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A FEW WAYS OF CALCULATING THE SIMILARITY PARAMETER κ^* FOR REAL GASES

W. Lorenz-Meyer

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16. Abstract

In connection with the question on the applicability of test results obtained from cryogenic wind tunnels to the large-scale model the similarity parameter $\kappa^* = 1 + (\delta a^2/\delta i)_s$ of various authors is referred to. A simple method is given for calculating κ^*. From the numerical values obtained it can be deduced that nitrogen behaves practically like an ideal gas when it is close to the saturation point and in a pressure range up to 4 bar. The influence of this parameter on the pressure distribution of a supercritical profile confirms this finding.

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A FEW WAYS OF CALCULATING THE SIMILARITY PARAMETER κ^* FOR REAL GASES

W. Lorenz-Meyer
German Research and Test Center for Aerospace Travel

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Notation /2

a	Speed of sound
A_i	Constants of the Beattie-Bridgeman equation ($i = 1,2...5$)
C_A	Lift coefficient
C_p	Pressure coefficient (* critical pressure coefficient)
c_p	Specific heat at constant pressure
c_v	Specific heat at constant volume
i	Enthalpy
ℓ	Depth of profile
Ma_∞	Mach number of undisturbed oncoming flow
N_i	Constants of the Jacobsen equation ($i = 1,2...5$)
p	pressure
R	Gas constant (for nitrogen = 896.82 Nm/kg K)
s	entropy
T	Temperature
t	Thickness of profile
U_∞	Velocity of the undisburbed oncoming flow
u,v	Disturbance velocities
x,y	Coordinates
α	Angle of attack, expansion coefficient
Γ	Hayes' similarity parameter
κ^*	Similarity parameter
κ	Ratio of specific heats c_p/c_v
ρ	Density
Φ,ϕ	Potential

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1. Introduction

The Reynolds number is an important simulation parameter for applying results obtained from small-scale wind tunnel models to the full scale model. Since especially in the transonic speed range the laws of similarity are not sufficiently known, very serious efforst have been made for years to increase the Reynolds number in wind tunnel experiments. The two traditional ways of increasing the Reynolds number -- namely increasing the static pressure or increasing the linear dimensions -- soon led to limits determined by costs and the sturdiness of the model. Very early on it was recognized by R. Smelt [1] that a reduction in temperature could bring about a considerable improvement. This idea was first taken up again in 1972 by M. J. Goodyer and R. A. Kilgore [2,3]. In a pilot tunnel constructed by NASA at Lnagley Field the technical feasibility of wind tunnel experiments at rest temperatures of 90K-120K was impressively demonstrated. However, since the flow medium (as a rule nitrogen) /3 no longer behaves like an ideal gas in the vicinity of the liquifaction limit, the applicability of test values again came into question. Studies by J. Adcock, R. A. Kilgore and E. J. Ray [4] showed, however, that in the generally used gas-dynamic calculations the deviations from the behavior of an ideal gas are only very small. In particular, from values tabulated by R. T. Jacobsen and R. B. Stewart [5], Adcock determined a pseudo adiabatic exponent, $\alpha = a^2\rho/p$, for typical experimental conditions. This deviated only slightly from a value of 1.40. Further studies, especially by H. Viviand [6] soon showed that neither the ratio of the specific heats κ nor the pseudo adiabatic exponent α is a determining factor for the similarity of the flow. The following value was given by Viviand:

$$\kappa^* = 1 + (\partial a^2/\partial i)_s$$

this replaces the κ in the transonic potential equation. This essentially universal similarity parameter was given by W. D. Hayes [7] as early as 1954, however in a somewhat different form:

$$\Gamma = (\kappa^* + 1)/2 = (\partial \rho a/\partial \rho)_s \cdot /a) .$$

Studies on the parameter Γ, especially with respect to compressible flows of real gases and liquids, were carried out by P. A. Thompson [8], P. A. Thompson and K. C. Lambrakis [9] and P. A. Thompson and B. A. Sullivan [10].

In this paper, after a brief refresher discussion of how κ^* is derived [6], an equation will be given which permits a simple calculation of κ^* from thermal and caloric parameters of state. Numerical values for κ^* are obtained by means of the Beattie-Bridgeman equation of state [11] and from [5] for nitrogen as the working medium. Applications of the Beattie-Bridgeman equation to various transonic flows are given by B. W. Wagner and W. Schmidt [12]. The influence of the similarity parameter on transonic flows is shown here by the flow around a supercritical profile.

