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# A SOLUTIOH TO THE SURFACE IHTERSECTION PROBLEM 



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## PREFACE

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### 1.0 INTRODUCTION

The subject of geometric modeling is receiving considerable attention from the industrial community not only within the United States, but throughout the world. Current thinking points toward the establishment of an application-independent geometric model within a data base framework. The modfil can then be accessed and used by any number of technologies within both engineering and manufacturing. This basic model may be used to create application-specific models such as a Nastran finite element model or a model appropriately paneled for aerodynamic analysis. Such a model may also be used by manufacturing to support their numerically controlled operations.

Thus fer a universally acceptable geometric model has not been found. However, certain attributes appear to be getting strong backing. First, the model should support the use of Boolean operators, i.e., UNION, INTERSECTION, and DIFFERENCE. A second property of the model is that the bounding surfaces be described parametrically. The detailed mathematical form of the surface representation has yet to be resolved.

The use of the Boolean operators is attractive since it allows the user to construct a complex mocel by appropriately combining a series of simple models. The use of these operators leads to the concept of imiplicitly and explicitly defined surfaces. With an explicitly defined model, the surface area may be computed by simply summing the surface areas of the bounding surfaces. For an implicitly defined model, the surface area computation must deal with active and inactive regions. To illustrate this, consider a simple wing-body combination. To model the combination explicitly, the modeler must first capture the wing-body intersection line which will act as a boundary curve for both the wing and the fuselage. Since the two components are generally designed separately, it would be advantageous to model them separately. Requiring the intersection curve to serve as the surface boundary curve would probably lead to local distortion in both the wing
and the fuselage. In the impiicit modeling scheme, both components are modeled separately with no consideration of the intersection curve. !hen combined in their desired orientation, the intersection curve can be determined numerically. The intersection curve serves as the boundary between active and inactive regions. For the wing, the portion contained within the fuselage is considered inactive. For the fuselage the hole made by the wing is inactive. Thus, to compute the surface area of the combination, the area of the inactive regions must be disregarded. The advantage of the implicit system can be appreciated if one considers the simple exercise of moving the wing forward by an inch after the model has been created. With an explicit mode\}, the entire job must be redone, while in the case of an implicit model, only the intersection curve need be regenerated (by the computer, not the user).

Examination of the seftware required to support implicitly defined models identifies the surface intersection capability as a critical item. The purpose of this report is to describe a particular solution to the surface intersection problem. Included will be an overall description of the scratedy employed plus a detailed presentation of the analytical and numerical considerations required to implement it. Finally, a description of the resulting software will be given, including user information and examples. thile the solution is implemented for the specific case of parametric cubic surface representation, it is felt that the solution should be equally applicable to other mathematical forms.

### 2.0 SURFACE INTERSECTIONS - AN OVERVIEN

The purpose of this section is to describe a computational scheme for obtaining the intersection between two surfaces. Since this problem is quite difficult and the required analysis quite complicated, it was decided to begin the discussion with an overview of the strategy as well as the definition of terms to be used later in the detailed breakdown of the various computational phases. To begin with, the
term surface needs to be more accurately defined. Within the context of the present problem, a surface $S$ is defined to be a rectangular mesh of parametrically defined patches which approximate or represent a surface in physical space. Figure 2.1 shows a typical surface composed of a $4 \times 5$ mesh of patches. The figure also indicates a patch numbering convention which is important since the patches must be stored and retrieved in some logical order. The numbering convention also establishes connectivity between the individual patches. An additional convention which is not obvious from the figure is that the vector product between the u-derivative and the v-derivative results in an outward surface normal.


Consider now the problem of finding the intersection between two surfaces, $S_{1}(u, v)$ and $S_{2}(s, t)$. The calculation may be decomposed into three distinct phases: the hunting phase, the tracing phase, and the sorting phase. The overall strategy is necessitated by the requirement that the calculation be capable of handling relatively complex surfaces. The hunting phase locates discrete starting points required for the curve tracing operation. The
tracing phase creates strings of points lying on the surface intersection. Finally, the sorting phase orders the point strings and separates them into disjoint segments or loops.

### 2.1 Hunting Phase

The purpose of the hunting phase is to locate a number of points on the intersection which serve as starting points for the tracing phase. At least one point on each loop of the intersection curve must be found during this phase, or that particular loop will go undetected. This type of situation suggests the need for some sort of refinement mechanism which cari be made to adjust to the most complex of cases. The capacity for refinement is obtained by introducing the hunting grid on the primary surface $S_{1}$. Figure 2.2 shows a typical primary surface with such a hunting grid superimposed. By specifying a single integer $N_{G}$, a grid is defined on $S_{1}$. The example intersection is composed of two loops. It would appear that a smaller value of $N_{G}$ could have been used without losing either of the loops. However, if a sufficiently small $N_{G}\left(N_{G}=3\right)$ were chosen, the loop located in the upper left corner could escape detection. The rectangular regions defined by the hunting grid are numbered in the same manner as are the conventional surface patches (indicated by circled numbers in Figure 2.2). The hunting phase consists of taking each curve defined by the grid and obtaining its intersection with $S_{2}$. By taking first the curves of constant $u$ and then the curves of const it $v$ the intersection points are located in a sequence indicated by their numbering in Figure 2.2.

The problem of determining the intersection of a given curve with $S_{2}$ requires additional comment. As in the overall problem, the major concern here is to not miss any intensection points. This problem is somewhat anelogous to determining the roots of a polynomial. The scheme must be able to isolate the roots and then determine them. The approach used to isolate
the intersection points is a refinement of the hunting grid philosophy. The curve is segmented by selecting an integer $M_{G}$ whose role is analogous to that of $N_{G}$ in the hunting grid. $M_{G}$ should be sufficiently large that two intersection points do not occur within the same segment. In keeping with the polynomial analog, a function is sought that will identify the presence of an intersection point by exhibiting a sign change between consecutive nodes. Consider the vector function:

$$
\vec{V} \equiv \vec{P}_{C}-\vec{P}_{S}=(F, G, H)
$$

which represents the directed distance between a point on $S_{2}$ and a point on the curve $C$. For an intersection to occur between nodes $i$ and $i+i$ on $C$, it is necessary that


FIGURE 2.2 - Example of Hunting Grid ( $\mathrm{NG}_{\mathrm{G}}=6$ )

The marching procedure is illustrated in Figure 2.3


FIGURE 2.3-Marching Procedure for Curve/Surface Intersection

This seemingly simple concept requires one final assumption. A point on the curve $\vec{P}_{C}$ is compared to the point on the surface $\vec{P}_{S}$ which is closest to $\overrightarrow{\mathrm{P}} \mathrm{C}$. Thus, the vector $\vec{V}$ has three components in parametric space: the curve parameter ( $u$ or $v$ ) and the two surface parameters $s$ and $t$. Since the effectiveness of this algorithm is not dependent on tios precise determination of $s$ and $t$ such that $|\vec{v}|$ in a mininium, an approximate scheme may be used. Specifying a value of the curve parameter ( $u$ or $v$ ) determines a point $\overline{\mathrm{P}} *$. The use of the surface inversion algorithm for a small number of iterations will satisfy the requirement for the approximate closest distance from a point to the surface $\vec{P}_{s}(s, t)-\vec{P} *=0$. Having isolated an intersection point,

$$
\text { i.e. }(u \text { or } v)_{i} \leq(u \text { or } v) \leq(u \text { or } v)_{i+1}
$$

the full three-variable problem ( $\vec{V}=0$ ) is then solvea subject to the indicated constraint on the curve parameter. The hunting phase will produce a table of intersection points lying on the grid boundaries of $S_{1}$. The ordering of this table obviously
depends on the order in which the grid curves are taken as well as the direction of march along these curves. To illustrate this, the grid intersection points are numbered in Figure 2.2 as they would appesr in the table.

### 2.2 Tracing Phase

The hunting phase produced a number of discrete intersection points lying on the boundaries of the hunting grid. As shown in the example (Figure 2.2) the ordering of these points is not optimum regarding the further enrichment of the data to form the complete intersection. The intersection curves are traced across the interior regions of the hunting grid in the following manner. First the regions are analyzed one at a time. The ordering is logically similar to the patch numbering scheme and is indicated in Figure 2.2 by circling the region numbers. In eac; region, the boundaries are compared with the tabulated points. When a match is found, the portion of the particular loop containing that boundary point and lying within that region is traced, usually ending at another boundary point. Thus, the direction of trace across a region is established by which endpoint occurs first in the table. The ending point is excluded from the rest of the search (for that region) in order to prevent multiple tracing of the same segment. In this manner, regions containing more than one intersection segment (more than two boundary points) will be correctly handled.

At the starting point (for a given region) the tangent vector to the intersection curve may be obtained by taking the vector product between the normals to surfaces $S_{1}$ and $S_{2}$. The order of the vector product is established such that the tangent vector points toward the interior of the region. Tilis tangent vection provides the deriva-
tives of the four parameters ( $u, v ; s, t$ ) with respect to arc length along the intersection curve. Using a step siz based on curvature and its relation to slope change along the intersection curve, all four parameters are incremented, e.g.

$$
u_{i+1}=u_{i}+\left(\frac{d u}{d \sigma}\right)_{i} \Delta \sigma_{i}
$$

This step places the next point in the neighborhood of the intersection curve and a secondary iterative convergence is required to achieve the desired tolerance. Since there are four unknowns ( $u, v, s, t$ ) and only three equations, one of the unknowns must be held constant. By selecting one of the four possible groupings of unknowns, the system is made determinate and a converged solution obtained quite rapidly using Newton's method. This two level marching process is continued until a region boundary is encountered.

