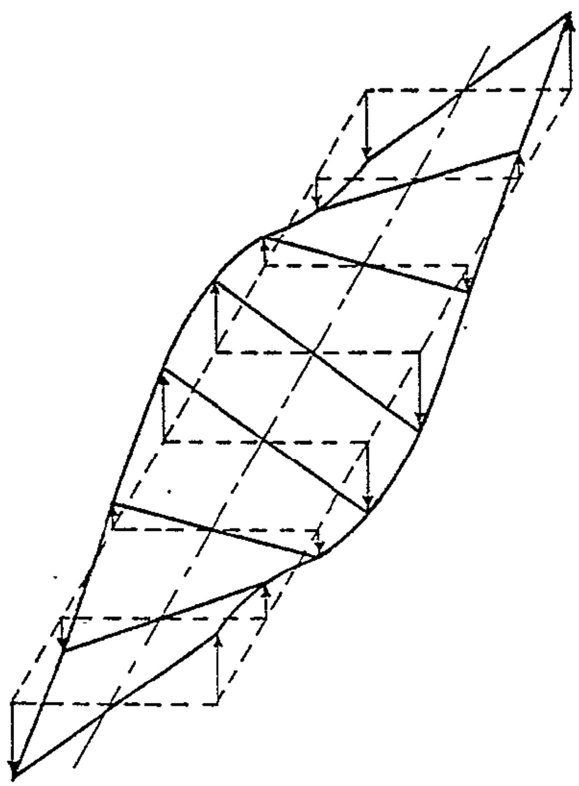

Figure 3c. Mode No. 4, second torsion.

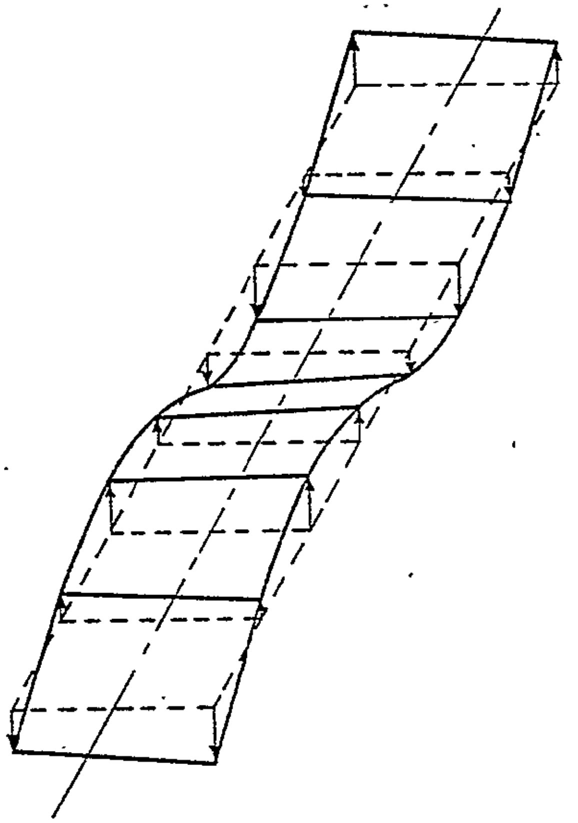

Figure 3d. Mode No. 5, second bending.

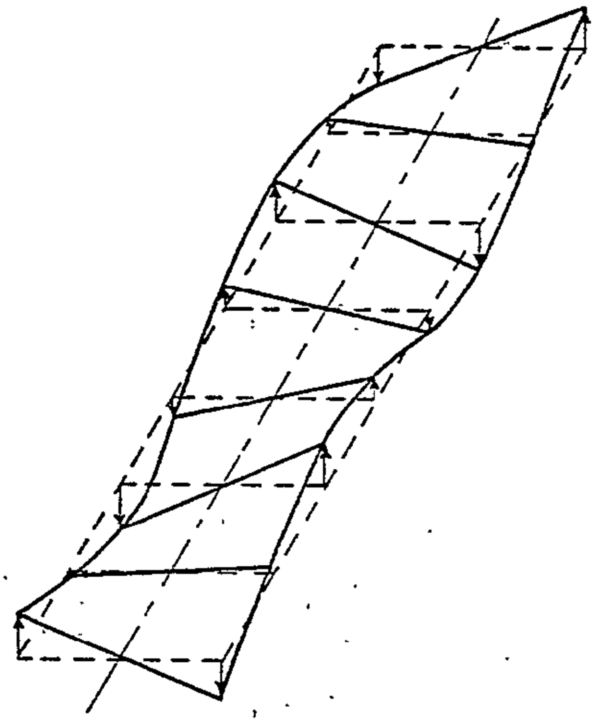

Figure 3e. Mode No. 6, third torsion.