2. Derivation of κ^* /4

Several ways of deriving κ^* will be discussed in this section section. In so doing, we will follow the line of thought of Viviand [6].

The following initial equations are used:
the potential equation

$$\left(1 - \left(\frac{\Phi_x}{a}\right)^2\right)\Phi_{xx} + \left(1 - \left(\frac{\Phi_y}{a}\right)^2\right)\Phi_{yy} - 2\,\frac{\Phi_x \Phi_y}{a^2}\,\Phi_{xy} = 0 \quad , \qquad (1)$$

the energy equation

$$i + (\Phi_x^2 + \Phi_y^2)/2 = i_\infty + U_\infty^2/2 \qquad (2)$$

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and an equation of state

$$a^2 = a^2(i, s) \tag{3}$$

Using the theory of small disturbances

$$\Phi_x = U_\infty + u \quad , \quad \Phi_y = v \quad , \quad a \approx a_\infty \tag{4}$$

one obtains the following equation for isentropic flows:

$$a^2 - a_\infty^2 = \left(\frac{\partial a^2}{\partial i}\right)_s (i - i_\infty) + \ldots\ldots \tag{5}$$

and with Eq. (2) one obtains the following equation

$$\left(\frac{a}{a_\infty}\right)^2 = 1 - \left(\frac{\partial a^2}{\partial i}\right)_s \mathrm{Ma}_\infty^2 \cdot \frac{u}{U_\infty} \quad . \tag{6}$$

Applying the theory of small disturbances to the first factor of Eq. (1) gives us the following:

$$1 - \left(\frac{\Phi_x}{a}\right)^2 = 1 - \left(\frac{a}{a_\infty}\right)^2 \mathrm{Ma}_\infty^2 \left(1 + 2\,\frac{u}{U_\infty}\right) \quad . \tag{7}$$

Substituting Eq. (6), we get:

$$1 - \left(\frac{\Phi_x}{a}\right)^2 = 1 - \mathrm{Ma}_\infty^2 - \mathrm{Ma}_\infty^2 \left(2 + \mathrm{Ma}_\infty^2 (\varkappa^* - 1)\right) \frac{u}{U_\infty} \tag{8}$$

with

$$\varkappa^* = 1 + \left(\frac{\partial a}{\partial i}\right)_s \quad . \tag{9}$$ /5

Thus the potential equation reads as follows:

$$\left(1 - \mathrm{Ma}_\infty^2 - \mathrm{Ma}_\infty^2 \left(2 + \mathrm{Ma}_\infty^2 (\varkappa^* - 1)\right) \varphi_x\right) \varphi_{xx} + \varphi_{yy} = 0 \quad . \tag{10}$$

This equation is formally identical to the transonic potential equation for ideal gases. In the case of the real gas only the ratic of the specific heat must be replaced by κ^* according to Eq. (9).

In dealing with tabular figures (e.g. [5]), the numerical determination of κ^* runs into considerable difficulties. Therefore, in the following section the attempt will be made to find a more convienent analytical form for κ^*.

The first law of thermodynamics yields the following for isentropic changes of state:

$$\rho \, di = dp \quad , \tag{11}$$

so that Eq. (9) can be written in the following form:

$$\varkappa^* = 1 + \rho\left(\frac{\partial a^2}{\partial p}\right)_s \quad . \tag{12}$$

If one takes a^2 as a function of p and T, one obtains

$$\left(\frac{\partial a^2}{\partial p}\right)_s = \left(\frac{\partial a^2}{\partial p}\right)_T + \left(\frac{\partial a^2}{\partial T}\right)_p \cdot \left(\frac{\partial T}{\partial p}\right)_s \quad . \tag{13}$$

with the identity (cf. E. Schmidt [13], ch. 11)

$$\left(\frac{\partial T}{\partial p}\right)_s = \frac{c_p - c_v}{c_p} \Big/ \left(\frac{\partial p}{\partial T}\right)_\rho \tag{14}$$

one obtains for Eq. (13)

$$\left(\frac{\partial a^2}{\partial p}\right)_s = \left(\frac{\partial a^2}{\partial p}\right)_T + \frac{c_p - c_v}{c_p}\left(\frac{\partial a^2}{\partial T}\right)_p \Big/ \left(\frac{\partial p}{\partial T}\right)_\rho \tag{15}$$

or for κ^*

$$\varkappa^* = 1 + \rho\left\{\left(\frac{\partial a^2}{\partial p}\right)_T + \frac{\varkappa - 1}{\varkappa}\left(\frac{\partial a^2}{\partial T}\right)_p \Big/ \left(\frac{\partial p}{\partial T}\right)_\rho\right\} \quad . \tag{16}$$ /6

