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EQUAL PERIOD ORBIT DETERMINATION FOR THE SOLAR SYSTEM PLANETS

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16. ABSTRACT

Rendezvous and docking maneuvers by spacecraft often make use of equi-period orbits. For example, a spacecraft in circular orbit about the Earth's Moon may put an observation module into an eliptical orbit which lowers the perigee for a closer observation of the lunar surface. Use of equi-period orbits will bring the observation module back to rendezvous with the other module which has remained in circular orbit. There will be one rendezvous opportunity each revolution.

This report presents the determination of a set of equations for equi-period orbits work and a set of figures for quick reference.

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (Concluded)

EQUAL PERIOD ORBIT DETERMINATION FOR THE SOLAR SYSTEM PLANETS

INTRODUCTION

Rendezvous and docking maneuvers by spacecraft often make use of equi-period orbits. For example, a spacecraft in circular orbit about the Earth's Moon may put an observation module into an elliptical orbit which lowers the perigee for a closer observation of the lunar surface. Use of equi-period orbits will bring the observation module back to rendezvous with the other module which has remained in circular orbit. There will be one rendezvous opportunity each revolution.

Only the application of an impulse to the spacecraft in the proper attitude is required to change between circular and elliptical equi-period orbits. The orbit transfer is accomplished by applying sufficient impulse to change the velocity by:

$$dv = R_o\sqrt{2g_o}\left[\frac{1}{R_o + h}\left(1 - \left[\frac{(R_o + h)^2 - (h - h_p)^2}{(R_o + h)}\right]^{1/2}\right)\right]^{1/2} . \quad (1)$$

The equi-period orbit velocity vector will be at an angle θ degrees from the circular velocity:

$$\theta = \text{arc tan} \frac{\dfrac{h}{R_o} - \dfrac{h_p}{R_o}}{\left[1 + \dfrac{2h}{R_o} + \dfrac{2hh_p}{R_o^{\,2}} - \dfrac{h_p^{\,2}}{R_o^{\,2}}\right]^{1/2}} . \quad (2)$$

The equations in this report are straightforward and are used to generate Figures 1 through 20. The constants for these calculations are presented in the Appendix. All figures are for the drag free case and are duly noted.

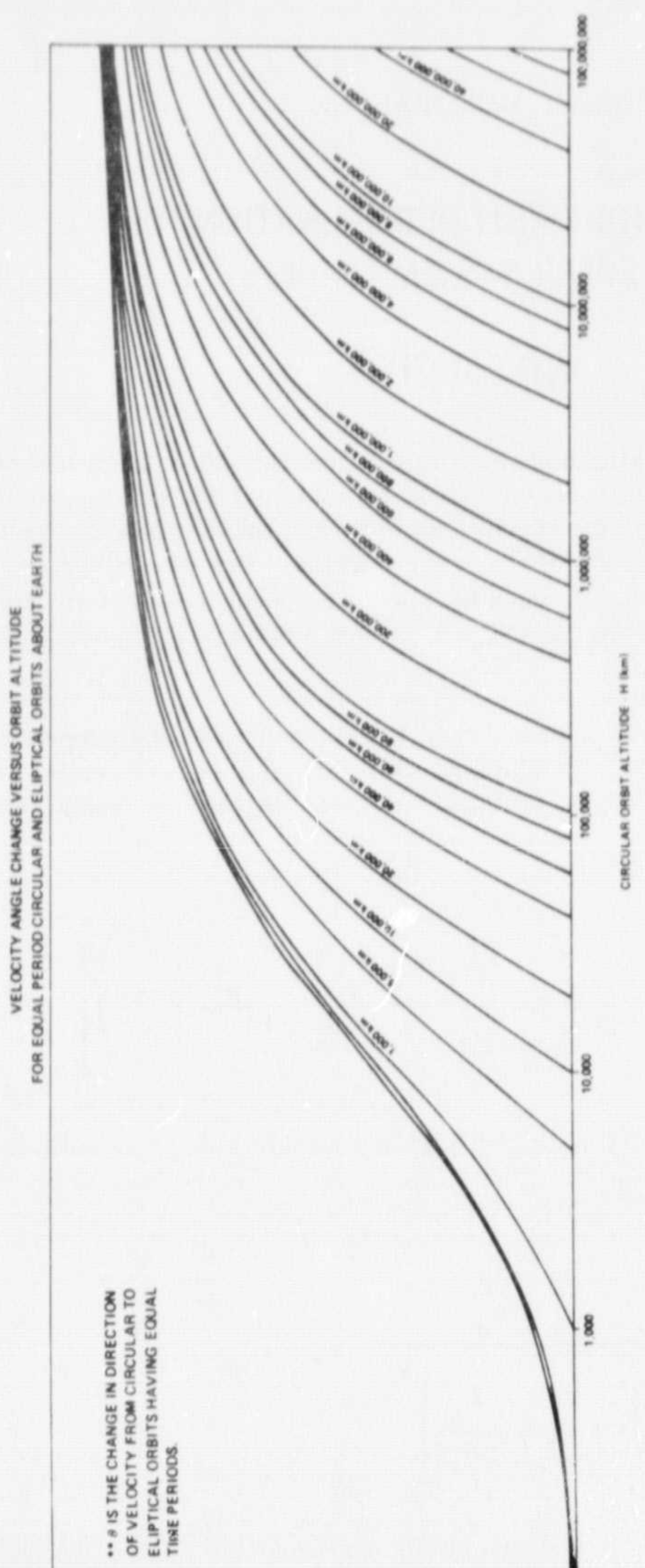

Figure 1. Velocity angle change (θ) for Earth.

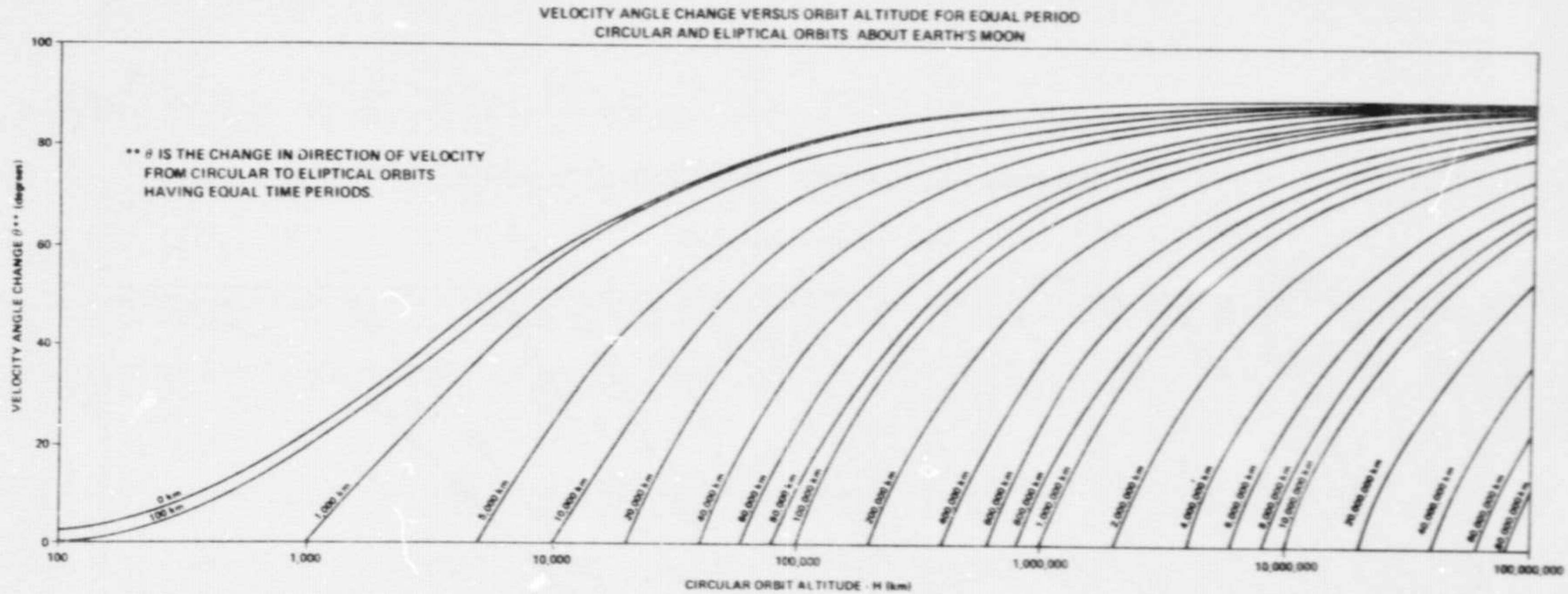

Figure 2. Velocity angle change (θ) for Earth's Moon.

