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THE ROLE OF TIME-HISTORY EFFECTS IN THE FORMULATION OF THE AERODYNAMICS OF AIRCRAFT DYNAMICS

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SUMMARY

The scope of any aerodynamic formulation proposing to embrace a range of possible maneuvers is shown to be determined principally by the extent to which the aerodynamic indicial response is allowed to depend on the past motion. Starting from the linearized formulation, in which the indicial response is independent of the past motion, two successively more comprehensive statements about the dependence on the past motion are assigned to the indicial response (1) dependence only on the recent past and (2) dependence additionally on a characteristic feature of the distant past. The first enables the rational introduction of nonlinear effects and accommodates a description of the rate-dependent aerodynamic phenomena characteristic of airfoils in low-speed dynamic stall; the second permits a description of the double-valued aerodynamic behavior characteristic of certain kinds of aircraft stall. An aerodynamic formulation based on the second statement, automatically embracing the first, may be sufficiently comprehensive to include a large part of the aircraft's possible maneuvers. The results suggest a favorable conclusion regarding the role of dynamic stability experiments in flight dynamics studies.

LIST OF SYMBOLS

C_m	pitching-moment coefficient, $\frac{\text{pitching moment}}{qSl}$
$G[\sigma(\xi)]$	functional notation: value at $\xi = t$ of a time-dependent function which depends on all values taken by the argument function $\sigma(\xi)$ over the time interval $0 \leq \xi \leq t$
I	moment of inertia about the pitching axis
l	reference length
q	dynamic pressure, $\frac{1}{2}\rho V^2$
S	reference area
t	time
V	magnitude of flight velocity vector
σ	angle of attack, Fig. 1
ρ	atmospheric density
ω	frequency of harmonic oscillatory motion

1. INTRODUCTION

One of the difficult problems in aircraft flight dynamics is that of formulating an aerodynamic force and moment system with sufficient scope to cover the wide range of maneuvers typical of modern aircraft (Refs. 1,2). What is the nature of the problem?

Consider the questions that arise in the prediction of a maneuver from a known initial state. Let an essentially rigid aircraft with known inertial properties undergo an arbitrary motion. At a certain time t_0, allow a measurement of the aircraft's state (i.e., its linear and angular velocity components) and its aerodynamic response (i.e., the aerodynamic force and moment). Given this information at t_0, what is needed to predict the aircraft's motion over the succeeding increment of time? The ability to carry the motion forward over the first increment of time implies, of course, the ability to predict the entire subsequent motion. What is needed principally is a form for the incremental changes in the aerodynamic force and moment, that is, the indicial response, over the increment of time. Assigning an adequate form constitutes the problem of formulation. The difficulty of the problem arises in assigning a form that applies not only to the motion under study, but to all of the other motions of which the aircraft is capable, and which might have occurred prior to t_0. This way of describing the difficulty allows one to appreciate the great virtue of a linearized version of the aerodynamic indicial response. Invoking linearity supposes that the aerodynamic indicial response is independent of anything that happened prior to the origin of the response. Thus, the calculation can be carried forward without any acknowledgment whatever of the motion prior to t_0. Although there are flow regimes where use of the linearized formulation can be justified (e.g., attached flows with small perturbations), these regimes do not embrace the whole range of flows that a modern aircraft may experience. A formulation applicable to the remaining regimes must be freed of the limitation imposed by linearization. This means, of course, that the aerodynamic indicial response must be allowed to depend on the past motion.

In a series of papers (cf. Ref. 3 for a connected account), the authors have tried to show how concepts from functional analysis could be used to construct a mathematical framework allowing a general

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**Research Scientist, Computational Fluid Dynamics Branch dependence of the aerodynamic indicial response on the past motion. Having a rigorous framework has enabled the introduction of rational approximations which, in effect, limit the dependence on the past motion to some definite property, applicable to an appropriate class of flows. One can argue in favor of limited statements about the dependence on the past motion as follows: Since the linearized formulation has found application to a certain class of flows, a formulation based on a limited statement, which includes the linearized formulation as a special case, must find application to a wider class of flows. A sequence of successively more comprehensive statements, each embracing all of the preceding ones, must eventually reach a stage where the resulting formulation is applicable to a sufficiently wide class of flows to accommodate a description of all of the motions of interest. It remains to discover whether this stage can be reached well short of having to account for the whole past motion in detail for any of the motions of interest.

Thus, the role assigned to time-history effects, that is, the statement about the dependence on the past motion, constitutes a determining criterion by which the merits and shortcomings of any aerodynamic formulation may be judged. The purpose of this report is to investigate how far the first few statements go toward fulfilling the goal of a sufficiently comprehensive statement. Starting from the linearized formulation, in which the indicial response is independent of the past motion, two successively more comprehensive statements about the dependence on the past motion are assigned to the indicial response (1) dependence only on the recent past and (2) dependence additionally on a characteristic feature of the distant past. The successive statements allow the effects of successively larger bodies of aerodynamic phenomena to be acknowledged within the scopes of the resulting formulations. The first enables the rational introduction of nonlinear effects and accommodates a description of the rate-dependent aerodynamic phenomena characteristic of airfoils in low-speed dynamic stall; the second permits a description of the double-valued aerodynamic behavior characteristic of certain kinds of aircraft stall. It is suggested that an aerodynamic formulation based on the second statement, automatically including the first, may be of sufficient scope to embrace a large part of the aircraft's possible maneuvers. Implications of the results with regard to dynamic stability experiments are discussed.

2. DEFINITION OF MANEUVER

To focus directly on the question of time-history effects, it is advisable to avoid the complications introduced by coordinate systems and motions with multiple degrees of freedom. In all of the study to follow, the aircraft's maneuver is restricted to be planar with only a single degree of freedom. Extension to more general motions will be straightforward, paralleling that described in Ref. 3.

Let the aircraft be in level steady flight prior to time zero. At time zero let it begin an arbitrary pitching maneuver during which the center of gravity continues to follow a rectilinear path at constant velocity V. Hence flight-path properties such as dynamic pressure, Mach number, and Reynolds number remain constant throughout the maneuver. The pitching maneuver is defined by the angle of attack σ (Fig. 1), the angle between the aircraft's longitudinal axis and the velocity vector. The motion, of course, may be specified to reproduce that of a wind-tunnel model in an oscillations-in-pitch experiment.