For an ideal gas one immediately obtains

$$\left(\frac{\partial a^2}{\partial p}\right)_T = 0 \quad , \quad \left(\frac{\partial a^2}{\partial T}\right)_p = \varkappa \cdot R \quad , \quad \left(\frac{\partial p}{\partial T}\right)_\rho = \rho \cdot R$$

$$\varkappa^* = \varkappa \quad . \tag{17}$$

Assuming that a^2 is a function of the thermal parameters of state ρ and T, another form for κ^* can be determined in a similar manner which has been given by E. Wedemeyer [14]. This reads as follows:

$$\varkappa^* = 1 + \frac{\rho}{a^2}\left\{\left(\frac{\partial a^2}{\partial \rho}\right)_T + \frac{T}{c_v \rho^2}\left(\frac{\partial p}{\partial T}\right)_\rho \left(\frac{\partial a^2}{\partial T}\right)_\rho\right\} \quad . \tag{18}$$

Eq. (16) was developed in order to be able to determine κ^* from tables in which the calculated parameters of state are given as a function of p and T. Eq. (18), by contrast, is directed more towards calculation from equations of state of the form $p = f(\rho, T)$. In the next section a few ways of calculating κ^* from equations of state will be demonstrated. The numerical values determined from tables [5] are also given.

3. Calculation of κ^*

3.1 General Remarks

For the most part, equations of state are given in the form

$$p = p(\rho, T) \tag{19}$$

In order to calculate κ^* it is necessary to know the speed of sound a^2, the specific heats c_p and c_v and the corresponding derivatives of the pressure and the speed of sound. To this end the following equations are used [5,13]:

$$c_v = (c_v)_{id} - \int_0^{\rho} T\left(\frac{\partial^2 p}{\partial T^2}\right)_{\rho} \frac{d\rho}{\rho^2} \qquad (20)$$ [1]

$$c_p = c_v + \frac{T}{\rho^2}\,\frac{(\partial p/\partial T)_{\rho}^2}{(\partial_p/\partial \rho)_T} \qquad (21)$$

$$a^2 = \frac{c_p}{c_v}\left(\frac{\partial p}{\partial \rho}\right)_T \quad . \qquad (22)$$

3.2 The Beattie-Bridgeman Equation of State

The Beattie-Bridgemean equation of state was given for the first time in 1927 [11]. It can be used for many gases. Up to gas densities of $\rho = 140\ \mathrm{kg/m^3}$ the gas conditions calculated with this equation are in excellent agreement with observations. It has 5 constants and is written as follows:

$$p = \rho \cdot R \cdot T(1 - \rho A_5/T^3)(1 + A_2\rho(1 - A_4\rho)) - A_1\rho^2(1 - A_3\rho) \quad . \qquad (23)$$

The constants are taken from [11]. In the MKS system for nitrogen they are as follows:

$$A_1 = 173.6 \qquad A_2 = 1.8013\cdot 10^{-3}$$

$$A_3 = 9.342\cdot 10^{-4} \ , \quad A_4 = -\ 2.467\cdot 10^{-4} \ , \quad A_5 = 1499.0$$

$$(\rho[\mathrm{kg/m^3}] \ , \quad R[\mathrm{Nm/kg\,K}] \ , \quad T[\mathrm{K}] \ , \quad p[\mathrm{N/m^2}]) \quad .$$

The necessary derivatives can be given immediately:

$$\left(\frac{\partial p}{\partial T}\right)_{\rho} = \rho \cdot R(1 + 2\rho A_5/T^3)(1 + A_2\rho(1 - A_4\rho)) \qquad (24)$$

---

1. $(c_v)_{id} = \frac{5}{2}\cdot R = 741,99\ \mathrm{Nm/kg\,K}$ (for nitrogen).

$$\left(\frac{\partial p}{\partial \rho}\right)_T = R\cdot T(1 - \rho A_5(2 + 3A_2\rho)/T^3 + A_2\rho(2 - 3\rho A_4 + 4\rho^2 A_4 A_5/T^3)) \tag{25}$$
$$- A_1\rho(2 - 3A_3\rho)$$

$$c_v = (c_v)_{id} + 6\rho R A_5(1 + A_2\rho(\tfrac{1}{2} - \tfrac{1}{3}\rho A_4))/T^3 \quad . \tag{26}$$