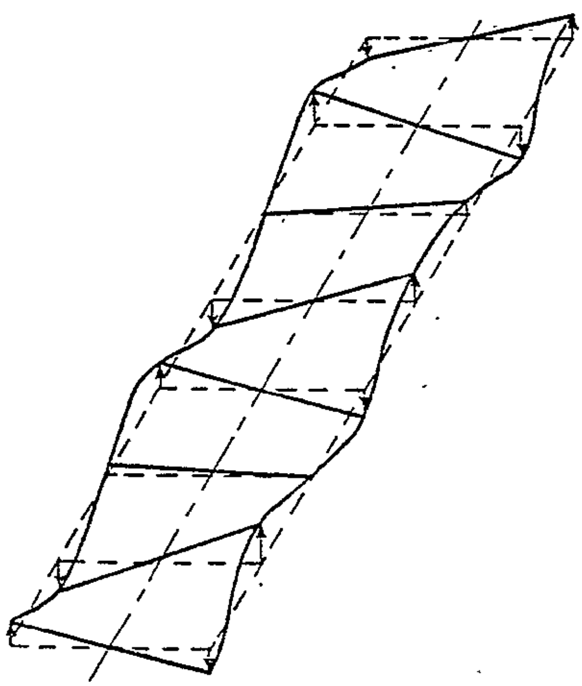

Figure 3f. Mode No. 10, fifth torsion.