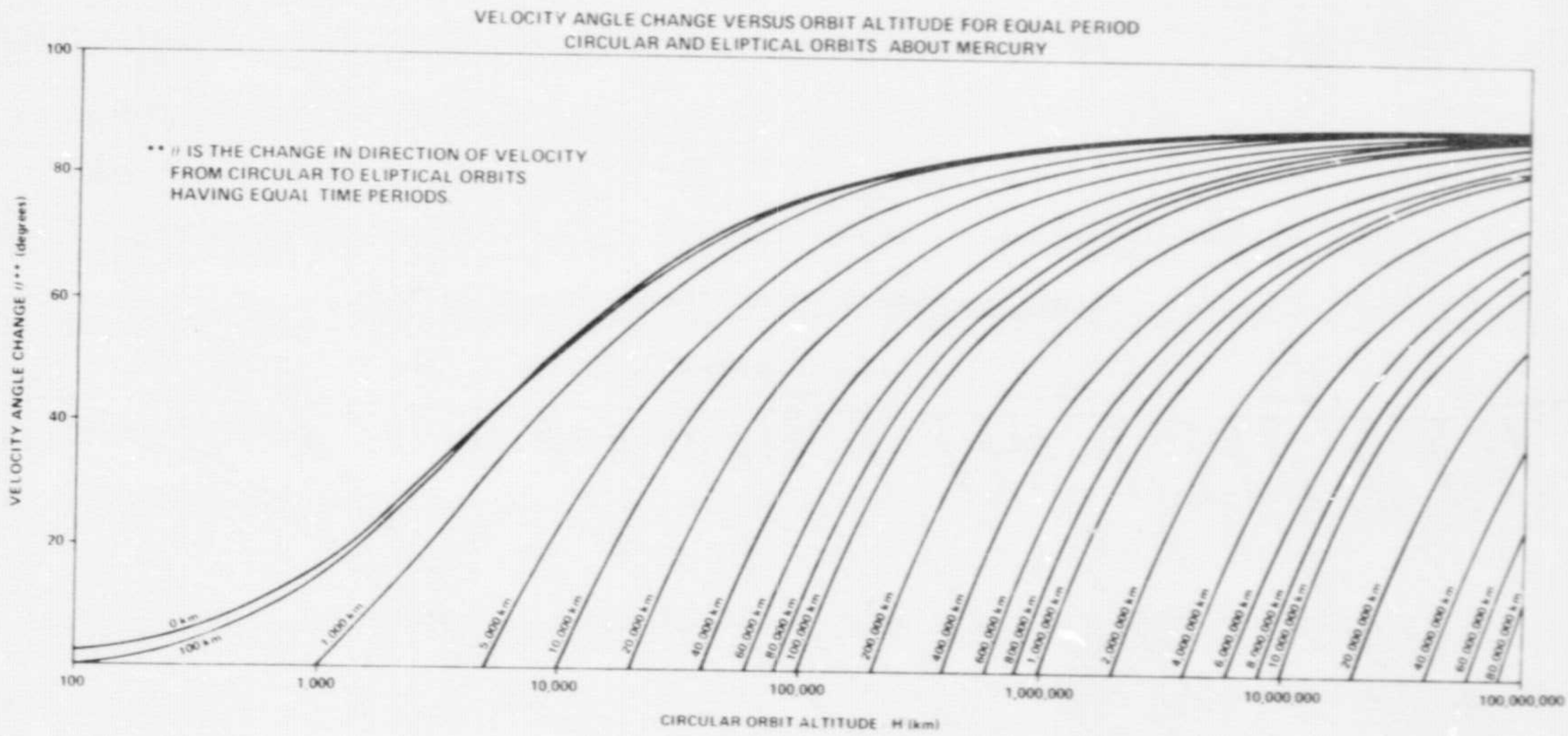

Figure 3. Velocity angle change (θ) for Mercury.

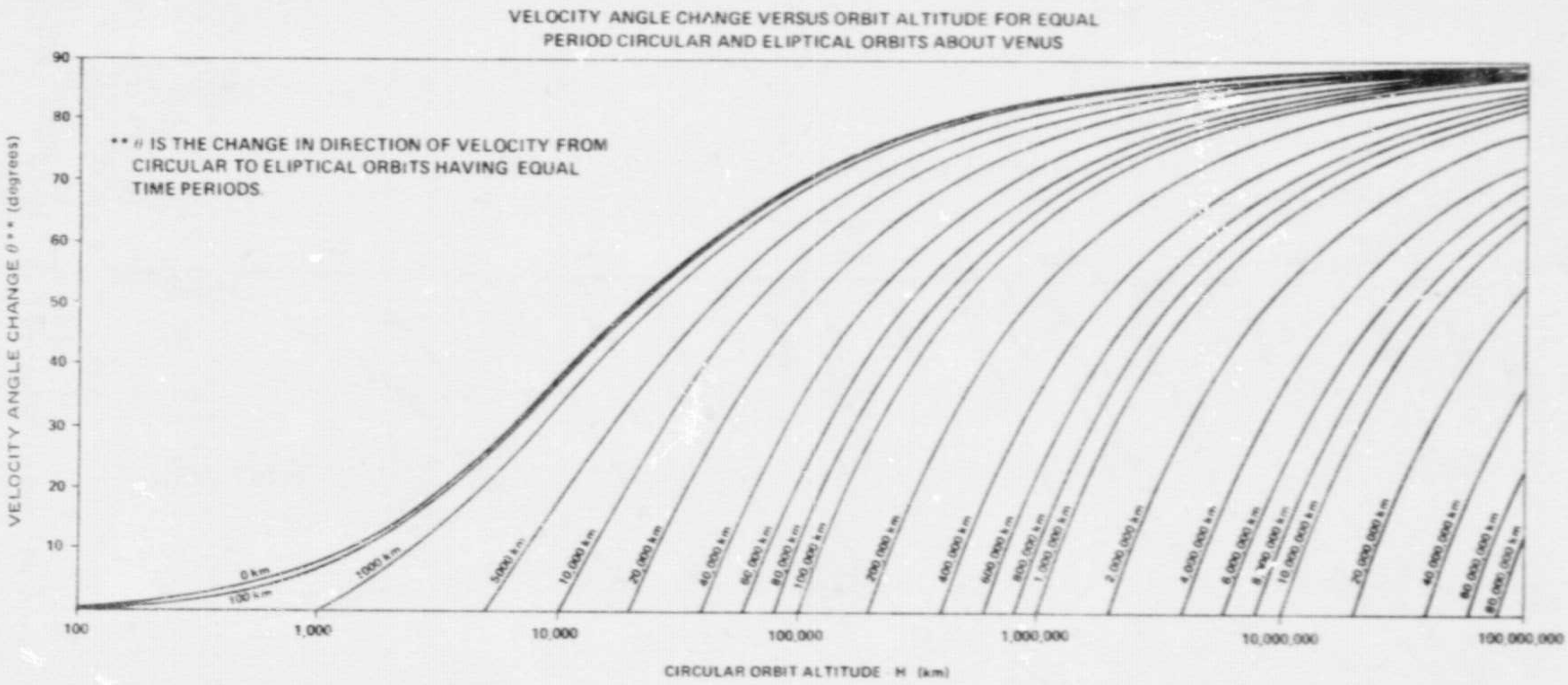

Figure 4. Velocity angle change (θ) for Venus.

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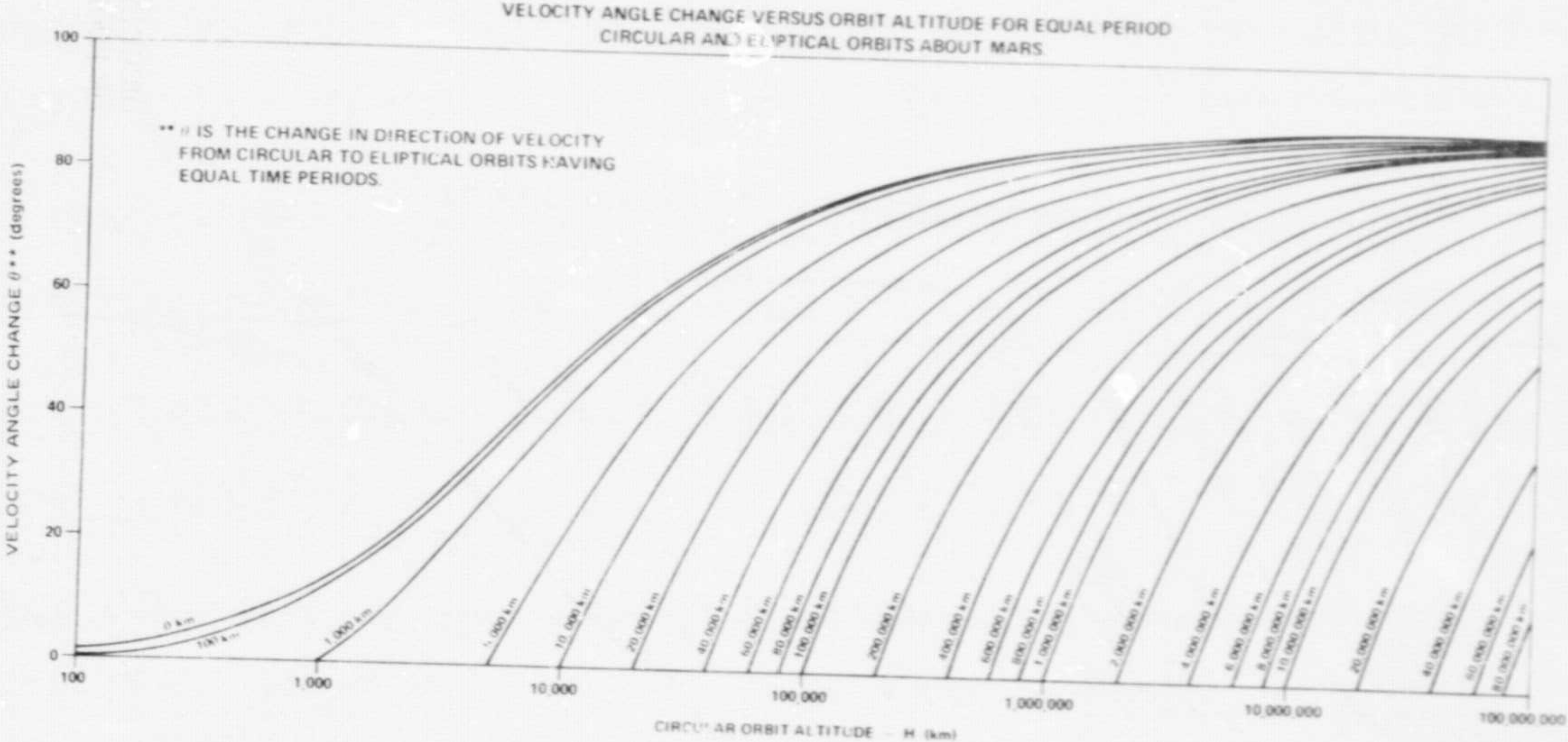

Figure 5. Velocity angle change (θ) for Mars.