<u>3.4 The Equation of State after R. T. Jacobsen</u> /8

The tables by Jacobsen [5] are based on an equation of state with 39 constants which were determined from values measured in a pressure range of p = 0.1 to 10,000 bar and a temperature range of T = 63 to 2,000 K. However, for the operation of a cryogenic wind tunnel the planned pressure range and temperature range are much smaller (p up to 8 bar, T up to 300 K) so that here it will be attempted to get by only with 5 constants. Such an equation, according to [5], runs as follows:

$$p = \rho R T + \rho^2(N_1\cdot T + N_2\sqrt{T} + N_3 + N_4/T + N_5/T^2) \quad . \tag{27}$$

In the MKS system the constants for nitrogen are as follows:

$N_1 = 0.17572$ $\quad N_2 = 0.13819\cdot 10^{-}$ $\quad N_3 = -0.31495\cdot 10^3$

$N_4 = 0.44060\cdot 10^4$, $\quad N_5 = -0.54611\cdot 10^6$,

$(\rho\,[kg/m^3]$, $R\,[Nm/kg\,K]$, $T\,[K]$, $p\,[N/m^2]$.

The simple derivatives turn out to be the following:

$$\left(\frac{\partial p}{\partial T}\right)_\rho = \rho\cdot R + \rho^2(N_1 + 0.5\,N_2/\sqrt{T} - N_4/T^2 - 2N_5/T^3) \tag{28}$$

$$\left(\frac{\partial p}{\partial \rho}\right)_T = R\cdot T + 2\rho(N_1\cdot T + N_2\sqrt{T} + N_3 + N_4/T + N_5/T^2) \qquad (29)$$

$$c_v = (c_v)_{id} + \rho(0,25\, N_2/\sqrt{T} - 2N_4/T^2 - 6N_5/T^3) \quad . \qquad (30)$$

3.4 Calculation of κ^2 [sic]

In order to calculate values for κ^* from the equations of state given in section 3.2 and 3.3 the following procedure is selected. For a given pair of values (p, T) the density ρ is determined iteratively from Eq. (23) or (27). Then these values are determined:

$$\frac{\partial p}{\partial T}(p, T)\ ;\ \frac{\partial p}{\partial \rho}(p, T);\ c_v(p, T)\ ;\ c_p(p, T) \quad \text{and} \quad a^2(p, T)$$

The derivatives $\delta a^2/\delta t$ and $\delta a^2/\delta p$ are determined numerically with the help of Spline polynomials. Then κ^* can be calculated directly from Eq. (16). Some results are shown in the figures /9 given at the end of this paper. Figs. 1 and 2 show the derivatives of the speed of sound according to temperature and pressure based on equations of state (23) and (27). If one formed the corresponding derivatives from [5], then deviations appear which suggest that the table is not sufficiently precise. Therefore, it is too inaccurate for calculating κ^*. The perceptible differences between Beattie-Bridgeman (Eq. (23)) and Jacobsen (Eq. (27)) clearly show that even these equations do not yet describe the gas conditions with sufficient accuracy.

Fig. 3 shows the values for κ^* calculated from the two equations of state in comparison with a few values determined directly from the table.

The following conclusions can be drawn from Fig. 3:

- In the interesting range for a cryogenic wind tunnel the similarity parameter κ^* deviates only slightly from 1.40.
- At T ≈ 105 K nitrogen behaves like an ideal gas.
- The real gas effects are more strongly influenced by the pressure than by the temperature.
- Differences depending on the equation of state selected are manifested only in the vicinity of the saturation point.
- The values determined from the table are not accurate enough.

4. Applications

In conclusion, we will briefly discuss the possible implications of a change in the similarity parameter.

With the example chosen here, the pressure distribution with a mach number $Ma_\infty = 0.75$ for the oncoming flow and an angle of attack of $\alpha = 0°$ is calculated according to the method of Garabedian and Korn [16,17] for a supercritical profile which was developed by Messerschmitt-Bölkow-Blohm and Co. [15]. In so doing, the similarity parameter κ^* is varied within the limits encountered in practice (Fig. 3). Fig. 4 shows the pressure distribution for $\kappa^* = 1.4$. Fig. 5 shows the recompression range in which the κ^* influence is detectable. It /10
is obvious that the effect of a variation in κ^* is small both with respect to the pressure distribution and with respect to lift.