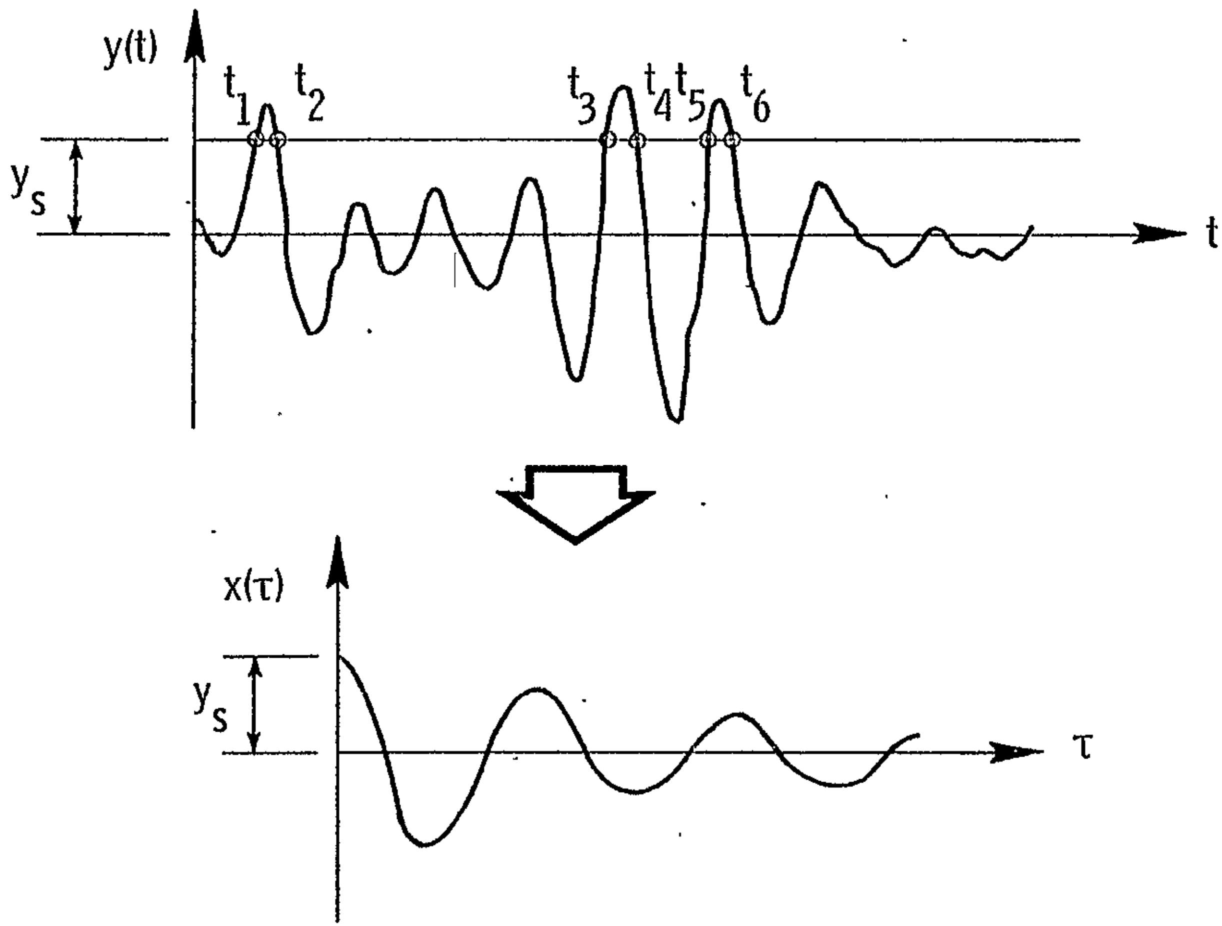

Figure 4. One-station random and randomdec responses.