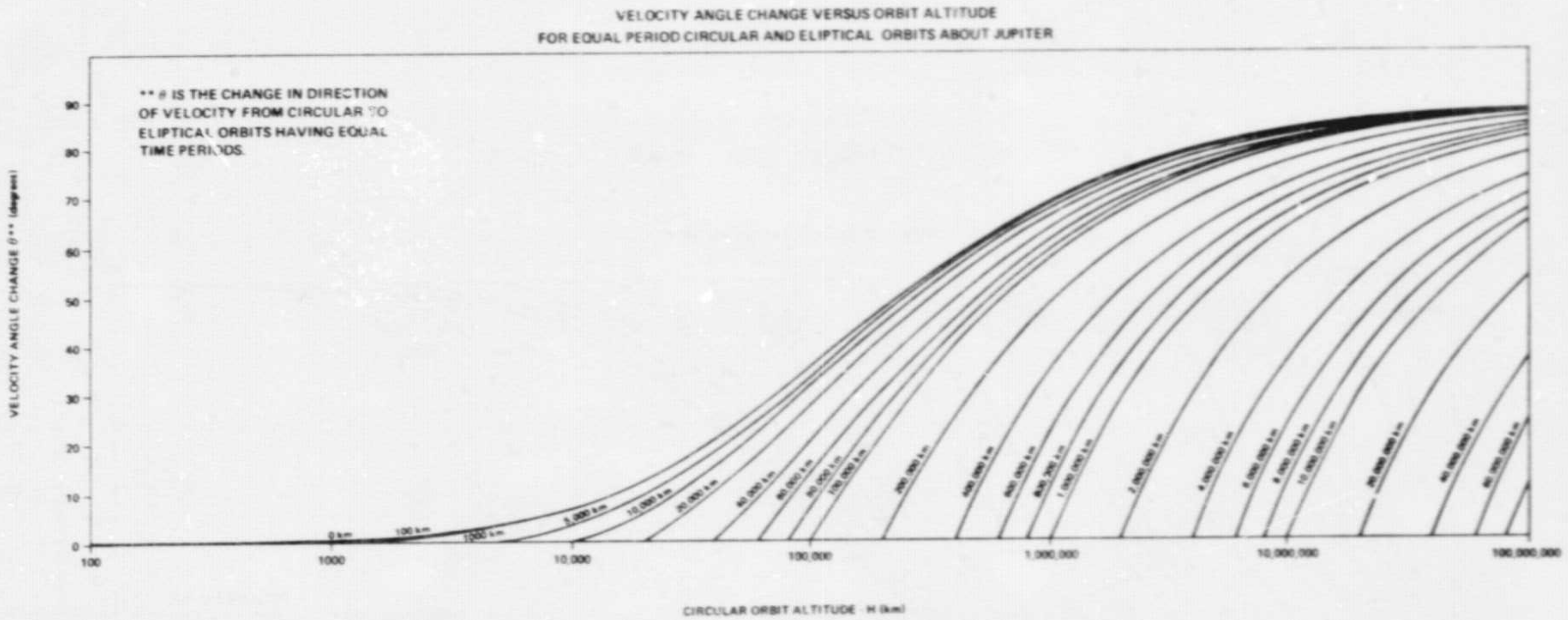

Figure 6. Velocity angle change (θ) for Jupiter.

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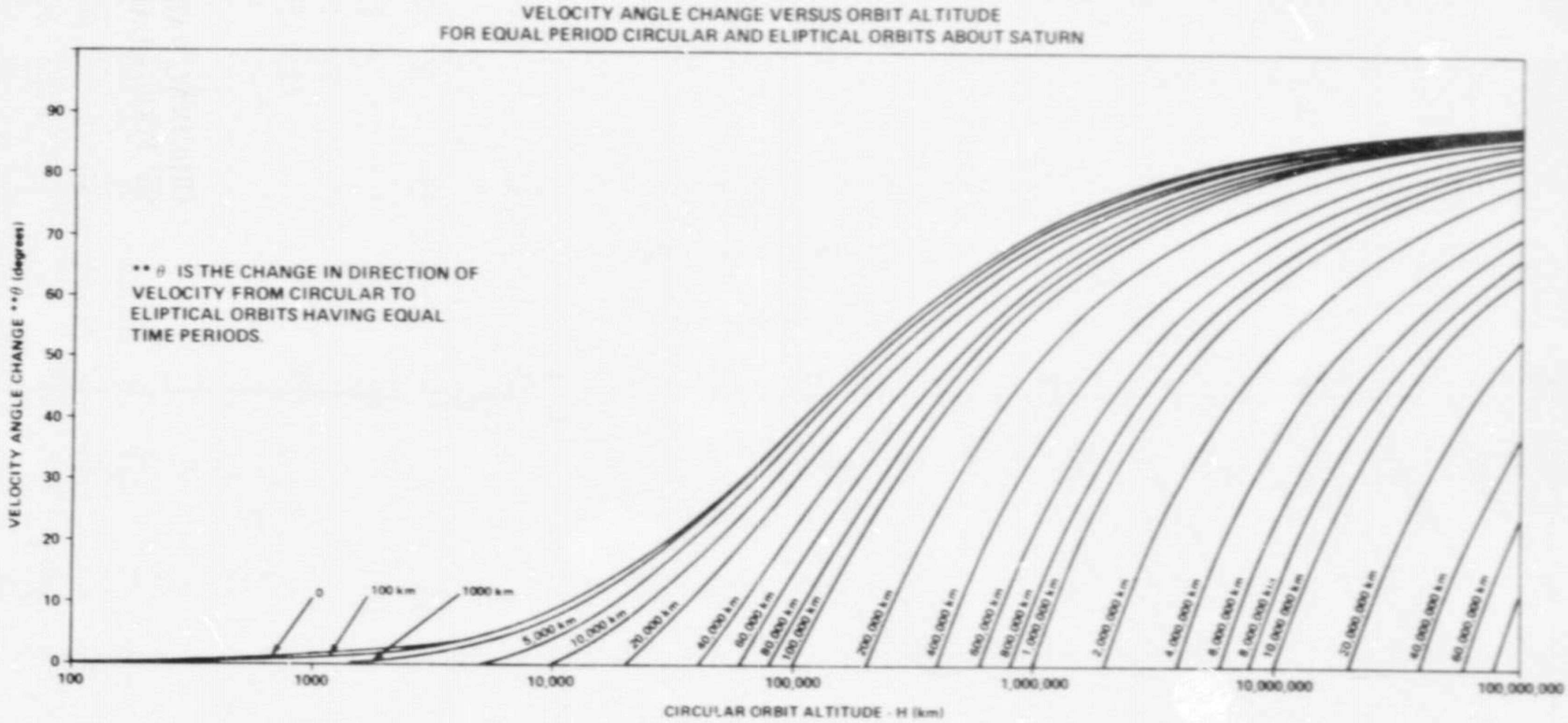

Figure 7. Velocity angle change (θ) for Saturn.

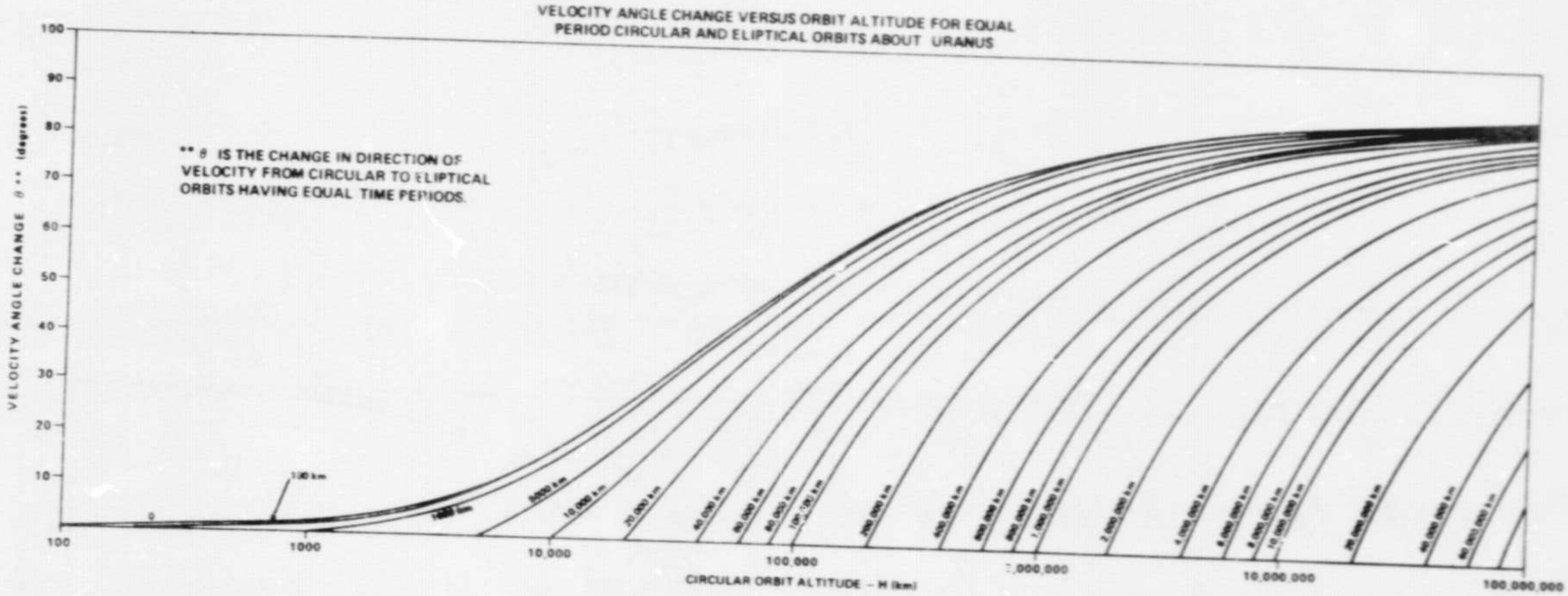

Figure 8. Velocity angle change (θ) for Uranus.

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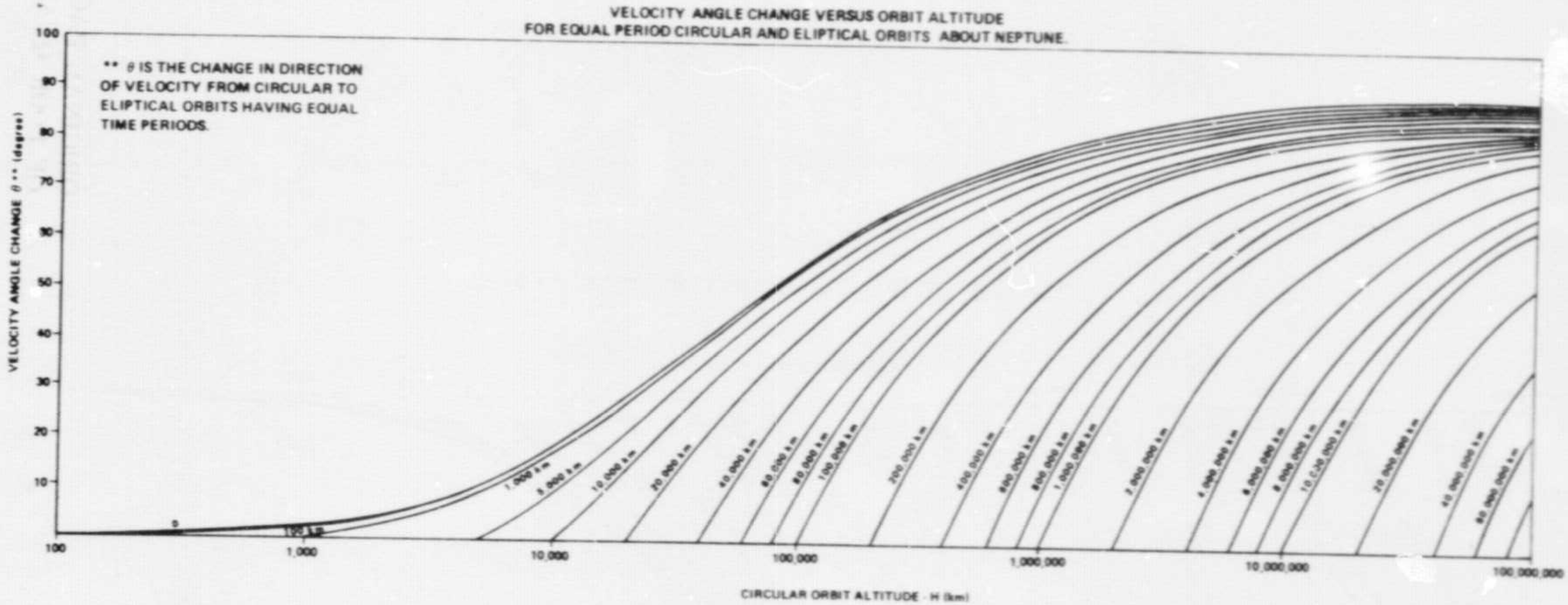

Figure 9. Velocity angle change (θ) for Neptune.

