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National Aeronautics and Space Administration

Lyndon B. Johnson Space Center
Houston, Texas

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SHUTTLE PROGRAM

IUS PRERELEASE ALINEMENT

By F. A. Evans
IBM

JSC Task Monitor: Alan D. Wylie

Approved: Emil R. Schiesser
Emil R. Schiesser, Chief
Mathematical Physics Branch

Approved: Ronald L. Berry
Ronald L. Berry, Chief
Mission Planning and Analysis Division

Mission Planning and Analysis Division

National Aeronautics and Space Administration

Lyndon B. Johnson Space Center

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INTRODUCTION

IUS PRERELEASE ALIGNMENT

PREFACE

On 15 March 1977, a splinter meeting of the Shuttle users ICD meeting was held during which attention was directed toward the errors involved in transferring the Orbiter IMU alignment to the IUS guidance system. Questions arose regarding what errors were pertinent, their nature, and the alignment transfer accuracy achievable. Failure to align with sufficient accuracy apparently implied a need to install a star tracker on the IUS.

Boeing had assumed a per-axis alignment transfer accuracy of 6.3 $\widehat{\text{min}}$ (3σ) in connection with navigation error analysis of four different IUS reference missions (reference 1). This value was based on the understanding that the Orbiter IMU per axis alignment errors would not exceed 6.0 $\widehat{\text{min}}$ (3σ) at the time of alignment transfer. After it was purported at the 15 March splinter meeting that the Orbiter alignment error might significantly exceed 6.0 $\widehat{\text{min}}$, with the implication that the alignment transfer error would significantly exceed 6.3 $\widehat{\text{min}}$, Boeing stated that a star tracker would be required on the IUS in order to achieve the reference mission requirements (reference 2). Thus, the 6.3 $\widehat{\text{min}}$ alignment transfer accuracy appears to stand as the IUS IMU alignment accuracy requirement.

NASA/JSC took the action to evaluate Orbiter/IUS alignment transfer. The first document (reference 3) under this action, titled "Orbiter In-Orbit Alignment Accuracy", dated 21 September 1977, addressed the question of the Orbiter's alignment accuracy, believed at the beginning of the task to be the major contributer to the overall alignment transfer error. The subject document, "IUS Prerelease Alignment", reports the results of analyzing alignment transfer accuracy. This second document shows that Orbiter in-orbit alignment accuracy is not a factor affecting IUS alignment accuracy, if certain procedures are followed.

The basic analysis results are as follows.

- Alignment of the Orbiter, per OFT procedures, followed by separate Orbiter/IUS alignment transfer procedures, just meets the IUS alignment accuracy requirement of 6.3 $\widehat{\text{min}}$, if the elapsed time between Orbiter alignment start and alignment transfer completion is 20 minutes or less, the Orbiter alignment stars are essentially 90 degrees apart, and star images are restricted to the central 4 x 4 degrees of the star tracker's field-of-view

- The 6.3 $\widehat{\text{min}}$ accuracy requirement is easily met by combining the Orbiter in-orbit alignment procedure, modified to remove sensor misalignment bias errors, and the Orbiter/IUS alignment transfer. In this case, expected IUS alignment accuracy is 1 6 $\widehat{\text{min}}$ or better.

The analysis results are more fully summarized in the next section.

SUMMARY OF RESULTS

The analysis initially assumed that the Orbiter alignment and the Orbiter/IUS alignment transfer are performed as two separate procedures. It was then discovered that combining the procedures would lead to a substantial improvement in IUS alignment accuracy. Results are summarized below first for the combined procedure and then for the separate procedures.

Combined Orbiter Alignment and Orbiter/IUS Alignment Transfer Procedure

Combining the Orbiter in-orbit alignment procedure, modified to remove body-fixed sensor misalignments, and the Orbiter/IUS alignment transfer procedure, as prescribed below, leads to an IUS per-axis alignment accuracy of 1.6 $\widehat{\text{min}}$ (3σ) or better. The combined procedure is as follows:

(1) Orbiter takes alignment sighting on star #1, using either of its two star trackers, recording star tracker and Orbiter IMU gimbal angle measurements. IUS attitude from the IUS strap-down IMU system is simultaneously recorded in IUS flight computer.

(2) Orbiter rotates 180 degrees about star #1 line of sight (LOS) and then takes another alignment sighting (same star tracker) on star #1, again recording star tracker and Orbiter IMU gimbal angle measurements. IUS attitude is again simultaneously recorded in the IUS flight computer.

(3) The first and second set of Orbiter measurements are averaged, removing the body-fixed sensor misalignment effects from star #1 measurements. In addition, the eigenvector (eigenvector #1) associated with the 180 degree rotation is computed in both the Orbiter and IUS flight computers. The eigenvector essentially represents the axis of rotation.

(4) The Orbiter selects alignment star #2 and repeats (1), (2), and (3), using either of its two star trackers. This yields an averaged star measurement on star #2 (Orbiter computer) and eigenvector #2 (in Orbiter and IUS computers).

(5) The Orbiter measurements, expressed in the Orbiter's IMU stable member inertial coordinate system, and the IUS measurements, expressed in the IUS inertial coordinate system (orientation unknown at this point), are jointly processed. (The implementation approach for computer processing of the measurements has not been definitely established at the present time, but it is understood that the Orbiter measurements will be supplied to the IUS flight computer where the alignment computation will take place.) The end result of the processing is a 3 x 3 matrix transformation that relates the IUS unknown inertial coordinate frame to the desired IUS inertial navigation coordinate frame (such as the M50 coordinate frame). Applying the matrix transformation to IUS body attitude (one shot computation) constitutes the IUS alignment.

If the alignment stars are 90 degrees apart, the per-axis error of the IUS alignment is 1.6 $\widehat{\text{min}}$ (3σ). If the Orbiter star tracker measurements are restricted to the central 4 x 4 degree field of view (full field of view is 10 x 10 degrees), then the per-axis error is 1.0 $\widehat{\text{min}}$ (3σ).

Table 1. IUS Alignment Accuracy, Combined Procedure

Star Tracker FOV	IUS Alignment Accuracy (3σ)
10 x 10 deg	1.6 $\widehat{\text{min}}$
4 x 4 deg	1.0 $\widehat{\text{min}}$

Note: Alignment stars are 90 degrees apart

The IUS alignment error is due to (1) the Orbiter star tracker and (2) the IUS IMU gyros. Orbiter IMU readout, drift, and alignment errors essentially do not impact the IUS alignment accuracy, given the procedures outlined above.

The average per-axis alignment error degrades by the factor $K = (1 + 2\csc^2 A)^{\frac{1}{2}}/\sqrt{3}$, where A is the subtended angle between the alignment stars. When A = 90 degree K = 1. For 60 degrees $\leq A \leq 120$ degrees, $K \leq 1.1$.

Separate Orbiter Alignment and Orbiter/IUS Alignment Transfer Procedures

When the Orbiter alignment and the Orbiter/IUS alignment transfer are accomplished via separate maneuvers and procedures, then the Orbiter IMU readout, drift, and alignment errors directly impact the IUS alignment accuracy. Two cases were analyzed with the following results.

Case A

The Orbiter is aligned per OFT in-orbit alignment procedure (reference 4), thus the error effects of Orbiter body-fixed sensor misalignments are not removed. Alignment is subsequently transferred to the IUS via rotations about two axes 90 degrees apart in inertial space. IUS alignment accuracy is presented below for (1) rotation magnitudes of 90 and 180 degrees and (2) Orbiter star measurements over the full 10 x 10 degree tracker field of view (FOV) and restricted to the central 4 x 4 degree FOV. The effects of Orbiter gyro drift (.035 deg/hr, 1σ), which depend on elapsed time after Orbiter alignment, are also indicated.

Table 2. Case A IUS Alignment, Orbiter Sensor Body-Fixed Misalignment Errors Not Removed*

Elapsed Time*	Alignment Transfer Rotation Magnitude	IUS Per-Axis Alignment Accuracy (3σ)	
		10 x 10 deg FOV	4 x 4 FOV
0 min	90 deg	6.3 $\widehat{\text{min}}$	6.0 $\widehat{\text{min}}$
	180 deg	6.1 $\widehat{\text{min}}$	5.8 $\widehat{\text{min}}$
20 min	90 deg	6.7 $\widehat{\text{min}}$	6.4 $\widehat{\text{min}}$
	180 deg	6.5 $\widehat{\text{min}}$	6.2 $\widehat{\text{min}}$
40 min	90 deg	7.6 $\widehat{\text{min}}$	7.3 $\widehat{\text{min}}$
	180 deg	7.4 $\widehat{\text{min}}$	7.2 $\widehat{\text{min}}$
60 min	90 deg	8.9 $\widehat{\text{min}}$	8.7 $\widehat{\text{min}}$
	180 deg	8.8 $\widehat{\text{min}}$	8.6 $\widehat{\text{min}}$

* Note

- o Elapsed time is period between start of Orbiter alignment and end of Orbiter/IUS alignment transfer

- o Angle between Orbiter alignment stars assumed 90 degrees.

Case B

The orbiter is aligned per OFT procedure modified to remove sensor body-fixed misalignment bias errors from the Orbiter star measurements. (This is the procedure described in step (2) of the combined alignment transfer procedure described earlier.) Alignment is subsequently transferred to the IUS via rotation about two axes 90 degrees apart in inertial space. The IUS alignment accuracy is presented below.

Table 3. Case B IUS Alignment, Orbiter Sensor Body-Fixed Misalignment Errors Removed*

Elapsed Time*	Alignment Transfer Rotation Magnitude	IUS Per-Axis Alignment Accuracy (3σ)	
		10 X 10 deg FOV	4 x 4 deg FOV
0 min	90 deg	3.7 min	3.4 min
	180 deg	3.1 min	2.8 min
20 min	90 deg	4.3 min	4.0 min
	180 deg	3.7 min	3.5 min
40 min	90 deg	5.6 min	5.4 min
	180 deg	5.2 min	5 0 min
60 min	90 deg	7.3 min	7.2 min
	180 deg	7.0 min	6.9 min

*Note:

- o Elapsed time is as defined for Table 2
- o Angle between Orbiter alignment stars assumed 90 degrees

For Case B the alignment transfer rotations are maneuvers separate from the 180 degree bias removal rotations about the Orbiter alignment star LOS's. It would seem that if the bias removal procedure is adopted, there would be little reason not to combine the Orbiter alignment and Orbiter/IUS alignment transfer procedures. Combining the procedures reduces the amount of maneuvering time required, and it improves the IUS alignment accuracy to that reported in Table 1.

ALIGNMENT TRANSFER: THE BASIC IDEA

For many IUS missions, the Orbiter will transport the IUS into orbit in a powered down state. One of the necessary steps in preparing the IUS for release is aligning the IUS's strapdown inertial guidance unit (IMU) to the basic inertial coordinate frame of reference chosen for the mission. Targeting data and the gravity model are stored in the IUS flight computer in such a basic reference frame. We can assume this inertial frame to be the same as the Orbiter's, i.e., the M50 inertial frame, without loss of generality.

At some point prior to IUS release from the Orbiter, the IUS's IMU and flight computers are powered up. At this time, the flight computer has no idea of the IUS's orientation relative to the M50 coordinate frame. The IUS flight computer does begin, however, to track IUS attitude relative to the inertial attitude existing at the instant the attitude computations were initiated. Thus, the IUS has an inertial reference frame, but it doesn't know the frame orientation relative to the M50 frame. The IUS inertial frame, at this point, is unknown.

Aligning a strapdown IMU consists of determining the orientation of the IUS's unknown inertial frame relative to the M50 frame. Then, the accelerations sensed along the body axes by the IUS's IMU can be expressed in M50 coordinates and combined with the gravity model accelerations as the basic inputs to the navigation computations. The alignment itself is expressed mathematically in the IUS flight computer as a 3 x 3 matrix transformation. The alignment procedure addresses the problem of determining the 3 x 3 matrix transformation via appropriate sensor measurements and vehicle maneuvers.

The basic idea behind transferring Orbiter alignment to the IUS is the following. The Orbiter, with the IUS firmly attached in the payload bay, performs rotations about two different spatial axes. The rotations are jointly sensed by the Orbiter and the IUS, affording two common lines of reference in inertial space. For the Orbiter, the two reference directions are expressed in the Orbiter's inertial coordinate frame. For the IUS, the same two reference directions are expressed in the IUS's unknown inertial frame. Since the reference directions are common to both the Orbiter and the IUS, it becomes a simple matter to compute the orientation of the IUS unknown frame with respect to Orbiter's frame.

If the Orbiter alignment and the Orbiter/IUS transfer maneuvers take place separately, then the alignment transfer error will be the sum of the Orbiter alignment, Orbiter IMU, and the IUS IMU errors. It will be shown that the Orbiter in-orbit alignment procedure and the Orbiter/IUS transfer alignment procedure can be combined with the consequence that only the Orbiter star tracker errors and the IUS IMU errors impact the IUS alignment. For this latter situation, it turns out that IUS alignment is substantially more accurate than the Orbiter's alignment.

Analysis Approach

At the present time, Orbiter/IUS alignment transfer procedures and calculations have not been explicitly baselined. Various approaches appear feasible. For this analysis, the liberty was taken to adopt a simple deterministic computational approach. The equations involved are derived in the text of this report. Such an approach is easy to understand, provides a framework upon which the error analysis can be performed, and furnished an accuracy benchmark against which other approaches can be compared.

The basic error sources considered in the analysis are the sensor error sources, discussed in a later section. These are the errors associated with the Orbiter IMU's and star trackers and the IUS's strapdown IMU. Two other potential error sources exist, but at the present time quantitative data is not available to evaluate their significance. Thus, they were not included in the analysis. These error sources are:

- o Data processing system implementation (mechanization) errors. The principal error source here would be timing errors associated with time tagging of measurement data. This error source is not significant, if the Orbiter and IUS master time references are known relative to each other within several milliseconds.

- o Variations of IUS orientation relative to the Orbiter, when the IUS is attached to the Orbiter payload bay. Accurate Orbiter/IUS alignment transfer is predicated upon the assumption that the Orbiter and IUS rotate as a single unit during alignment transfer maneuvers. Strictly speaking, changes in the IUS's navigation base relative to the Orbiter's navigation base, from measurement to measurement, will introduce errors into the alignment transfer process. For example, a .1 degree relative shift in IUS navigation base orientation, say due to body flexing or Orbiter/IUS attach points that are not rigid, might yield a 6 min error in the IUS alignment.

It would seem very important to verify, or be quite confident, that the IUS relative orientation does not change significantly from measurement to measurement.

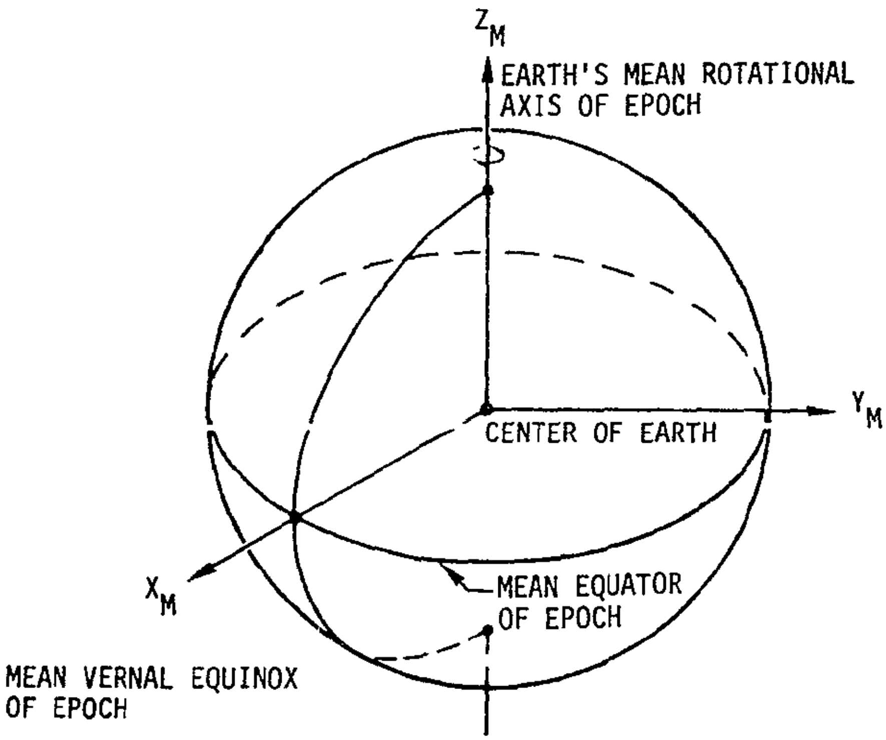
This document will first address IUS alignment transfer via separate Orbiter alignment and Orbiter/IUS alignment transfer procedures. Building on the analysis results and procedures developed thereby, the analysis will then address IUS direct alignment via combined procedures.

Comment on the Appendices

Appendix A is a detailed discussion of eigenvector calculations pertaining to IUS alignment. The other appendices (B,C,D, and E) are sections from the first report under this analysis task, "In-Orbit Alignment Accuracy", reference 3. They are included for completeness and convenience of the reader, since the subject report makes a number of references to the first report.

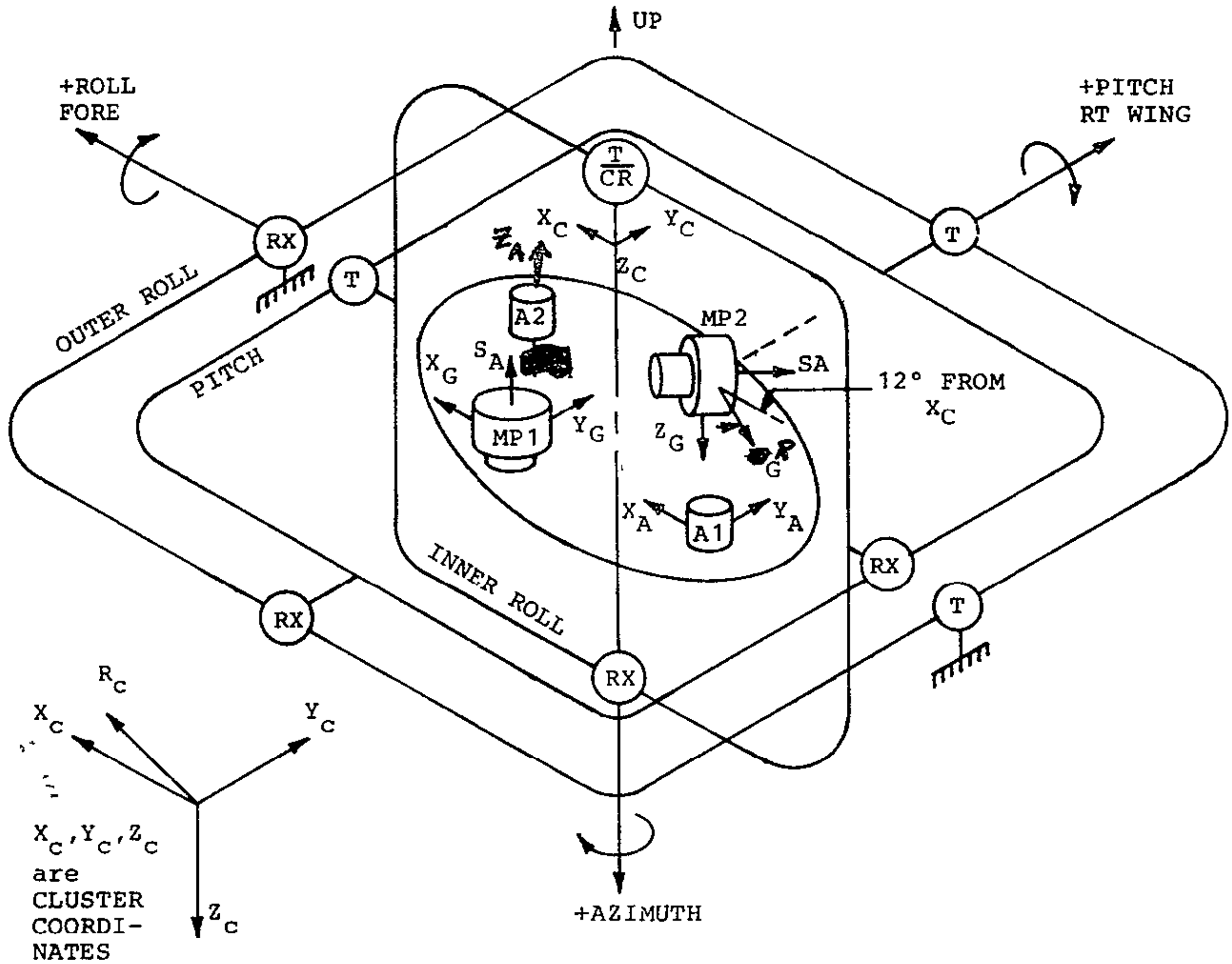
DEFINITIONS

(From Rockwell International Functional Subsystem Software Requirement Documents)



NAME:	Aries mean of 1950, Cartesian, coordinate system
ORIGIN	The center of the earth
ORIENTATION:	The epoch is the beginning of Besselian Year 1950 or Julian ephemeris date 2433282.423357
	The X_M-Y_M plane is the mean earth's equator of epoch.
	The X_M axis is directed towards the mean vernal equinox of epoch.
	The Z_M axis is directed along the earth's mean rotational axis of epoch and is positive north.
	The Y_M axis completes a right-handed system.
CHARACTERISTICS.	Inertial, right-handed Cartesian system.