### 2.3 Ordering Phase

The example shown in Figure 2.2 illustrates the need to sort and order the various point strings into loops. The order in which the segment strings were produced is indicated by the lower case letters. l:hile this operation might be regarded as mere bookkeeping, it is significant enough to warrant discussion. The result of performing the first two phases is a collection of $N$ point strings ( $N=10$ in Figure 2.2). With regard to sorting: the only significant points are the first and last points in each segment string. Beginning with the first string, the start and end point of the current loop (containing that string) are compared to the remaining strings. When a match is found, that string number is saved in an ordering array as a signed integer. The minus sign is used to indicate that a particular string requires reversal when it is concatenated to form a loop. In the example shown in Figure 2.2, the ordering arrays are
as follows:

$$
\begin{aligned}
& \text { LOOP 1: } a,-b,-c, d,-e,-f \\
& \text { LOOP 2: }-h, g,-i ; j
\end{aligned}
$$

Some further explanation is required regarding LOOP 2. When LOOP 1 is completed, only four segments remain: $g, h, i$, and $k$ in that order. The scan to order LOOP 2 begins with $g$ which has boundary point 1 as its beginning and point 10 as its end. The scan proceeds to segment $h$ where a match is found with the start point of LOOP 2. Since the direction of LOOP 2 has already been established by $g$, $h$ must be reversed and should precede $g$ in the ordering table. At this stage, LOOP 2 has point 11 as its start and point 11 as its end. The top end of 1 matches the top end of LOOP 2 so it must be reversed and point 2 becomes the too end of LOOP 2. Finally the bottom of $k$ matches the top of LOOP 2. At this stage, there are no segments not accounted for and the sorting phase is complete.

One final consideration of the sorting phase is illustrated in Figure 2.4. LOOP 1 is started by a running from points 2 to 4. When $b$ is compared to LOOP 1 it is found to be disjoint. At the end of the first pass, the ordering array for LOOP 1 is $a,-c, d$, with $b$ considered as disjoint. This failure is a result of the order of the search and is easily remedied by performing a second pass prior to starting on LOOP 2 and would append $-b$ to the ordering array for LOOP 1.


FIGURE 2.4 - Sorting Example

### 3.0 SURFACE INTERSECTIONS - MATHEMATICAL DETAILS

Most of the mathematical manipulations involved with solving the surface intersection problem concern the solution of three simultaneous equations - one for each of the Cartesian coordinates of our threedimension physical space. The distinguishing feature in these subproblems is usually the number of unknown variables. The general surface intersection problem has four unknowns - the parametric conrdinates of each surface $\{u, v: s, t\}$. Since the solution is a space curve, the solution actually involves determining these four unknowns as a function of single parameter, e.g. the arc length $\sigma$. This problem might be contrasted to that of determining the point on a space curve closest to a given point. in space. The latter problem involves only one unknown - the parametric coordinate of the point on the curve. The fullowing subsections will identify the various subproblems occurring in the general intersection problem and provide the mathematical basis for their solution.
3.1 Determination of Parametric Derivatives from a Tangent Vector

It was stated in Section 2.2 that the vector product of the two surface normals at a point on the intersection curve produces a vector tangent to that curve. Furthermore, this tangent vector may be used to obtain the derivatives of the parametric coordinates with respect to arc length along the intersection curve. Let the tangent vector $\overrightarrow{\mathrm{F}}$ be given by

$$
\vec{T}=\vec{N}_{1} \times \vec{N}_{2}=\frac{\partial \vec{P}_{1}}{\partial u} \times \frac{\partial \vec{P}_{1}}{\partial v} \times \frac{\partial \vec{P}_{2}}{\partial S} \times \frac{\vec{P}_{2}}{\partial t}
$$

and $\vec{T} \equiv \vec{T} /|\vec{T}|$.
The unit vector $\hat{T}$ satisfies the equation

$$
\hat{\mathrm{T}}=\frac{\mathrm{dP}}{\mathrm{~d} \sigma}
$$

which for $S_{1}$ may be expanded as

$$
\hat{T}=\frac{\vec{P}}{\partial u} u^{l}+\frac{\partial \vec{P}}{\partial u} v^{l}
$$

where the primes are used to denote derivatives with respect to arc length $\sigma$. A similar expression can be written for $S_{2}$ in terms of $s^{1}$ and $t^{1}$.

Each of these vector equations represents a set of three nonlinear algebraic equations in two unknowns. Thus, one would expect a degree of freedom in their solution, i.e., any set of two equations should suffice. By taking the appropriate vector products, $u^{1}$ and $v^{1}$ may be isolated as follows

$$
\begin{aligned}
& u^{l}=-\frac{\vec{P}}{\partial v} \times \vec{T} / \vec{N}_{1} \\
& \text { where } \quad \vec{N}_{I} \equiv \frac{\partial \vec{P}}{\partial u} \times \frac{\partial \vec{P}}{\partial v} \\
& v^{l}=\frac{\partial \vec{P}}{\partial u} \times \vec{T} / \vec{N}_{1}
\end{aligned}
$$

Some explanation is required for the above equation in order to properly interpret the quotient of two vectors as a scalar. The equations are evaluated by selecting any of the three vector components which is where the added degree of freedom comes in; e.g.

$$
u=-\frac{\left(\frac{\partial y}{\partial v} \frac{d z}{d \sigma}\right)-\left(\frac{\partial z}{\partial v} \frac{d y}{d \sigma}\right)}{\left(\frac{\partial y}{\partial u} \frac{\partial z}{\partial v}\right)-\left(\frac{\partial y}{\partial v} \frac{\partial z}{\partial u}\right)}
$$

where the $x$-component was selected. Barring the presence of more important considerations, the component of $\vec{N}$ having
the largest magnitude will be selected.

### 3.2 Surface Inversion

The problem of inverting the surface equations to solve for the parametric coordinates of a given point in physical space is similar to the previously discussed parametric derivative determination in that it is also overspecified. The set of vector equations which must be solved is given by

$$
\vec{V} \equiv \vec{P}(u, v)-\vec{P} *=0,
$$

three equations in two unknowns. Since these equations may not he inverted analytically, an iterative scheme is required. The first variation of the above equation may be written as

$$
\delta \vec{V}=\frac{\partial \vec{P}}{\partial u} \delta u+\frac{\partial \vec{P}}{\partial v} \delta \vec{v}
$$

Given an initial guess (ui, vi) such that $\vec{V}(u i, v i)=\vec{V} i$, the variations in the above equation may be interpretted as

$$
\vec{v}_{i+1}-\vec{v}_{i}=\left(\frac{\partial P}{\partial u}\right)_{i}\left(u_{i+1}-u_{i}\right)+\left(\frac{\partial \vec{P}}{\partial v}\right)_{i}\left(v_{i+1}-v_{i}\right)
$$

Setting $\vec{V}_{i+1}=0$ and taking the appropriate vector products,

$$
\begin{aligned}
& u_{i+1}=u_{i}+\vec{A}_{i} / \vec{N}_{i} \\
& v_{i+1}=v_{i}-\vec{B}_{i} / \vec{N}_{i} \\
& \vec{A}_{i}=\frac{\partial P}{\partial v} i \times \vec{v}_{i} \\
& \vec{B}_{i}=\frac{\partial P}{\partial u} i \times \vec{v}_{i} \\
& \vec{N}_{i}=\frac{\partial P}{\partial u} i \times \frac{\partial \vec{P}_{j}}{\partial v}
\end{aligned}
$$

In the above expressions, the ratio of two vectors is interpretted in the same way as it was in Section 3.1. If $\vec{p}_{*}$ lies on the surface, the solution will tend to converge to the point on the surface closest to $\vec{p}_{*}$. In the latter case, the cutoff criterion may be as simple as exceeding a fixed number of iterations or possibly

$$
\left|\vec{v}_{i+1}-\vec{v}_{i}\right| \leq \varepsilon
$$

### 3.3 Curve-Surface Intersection

The curve-surface intersection problem is the basis for the hunting piase. It differs from the above two problems in that it results in a fully specified system of three equations in three unknowns. Let the curve be given as $\vec{P}_{C}(v)$ and the surface by $\vec{P}_{s}(s, t)$. The system of equations is given by

$$
\vec{V}=\vec{P}_{C}(v)-\vec{P}_{S}(s, t)=0
$$

The first variation of this equation is written as

$$
\delta \vec{V}=\frac{\partial \vec{P}}{\partial v} \delta \delta v-\frac{\partial \vec{P} s}{\partial s} \delta s-\frac{\partial \vec{P} s}{\partial t} \delta t
$$

The variations in the parameters may be isolated by taking appropriate vector and scalar products. First, for example