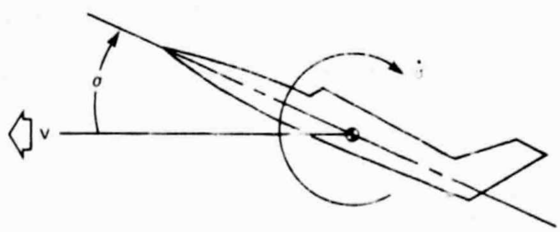

Fig. 1. Single-degree-of-freedom pitching maneuver.

Thus, focusing on this motion will facilitate a later discussion of the implications of the results with regard to wind-tunnel experiments.

3. FORMATION OF INDICIAL PITCHING-MOMENT RESPONSE AND INTEGRAL FORMS

Since it will be necessary in later sections to consider the influence of random fluctuations, the formation of the indicial pitching-moment **response** will be described in a way that acknowledges their presence. Two motions have to be considered (cf. Fig. 2). First, beginning at $\xi = 0$, the aircraft is made to undergo the motion under study $\sigma(\xi)$. At a certain time τ the motion is constrained so that the value of σ at time τ, that is, $\sigma(\tau)$, remains constant thereafter. The pitching moment corresponding to this motion is measured at a time t. Now if $\sigma(\tau)$

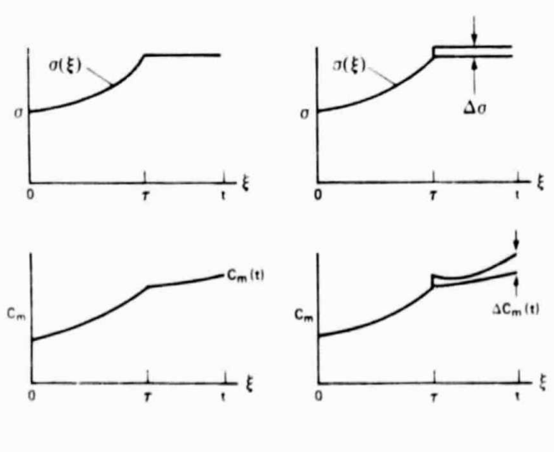

Fig. 2. Formation of indicial response.

is sufficiently large so that, for example, flow separation occurs in the course of a maneuver, then as a result of the ensuing fluctuations in the flow, any single measurement of the pitching moment at time t will include a random component. This circumstance calls for repeating the maneuver and the corresponding measurement at time t many times and taking the arithmetic mean of the measurements. If the fluctuating part of the response is truly random, its contribution to the measurement at time t should cancel in the mean, and the resulting mean value should be representative of the deterministic part of the response. It will be assumed that this is true for any time t, and that as a result, a deterministic part of the response will exist that is continuous for all ξ in the interval $0 < \xi < t$. Second, the aircraft is made to execute precisely the same motion, beginning at $\xi = 0$ and constrained in the same way at $\xi = \tau$, except that at the latter time, σ is given an incremental step $\Delta\sigma$ over its value at $\xi = \tau$. Hence, for all time subsequent to τ, σ is equal to $\sigma(\tau) + \Delta\sigma$. The pitching moment corresponding to this motion is again measured at time t. Just as before, the second maneuver and the corresponding measurement at time t must be repeated many times, and the arithmetic mean of the measurements taken to be the deterministic part of the response. The difference between mean values for the two measurements, $\Delta C_m(t)$, is divided by the incremental step $\Delta\sigma$. The limit of this ratio (if it exists) as the magnitude of the step approaches zero is called the indicial pitching-moment response at time t per unit step change in σ at time τ. Since the two maneuvers prior to $\xi = \tau$ are identical (in the mean), the ratio must be identically zero for $0 < \xi < \tau$. At $\xi = \tau$ a discontinuity in the ratio is permissible, reflecting the discontinuous change in σ. For all $\xi > \tau$ the ratio is assumed to be continuous. With the understanding that the pitching-moment response to each maneuver and at each time t is the result of an ensemble average of measurements, the indicial pitching-moment response is defined as:

$$\lim_{\Delta\sigma \to 0} \frac{\Delta C_m(t)}{\Delta\sigma} = C_{m_\sigma}[\sigma(\xi);t,\tau] \tag{1}$$

As the functional notation indicates, the indicial response is allowed to depend in an unspecified way on the entire history of the motion $\sigma(\xi)$.

When the assumptions leading to the definition of a deterministic indicial response can be said to hold within each increment of the stepwise representation of an arbitrary motion $\sigma(t)$, the pitching-moment response $C_m(t)$ to the motion $\sigma(t)$ follows from a summation of incremental responses over the time interval 0 to t:

$$C_m(t) = C_m(0) + \int_0^t C_{m_\sigma}[\sigma(\xi);t,\tau] \frac{d\sigma}{d\tau} d\tau \tag{2}$$

This is the general integral form for $C_m(t)$ corresponding to an arbitrary motion $\sigma(t)$. The form is essentially exact, but its further use without approximation is exceedingly difficult. The nature of the difficulty becomes clear if one writes the equation of motion for the single-degree-of-freedom pitching motion $\sigma(t)$, and asks for a solution of $\sigma(t)$ for specified initial conditions. The equation of motion is:

$$\ddot{\sigma}(t) = \left(\frac{qS\ell}{I}\right) \left\{C_m(0) + \int_0^t C_{m_\sigma}[\sigma(\xi);t,\tau] \frac{d\sigma}{d\tau} d\tau\right\} \tag{3}$$

Since the indicial response within the integral is a functional, dependent in general on the whole past motion $\sigma(\xi)$, it is unknown when σ is unknown. Thus, both the indicial response and the motion must be found simultaneously, an awesome prospect. Cases can be envisioned (e.g., massively ablating reentry vehicles) where the mutual dependence between the motion and the indicial response cannot be uncoupled. However, for rigid aircraft of fixed shape, the past success of the linearized formulation lends credence to the belief that the interdependence may be at least partially uncoupled. Now if the indicial response were somehow known, Eq. (3) would become an integro-differential equation of the Volterra type for which solutions can be found by known, albeit numerical, techniques. The indicial response could be considered known, at least in principle, if its dependence on the past motion were specified in a way that allowed it to be an identifiable member of a collection of indicial responses, all of which had been obtained beforehand from, for example, a suitable series of experiments. Thus, Eq. (3) can be made tractable by assigning to the indicial response appropriate statements about how it depends on the past motion which allow it to be determined in advance. Let it be noted, however, that since every statement assigned to the indicial response will multiply the number of responses in the collection required to be known in advance, it becomes imperative to make the least number of statements possible. In succeeding sections, two such statements will be introduced, which it is hoped, may suffice to cover the cases of interest.