Figure 4.2.1-7. Aries Mean of 1950 Cartesian Coordinate System



NOTES: X_G = X GYRO I.A. } VERTICAL GYRO
Y_G = Y GYRO I.A.

Z_G = Z GYRO I.A. } AZIMUTH GYRO
R_G = RED GYRO I.A

Z_A = Z ACCEL SENS AXIS } SINGLE AXIS ACCEL

X_A = X ACCEL SENS. AXIS } DUAL AXIS ACCEL.
Y_A = Y ACCEL SENS. AXIS

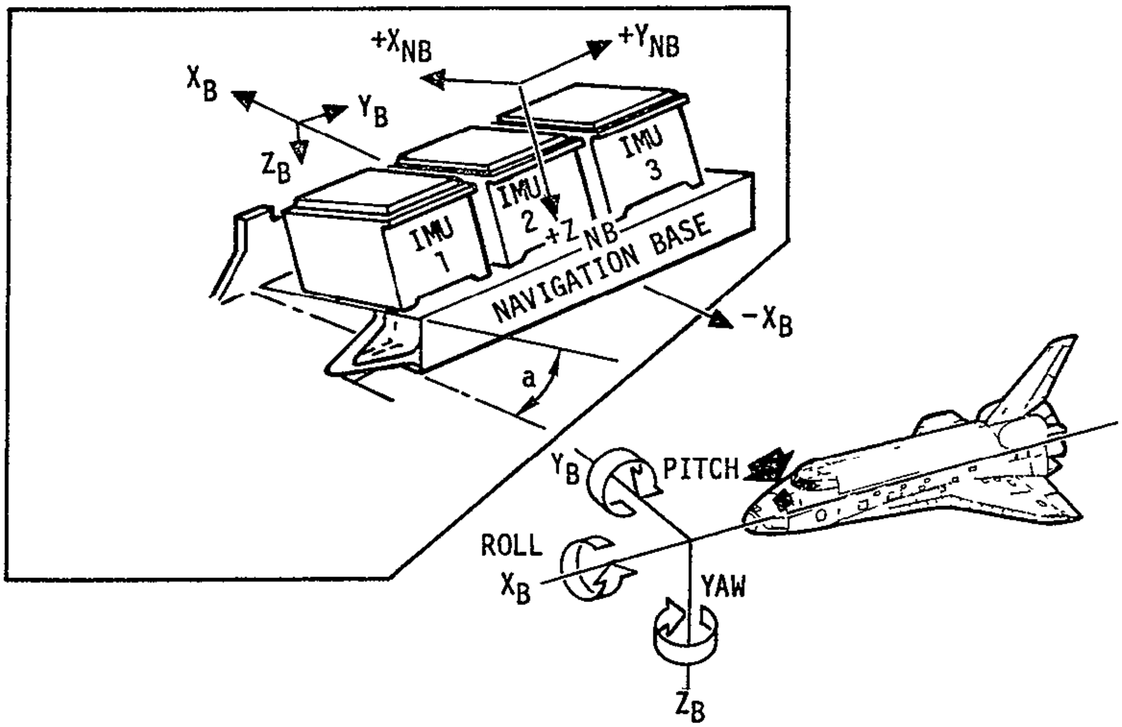
(T) = GIMBAL TORQUER

(RX) = GIMBAL ANGLE RESOLVER

Figure 4.2.1-4. Stable Member (IMU Cluster) Coordinate System (Sheet 1 of 2)

NAME	STABLE MEMBER (IMU)
ORIGIN	THE INTERSECTION OF THE INNERMOST GIMBAL AXIS AND THE MEASUREMENT PLANE OF THE XY TWO AXIS ACCELEROMETER
ORIENTATION	THE Z_C AXIS IS COINCIDENT WITH THE INNERMOST GIMBAL AXIS
	THE X_C AXIS IS DETERMINED BY THE PROJECTION OF THE X ACCELEROMETER INPUT AXIS (IA) ONTO A PLANE ORTHOGONAL TO Z_C Y_C COMPLETES A RIGHT-HANDED TRIAD.
	IN A PERFECT IMU, WITH ALL MISALIGNMENTS ZERO, THESE RELATIONSHIPS HOLD
	THE X ACCELEROMETER AND X GYRO IAS ARE PARALLEL TO THE X_C AXIS
	THE Y ACCELEROMETER AND Y GYRO IAS ARE PARALLEL TO THE Y_C AXIS
	THE Z ACCELEROMETER AND Z GYRO IAS ARE PARALLEL TO THE Z_C AXIS
CHARACTERISTICS	NONROTATING, RIGHT-HANDED, CARTESIAN SYSTEM
	THE REFERENCE ALIGNMENT FOR THE GIMBAL CASE SHALL BE DEFINED WITH THE FOUR GIMBAL ANGLES AT ZERO AND WITH THE VEHICLE IN A HORIZONTAL POSITION. IN A PERFECT IMU, WITH ALL MISALIGNMENTS ZERO AND WITH ALL GIMBAL ANGLES AT ZERO, THE FOLLOWING RELATIONSHIPS HOLD
	THE OUTER ROLL AXIS AND THE X_C AXIS WILL BE PARALLEL TO X_{NB}. POSITIVE X_C WILL BE IN THE FORWARD DIRECTION POSITIVE ROLL GIMBAL ANGLES WILL BE IN THE SENSE OF A RIGHT-HANDED ROTATION OF THE GIMBAL CASE RELATIVE TO THE PLATFORM ABOUT THE PLUS OUTER ROLL AXIS
	THE PITCH AXIS AND Y_C WILL BE PARALLEL TO Y_{NB} POSITIVE Y_C WILL BE TO THE RIGHT OF AN OBSERVER LOOKING FORWARD IN THE VEHICLE. POSITIVE PITCH GIMBAL ANGLES WILL BE IN THE SENSE OF A RIGHT-HANDED ROTATION OF THE GIMBAL CASE RELATIVE TO THE PLATFORM ABOUT THE PLUS PITCH AXIS
	THE INNER ROLL AXIS WILL BE PARALLEL TO THE OUTER ROLL AXIS, WITH THE SENSE OF ROTATION THE SAME AS FOR THE OUTER ROLL AXIS
	THE AZIMUTH AXIS AND Z_C WILL BE PARALLEL TO Z_{NB} POSITIVE Z_C WILL BE DOWN RELATIVE TO AN OBSERVER STANDING IN THE VEHICLE. POSITIVE AZIMUTH GIMBAL ANGLES WILL BE IN THE SENSE OF A RIGHT-HANDED ROTATION OF THE GIMBAL CASE RELATIVE TO THE PLATFORM ABOUT THE PLUS AZIMUTH AXIS.
	X_{NB}, Y_{NB}, Z_{NB} ARE CARTESIAN COMPONENTS OF THE NAVIGATION BASE COORDINATE SYSTEM

Figure 4.2.1-4. Stable Member (IMU Cluster) Coordinate System (Sheet 2 of 2)



NAME:	Navigation base system
ORIGIN:	At the mutual intersection of:

(A) A plane parallel to the orbiter plane of symmetry, 14 inches left of center

*(B) Plane of top surfaces of mounting pads for IMU 1 (left)

*(C) Plane of vertical surfaces of aft pads for IMU 1

ORIENTATION: Y_{NB} lies along the intersection of planes (B) and (C), positive out the orbiter right wing.

X_{NB} lies in plane (B), perpendicular to Y_{NB} and positive forward.

Z_{NB} completes the right-handed system.

CHARACTERISTICS: Rotating, right-handed Cartesian

*As determined by an alignment fixture

Figure 4.2.1-3. Navigation Base Coordinate System

ENTER TNB_CL

COMPUTE RESOLVER READINGS IN RADIANS

OR = (ORANG)(1/RADDEG) *

P = (PANG)(1/RADDEG)

AZ = (AZANG)(1/RADDEG)

IR = (IRANG)(1/RADDEG)

* WHEN CALLED BY STAR TRACKER SOP ORAVG21, PAVG21, AZAVG21, AND IRAVG21 REPLACE ORANG, PANG, AZANG, AND IRANG

FORM NAV BASE TO CLUSTER TRANSFORMATION.

$$\left[T_{NB}^{PC}\right] = \begin{bmatrix} CAZ & -SAZ & 0 \\ SAZ & CAZ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & CIR & -SIR \\ 0 & SIR & CIR \end{bmatrix} \begin{bmatrix} CP & 0 & SP \\ 0 & 1 & 0 \\ -SP & 0 & CP \end{bmatrix}$$

$$\begin{bmatrix} 1 & -DP & 0 \\ DP & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & COR & -SOR \\ 0 & SOR & COR \end{bmatrix} \left[T_{NB}^{RL}\right]$$

RETURN

Figure 4 6.2.8-1. TNB_CL Flow Diagram

4.66 STAR TRACKER SUBSYSTEM OPERATING PROGRAM

This section defines the detailed functional requirements and formulations for the Star Tracker (ST) Subsystem Operating Program (SOP). The ST SOP defines software associated with ST moding, self-test, failure identification, star tracking, target tracking and IMU-to-ST alignment.

4.66.1 ST REQUIREMENTS OVERVIEW

4.66.1.1 ST Functional Overview

The Orbiter ST is a strapped-down, wide field of view (FOV) image-dissector electro optical tracking device. It is used to obtain precise angular measurements of selected stars and sun illuminated orbiting elements (targets).

Two ST's are physically located on the Orbiter navigation base. The -Y axis ST centerline is approximately 10.5° from the Orbiter -Y axis and the -Z axis ST centerline is approximately 3° from the Orbiter -Z axis. Figure 4.66.1.1-1 depicts ST and IMU placement on the navigation base. The ST mounts on the underside of the navigation base while a light shade and viewing window are mounted on top of the navigation base. The ST interfaces with the GPC via the serial digital multiplex/demultiplex input/output data channel.

4.66.1.1.1 ST Performance Characteristics

The Orbiter ST's exhibit the following performance characteristics and operating features:

Sensitivity - The ST's will acquire and track the 153 brightest stars based upon the S-20 star intensity scale. The ST sensitivity threshold can be adjusted via GPC control.

Field of View - The ST's total field of view is a minimum 10 degree by 10 degree square. The ST's can also be commanded to search a 1 degree by 1 degree square field within the field of view.

Accuracy - The ST's total error in measurement of star or target angles does not exceed 1 arc minute (1 sigma). Star or target magnitude measurement errors do not exceed an absolute maximum of 0.6 magnitude.

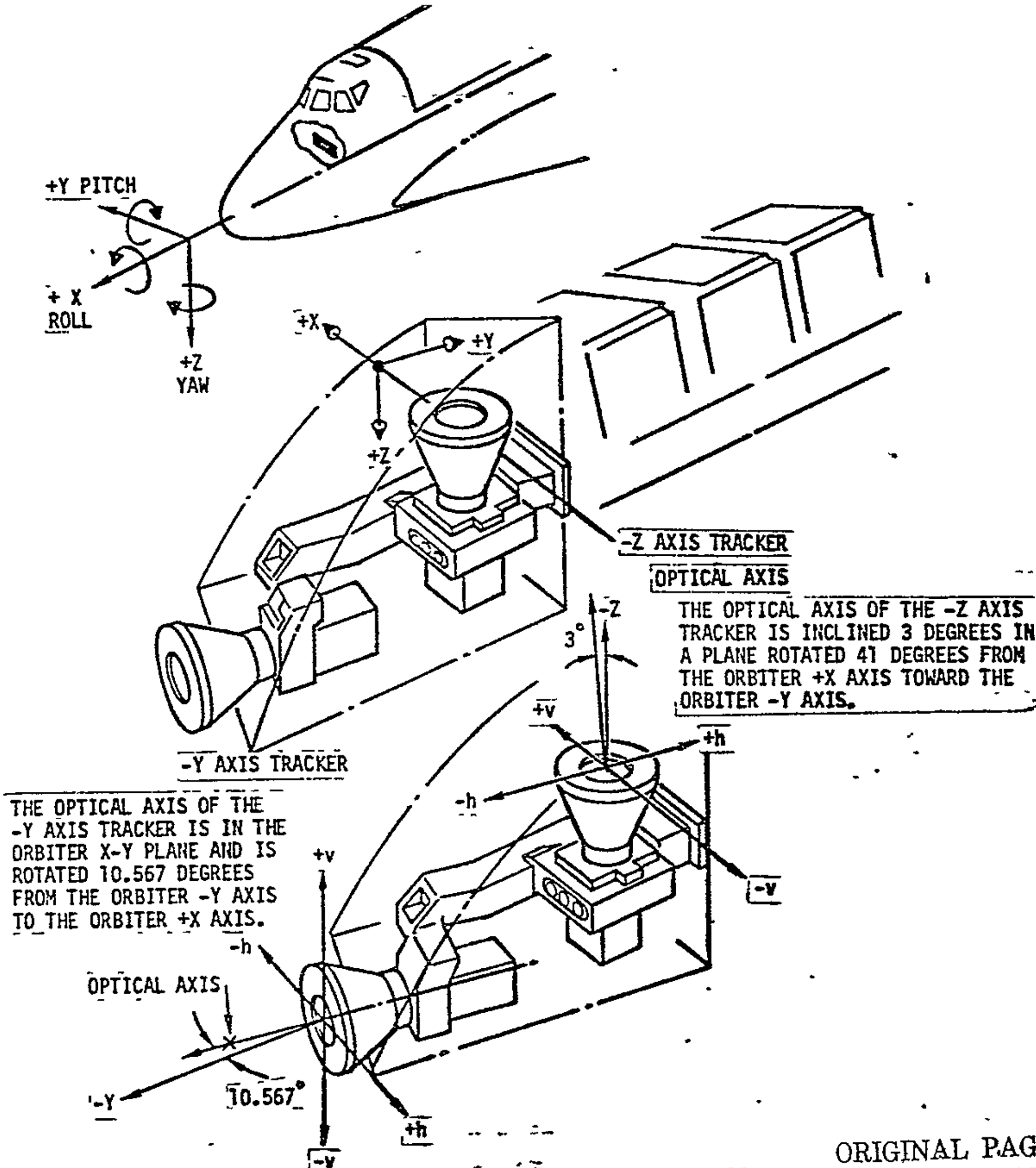


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Figure 4.66.1.1-1. ST_IMU_NAV Base Orientation

<u>ERROR</u>

<u>MODELS</u>

ERROR MODELS

The sensor errors pertinent to alignment transfer are presented in this section. Table 4 lists the Orbiter IMU errors. These errors were discussed in some detail in the first report of this task (reference 3). This discussion is repeated for the convenience of the reader in Appendix B to this report.

The Orbiter alignment error analyzed in reference 3 (also discussed in Appendices C and E) is presented in Table 5. Two situations are addressed:

- o Orbiter is aligned in accord with OFT in-orbit alignment procedures, reference 4.
- o Orbiter is aligned per OFT procedure modified to remove the effects of body fixed bias effects in the star tracker and IMU measurements. This is accomplished for each of two star sightings by (1) taking star measurement, (2) rotating Orbiter 180 degrees around LOS to star, (3) taking the second star measurement, (4) averaging the two measurements to remove the bias effects. This modified alignment procedure is described and analyzed in reference 3 (also discussed in Appendix D).

It is seen that removing the bias effects from the star sightings materially reduces the alignment error.

Table 6 presents the IUS IMU error sources significant to the Orbiter/IUS alignment transfer. This error model is based on information received from the Boeing Company. Note that Table 6 does not include such errors as "nav base to IMU alignment error." This is not because the IUS IMU is perfectly aligned on its navigation base plate. It is because, rather, we are not depending a priori on any particular orientation of the IMU relative to its nav base when performing the Orbiter/IUS alignment transfer procedure. If the IUS IMU alignment were carried out using an IUS star tracker, then the precision of the mounting alignments of both the IUS IMU and IUS star tracker would become significant.

Redundancy

The IUS IMU design contains redundant gyros (and accelerometers). Such redundancy, when taken into account, should lead to a net reduction in IMU error. This analysis will not address redundancy effects, since the IUS IMU turns out to be a minor error contributor to Orbiter/IUS alignment transfer.

TABLE 4

ALIGNMENT ERROR MODEL

ERROR SOURCE	SYMBOL *	VALUE/AXIS (1σ)	
Orbiter IMU			
Gyro Drift		.035 deg/hr	
Nav Base Ref to Mounting Pads	$\delta\theta_{IPN}$	60 $\widehat{\text{sec}}$	Body Fixed Biases
IMU Case to Pads	$\delta\theta_{CMP}$	20 $\widehat{\text{sec}}$	
Case to Outer Roll Gimbal	$\delta\theta_{COR}$	28 $\widehat{\text{sec}}$	
Non-Orthogonality / Resolver	$\delta\theta_{R}$	30 $\widehat{\text{sec}}$ / 44 $\widehat{\text{sec}}$ (53 $\widehat{\text{sec}}$ RSS)	68 $\widehat{\text{sec}}$ RSS
Star Tracker			
Horizontal, Vertical Measurements	$\delta\theta_{ST}$	42 $\widehat{\text{sec}}$	
Tracker to Nav Base Ref	$\delta\theta_{NST}$	60 $\widehat{\text{sec}}$	Body Fixed Bias
RSS (not including gyro drift)	ε	114 $\widehat{\text{sec}}$ ($1\ \sigma$) 5.7 $\widehat{\text{min}}$ ($3\ \sigma$)	

* Note These symbols are employed in the error analysis, reference 3.

Table 5. Orbiter IMU Alignment Error

Alignment Error, per OFT Procedures	114 sec (1σ) 5.7 min (3σ)
Alignment Error, per Modified Procedures	48 sec (1σ) 2.4 min (3σ)

Table 6. IUS IMU Gyro Error Model (1σ), per Axis

Uncompensated Random Drift		.009°/hr
90 day bias uncertainty	.007°/hr	
Thermal cycle stability	.005°/hr	
Shutdown repeatibility	.002°/hr	
Continuous operating random	.001°/hr	
Scale Factor Error		45 ppm
90 day uncertainty	25 ppm	
Linearity	37 ppm	
Misalignment Stability		10 sec

Error Analysis Technique

All the various error sources result in small angular errors, which are small rotations. As discussed in reference 3, small rotations can be expressed, to first order, as vectors. The vector magnitude is the angular error magnitude in radians. The vector orientation is the axis about which the angular error takes place. In general, individual angular error vectors have different magnitudes and orientations. The total error (vector) is the vector sum of the individual error vectors.

Each angular error vector can obviously be resolved in X, Y, and Z components in any particular coordinate system of interest.

For a given error source, we will assume the per-axis (X, Y, and Z) error statistics to be the same and also uncorrelated from axis to axis. This seems to be a reasonable assumption, based on the data at hand. In general, the error ellipsoid corresponding to such an error distribution is a sphere. Thus, the per-axis error statistics are invariant to coordinate frame transformations. We can, therefore, assume that the per-axis error statistics presented in Tables 4, 5, and 6 pertain to the same coordinate frame, with said frame being whatever coordinate frame we choose to perform the error analysis.

The star tracker is an exception to the above since its "error ellipsoid" is really an ellipse in the plane perpendicular to the tracker boresight axis. This will not present a problem in the analysis of per-axis errors, since we will choose the analysis coordinate frame Z axis to be coincident with the tracker boresight axis. The tracker measurement errors (X and Y axis errors) are then directly additive to the other error source X and Y axis errors. Z axis errors from any source (angular errors around the tracker boresight axis) to first order have little effect on alignment accuracy.

