NASA TECHNICAL MEMORANDUM NASA TM-75256

PARTIAL DISCHARGES IN THE TRANSFORMERS DUE TO INDUCED VOLTAGES

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Translation of "Castecne vyjoje v transformatoru pri indukovanem napeti" supplement, Elektrotechniky Obzor, vol. 60, no. 3, March, 1971, pp.117-119.

(NASA-TM-75256) PARTIAL DISCHARGES IN THE TRANSFORMERS DUE TO INDUCED VOLTAGES (National Aeronautics and Space Administration) 8 p HC A02/MF A01 CSCL 09C N78-18310 Unclas G3/33 05391

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 FEBRUARY 1978

STANDARD TITLE PAGE

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-75256		

4. Title and Subtitle	5. Report Date
PARTIAL DISCHARGES IN THE TRANSFORMERS DUE TO INDUCED VOLTAGE (SUPPLEMENT)	February 1978
	6. Performing Organization Code
7. Author(s) A. Veverka	8. Performing Organization Report No.
	10. Work Unit No.
9. Performing Organization Name and Address Leo Kanner Associates Redwood City, California 94063	11. Contract or Grant No. NASw-2790
	13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address National Aeronautics And Space Administration, Washington, D.C. 20546	14. Sponsoring Agency Code

15. Supplementary Notes

Translation of "Castecne vyboje v transformatoru pri indukovanem napeti" supplement, Elektrotechniky Obzor, vol. 60, no. 3, March, 1971, pp. 117-119.

16. Abstract

The supplement furnishes a thoretical proof containing the experimentally determined advantage for the connection of capacitance C_0 between the insulated frame and ground according to the arrangement presented in Fig. 4 in the article.

17. Key Words (Selected by Author(s))	18. Distribution Statement
	Unclassified-Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified		

PARTIAL DISCHARGES IN THE TRANSORMERS DUE TO INDUCED VOLTAGE
(SUPPLEMENT)

A. Veverka

Supplement /117*

For a rough comparison of the operation of a transformer winding connection during an investigation of the transient phenomena caused by a discharge, we consider the relations at the unit voltage jump introduced between the input to the winding and frame as shown in Fig. 9. For simplicity, we investigate Wagner's substitution diagram for a coil winding. The frame is insulated and connected across capacitance C_0 to the ground. C,K,L, are the capacitance to the ground; the capacitance between the windings and the winding inductance, all referred to unit axial length of the winding. Here, the following are valid (u is the voltage between the point of the winding and the ground, u_0 is the voltage between the insulated frame and ground).

$$-\frac{\partial i_L}{\partial x} - \frac{\partial i_K}{\partial x} = C\,\frac{\partial (u-u_0)}{\partial t} \qquad (5)$$

$$i_K = -K\,\frac{\partial^2 u}{\partial x\,\partial t} \qquad (6)$$

$$-\frac{\partial u}{\partial x} = L\,\frac{\partial i_L}{\partial t} \qquad (7)$$

$$C_0\,\frac{\partial u_0}{\partial t} = \int_0^l C\,\mathrm{d}x\,\frac{\partial (u-u_0)}{\partial t} \qquad (8)$$

In the transformed range (Carson-Wagner) we obtain from equations (5), (6) and (7)

$$\frac{\mathrm{d}^2 U}{\mathrm{d}x^2} = \frac{p^2 LC}{1+p^2 LK}\,(U-U_0) \qquad (9)$$

The general solution of this equation is:

$$U = A\,e^{p\frac{\sqrt{LC}}{\sqrt{1+p^2LK}}x} + B\,e^{-p\frac{\sqrt{LC}}{\sqrt{1+p^2LK}}x} + U_0 \qquad (10)$$

* Numbers in the margin indicate pagination in the foreign text.

The integration constants are determined from the boundary conditions

$$x = 0 \; . \; . \; . \; U - U_0 = 1$$
$$x = l \; . \; . \; . \; . \; . \; . \; . \; U = 0$$

Having determined the integration constants and substituted them in equation (10), we obtain, after appropriate transformations,

$$U = \frac{-U_0 \sinh p \dfrac{\sqrt{LC}}{\sqrt{1+p^2LK}}\, x + \sinh p \dfrac{\sqrt{LC}}{\sqrt{1+p^2LK}}\,(l-x)}{\sinh p \dfrac{\sqrt{LC}}{\sqrt{1+p^2LK}}\, l} + U_0 \qquad (11)$$