Finally, it is recognized that counting on the availability of a collection of indicial responses may be somewhat unrealistic in view of the difficulty of experimentally determining an indicial response. In a later section, however, it will be shown how the results from suitably designed oscillations-in-pitch experiments, which, it may be presumed with greater reason, are technically feasible, may be used in place of the integral in Eq. (3). Thus, for the availability in principle of a collection of indicial responses may be read the availability of an equivalent collection of results from oscillations-in-pitch experiments.

4. DEPENDENCE OF THE INDICIAL RESPONSE ON THE RECENT PAST

First, Eq. (1) for the indicial pitching-moment response will be put in an equivalent form that will suggest a first statement about its dependence on the past motion. If $\sigma(\xi)$ can be considered to be analytic in a neighborhood of $\xi = \tau$ (corresponding to the most recent past for an indicial response with origin at $\xi = \tau$), in principle, its history can be reconstructed from a knowledge of all of the coefficients of its Taylor series expansion about $\xi = \tau$. Then, since $\sigma(\xi)$ is equally represented by the coefficients of its expansion, the functional, with its dependence on $\sigma(\xi)$, can be replaced without approximation by a function with a dependence on all of the coefficients of the expansion of $\sigma(\xi)$ about $\xi = \tau$; that is,

$$C_{m_\sigma}[\sigma(\xi);t,\tau] = C_{m_\sigma}(t - \tau;\sigma(\tau),\dot{\sigma}(\tau),\ddot{\sigma}(\tau), . . .) \tag{4}$$

The additional replacement of a dependence on elapsed time $t - \tau$ rather than on t and τ separately is justified within the specification already invoked of constant flight-path properties.

Now it is argued that a class of flows exists for which disturbances originating at times far removed from the vicinity of $\xi = \tau$ will have died out before they are able to influence events in the vicinity of $\xi = \tau$. In such cases, it can be assumed that the indicial response will have "forgotten" long-past events, and so will depend only on events occurring in the most recent past. Therefore, to the extent that the indicial response can be influenced by the past motion, the form of the past motion just prior to the origin of the step might just as well have existed for all earlier times. Accordingly, only the first few coefficients of the expansion of $\sigma(\xi)$ need be retained to characterize correctly the most recent past, which is all the indicial response is assumed to remember. Retaining the first two coefficients of $\sigma(\xi)$, for example, implies matching the true past history of σ in magnitude and slope at the origin of the step, thereby approximating $\sigma(\xi)$ by a linear function of time $\sigma(\xi) = \sigma(\tau) - \dot{\sigma}(\tau)(\tau - \xi)$. With an approximation of this order in force in Eq. (1), the integral form replacing Eq. (2) and the right-hand side of Eq. (3) becomes

$$C_m(t) = C_m(0) + \int_0^t C_{m_\sigma}\big(t - \tau;\sigma(\tau),\dot{\sigma}(\tau)\big)\frac{d\sigma}{d\tau}\,d\tau \tag{5}$$

The steady-state value of the indicial response can be put in evidence with the additional (consistent) assumption that events in the recent past, that is for $\xi < \tau$, will again be far removed and so, forgotten, so far as the indicial response is concerned when $t - \tau \to \infty$. This means that the steady-state value of the indicial response will depend only on local conditions, that is, on the constant value of $\sigma(\tau)$. The latter behavior is put in evidence by the substitution

$$C_{m_\sigma}\big(t - \tau;\sigma(\tau),\dot{\sigma}(\tau)\big) = C_{m_\sigma}\big(\infty;\sigma(\tau)\big) - F\big(t - \tau;\sigma(\tau),\dot{\sigma}(\tau)\big) \tag{6}$$

where $C_{m_\sigma}(\infty;\sigma(\tau))$ is the steady-state value of the indicial response. Notice that it must be a single-valued function of $\sigma(\tau)$. The function $F(t - \tau;\sigma(\tau),\dot{\sigma}(\tau))$ is called the deficiency function; it approaches zero as $t - \tau \to \infty$ and, in practice, will be essentially zero for all elapsed time $t - \tau$ larger than a relatively small value t_a. When Eq. (6) is substituted in Eq. (5), the steady-state term multiplied by $(d\sigma/d\tau)d\tau$ forms a perfect differential which can be integrated. The resulting formulation for $C_m(t)$ becomes

$$C_m(t) = C_m\big(\infty;\sigma(t)\big) - \int_0^t F\big(t - \tau;\sigma(\tau),\dot{\sigma}(\tau)\big)\,\frac{d\sigma}{d\tau}\,d\tau \tag{7}$$

Here, $C_m(\infty;\sigma(t))$ is the pitching-moment coefficient that would be measured in a steady flow with σ fixed at the instantaneous value $\sigma(t)$. Again, note that $C_m(\infty;\sigma(t))$ must be a single-valued function of σ according to this formulation.

Equation (7) actually includes three increasingly comprehensive formulations, each of which may be applicable in appropriate circumstances. The simplest, of course, is the linear formulation, for which the indicial response is said to be independent of both $\sigma(\tau)$ and $\dot{\sigma}(\tau)$. The resulting simplification is reflected in the equation of motion, Eq. (3), which becomes

$$\ddot{\sigma}(t) = \left(\frac{qSl}{I}\right)\left\{\sigma(t)C_{m_\sigma}(\infty) - \int_0^t F(t - \tau)\,\frac{d\sigma}{d\tau}\,d\tau\right\} \tag{8}$$

The equation is linear and the integral term is of the convolution type, which enables an immediate solution for σ by the aid of Laplace transforms. A considerable additional virtue is that the collection of indicial responses required to be known in advance consists of one member.

