APPROXIMATE DYNAMIC MODEL OF A TURBOJET ENGINE

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(NASA-TM-75263) APPROXIMATE DYNAMIC MODEL OF A TURBOJET ENGINE (National Aeronautics and Space Administration) 9 p HC A02/MF A01 CSCL 21E N78-19159 Unclas G3/07 09251

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 MARCH 1978

STANDARD TITLE PAGE

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-75263		

4. Title and Subtitle	5. Report Date
APPROXIMATE DYNAMIC MODEL OF A TURBO-JET ENGINE	March 1978
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
O. A. Artemov	10. Work Unit No.
9. Performing Organization Name and Address	11. Contract or Grant No.
Leo Kanner Associates Redwood City, California 94063	13. Type of Report and Period Covered
12. Sponsoring Agency Name and Address	Translation
National Aeronautics and Space Administration, Washington, D.C. 20546	14. Sponsoring Agency Code

15. Supplementary Notes

Translation of "Priblizhennaya dinamicheskaya model' turboreaktivnogo dvigatelya," Samoletostroyeniye - Tekhnika vozdushnogo flota, No. 40, 1976. pp. 34-37 (A77-32705)

16. Abstract

An approximate dynamic nonlinear model of a turbojet engine is elaborated as a tool in studying the aircraft control loop, with the turbojet engine treated as an actuating component. Approximate relationships linking the basic engine parameters and shaft speed are derived to simplify the problem, and to aid in constructing an approximate nonlinear dynamic model of turbojet engine performance useful for predicting aircraft motion.

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17. Key Words (Selected by Author(s))	18. Distribution Statement
	Unclassified-Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified		

NASA-HQ

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The turbojet engine (TJE) is being increasingly utilized as a component of the aircraft control loop. Its applications include velocity control in conventional aircraft and attitude control in VTOL craft [1,2]. The use of ordinary linearized TJE controls [sic -- should apparently be "TJE equations"] [3] introduces substantial errors, since the parameters of the TJE as a system change continuously during control, and the small-parameter principle is no longer applicable. Moreover, the equations indicated involve many internal parameters of the engine itself; but including these parameters in the complete control-loop equations when studying the aircraft control loop complicates the problem considerably. The object of the present paper is to obtain an approximate dynamic nonlinear model of the TJE which can be used in the study of aircraft motion. /34*

The complete system of equations [3] describing TJE motion is strictly nonlinear. Furthermore, the very form of the equations changes with the transition from subcritical to supercritical nozzle exit velocities. For this reason we have based our work on approximate functions relating the basic engine parameters to the shaft speed n, which are used in calculating transient processes [3].

We shall make the following assumptions:

1. The TJE used as an element of the control loop must be as fast-acting as possible; this is achieved by increasing the range of the pre-turbine gas temperature T_3^*. It is assumed

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* Numbers in the margin indicate pagination in the foreign text.

that the role of the controller is reduced to placing an upper limit on the maximum temperature T^*_{3max} permissible from the standpoint of turbine-blade strength, and a lower limit on the minimum temperature T^*_{3min} from the standpoint of fuel combustion stability. Such a limitation is also justified if engine performance is actually controlled according to some other parameter.

2. The TJE used as an element of the control loop must be stable throughout the range of shaft speeds from n_{min} to n_{max}. This stability is achieved by creating a negative shaft-speed feedback. The TJE equations are written as follows:

$$I\frac{\partial n}{\partial t}=\frac{G_C}{n}\left(118\,T^*_3 e_T \eta_T - 102{,}5\,T^*_1 e_C \frac{1}{\eta_C}\right); \quad (1)$$

$$\tilde{T}^*_3 - T^*_1\left(1+\frac{e_C}{\eta_C}\right)=\frac{H_u \eta_{cc}}{c_p G_C}\tilde{G}_T; \quad (2)$$

$$\tilde{G}_T = G_T^{\Delta\alpha}\Delta\alpha + G_T^n n; \quad (3)$$

$$\Delta\alpha = \alpha - Kr \quad (4)$$

$$T^*_3(t)=\begin{cases} T^*_{3max}, & \text{if } \tilde{T}^*_3 \geqslant T^*_{3max},\\ \tilde{T}^*_3(t-\tau), & \text{if } T^*_{3min} < \tilde{T}^*_3 < T^*_{3max},\\ T^*_{3min}, & \text{if } \tilde{T}^*_3 < T^*_{3min}; \end{cases} \quad (5)$$