5. Summary

A formula is given for the determining parameter $\kappa^* = 1 + (\delta a^2/\delta i)_s$ used for estimating real gas effects. The formula makes it possible to determine this parameter both from tables and from equations of state ($p(\rho,T)$). The equations of state of

Beattie-Bridgeman and Jacobsen are applied to the formula. Neither equation, however, permits a sufficiently exact description of the gas conditions. Using the calculated pressure distribution of a supercritical profile, the effect of a variation in κ^* within the limits encountered in practice is demonstrated. The results obtained permit the conclusion that nitrogen, in the pressure range considered and in the vicinity of the saturation point shows only slight real gas effects.

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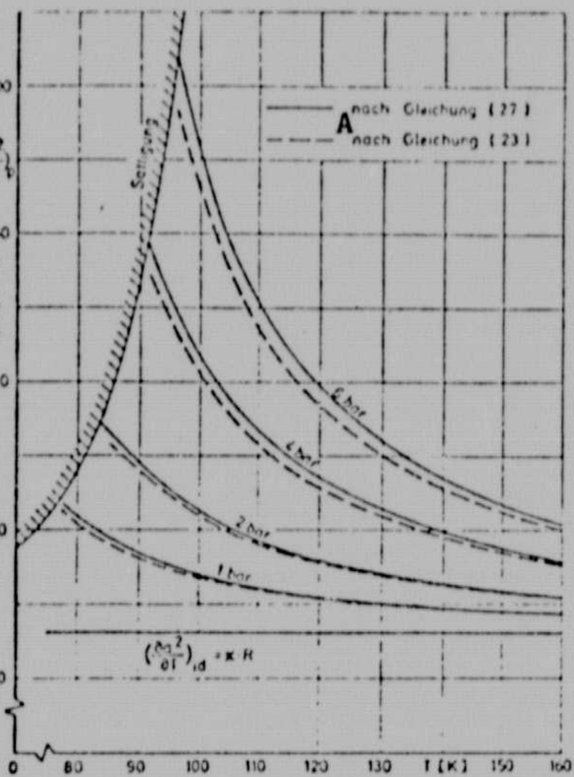

Fig. 1. $\delta a^2/\delta T$ at constant pressure (for nitrogen)
Key: A) According to equation

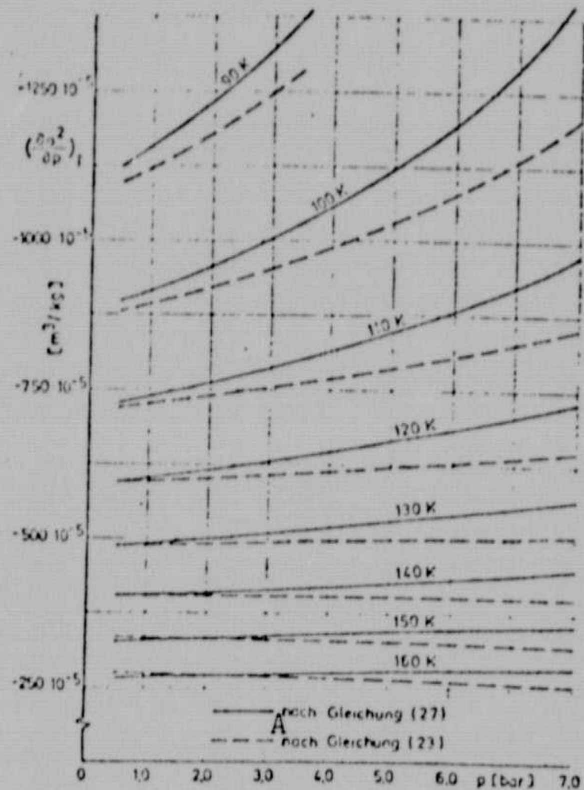

Fig. 2. $\delta a^2/\delta p$ at constant temperature (for nitrogen)
Key: A) According to equation