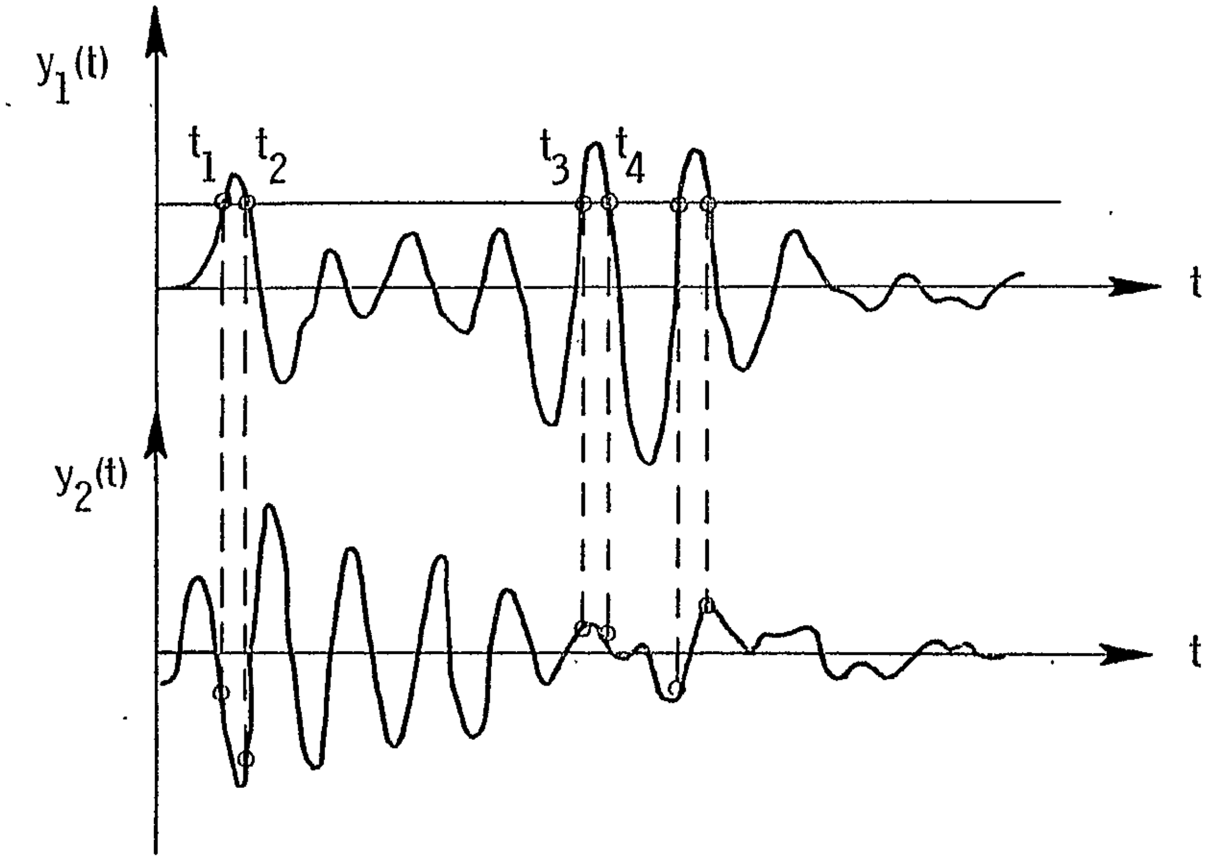

Figure 5. Two-station random responses.