PART 2

ORBITER/IUS ALIGNMENT TRANSFER

SEPARATE ORBITER ALIGNMENT AND ORBITER/IUS ALIGNMENT PROCEDURES

EIGENVECTOR INERTIAL REFERENCE DIRECTION

When the Orbiter/IUS performs an alignment transfer rotation, the angular velocity vector at any instant of time represents a reference direction in inertial space. If the angular velocity vector were sensed jointly by the Orbiter and the IUS at the same time point, a common inertial reference direction would then be known. In practice, because of sensor errors and the nature of the sensor data (e.g., the Orbiter IMU provides angles, not rates), it would probably be necessary to collect IMU data over some period of time. The inertial reference direction would then be estimated from the data by some technique such as Kalman filtering.

The analysis of this document does not attempt to work with angular rates. The basic measurement data is assumed to be body attitudes existing (1) at the beginning of the rotation maneuver and (2) at the end of the rotation manuever. This attitude data is directly available to both the Orbiter and the IUS flight computers.

There are an infinite number of ways that the Orbiter/IUS could reorient from the initial attitude to the final attitude. In general, the instantaneous axis of rotation would vary throughout the maneuvering. However, there exists one "ideal" rotation of 180 degrees or less about fixed axis which would accomplish the given reorientation. This fixed axis, which can be easily calculated as a function of the initial and final body attitudes, will be taken as the inertial reference direction. This axis is independent of the actual maneuvering employed to reorient from the initial attitude to ending attitude.

Mathematically, body attitude orientations are represented by orthogonal matrix transformations. An orthogonal transformation has one independent eigenvector whose direction, it turns out, corresponds to the fixed "ideal" axis of rotation. Thus, the inertial reference direction is represented by the eigenvector of the 3 x 3 matrix transformation relating the initial and ending body attitudes.

Representation of Body Attitude

In this analysis vehicle body attitudes will be represented by 3 x 3 matrices of body axis direction cosines. Let A be such a matrix.

$$A = \left[\begin{array}{c|c|c} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{array}\right]$$

The three columns of A are three orthonormal unit vectors representing respectively the roll, pitch, and yaw body axes (left to right) as resolved in the given coordinate frame. Component a_{ij} is the i^{th} component (X, Y, or Z) of the j^{th} unit vector (roll, pitch or yaw). It turns out that A is an orthogonal matrix, with positive determinant.

Let A_1 represent the initial body orientation, expressed in a selected inertial coordinate frame, and A_2 represent the ending body orientation, expressed in the same frame. We define a matrix transformation C relating A_1 and A_2. By definition we have:

$$A_2 \triangleq CA_1$$

C always exists, since A^{-1} always exists; for an orthogonal matrix, the inverse matrix is the transposed matrix ($A^{-1} = A^t$). Thus:

$$C = A_2 A_1^t$$

C is seen to be an orthogonal transformation since it is the product of two orthogonal transformations. By definition of an eigenvector we have:

$$C\underline{d} \triangleq \underline{d}$$

where $\underline{d}$ is the eigenvector of C. Likewise it can be seen that:

$$C^t\underline{d} = \underline{d}$$

Subtracting yields:

$$(C^t - C)\underline{d} = 0$$

Writing this equation in component form yields:

$$\begin{bmatrix} 0 & c_{21} - c_{12} & c_{31} - c_{13} \\ c_{12} - c_{21} & 0 & c_{32} - c_{23} \\ c_{13} - c_{31} & c_{23} - c_{32} & 0 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence:

$$\frac{d_2}{d_3} = \frac{c_{31} - c_{13}}{c_{12} - c_{21}}$$

$$\frac{d_1}{d_3} = \frac{c_{23} - c_{32}}{c_{12} - c_{21}}$$

$$\frac{d_1}{d_2} = \frac{c_{23} - c_{32}}{c_{31} - c_{13}}$$

We see that the equations above are satisfied by:

$$d_1 = k\ (c_{32} - c_{23})$$

$$d_2 = k\ (c_{13} - c_{31})$$

$$d_3 = k\ (c_{21} - c_{12})$$

where k is a constant. Since $\underline{d}$ has unit length,

$$k = \left[(c_{32} - c_{23})^2 + (c_{13} - c_{31})^2 + (c_{21} - c_{12})^2\right]^{-\frac{1}{2}}$$

Thus the eigenvector of C can be determined by picking the appropriate components out of the matrix C, computing k, and then forming $\underline{d}$.

We will see in the error analysis that a mathematical singularity exists in the above solution for rotations of exactly 180 degrees. This does not preclude using this solution in the error analysis, which we wish to do because of its simplicity of form. In Appendix A, alternate methods for computing $\underline{d}$ are presented.

ALIGNMENT TRANSFER EQUATIONS, UNCOMBINED PROCEDURES

We now develop the equations for determining IUS alignment wherein the Orbiter alignment and IUS alignment procedures are not combined.

Body Cosine Matrices

We express the Orbiter's attitude matrix A in Orbiter IMU stable member (present cluster) inertial coordinates.

$$A = T_{RL}^{PC} \; T_{NB}^{RL}$$

where

T_{NB}^{RL} = orthogonal transformation (3 x 3), nav base reference to IMU outer roll gimbal. Known nominally, constant during rotation maneuvers.

T_{RL}^{PC} = orthogonal transformation (3 x 3), outer roll gimbal to present cluster (stable member) frame.

A = Orbiter body attitude matrix in stable member frame.

The equation above expresses the Orbiter's attitude in terms of IMU gimbal angles, since T_{RL}^{PC} is a function of the gimbal angle readings.

Corresponding to A is the IUS attitude matrix U, as expressed in the unknown IUS inertial coordinate frame.

First Rotation Maneuver

The Orbiter/IUS performs its first of two rotation maneuvers. In doing so, the Orbiter transitions from attitude A_1 to attitude A_2. Likewise the IUS transitions from U_1 to U_2.

For the Orbiter, we compute the ideal rotation matrix C (discussed earlier) from A_1 and A_2 with

$$A_1 = T_{RL}^{PC_1} \; T_{NB}^{RL} \qquad \text{initial orientation}$$

$$A_2 = T_{RL}^{PC_2} \; T_{NB}^{RL} \qquad \text{final orientation}$$

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Since $$C = A_2 A_1^t \qquad \text{we have}$$

$$C = T_{RL}^{PC_2} \; T_{PC_1}^{RL} \qquad \text{(stable member frame)}$$

Note that T_{NB}^{RL} is eliminated in the computations. Over the period of the rotation maneuvers, T_{NB}^{RL} can be considered constant (reference 3). This assumes the <u>same</u>

IMU provides the angle measurements determining A_1 and A_2; i.e., Orbiter redundancy management does not switch IMU's during a rotation maneuver.

From C, the eigenvector $\underline{d}_1$ (corresponding to the first rotation axis) is computed using the appropriate equations presented later in this document.

Using the Orbiter's alignment matrix T_{PC}^{M50} (determined by Orbiter alignment procedures), which transforms from the stable member frame to the M50 frame, $\underline{d}_1$ is transformed to M50 coordinates.

$$\underline{e}_1 = T_{PC}^{M50}\ \underline{d}_1$$

For the IUS, compute ideal rotation matrix F from U_1 and U_2 .

$$F = U_2\ U_1^t$$

From F, the eigenvector $\underline{g}_1$ is computed, using the appropriate equations presented later in this document. $\underline{e}_1$ and $\underline{g}_1$ are nominally the same vector quantity in inertial space. However, they are expressed in different coordinate frames.

Second Rotation Maneuver

The Orbiter/IUS performs the second rotation maneuver. Employing the same type calculations as above yields $\underline{e}_2$ and $\underline{g}_2$, the rotation eigenvector expressed in the M50 frame and the IUS unknown frame respectively. We assume $\underline{e}_1$ and $\underline{e}_2$ are not colinear (accordingly likewise $\underline{g}_1$ and $\underline{g}_2$).

Computation of IUS Alignment

Form the following orthonormal triads:

$$e_3 = \text{unit}\left[\underline{e}_1 \times \underline{e}_2\right] \qquad \underline{g}_3 = \text{unit}\left[\underline{g}_1 \times \underline{g}_2\right]$$

$$\underline{e}_2' = \underline{e}_3 \times \underline{e}_1 \qquad \underline{g}_2' = \underline{g}_3 \times \underline{g}_1$$

Form the following matrices E and G from column vectors $\underline{e}_1$, $\underline{e}_2'$, $\underline{e}_3$ and $\underline{g}_1$, $\underline{g}_2'$, $\underline{g}_3$.

$$E = \left[\underline{e}_1\ \ \underline{e}_2'\ \ \underline{e}_3\right] \qquad G = \left[\underline{g}_1\ \ \underline{g}_2'\ \ \underline{g}_3\right]$$

The desired IUS alignment transformation matrix is T_U^{M50}, which transforms any vector in the IUS unknown frame to the M50 frame. Clearly, T_U^{M50} transforms $\underline{g}_1$ to $\underline{e}_1$, $\underline{g}_2'$ to $\underline{e}_2'$, and $\underline{g}_3$ to $\underline{e}_3$. Hence:

$$E = T_U^{M50} G$$

Solving for T_U^{M50} yields:

$$T_U^{M50} = EG^t$$

IUS alignment takes place by transforming the IUS body attitudes U from the IUS unknown frame to M50 coordinates.

$$K = T_U^{M50} U$$

where K is the IUS body attitude matrix expressed in M50 coordinates. This is a one-time computation at some point in time subsequent to determining T_U^{M50}. Once the transformation is made, the IUS flight computer will proceed to update K in the M50 frame, using its strapdown IMU inputs.

As will be seen in the error analysis, the errors in T_U^{M50} per this uncombined procedure are due to the Orbiter alignment error in T_{PC}^{M50}, the errors in determining $\underline{d}_1$ and $\underline{d}_2$ due to IMU readout errors, and the errors in determining $\underline{g}_1$ and $\underline{g}_2$ due to IUS IMU errors.

Comment on Orbiter IMU Redundancy Management

The Orbiter has three IMU's. The on-board redundancy management function middle-value selects one IMU among the three IMU's for input to the flight computers. As stated above, it would be important that the same IMU provide the measurements prior to and at the completion of a rotation maneuver. It is understood that the crew can control IMU selection via the flight computer keyboard. Thus, the crew would inhibit the redundancy management switching function during the period beginning just prior to taking the first measurements and continuing until after taking the second measurement.

It is not necessary that the same IMU be employed for different rotation maneuvers. $\underline{d}_2$ can be determined using a different IMU. However, the appropriate alignment transformation T_{PC}^{M50} must be employed in the computation of $\underline{e}_2$. There is a different T_{PC}^{M50} transformation corresponding to each IMU, unless the IMU stable platforms are aligned and torqued to the same inertial orientation.

PART 3

ERROR ANALYSIS

ORBITER/IUS ALIGNMENT TRANSFER

ERROR ANALYSIS OF ORBITER EIGENVECTOR

To begin the alignment transfer error analysis, the rotation eigenvector pointing error due to Orbiter IMU errors will be determined. We recall that the eigenvector $\underline{d}$ is computed from rotation matrix C, as follows:

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

$$\underline{c} = (c_{32} - c_{23}, c_{13} - c_{31}, c_{21} - c_{12})$$

$$k = \left[(c_{32} - c_{23})^2 + (c_{13} - c_{31})^2 + (c_{21} - c_{12})^2\right]^{-\frac{1}{2}}$$

$$\underline{d} = k\underline{c}$$

The rotation matrix C is calculated as follows (derived in an earlier section):

$$C = A_2 A_1^t = T_{RL}^{PC2}\, T_{NB}^{RL}\, (T_{RL}^{PC1}\, T_{NB}^{RL})^t$$

$$C = T_{RL}^{PC2}\, \underbrace{T_{NB}^{RL}\, T_{RL}^{NB}}_{I}\, T_{PC1}^{RL} = T_{RL}^{PC2}\, T_{PC1}^{RL}$$

It is seen that T_{NB}^{RL} is eliminated in the calculation of C. The errors associated with T_{NB}^{RL} are the Orbiter body-fixed IMU errors listed in Table 4 ($\delta\theta_{IPN}$, $\delta\theta_{CMP}$, $\delta\theta_{COR}$). These are the geometrical misalignments of the Orbiter nav base and IMU. Thus, these errors do not affect the accuracy of C.

Introducing error perturbations yields:

$$C + \delta C = (T_{RL}^{PC2} + \delta T_{RL}^{PC2})\ (T_{RL}^{PC1} + \delta T_{RL}^{PC1})^t$$

The errors δT_{RL}^{PC2} and δT_{RL}^{PC1} are IMU readout random errors due to the gimbals and resolvers. They can be expressed as the effects of small error rotations of T_{RL}^{PC2} and T_{RL}^{PC1}.

$$\delta T_{RL}^{PC2} = \delta R_2 T_{RL}^{PC2}$$

$$\delta T_{RL}^{PC1} = \delta R_1 T_{RL}^{PC1}$$

where

$$\delta R_1 = \begin{bmatrix} 0 & -\gamma_1 & \beta_1 \\ \gamma_1 & 0 & -\alpha_1 \\ -\beta_1 & \alpha_1 & 0 \end{bmatrix} \qquad \delta R_2 = \begin{bmatrix} 0 & -\gamma_2 & \beta_2 \\ \gamma_2 & 0 & -\alpha_2 \\ -\beta_2 & \alpha_2 & 0 \end{bmatrix}$$

and α_1, β_1, γ_1 and α_2, β_2, γ_2 are small angular (readout) errors about the X, Y, and Z coordinate axes respectively.

Substituting yields:

$$C + \delta C = (I + \delta R_2)\, T_{RL}^{PC2}\, T_{PC1}^{RL}\, (I + \delta R_1)^t$$

Since $(I + \delta R_2)^t = I - \delta R_1$, we have

$$C + \delta C = (I + \delta R_2)\, C\, (I - \delta R_1)$$

and, to first order,

$$\delta C = \delta R_2 C - C \delta R_1$$

We will now choose the coordinate frame in which the eigenvector error $\delta \underline{d}$ will be analyzed. We choose the coordinate frame such that the Z axis corresponds to the rotation axis of C; i.e., C represents a rotation around the Z axis through angle θ. Figure 1 illustrates the situation. The normalization factor k and the eigenvector $\underline{d}$ are computed as derived above.

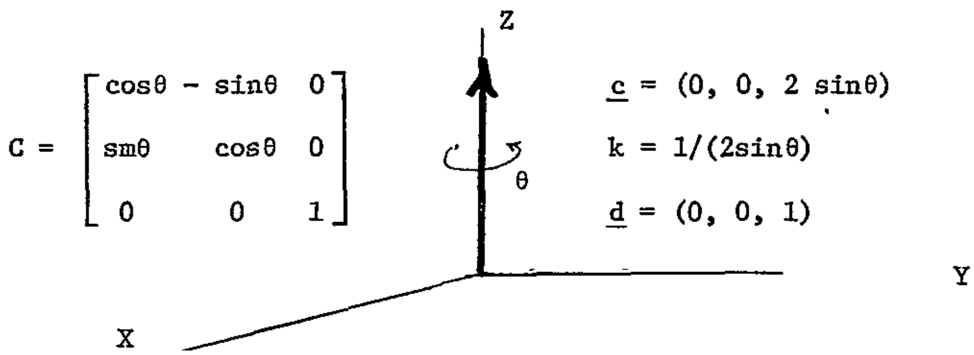

Figure 1. Eigenvector $\underline{d}$ Error Analysis Frame

We now compute δC in terms of the IMU errors $\alpha_1, \beta_1, \gamma_1$, and α_2, β_2, γ_2. Substituting in the expression $\delta C = \delta R_2 C - C \delta R_1$ yields

$$\delta R_2 C = \begin{bmatrix} 0 & -\gamma_2 & \beta_2 \\ \gamma_2 & 0 & -\alpha_2 \\ -\beta_2 & \alpha_2 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\gamma_2 \sin\theta & -\gamma_2 \cos\theta & \beta_2 \\ \gamma_2 \cos\theta & -\gamma_2 \sin\theta & -\alpha_2 \\ -\beta_2 \cos\theta + \alpha_2 \sin\theta & \beta_2 \sin\theta + \alpha_2 \cos\theta & 0 \end{bmatrix}$$

$$C\delta R_1 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -\gamma_1 & \beta_1 \\ \gamma_1 & 0 & -\gamma_1 \\ -\beta_1 & \alpha_1 & 0 \end{bmatrix} = \begin{bmatrix} -\gamma_1 \sin\theta & -\gamma_1 \cos\theta & \beta_1 \cos\theta + \alpha_1 \sin\theta \\ \gamma_1 \cos\theta & \gamma_1 \sin\theta & \beta_1 \sin\theta - \alpha_1 \cos\theta \\ -\beta_1 & \alpha_1 & 0 \end{bmatrix}$$

Hence

$$\delta C = \begin{bmatrix} -(\gamma_2 - \gamma_1)\sin\theta & -(\gamma_2 - \gamma_1)\cos\theta & \beta_2 - \beta_1 \cos\theta - \alpha_1 \sin\theta \\ (\gamma_2 - \gamma_1)\cos\theta & -(\gamma_2 - \gamma_1)\sin\theta & -\alpha_2 - \beta_1 \sin\theta + \alpha_1 \cos\theta \\ \beta_1 - \beta_2 \cos\theta + \alpha_2 \sin\theta & -\alpha_1 + \beta_2 \sin\theta + \alpha_2 \cos\theta & 0 \end{bmatrix}$$

The error in $\underline{d}$, from the expression $\underline{d} = k\underline{c}$ above, turns out to be:

$$\delta\underline{d} = k\delta c - (\underline{d} \cdot k\delta\underline{c})\underline{d}$$

Since $\delta\underline{c} = (\delta c_{32} - \delta c_{23}, \delta c_{13} - \delta c_{31}, \delta c_{21} - \delta c_{12})$, then it follows,

$$\delta c_X = (\alpha_2 - \alpha_1)(1 + \cos\theta) + (\beta_1 + \beta_2)\sin\theta$$

$$\delta c_Y = (\beta_2 - \beta_1)(1 + \cos\theta) - (\alpha_1 + \alpha_2)\sin\theta$$

$$\delta c_Z = 2(\gamma_2 - \gamma_1)\cos\theta$$

Substituting $\delta\underline{c}$ in the expression for $\delta\underline{d}$ yields:

$$\delta d_X = k\delta c_X$$

$$\delta d_Y = k\delta c_Y$$

$$\delta d_Z = 0$$

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The result $\delta d_Z = 0$ should not be surprising since $\underline{d}$ by definition is a unit vector pointing in the Z axis direction.

To complete the error analysis of $\underline{d}$, we determine the error statistics of δd_X and δd_Y in terms of the IMU error statistics. Treating α_1, β_1, γ_1 and α_2, β_2, γ_2 as uncorrelated random errors with zero mean yields the following mean square errors:

$$\overline{\delta c_X^2} = (\overline{\alpha_2^2} + \overline{\alpha_1^2})(1 + \cos\theta)^2 + (\overline{\beta_1^2} + \overline{\beta_2^2})\sin^2\theta$$

$$\overline{\delta c_Y^2} = (\overline{\beta_2^2} + \overline{\beta_1^2})(1 + \cos\theta)^2 + (\overline{\alpha_1^2} + \overline{\alpha_1^2})\sin^2\theta$$

$$\overline{\delta c_Z^2} = 0$$

Also,

$$\overline{\delta c_X \delta c_Y} = (\overline{\alpha_1^2} - \overline{\alpha_2^2} + \overline{\beta_2^2} - \overline{\beta_1^2})(1 + \cos\theta)\sin\theta$$

From Table 4, the standard deviations of α_1, β_1, γ_1 and α_2, β_2, γ_2 are each equal to $\delta\theta_R$. Thus,

$$\overline{\delta c_X^2} = \overline{\delta c_Y^2} = 4(\delta\theta_R)^2(1 + \cos\theta)$$

$$\overline{\delta c_X \delta c_Y} = 0$$

Computing $\overline{\delta d_X^2}$ and $\overline{\delta d_Y^2}$ yields:

$$\overline{\delta d_X^2} = \overline{\delta d_Y^2} = k^2 \overline{\delta c_X^2} \doteq \frac{4(\delta\theta_R)^2(1 + \cos\theta)}{4 \sin^2\theta} = \frac{(\delta\theta_R)^2}{1 - \cos\theta}$$

Thus, the per-axis standard deviation of $\delta\underline{d}$ is seen to be:

$$\text{dev}\delta d_X = \text{dev}\delta d_Y = \delta\theta_R(1 - \cos\theta)^{-\frac{1}{2}}$$

We note the following:

- o For small θ, the per-axis error in $\underline{d}$ is very large.
- o When the rotation maneuver turns through $\theta = 90$ degrees, the per-axis error in $\underline{d}$ equals $\delta\theta_R$, the IMU readout error.
- o When the rotation maneuver turns through $\theta = 180$ degrees, the per-axis error is $\delta\theta_R/\sqrt{2}$. This is the minimum error, as a function of θ.