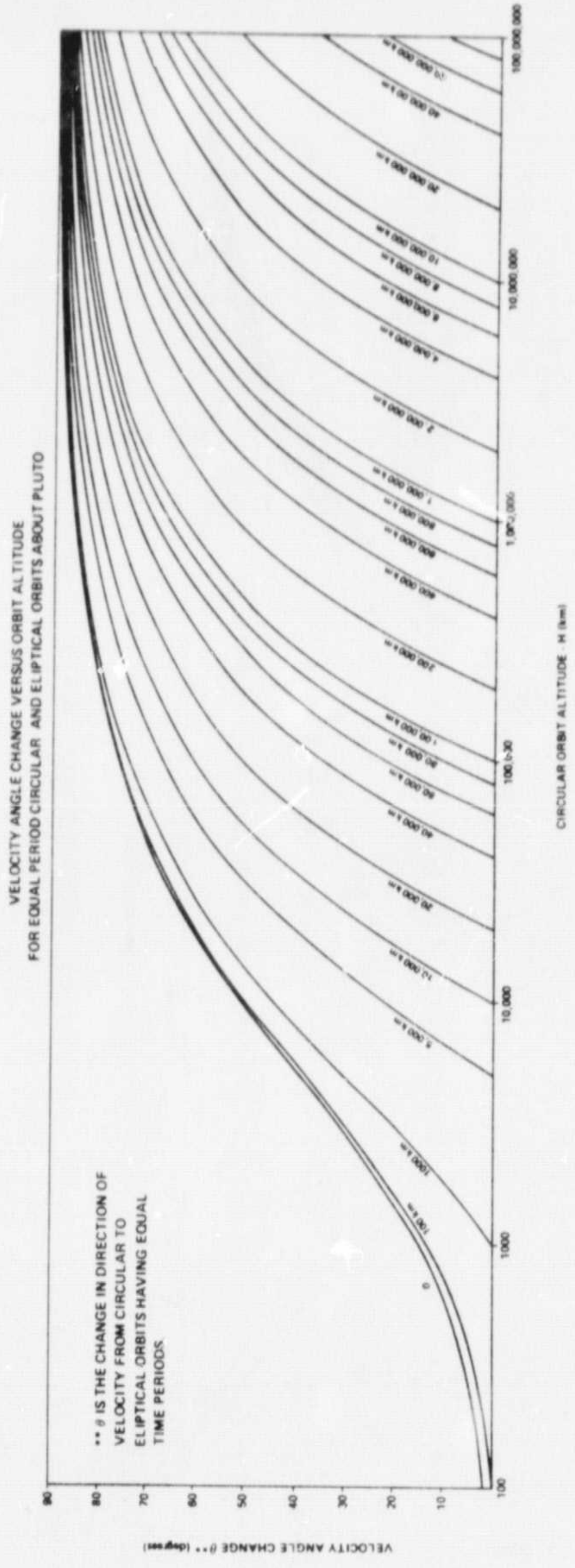

Figure 10. Velocity angle change (θ) for Pluto.

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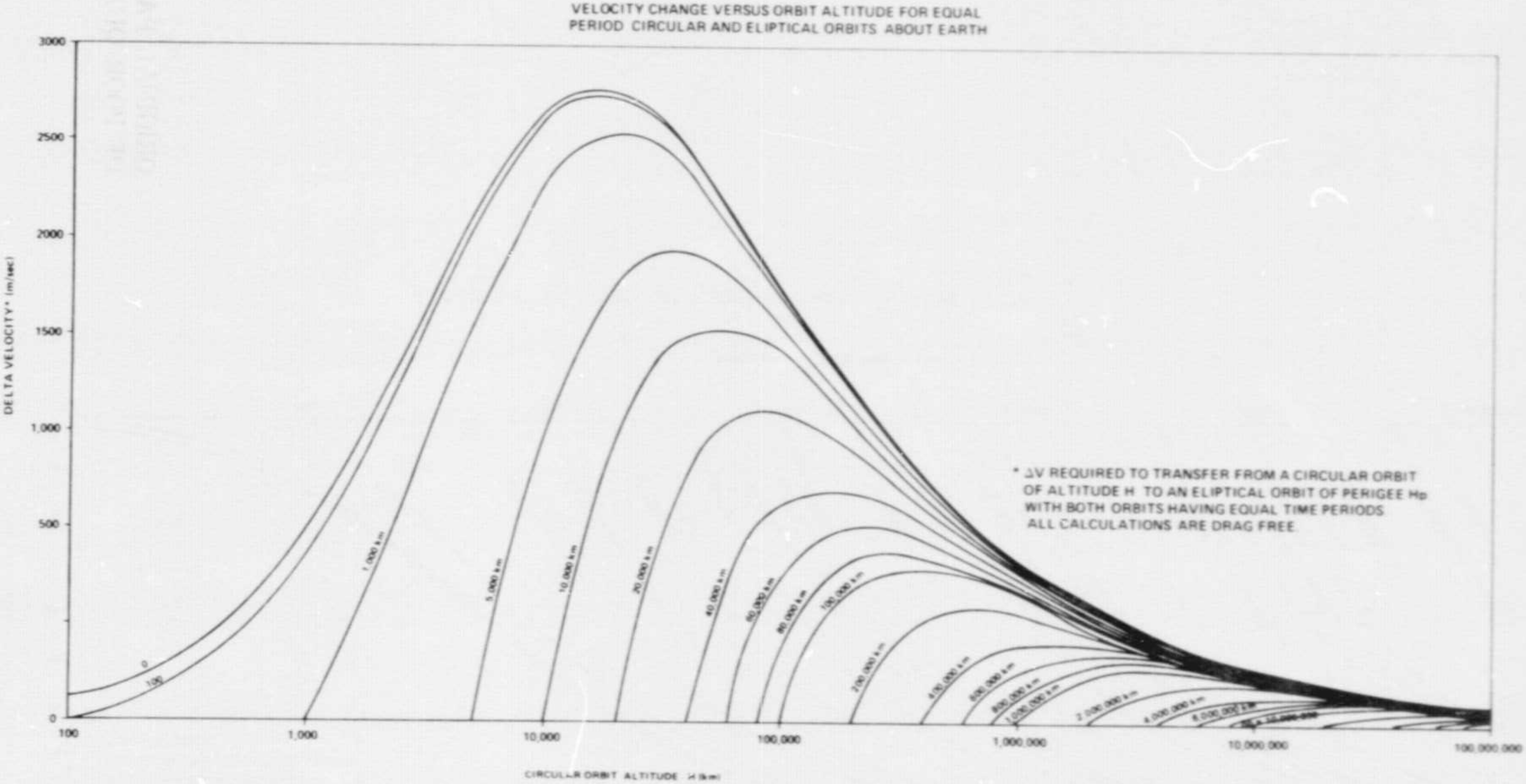

Figure 11. Velocity change (ΔV) for Earth.

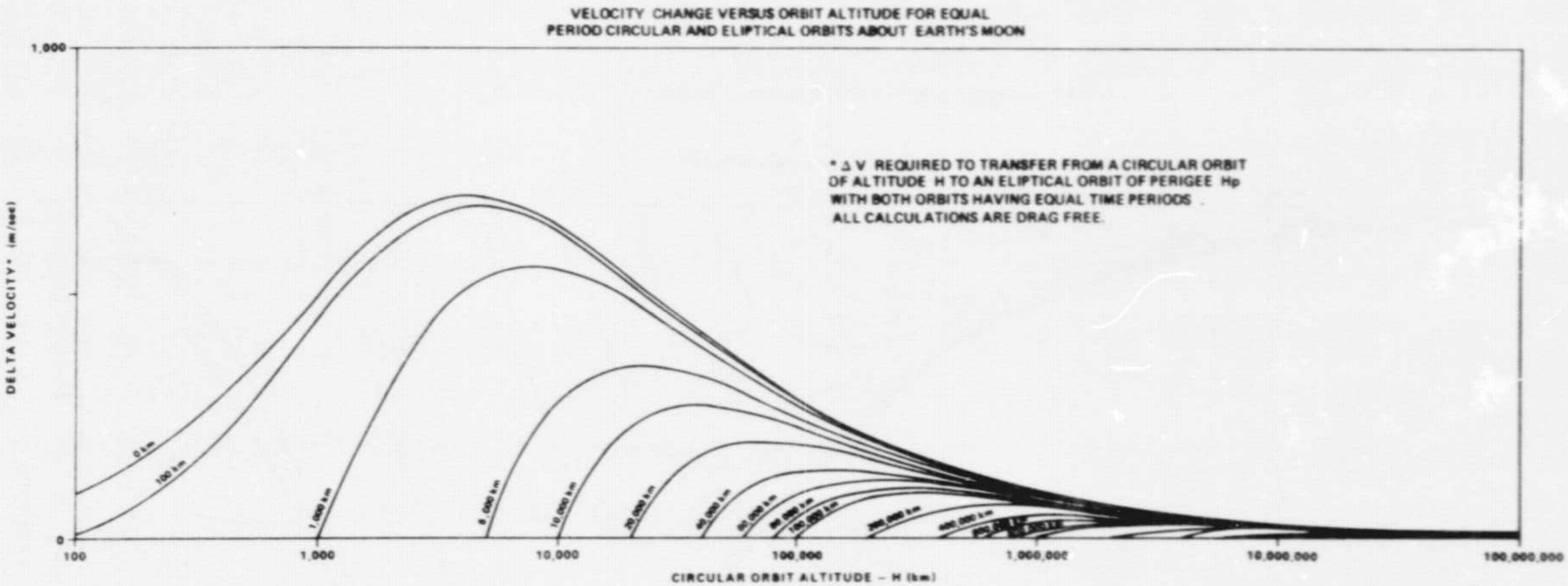

Figure 12. Velocity change (ΔV) for Earth's Moon.