The second formulation, applicable in particular to slowly varying motions, is obtained by omitting the dependence on $\dot{\sigma}(\tau)$ from the indicial response. Omitting this dependence in Eq. (6) means that, so far as the indicial response is concerned, the motion prior to the origin of a step is being approximated by the time-invariant motion $\sigma(\xi) = \sigma(\tau)$. The equation of motion, Eq. (3), becomes

$$\ddot{\sigma}(t) = \left(\frac{qSl}{I}\right)\left\{C_m\big(\infty;\sigma(t)\big) - \int_0^t F\big(t - \tau;\sigma(\tau)\big)\,\frac{d\sigma}{d\tau}\,d\tau\right\} \tag{9}$$

a nonlinear Volterra integro-differential equation, solvable by numerical techniques. Here, the collection of indicial responses required to be known in advance must consist of members corresponding to a range of values of $\sigma(\tau)$. This formulation also lends itself to reduction to a form correct to the first order in frequency, resulting in a nonlinear generalization of the classical stability derivative formulation. The form has been studied at length in the authors' previous work (Ref. 3). Approximation at the level of the second formulation thus enables the rational introduction of nonlinear effects.

Finally, the third, most comprehensive formulation is that represented in full by Eq. (7). This form is of sufficient scope to allow the treatment of motions involving hysteresis effects caused by rate-dependent aerodynamic phenomena. It is believed to be applicable, for example, to the complex set of aerodynamic phenomena characteristic of airfoils in low-speed dynamic stall (Refs. 4-6). Retaining a dependence on $\dot{\sigma}(\tau)$ allows assigning different indicial responses to a step at a single value of $\sigma(\tau)$, depending on the magnitude and sign of $\dot{\sigma}(\tau)$. It is possible, for example, to distinguish between indicial responses where σ was increasing or decreasing prior to the step. The motion that would be required to obtain the indicial response experimentally with positive $\dot{\sigma}(\tau)$ is essentially the same as the maneuver that has been used to study the overshoot in lift that occurs following a rapid pitch-up (Ref. 7). It would also be necessary to carry out experiments involving pitch-down maneuvers to allow for the possibility that the indicial response for a given $\sigma(\tau)$ and a negative value of $\dot{\sigma}(\tau)$ will be different from that for a positive $\dot{\sigma}(\tau)$ of the same magnitude. Here, the collection of indicial responses required to be known in advance must embrace a range of values of $\sigma(\tau)$ and a range of both positive and negative values of $\dot{\sigma}(\tau)$.

While Eq. (7) goes some distance toward fulfilling the goal of a sufficiently comprehensive formulation, it is still incapable of accounting for the existence of multivalued aerodynamic phenomena that need not depend on the pitching rate. This is evident, in particular, in the representation of the steady-state aerodynamic pitching moment $C_m(\infty;\sigma(t))$, and its derivative $C_{m_\sigma}(\infty;\sigma(t))$, which must be single-valued functions of σ. Admitting the possibility of multivalued aerodynamic responses, not necessarily dependent on the pitching rate, will require an acknowledgment of the influence of the distant past on the indicial response.

5. DEPENDENCE OF THE INDICIAL RESPONSE ON THE DISTANT PAST

In the preceding treatment, on the assumption that events in the distant past should be incapable of influencing the indicial response, the indicial response functional $C_{m_\sigma}[\sigma(\xi);t,\tau]$ was replaced by a function $C_{m_\sigma}(t - \tau;\sigma(\tau),\dot{\sigma}(\tau))$ which depends only on the magnitude and slope of the past motion $\sigma(\xi)$ at the origin of the step. This replacement can be viewed either as an approximation of the actual motion $\sigma(\xi)$ by a linearly varying motion $\sigma(\xi) = \sigma(\tau) - \dot{\sigma}(\tau)(\tau - \xi)$ for all past time, or, as a substitution applicable only in the vicinity of $\xi = \tau$ with the implicit understanding that the distant-past motion, being immaterial to the indicial response, can be assigned at will. In what follows, the latter interpretation will be the desired one to the extent possible. However, the presence of fluctuations appears to represent a condition where events occurring in the distant past (e.g., the initiation of fluctuations due to flow separation) can affect the present evolution of the indicial response. For suppose that certain distant-past motions, but not others, can initiate fluctuations that persist up to the measuring time, and that the indicial response measured in the presence of fluctuations can differ from the measurement in the absence of fluctuations. Then this must be taken into account somehow, and in a way that does not require assigning the actual distant-past motion to every indicial response.

5.1 Reformulation of Pitching-Moment Response

It is convenient to consider first a typical motion qualifying as the first of the two motions required to form the indicial response (cf. Fig. 3). The motion for values of $\xi < \tau$ is given by $\sigma(\xi)$; it is held constant at $\sigma(\tau)$ for all $\xi > \tau$. The time $\xi = t$ is the time at which the pitching moment is measured. The whole time period prior to $\xi = \tau$ will be called the past relative to any value of $\xi = t > \tau$. Let the past be divided into two parts so that there is an interval T just prior to $\xi = \tau$. The interval of duration T will be called the recent past and the remaining interval the distant past. Let T be chosen sufficiently large so that under normal circumstances, the particular form of the motion $\sigma(\xi)$ in the distant past is immaterial to the pitching-moment response for values of $\xi = t > \tau$. This is the condition that has been used in the preceding section.

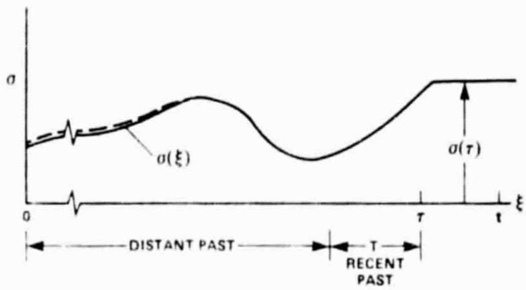

Fig. 3. Recent past and distant past.