Furthermore

$$
\frac{\partial \vec{P}_{S}}{\partial v} \cdot\left(\frac{\partial \vec{P}_{s}}{\partial s} \times \delta \vec{v}\right)=\frac{\partial \vec{P}_{c}}{\partial v} \cdot\left(\frac{\partial \vec{P}_{s}}{\partial s} \times \frac{\partial \vec{P}^{2} c}{\partial v}\right) \delta 0-\frac{\partial \vec{P}_{c}}{\partial v} \cdot\left(\frac{\partial \vec{P}_{s}}{\partial s} \times \frac{\partial \vec{P}_{s}}{\partial t}\right) \text { ot }
$$

In a like manner, the iteration is supported by the equations

$$
\begin{aligned}
& v_{i+1}=v_{i}-\left[\frac{\partial \vec{P}_{s i}}{\partial s} \cdot\left(\frac{\partial \vec{P}_{s} i}{\partial t} \times \vec{v}_{i}\right)\right] / D \\
& s_{i+1}=s_{i}+\left[\begin{array}{l}
\partial \vec{P}_{s i} \\
\partial t
\end{array}\left(\frac{\partial \vec{P}_{c}}{\partial v} \times \vec{v}_{i}\right)\right] / D \\
& t_{i}=t_{i}+\left[\frac{\partial \vec{P}_{c} i}{\partial v} \cdot\left(\frac{\partial \vec{P}_{s} i}{\partial s} \times \vec{v}_{i}\right)\right] / D
\end{aligned}
$$

where

$$
D=\frac{\partial \vec{P}^{2}}{\partial V} c i \cdot\left(\frac{\partial \vec{P}^{2}}{\partial s} s i \times \frac{\partial \vec{P}_{s}}{\partial t} i\right\rangle=\frac{\partial \vec{P}^{2}}{\partial V} c i \cdot \vec{N}_{i}
$$

It should be noted that since this system is fully specified, the redundancy as characterized by the ratio of vectors is no longer present.

### 3.4 Surface-Surface Intersection

In Section 2.2 it was iniplied that final convergence in the tracing phase involved an underspecified system, namely three equations in four unknowns.

$$
\vec{V}=\vec{p}_{1}(u, v)-\vec{p}_{2}(s, t)=0
$$

It was also stated that this problem was resolved by holding one of the unknowns constant. This would reduce the problem to a curve-surface intersection already covered in Section 3.3. The criterion used to determine which variable to fix is of some interest. The mast olvious criterion would be to fix the variable which had the smallest derivative with respect to arc length at the previous converged point. A second approach begins by observing the similarity between the overspecified and the underspecified probiem. In both
cases a decision must be made before the computation may proceed. In the overspecified problem the decision was based on maximizing the magnitude of the denominator. By observing that any decision in the underspecified problem reduces it to a fully specified problem whose solution also involves a denominator, the same criterion could be applied. The variable is fixed that provides the maximum denominator in the resulting fully specified problem. This criterion is primarily an intuitive one with the only obvious advantage being avoidance of numerical problems associated with small $|D|$. The resulting four sets of equations are as follows.
I. $\quad u=$ constant: $u_{i+1}=u_{i}$

$$
\begin{aligned}
& v_{i+1}=v_{i}-\left[\frac{\partial \vec{P}_{2 i}}{\partial S} \cdot\left(\frac{\partial \vec{P}_{2 j}}{\partial \dot{t}} \times \vec{V}_{i}\right)\right] / D_{u} \\
& s_{i+1}=s_{i}+\left[\frac{\partial P_{2 i}}{\partial t} \cdot\left(\frac{\partial P_{l i}}{\partial V} \times \vec{V}_{i}\right) / / D_{U}\right. \\
& t_{i+1}=t_{i}+\left[\frac{\partial \vec{P}_{i j}}{\partial v} \cdot\left(\frac{\partial \vec{P}_{2 i}}{\partial S} \times \vec{V}_{i}\right) / / D_{u}\right. \\
& D_{U}=\frac{\overrightarrow{\partial P}}{\partial V} \cdot\left(\begin{array}{cc}
\overrightarrow{\partial P} & \vec{~} \\
\partial S
\end{array} \frac{\partial P}{\partial t}\right) \\
& \mathbf{v}_{\mathrm{i}+1}=\mathbf{v}_{\boldsymbol{i}} \\
& s_{i+}=s_{i}+\left[\frac{\vec{P}_{2 j}}{\partial t} \cdot\left(\frac{\vec{P}_{d j}}{\partial u} \times \vec{V}_{i}\right)\right] / D_{v} \\
& t_{i+1}=t_{i}+\left[\left.\frac{\overrightarrow{\partial P}_{i j}}{\partial u} \cdot\left(\frac{\partial \vec{P}_{2 j}}{\partial S} \times \vec{V}_{i}\right) \right\rvert\, / D_{v}\right.
\end{aligned}
$$

$$
\begin{aligned}
& D_{V}=\frac{\partial \vec{P}_{1 i}}{\partial u} \cdot\left(\frac{\partial \vec{P}_{2 i}}{\partial s} \times \frac{\partial \vec{P}_{2 i}}{\partial t}\right) \\
& \text { iII. } s=\text { constant: } u_{i+1}=u_{i}+\left[\frac{\partial \vec{P}_{11}}{\partial v} \cdot\left(\frac{\partial \vec{P}_{2 i}}{\partial t} \times \vec{v}_{i}\right)\right] / D_{s} \\
& v_{i+1}=v_{i}+\left[\frac{\partial \vec{p}_{2 j}}{\partial t} \cdot\left(\frac{\partial \vec{P}_{j i}}{\partial u} \times \vec{v}_{i}\right)\right] / D_{s} \\
& s_{i+1}=s_{i} \\
& t_{i+1}=t \cdot-\left[\left.\frac{\partial \vec{P}_{l i}}{\partial u} \cdot\left(\frac{\partial \vec{P}_{l i}}{\partial v} \times \vec{V}_{i}\right)\right|_{/ D_{s}}\right. \\
& D_{S}=-\frac{\partial \vec{P}_{i j}}{\partial t} \cdot\left(\frac{\partial \vec{P}_{j i}}{\partial u} \times \frac{\partial \vec{P}_{j i}}{\partial v}\right)
\end{aligned}
$$

IV. $\quad t=$ constant: $u_{i+1}=u_{i}+\left\{\left.\frac{\partial \vec{P}_{1 i}}{\partial v} \cdot\left(\frac{\partial \vec{P}_{2 i}}{\partial s} \times \vec{V}_{i}\right) \right\rvert\, / D_{t}\right.$

$$
\begin{aligned}
& v_{i+1}=v_{i}+\left[\frac{\partial \vec{P}_{2 i}}{\partial s} \cdot\left(\frac{\partial \vec{P}_{1 i}}{\partial u} \times \vec{v}_{i}\right)\right] / \Gamma_{t} \\
& s_{i+1}=s_{i}-\left[\frac{\partial \vec{P}_{1 i}}{\partial u} \cdot\left(\frac{\partial \vec{P}_{1 i}}{\partial v} \times \vec{v}_{i}\right)\right] / \Lambda_{t} \\
& t_{i+1}=t_{i} \\
& D_{t}=-\frac{\partial \vec{P}_{2 i}}{\partial s} \cdot\left(\frac{\partial \vec{P}_{i j}}{\partial u} \times \frac{\partial \vec{P}_{1 i}}{\partial v}\right)
\end{aligned}
$$

### 3.5 Step Size Selection for Tracing Phase

At a point on the intersection curve, the curvature may be expressed in terms of the angular rate of change.

$$
K=-\frac{d \theta}{d \sigma}
$$



FIGURE 3.1-Step Size Selection
By selecting a nominal value of the angular change, the step size $\delta \sigma$ can be obtained from $\delta \sigma=\frac{\delta \theta}{|\mathrm{K}|}$. The tangent vector to the intesection curve $\vec{T}$ is defined by $\vec{T}=\varepsilon \vec{N}_{1} \times \vec{N}_{2}$, where

$$
\begin{aligned}
& \vec{N}_{1}=\frac{\partial \vec{P}_{1}}{\partial u} \times \frac{\partial \vec{P}_{1}}{\partial V}=\text { a vector normal to } S_{1}, \\
& \vec{N}_{2}=\frac{\partial \vec{P}_{2}}{\partial S} \times \frac{\partial \vec{p}_{2}}{\partial t}=\text { a vector normal to } S_{2}, \\
& \varepsilon= \pm 1=\text { a scalar multiplier to correct the sense of } \\
& \text { the tangent vector. }
\end{aligned}
$$

The curvature $|K|$ may be defined as $\left|\frac{d \hat{T}}{d \sigma}\right|$ where $\hat{T}=\vec{T} /|\vec{T}|$. Expanding this expression,

$$
\frac{d \hat{T}}{d \sigma}=\frac{\vec{T}}{|\vec{T}|}\left|\frac{d t}{d \sigma}-\left(\vec{T} \cdot \frac{d \vec{T}}{d \sigma}\right) \hat{T}\right|
$$

Since $\vec{T}$ is a function of all four independent variables ( $u, v ; s, t$ ),
$\frac{1}{\varepsilon} \frac{d \vec{T}}{d \sigma}=\left(\frac{\partial \vec{N}_{1}}{\partial u} \times \vec{N}_{2}\right) u^{1}+\left(\frac{\partial \vec{N}_{1}}{\partial v} \times \vec{N}_{2}\right) v^{1}+\left(\vec{N}_{1} \times \frac{\partial \vec{N}_{2}}{\partial s}\right) s^{1}+\left(\vec{N}_{1} \times \frac{\partial \vec{N}_{2}}{\partial t}\right) t^{1}$