- $\underline{c}$ approaches 0 as θ approaches 180 degrees. The particular calculations employed above to compute $\underline{c}$ from the components of C lead to a mathematical singularity at θ = 180 degrees. Such a computation of $\underline{c}$ mechanized in a computer would "blow up" for θ within a fraction of a degree of 180 degrees due to computer quantization, creating large errors in the computation of $\underline{d}$. We will see later that other equations can be employed to compute $\underline{c}$ from C. Employing these different calculations does not change the error analysis results above.

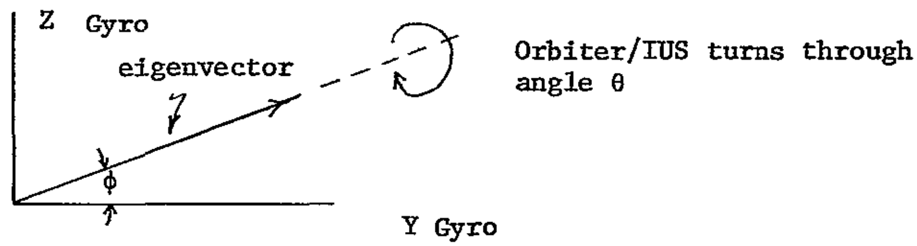
ERROR ANALYSIS OF IUS EIGENVECTOR

For the IUS, the measurement errors consist principally of the gyro misalignment, scale factor and gyro drift errors. These errors are not random from measurement to measurement but are, rather, unknown parameters in their effects. The error expression derived for the Orbiter eigenvector does not apply to the IUS eigenvector. However, to first order, the IUS eigenvector error may be easily determined.

IUS Gyro Scale Factor Error Effect

The scale factor error causes an angular error proportional to the angular change sensed by the IMU. The IUS eigenvector pointing error resulting from IUS gyro scale factor error is on the order of 6 $\widehat{\text{sec}}$ (1σ). This is shown immediately below. To simplify the analysis, the eigenvector (axis of rotation) is assumed to lie in a plan containing the IUS's pitch and yaw axes. We also assume nonredundant IUS gyros, respectively oriented along the IUS pitch, yaw and roll axes.

Let k_Y and k_Z represent the pitch and yaw gyro scale factor errors.



where

θ is the rotation angle of the alignment transfer maneuver

ϕ is the angle between the eigenvector and the pitch axis

Then, the rotations θ_Y and θ_Z sensed about the Y and Z axes are

$$\theta_Y = (1 + k_Y)\theta\cos\phi$$

$$\theta_Z = (1 + k_Z)\theta\sin\phi$$

The eigenvector angular error $\delta\phi$ is calculated as follows.

$$\tan(\phi + \delta\phi) = \frac{(1 + k_Z)\theta\sin\phi}{(1 + k_Y)\theta\cos\phi} = \left(\frac{1 + k_Z}{1 + k_Y}\right)\tan\phi$$

This results (to first order) in

$$\delta\phi = \left(\frac{k_Z - k_Y}{2}\right)\sin 2\phi$$

Notice that $\delta\phi$ is independent of θ.

Setting $k_{SF}^2 \triangleq \overline{k_Y^2} \triangleq \overline{k_Z^2}$, yields

$$\mathrm{Dev}\,\delta\phi = \frac{k_{SF}}{\sqrt{2}} \sin 2\phi$$

So, with k_{sf} = 45 ppm (1σ), then

$$\mathrm{Dev}\ \delta\phi = 6.6 \sin 2\phi\ \widehat{\sec}$$

We note the error is maximum when ϕ = 45 degrees. If ϕ = 0 degrees or 90 degrees, the error effect is zero. The scale factor error (in these cases) does not affect the sensed axis of rotation. We will assume that the IUS gyro scale factor error effect on IUS alignment is 6 $\widehat{\sec}$ (1σ).

IUS Gyro Drift

The effect of IUS gyro drift depends on the elapsed time during the alignment process. If we assume 10 minutes is required to complete the alignment maneuvers, then the per axis attitude error caused by gyro drift is about 6 $\widehat{\sec}$ (1σ).

IUS Gyro Misalignment

The gyro misalignment directly affects the sensed eigenvector pointing direction. Thus, gyro misalignment introduces 10 $\widehat{\sec}$ (1σ) per-axis error into the eigenvector per-axis error.

IUS Eigenvector Error

The per-axis IUS eigenvector angular error is the RSS of the three principal error sources discussed above. The RSS value is 13 $\widehat{\sec}$ (1σ). The IUS eigenvector error does not depend on the amount of Orbiter/IUS rotation, assuming some minimum amount of rotation necessary to eliminate the effects of small, random error sources.

Different IUS IMU Models

At the time this report was written, Boeing was considering two procurement sources for the IUS IMU. The error models are somewhat different. The differences, however, do not significantly change the analysis results.

ALIGNMENT TRANSFER ERROR

The alignment transfer error is the sum of the Orbiter alignment error, Orbiter gimbal errors, and IUS IMU errors. The alignment transfer error is computed as the RSS of the individual errors below.

Table 7. Per-Axis IUS Alignment Transfer Error

Case	Orbiter Alignment Error*	Transfer Rotation Angle	Orbiter Eigen-Vector Error	IUS Eigen-Vector Error	IUS Alignment Error	
					1σ	3σ
A	114 sec (Alignment per OFT Procedure)	90 deg	53 sec	13 sec	126 sec	6.3 min
		180 deg	38 sec	13 sec	121 sec	6.1 min
B	48 sec (Modified procedure, biases removed)	90 deg	53 sec	13 sec	73 sec	3.7 min
		80 deg	38 sec	13 sec	63 sec	3.1 min

*Does not include Orbiter IMU drift error effects.

The table above does not account for the Orbiter's gyro drift error (.035 deg/hr., 1σ) accumulating between the times of Orbiter alignment and the Orbiter/IUS alignment transfer. Below, Tables 2 and 3 (repeated from the results summary) show the effects of Orbiter gyro drift. The effect of restricting star measurements (Orbiter alignment) to the 4 x 4 degree FOV is also shown.

Table 2. Case A: IUS Alignment, Orbiter Sensor Body-Fixed Misalignment Errors Not Removed*

Elapsed Time *	Alignment Transfer Rotation Magnitude	IUS Per-Axis Alignment Accuracy (3σ)	
		10 x 10 deg FOV	4 x 4 deg FOV
0 min	90 deg 180 deg	6.3 $\widehat{min}$ 6.1 $\widehat{min}$	6.0 $\widehat{min}$ 5.8 $\widehat{min}$
20 min	90 deg 180 deg	6.7 $\widehat{min}$ 6.5 $\widehat{min}$	6.4 $\widehat{min}$ 6.2 $\widehat{min}$
40 min	90 deg 180 deg	7.6 $\widehat{min}$ 7.4 $\widehat{min}$	7.3 $\widehat{min}$ 7.2 $\widehat{min}$
60 min	90 deg 180 deg	8.9 $\widehat{min}$ 8.8 $\widehat{min}$	8.7 $\widehat{min}$ 8.6 $\widehat{min}$

* Note:

- Elapsed time is period between start of Orbiter alignment and end of Orbiter/IUS Alignment transfer.
- Angle between Orbiter alignment stars assumed 90 degrees

Table 3. Case B:IUS Alignment, Orbiter Sensor Body-fixed Misalignment Errors Removed *

Elapsed Time*	Alignment Transfer Rotation Magnitude	IUS Per-Axis Alignment Accuracy (3σ)	
		10 x 10 deg FOV	4 x 4 deg FOV
0 min	90 deg	3.7 min	3.4 min
	180 deg	3.1 min	2.8 min
20 min	90 deg	4.3 min	4.0 min
	180 deg	3.7 min	3.5 min
40 min	90 deg	5.6 min	5.4 min
	180 deg	5.2 min	5.0 min
60 min	90 deg	7.3 min	7.2 min
	180 deg	7.0 min	6.9 min

*Note:

- Elapsed time is as defined for Table 2
- Angle between Orbiter alignment stars assumed 90 degrees

PART 4

DIRECT IUS ALIGNMENT PROCESS

DIRECT IUS ALIGNMENT VIA COMBINED PROCEDURES

In the Summary of Results, the combined procedure was given for Orbiter alignment, body-fixed bias removal, and IUS alignment via rotation maneuvers. In essence, the Orbiter/IUS rotates 180 degrees about the LOS to each alignment star, simultaneously recording Orbiter star measurements and IMU attitude measurements (both Orbiter and IUS) before and after each rotation. The IUS alignment accuracy resulting is better than the Orbiter's alignment accuracy and, in fact, is independent of the Orbiter's alignment.

Rationale for the Combined Maneuver

Imagine that the flight crew were able to orient the Orbiter so that the star image always fell exactly in the same location in the star tracker's FOV (e.g., directly on the tracker's boresight axis). Then, the Orbiter's eigenvector produced by the 180 degree maneuver around the star LOS would be exactly colinear with the star LOS. In the IUS computer, the IUS eigenvector, although expressed in the unknown IUS inertial coordinate frame, would also point exactly at the selected star.

Since the star's inertial coordinates are known in the desired inertial navigation coordinate frame (e.g., M50) from star catalogue data, it becomes a simple matter to calculate directly the IUS alignment transformation relating the two coordinate frames. This would be done without regard to the Orbiter's IMU. The only errors bearing on the IUS alignment achieved thusly would be the Orbiter star tracker measurement error and the IUS IMU errors. The resulting IUS alignment error would be about 1 $\widehat{\text{min}}$ (3σ) per axis, which is considerably less than the Orbiter's alignment error. (This will be shown later.)

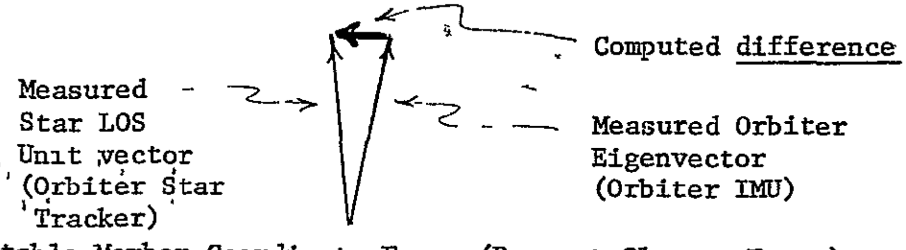
In the actual situation, we do not expect the star image to always fall exactly in the same FOV location. However, the image will always fall somewhere in the FOV and hence the eigenvector produced by the 180 degree rotation maneuver will not deviate by more than 5 degrees maximum per axis from the star LOS unit vector.

Now, it would seem since the measured star LOS unit vector and the rotation eigenvector are <u>almost</u> colinear (within a few degrees) that information is available to essentially eliminate Orbiter IMU errors from the IUS IMU alignment problem. It turns out that this is the case, as discussed next.

Computational Approach Taken

We begin with the fact that the star LOS unit vector and the rotation eigenvector point essentially in the same direction. The following steps are then taken:

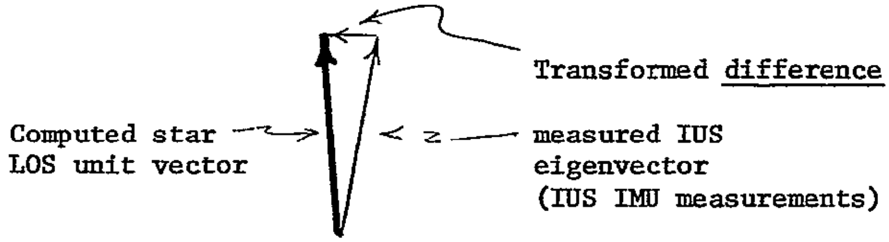
(1) Compute the <u>difference</u> vector (small) between the star LOS unit vector and the eigenvector, in Orbiter stable numbers coordinate.



Orbiter Stable Member Coordinate Frame (Present Cluster Frame)

(2) Transform the <u>difference</u> to the IUS unknown frame using transformation T_{PC}^{U} (present cluster to unknown frame). This transformation is determined from the Orbiter and IUS eigenvectors expressed in Orbiter IMU stable member and IUS IMU unknown frame coordinates respectively.

(3) Add the transformed <u>difference</u> to the IUS eigenvector, to compute the star LOS unit vector in the IUS unknown frame coordinates.



IUS Unknown Coordinate Frame

(4) Use star LOS unit vector (U frame) and corresponding star catalogue data (e.g., M50) to compute the desired IUS alignment transformation (e.g., T_{U}^{M50}). This completes the alignment determination.

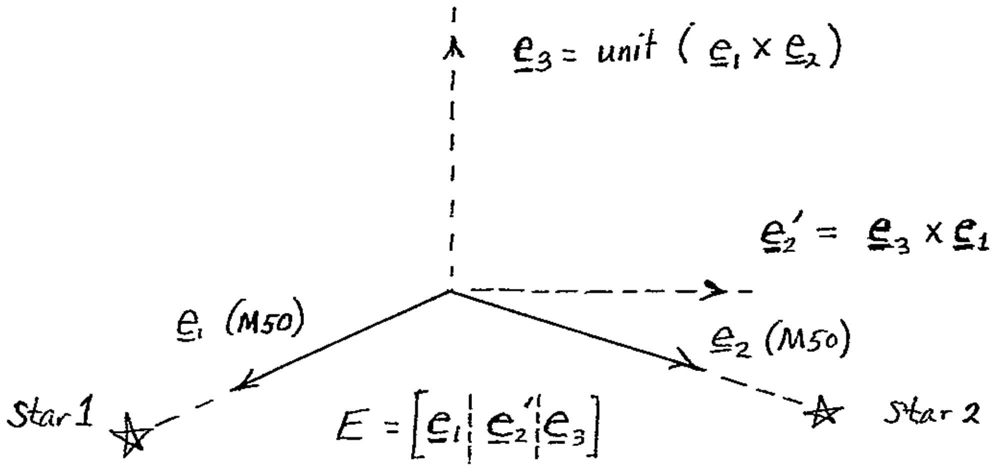
We see that the Orbiter IMU errors enter only via the transformed difference vector. Because this difference is small, less than a tenth of the magnitude of the unit vectors, the effects of Orbiter IMU measurement errors for all practical purposes are eliminated. This will be shown in the error analysis. Also, it is seen that Orbiter IMU alignment or lack thereof is not a consideration. All Orbiter measurements are expressed in stable member coordinates; knowledge of the alignment of the IMU stable member relative to the M50 frame is not required.

<u>IUS Alignment Calculations, Combined Procedures</u>

Let us now proceed through the steps listed above, employing the needed calculations. We will assume, without any loss of generality, that the desired IUS basic navigation frame is M50.

We will express Orbiter and IUS inertial attitudes as 3 x 3 matrices of body axis cosines. The columns of each matrix are formed by vehicle X, Y, and Z axis unit vectors respectively. For the Orbiter, the X, Y, and Z axes (unit vectors) are expressed in the IMU stable member frame. This matrix exists in the Orbiter flight computer and is denoted T_{NB}^{PC}, the transformation from the nav base reference axes to the present cluster (stable member) frame. In the IUS flight computer a similar quantity would exist (probably expressed as quaternions) relative to the IUS unknown inertial coordinate frame. For this analysis we will denote the IUS inertial attitude as the 3 x 3 matrix U.

Compute 3 x 3 E matrix from star 1 and star 2 M50 coordinates (Star catalogue data).



o Sight on the first star and take the following measurements

$\underline{m}_{11}$ --------- star LOS unit vector, in stable member coordinates, first measurement first star.

T_{NB}^{PC11} ------- Orbiter attitude, first measurement first star, stable member coordinates.

U_{11} --------- IUS attitude, first measurement first star, IUS unknown frame.

o Rotate around the first star LOS 180 degrees and repeat the measurements

$\underline{m}_{12}$ --------- Star LOS unit vector, in stable member coordinates, second measurement, first star.

T_{NB}^{PC12}-------- Orbiter attitude, second measurement first star, stable member coordinates

U_{12} --------- IUS attitude, second measurement first star, IUS unknown frame.

o Compute with first star measurements

$$\underline{m}_1 = \text{unit}\left[\tfrac{1}{2}(\underline{m}_{11} + \underline{m}_{12})\right] \qquad \text{(bias removal)}$$

$$C_1 = T_{NB}^{PC12}\left[T_{NB}^{PC11}\right]^t$$

$$F_1 = U_{12}\left[U_{11}\right]^t$$

o From C_1 and F_1, compute (see Appendix A) first rotation eigenvectors $\underline{d}_1$ and $\underline{g}_1$.

$\underline{d}_1$ ——— Orbiter rotation eigenvector, first 180 degree rotation, stable member coordinates

$\underline{g}_1$ ——— IUS rotation eigenvector, same rotation as for $\underline{d}_1$, IUS unknown frame

Averaging $\underline{m}_{11}$ and $\underline{m}_{12}$ removes Orbiter body fixed biases from the star measurements (Appendix D). C_1 is the orthogonal transformation, in stable member coordinates, linking the initial and final Orbiter attitudes relative to the 180 degree rotation around the star LOS.

F_1 is the orthogonal transformation, in IUS unknown frame coordinates linking the initial and final IUS attitudes relative to the same 180 degree rotation. It is from C_1 and F_1 that the Orbiter and IUS eigenvectors $\underline{d}_1$ and $\underline{g}_1$ are determined, as shown in Appendix A. Although expressed in different coordinate systems, $\underline{d}_1$ and $\underline{g}_1$ point in the same direction in inertial space, given that the IUS position relative to the Orbiter is unchanged from measurement to measurement.

o Sight on the second star and take the following measurements (simultaneously):

$\underline{m}_{21}$ ——— star LOS unit vector, in stable member coordinates, first measurement second star

T_{NB}^{PC21} ——— Orbiter attitude, first measurement second star, stable member coordinates

U_{21} ——— IUS attitude, first measurement second star, IUS unknown frame

o Rotate around the second star LOS 180 degrees and repeat the measurements

$\underline{m}_{22}$ ——— star LOS unit vector, in stable member coordinates, second measurement, second star

T_{NB}^{PC22} ——— Orbiter attitude, second measurement second star, stable member coordinates

U_{22} ——— IUS attitude, second measurement second star IUS unknown frame

o Compute with second star measurements

$$\underline{m}_2 = \text{unit}\left[\tfrac{1}{2}(\underline{m}_{21} + \underline{m}_{22})\right]$$

$$C_2 = T_{NB}^{PC22}\left[T_{NB}^{PC21}\right]^t$$

$$F_2 = U_{22}\left[U_{21}\right]^t$$

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o From C_2 and F_2, compute second rotation eigenvectors $\underline{d}_2$ and $\underline{g}_2$ (Appendix A).