How can motion in the distant past influence the measurement for the pitching moment at time t? It is clear that it will do so if it is capable of changing the nature of the flow in the recent past, to which the measurement is certainly sensitive. This is possible if motion in the distant past has initiated fluctuations that persist into the recent past. The following is an explicit argument. Suppose that the pitching moment is measured at time t for the motion shown in Fig. 3. (Recall that the measurement is actually the ensemble average of repeated measurements at t corresponding to repetitions of the same motion.) With everything else remaining the same, make an infinitesimal change in the distant-past motion as shown by the dotted curve in Fig. 3. After many repetitions of this motion and the corresponding measurement for the pitching moment at time t, compare the ensemble-averaged result with that of the original motion. Suppose first that there is no difference between the two measurements. This, of course, is the expected result according to the preceding formulation. However, there are two conditions under which it should hold: (1) neither distant-past motion has initiated fluctuations that persist into the recent past or (2) both distant-past motions have initiated fluctuations that attain similar statistical properties over the identical recent past, thereby altering the ensemble-averaged flow over the recent-past motion in the same way. Both conditions can be incorporated within a single assertion that in neither case did only one motion initiate fluctuations persisting into the recent past. Now suppose that the two measurements differ at time t. Then the infinitesimal change in the distant-past motion must have exceeded a critical condition for the initiation of fluctuations, and the persistence of these fluctuations into the recent past must have changed the form of the flow in the recent past in ensemble average. The argument of exceeding a critical condition is equally valid for cases in which an infinitesimal change in the distant-past motion stops previously existing fluctuations from persisting into the recent past. Thus, if the distant-past motion is able to influence the measurement at t, the cause is attributable to the existence of a critical condition for the initiation or cessation of persistent fluctuations.

Finally, consider a second infinitesimal change in the distant-past motion that also exceeds the critical condition. Consistent with condition (2) already noted, it is argued that the pitching-moment response to this motion at time t will be the same as the response to the previous motion which had also exceeded the critical condition. The argument is that relative to values of $\xi = t > \tau$, there is sufficient time over the duration T of the recent past for fluctuations originating in the distant past to attain statistical properties that no longer depend on their origin, but rather depend only on their experience over the recent past. Since the recent-past motion is the same for both distant-past motions, the statistical properties of their respective fluctuations should have become the same by the time they have reached the vicinity of $\xi = \tau$. With identical statistical properties, ensemble averaging of repetitions of the pitching-moment responses to the two motions should then yield the same result at $\xi = t$. Now the same argument should hold for any distant-past motion that has initiated fluctuations persisting into the recent past. According to this argument then, the pitching-moment response at time t for a given past motion can be duplicated by the pitching-moment response for a motion whose form in the distant past is assigned at will, so long as it is known that both have initiated fluctuations persisting into a same recent past. Likewise, a distant-past motion whose fluctuations do not persist into the recent past can be replaced, according to the argument, by one assigned at will, so long as fluctuations originating in the latter motion also do not persist into the recent past. The argument translates into a mathematical statement that the pitching-moment response at $\xi = t > \tau$ must be a unique functional of the recent-past motion and additionally, must depend on a parameter that designates by say, one of two numbers, whether fluctuations originating in an otherwise arbitrary distant-past motion do or do not persist into the recent past. Accordingly, the pitching-moment response is written in the functional form

$$C_m(t) = C_m[\sigma(\xi);t,\tau,\lambda(\tau)] \tag{10}$$

where it will be understood that the range of ξ over which there is a functional dependence on $\sigma(\xi)$ is now restricted to cover the recent past only, that is, $\tau - T \leq \xi \leq \tau$, and where λ designates the type of distant-past motion, otherwise arbitrary, by one of two numbers; that is,

$$\left.\begin{array}{ll}\lambda(\tau) = 0: & \text{fluctuations originating in distant past do not persist into recent past;}\\ \lambda(\tau) = 1: & \text{fluctuations originating in distant past persist into recent past.}\end{array}\right\} \tag{11}$$

Notice in Eqs. (10) and (11) that although λ can take only one of two values, nevertheless it is denoted a function of τ. This is because the event characterized by λ must originate in the distant past, and by definition, what is called the distant past depends on τ. That is, an event is said to occur in the distant past if the time ξ at which it occurs satisfies $\tau - \xi > T$.

5.2 Reformulation of Indicial Response and Integral Form

Now it is necessary to consider the role of fluctuations in the formation of the indicial response. Consider again the motion illustrated in Fig. 3, for which the pitching-moment response is of the form Eq. (10). Just as before, with everything else remaining the same, make a small change $\Delta\sigma$ in the constant value $\sigma(\tau)$. Repetitions of this motion and the corresponding measurement at time t yield an ensemble-average value for the pitching moment at time t. The procedure is repeated for successively smaller values of $\Delta\sigma$, as many times as necessary, to carry out the limiting process indicated formally in Eq. (12):

$$\begin{aligned}\lim_{\Delta\sigma\to 0} \frac{\Delta C_m(t)}{\Delta\sigma} &= \lim_{\Delta\sigma\to 0}\left\{\frac{C_m[\sigma_2(\xi);t,\tau,\lambda(\tau)] - C_m[\sigma_1(\xi);t,\tau,\lambda(\tau)]}{\Delta\sigma}\right\}\\ &\equiv C_{m_\sigma}[\sigma(\xi);t,\tau,\lambda(\tau)]\end{aligned} \tag{12}$$

where

$$\begin{aligned}\sigma_1(\xi) &= \sigma(\xi)\ ; && 0 \leq \xi \leq \tau\\ &= \sigma(\tau)\ ; && \xi \geq \tau\\ \sigma_2(\xi) &= \sigma(\xi)\ ; && 0 \leq \xi \leq \tau\\ &= \sigma(\tau) + \Delta\sigma\ ; && \xi \geq \tau\end{aligned}$$

The result (if it exists) is the definition of the indicial response. Suppose first that the limit exists, and moreover, that the pitching-moment response to the second motion is very similar in character to that of the first. Again, there are two conditions: (1) neither motion $\sigma_1(\xi)$ nor $\sigma_1(\xi)$ contains flow fluctuations or (2) both motions $\sigma_1(\xi)$ and $\sigma_2(\xi)$ contain flow fluctuations of similar statistical character. Both conditions are incorporated within a single assertion that in neither case did the infinitesimal change $\Delta\sigma$ either initiate fluctuations or stop prior fluctuations. Now suppose that the ensemble-averaged pitching-moment response to the second motion differs from that of the first in such a marked way that the limiting process $\lim_{\Delta\sigma\to 0}\left(\Delta C_m(t)/\Delta\sigma\right)$ almost fails to converge. In this event, it will be said that the infinitesimal change $\Delta\sigma$ has either initiated fluctuations or stopped prior fluctuations. In other words, if the infinitesimal change $\Delta\sigma$ required to form the indicial response itself either initiates fluctuations or stops prior fluctuations, the evidence of this will be a marked, almost discontinuous, change in the indicial response. This abrupt change will be evidenced, in particular, in the steady-state value of the indicial response. While the cause of an abrupt change in the indicial response most often can be attributed to the initiation or termination of fluctuations, let it be noted that nothing in the analysis prevents associating the abrupt change with other, perhaps nonfluctuating, phenomena as well; for example, an abrupt shift in the pattern of flow separation or an abrupt shift in the location of a shock wave (Ref. 8). All that the analysis requires in principle is the existence of two distinct regimes of flow separated by critical conditions. Thus, the phrase "initiation (or termination) of fluctuations" may stand for any flow phenomenon leading to an abrupt change in the indicial response.