The parametric derivatives ( $u^{1}, v^{1}, s^{l}, t^{l}$ ) are derivable from $\hat{T}$ (see Section 3.1). Expanding this equation yields

$$
\begin{aligned}
\frac{1}{\varepsilon} \frac{d T}{d \sigma}= & {\left[\left(\frac{\partial \vec{p}_{1}}{\partial u^{2}} \times \frac{\partial \vec{P}_{1}}{\partial v}+\frac{\partial \vec{P}_{1}}{\partial u} \times \frac{\partial^{2} \vec{p}_{1}}{\partial u \partial v}\right) \times\left(\frac{\partial \vec{P}_{2}}{\partial s} \times \frac{\partial \vec{P}_{2}}{\partial t}\right)\right] u^{1}+} \\
& {\left[\left(\frac{\partial \vec{p}_{1}}{\partial \vec{p}_{1}} \times \frac{\partial \vec{P}_{1}}{\partial v}+\frac{\partial \vec{P}_{1}}{\partial u} \times \frac{\partial^{2} \vec{p}_{1}}{\partial v^{2}}\right) \times\left(\frac{\partial \vec{P}_{2}}{\partial s} \times \frac{\partial \vec{P}_{2}}{\partial t}\right)\right] v^{1}+} \\
& {\left[\left(\frac{\partial \vec{P}_{1}}{\partial u} \times \frac{\partial \vec{P}_{1}}{\partial v}\right) \times\left(\frac{\partial^{2} \vec{p}_{2}}{\partial S^{2}} \times \frac{\partial \vec{P}_{2}}{\partial t}+\frac{\partial \vec{P}_{2}}{\partial S} \times \frac{\partial^{2} \vec{p}_{2}}{\partial S \partial t}\right)\right] s^{1}+} \\
& {\left[\left(\frac{\partial \vec{P}_{1}}{\partial u} \times \frac{\partial \vec{P}_{1}}{\partial v}\right) \times\left(\frac{\partial \vec{P}_{2}}{\partial s \partial t} \times \frac{\partial \vec{P}_{2}}{\partial t}+\frac{\partial \vec{P}_{2}}{\partial s} \times \frac{\partial^{2} \vec{p}_{2}}{\partial t^{2}}\right)\right] t^{1} }
\end{aligned}
$$

Thus, the step size may be astimated based entirely on the local surface properties.
3.6 Interpolation of the Intersection Point String

The surface intersection computation yields an ordered string of points lying on the intersection curve (within a given tolerance). The string is subdivided into closed curves or loops. The problem to be considered here is that of interpolating this output data and it is sufficient for this purpose to consider a single loop. The number of output points is controlled by the spacing constraints on maximum step size and maximum slope change. The values of these constraining parameters depend heavily on the use intended for the output data. Although a comprehensive study regarding the use of the output data in a variety of applications is beyond the scope of the present effort, it is felt that the issue should be discussed.

Let us begin with the premise that most computer programs are somewhat limited by available storage. The intersection calculation is capable of producing a large amount of output
data. Consider the case of a single loop intersection that produces 100 output points. A single point consists of the following data:

1) ( $x, y, z$ ) - physical coordinates of the point
2) ( $u, v, s, t)$ - parametric coordinates of the point
3) ( $\frac{d u}{d \sigma}, \frac{d v}{d \sigma}, \frac{d s}{d \sigma}, \frac{d t}{d \sigma}$ ) - derivatives of the parametric coordinates with respect to arc length along the intersection curve.

Thus the example case provides 1100 words of data. Obviously some of the data is redundant and could be derived from the other data. There would appear to be a good argument for tailoring the output to fit the application. If a standard, nonredundant output were used, e.g. the physical coordinates, applications which required the other data would have to pay the computational penalty.

Regardless of the particular set of data selected, a decision must be made concerning its output format. In particular, should the data be returned as discrete points or should it be curve fit? Once again, the application will probably provide the answer. Consider the example case where it has been decided to work with the intersection in physical space. The discrete point format would provide 300 words of data. An PC fit of the data, using 10 segments in order to satisfy some tolerance constraint, would produce 120 words of data. Clearly some compromise must be made between core requirements, execution speed and accuracy.

If the application calls for the parametric coordinates there are additional options available if curve fits are desired. All four parametric variable could be defined as functions of a fifth parametric variable monotonically related to arc length along the intersection curve. In this case, the
tolerancing can be carried out in physical space with the error at any given point given as

$$
\delta=|\vec{P}(u, v)-\vec{Q}(s, t)|
$$

where ( $u, v, s, t$ ) are taken at the same value of the independent parameter. A second approach was to treat each of the pairs, ( $u, v$ ) and ( $s, t$ ), as separate problems to be fit independently. The entire fitting and tolerancing is carried out in the parametric plane so that the tolerancing loses its relationship with the physical intersection curve. The first approach provides interpolation with physically meaningful precision, but generally renuires a large number of segments resulting in a large nuantity of data. The second approach sacrifices the controlled precision but significantly reduces the amount of data which must be saved. The question as to which approach is best is highly anplication dependent and really needs further investioation. The second approach is currently incorporated in the subject program.

### 4.0 SOFTWARE DESCRIPTION

The purpose of this section is to describe the computer program written to implement the strategies outlined in the previous sections. The code was written in FORTRAN IV for implementation on the IBM 370 timesharing system, TSO. The input to the program has two forms. First the actual coefficient data for the two surfaces to be operated upon is supplied in the form of formatted input files which are automatically read in by the main program. The second group of data constitutes a set of prompted input parameters to be entered from the terminal by the user. The second set of data consists mostly of tolerances, step sizes, or option flags which are likely to vary from case to case. The program output is written to an output file whose later disposition is a user option.
4.1 TSO Command List

The use of a CLIST attempts to remove the IB: job control language burden from the user by submerging it. The purpose
of the CLIST in the present case is to handle the allocation of the various files used by the program during execution. The CLIST has two positional parameters plus an optional keyword parameter. The CLIST is invoked by the following statement:

EXECUTE NAME 'INT INL OUTPUT(NAMEOUT)'
where NAME is the name given to the CLIST, IN1 and IN2 are the names of the data sets containing the coefficient data for the two surfaces, and NAMEOUT is the name to be given to the output data set. If the optional keyword parameter is not specified, the output data set is given the default name OUT.DATA. For completeness, the CLIST is as follows.

PROC 2 IN1 IN2 OUTPUT(OUT)
FREEALL
ATTR FI BUFNO (1)
ALLOC F(FTOTFOO1) DA(\&IN1, ,DATA) SHR USING(F1)
ATTR F2 BUFNO(1)
ALLOC F(FTO2F001) DA(\&IN2..DATA) SHR USING(F2)
ATTR F3 BUFNO (1)
ALLOC F(FT03F001) USING(F3) NEW BL(3200) SP(20,10)
ATTR F4 BUFMO(1) REC(F B) LR(140) BL(1630)
ALLOC F(FT04F001) DA(\&OUTPUT.,DATA) USING(F4) NEW BL(1680) SP(10,10)
ALLOC F(FT05F001) DA(*)
ALLOC F(FT06F001) DA(*)
CALL NAME LOAD (TEMPNAME)
FREE F(FTOTFOOT)
FREE F(FTO2F001)
FREE F(FTO3F001)
FREE F(FT04F001)
END

### 4.2 Surface Data Files

The surface data consists primarily of parametric bicubic patch coefficients in geometric form. The patches are ordered as indicated in Figure 2.1. For completeness, a parametric bicubic patch may be represented in geometric form by a matrix ot vectors $\vec{B}$ which may be written as

$$
\left.\vec{B}=\left[\begin{array}{lll}
\vec{P}(0,0) & \vec{P}(0,1) & \frac{\partial \vec{P}}{\partial v}(0,0) \\
\vec{\partial}(1,0) & \frac{\partial \vec{F}}{\partial v}(0,1) \\
\vec{P}(1,1) & \frac{\partial \vec{P}}{\partial v}(1,0) & \frac{\partial \vec{P}}{\partial v}(1,1) \\
\frac{\partial \vec{P}}{\partial u}(0,0) & \frac{\partial \vec{P}}{\partial u}(0,1) & \frac{\partial \vec{P}}{\partial u \partial v}(0,0)
\end{array}\right) \frac{\partial^{2} \vec{P}}{\partial u \partial v}(0,1)\right]\left[\begin{array}{ll}
\frac{\partial \vec{P}}{\partial u}(1,0) & \frac{\partial \vec{P}}{\partial u}(1,1) \\
\frac{\partial \vec{P}}{\partial u \partial v}(1,0) \frac{\partial \vec{P}}{\partial u \partial v}(1,1)
\end{array}\right]
$$

The first line of data in the surface data file contains the following information

```
NP = number of patches
ANAM(3) = three word hollerith title
NU = number of patch columns (see Figure 2.1)
NV = number of patch rov's (see Figure 2.1)
```

The second block of information contained in the surface data file is a transformation matrix which is used to orient the surface in space. The matrix is a $3 \times 4$ array whicil may be partitioned into a $3 \times 3$ rotation matrix $R$ and a $3 \times 1$ translation vector $\mathcal{F}$. Since the geometric form of the patch coefficients consists of vectors, the surface may be transformed by transforming the defining vectors. The only noteworthy' feature of implementing the transformation is that all vectors are rotated, but only the corner point position vectors are translated. The transfomation matrix is uritten using FORTRAil (3020.13).
MCDONNELL DOUGLAS CORPORATION

The remainder of the data consist; of actual patch cogfficient data. Each card image is written using FORHAT (3020.13,215,6x, A4). Each card image contains one of the vector elements of the $\vec{B}$ matrix. The two integers indicate the element number and the patch number, while the hollerith text maybe used for identification purposes. The element number varies from 1 through 16 listing the complete $\vec{B}$-matrix, column by column.