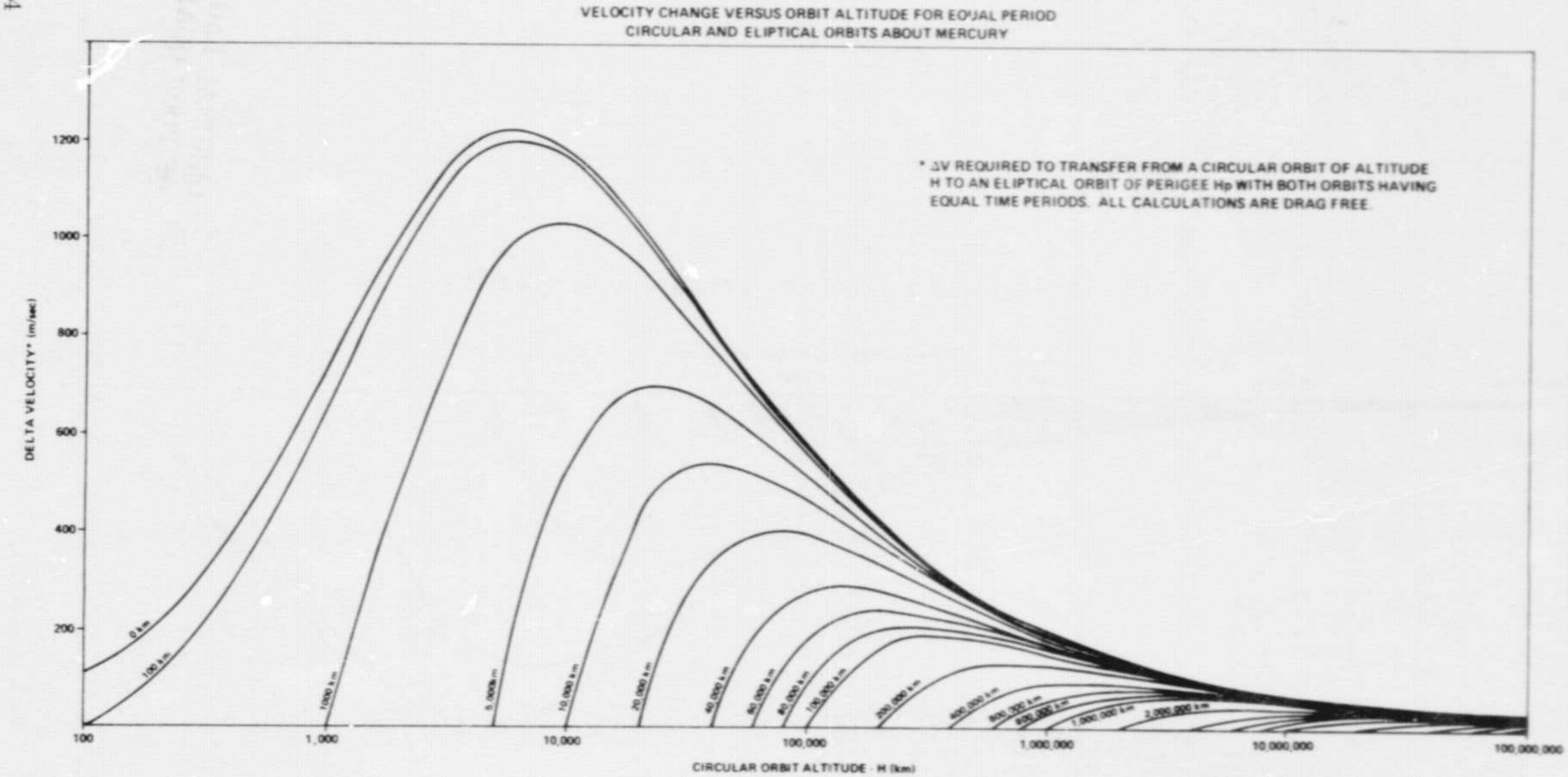

Figure 13. Velocity change (ΔV) for Mercury.

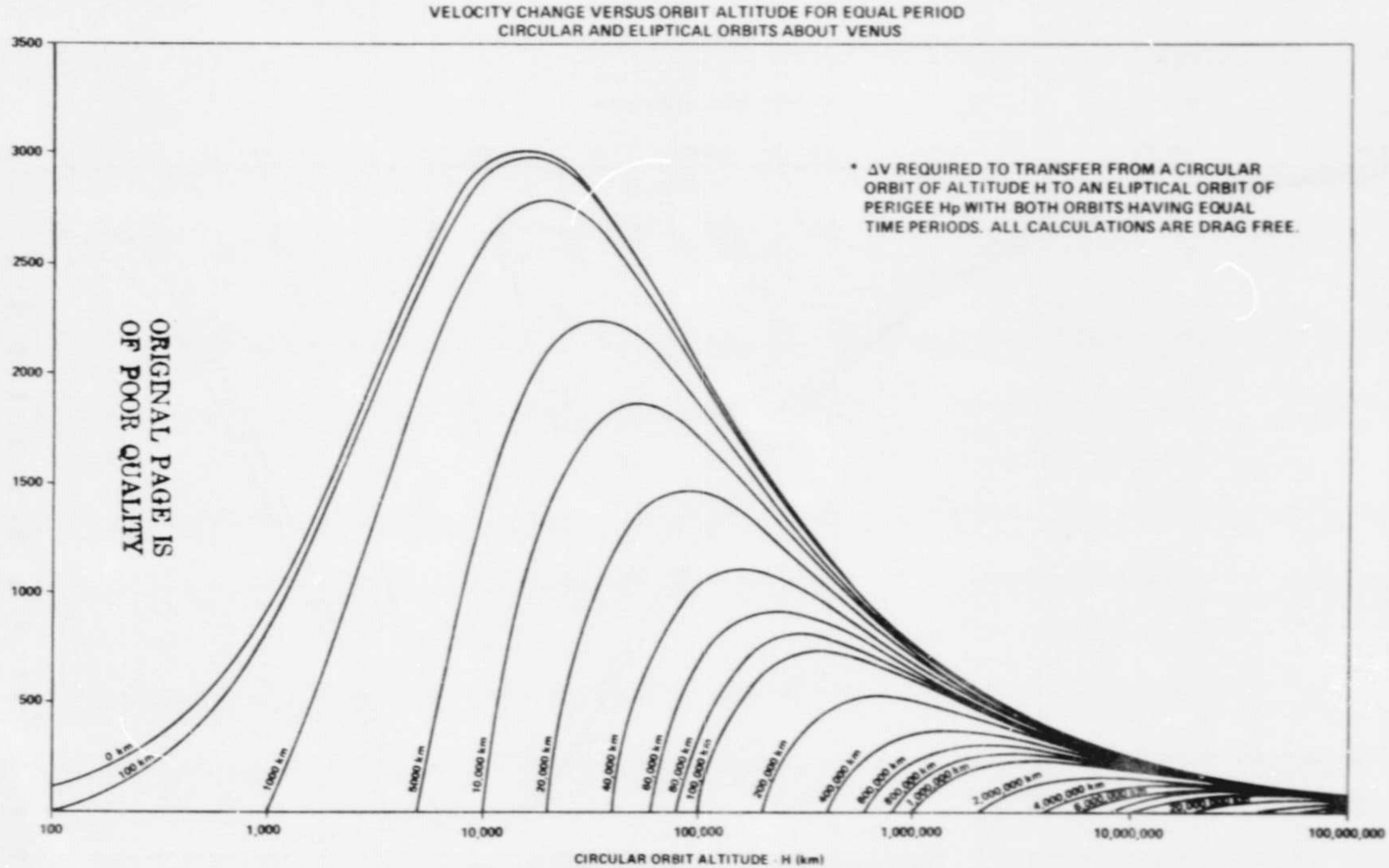

Figure 14. Velocity change (ΔV) for Venus.