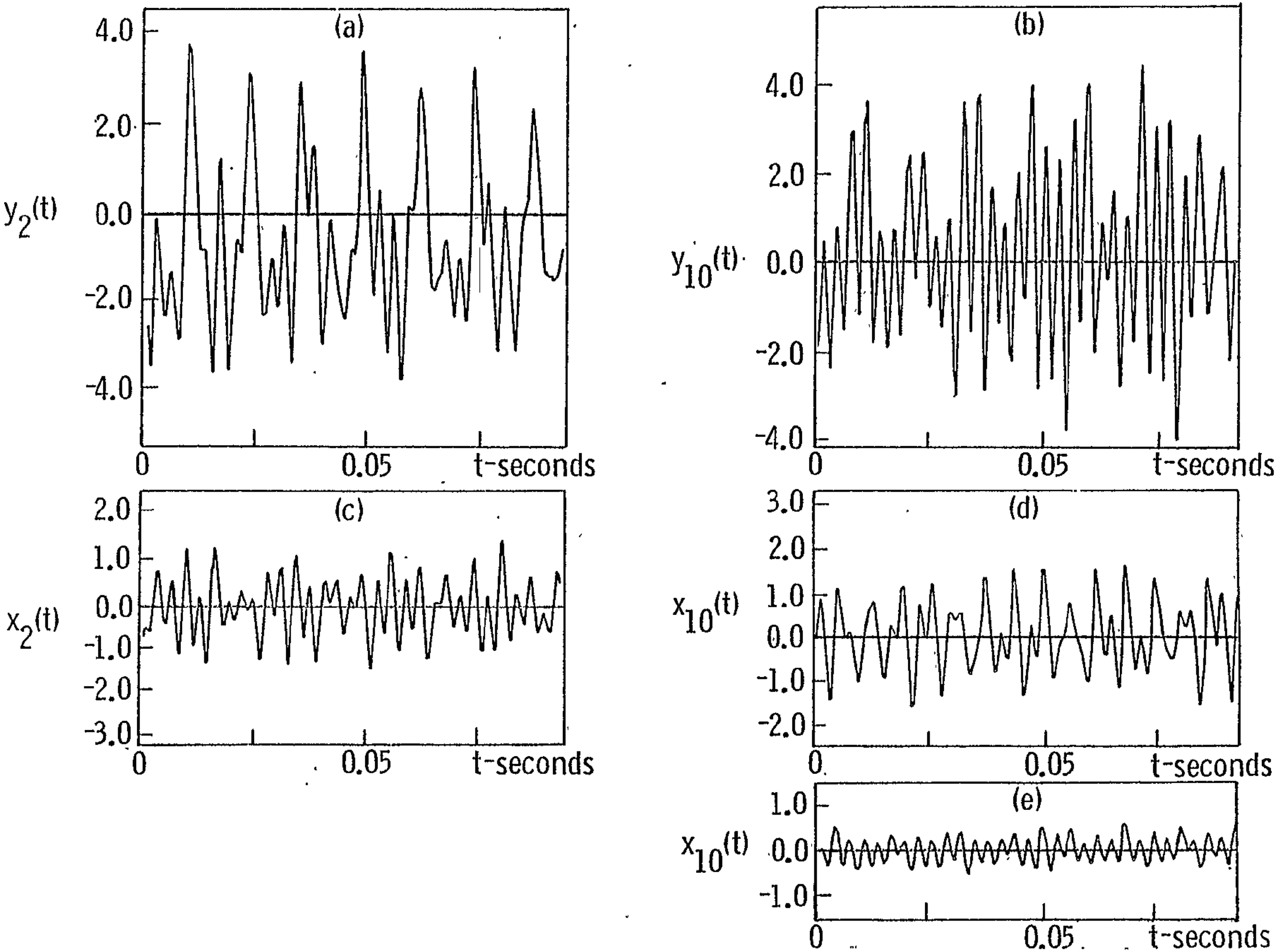

Figure 6. Random and randomdec free decay response of selected stations on the payload model.

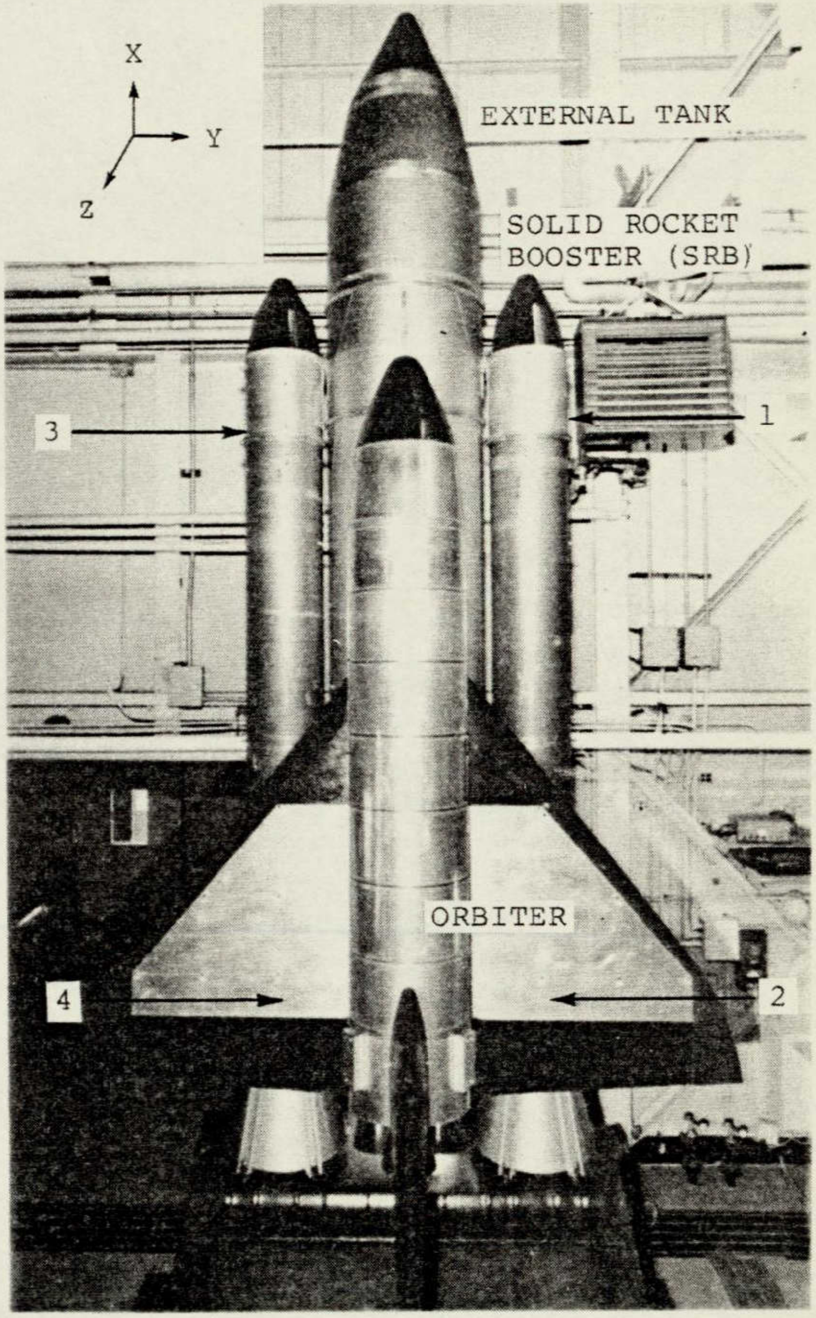

Figure 7. 1/8-scale space shuttle model.