### 4.3 Interactive Data

The prompted inputs are supplied by the user as unformatted, list-directed inputs. The following is a list of the prompts supplied by the program along with a brief explanation of the requested data.

1) "INPUT TOLEPANCE" - The tolerance is supplied in units of the surface definition, e.g., inches or centimeters. This number controls the accuracy of each calculated point relative to the true intersection curve.
2) "INPUT SMAX AND TMAX" - SMAX is the maximum allowable step size in arc length between the calculated intersection points. This number acts as a constraint on the automatic step stize selertor algorithm described in Section 3.5. TMAX is the desired slope change between consectitive calculated points. In the notation of Section 3.5, $\delta \theta=$ TMAX.
3) "IMPUT NGRID AMD NSTEP" - NGPID establishes the hunting grid described in Section 2.1 (in the notation of Section 2.1. NGRID $\equiv N_{G}$ ). NSTEP establishes the number of steps along each curve (in the notation of Section 2.1, NSTEP $\equiv "_{T_{9}}$ ).
4) "Input output flag (1 ge iof le 3)"

$$
\begin{aligned}
\text { IOF }=1 & \text { - the output array consists of } \\
& \text { ordered quadrunles representing } \\
& \text { the parametric coordinates of the }
\end{aligned}
$$

intersection points.
10F=2 - the output array consists of parametric cubic curve fit coefficients (neometric form) for the parametric coordinates along the intersection curve. The curve segments are ordered such that all of the loops for surface 1 are given followed by all of the loops for surface 2 .

10F=3 - the nutput array consists of ordered triples giving the physical coordinates along the intersection curve.

### 4.4 Output Data

The output data file consists of two parts. The first card image is written using FORMAT (1215) and consists of the following integer data

```
NW = number of words of output data
LOOP = number of loons in intersection (\leq3)
NPL(3) = numbt.' of points in each loop
MCL}(3,2)= number of curve segments for each loo
    for each surface (IOF=2)
```

The second portion of the output file consists of the array $A O(I), I=1$, NW. For $10 F:=1$ or 2 , the output format is (IH , 6 E 12 5). The output array may be decoded as follows

IOF=1 - Each set of four numbers represents a point ( $u, v, s, t$ )

10F=2 - As mentioned above, the intersection curves on surface 1 precede the curves on surface 2. gach segment is defined by a group of 8 numbers, i.e. $u_{0}, v_{n}, u_{1}, v_{1}, u_{0}^{0}, v_{0}^{o}, u_{i}^{i}, v_{i}^{i}$
where the primes indicate derivatives and the subscript denotes the value of the parametric independent variable.

IOF=3 - Each set of three numbers represents a point in physical space ( $x, y, z$ ).

### 5.0 EXAMPLES

Two examples were chosen in order to verify the software and to serve as benchmark cases. The first example involves a simplified niouciel of a wing and a fuselane. Due to nomal aircraft symmetry, only half of the configuration was modeled. The first example would then produce a single, closed loop intersection curve. In order to test the program for multi-loop intersections, the second example was taken as a pair of orthogonal circular cylinders of different diameter.

### 5.1 Wing-Fuselage Intersection (Example 1)

The wing-fuselage combination selected for the first example is shown in Figure 5.1. The fuselage surface (symmetrical half) was modeled as a $4 \times 3$ mesh of parametric cubic patches. The input file for the fuselage surface is given by Table 5.1. The wing surface was modeled as a $6 \times 2$ mesh of patches and its input file is shown by Table 5.2. The proaram was run with the following set of inouts: TOLERANCE $=0.0001$ inches, SMAX $=2.0$ inches, THAX $=0.1$ radian, $\operatorname{MGRID~}=3$, NSTEP $=10$. The output file for $\mathrm{IOF}=1$ is given by Table 5.3. Figure 5.2 shows the intersection plotted in the parametric plane for both surfaces. This example illustrates a rather important aspect of the hunting phase, nameiy which surfaces is designated as the primary surface. If the wing were translated vertically by a small distance, and if the fuselaṇe were taken as the primary with NGRID=3, no intersection points would be found. To guard against this possibility and to remove the burden of designating one surface as primary, both surfaces are run

FIGURE 5.1-Uing-Body Combination


FIGURE 5.2 - Ming-Fuselage Intersection
through the hunting phase in the primary role. The surface locating the most intersection points is then carried throunh as the primary surface for the remainder of the computation.

### 5.2 Cylinder - Cylinder Intersection (Example 2)

This example was selected for its simplicity in illustrating the multi-loop capability of the program. Figure 5.3 depicts the cylinder geometry.


FIGURE 5.3 - Intersecting Cylinders
$s$


Each cylinder is modeled as an $8 \times 1$ mesh of parametric cubic patches. The surface input files for CYLINDERI and CYLINDER2 are given by TABLES 5.4 and 5.5. The program was run using the same input parameters as the first example with the output file for $10 F=1$ given in Table 5.6. Figure 5.4 illustrates the solution in the parametric plane.

| 12 | AMSTPOO: SUA | ACE 1 | FusElage |  |
| :---: | :---: | :---: | :---: | :---: |
| 0.100 | $00000000000+31$ | 0.0 |  | 0.0 |
| 0.0 |  | 0.10 | $00000000000+01$ | 0.0 |
| 0.0 |  | 0.0 |  | 0.10 |
| 0.0 |  | 0.0 |  | 0.0 |


|  |  | 00001 <br> 00002 <br> 00003 <br> 00004 <br> 00005 |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
| 1 | 1 | 00008 |
| 2 | 1 | 00007 |
| 3 | 1 | 00008 |
| 4 | 1 | 00009 |
| 5 | 1 | 00010 |
| 6 | 1 | 00011 |
| 7 | 1 | 00012 |
| 8 | 1 | 00013 |
| 9 | 1 | 00014 |
| 10 | 1 | 00015 |
| 11 | 1 | 00016 |
| 12 | 1 | 00017 |
| 13 | 1 | 00018 |
| 14 | 1 | 00019 |
| 15 | 1 | 00020 |
| 16 | 1 | 00021 |
| 1 | 2 | 00022 |
| 2 | 2 | 00023 |
| 3 | 2 | 00024 |
| 4 | 2 | 00025 |
| 5 | 2 | 00026 |
| 6 | 2 | 00027 |
| 7 | 2 | 00028 |
| 8 | 2 | 00029 |
| 9 | 2 | 00030 |
| 10 | 2 | 00031 |
| 11 | 2 | 00032 |
| 12 | 2 | 00033 |
| 13 | 2 | 00034 |
| 14 | 2 | 00035 |
| 15 | 2 | 00036 |
| ! 6 | 2 | 00037 |
| 1 | 3 | 00038 |
| 2 | 3 | 00039 |
| 3 | 3 | 00340 |
| 4 | 3 | 00041 |
| 5 | 3 | 00042 |
| 6 | 3 | 00043 |
| 7 | 3 | 00944 |
| 8 | 3 | 00045 |
| 9 | 3 | 00046 |
| 10 | 3 | 00047 |
| 11 | 3 | 00048 |
| 12 | 3 | 00949 |
| 13 | 3 | 00050 |
| 14 | 3 | 00951 |
| 15 | 3 | $0.005 ?$ |