$\underline{d}_2$ ——— Orbiter rotation eigenvection, second 180 degree rotation stable member coordinates

$\underline{g}_2$ ——— IUS rotation eigenvector, same rotation as for $\underline{d}_2$, IUS unknown frame

o Form 3x3 D matrix from $\underline{d}_1$ and $\underline{d}_2$.

$$\underline{d}_3 = \text{unit } (\underline{d}_3 \times \underline{d}_1)$$

$$\underline{d}_2' = \underline{d}_3 \times \underline{d}_1$$

$$D = \left[\underline{d}_1 \,\middle|\, \underline{d}_2' \,\middle|\, \underline{d}_3\right]$$

o Form 3x3 G matrix from $\underline{g}_1$ and $\underline{g}_2$.

$$\underline{g}_3 = \text{unit } (\underline{g}_1 \times \underline{g}_2)$$

$$\underline{g}_2' = \underline{g}_3 \times \underline{g}_1$$

$$G = \left[\underline{g}_1 \,\middle|\, \underline{g}_2' \,\middle|\, \underline{g}_3\right]$$

o Compute <u>difference</u> vectors $\Delta\underline{d}_1$ and $\Delta\underline{d}_2$.

$$\Delta\underline{d}_1 = \underline{m}_1 - \underline{d}_1$$

$$\Delta\underline{d}_2 = \underline{m}_2 - \underline{d}_2$$

The $\Delta\underline{d}_1$ and $\Delta\underline{d}_2$ are the differences between the star LOS unit vectors and the eigenvectors, in stable member coordinates.

o Compute transformation T^U_{PC} relating the Orbiter IMU stable member (present cluster) frame to the IUS unknown frame. We know by definition that T^U_{PC} transforms $\underline{d}_1$ to $\underline{g}_1$, $\underline{d}_2'$ to $\underline{g}_2'$, and $\underline{d}_3$ to $\underline{g}_3$. Thus T^U_{PC} transforms D to G.

$$G = T^U_{PC} D$$

Solving for T^U_{PC} yields

$$T^U_{PC} = GD^t$$

o Transform <u>differences</u> $\Delta\underline{d}_1$ and $\Delta\underline{d}_2$ to the IUS unknown frame (call the results $\Delta\underline{g}_1$ and $\Delta\underline{g}_2$)

$$\Delta\underline{g}_1 = T^U_{PC} \Delta\underline{d}_1$$

$$\Delta\underline{g}_2 = T^U_{PC} \Delta\underline{d}_2$$

- Compute star LOS unit vectors $\underline{h}_1$ and $\underline{h}_2$

  $\underline{h}_1 = \text{unit}\ (\Delta\underline{g}_1 + \underline{g}_1)$

  $\underline{h}_2 = \text{unit}\ (\Delta\underline{g}_2 + \underline{g}_2)$

  The above yields computed star 1 and star 2 LOS unit vectors in IUS unknown frame coordinates

- Form 3 x 3 H matrix from $\underline{h}_1$ and $\underline{h}_2$.

  $\underline{h}_3 = \text{unit}\ (\underline{h}_1 \times \underline{h}_2)$

  $\underline{h}_2' = \underline{h}_3 \times \underline{h}_1$

  $H = [\underline{h}_1 \mid \underline{h}_2' \mid \underline{h}_3]$

- Compute the IUS alignment transformation T_U^{M50} relating the IUS unknown frame to the M50 coordinate system. We know that T_U^{M50} by definition transforms $\underline{h}_1$ to $\underline{e}_1$, $\underline{h}_2'$ to $\underline{e}_2'$, and $\underline{h}_3$ to $\underline{e}_3$. Thus, T_U^{M50} transforms H to E.

  $$E = T_U^{M50} H$$

  Solving for T_U^{M50} yields

  $$T_U^{M50} = E\ H^t$$

  T_U^{M50} is the desired result of the alignment procedures. The actual alignment is carried out subsequently by transforming IUS inertial attitude U from IUS unknown frame coordinates to M50 coordinates using T_U^{M50}. This is a one-time computation. The IUS strapdown inertial system will then proceed to update IUS inertial attitude in the M50 system.

PART 5

ERROR ANALYSIS

DIRECT IUS ALIGNMENT

ERROR ANALYSIS OF DIRECT IUS ALIGNMENT

From the equations above, we see that the IUS alignment error depends on the error in $\underline{h}$, the computed star LOS unit vector expressed in IUS unknown frame coordinates. The equations are:

$$\underline{h} = \Delta\underline{g} + \underline{g}$$

$$\Delta\underline{g} = T_{PC}^{U}\,\Delta\underline{d}$$

$$\Delta\underline{d} = \underline{m} - \underline{d}$$

where

$\underline{m}$ -----star LOS unit vector, average of two measurements, Orbiter stable member coordinates

$\underline{d}$, $\underline{g}$ --Orbiter and IUS jointly sensed eigenvectors, in Orbiter stable member and IUS unknown frames, respectively

T_{PC}^{U} ---Transformation from Orbiter stable member frame to IUS unknown frame

We will assume the star catalogue data to be error free. We will also assume, for the present, that the two alignment stars are 90 degrees apart.

The error equation for $\underline{h}$ is:

$$\delta\underline{h} = \delta\underline{g} + \left[\delta T_{PC}^{U}\right]\Delta\underline{d} + T_{PC}^{U}\delta\Delta\underline{d}$$

Determining the errors $\delta\underline{g}$, δT_{PC}^{U}, and $\delta\Delta\underline{d}$ will allow determination of the IUS alignment error.

Error $\delta\underline{g}$

The error $\delta\underline{g}$ is due entirely to the IUS IMU. This error has already been determined to be 13 $\widehat{\text{sec}}$ (1δ) per axis.

Error δT_{PC}^{U}

The per axis error δT_{PC}^{U} has already been determined. It is the RSS of the Orbiter eigenvector error (38 $\widehat{\text{sec}}$, 1σ) and the IUS eigenvector error (13 $\widehat{\text{sec}}$, 1σ), which is 40 $\widehat{\text{sec}}$. The effect of δT_{PC}^{U} is to rotate $\Delta\underline{d}$ 40 $\widehat{\text{sec}}$ per axis.

Since $\Delta\underline{d}$ is small compared to $\underline{g}$ (a tenth or less), the error $\delta T_{PC}^{U}\Delta\underline{d}$ is less than 4 $\widehat{\text{sec}}$ (1σ) per axis. The smaller $\Delta\underline{d}$, the smaller the error effect.

Error $\delta\Delta d$

Both $\underline{m}$ and $\underline{d}$ incorporate the same IMU measurement. In addition, $\underline{m}$ incorporates the star tracker measurements. It would appear, since the IMU measurement errors are common to both $\underline{m}$ and $\underline{d}$, that in forming the difference $\Delta\underline{d}$ the IMU errors might cancel, leaving only the star tracker error effects. This is indeed the case, as demonstrated below.

We will build on the error analysis already completed determining the error in the eigenvector $\underline{d}$. It was determined that

$$\delta d_X = \frac{1}{2\sin\theta}\left[(\alpha_2 - \alpha_1)(1 + \cos\theta) + (\beta_1 + \beta_2)\sin\theta\right]$$

$$\delta d_Y = \frac{1}{2\sin\theta}\left[(\beta_2 - \beta_1)(1 + \cos\theta) - (\alpha_1 + \alpha_2)\sin\theta\right]$$

$$\delta d_Z = 0$$

where

α_1, β_1 -----X and Y axis angular measurement errors due to Orbiter IMU gimbals and resolver, first measurement (prior to 180 degree rotation)

α_2, β_2 -----X and Y axis angular measurement errors, second measurement (taken after the 180 degree rotation).

The above results were determined in a coordinate frame in which the Z axis is oriented along eigenvector $\underline{d}$. Employing the measurement errors α_1, β_1, α_2, β_2, the averaged star tracker measurement error $\delta\underline{m}$ will be calculated. This will be followed by the calculation of $\delta\Delta\underline{d}$. To simplify the analysis, we will assume that $\underline{m}$ also points in the Z axis direction. To first order, this will not affect the error analysis since $\underline{m}$ and $\underline{d}$ are only a few degrees apart.

$\underline{m}$ is the average of two measurements (in stable member coordinates) which will be labeled here as $\underline{m}_1$ and $\underline{m}_2$. Thus

$$\underline{m} = \tfrac{1}{2}\,(\underline{m}_1 + \underline{m}_2)$$

$$\delta\underline{m} = \tfrac{1}{2}\,(\delta\underline{m}_1 + \delta\underline{m}_2)$$

Averaging $\underline{m}_1$ and $\underline{m}_2$ removes the Orbiter body fixed sensor biases from $\underline{m}$ (as explained in Appendix D). We need consider only the random errors. These are the star tracker measurement errors (denoted $\delta\theta_{ST}$ in the error model, Table 4) and the IMU gimbal/resolver error (denoted $\delta\theta_R$, Table 4). Thus, the errors $\delta\underline{m}_1$ and $\delta\underline{m}_2$ are the random angular error effects of the Orbiter IMU and star tracker measurements. Using nomenclature previously employed (page 30), the errors can be written:

$$\delta\underline{m}_1 = \delta R_1\underline{m}_1$$

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$$\delta\underline{m}_2 = \delta R_2\underline{m}_2$$

where δR_1 and δR_2 are now the rotation error matrices (3 x 3) due to the Orbiter IMU and star tracker errors, first and second measurements respectively.

The per-axis measurement errors corresponding to δR_1 and δR_2 are written below.

First measurement:

X-axis angular error = $\alpha_1 + \alpha_3$

Y-axis angular error = $\beta_1 + \beta_3$

(Z-axis angular error we ignore since $\underline{m}_1 = (0, 0, 1)$)

where

α_1, β_1 --- IMU errors (radians)

α_3, β_3 --- star tracker errors (radians)

Second measurement:

X-axis angular error = $\alpha_2 + \alpha_4$

Y-axis angular error = $\beta_2 + \beta_4$

(Z-axis angular error we ignore since $\underline{m}_2 = (0, 0, 1)$)

where

α_2, β_2 --- IMU errors (radians)

α_4, β_4 --- star tracker errors (radians)

Thus

$$\delta R_1 = \begin{bmatrix} 0 & 0 & (\beta_1 + \beta_3) \\ 0 & 0 & -(\alpha_1 + \alpha_3) \\ -(\beta_1 + \beta_3) & (\alpha_1 + \alpha_3) & 0 \end{bmatrix}$$

$$\delta R_2 \quad \begin{bmatrix} 0 & 0 & (\beta_2 + \beta_4) \\ 0 & 0 & -(\alpha_2 + \alpha_4) \\ -(\beta_2 + \beta_4) & (\alpha_2 + \alpha_4) & 0 \end{bmatrix}$$

We have, to first order,

$$\delta\underline{m} = \tfrac{1}{2}(\delta\underline{m}_1 + \delta\underline{m}_2) = \tfrac{1}{2}(\delta R_1 + \delta R_2)\underline{m}$$

Thus

$$\delta\underline{m} = \tfrac{1}{2} \begin{bmatrix} (\beta_1+\beta_2+\beta_3+\beta_4) \\ -(\alpha_1+\alpha_2+\alpha_3+\alpha_4) \\ 0 \end{bmatrix}$$

Now calculate $\underline{\delta\Delta d} = \delta\underline{m} - \delta\underline{d}$.

$$\delta\Delta d_X = \tfrac{1}{2}(\beta_1+\beta_2+\beta_3+\beta_4) - \frac{1}{2\sin\theta}\left[(\alpha_2-\alpha_1)(1+\cos\theta) + (\beta_1+\beta_2)\sin\theta\right]$$

$$\delta\Delta d_Y = -\tfrac{1}{2}(\alpha_1+\alpha_2+\alpha_3+\alpha_4) - \frac{1}{2\sin\theta}\left[(\beta_2-\beta_1)(1+\cos\theta) - (\alpha_1+\alpha_2)\sin\theta\right]$$

$$\delta\Delta d_Z = 0$$

Combining terms yields

$$\delta\Delta d_X = \frac{\beta_3+\beta_4}{2} + \frac{(\alpha_1-\alpha_2)(1+\cos\theta)}{2\sin\theta}$$

$$\delta\Delta d_Y = -\frac{\alpha_3+\alpha_4}{2} + \frac{(\beta_1-\beta_2)(1+\cos\theta)}{2\sin\theta}$$

$$\delta\Delta d_Z = 0$$

The mean square errors are:

$$\overline{\delta\Delta d_X^2} = \frac{\overline{\beta_3^2}+\overline{\beta_4^2}}{4} + \frac{\overline{\alpha_1^2}+\overline{\alpha_2^2}}{4}\left[\frac{1+\cos\theta}{1-\cos\theta}\right]$$

$$\overline{\delta\Delta d_Y^2} = \frac{\overline{\alpha_3^2}+\overline{\alpha_4^2}}{4} \quad \frac{\overline{\beta_1^2}+\overline{\beta_2^2}}{4}\left[\frac{1+\cos\theta}{1-\cos\theta}\right]$$

where it is assumed that $\alpha_1,\beta_1,\alpha_2,\beta_2,\alpha_3,\beta_3,\alpha_4,\beta_4$ are uncorrelated with zero means. From the error model, Table 4, we have

$$\delta\theta_{ST}^2 = \overline{\alpha_3^2} = \overline{\beta_3^2} = \overline{\alpha_4^2} = \overline{\beta_4^2}$$

$$\delta\theta_R^2 = \overline{\alpha_1^2} = \overline{\beta_1^2} = \overline{\alpha_2^2} = \overline{\beta_2^2}$$

Thus

$$\mathrm{dev}(\delta\Delta d_X) = \mathrm{dev}(\delta\Delta d_Y) = \left\{\frac{\delta\theta_{ST}^2}{2} + \frac{\delta\theta_R^2}{2}\left[\frac{1+\cos\theta}{1-\cos\theta}\right]\right\}^{\frac{1}{2}}$$

$$\mathrm{dev}(\delta\Delta d_Z) = 0$$

The eigenvector rotation angle is 180 degrees. For $\theta=180$, $(1+\cos\theta)/(1-\cos\theta) = 0$; the per axis error in $\delta\Delta\underline{d}$, to first order, is due only to the star tracker error $\delta\theta_{ST}$. The IMU errors "cancel out".

$$\mathrm{dev}(\delta\Delta d_X) = \mathrm{dev}(\delta\Delta d_Y) = \frac{\delta\theta_{ST}}{\sqrt{2}} \qquad (\theta = 180 \text{ deg})$$

The error model (Table 4) gives $\delta\theta_{ST}$ the value 42 $\widehat{\sec}$ ($1\,\sigma$). We will also consider the situation where star measurements are restricted to the central 4 x 4 degree FOV of the star tracker, which it is understood halves the error. Thus:

Table 8. $\underline{\Delta d}$ per-Axis Error Value

FOV	$\mathrm{dev}(\delta\Delta d_{X \text{ or } Y})$ $\theta = 180$ deg
10 x 10 deg	30 $\widehat{\sec}$ (1δ)
4 x 4 deg	15 $\widehat{\sec}$ (1δ)

It is recalled that $\delta\Delta\underline{d}$ effects IUS alignment accuracy in the term T^U_{PC} $(\delta\Delta\underline{d})$. Since T^U_{PC} is an orthogonal transformation, the per-axis error T^U_{PC} $(\delta\Delta\underline{d})$ remains the same as given in Table 8 immediately above for $\delta\Delta\underline{d}$.

IUS ALIGNMENT ERROR, DIRECT ALIGNMENT

From above, it was determined that the error in $\underline{h}$, the computed star LOS unit vector expressed in IUS unknown frame coordinates, is

$$\delta \underline{h} = \delta \underline{g} + \underbrace{\left[\delta T^{U}_{PC}\right] \Delta \underline{d} + T^{U}_{PC}(\delta \Delta \underline{d})}_{\delta \Delta \underline{g}}$$

A rigorous treatment at this point would account for the correlations between (1) $\delta \Delta \underline{d}$ and δT^{U}_{PC} and (2) δT^{U}_{PC} and $\delta \underline{g}$, since $\underline{d}$ and $\underline{g}$ were employed to calculate T^{U}_{PC}. For our purposes, we ignore the correlation because the major error sources ($\delta \underline{g}$ and $\delta \Delta \underline{d}$) are uncorrelated. These errors are due to the star tracker and IUS IMU respectively.

We consider the selected alignment stars to be 90 degrees apart. Hence, the per-axis IUS alignment error equals the per-axis error in $\delta \underline{h}$ (Appendix C). We thus calculate the IUS alignment error as the RSS of the component errors.

The results are presented immediately below.

Table 9. IUS Alignment Error (Alignment Stars 90 Degrees Apart)

Star Tracker FOV	$\delta \underline{g}$	δT^{U}_{PC} Δd	$T^{U}_{PC}(\delta \Delta \underline{d})$	IUS Alignment Error	
				1σ	3σ
10x10 deg	13 sec	4 sec	30 sec	33 sec	1.6 min
4x 4 deg	13 sec	2 sec	15 sec	20 sec	1.0 min
	IUS IMU Error Effect	Transformation Error Effect	Star Tracker Error Effect		

If the alignment stars are not 90 degrees apart, then the alignment degrades somewhat, as explained in Appendix C. For alignment star angles within 90 $\pm$ 30 degrees, the average degradation is less than ten percent.

PART 6

STRAWMAN MECHANIZATION

DIRECT IUS ALIGNMENT PROCEDURE

EQUATION MECHANIZATION. DATA INTERFACE

The equations developed above were examined for on-board implementation in case such were considered. A brief investigation showed implementation should be straightforward. The description below represents a possible implementation approach. Figure 2 presents a block diagram of the Orbiter/IUS data processing interface.

Software Mechanization

It is assumed that the alignment equations would be processed in the IUS computer. Orbiter data would flow from the Orbiter GN&C flight computer to the Orbiter Systems Management (SM) flight computer to the IUS flight computer.

A small software program would be required in the SM computer to control the alignment transfer. This program would accept keyboard inputs from the crew, notify the IUS computer of the impending alignment, monitor the GN&C computer for the start of each data collection period, signal the IUS computer to take IUS data at the appropriate times, receive and transfer Orbiter data to the IUS flight computer, and provide CRT displays for crew control.

The only new software required in the GN&C computer would be a flag in the COMPOOL data base, set and reset at the initiation and completion of each Orbiter data collection period (lasting 3.2 seconds). The SM computer would monitor this flag every 160 ms, when in the IUS alignment mode.

Operational Sequence

The IUS direct alignment operational sequence would be the following:

- o Crew commands Orbiter IMU in-orbit alignment via the GN&C computer.
- o Before the first star sighting is taken, the SM computer is placed in the IUS alignment mode by keyboard command.
- o Thereafter, four collections of Orbiter and IUS data sets take place, as the Orbiter maneuvers and takes four star tracker sightings (on two stars) in accord with the direct IUS alignment procedures of this document. Data would be automatically taken and transferred to the IUS flight computer.
- o The SM computer would provide appropriate outputs to the CRT display for crew monitoring of the alignment process.

Data Interfaces

Orbiter data supplied to the IUS computer would be that indicated below. This data is located in the Orbiter GN&C flight computer COMPOOL data base. The nomenclature is that defined in the IMU SOP, reference 4.