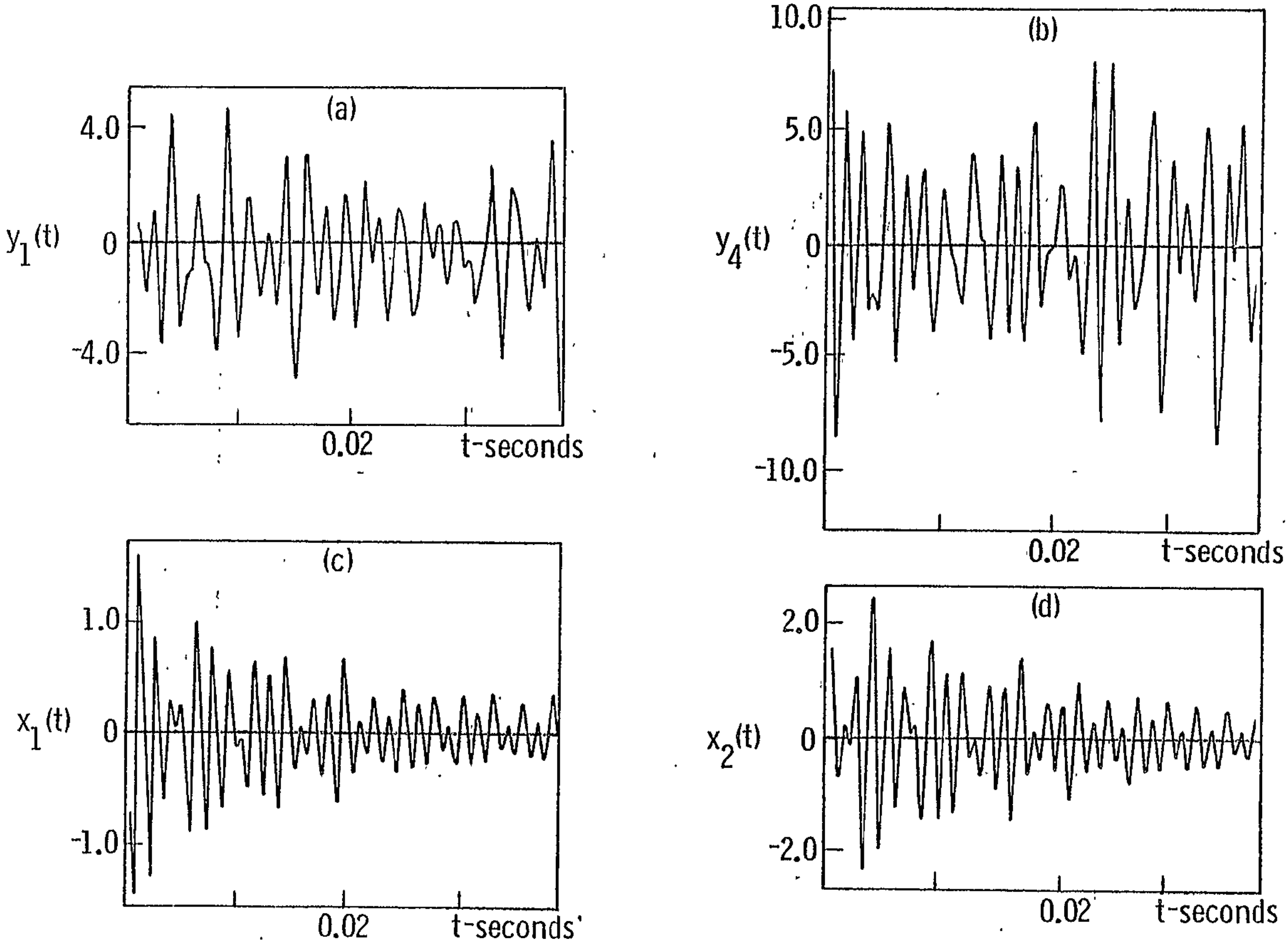

Figure 8. Random and randomdec free decay response of selected stations on the shuttle model.