$-0.17038177474860-32-0.86736173798840-18-0.30814879110200-32$ $-0.138777815106 \mathrm{RD}-16-0.96732301297410-18-0.13977781510680-16$ $-0.1353954159478 D-160.23693614$ A4291D-16-0.4300191095336D-16 $-0.1286234447609 \mathrm{AD}-160.16924083750730-170.9773060184701 \mathrm{D}-17$ 0.173811472 PRP6D-17 $0.16000000000000+020.27000000000000+01$ $0.19092000000000+910.16000000000000+020.19092900000000+01$ $0.21935476362410+040.27237326977180-14-0.14613231017670-01$ $0.15046196284490+01-0.3469446951950 \mathrm{D}-17-0.1500426049154 \mathrm{D}+01$ $-0.10583566613330-160.15748201651030+02 \quad 0.41763750419300+01$ $0.29531979262530+010.15748201471500+020.29531979262530+01$ $0: 33937757026690+010.30527985125170-06-0.22702361195660-01$ $0.23269667499100+010.21805801160510-07-0.2320489400848 \mathrm{D}+01$ $0.65586372230720-170.16162160157690+020.16419954368560+01$ $0.11892810275180+010.16162160133730+020.11892810275180401$ $0.13670770182710+0110.28650448719250-07-0.8956321374306 \mathrm{D}-02$ $0.93712693762270+90 \quad 0.2046476767450 \mathrm{D}-0 \mathrm{O}-0.93449874012920+00$ -0.138777 A15106RD-16-0.8673230129741D-13-0.13R777R151068D-16 $-0.61629758220390-32-0.86736173798940-18 \quad 0.12095915646460-31$ -0.128623444760RD-16 0.16924083750730-17 0.97730601847010-17 $-0.56051118350630-31-0.1640741$ 月49741D-30 0.13791540129630-16 $0.19092000000000+010.1600000000000 D+020.19092000000000+01$ $0.27000000000000+110.1600000000000 \mathrm{D}+020.44692635926930-15$ $0.15046196284490+01-0.34694469519500-17-0.15004260491540+01$ $-0.4174265468233 D-150.78409501114150-15-0.21202999439000+01$ $0.2953197926253 \mathrm{D}+0 \mathrm{i}$ : $0.1574820147150 \mathrm{D}+020.29531979262530+01$ $0.41763750419300+010.15748201651030+020.1453$ R466A4D19D-14 $0.23269667499100+01 \quad 0.2!R 05 R 01160510-07-0.2329489400$ A48D+01 $0.99900063508410-150.27339157466310-14-0.32902209729650+01$ $0.11892910275 \mathrm{~F} \overline{\mathrm{Nu}}+01 \mathrm{0} 0.1616216013373 \mathrm{O}+020.1189291027518 \mathrm{O}+01$ $0.16819854368560+010.1616916015769 \mathrm{D}+02-0.8394777791803 \mathrm{D}-16$ $0.93712693762270+970.20464767674500-08-0.93449874012920+00$ $0.63954403589180-150.6409604587745 \mathrm{D}-14-0.13212960554940+01$ -0.6162975 R2? $039 \mathrm{D}-72-0.86776173798840-18$ 0.1209591564646D-31 -0.138777 A15106AD-18-0.86732301297410-18 0.138777A151068D-16 $-0.56051118350670-31-0.16407418493410-300.13291540129630-16$ 0.1286334447609D-18-0.169240A375077D-170.9773060184701D-17 $0.27000000000000+910.16000000000000+070.44693635926930-15$ $0.19092000000000+010.16000000000000+0 ?-0.1909 ? 000000000+01$ $-0.41742654682730-156.78409501114150-15-0.21292999439000+01$ $-0.15046196284490+n 1-0.77715611773760-15-0.15004260491540+01$ $0.41763750419700+210.15748701651070+020.1453946694019 \mathrm{D}-14$ $0.79571979262570+91 \quad 0.1574870147151010+07-0.7953197926253 \mathrm{D}+01$ $0.99900063508410-150.77739157466310-14-0.32802709729650+01$ $-0.77269667499100+01-0.2180579970$ ?77D-07-0.7720489400848D+01 $0.1681985436 \mathrm{AF} 6 \mathrm{D}+910.16: 47160157690+07-0.87947777918030-16$


$-0.9371269376727 D+00-0.20464594634650-08-0.93449874012920+00$ $-0.138777815106 \mathrm{BD}-16-0.86732301297410-180.1787778151068 \mathrm{D}-16$ $0.14217004870070-31-0.86736173798840-18-0.30814879110200-32$ $0.1286234447608 D-16-0.1692408275073 D-170.97730601847010-17$ $0.1353954159478 \mathrm{D}-16-0.2769361484291 \mathrm{D}-16-0.4300191095336 \mathrm{D}-16$ $0.1909200000000 D+01 \quad 0.16000000000000+02-0.19092000000000+01$ $0.13903808659950-140.16000000000000+0 \geq-0.27000000000000+01$ -0.150461962 A449D+01-0.7771561172376D-15-0.1500426049154D+01 $-0.21935476362410+01-0.26645352591000-14-0.14617231017670-01$ $0.29531979262530+010.15748 \geq 01471500+02-0.29531979262530+01$ $0.20553538078430-150.1574 R 201651030+02-0.41763750419300+01$ $-0.2326966749910 \mathrm{D}+01-0.2180579920237 \mathrm{D}-07-0.232048940084 \mathrm{RD}+01$ $-0.33937757026690+01-0.305279850 .79940-06-0.22702361195680-01$ $0.118928102751 \mathrm{BD}+010.1616216013377 \mathrm{D}+02-0.118928102751 \mathrm{BD}+01$ $0.16452484067560-140.16162160157690+02-0.16919854368560401$ $-0.93712693762270+00-0.2046458463465 \mathrm{D}-08-0.9344987401292 \mathrm{D}+00$ $-0.13670770182710+01-0$. ? 26504543 A439D-07-0. $89563213743020-02$ $0.1738114728286 \mathrm{D}-17 \quad 0.16000000000000+020.27000000000300+01$ $0.1909200000000 D+01 \quad 0.1600000000000 D+020.19092000500000+01$ $0.21935476362410+010.27237326977180-14-0.14613231017670-01$ $0.15046196284490+01-0.3469446951950 D-17-0.15004260491540+01$ $0.34681752449440-17 \quad 0.32905000000007+020.38000000000000+01$ $0.2687000000000 \mathrm{D}+01 \quad 0.32905000000000+020.2687000000000 \mathrm{D}+01$ $0.3087572360395 \mathrm{D}+01-0.3226672401491 \mathrm{D}-13-0.2074350912606 \mathrm{D}-01$ $0.21173773182150+01-0.18700319671030-14-0.2111489357288 \mathrm{D}+01$ $0.6843036024970 \mathrm{D}-170.16862991569550+020.1754920503514 \mathrm{D}+01$ $0.12408512070910+010.1686299154455 D+020.12408512070910+01$ $0.14263568736550+010.29$ ค92903744530-07-0.93446897169400-02 $0.97776308949720+000.21352170936130-08-0.97502092683190+00$ $-0.31690233071230-17 \quad 0.16941674748180+02 \quad 0.49379927503340+00$ $0.3491795715505 \mathrm{D}+000.1694167475367 \mathrm{D}+020.3491795715505 \mathrm{D}+00$ $0.4006560712960 \mathrm{D}+00-0.1712694839741 \mathrm{D}-07-0.2 \mathrm{P} 24234211722 \mathrm{D}-02$ $0.27533392275070+00-0.12233526711520-08-0.2745807576166 \mathrm{D}+00$ $0.19092000000000+010.16000003000000+020.19092000000000+01$ $0.27000000000000+01 \quad 0.16000000000000+020.446926 .35926930-15$ $0.15046196284490+01-0.34694469519500-17-0.15004260491540+01$ $-0.41742654682330-150.78409501114150-15-0.21202999439000+01$ $0.26870000000000+010.32905000000000+070.26870000000000 * 01$ $0.38000000000000+01 \quad 0.32905000000000+02 \quad 0.44692217026400-15$ $0.21173773182150+01-0.18700319071070-14-0.2111489357288 D+01$ $0.2527199630957 \mathrm{D}-15 \quad 0.1221245327089 \mathrm{C}-14-0.2984354595898 \mathrm{C}+01$ $0.12408512070910+010.16862991544550+i) 20.12408512070910+01$ $0.17549205035140+010.16 R 67991569550+02-0.97587962098050-16$ $0.97776308949720+010.2135 ? 170876170-08-0.97502097687190+00$ $0.66727625393990-150.66875409643730-14-0.1378496$ A319860+01 $0.3491795715505 D+1300.16941674757670+020.34917957155050+00$ $0.49379927503340+000.1694167474 \mathrm{R1RD}+0 \geq-0.69548$ R9R43403D-17 $0.27537397275070+00-0.12773536711520-08-0.27458075761660+00$ $0.99681320 R 71540-160.10104675994120-17-0.3 A 744983176700+00$ $0.27000000000000+010.1600000000000 \mathrm{D}+0$ ? $0.44 \mathrm{F9} 263592693 \mathrm{D}-15$ $0.19092000000000+010.16000003000000+02-0.19092000000000+01$ $-0.41742654687330-150.78409501114150-15-0.71202999439000+01$ $-0.15046196284490+01-0.77715611777760-15-0.1500426049154 \mathrm{D}+01$