$$R = G_C R_{sp}(T^*_3, e_C), \quad (6)$$

where n is the angular velocity of the TJE shaft, rad/sec; g_c is the air flow rate through the compressor, kg/sec; I is the inertial moment of the shaft, kg·m·sec²; η_τ is the turbine efficiency; η_c is the compressor efficiency; 118, 102.5 are coefficients, (kg·m)/(kg·deg); T^*_1 is the temperature at the compressor inlet, deg; T^*_3 is the gas temperature before the turbine, deg; H_u is the heating value of the fuel, kcal/kg; η_{cc} is the efficiency of the combustion chamber; c_p is the /35

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heat capacity of the gas at constant pressure, kcal/(kg·deg); G_τ is the fuel consumption, kg/sec; α is the position of the control element; K is the feedback coefficient; R is engine thrust, kg; R_{sp} is the specific engine thrust, (kg/sec)/kg; τ is the delay of fuel combustion in the combustion chamber, sec; t is real time, sec; $e_c - \pi_c^{*(k-1)/k}$ is one; $e = 1 - \pi^{*-(k_2-1/k_2)}$; π_c^* is the degree of pressure increase in the compressor; π_τ^* = the degree of gas expansion in the turbine; and k, k_3 are the specific heat ratios of the air and gas, respectively.

According to [3] the values of e_c, e_τ and G_c are the following functions of n:

$$e_c = e_c^n n;\ e_\tau = \begin{cases} e_\tau^n n, & \text{if } n < n_{cr} \\ e_\tau^n n_{cr} = \text{const}, & \text{if } n \geqslant n_{cr} \end{cases}$$

and $G_c = G_c^n n$. The tilde sign used in eqn. (1-6) indicates the values that would be assumed by T_3^* and G were it not for the limits imposed by $T_{3\,max}^*$ and $T_{3\,min}^*$. As the system of equations shows, the motion of the turbojet engine is divided into two qualitatively different motions: a "greater" motion, when T_3^* = const and only the shaft speed changes; and a "lesser" motion, when T_3^* changes relatively rapidly and n changes slowly. By using this division of motions we obtain a further simplification of the equations.

The steady value of T_3^*, i.e. the value for which $\frac{dn}{dt} = 0$, is a function of the form

$$T_{31}^* = \begin{cases} T_{30}^*, & \text{if } n \leqslant n_{cr} \\ T_{30}^* + \dfrac{T_{3max}^* - T_{30}^*}{n_{max} - n_{cr}}(n - n_{cr}), & \text{if } n > n_{cr} \end{cases} \qquad (7)$$

In the control regime, i.e. in the case of large control element displacements α, it is convenient to average the right-hand side of eqn. (1) with respect to n, since this side varies linearly with T_3^* at a constant n. We can then rewrite eqn. (1) in relative values as follows:

$$\frac{\partial \bar{n}}{\partial \bar{t}} = \bar{T}_3^* - \bar{T}_{31}^*(\bar{n}) - \bar{T}_3^* - 1 - \Delta \bar{T}_{31}^*(\bar{n}), \tag{8}$$

where $\bar{n} = \frac{n}{n_{max}}$; $\bar{t} = \frac{t}{\vartheta}$; $\bar{T}_3^* = \frac{T_3^*}{T_0^*}$;

$$\frac{1}{\vartheta} \frac{G_C^{\bar{n}}\, 118\, \tau_T\, T_{30\bar{n}_{min}}^* \int^1 [\bar{T}_{3max}^* e_T(\bar{n}) - \bar{T}_{3min}^* e_T(\bar{n})]\, d\bar{n}}{n_{max}^2\, I(\bar{T}_{3max}^* - \bar{T}_{3min}^*)(1 - \bar{n}_{min})}.$$

Eqn. (5) assumes the form /36

$$\bar{T}_3^* - 1 = \begin{cases} \bar{T}_{3max}^* - 1, & \text{if } \quad \bar{\tilde{T}}_3^* \geqslant \bar{T}_{3max}^*, \\ \bar{\tilde{T}}_3^*(\bar{t} - \bar{\tau}) - 1, & \text{if } \quad \bar{T}_{3min}^* < \bar{\tilde{T}}_3^* < \bar{T}_{3max}^*, \\ \bar{T}_{3min}^* - 1, & \text{if } \quad \bar{\tilde{T}}_3^* \leqslant \bar{T}_{3min}^*. \end{cases} \tag{9}$$