Now we will also apply the last equation (8). In the transformed range, we obtain for the voltage at C_0:

$$U_0 = \int_0^l \frac{C}{C_0} \cdot \frac{-U_0 \sinh p \dfrac{\sqrt{LC}}{\sqrt{1+p^2LK}}\, x + \sinh p \dfrac{\sqrt{LC}}{\sqrt{1+p^2LK}}\,(l-x)}{\sinh p \dfrac{\sqrt{LC}}{\sqrt{1+p^2LK}}\, l}\, dx \qquad (12)$$

Evaluating the integral, we obtain after appropriate transformations:

$$U_0 = \frac{\dfrac{Cl}{C_0}}{\dfrac{p\sqrt{LC}\,l}{\sqrt{1+p^2LK}} \coth \dfrac{p\sqrt{LC}\,l}{2\sqrt{1+p^2LK}} + \dfrac{Cl}{C_0}} \qquad (13)$$

for $p \to \infty$

$$U_{0,\infty} = \frac{\dfrac{Cl}{C_0}}{2\left(\sqrt{\dfrac{C}{K}} \cdot \dfrac{l}{2}\right) \coth \sqrt{\dfrac{C}{K}} \cdot \dfrac{l}{2} + \dfrac{Cl}{C_0}} \qquad (14)$$

and for p = 0 /118

$$U_{0,0} = \frac{\frac{Cl}{C_0}}{2 + \frac{Cl}{C_0}} \tag{15}$$

Let us calculate

$$\frac{U_{0,0} - U_{0,\infty}}{U_{0,0}}$$

$$\frac{U_{0,0} - U_{0,\infty}}{U_{0,0}} = \frac{2\left[\left(\sqrt{\frac{C}{K}} \cdot \frac{l}{2}\right)\coth\sqrt{\frac{C}{K}} \cdot \frac{l}{2} - 1\right]}{2\left(\sqrt{\frac{C}{K}} \cdot \frac{l}{2}\right)\coth\sqrt{\frac{C}{K}} \cdot \frac{l}{2} + \frac{Cl}{C_0}} \tag{16}$$

and compare the value with the analogous expression for a connection according to Fig. 10.

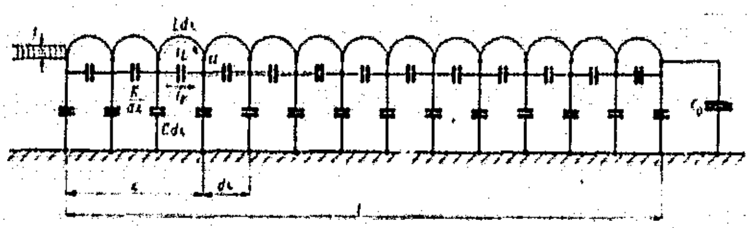

Fig. 10

Here the following equations hold:

$$-\frac{\partial i_L}{\partial x} - \frac{\partial i_K}{\partial x} = C\,\frac{\partial u}{\partial t} \tag{17}$$

$$i_K = -K\,\frac{\partial^2 u}{\partial x\,\partial t} \tag{18}$$

$$-\frac{\partial u}{\partial x} = L\,\frac{\partial i_L}{\partial t} \tag{19}$$

In the transformed (Carson-Wagner) range, we obtain the equations

$$-\frac{dI_L}{dx} - \frac{dI_K}{dx} = pCU \tag{20}$$

$$I_K = -pK\,\frac{dU}{dx} \tag{21}$$

$$-\frac{dU}{dx} = pLI_L \qquad (22)$$

and from them:

$$\frac{d^2U}{dx^2} = \frac{p^2LC}{1 + p^2LK}\,U \qquad (23)$$

The general solution of the last equation is

$$U = A\,e^{p\frac{\sqrt{LC}}{\sqrt{1+p^2LK}}x} + B\,e^{-p\frac{\sqrt{LC}}{\sqrt{1+p^2LK}}x} \qquad (24)$$

The integration constants are determined from the boundary conditions

$$x = 0\,.\,.\,.\,.\,.\,.\,U = 1$$

$$x = l\,.\,.\,.\,.\,.\,.\,I_K + I_L = -\left(pK + \frac{1}{pL}\right)\left(\frac{dU}{dx}\right)_{x=l} = pC_0U_{x=l}$$