To complete the reformulation, it is necessary to consider the summation process leading to an integral form for the pitching-moment response to an arbitrary motion. Although, as noted, the possibility is allowed of a near-discontinuity with respect to σ in the indicial response, this possibility will exist only at certain isolated values of σ where the exceeding of a critical condition by the infinitesimal change $\Delta\sigma$ either initiates or stops fluctuations. Since the nearly singular behavior of the indicial response thus will be confined to discrete events in the history of σ, these events will not invalidate the general applicability of the summation procedure. Then, as before, the result of summation yields for $C_m(t)$:

$$C_m(t) = C_m(0) + \int_0^t C_{m_\sigma}[\sigma(\xi);t,\tau,\lambda(\tau)] \frac{d\sigma}{d\tau} d\tau \tag{13}$$

The form differs from the one presented earlier (Eq. (2)) in two respects: (1) the functional dependence of the indicial response on $\sigma(\xi)$ extends only over the recent past; (2) the indicial response depends additionally on the parameter λ, designating the <u>type</u> of distant-past motion that is to be attached to the recent-past motion.

Finally, the same argument that was used before can be invoked to replace the indicial response functional in Eq. (13) by a function dependent on a limited number of parameters, rather than on all values of $\sigma(\xi)$ over the interval T of the recent past. If it is assumed again that the recent-past motion is adequately represented by the first two terms of its Taylor series expansion about $\xi = \tau$, then Eq. (13) becomes

$$C_m(t) = C_m(0) + \int_0^t C_{m_\sigma}\left(t - \tau;\sigma(\tau),\dot\sigma(\tau),\lambda(\tau)\right) \frac{d\sigma}{d\tau} d\tau \tag{14}$$

Again, it can be argued that as the indicial response approaches its steady-state value with increasing values of $t - \tau$, it must become independent of any particular recent-past motion (characterized by $\dot\sigma(\tau)$ in Eq. (14)), since the statistical properties of disturbances originating in the recent past will have become independent of their origin as $t - \tau \to \infty$. The dependence on λ remains, however, and this means that the steady-state value of the indicial response may now be a <u>double-valued</u> function of $\sigma(\tau)$, corresponding to the two possible values of λ. The substitution

$$C_{m_\sigma}\left(t - \tau;\sigma(\tau),\dot\sigma(\tau),\lambda(\tau)\right) = C_{m_\sigma}\left(\infty;\sigma(\tau),\lambda(\tau)\right) - F\left(t - \tau;\sigma(\tau),\dot\sigma(\tau),\lambda(\tau)\right) \tag{15}$$

puts the steady-state value of the indicial response in evidence. If Eq. (15) is substituted in Eq. (14) and the interval $0 < \tau < t$ is divided into segments, each of which contains only a single value of λ, then the steady-state term can be integrated over each of the segments. The intermediate terms always cancel, however, so that the formulation for $C_m(t)$ takes the form:

$$C_m(t) = C_m\left(\infty;\sigma(t),\lambda(t)\right) - \int_0^t F\left(t - \tau;\sigma(\tau),\dot\sigma(\tau),\lambda(\tau)\right) \frac{d\sigma}{d\tau} d\tau \tag{16}$$

As before, $C_m(\infty;\sigma(t),\lambda(t))$ is the pitching-moment coefficient that would be measured in a steady flow with σ fixed at the instantaneous value $\sigma(t)$. Like its derivative, it may now be a <u>double-valued</u> function of σ corresponding to the two possible values of λ.

5.3 Decision Logic for Choice of λ

It remains to determine a logic for assigning the appropriate value of λ to an indicial response. Recall that the influence of the distant-past motion on an indicial response at current time, characterized by λ, was associated with the exceeding of a critical condition in the distant past which either initiated or terminated persistent fluctuations. It was noted also that at the time of exceeding a critical condition the initiation or cessation of fluctuations would be evidenced by a marked change in the behavior of the indicial response, and in particular the steady-state value of the indicial response, accompanying an infinitesimal change in σ. This suggests that a suitable experiment for determining where the onset and cessation of fluctuations occur is simply the experiment that would be required in any case to determine the steady-state pitching moment as a function of angle of attack. A suitable program for changing the angle of attack is illustrated in Fig. 4, where, at each level, sufficient time is allowed before measuring the pitching moment for the pitching-moment response to reach a steady state (in the mean). Advancing through a series of increasing angles of attack, and then similarly, through a series of decreasing angles of attack, should allow determining whether double-valued behavior of the steady-state pitching moment is possible. Suppose that the result of measurements for the steady-state pitching-moment coefficient resembles that shown on Fig. 5(a). The curve has two distinct branches, reflecting the nonfluctuating and fluctuating regimes of flow. A region of overlap, $\sigma_R < \sigma < \sigma_S$, exists in which the pitching-moment coefficient is double-valued. As illustrated schematically in Fig. 5(b), passage from one regime to the

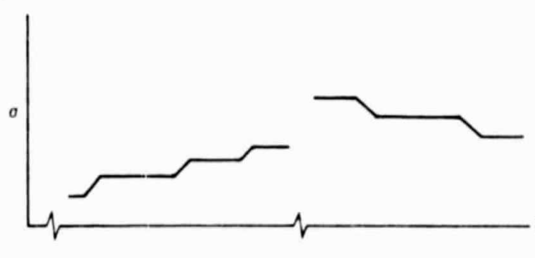

Fig. 4. Angle-of-attack program for measuring steady-state pitching moment.