We can combine eqn. (2-4) thus:

$$\bar{\tilde{T}}_3^* - 1 = A \frac{1}{\bar{n}} \bar{\alpha} + B\bar{n} + C, \tag{10}$$

where $\bar{\alpha} = \frac{\alpha}{n_{max}}$; $A = \frac{H_u \tau_{cc} G_T^{\bar{\Delta}\alpha}}{c_p G^{\bar{n}} T_{80}^*}$; $B = \bar{T}_1^* \frac{e_C^{\bar{n}}}{\tau_C}$; $C = A(G_T^{\bar{n}} - G_T^{\bar{\Delta}\alpha} K)$.

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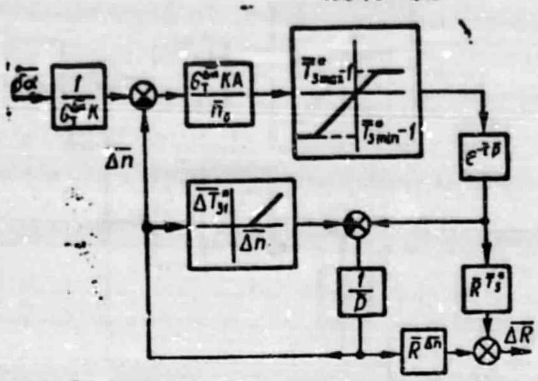

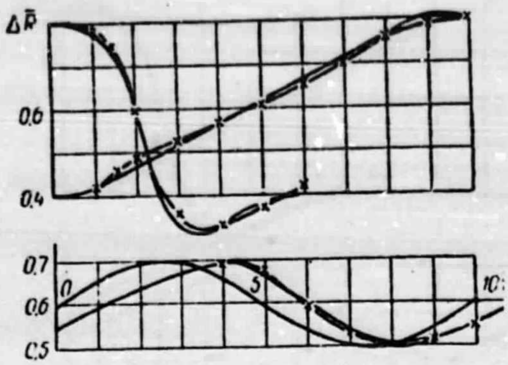

Fig. 1. Structural diagram of turbojet engine.

Fig. 2. Time behavior of TJE.
--- = calculated; —— = experimental.

We next linearize eqn. (10):

$$\tilde{\overline{T}}{}^*_3 - 1 = \left[A\,\frac{1}{(-\overline{n}_0^2)}\,\overline{\alpha}_0 + B\right]\Delta\overline{n} + A\,\frac{1}{\overline{n}_0}\,\delta\overline{\alpha}. \qquad (11)$$

where $\overline{\alpha}_0$ and $\overline{n}_0$ correspond to the initial steady state of the system and are related by the expression $\Delta\overline{T}^*_{31}(\overline{n}_0) = A\frac{1}{\overline{n}_0}\overline{\alpha}_0 + B\overline{n}_0 + C,$ which, due to the smallness of $\Delta\overline{T}^*_{31}(\overline{n}_0)$ and $B\overline{n}_0$ compared with C, can be rewritten in approximate form:

$$A\,\frac{\overline{\alpha}_0}{\overline{n}_0} \simeq -C. \qquad (12)$$

In relative values, the linearized eqn. (6) assumes the form

$$\Delta\overline{R} = \overline{R}^{\Delta\overline{n}}\,\Delta\overline{n} + \overline{R}^{\Delta\overline{T}^*_3}\,\Delta\overline{T}^*_3, \qquad (13)$$

where $\Delta\overline{R} = \dfrac{\Delta R}{R_{max}}$.

The structural scheme corresponding to eqn. (7-9) and (11-13) is shown in Fig. 1. The behavior of such a system in the aircraft control loop is analyzed using methods of the nonlinear theory of automatic control. In most cases, system parameters can be determined with sufficient accuracy only by experimentation. It is therefore advisable to use as the parameters of the approximate TJE dynamic model the parameters obtained by approximating the experimental curves for a full-scale TJE model. Fig. 2 shows the experimentally-determined responses of the TJE to a graded signal and sinusoidal actuation. For comparison, the analogous characteristics calculated from eqn. (7-9), (11) and (13) with approximated coefficients are also shown. It is evident that the proposed approximate dynamic model of the TJE provides a description of engine behavior which is useful for studying the aircraft control loop. /37

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