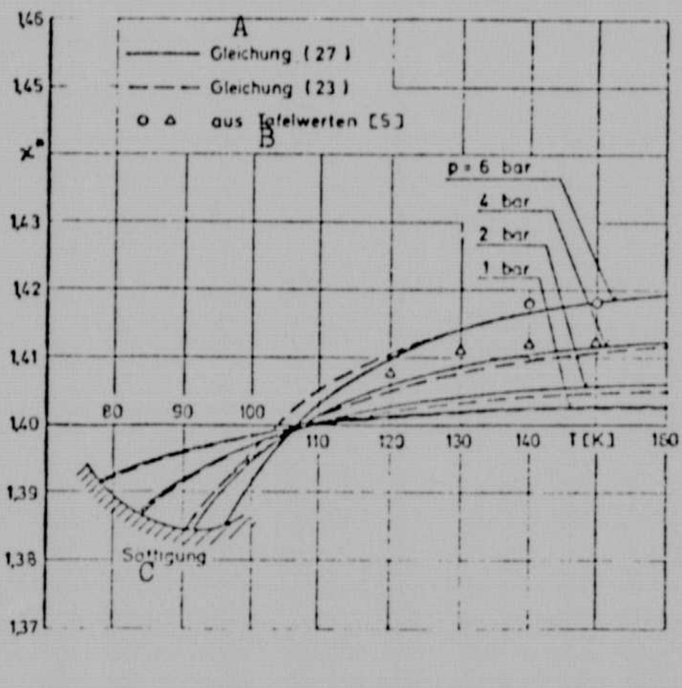

Fig. 3. * as a function of temperature and pressure (for nitrogen).
Key: A) Equation
B) From tabulated values
C) Saturation

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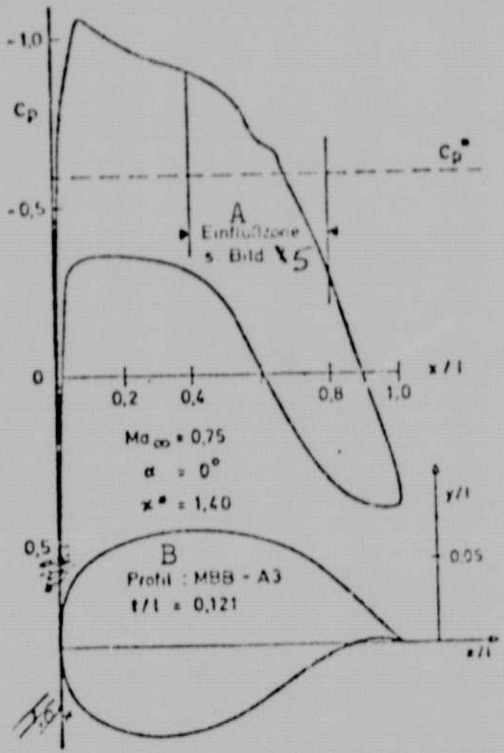

Fig. 4. Calculated pressure distribution on a supercritical profile MBB-A3 with $\kappa^* = 1.40$ (ideal gas)
Key: A) Zone of influence, see Fig. 5
B) Profile

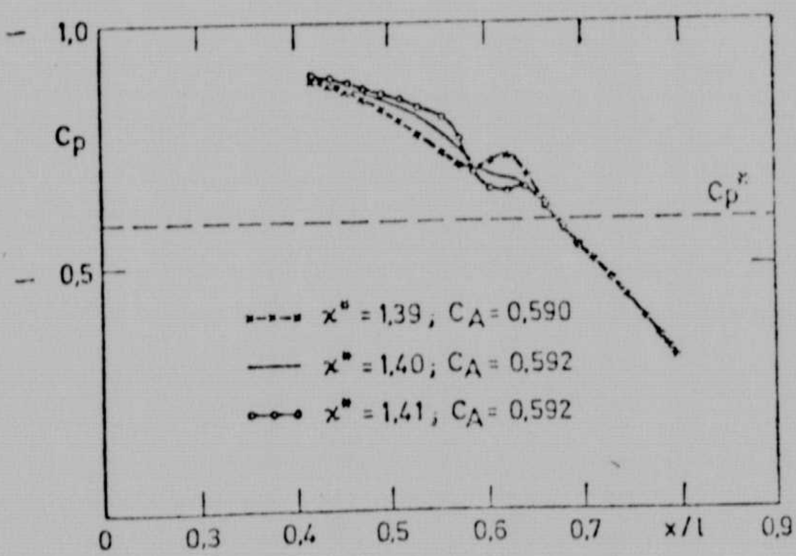

Fig. 5. Pressure distribution on the suction side of the MBB-A3 profile (segment) for various κ^* values.