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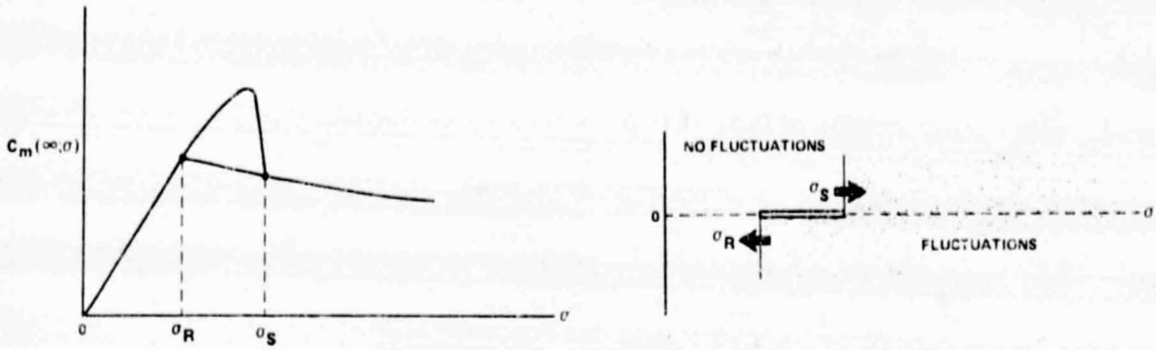

(a) Pitching-moment coefficient. (b) Regimes of flows.

Fig. 5. Steady-state pitching-moment coefficient with region of double-valued dependence on angle of attack.

other is barred (indicated by the double line) except by traversing the critical points $\sigma = \sigma_S$, $\sigma = \sigma_R$ in the directions indicated by the arrows. Then, having the values of σ_S and σ_R should suffice to determine a logic for the choice of λ.

Assume that a step-by-step calculation is being made of a maneuver, and that the calculation has advanced to the point $\xi = \tau$ (cf. Fig. 6). To continue the calculation one more step, it is necessary to assign the appropriate indicial response to the point $\xi = \tau$. Can it now be done? The form of the indicial response, Eq. (15), indicates the parameters that have to be known: $\sigma(\tau)$, $\dot{\sigma}(\tau)$, and $\lambda(\tau)$. Since $\sigma(\xi)$ is known for $\xi \leq \tau$, the values of $\sigma(\tau)$, $\dot{\sigma}(\tau)$ can be specified. It remains to determine whether $\lambda = 0$ or 1 to complete the specification. The following three questions are asked: (1) Is there at least one ξ_0 with $\tau - \xi_0 > T$ such that $\sigma(\xi_0) = \sigma_S$? (2) Is there at least one ξ_1 with $\tau - \xi_1 > T$ such that $\sigma(\xi_1) = \sigma_R$? (3) If the answers to (1) and (2) are yes, is $\min(\tau - \xi_1) > \min(\tau - \xi_0)$? Yes or no answers to the three questions determine the value of λ. Results are given in Table 1:

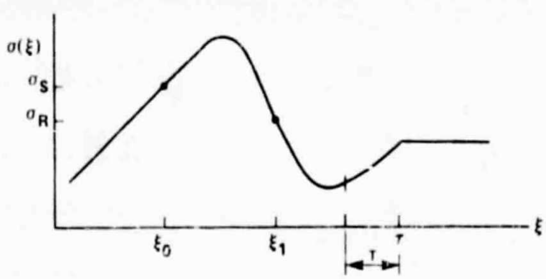

Fig. 6. Step-by-step calculation of a maneuver.

TABLE 1. DECISION LOGIC FOR λ

1	2	3	λ
No	---	---	0
Yes	No	---	1
Yes	Yes	No	0
Yes	Yes	Yes	1

6. IMPLICATIONS FOR DYNAMIC STABILITY EXPERIMENTS

The formulation Eq. (16) is considered to be the principal result of this study. Given the information required to determine λ, its scope may be sufficiently wide to embrace motions involving both nonlinear aerodynamic responses and the double-valued aerodynamic behavior evident in certain kinds of aircraft stall (Refs. 8-10). However, its usefulness in practice appears to hinge on the availability of a collection of indicial responses, forming the kernel of the integral term in Eq. (16). As noted earlier, the eventual availability of such a collection is very unlikely in view of the great difficulty of experimentally determining an indicial response. The purpose of this section will be to show that the usefulness of the formulation in fact is not contingent on the availability of indicial responses. It will be shown that the integral term in Eq. (16) is replaceable by results from a technically more feasible experiment: the oscillations-in-pitch experiment.

For an arbitrary motion $\sigma(t)$, the contribution to $C_m(t)$ of the integral term in Eq. (16) may be approximated as a finite sum of responses to discrete steps $\Delta\sigma(\tau)$. The form of the summation, that is, the integrand, is illustrated in Fig. 7. At each step, the deficiency function $F(t - \tau; \sigma(\tau)\dot{\sigma}(\tau), \lambda(\tau))$ dies out to zero as $t - \tau \to \infty$, and will be essentially zero for all $t - \tau$ larger than a relatively small value t_a. This is shown schematically in Fig. 7. It is clear that in the summation of responses at time t, only the responses in the interval $t - t_a < \tau < t$ yield measurable contributions at t. Thus, the form of the motion $\sigma(\tau)$ outside the interval $t - t_a < \tau < t$ is immaterial to the summation, except insofar as it determines the value of λ. On the assumption that the motion outside the interval is such that it ensures the correct value of λ, the motion within the interval $t - t_a < \tau < t$ can be approximated without serious error by any convenient substitute motion. If, for example, the substitute motion is harmonic, the resulting summation of responses at time t will be that corresponding to an equivalent harmonic oscillatory motion.

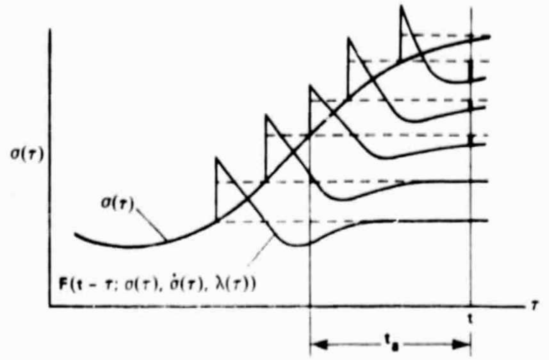

Fig. 7. Summation of indicial responses.