## 00053

00054 00054 00055 0005 00057 00058
$0.3800000000000 \mathrm{D}+010.3290500000000 \mathrm{D}+02.0 .44692217026400-15$ $0.26870000000000+010.32905000000000+02-0.26870000000000+0$ $0.2527199630957 \mathrm{D}-150.12212453270 \mathrm{RRD}-14-0.29843545958980+01$ $-0.211737731 \mathrm{R} 215 \mathrm{D}+010.1776356839400 \mathrm{D}-14-0.2111489357288 \mathrm{D}+01$ $0.17549205035140+010.16862991569550+02-0.87587962098050-16$ $0.12408512070910+010.16862991544550+02-0.12408512070910+01$ $0.5672762579399 \mathrm{D}-150.6687540964377 \mathrm{D}-14-0.1378496831986 \mathrm{D}+01$ $-0.97776308949720+00-0.21351979859220-08-0.9750209268319 \mathrm{D}+00$ $0.49379927503340+000.1694167474 \mathrm{Bl} 1 \mathrm{BD}+02-0.69548 \mathrm{B9B43403D-17}$ $0.3491795715505 D+00 \quad 0.16941674753620+02-0.34917957155050+00$ 0.996 11320R71540－16 0．101046258941 20－13－0．3874498312670D＋00 $-0.27533392275070+000.1227369833856 \mathrm{D}-0 \mathrm{~A}-0.2745807576166 \mathrm{D}+00$ $0.19092000000000+010.16000000000000+02-0.19092000000000+01$ $0.13903808659950-140.16000000000000+02-0.27000000000000+0$ $-0.1504619628449 D+01-0.77715611723760-15-0.1500426049154 \mathrm{D}+0$ $-0.21935476362410+01-0.26645352591000-14-0.1461323101767 \mathrm{D}-01$ $0.2687000000000 \mathrm{D}+01 \quad 0.32905000000000+02-0.26870000000000+01$ 0.695624113 RG67D－15 $0.32905000000000+02-0.38000000000000+01$ $-0.21173773182150+010.17763568394000-14-0.21114893572 \mathrm{BPD}+01$ $-0.3087572360395 D+010.29421709430400-13-0.2074350912606 \mathrm{D}-01$ $0.1240851207091 \mathrm{D}+010.16862991544550+02-0.12408512070910+0$ $0.1716590464532 \mathrm{D}-140.168 \mathrm{~A} 799156955 \mathrm{D}+02-0.1754920503514 \mathrm{D}+01$ $-0.9777630894972 \mathrm{D}+00-0.2175197985972 \mathrm{D}-0 \mathrm{O}-0.9750209268319 \mathrm{D}+00$ $-0.14263568736550+01-0.29892809255320-07-0.9344689716937 \mathrm{D}-02$ $0.34917957155050+000.16941674753620+02-0.34917957155050+00$ $0.1817400419198 D-150.1694167474 \mathrm{AlRD}+02-0.49379927503340+00$ $-0.27533392275070+000.12 ? 3369833856 \mathrm{D}-08-0.27458075761660+00$ $-0.4006560712960 \mathrm{D}+000.17126955617350-07-0.2824274211725 \mathrm{D}-02$ $0.34691752449440-170.37905000000000+020.3800000000000 \mathrm{D}+01$ $0.26870000000000+010.32905000000000+020.26870000000000+01$ $0.3087572360795 \mathrm{D}+01-0.32266724014910-13-0.20743509126060-01$ $0.21173773182150+01-0.18700319071030-14-0.7111489357288 \mathrm{C}+01$ $-0.86732199604230-1 R 0.81650000000000+020.20600000000000+0$ $0.14566000000000+010.81650000000000+020.14566000000000+01$ $0.1673831442859 D+010.35877550970150-13-0.1104410777057 \mathrm{D}-01$ $0.1147918066360 D+010.7792377854088 D-14-0.11447054083310+01$ -0.91324 R 2 RR7193D－17 $0.48 R 7248 R 99698 \mathrm{D}+070.1423029897007 \mathrm{D}+01$ $0.1006 ? 640855590+010.488 ? 248901 ? 67 D+020.1006265085559 D+01$ $0.17546099735230+01-0.49356410778990-07-0.81388732692110-02$ $0.7934568 \geq 252 \mathrm{ADD}+00-0.3525455612536 \mathrm{D}-0 \mathrm{R}-0.79128635254670+00$ $-0.23136491968140-160.4843770 ? 267970+0 ?-0.7179051282+310+01$ $-0.50770647730410+010.48473701940190+0 ?-0.50770647330410+01$ $-0.5833670084 ? 170+010.11897751699100-06$ 0．3R74599612559D－01
 $0.26870000000000+010.32905000000000+0 ? 0.76870000000000+01$ $0.3800000000000 \mathrm{D}+010.3290500000000 \mathrm{D}+020.4469221702640 \mathrm{D}-15$ $0.21177777182150+01-0.18700719071030-14-0.21114997572890+01$ 0． $2527199630 R 57 D-15$ 0．12？17457270RRD－14－0．？9R4754595R9AD＋01 $0.14566000000000+010.81650000000000+020.14566000000000+01$ $0.20600000000000+01$ 0．R1650000000000＋0？ $0.44693635992640-15$
 $0.4559015835398 D-15-0.50515147670440-14-0.16179793323200+01$ 0．1006？650R5559D＋01 0．4RR？？48901767D＋0つ 0．10062650R5959D＋01

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0015 ？ 00153 00154 00155 00156 00157
00159
$0.1423029897007 \mathrm{D}+01.0 .4882248899698 \mathrm{D}+02-0.2004258952564 \mathrm{D}-16$ $0.7934569225280 \mathrm{D}+00-0.3525455612536 \mathrm{D}-0 \mathrm{R}-0.7912963525467 \mathrm{D}+00$ $0.2872614581376 \mathrm{D}-15 \quad 0.29119493430670-13-0.11165522538410+01$ $-0.507706473304 \mathrm{i} D+010.4843370194019 \mathrm{D}+02 \mathrm{D}-0.50777647330410+01$ $-0.71790512824310+01 \quad 0.48433702267970+02-0.56351979511060-18$ $-0.4000139378942 \mathrm{D}+010.8495147221915 \mathrm{D}-080.39890045238700+01$
$0.12305430013510-130.2646678338856 \mathrm{D}-120.56387692058780+01$
$0.38000000000000+010.32905000000000+020.44692217026400-15$
$0.26870090000000+01 \quad 0.32905000000000+02-0.26870000000000+01$ $0.25271996308570-15 \quad 0.12212453270 R A D-14-0.2984354595898 D+01$ $-0.2117377318215 \mathrm{D}+01 \quad 0.1776356839400 \mathrm{D}-14-0.21114893572880+01$ $0.20600000000000+01 \quad 0.81650000000000+020.44693635992640-15$ $0.1456600000000 \mathrm{D}+010.8165000000000 \mathrm{D}+02-0.14566000000000+01$ $0.455907593539 R D-15-0.50515147620440-14-0.16178393323200+01$ $-0.1147916066360 D+01-0.10658141036400-13-0.1144705408331 D+01$ $0.1423029897007 \mathrm{E} 401 \quad 0.48822489996980+02-0.2004258952564 \mathrm{D}-16$ $0.1006265085559 \mathrm{D}+01 \quad 0.48 \mathrm{R} 2248901267 \mathrm{D}+02-0.1006265085559 \mathrm{D}+01$ $0.2872614581376 \mathrm{D}-150.2911949343067 \mathrm{D}-13-0.1116552253841 \mathrm{D}+01$ $-0.7934569225280 D+000.3525505071986 \mathrm{D}-08-0.7912863525467 \mathrm{D}+00$ $-0.71790512824310+010.48433702267970+02-0.56351979511060-18$ $-0.50770647330410+01 \quad 0.48433701940190+020.50770647330410+01$ $0.1230543001351 \mathrm{D}-130.264667833 \mathrm{AR560}-120.563976920587 \mathrm{BD}+01$ $0.4000139778942 D+01-0.84948480700900-080.39890645238700+01$ $0.26872000000000+010.32905000000000+02-0.26970000000000+0$ $0.6956241138667 \mathrm{D}-150.32905000000000+02-0.38000000000000+0$ $-0.211737731 R 215 D+0110.1776356839400 \mathrm{D}-14-0.2111489357288 \mathrm{CO}+01$ $-0.3087572760395 \mathrm{D}+010.2842170943040 \mathrm{D}-13-0.2074350912608 \mathrm{D}-01$ $0.14566000000000+010.81650000000000+02-0.14566000000000+01$ $0.2341876692569 D-150.81650000000000+02-0.20600000000000+01$ $-0.11479160663600+01-0.10658141036400-13-0.1144705408331 D+0$ $-0.16738314428590+01-0.42632564145610-13-0.11044107770570-01$ $0.10062650855590+010.4$ RR22489012670 二 斤 2-0.10962650R55590+01 $0.52373817857430-150.48 R 2248999698 \mathrm{D}+02-0.14230298970070+01$ $-0.793456 R 2252800+000.35255050719860-08-0.79128635254670+00$ $-0.11546099735230+010.4935643108541 \mathrm{D}-07-0.8138873268221 \mathrm{D}-02$ $-0.50770647730410+010.48433701940190+020.50770647330410+01$ $-0.67922107121170-14 \quad 0.48433702267970+020.7179051282431 D+01$ $0.40001797789420+01-0.84948480700900-080.79890045238700+01$ $0.58336709842170+01-0.11892758007230-060.3874589612561 D-01$

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00169 00170 00171 00171 00172 00173 00174 00175 00176 00177 00178 00178 00179 00180 00181 00182 00183 00184 00184
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00023 00023 00024 00025 00026 00027 00028 00028 00029 00030 00031 00032 00033 00033 00034 00035 00036 00037 00038 00039 00039 00040 00042 00043 000914 000414
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## 00097