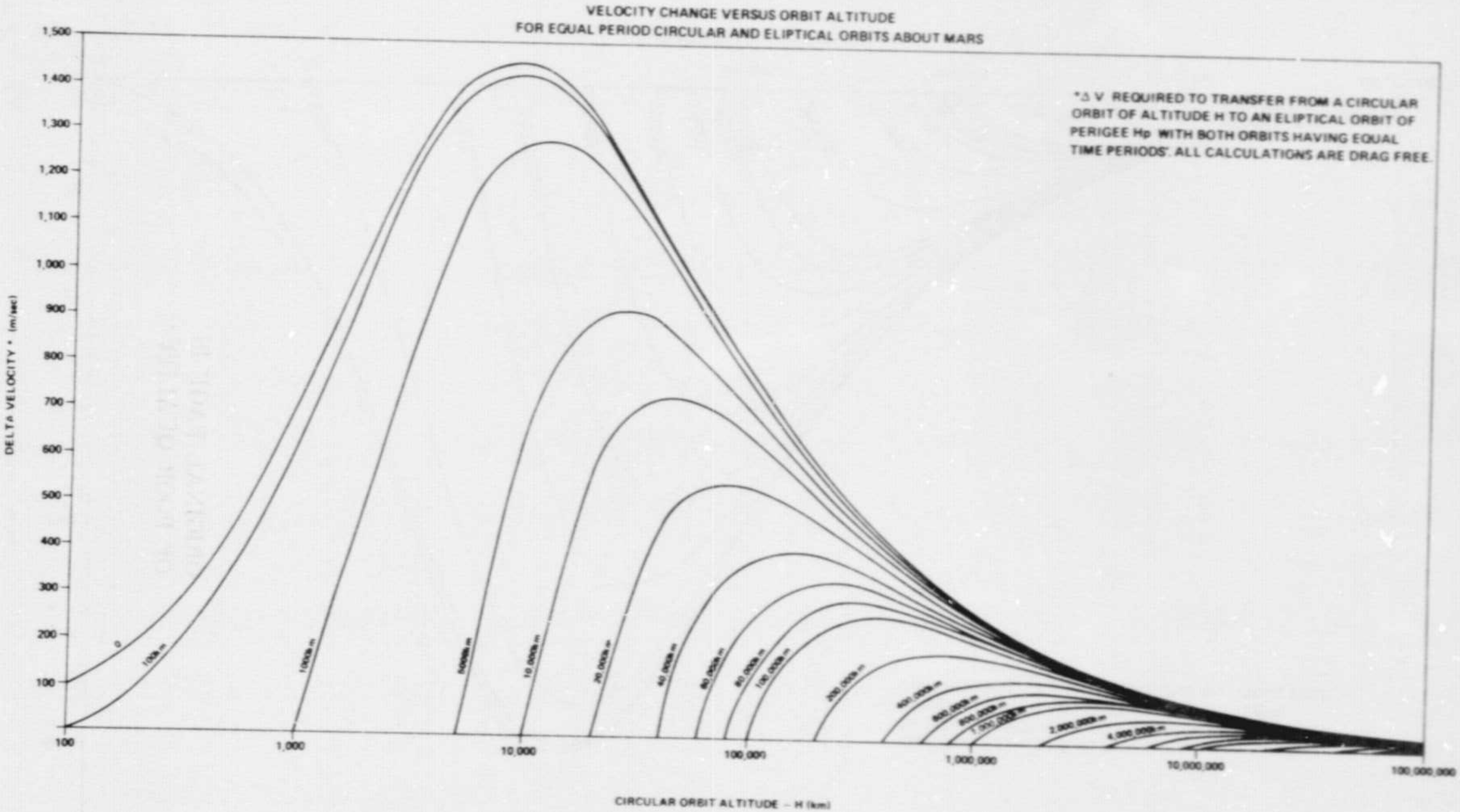

Figure 15. Velocity change (ΔV) for Mars.

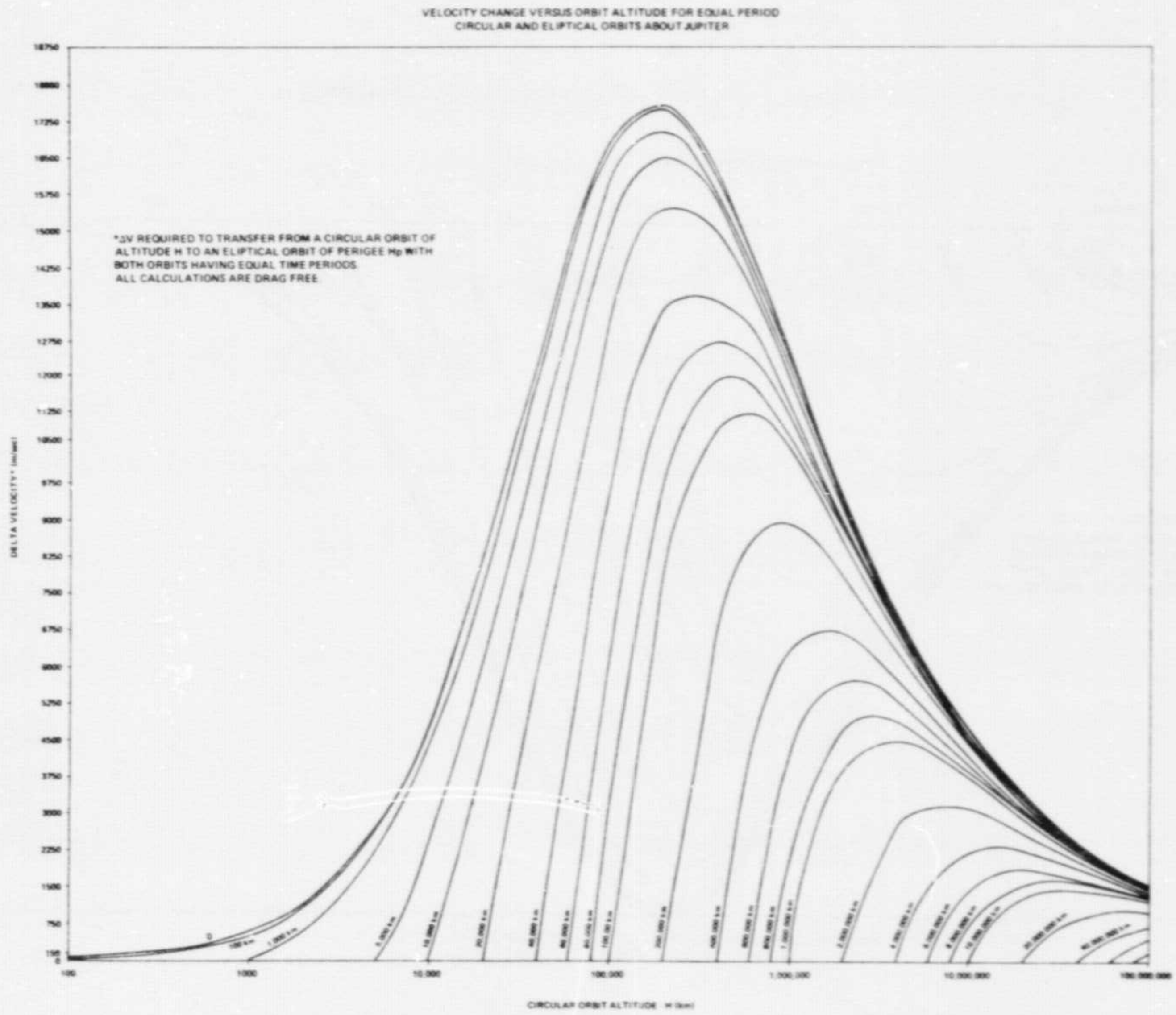

Figure 16. Velocity change (ΔV) for Jupiter.

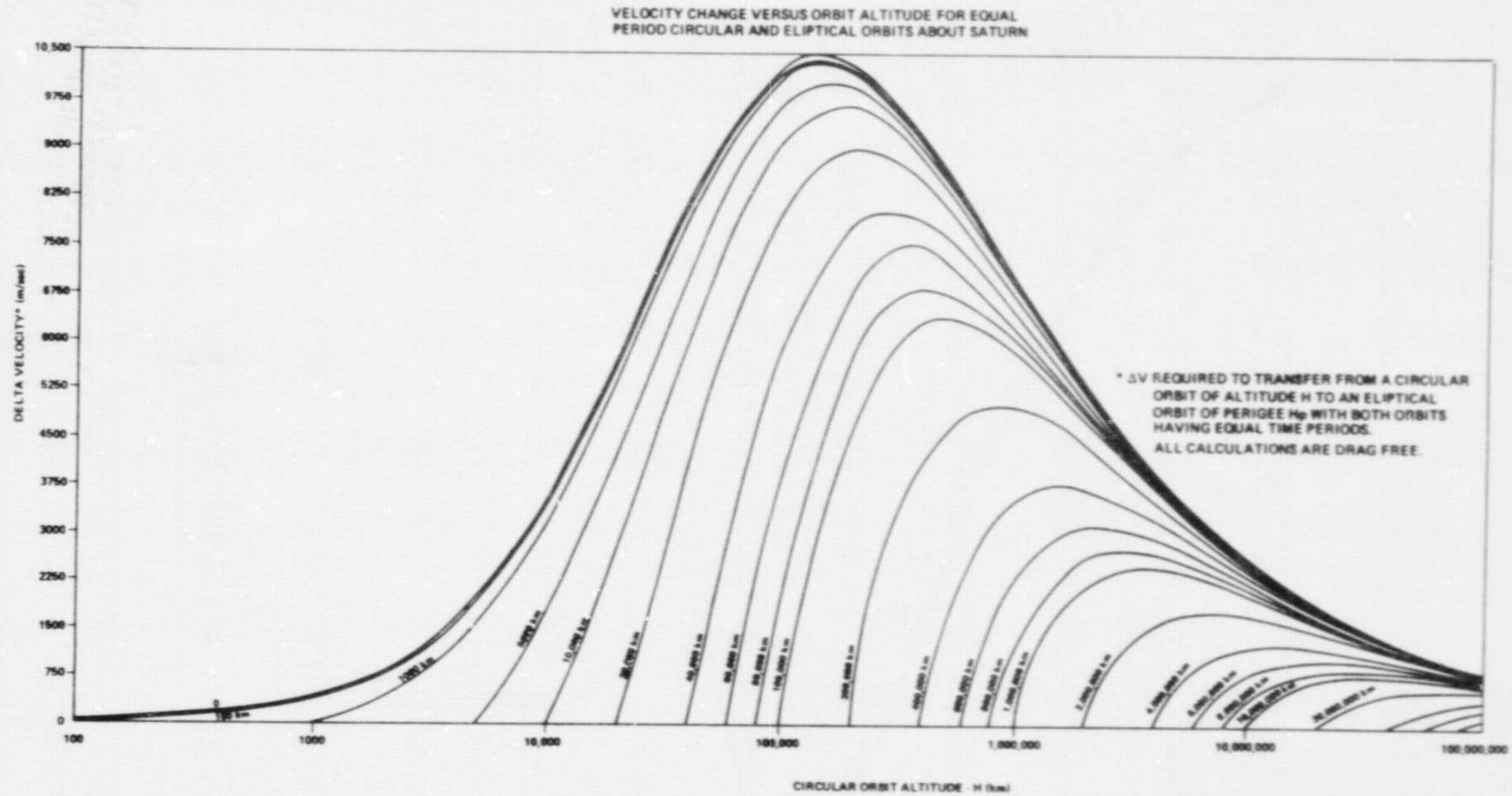

Figure 17. Velocity change (ΔV) for Saturn.