An arbitrary harmonic motion $\sigma_h(\tau)$ about a constant mean provides three arbitrary constants (assuming that the frequency ω is chosen to match a characteristic frequency of the actual motion) which may be chosen to match three properties of the actual motion. It is especially important to match the actual motion in the immediate vicinity of $\tau = t$ where the contributions to the summation are largest. If it is chosen to match the actual values of $\sigma(t)$, $\dot{\sigma}(t)$, and $\ddot{\sigma}(t)$ by a harmonic motion $\sigma_h(\tau)$, then the form of the harmonic motion is given by

$$\sigma_h(\tau) = \left[\sigma(t) + \frac{\ddot{\sigma}(t)}{\omega^2}\right] - \frac{\dot{\sigma}(t)}{\omega}\sin\omega(t-\tau) - \frac{\ddot{\sigma}(t)}{\omega^2}\cos\omega(t-\tau) \tag{17}$$

so that

$$\dot{\sigma}_h(\tau) = \dot{\sigma}(t)\cos\omega(t-\tau) - \frac{\ddot{\sigma}(t)}{\omega}\sin\omega(t-\tau) \tag{18}$$

Substituting Eqs. (17) and (18) for $\sigma(\tau)$, $\dot{\sigma}(\tau)$ in the integral term in Eq. (16) yields for the integral term (letting the lower limit be $t - t_a$ for consistency):

$$J(t) = -\dot{\sigma}(t)\int_{t-t_a}^{t} F\big(t-\tau;\sigma_h(\tau),\dot{\sigma}_h(\tau),\lambda(\tau)\big)\cos\omega(t-\tau)d\tau + \frac{\ddot{\sigma}(t)}{\omega}\int_{t-t_a}^{t} F\big(t-\tau;\sigma_h(\tau),\dot{\sigma}_h(\tau),\lambda(\tau)\big)\sin\omega(t-\tau)d\tau \tag{19}$$

or, with a change of variable,

$$J(t) = -\dot{\sigma}(t)\int_{0}^{t_a} F\big(u;\sigma_h(t-u),\dot{\sigma}_h(t-u),\lambda(t-u)\big)\cos\omega u\,du + \frac{\ddot{\sigma}(t)}{\omega}\int_{0}^{t_a} F\big(u;\sigma_h(t-u),\dot{\sigma}_h(t-u),\lambda(t-u)\big)\sin\omega u\,du \tag{20}$$

This is the contribution to $C_m(t)$ from the integral term, and it is the same contribution that would be obtained from an oscillations-in-pitch experiment for an oscillation constructed according to Eq. (17). This means matching $\sigma(t)$, $\dot{\sigma}(t)$, and $\ddot{\sigma}(t)$ requires that an equivalent harmonic motion have a mean value equal to $[\sigma(t) + \ddot{\sigma}(t)/\omega^2]$ and an amplitude equal to $\{[\dot{\sigma}(t)/\omega]^2 + [\ddot{\sigma}(t)/\omega^2]^2\}^{1/2}$. The contribution from the term multiplied by $\ddot{\sigma}(t)$ in Eq. (20) is actually of second order in ω and probably negligible for the very low reduced frequencies typical of most aircraft motions. For rapid maneuvers, however, (e.g., a rapid pitch-up maneuver) it may be necessary to retain the term, since it accounts for the very large deviation from the steady-state aerodynamic contribution (i.e., $C_m(\infty;\sigma(t),\lambda(t))$ in Eq. (16) at $\sigma = \sigma_{max}$ (where $\dot{\sigma} \equiv 0$) which has been observed in wind-tunnel experiments with oscillating airfoils in the low-speed dynamic stall regime (Refs. 4,5).

The result represented by Eq. (20) mathematically expresses the major theme of this study. However, couched in more general terms, it leads to an almost self-evident conclusion that should hold in the general case: the instantaneous aerodynamic force and moment corresponding to an arbitrary motion can be duplicated with the instantaneous force and moment corresponding to an assigned motion, so long as both motions are essentially the same in the recent past relative to the instant, and are in the same flow regime determined by the otherwise immaterial distant-past motion.

7. CONCLUDING REMARKS

The scope of any aerodynamic formulation proposing to embrace a range of possible aircraft maneuvers has been shown to be determined principally by the extent to which the aerodynamic indicial response is allowed to depend on the past motion. Allowing the indicial response to depend only on motion in the recent past resulted in an aerodynamic formulation enabling the rational introduction of nonlinear effects and a description of the rate-dependent aerodynamic phenomena characteristic of airfoils in low-speed dynamic stall. Allowing the indicial response to depend additionally on a characteristic feature of motion in the distant past, that is, the initiation or termination of persistent fluctuations, resulted in a more comprehensive formulation permitting a description of the double-valued aerodynamic behavior characteristic of certain kinds of aircraft stall. The scope of the latter formulation should be sufficiently wide to include any pitching maneuver having no more than two distinct regimes of flow, separated by critical conditions. Straightforward extensions of the formalism already developed should yield formulations permitting a description of any number of flow regimes and embracing motions with multiple degrees of freedom. A general conclusion that can be drawn from this study, favorable regarding the role of dynamic stability experiments, is the following: the instantaneous aerodynamic force and moment corresponding to an arbitrary motion can be duplicated with the instantaneous force and moment corresponding to an assigned motion, so long as both motions are essentially the same in the recent past relative to the instant, and are in the same flow regime determined by the otherwise immaterial distant-past motion.

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15. Supplementary Notes

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16. Abstract

The scope of any aerodynamic formulation proposing to embrace a range of possible maneuvers is shown to be determined principally by the extent to which the aerodynamic indicial response is allowed to depend on the past motion. Starting from the linearized formulation, in which the indicial response is independent of the past motion, two successively more comprehensive statements about the dependence on the past motion are assigned to the indicial response (1) dependence only on the recent past and (2) dependence additionally on a characteristic feature of the distant past. The first enables the rational introduction of nonlinear effects and accommodates a description of the rate-dependent aerodynamic phenomena characteristic of airfoils in low-speed dynamic stall; the second permits a description of the double-valued aerodynamic behavior characteristic of certain kinds of aircraft stall. An aerodynamic formulation based on the second statement, automatically embracing the first, may be sufficiently comprehensive to include a large part of the aircraft's possible maneuvers. The results suggest a favorable conclusion regarding the role of dynamic stability experiments in flight dynamics studies.

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