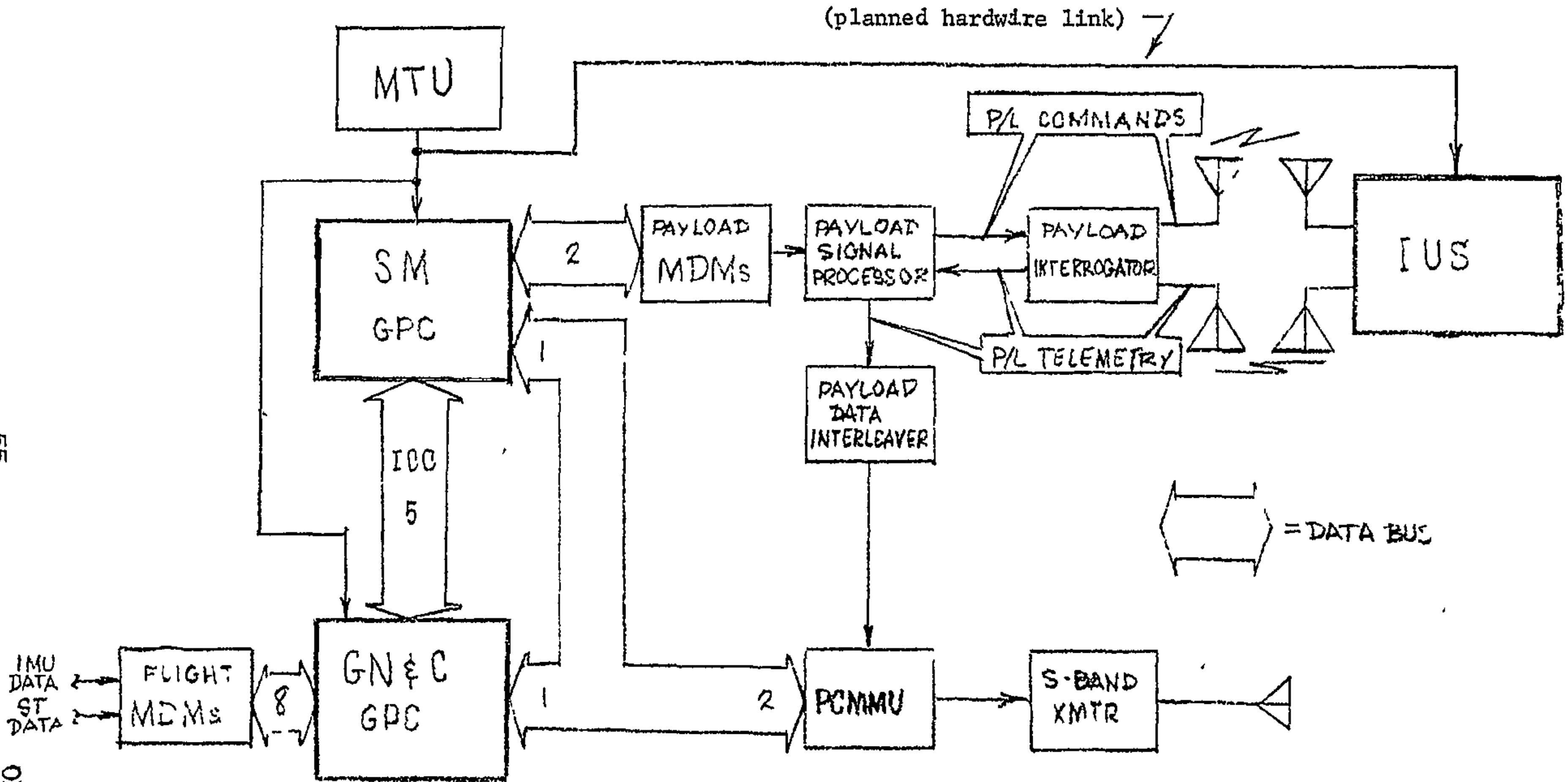

Figure 6. ON-ORBIT DATA COLLECTION HARDWARE INTERFACES

- o I_STAR_SEL1, I_STAR_SEL2....unit LOS vectors from the center of the earth to alignment stars #1 and #2 respectively, M50 coordinates.

- o I_STAR_SELC(I,J).....Orbiter star measurement LOS unit vector, in present cluster (stable members) coordinates, J^{th} IMU (1, 2, or 3), for the I^{th} star (1 or 2). This quantity is the averaged result of 21 star tracker measurements taken 160 milliseconds apart, representing data spread over 3.2 seconds.

  $T_{NB}^{PC(J)}$......3x3 transformation matrix, Orbiter nav base reference axes to present cluster coordinates. This quantity is the averaged result of 21 Orbiter IMU readings, J^{th} IMU (1, 2, or 3), taken at the same time as the 21 star tracker measurements above.

- o Time tag...data time tag for measured quantities above.

GN&C Computer

The GN&C computer would set a flag in COMPOOL when the star tracker measurements begin. Specifically, the flag is set when star tracker software DATA_FILR routine is called by the GN&C operating system (FCOS).

Orbiter SM Computer

The IUS alignment process would be controlled via the Orbiter systems management (SM) flight computer. The SM computer would alert the IUS computer that the alignment transfer is about to take place. SM keyboard entry would determine which Orbiter IMU (J=1,2, or 3) would provide data to the IUS.

The SM computer has the capability of accessing SM selected GN&C COMPOOL data via the ICC_SSIP software module. The GN&C data is transferred to the SM computer via inter-computer channels (ICC). This data is then quickly transferred by the SM computer to the IUS computer via payload interface.

The SM computer would do the following basic things:

- o Cyclically (every 160 ms via ICC) receive DATA_FILR flag.

- o When flag is set, command the IUS computer to take IUS attitude data.

- o Request,receive, and transmit alignment data from GN&C to IUS.

  (1) I_STAR_SEL(I)

  (2) I_STAR_SELC(I,J)

  (3) $T_{NB}^{PC(J)}$

  (4) TIME TAG, for (2) and (3)

IUS Computer

The IUS computer would collect the Orbiter/IUS data and perform the alignment calculations, doing the following basic things:

- o Initiate IUS body attitude data collection (body attitudes and time tags) following receipt of command from the SM computer. This data would be collected, say, over a two second period every .2 second for filtering and interpolation purposes.
- o Receive Orbiter alignment data from the SM computer.
- o Interpolate the IUS data collected to the Orbiter time tag point.
- o Calculate the IUS IMU alignment matrix. If the IUS basic navigation coordinate frame is other than M50, the IUS computer would need a fixed 3x3 transformation matrix to transfer I_STAR_SEL(I) to the desired coordinate frame.

Timing Considerations

The mechanization approach above has no critical timing requirements. Data staleness is not a problem since I_STAR_SELC(I,J) and $T_{NB}^{PC(J)}$ are time tagged. I_STAR_SEL(I) are constants (vectors) and they form part of the GN&C computer I-Load (pre-mission data load).

The approach above does assume that the Orbiter and IUS flight computers operate with a common time base, since time tags are involved. It is understood, unofficially, that there will be a hardwire connection between Orbiter and IUS master timing units such that the respective time bases will be significantly less than a millisecond apart. This accuracy is entirely adequate.

The only real-time requirement is that each IUS data collection period (2 seconds suggested above) fall within the Orbiter 3.2 second data collection time span. Thus, the IUS data collection should begin no later than .75 seconds after the Orbiter data collection begins. The brief investigation conducted to date indicates that the IUS computer can be notified about .33 seconds (maximum delay) after the Orbiter data collection process is initiated. Thus, there appears to be plenty of time for the IUS computer to initiate its data collection process. Subsequent processing of the Orbiter and IUS collected data by the IUS computer is non-cyclic and non-time critical.

PART 7

CONCLUSIONS

CONCLUSION

The basic conclusions of this report are:

- o IUS alignment can be achieved with an error much less than 6.3 $\widehat{\text{min}}$ (3σ), independent of the Orbiter IMU alignment error. This is accomplished by combining the OFT in-orbit IMU alignment procedure, modified to remove Orbiter sensor biases, and the IUS alignment transfer procedure. The accuracy achieved thereby is estimated to be about 1.6 $\widehat{\text{min}}$ (3σ).

- o Accuracy can be improved to about 1.0 $\widehat{\text{min}}$ (3σ) by restricting star images to the central 4x4 degree star tracker field of view.

- o The recommended rotation maneuvers do not appear difficult to perform.

- o On-board implementation of the recommended IUS alignment approach, were such to be considered, appears to be straightforward. Impact to the Orbiter/IUS interface appears to be minimum.

REFERENCES

Reference 1 "Interim Upper Stage Flight Operations/Mission Analysis," Boeing Document D290-10007-1, Oct. 12, 1976

Reference 2 "Subsystem Design Analysis Report, Guidance and Navigation Analysis," Boeing Document D290-10102-1, March 16, 1977

Reference 3 "Orbiter In-Orbit Alignment Accuracy," IBM/Houston Document Res 17-2, Sept. 21, 1977

Reference 4 Space Shuttle Orbital Flight Test, Level C; Functional Sub-System Software Requirements; "Guidance, Navigation, and Control, Part E, Subsystem Operating Programs, Inertial Guidance Unit;" Rockwell International SD76-SH-0013

Reference 5 "Shuttle IMU Math Model for Real Time Simulation;" Internal Letter No. 392-240-76-231, Rockwell International, June 30, 1976

APPENDIX A

ORTHOGONAL TRANSFORMATION

EIGENVECTOR COMPUTATION

APPENDIX A

Determination of Orthogonal Transformation Eigenvector

Let C be a 3 x 3 orthogonal transformation (rotation matrix). Let $\underline{x}$ be an eigenvector of C. Since the eigenvalue of an orthogonal transformation is $\underline{1}$, then by definition

$$C\underline{x} = \underline{x}$$

or,

$$(C-I)\underline{x} = 0$$

Writing the equation above in component form yields

$$\begin{bmatrix} c_{11}-1 & c_{12} & c_{13} \\ c_{21} & (c_{22}-1) & c_{23} \\ c_{31} & c_{32} & (c_{33}-1) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The above is a linear system of 3 homogeneous equations in 3 unknowns (x_1, x_2, x_3) of rank 2. The solution comes about immediately from a well-known property of determinants.

Solution

Let $[a_{ij}]$ be an nxn matrix with elements a_{ij}. Let the cofactors of a_{ij} be denoted A_{ij}. A property of determinants (termed the Laplace development) is

$$\sum_{j=1}^{n} a_{ij} A_{kj} = \begin{cases} \det [a_{ij}] & i = k \\ 0 & i \neq k \end{cases}$$

Denoting the matrix of cofactors as $[A_{ij}]$, we can write the above as

$$[a_{ij}] [A_{ij}]^t = [a_{ij}][A_{ji}] = \det [a_{ij}] I$$

where I is the nxn identify matrix.

Applying the above, we let $[a_{ij}]$ = C - I. Since C - I is of rank 2, then det $[a_{ij}]$= 0. Thus, in this particular case,

$$[a_{ij}][A_{ji}] = 0$$

It is seen that any column of A_{ji} (i.e., any row of A_{ij}) is a solution to the linear homogeneous equation $\left[a_{ij}\right] \underline{x} = 0$. Thus we can write the solutions

$$x_1 = kA_{i1}$$

$$x_2 = kA_{i2}$$

$$x_3 = kA_{i3}$$

where k is any arbitrary constant, and i = 1, 2, 3.

The theory of linear homogeneous equations tells us that when the rank = n - 1, there exists exactly one independent solution. Therefore, the ratios x_j/x_k are uniquely determined; the solutions obtained for different values of i above are identical in terms of the ratios, although the computations are different.

We now write down the three alternative ways to compute the solution of $(C - I)\underline{x} = 0$, corresponding to i = 1, 2, 3. Each alternative consists of computing the appropriate cofactors. The cofactor of an element of C-I is determined by striking out the row and column containing the element, forming the determinant of the 2 x 2 submatrix remaining, and assigning the polarity $(-1)^{i+j}$. Let k = 1.

<u>Alternative 1</u> (let i = 1):

$$x_1 = (c_{22} - 1)(c_{33} - 1) - c_{23}\ c_{32}$$

$$x_2 = - c_{21}(c_{33} - 1) + c_{23}\ c_{31}$$

$$x_3 = c_{21}c_{32} - (c_{22} - 1)c_{31}$$

<u>Alternative 2</u> (let i = 2):

$$x_1 = c_{12}(c_{33} - 1) + c_{13}c_{32}$$

$$x_2 = (c_{11} - 1)(c_{33} - 1) - c_{13}c_{31}$$

$$x_3 = -c_{32}(c_{11} - 1) + c_{12}c_{31}$$

<u>Alternative 3</u> (let i = 3):

$$x_1 = c_{12}c_{23} - c_{13}(c_{22} - 1)$$

$$x_2 = -c_{23}(c_{11} - 1) + c_{13}c_{21}$$

$$x_3 = (c_{11} - 1)(c_{22} - 1) - c_{12}c_{21}$$

The solutions $\underline{x}$ above must be normalized in order to form eigenvector $\underline{d}$ of unit length. The normalization factor $k = |\underline{x}|^{-1}$ is, in general, different depending on the computational alternative chosen.

Of particular interest are the possible singularities which might arise during the computations. In this case, the matrix C given is such that under the computational alternative chosen, the trivial solution $\underline{x} = 0$ results. This occurrence was noted earlier in the text of this report, page 33. However, since exactly one independent non-trivial solution exists, at least one of the computational alternatives above must provide finite, non-trivial $\underline{x}$.

Take the situation from page 30 where C represents a rotation around the Z coordinate axis.

$$C = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

When $\Theta = 180$ degrees, the computational method employed (which is different from the three alternatives above) yielded $\underline{x} = 0$, which is a solution, but not the non-trivial solution sought. Let us apply the three alternative computations of this section. The results are as follows:

Alternative 1: $\underline{x} = 0$ (trivial solution)

Alternative 2: $\underline{x} = 0$ (trivial solution)

Alternative 3: $\underline{x} = (0, 0, 4)$

Alternative 3 yields unit eigenvector $\underline{d} = (0, 0, 1)$, which is the proper solution since we postulated C to be a rotation about the Z coordinate axis.

<u>Computational Procedure</u>

A sure way to avoid the singular solution $\underline{x} = 0$ is to compute $\underline{x}$ using <u>each</u> of the three alternatives. At least one solution will be non-trivial. If at least two solutions are non-trivial we could choose the solution $\underline{x}$ having the largest magnitude $|\underline{x}|$. This minimizes the effects of computer roundoff errors.

<u>Which Way Does x Point?</u>

The non-trivial solution $-\underline{x}$ satisfies $(C - I)\underline{x} = 0$ as well as the non-trivial solution $\underline{x}$. Both $\underline{x}$ and $-\underline{x}$ lie along the axis of rotation (eigenaxis) represented by C but point in opposite directions. In applying the computational procedure discussed above, the question arises "which way along the eigenaxis does the computed solution $\underline{x}$ point?" The answer is important, because we want the jointly sensed and computed eigenvectors (Orbiter and IUS) to point in the same direction in inertial space, not opposite directions. Also, when combining the Orbiter and IUS alignment procedures, we want the jointly sensed eigenvectors to point toward the alignment star, not away from it.

Since the solution to $(C - I)\underline{x} = 0$ is not uniquely determined as to its direction along the eigenaxis, an additional step must be taken. A relatively simple procedure resolving the question of "which direction?" is described below.

For the sake of simplicity, it is assumed that the IUS is mounted in the Orbiter payload bay with its body axes (roll, pitch, yaw) roughly parallel to the Orbiter's body axes (say within 10 degrees). The basis of the procedure is to (a) transform the jointly sensed eigenvectors $\underline{d}$ (Orbiter stable member frame) and $\underline{g}$ (IUS unknown frame) to the respective body axis frames and then (b) make some comparisons. Since the Orbiter and IUS body axes are assumed roughly aligned, the transformed eigenvectors, in respective body axis coordinates, should be roughly equivalent. The magnitude of the difference between the transformed eigenvectors should be significantly less than 1. If not, a reversal in direction is required.

The following quantities are involved, all of which have already been employed in the main text analysis of the IUS alignment.

$\underline{d}$ ---- Orbiter eigenvector, stable member coordinates

$\underline{g}$ ---- IUS eigenvector (jointly sensed with $\underline{d}$), IUS unknown frame coordinates

T_{NB}^{PC2} --- transformation, Orbiter nav base to stable member coordinates, at time of 2nd measurement taken at completion of eigenvector rotation maneuver -- the columns of this matrix are the Orbiter body axis direction cosines relative to the stable member frame.

U_2 --- IUS body axis matrix at same time point ----the columns of this matrix are the IUS body axis direction cosines relative to the IUS unknown frame

$\underline{m}$ ---- unit vector pointing toward alignment star, Orbiter stable member frame

$\underline{\Delta d}$ ---- $\underline{\Delta d} = \underline{m} - \underline{d}$; hence $|\underline{\Delta d}| << 1$

Eigenvector Direction Resolution Subprocedure

The subprocedure begins at the point where the difference $\underline{\Delta d} = \underline{m} - \underline{d}$ is computed.

(1) Correct the direction of $\underline{d}$, if required.

If: $|\underline{\Delta d}| > 1$, set $\underline{d} \leftarrow -\underline{d}$

Otherwise: $\underline{d}$ already has correct direction

(2) Transform $\underline{d}$ to Orbiter nav base coordinates

$$\underline{d}' = \left[T_{NB}^{PC2}\right]^t \underline{d}$$

(3) Transform $\underline{g}$ to IUS body axis coordinates

$$\underline{g}' = \left[U_2\right]^t \underline{g}$$

(4) Correct the direction of $\underline{g}$, if required.

If: $|\underline{d}' - \underline{g}'| > 1$, set $\underline{g} \leftarrow -\underline{g}$

Otherwise: $\underline{g}$ already has correct direction

This completes the subprocedure; $\underline{d}$ and $\underline{g}$ now have correct orientations in their respective inertial frames. If the star measurement is not involved (separate Orbiter alignment and Orbiter/IUS alignment transfer), simply start the subprocedure at step (2).

If it turns out that the Orbiter body axes and IUS body axes in the payload bay are not roughly parallel, the transformation relating these axes can be easily determined (prior to flight) within the accuracy needed (say, within 10 degree accuracy). This transformation, call it T_{ORB}^{IUS}, would be used to transform $\underline{d}'$ to $\underline{d}''$, where d'' is the Orbiter eigenvector expressed (roughly) in IUS body axis coordinates. The $\underline{d}''$, instead of $\underline{d}'$, would then be used in step (4) above.

APPENDIX B

ORBITER IMU AND STAR TRACKER ERROR MODEL DISCUSSION

ERROR MODEL

Each of the three IMU's consists of a stable platform interconnected to its case through four gimbals. Each IMU case is mounted on a common baseplate called the navigation base. Also mounted on the navigation base are two star trackers. The body fixed orientation of each sensor case relative to the navigation base is carefully measured.

The principal function of the stable platform is to maintain a fixed, known orientation in inertial space, regardless of attitude changes by the Orbiter vehicle. Gyroscopes on the platform sense angular motion of the stable platform and act through servo loops and gimbal motors to maintain the platform orientation fixed in inertial space. The orientation of the Orbiter's navigation base relative to the stable platform is determined by simultaneously measuring the four IMU gimbal angles (gimbal resolves). These angles, plus the known orientation of the IMU case relative to the navigation base, allow the determination of the Orbiter's attitude relative to inertial space.

Complete error models of the IMU and star tracker address every error source important to overall operational capability. An example of a complete, detailed error model is Reference 25. For the purpose of analyzing in-orbit alignment and alignment stability, only a subset of these error sources is needed. These error sources turn out to be the uncompensated errors associated with (1) sensor geometrical orientation errors relative to the navigation base, (2) IMU gimbal and resolver errors, (3) gyro drift rate errors, and (4) star tracker measurement errors.

Table B1 presents the error model employed in this analysis. The primary data source is the error budget presented by Rockwell International (RI) at the 15 March splinter meeting. These errors have been further broken down (Table B2) into error types in accord with reference 5. Discussion of these errors follows.

TABLE B1

ALIGNMENT ERROR MODEL

ERROR SOURCE	SYMBOL *	VALUE/AXIS (1σ)	
Orbiter IMU			
Gyro Drift		.035 deg/hr	
Nav Base Ref. to Mounting Pads	$\delta\theta_{IPN}$	60 $\widehat{\text{sec}}$	Body Fixed Biases
IMU Case to Pads	$\delta\theta_{CMP}$	20 $\widehat{\text{sec}}$	
Case to Outer Roll Gimbal	$\delta\theta_{COR}$	28 $\widehat{\text{sec}}$	
Non-Orthogonality Resolver	$\delta\theta_R$	30 $\widehat{\text{sec}}$ 44 $\widehat{\text{sec}}$ } 53 $\widehat{\text{sec}}$ RSS	68 $\widehat{\text{sec}}$ RSS
Star Tracker			
Horizontal, Vertical Measurements	$\delta\theta_{ST}$	42 $\widehat{\text{sec}}$	
Tracker to Nav Base Ref.	$\delta\theta_{NST}$	60 $\widehat{\text{sec}}$	Body Fixed Bias
RSS (not including gyro drift)	ε	114 $\widehat{\text{sec}}$ ($1\ \sigma$) 5.7 $\widehat{\text{min}}$ ($3\ \sigma$)	

* Note. These symbols are employed in the error analysis.

TABLE B2

BREAKDOWN OF IMU CASE TO STABLE MEMBER ERRORS

ERROR SOURCE				ERROR VALUE/AXIS (1σ)
a Pitch to Outer Roll Gimbal Non-Orthogonality				30 $\widehat{\text{sec}}$
b Resolvers				44 $\widehat{\text{sec}}$
Component Errors	Offset		30 $\widehat{\text{sec}}$	
	Random bias		12 $\widehat{\text{sec}}$	
	Sinusoids			
	1 speed	7.6	29 $\widehat{\text{sec}}$	
	8 speed	19		
	9 speed	4.2		
	16 speed	20		
	Quantization (20 $\widehat{\text{sec}}$)		6 $\widehat{\text{sec}}$	
	RSS subtotals		44 $\widehat{\text{sec}}$	
c Case to Outer Roll Gimbal Geometrical misalignment				28 $\widehat{\text{sec}}$
RSS (1σ)				60 $\widehat{\text{sec}}$

The Geometrical Orientation Errors

The body fixed geometrical misalignments of the IMU's and the star trackers adversely affect the precision with which the star sighting vectors can be transformed into the IMU stable platform inertial reference frame.