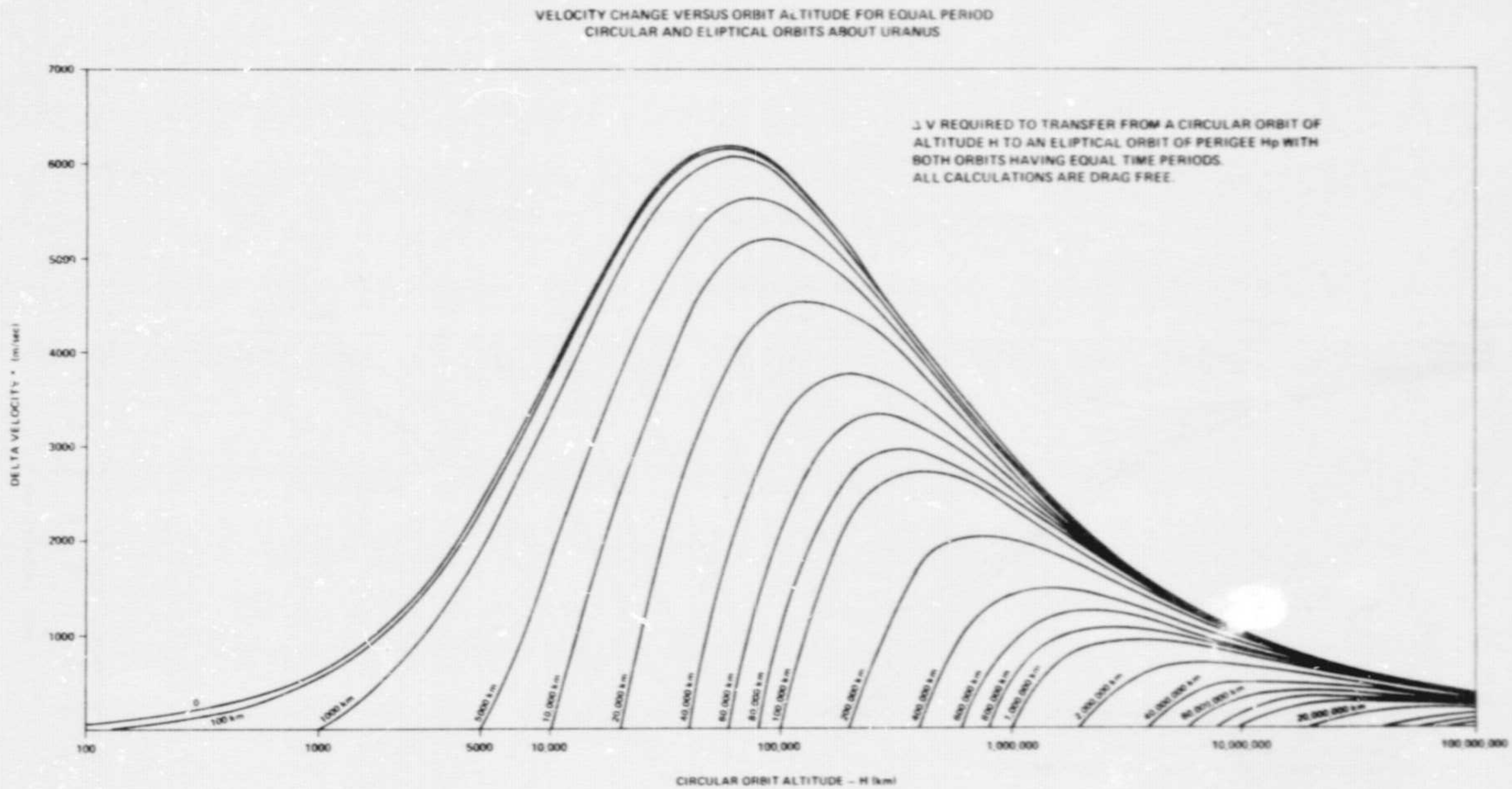

Figure 18. Velocity change (ΔV) for Uranus.

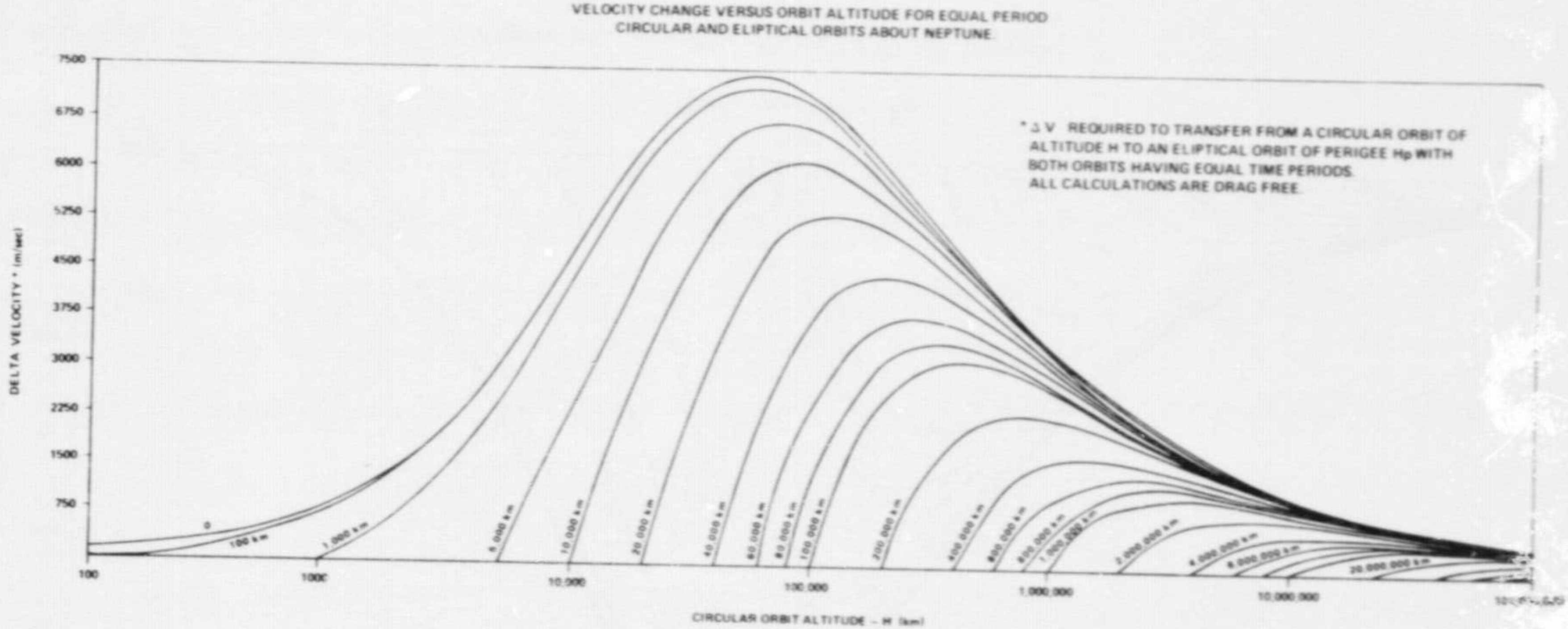

Figure 19. Velocity change (ΔV) for Neptune.

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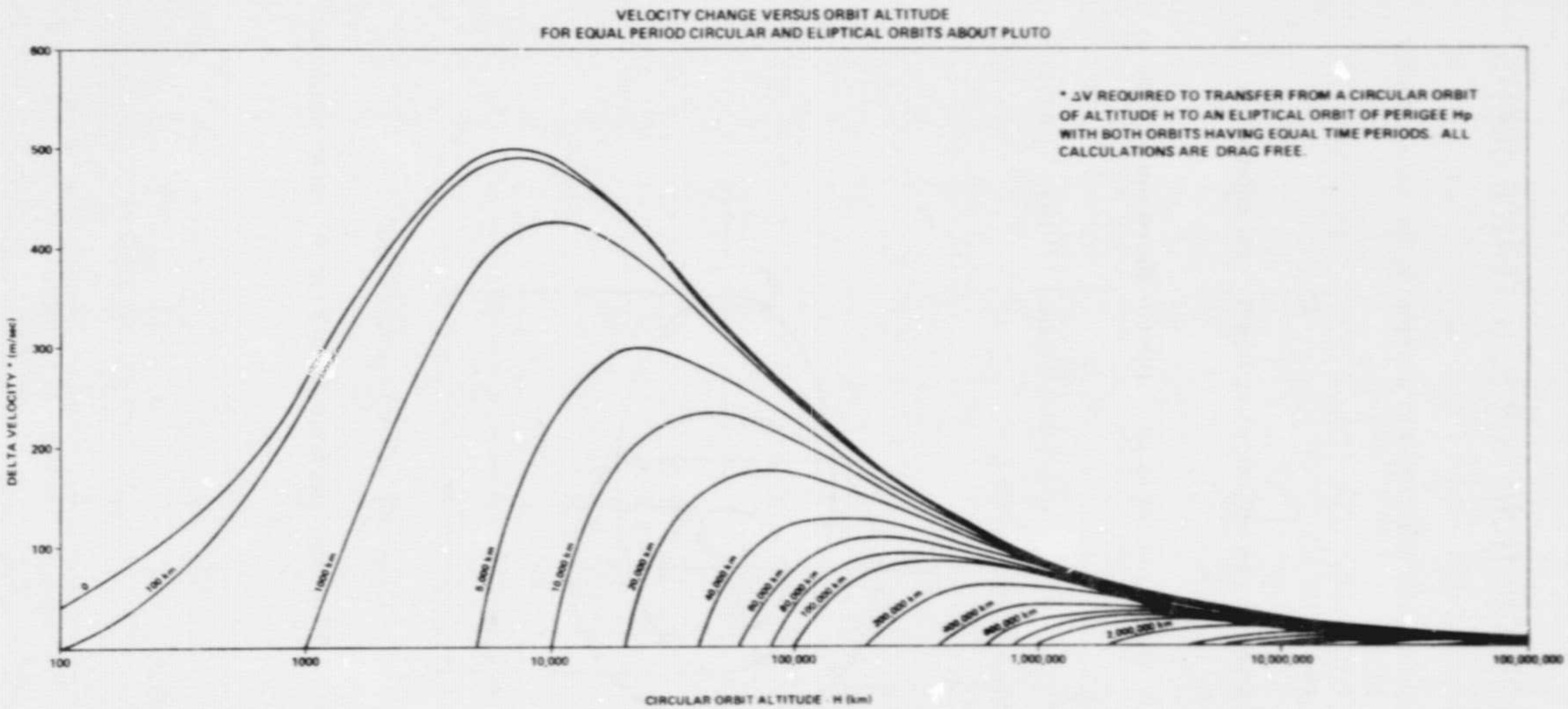

Figure 20. Velocity change (ΔV) for Pluto.

DEVELOPMENT OF EQUATIONS FOR EQUI-PERIOD ORBITS

The following assumptions were used in developing the equations for equi-period orbits (Fig 21):

(a) A central force field.

(b) A uniform homogeneous sphere of radius R_o and acceleration of gravity g_o.

(c) A body M_1 in circular orbit at an altitude h above the surface of the sphere.

(d) A body M_2 in an elliptical orbit with perigee altitude h_p and apogee altitude h_a above the surface of the sphere.

