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## A SOLUTION TO THE PROBLEM OF GAR RANGE CURVATURE

R. K. Raney<br>Canada Centre for Remote Sensing<br>Ottawa, Ontario

## ABSTRACT

When synthetic aperture radar systems are pushed to attain finer resolution at larger ranges than was previously the case for remote sensing purposes, the geometric signal aberration known as range curvature arises. Known techniques for correcting range curvature are exact at only one selected range, thus forcing neighboring ranges to use the same correction as an approximation. (For Seasat A, four separate correction filters would be required over the design swath width.) This paper outlines a solution to the problem that is exact at all ranges, thus simplifying and improving the image processing for such systems.

## 1. INTRODUCTION

Synthetic aperture radar systems achieve beam sharpened azimuth resolution by coherently exploiting the incremental changes in range between the sensor and an object as it moves through the beam, as shown in Figure 1. Range curvatore arises if the differential range $\delta \mathrm{R}$ that is beneficial becomes larger than (half of) the range resolution $\mathrm{pr}_{\mathrm{r}}$, at which time the resulting signal accumulates an aberration that may prevent the desired range resolution from being achieved.

The radar/object range as a function of time is given by

$$
\begin{equation*}
R(t)=R o+\frac{1}{2} \frac{(V t)^{2}}{R_{o}} \tag{1}
\end{equation*}
$$

using the first two terms of the Taylor series expansion. The second term of Eqn. 1 (the differential range) takes on its maximum value for the largest value of aspect angle $\theta$ which we let be $\theta_{\mathrm{M}}$.

$$
\begin{equation*}
\operatorname{Max} \delta R=\frac{3}{2} R_{0} \theta^{2} M \tag{2}
\end{equation*}
$$

which is linearly increasing with range. Likewise, the achievable azimuth resolution Pa is proportional to $\theta_{\overline{\mathrm{M}}}{ }^{1}$, so we see that range curvature is compounded by greater ranges on finer resolution.*
*In the broadside case, $\theta_{M}=\frac{\lambda}{4 \rho}$ so that range curvature,

which arises if | Max $\delta R$ |
| :--- |
| $>\frac{3}{2} \rho_{r}$ |

| imposes the SAR constraint |
| :--- |
| that if |$R_{0} \frac{\lambda^{2}}{1602_{a}}>\rho_{r} \quad$ then range curvature should

be corrected.

Previous techniques used to correct for range curvature are given an excellent summary in Leith (1). These techniques have in common a recognition of the range dependence of curvature described by Eqn. 1. However, it turns out that for optical implementations it is more convenient to work either with the range Fourier transform of (the ensemble of the) functions represented by Eqn. 1, or with the two dimensional transform. In these approaches, the (transformed) range data are dispersed and superimposed, thus one has no choice but to choose a correction that is exact at only one range, and to accept the consequential approximation that results for adjacent ranges. Implementation may be elegant, but in effect a compromise.

## 2. THE SOLLUTION

The key to the solution of the range curvature problem rests in recognition of the coordinate system in which the aberration arises within the sensor/object space itself. From Figure 1, the incremental curvature is a single valued function determined by (i) the (instantaneous slant) range to an object, and (ii) its position (with respect to same reference angle) within the real antenna beam in the so-called azimuth coordinate. Operationally, a convenient azimuth position reference is the zero-Doppler line,* and in the simple case modelled here, there is a one-to-one equivalence between angular bearing and Doppler frequency for given object within the antenna pattern at a particular point in time. Indeed, Doppler frequency $f_{d}$ is given by

$$
\begin{equation*}
f_{d}(t)=\frac{2}{\lambda} \quad \frac{d}{d t} R(t) \tag{3}
\end{equation*}
$$

so that from Eqn. 1 we have

$$
\begin{equation*}
f_{d}(t)=\frac{2 V^{2} t}{\lambda R_{o}} \tag{4}
\end{equation*}
$$

which relates for each and every object the time history and its Doppler history as it traverses the beam.

The signal history of relevance to the correction problem is to be found in the range image/azimuth Doppler plane. Thus, on reception of SAR data, we require two transformations: range focusing and azimuth frequency (Fourier) transformation. The loci of signals in the ( $\mathrm{r}, \mathrm{f}_{\mathrm{d}}$ ), plane is given by Eqn. 1 , with a linear change in variables according to Eqn. 4, yeilding

$$
\begin{equation*}
R\left(f_{d}\right)=R_{0}\left[1+\frac{1}{2}\left(\frac{\lambda}{2 V}\right)^{2} f_{d}\right] \tag{5}
\end{equation*}
$$

as plotted in Figure 2.
This apparently trivial step has many ramifications. First, it is conditional on there being sufficient signal structure (large time-bandwidth product) so that the one-one mapping from time to frequency is valid. This is the case for Seasat A, and other SAR devices in the class we are considering. Second, the correction to be accomplished is explicitly represented by the second term of Eqn. 5, which is range and Doppler dependent. Since it is gently quadratic in Doppler and linear in range, it may be realized by optical or digital devices, and is exact at all ranges so compensated. Third, the range/ Doppler plane(surprisingly!) does not occur in conventional anamorphic telescopic optical processors normally used for SAR, although there is no fundamental reason forbidding it! Thus, this implementation would require modification of the current methods, although substantially the same optical elements could be employed. Fourth, in the event several sub-images are to be integrated, each

[^0]from a different aspect angle within the real antenna beam, implementation of the ( $r, f_{d}$ ) range curvature correction as suggested herein automatically brings about the required registration of these multi-look images, thereby solving another problem facing these systems.

## 3. CONCLUSIONS

Use of the range image/azimuth Doppler plane allows full correction of the SAR range curvature problem, correct at all ranges and in such a way that multilook images derived from sub-apertures become automatically registered. The approach is realizable either optically or digitally, although analysis shows that for the Seasat case, in which the model is more complex than that shown here, digital methods are more flexible.

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REFERENCE
(1) E.N. Leith JOSA 63 No.2, Feb. 1973;pp 119-126.


Figure 1. Geometry. Coherent Pulse Ranging System in the sensor/obiect trajectory plane. $t=0$ at the abeam or $0=0$ position.


Figure 2. Loci of scatterer range plotted as a function of Doopler frequency shift, for a variety of reference ranges Ro.


[^0]:    ${ }^{*}$ Note that for the Seasat case the "zero-Doppler line is neither perpendicular to the satellite velocity vector, nor is it a straight line in the slant range plane.