These errors stem from (1) the precision with which the sensor case orientations on the navigation base can be optically measured prior to lift-off and (2) on the flexing of the navigation base in orbit. The flex is due principally to the heat load imposed on the navigation base by the IMU's and star tracker's power supplies. During the period of time between the Orbiter star alignment and the alignment transfer to the IUS, the flex can be considered constant.

An additional body fixed misalignment (per reference 2) is the IMU case-to-outer-roll-gimbal error, also listed in Table B1.

Star Tracker Errors

The star tracker error consists of two basic components. The noise equivalent error is random, being about 20 $\widehat{\text{sec}}$ (1σ). The second error is a function of the position of the star image in the star tracker field of view. The star tracker procurement specification stipulates that the star tracker shall measure the sensed star LOS with a total error not exceeding 60 $\widehat{\text{sec}}$ ($1\ \sigma$) over the entire field of view. The per-axis requirement, normal to the tracker's boresight axis would therefore be $60/\sqrt{2}$, or about 42 $\widehat{\text{sec}}$.

Although the star tracker is believed to perform somewhat better than the specification value, the 42 $\widehat{\text{sec}}$ value will be used in this analysis. Discussion with W. Swingle, NASA/JSC, indicated that star tracker accuracy improves when the star sightings take place near the center of the star tracker field of view. The star tracker field is a 10 x 10 degree square, the center of which being the star tracker's boresight axis. If star sightings are restricted to the central 4 x 4 degree field, star tracker measurement errors are halved.

IMU Gimbal and Resolver Errors

These error source values are based on reference 5 data. In general the effect of these errors is a function of gimbal angle.

Referring to Page 7, where the pitch resolver angle is zero, the pitch to outer roll gimbal non-orthogonality error of 30 $\widehat{\text{sec}}$ can be visualized as a slight rotation of the pitch gimbal about the stable member azimuth axis. This introduces an error in the IMU azimuth readings. For non-zero pitch gimbal angles, this non-orthogonality also affects the outer roll gimbal resolver readings.

The resolver readings themselves are subject to several errors. The offset error is the error existing when the gimbal resolver reading should be zero. The resolver readout errors, over the range of gimbal angles readings (0 to 360 degrees) can be expressed as a random bias and a combination of sinusoidal harmonics whose magnitude and phase are random variables. The 1σ magnitudes are given in Table B2. The phases can be considered uniformly distributed between 0 and 360 degrees.

Resolver readout quantization is 20 $\widehat{\text{sec}}$. The maximum quantization error is therefore 10 $\widehat{\text{sec}}$. Treating the quantization error as uniformly distributed leads to a 1σ quantization error of 10 $\widehat{\text{sec}}/\sqrt{3}$, or about 6 $\widehat{\text{sec}}$.

The gimbal and resolver errors affect the IMU alignment systematically as a function of the gimbal angles. If all gimbal angles changed significantly between two readings, the readout errors might be considered independent random errors. If the change were small, then the readout errors would tend to behave as biases. In general, when the Orbiter changes its attitude, some of the gimbals might change significantly while other gimbal angles might change very little. Thus, the nature of these errors varies somewhat unpredictably. However, since both the in-orbit alignment and alignment transfer maneuvers involve large changes in Orbiter attitude, for this analysis the gimbal and resolver errors will be assumed independent random errors.

Effect of Timing Errors

Table B3 lists timing errors associated with the in-orbit alignment of the IMU. Only three error sources exceed 1 millisecond (ms). Of these, the star tracker data staleness error ranging from 10 ms to 52 ms predominates. The in-orbit alignment equations mechanized in the flight computer average 21 data points of star tracker and IMU gimbal angle data taken 160 ms apart. Thus, the averaged staleness error is about 30 ms.

Timing errors affect the alignment only if the Orbiter is rotating during the star sightings. If we assume, for example, the Orbiter is rotating 0.1 deg/$\widehat{\text{sec}}$ during the measurements, an average error of .003 degrees or 11 $\widehat{\text{sec}}$ could be induced in the alignment. Presumably, the Orbiter's angular velocity during the star measurements will be low, on the order of 0.1 deg/$\widehat{\text{sec}}$ or less. Hence, this analysis will ignore the effects of equipment timing errors.

TABLE B3

ON-ORBIT IMU ALIGNMENT TIMING ERRORS

ERROR SOURCE	MAGNITUDE	COMPENSATABLE	
GPC time tag error	< 4 ms	Partially	
IMU data input period skew	< 16.5 μs	No	
IMU/star tracker data input deviation	1782 μs	Yes	
Star tracker data input period skew	< 16.5 μs	No	
IMU 8X resolver data staleness	5.608 ms	Yes	
IMU 8X resolver data staleness uncertainty	< 100 μs	No	
IMU 8X resolver data skew	< 100 μs	No	
IMU 1X resolver data skew	< 12 ms	No*	(No error effect)
Star tracker data staleness uncertainty	10 to 52 ms	No	

* Since only the three most significant digits of these resolvers are used to determine the octant of the IMU gimbal angle, no compensation is necessary.

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APPENDIX C

ORBITER IMU ALIGNMENT PROCESS

AND

ERROR ANALYSIS

ALIGNMENT PROCESS

The purpose of the alignment is to determine the transformation T_{PC}^{M50} from the stable member (present cluster) frame to the M50 inertial reference frame. To obtain T_{PC}^{M50}, sightings are made on two different stars with a star tracker. The star tracker measurements, horizontal and vertical deflection angles relative to the star tracker boresight axis (body-fixed), are combined with the IMU gimbal angles and known geometrical orientations between the star tracker and the IMU to compute unit star vectors in the IMU stable member coordinate system. Unit star vectors are available in the M50 coordinate system via a star catalogue. Therefore, let

$\underline{r}$, $\underline{s}$, be star vectors in M50 (published data)

$\underline{m}$, $\underline{n}$, be measured star vectors (same stars as $\underline{r}$, $\underline{s}$) in IMU stable member coordinates.

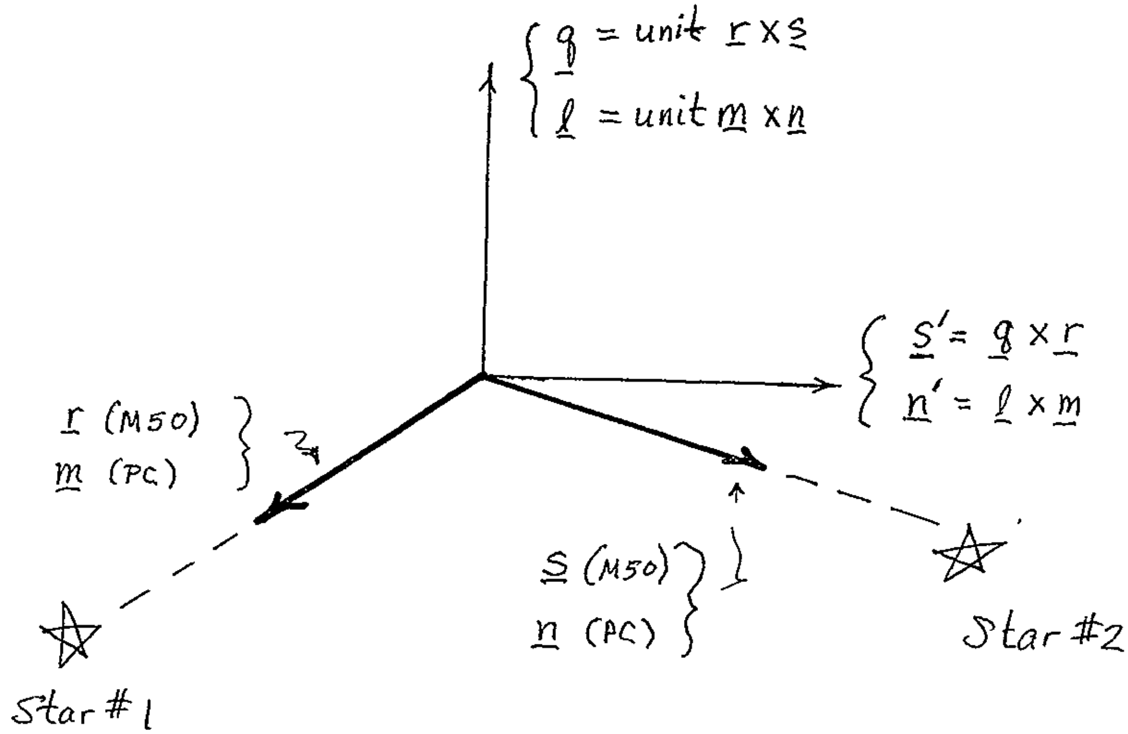

Figure 2 Star LOS's

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To obtain T_{PC}^{M50}, compute as follows:

$$\underline{q} = \text{unit } (\underline{r} \times \underline{s})$$

$$\underline{s}' = \underline{q} \times \underline{r}$$

$\underline{r}$, $\underline{s}'$, $\underline{q}$ form orthonormal triad in M50

$$\underline{\ell} = \text{unit } (\underline{m} \times \underline{n})$$

$$\underline{n}' = \underline{\ell} \times \underline{m}$$

$\underline{m}$, $\underline{n}'$, $\underline{\ell}$ form orthonormal triad in IMU stable member coordinates.

Let matrix $S = \left[\underline{r}, \underline{s}', \underline{q}\right]$, where the indicated vectors form the columns of the matrix. S is an orthogonal matrix (rotation transformation).

Let matrix $M = \left[\underline{m}, \underline{n}', \underline{\ell}\right]$. M is an orthogonal matrix. T_{PC}^{M50} obviously transforms $\underline{m}$ to $\underline{r}$, $\underline{n}'$ to $\underline{s}'$, and $\underline{\ell}$ to $\underline{q}$. Therefore:

$$S = T_{PC}^{M50} M$$

T_{PC}^{M50} is found by solving the matrix equation above.

$$T_{PC}^{M50} = S\ M^t$$

where M^t is the transpose of M. For orthogonal matrices, the transpose is also the inverse transformation.

RELATION OF THE ALIGNMENT ERRORS TO THE MEASUREMENT ERRORS

The errors affecting the alignment accuracy are assumed to be due to the measurements by the star tracker and IMU and the mechanical alignment (geometrical orientation) of the star tracker to the IMU. Star catalogue errors are assumed negligible. Consider the figure below:

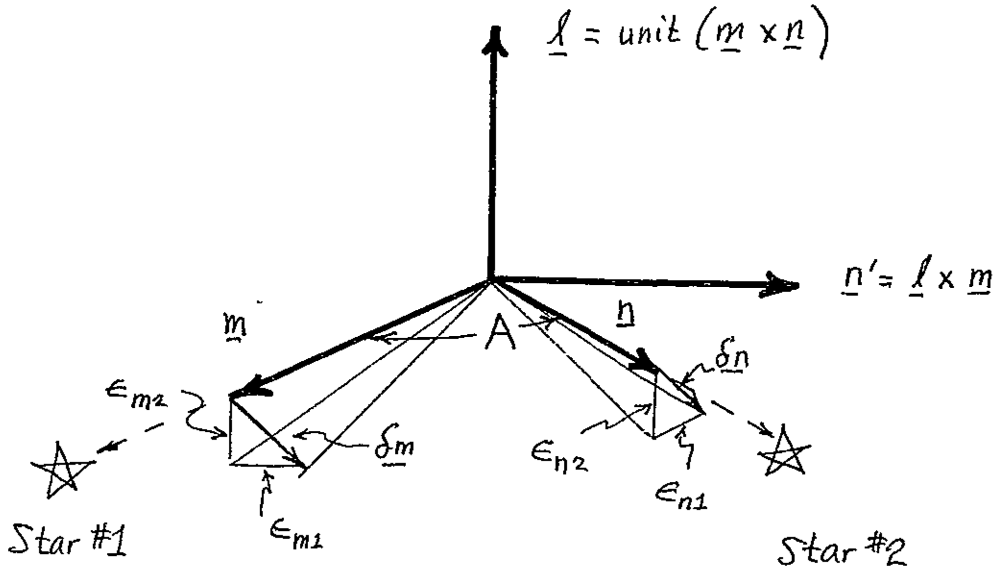

Figure 3 Measurement Errors

where:

per-axis measurement errors ϵ_{m1}, ϵ_{n1}, are in the inertially fixed plane formed by $\underline{m}$ and $\underline{n}$. ϵ_{n2}, ϵ_{n2} are perpendicular to the plane. A is the angle between $\underline{m}$ and $\underline{n}$. $\underline{\delta m}$ and $\underline{\delta n}$ represent the vector pointing errors in $\underline{m}$ and $\underline{n}$ respectively.

The star LOS vectors containing the measurement errors are $\underline{m} + \underline{\delta m}$ and $\underline{n} + \underline{\delta n}$. These unit vectors generate an orthonormal triad of vectors similar to $\underline{m}$, $\underline{n}'$, $\underline{l}$.

This second triad we denote $\underline{m} + \delta\underline{m}$, $\underline{n}' + \delta\underline{n}'$, $\underline{\ell} + \delta\underline{\ell}$. The two triads, almost coincident, are related to first order as follows:

$$\left[\begin{array}{c|c|c} \underline{m} + \delta\underline{m} & \underline{n}' + \delta\underline{n}' & \underline{\ell} + \delta\underline{\ell} \end{array}\right] = \begin{bmatrix} 1 & -\delta\phi_\ell & \delta\phi_{n'} \\ \delta\phi_\ell & 1 & -\delta\phi_m \\ -\delta\phi_{n'} & \delta\phi_m & 1 \end{bmatrix} \left[\begin{array}{c|c|c} \underline{m} & \underline{n}' & \underline{\ell} \end{array}\right]$$

$\delta\phi_m$, $\delta\phi_n$, $\delta\phi_\ell$ represent small angular rotations about $\underline{m}$, $\underline{n}'$, and $\underline{\ell}$ respectively.

The angular error vector $\delta\underline{\phi} = (\delta\phi_m, \delta\phi_{n'}, \delta\phi_\ell)$ is the Orbiter alignment error.

We see that the matrix error equation is:

$$\left[\begin{array}{c|c|c} \delta\underline{m} & \delta\underline{n}' & \delta\underline{\ell} \end{array}\right] = \begin{bmatrix} 0 & -\delta\phi_\ell & \delta\phi_{n'} \\ \delta\phi_\ell & 0 & -\delta\phi_m \\ -\delta\phi_{n'} & \delta\phi_m & 0 \end{bmatrix} \left[\begin{array}{c|c|c} \underline{m} & \underline{n}' & \underline{\ell} \end{array}\right]$$

The accuracy will be analyzed in the coordinate frame determined by the orthonormal triad $\underline{m}$, $\underline{n}'$, $\underline{\ell}$. Thus,

$\underline{m} = (1, 0, 0)$

$\underline{n} = (\text{Cos } A, \text{Sin } A, 0)$

$\underline{n}' = (0, 1, 0)$

$\underline{\ell} = (0, 0, 1)$

$\delta\underline{m} = (0, \varepsilon_{m1}, -\varepsilon_{m2})$

$\delta\underline{n} = (-\varepsilon_{n1} \text{ Sin } A, \varepsilon_{n1} \text{ Cos } A, -\varepsilon_{n2})$

We need to determine $\underline{\delta n}$ and $\underline{\delta \ell}$.

$$\underline{\ell} = \text{unit}\ (\underline{m} \times \underline{n}) = \frac{1}{\text{Sin } A} \quad (\underline{m} \times \underline{n})$$

$$\underline{\delta \ell} = - \text{Cot } A\ (\underline{m} \times \underline{n}\ ) + \text{Csc } A\ (\ \underline{\delta m} \times \underline{n} + \underline{m} \times \underline{\delta n}\ )$$

Substituting the vector components, performing the vector products, and combining terms give:

$$\underline{\delta \ell} = (\ \varepsilon_{m2},\ \varepsilon_{n2} \text{ Csc } A - \varepsilon_{m2} \text{ Cot } A,\ 0\ )$$

Next, we have

$$\underline{n}' = \underline{\ell} \times \underline{m}$$

$$\underline{\delta n}' = \underline{\delta \ell} \times \underline{m} + \underline{\ell} \times \underline{\delta m}$$

Substituting the vector components, performing the vector products and combining terms give:

$$\underline{\delta n}' = (\ -\varepsilon_{m1},\ 0,\ -\varepsilon_{n2}\ \text{ Csc } A + \varepsilon_{m2} \text{ Cot } A)$$

Substituting $\underline{\delta m}$, $\underline{\delta n}'$, $\underline{\delta \ell}$ and $\underline{m}$, $\underline{n}'$, and $\underline{\ell}$ in the matrix error equation above yields

$$\left[\begin{array}{c|c|c} 0 & -\epsilon_{m1} & \epsilon_{m2} \\ \epsilon_{m1} & 0 & \epsilon_{n2} \text{Csc} A - \epsilon_{m2} \text{Cot} A \\ -\epsilon_{m2} & -\epsilon_{n2} \text{Csc} A + \epsilon_{m2} \text{Cot} A & 0 \end{array}\right] = \begin{bmatrix} 0 & -\delta\phi_{\ell} & \delta\phi_{n'} \\ \delta\phi_{\ell} & 0 & -\delta\phi_{m} \\ -\delta\phi_{n'} & \delta\phi_{m} & 0 \end{bmatrix}$$

Thus:

$$\delta\underline{\phi} = \begin{bmatrix} \delta\phi_m \\ \delta\phi_{n'} \\ \delta\phi_\ell \end{bmatrix} = \begin{bmatrix} \varepsilon_{m2}\text{CotA} - \varepsilon_{n2}\text{CscA} \\ \varepsilon_{m2} \\ \varepsilon_{m1} \end{bmatrix}$$

It is seen that $\delta\phi_{n'}$ and $\delta\phi_\ell$ depend respectively on ε_{m2} and ε_{m1} These per-axis alignment errors exactly equal the per-axis measurement errors indicated. Small values of A degrade only the alignment error $\delta\phi_m$ about the m axis. The alignment errors $\delta\phi_{n'}$ and $\delta\phi_\ell$ about the other two reference axes n' and ℓ do not depend on A. As A approaches 90 degrees $\delta\phi_m$ approaches $-\varepsilon_{n2}$, so $\delta\phi_m$ tends also to exactly equal a per-axis measurement error when the subtended angle is large. Thus, with A near 90 degrees, the Orbiter alignment errors are essentially equivalent to the measurement errors, which, intuitively, should not come as a surprise.

Note the ε_{n1} has no effect on alignment accuracy. This is because the "first" star m was chosen as one of the orthonormal triad vectors, while the "second" star n was not, being replaced by n'.

In short, it is seen that the per-axis alignment error, for angle A in the vicinity of 90 degrees, essentially equals the per-axis measurement error. For Orbiter in-orbit alignment, the rule of thumb whereby the alignment error is computed as $\sqrt{2}$ times the measurement error is not valid.

Alignment Error Statistics

It will be seen later in this report, that the inertial per-axis measurement errors ε_{m1}, ε_{m2}, and ε_{n2} are sums of errors stemming from the star trackers, the IMU's and the geometrical misalignments of the IMU's relative to the star trackers. We will assume ε_{m1}, ε_{m2}, and ε_{n2} to be normally distributed (central limit theorem) and, based on the data at hand, independent of each other.

It will also be seen that the geometrical misalignments essentially are body fixed biases, during the period of an in-orbit alignment. Since ε_{m1}, ε_{m2}, and ε_{n2} are referenced to an inertial frame, the question of Orbiter attitude maneuver policy arises. If the Orbiter performs standardized orientations in inertial space when sighting on given stars, then the body fixed biases will map the same way relative to inertial space. Hence, ε_{m1}, ε_{m2}, ε_{n2} would have significant bias components for a given alignment procedure. We will, therefore, assume the measurement errors normally distributed about a bias, in general.