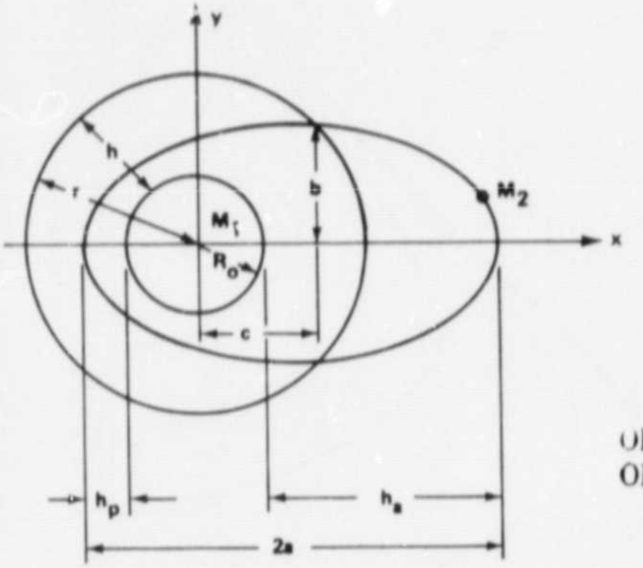

Figure 21. Two-body orbital configuration.

To impose equi-period orbits, the time period for M_1 must equal the time period for M_2:

$$\tau_{M_1} = \tau_{M_2} \quad . \tag{3}$$

The time period (τ_1) for M_1 in circular orbit is

$$\tau_{M_1} = \frac{2\pi\left(1 + \frac{h}{R_o}\right)^{3/2}}{\left(\frac{g_o}{R_o}\right)^{1/2}} \quad . \tag{4}$$

The time period for M_2 elliptical orbit is

$$\tau_{M_2} = \frac{2\pi}{\left(\frac{g_o}{R_o}\right)^{1/2}} \cdot \left(1 + \frac{h_p + h_a}{2R_o}\right)^{3/2} \quad . \tag{5}$$

Substituting equations (4) and (5) into equation (3) yields

$$\frac{2\pi\left(1 + \frac{h}{R_o}\right)^{3/2}}{\left(\frac{g_o}{R_o}\right)^{1/2}} = \frac{2\pi}{\left(\frac{g_o}{R_o}\right)^{1/2}}\left(1 + \frac{h_p + h_a}{2R_o}\right)^{3/2} \quad , \tag{6}$$

which simplifies to

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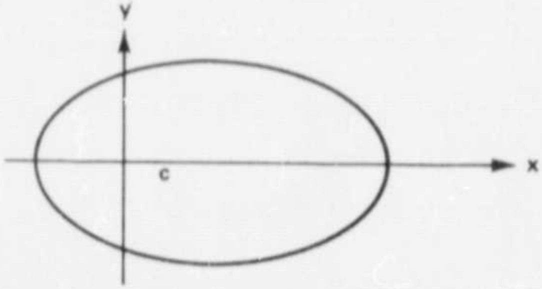
$$\frac{h}{R_o} = \frac{h_p + h_a}{2R_o} \quad . \tag{7}$$

and

$$h_a = 2h_a - h_p \quad . \tag{8}$$

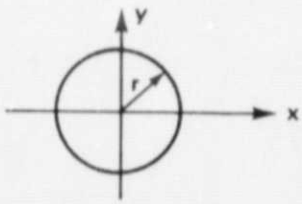
We now select as our coordinate system a cartesion coordinate with origin at the center of the spheroid. The equation for an ellipse with one focal point at the origin of the coordinate system is

$$b^2 (x-c)^2 + a^2y^2 = a^2b^2 \quad . \tag{9}$$



The equation for a circle with center at the origin of the coordinate system is

$$x^2 + y^2 = r^2 \quad . \tag{10}$$



Rewriting equation (8),

$$h = \frac{1}{2} (h_a + h_p) \quad . \tag{11}$$

From Figure 21, it can be seen that the radius (r) of the circular orbit is equal to $h + R_o$. Adding R_o to both sides of equation (11),

$$r = h + R_o = \frac{1}{2}(h_a + h_p) + R_o \quad . \tag{12}$$

From Figure 21, the major axis 2a is

$$2a = h_p + 2\,R_o + h_a \quad . \tag{13}$$

The semimajor axis a is

$$a = \frac{1}{2}(h_p + h_a) + R_o \quad . \tag{14}$$

Comparing equations (12) and (14), it can be seen that the semimajor axis of the ellipse is equal to the radius of the circle in terms of R_o, h_a, and h_p.

As shown in Figure 21, the circle and ellipse intersect at coordinate $X = c$ and $Y = b$. From the trigonometric relationship of Figure 22,

$$a = R_o + h \quad . \tag{15}$$

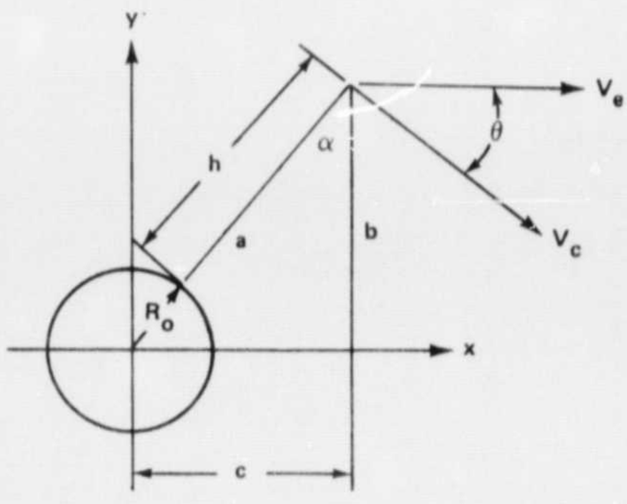


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Figure 22. Orbital vectors.

In Figure 22,

dv = change of velocity required to transfer from a circular orbit of altitude h to an equi-period orbit of perigee h_p

R_o = radius of body

h = altitude of circular orbit

h_p = perigee of elliptical orbit

θ = change in direction of velocity from circular to equi-period elliptical orbit

V_e = elliptical velocity

V_c = circular velocity.

From the trigonometric relationship of Figure 21,

$$a - c - R_o - h_p = o \quad . \tag{16}$$

Substituting equation (15) into equation (16),

$$R_o + h - c - R_o - h_p = o \quad , \tag{17}$$

which simplifies to

$$c = h - h_p \quad . \tag{18}$$

From the trigonometric relationship of Figure 22,

$$b = (a^2 - c^2)^{1/2} \quad . \tag{19}$$

Substituting equations (15) and (17) into equation (19) yields

$$b = [(R_o + h)^2 - (h - h_p)^2]^{1/2} \quad . \tag{20}$$

From the trigonometric relationship of Figure 22, it can be seen that a is perpendicular to V_c and b is perpendicular to V_e. Therefore,

$$\theta = \alpha = \text{arc tan} \frac{c}{b} \quad . \tag{21}$$

Substituting equations (18) and (20) into equation (21) yields

$$\theta = \text{arc tan} \frac{h - h_p}{[(R_o + h)^2 - (h - h_p)^2]^{1/2}} \quad . \tag{22}$$

Dividing equation (22) by R,

$$\theta = \text{arc tan} \frac{\frac{h}{R_o} - \frac{h_p}{R_o}}{\left[\frac{(R_o + h)^2}{R_o^{\,2}} - \frac{(h - h_p)^2}{R_o^{\,2}}\right]^{1/2}} \quad . \tag{23}$$

Expanding the denominator,

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$$\theta = \arc\tan \frac{\dfrac{h}{R_o} - \dfrac{h_p}{R_o}}{\left[\dfrac{R_o^{\ 2}}{R_o^{\ 2}} + \dfrac{2\,R_o\,h}{R_o^{\ 2}} + \dfrac{h^2}{R_o^{\ 2}} - \dfrac{h^2}{R_o^{\ 2}} - \dfrac{2\,h\,h_p}{R_o^{\ 2}} - \dfrac{h_p^{\ 2}}{R_o^{\ 2}}\right]^{1/2}} \quad , \tag{24}$$

which simplifies to

$$\theta = \arc\tan \frac{\dfrac{h}{R_o} - \dfrac{h_p}{R_o}}{\left[1 + \dfrac{2h}{R_o} + \dfrac{2h\,h_p}{R_o^{\ 2}} - \dfrac{h_p^{\ 2}}{R_o^{\ 2}}\right]^{1/2}} \quad . \tag{25}$$

From Figure 22, $V_e = V_c$ at points of intersection of circle and ellipse. Applying cosine law to Figure 22,

$$dv^2 = V_c^{\ 2} + V_c^{\ 2} - 2V_c^{\ 2} \cos\theta \quad . \tag{26}$$

Taking square root of both sides,

$$dv = \left(2V_c^{\ 2} - 2V_c^{\ 2} \cos\theta\right)^{1/2} \quad . \tag{27}$$

Simplifying,

$$dv = V_c\left(2 - 2\cos\theta\right)^{1/2} \quad . \tag{28}$$

From the trigonometric relationship of Figure 22,

$$\cos \theta = \cos \alpha = \frac{b}{a} \quad ; \tag{29}$$

therefore,

$$dv = V_c \left(2 - \frac{2b}{a}\right)^{1/2} \quad . \tag{30}$$

Substituting equation (15) for a and equation (20) for b into equation (30),

$$dv = V_c \left[\frac{2 - 2[(R_o + h)^2 - (h - h_p)^2]^{1/2}}{R_o + h}\right]^{1/2} \quad . \tag{31}$$

Substituting $(g_o R_o / R_o + h)^{1/2}$ for V_c,

$$dv = \left(\frac{g_o R_o^2}{R_o + h}\right)^{1/2} \left[2 - 2\left[\frac{(R_o + h)^2 - (h - h_p)^2}{R_o + h}\right]^{1/2}\right]^{1/2} \quad . \tag{30}$$

Collecting terms,

$$dv = R_o \sqrt{2 g_o} \left[\frac{1}{R_o + h} \left(1 - \left[\frac{(R_o + h)^2 - (h - h_p)^2}{R_o + h}\right]^{1/2}\right)\right]^{1/2} \quad . \tag{31}$$

APPENDIX. VALUES USED FOR TABLES

	Radius (km)	Acceleration of Gravity (m/sec/sec)
Earth	6 371	9.806
Earth's Moon	1 738	1.622
Mercury	2 421	3.60
Venus	6 161	8.69
Mars	3 332	3.73
Jupiter	69 893	26.0
Saturn	57 532	11.2
Uranus	23 701	9.39
Neptune	21 535	14.99
Pluto	2 867	0.5

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Herrick, Samuel and Baker, Robert: Gravitational and Related Constants for Accurate Space Navigation. The Eighth International Astronautical Congress, Barcelona, Spain, October 6, 1957.

APPROVAL

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By Wilbur H. Funston

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

J. T. POWELL
Director, Data Systems Laboratory