After the values of the integration constants have been determined and substituted in equation (24), we obtain after appropriate transformations

$$U = \frac{\sqrt{1+p^2LK}\,.\cosh p\frac{\sqrt{LC}}{\sqrt{1+p^2LK}}(l-x) + pC_0\sqrt{\frac{L}{C}}\sinh p\frac{\sqrt{LC}}{\sqrt{1+p^2LK}}(l-x)}{\sqrt{1+p^2LK}\,.\cosh p\frac{\sqrt{LC}}{\sqrt{1+p^2LK}}l + pC_0\sqrt{\frac{L}{C}}\sinh p\frac{\sqrt{LC}}{\sqrt{1+p^2LK}}l} \qquad (25)$$

At the end of the winding, i.e. for x = 1:

$$U_l = \frac{1}{\cosh p\frac{\sqrt{LC}}{\sqrt{1+p^2LK}}l + \frac{pC_0\sqrt{L/C}}{\sqrt{1+p^2LK}}\sinh p\frac{\sqrt{LC}}{\sqrt{1+p^2LK}}l} \qquad (26)$$

For $p \to \infty$

$$U_{l,\infty} = \frac{1}{\cosh\sqrt{\frac{C}{K}}\,l + \frac{C_0}{\sqrt{CK}}\sinh\sqrt{\frac{C}{K}}\,l} \qquad (27)$$

and for $p = 0$ $\quad U_{1\infty} = 1 \qquad (28)$

Let us calculate $\frac{U_{l,0} - U_{l,\infty}}{U_{l,0}}$;

$$\frac{U_{l,0} - U_{l,\infty}}{U_{l,0}} = \frac{\cosh\sqrt{\frac{C}{K}}\,l + \frac{C_0}{\sqrt{CK}}\sinh\sqrt{\frac{C}{K}}\,l - 1}{\cosh\sqrt{\frac{C}{K}}\,l + \frac{C_0}{\sqrt{CK}}\sinh\sqrt{\frac{C}{K}}\,l} \qquad (29)$$

and compute the ratio of expression (16) to (29). After /119 appropriate transformations we obtain

$$\xi = \frac{\frac{U_{0,0} - U_{0,\infty}}{U_{0,0}}}{\frac{U_{l,0} - U_{l,\infty}}{U_{l,0}}} = \frac{\frac{C_0}{Cl}\left(\sqrt{\frac{C}{K}}\cdot\frac{l}{2}\cosh\sqrt{\frac{C}{K}}\cdot\frac{l}{2} - \sinh\sqrt{\frac{C}{K}}\cdot\frac{l}{2}\right)\left(\cosh\sqrt{\frac{C}{K}}\,l + \frac{C_0}{Cl}\sqrt{\frac{C}{K}}\,l\sinh\sqrt{\frac{C}{K}}\,l\right)}{\left(\sinh\sqrt{\frac{C}{K}}\cdot\frac{l}{2} + \frac{C_0}{Cl}\sqrt{\frac{C}{K}}\,l\cosh\sqrt{\frac{C}{K}}\cdot\frac{l}{2}\right)^2\cdot\sinh\sqrt{\frac{C}{K}}\cdot\frac{l}{2}} \qquad (30)$$

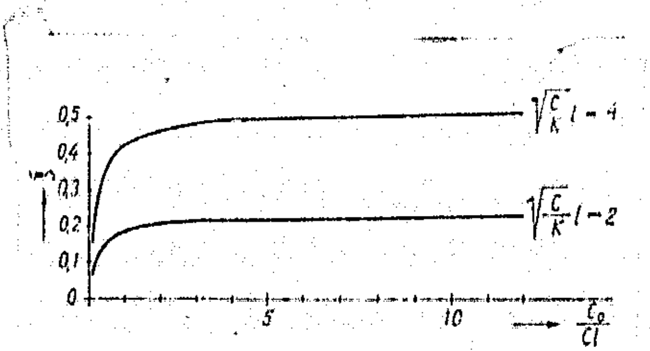

Fig. 11

Fig. 11 shows the plot of the ratio ξ versus C_0/Cl for two values of $\sqrt{\frac{C}{K}}\,l$, namely $\sqrt{\frac{C}{K}}\,l = 2$ and 4.

If it is evident that $\xi < 1$ and that ξ decreases with decreasing value of $\sqrt{\frac{C}{K}}\,l$, which is a well known characteristic for the initial voltage distribution in the transformer windings at a rectangular pulse. For a nonoscillating winding $\left(\sqrt{\frac{C}{K}}\,l \to 0\right) \ldots \xi \to 0.$

REFERENCES

1. Veverka, A., A. Hon and A. Bartok, "Measurement of partial discharges in transformers at an induced voltage," Elektrotechniky Obzor 59, 10, 515-517 (1970).