Denoting the error random component as r with standard deviation σ and the bias as b, we have:

$$\epsilon_{m1} = r_{m1} + b_{m1}$$

$$\epsilon_{m2} = r_{m2} + b_{m2}$$

$$\epsilon_{n2} = r_{n2} + b_{n2}$$

Hence, the mean square errors are:

$$\overline{\epsilon_{m1}^2} = \overline{r_{m1}^2} + \overline{b_{m1}^2} = \sigma_{m1}^2 + b_{m1}^2$$

$$\overline{\epsilon_{m2}^2} = \overline{r_{m2}^2} + \overline{b_{m2}^2} = \sigma_{m2}^2 + b_{m2}^2$$

$$\overline{\epsilon_{n2}^2} = \overline{r_{n2}^2} + \overline{b_{n2}^2} = \sigma_{n2}^2 + b_{n2}^2$$

Also:

$$\overline{\epsilon_{m1}\epsilon_{m2}} = b_{m1} b_{m2}$$

$$\overline{\epsilon_{m1}\epsilon_{n2}} = b_{m1} b_{n2}$$

$$\overline{\epsilon_{m2}\epsilon_{n2}} = b_{m2} b_{n2}$$

We now compute the mean square per-axis alignment errors. The result is.

$$\overline{\delta\phi_m^2} = \overline{\epsilon_{m2}^2} \cot^2 A - 2\overline{\epsilon_{m2}\epsilon_{n2}} \cot A \csc A + \overline{\epsilon_{n2}^2} \csc^2 A$$

$$\overline{\delta\phi_{n'}^2} = \overline{\epsilon_{m2}^2}$$

$$\overline{\delta\phi_\ell^2} = \overline{\epsilon_{n1}^2}$$

Substituting the measurement error statistics yields:

$$\overline{\delta\phi_m^2} = \sigma_{m2}^2 \cot^2 A + \sigma_{n2}^2 \csc^2 A + \left(b_{m1}\cot A - b_{n2}\csc A\right)^2$$

$$\overline{\delta\phi_{n'}^2} = \sigma_{m2}^2 + b_{m2}^2$$

$$\overline{\delta\phi_\ell^2} = \sigma_{m1}^2 + b_{m1}^2$$

Computing the error covariance matrix Cov $\left[\underline{\delta\phi}\,\underline{\delta\phi}^t\right]$ yields:

$$\mathrm{Cov}\left[\underline{\delta\phi}\,\underline{\delta\phi}^t\right] = \begin{bmatrix} \sigma_{m2}^2 \cot^2 A + \sigma_{n2}^2 \csc^2 A & \sigma_{m2}^2 \cot A & 0 \\ \sigma_{m2}^2 \cot A & \sigma_{m2}^2 & 0 \\ 0 & 0 & \sigma_{m1}^2 \end{bmatrix}$$

The variance of the total alignment error is the trace of the above matrix.

The data on hand does not suggest significantly different statistics for ε_{m1}, ε_{m2}, ε_{n2}. Thus, we will assume the following:

$$\overline{\epsilon_{m1}^2} = \overline{\epsilon_{m2}^2} = \overline{\epsilon_{n2}^2} \triangleq \overline{\epsilon^2}$$

$$\sigma_{m1} = \sigma_{m2} = \sigma_{n2} \triangleq \sigma$$

$$b_{m1}^2 = b_{m2}^2 = b_{n2}^2 \triangleq b^2$$

Hence;

$$\overline{\phi_m^2} = \sigma^2(\mathrm{Cot}^2A + \mathrm{Csc}^2A) + b^2(\pm\ \mathrm{Cot}\ A \pm \mathrm{Csc}A)^2$$

$$\overline{\phi_{n'}^2} = \sigma^2 + b^2$$

$$\overline{\phi_{\ell}^2} = \sigma^2 + b^2$$

In general, $A \neq 90$ degrees, so the alignment error about $\underline{m}$ is different from the alignment errors about $\underline{n}'$ and $\underline{\ell}$. For convenience, we now define the "averaged" per-axis alignment error to be $1/\sqrt{3}$ times the total alignment error. Thus, the root mean square (RMS) per-axis alignment error is defined as

$$\text{per-axis RMS error} = \frac{1}{\sqrt{3}}\left\{\sigma^2(1 + 2\mathrm{Csc}^2A) + b^2\left[2 + (\pm\mathrm{Cot}A\pm\mathrm{Csc}A)^2\right]\right\}^{\frac{1}{2}}$$

The standard deviation of the per-axis alignment error is:

$$\text{per-axis std. deviation} = \sigma\left(\frac{1 + 2\mathrm{Csc}^2A}{3}\right)^{\frac{1}{2}}$$

As A approaches 90 degrees, the RMS per-axis alignment error approaches $\sqrt{\sigma^2 + b^2}$, the RMS per-axis measurement error.

APPENDIX D

ORBITER SENSOR BODY FIXED

BIAS ERROR REMOVAL

BODY-FIXED BIAS REMOVAL

Because each star tracker boresight axis is fixed relative to the Orbiter and because the star tracker field of view is relatively narrow (horizontal and vertical maximum deflections are $\pm$ 5 degrees), then a simple procedure can be invoked to remove the body-fixed alignment errors, as follows.

After taking a star sighting simply rotate the Orbiter 180 degrees around the star LOS. Take a second reading (same star tracker!). For each reading, compute the star LOS unit vector in stable member coordinates. Average the two vectors by adding them together and dividing by 2. The effect of the body-fixed bias errors to first order is removed by the averaging process.

This may be clearly seen by considering the figure below, which represents the region of the celestial sphere in the star tracker field of view. Assume star 1 is in view. The unit vectors $\underline{m}$, $\underline{m}_1$, and $\underline{m}_2$ are directed from the viewer toward the field of view (into the page).

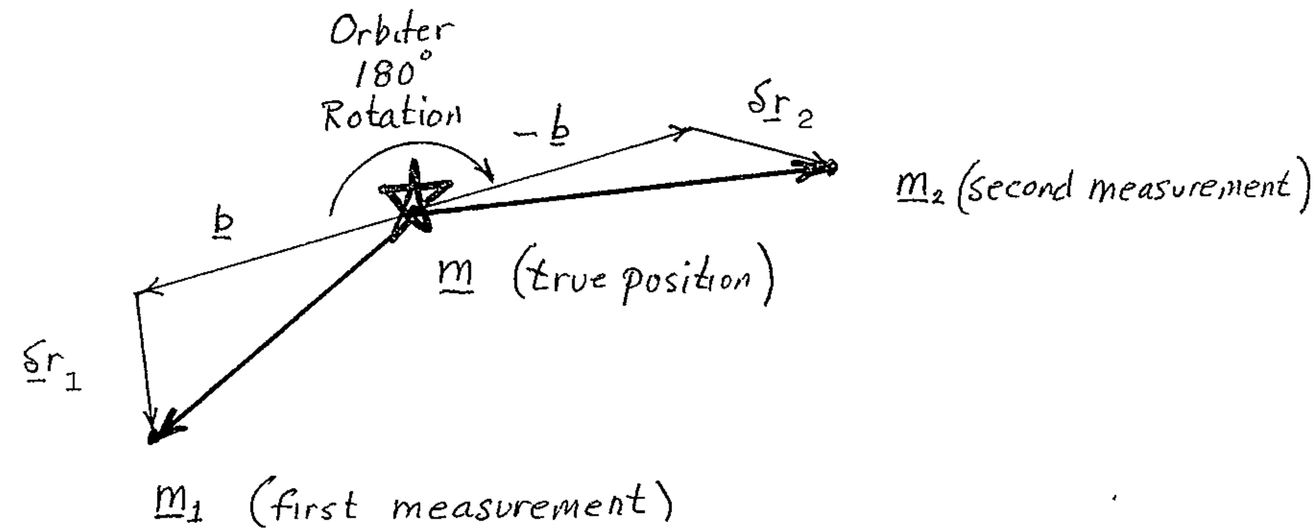

Figure D1 Bias Removal

Let $\underline{m}_1$ represent the first measurement, offset from the true value $\underline{m}$ by the body-fixed bias $\underline{b}$ and random error $\underline{\delta r}_1$. Let $\underline{m}_2$ be the second reading. Since the Orbiter has rotated 180 degrees around the LOS to Star 1, the bias effects on $\underline{m}_2$ are reversed. Thus, $\underline{m}_2$ is offset from the true value $\underline{m}$ by the reversed bias effect $-\underline{b}$ and random error $\underline{\delta r}_2$. The averaged measurement is

$$\underline{m}_{av} = \tfrac{1}{2}(\underline{m}_1 + \underline{m}_2)$$
$$\underline{m}_{av} = \tfrac{1}{2}(\underline{m} + \underline{b} + \underline{\delta r}_1 + \underline{m} - \underline{b} + \underline{\delta r}_2)$$
$$\underline{m}_{av} = \underline{m} + \tfrac{1}{2}(\underline{\delta r}_1 + \underline{\delta r}_2)$$

It is seen that the bias effect $\underline{b}$ is removed from $\underline{m}_{av}$. The per-axis standard deviation of $\underline{m}_{av}$ varies from 0 to σ (where σ is the per-axis deviation of both $\underline{\delta r}_1$ and $\underline{\delta r}_2$), depending on the correlation between $\underline{\delta r}_1$ and $\underline{\delta r}_2$.

For uncorrelated $\underline{\delta r}_1$ and $\underline{\delta r}_2$ the per-axis standard deviation of $\underline{m}_{av}$ is $\sigma/\sqrt{2}$.

The bias effect $\underline{b}$ could be estimated, if desired, by differencing measurements $\underline{m}_1$ and $\underline{m}_2$ and dividing by 2.

$$\underline{m}_1 - \underline{m}_2 = \underline{m} + \underline{b} + \underline{\delta r}_1 - \underline{m} + \underline{b} - \underline{\delta r}_2$$

$$\underline{b}_{est} = \tfrac{1}{2}(\underline{m}_1 - \underline{m}_2) = \underline{b} + \tfrac{1}{2}\,(\underline{\delta r}_1 - \underline{\delta r}_2)$$

The per-axis error in $\underline{b}_{est}$ would vary from 0 to σ, depending on the correlation. For uncorrelated $\underline{\delta r}_1$ and $\underline{\delta r}_2$, the per-axis deviation in the estimate $\underline{b}_{est}$ would be $\sigma/\sqrt{2}$.

Note that the 180 degree rotation need not be a precision maneuver. The rotation need not be strictly around the star LOS. It could be nominally around the tracker boresight axis, if desired. It is not necessary that the measurement after the rotation be taken with the star image in the same location in the tracker field of view as the first measurement. It is not important that the star be maintained in the tracker field of view during the maneuver. Also, rotation within ± 3 degrees of the nominal 180 degrees is entirely adequate, for practical purposes. Since the star trackers point essentially along the Orbiter's pitch and yaw axes respectively, the Orbiter would be rotating about principal axes of inertia during the rotations, rendering these 180 degree maneuvers relatively easy to perform.

It should also be noted that the procedure above does not necessarily remove angular bias errors about an axis coinciding with the star tracker boresight axis. However, angular error effects about the boresight axis are reduced by at least an order of magnitude since the star tracker measurements (horizontal, vertical deflections) are restricted to ± 5 degrees of the boresight axis. Hence, to first order, the angular bias error about the boresight axis has little significant impact on the Orbiter alignment accuracy.

APPENDIX E

ORBITER IMU IN-ORBIT

ALIGNMENT ACCURACY

ORBITER ALIGNMENT ACCURACY

Two sets of results are presented.

The first set of results, presented in Figure 5, pertains to the general case, where all error sources are treated as random with zero means.

The second set of results, presented in Figure 6, pertains to a single in-orbit alignment process with the geometrical misalignment biases removed by averaging two measurements per star, measurements taken before and after rotating the Orbiter 180 degrees around each star LOS. The remaining random errors are considered uncorrelated; hence the averaged measurement deviation is $\sigma/\sqrt{2}$ (68 $\widehat{sec}/\sqrt{2}$ = 48 $\widehat{sec}$).

For both sets, the effect of Orbiter IMU gyro drift (.035 deg/hr, 1σ) is added (in quadrature) to show how the in-orbit alignment degrades with time after it is initially achieved. Figures 5 and 6 are contours of error values, as functions of subtended star angle A and elapsed time after alignment takes place. Initial alignment error, as a function of star angle A, is indicated along the vertical axes, where the contours intercept the vertical axes at elapsed time equal zero. The allowable elapsed time, during which the Orbiter's per-axis attitude error is less than 6 $\widehat{min}$ ($3\ \sigma$), is determined by moving horizontally to the right (from the initial alignment error at time zero) until the 6 $\widehat{min}$ contour is intercepted.

Figure 5 shows that with the misalignment biases present in the star measurements, a maximum time of about 18 minutes is available (A = 90 deg) before the Orbiter attitude error exceeds 6 $\widehat{min}$. The star angle A must be within 90 $\pm$ 20 degrees in order that the initial alignment error not exceed 6 $\widehat{min}$.

Figure 6 shows that with the misalignment biases removed, a maximum time of over 52 minutes is available (A ≈ 90 deg) before the Orbiter attitude error exceeds 6 min. The range of allowable star angles is seen to be quite broad (90 $\pm$ 70 Deg.).

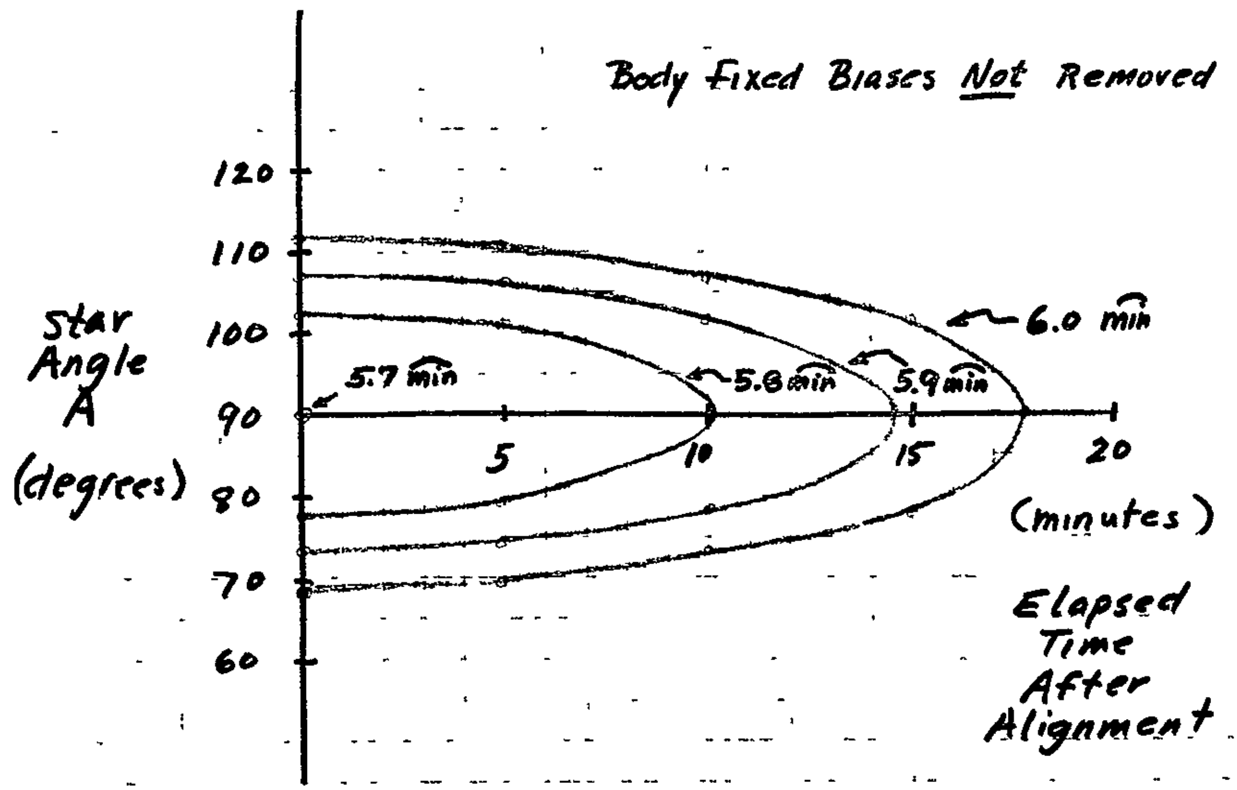

FIGURE 5

ORBITER ATTITUDE ERROR (3σ) CURVES for ALIGNMENT ERROR = 5.7 min (3σ)

Contours are function of:

- Star Angle A
- Elapsed Time after Aligning

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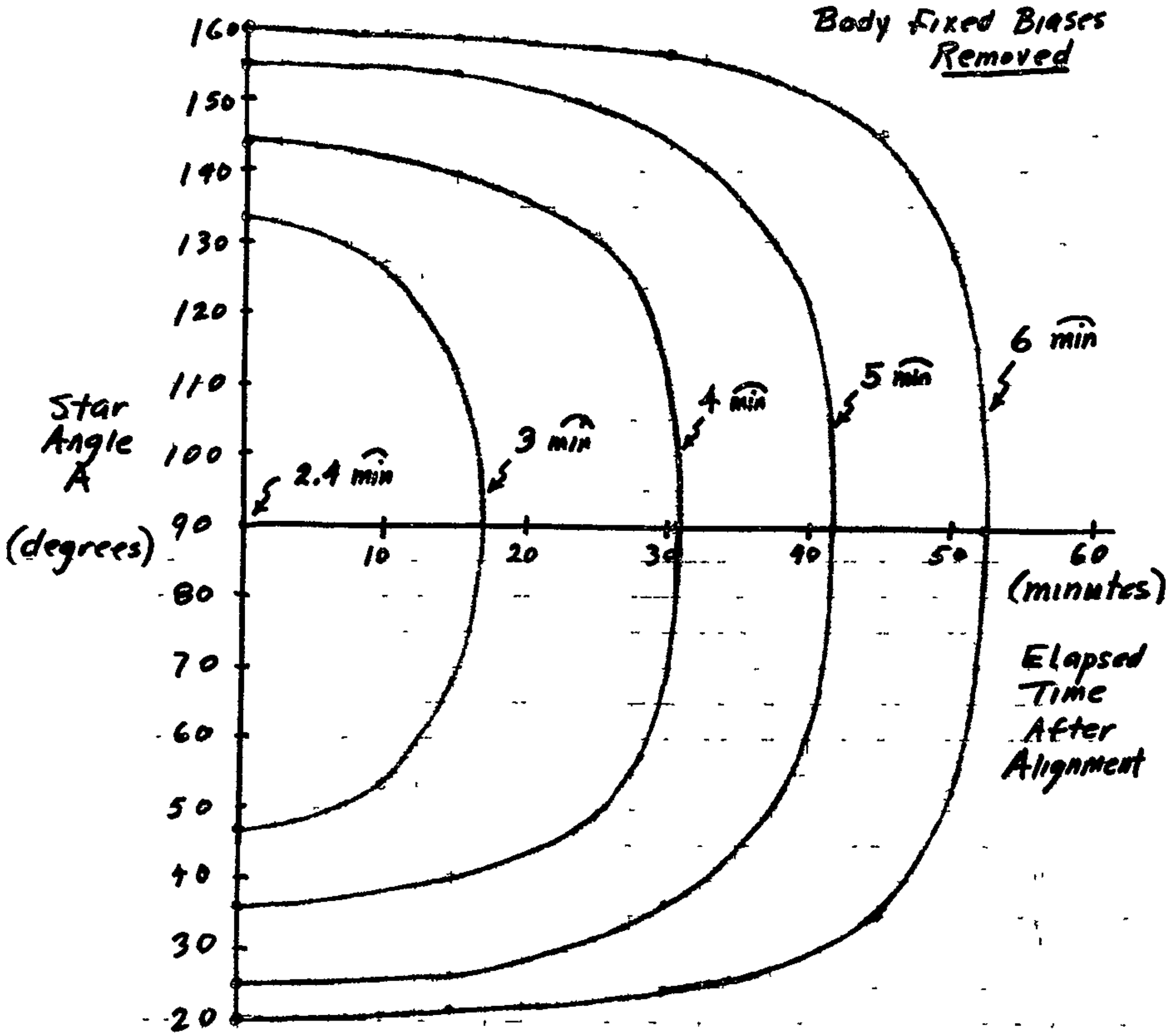

FIGURE 6

ORBITOR ATTITUDE ERROR (3σ) CURVES for ALIGNMENT ERROR = 2.4 min (3σ)

Contours are function of:

- Star Angle A
- Elapsed Time after Aligning

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