$1.80000000000000+01 \quad 3.07500000000000+01-1.3877787807814 \mathrm{D}-17$
$1.80000000000000+012.47500000000000+015.20749182624620-01$ $-1.3647469753706 \mathrm{D}-15-6.7990378914859 \mathrm{D}+00 \quad 7.4849447248738 \mathrm{D}-01$ $-4.0873132764907 \mathrm{D}-16-5.20096210851400+002.93003897761860-01$ 9. $3999999999996 \mathrm{D}+00$ 8. 22499999999940 $+00-1.3877787807814 \mathrm{D}-17$ $9.3999999999996 \mathrm{D}+009.00833333373270+00-6.79866988426560-02$ $-1.3647469753706 \mathrm{D}-15$ 8. 8755 ? $16916818 \mathrm{C}-01-9.7720111685848 \mathrm{D}-02$ 4. $7800782386678 \mathrm{D}-166.79014497500410-01-3 . \mathrm{B25} 32859994640-02$ 9. $3999999999996 \mathrm{D}+00$ 8. $2249999999994 \mathrm{D}+00-1.3877787807814 \mathrm{D}-17$ 9. $3999999999996 \mathrm{D}+00$ 9.0083333733327D $+00-6.7986698842656 \mathrm{D}-\mathrm{GZ}$ $-1.3647469753706 \mathrm{D}-15$ 8. 8765216916618D-01-9.77201116858480-02 4.7800782386078D-16 6.7901449750041D-01-3.8253285999464D-02 B. $6000000000004 \mathrm{D}+001.57416668606670+015$. RB73588146728D-01 $8.60000000000040+00 \quad 1.0315000000000 \mathrm{D}+01 \quad 3.3974841151908 \mathrm{D}-01$
4. $30255780000030-16-4.691 ? 9797028700+002.64290984380400-01$ $-1.60564340867310-15-7.47244587232440+00-1.11164571523170 * 00$ $1.80000000000000+012.47500000000000+015.20749182624620-01$ 1. $80000000000000+01 \quad 1.99500000000000+01 \quad 3.00514565471420-01$ $-3.26103136346870-16-4.149547492145 \mathrm{AD}+00 \quad 2.33770895275050-01$ $-1.43457744133610-14-6.60953443252 \geqslant 90+00-9.83273851310590-01$ 9. $3999999999996 \mathrm{D}+009.00833333333270+00-6.79$ R66988426560-02 $9.3999999999996 \mathrm{D}+009.6349999999994 \mathrm{D}+00-3.9$ ? $33846047656 \mathrm{D}-02$ 3. $1137485437176 \mathrm{D}-165.41746478141240-01-3.05200891053530-02$ $-3.02612656186290-158.62911434801560-011.28371863921100-01$ $9.39999999999960+009.00833333333270 \div 00-6.79866988426560-0 \bar{z}$ $9.39999999999960+00 \quad 9.63499999999940+00-3.92338460476560-02$ 3. $\mathrm{A} 1374854371760-165.41746478141240-01-3.05200891053530-02$ -3.0?612656186290-15 8.62911439801560-01 1.2R371863921100-01 $8.60000000000040+001.03150000000000+013.3974841151908 \mathrm{D}-01$ 8. $6000000000004 \mathrm{D}+008.95833333333360+00$ 2.4698257185492D-15 $-4.13371734596860-16-1.92377578691190+00-2.86192385616020-01$ -9. BR9616369873フD-17 0.

1. R0000000000000 + $011.99500000000000+013.00514565471420-01$
2. $20000000000000 \mathrm{D}+011.87500000000000 \mathrm{D}+012.19805237397950-15$ $-3.6933092500728 \mathrm{D}-15-1.70161986065 \mathrm{C}$ ? $\mathrm{O}+00-2.5314314206822 \mathrm{E}-01$ - R. $3243533473033 \mathrm{D}-17 \quad 0.0$ -3.24200277173020-01
$9.3999999999996 \mathrm{D}+009.6349999999994 \mathrm{D}+00-3.9233846047656 \mathrm{D}-02$ $9.39999999999960+009.79166666666600+00-2.71773344569690-16$ $-7.79074095324540-162.22155926252720-01 \quad 3.30492435477940-02$ 9.8896163698732D-17 0.0 4. 23261477975870-02
3. $3999999999996 \mathrm{D}+009.6349999999994 \mathrm{D}+00-3.9233846047656 \mathrm{D}-02$ 9. $39999999999960+009.79166666666600+00-2.7177334456969 \mathrm{D}-16$ $-7.79074095324540-162.22155926252720-01 \quad 3.3049243547940-02$ 9. 8R96163698772D-17 0.0 4. 2326147297587D-02
 8. $60000000000040+001.03150000000000+01-3.3974941151908 \mathrm{D}-01$ -9. 889616369877 ?D-17 0.0 $-3.66526424470610-01$
7.17175057613R2D-16 1.9737757869119D+00-?. A619238561602D-01 1.90000000000000+01 1.87500000000000 +01 2.19805?277397950-15 $1.80000000000000+011.99500000000000+01-3.00514565471420-01$ $-8,32435334730330-17 \quad 0.0$ -3. $24200277173020-01$
$1.79 R 67950957170-15 \quad 1.70161986665970+00-7.53147147068230-01$
$9.39999999999960+00$
$9.79166666666600+00-7.7!777344569690-16$

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00132 00133 00134 00135 00136 00137 00138 00139 00140 001 : 00142 00143 00144 00145 00146 00147 00148 00149 00150 00151 00152
00153 00154 00155 00156 00157 00158
$9.39999999999960+009.63499999999940+00 \quad 3.02338460476560-02$ 9.88961636987320-17 0.0
4. 232 в14729758
-1.1560564517710-15-2.2215592625272D-01 3. 3049243547795D-02 9. $39999999999960+009.79166666666600+00-2.71773344569690-16$ $9.3999999999996 \mathrm{D}+00 \quad 9.6349999999994 \mathrm{D}+00 \quad 3.92338460476560-02$ 9. 889616369873 ? D-17 0.0 4. 23261472975870-02
$-1.11560564517710-15-2.22155926252720-013.30492435477950-02$ 8. $6000000000004 \mathrm{D}+001.0315000000000 \mathrm{D}+01-3.3974941151908 \mathrm{D}-01$ A. $60000000000040+001.5741666666667 \mathrm{D}+01-5.8873588146728 \mathrm{D}-01$ 2. 7856945 RR4546D-15 7.4724458723?44D+00-1. $11164571523170+00$
$-1.7686949767966 \mathrm{D}-154.69129397028700+00 \quad 2.64290984380410-01$ 1. $80000000000000+01 \quad 1.99500000600000+01-3.00514565471420-01$ $1.8090000000000 D+01,2.47500000000000+01-5.2074918262462 \mathrm{D}-01$ $6.98634515834650-156.60953443252290+00-9.83273851310620-01$ $0.0 \quad 4.14954749714590+00 \quad 2.33770895275060-01$ $9.3999999999996 \mathrm{D}+00 \mathrm{9.61499999999940+00} \mathrm{\quad 3.9233846047656D-02}$ $9.3999999999996 \mathrm{D}+00$ 9.0083333333327D+00 6. $7986698842656 \mathrm{D}-02$ $-4.3333026931516 \mathrm{D}-15-8.6791147980157 \mathrm{D}-011.2877186392110 \mathrm{D}-01$ $1.41495598143730-15-5.4174647$ A14124D-01-3.05200891053550-02 $9.39995999999960+009.63499999999940+00 \quad 3.9233846047656 \mathrm{D}-02$ $9.39999999999960+00$ 9.0083333333327D+006.79866988426560-02
-4 . 33370269315160-15-8.62911439801570-01 1. 28371863921100-01 1. $41495598143730-15-5.4174647$ A14124D-01-3.05200891053550-02 8. $6000000000004 \mathrm{D}+001.57416666666670+01-5$. 8 R $735 \mathrm{BR146728BD-01}$ 8. $60000000000040+00$ ?. $25750000000000+01-1.20736753927990-15$ $-2.216847 R 7877460-15 \quad 5.87997660601440+00 \quad 3.31257178761390-01$ 1. $77347830301970-15$ 7.6月669006065?10+00 8.46?14584173220-01 $1.80000000000000+01 ? .47500000000000+01-5.20749182624620-01$ 1. $20000000000000+013.07500000000000+01-1.16573417585640-15$ 0.0 5. $20096 ? 10851400+00 \quad 2.93003892761870-01$ 0.0 6.799037R9148590+00 7.4R494472487370-01 $9.3999999999996 \mathrm{D}+009.0087337333327 \mathrm{D}+006.79866988426560-02$ 9. $3999999999996 D+00$ R. $37499999999940+004.16333634234430-17$ 1.77347830701970-15-6.79014497500410-01-3.82532R59994660-02 0.0
$-8.87652169166190-01-9.77201116858460-02$ $9.79999999999960+009.00877333733270+006.7986698842656 \mathrm{D}-02$ $9.39999999999960+00$ \&. $27499999999940+004.16333634234430-17$ $1.77747870301970-15-6.79014497500410-01-3.82537859994680-02$ 0.0 -R.R765?169166190-01-9.77201116858480-02
$\begin{array}{ccccccccccc}176 & 1 & 0 & 44 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0 & & & 0.474902450+00 & 0.200000000+01 & 0.229496740+01\end{array}$ $0.243703475+000.472376340+000.193684910+010 . \Sigma 25525700+01$ $0.500160170+000.467959460+000.1$ 1月RO37 $050+010.221566850+01$ $0.772336570+000.462189770+000.183162620+010.21762474 \mathrm{D}+01$ $0.106438900+010.455931520+000.179393710+010.213872650+01$ $0.146008050+010.449682240+000.177858780+010.209728640+01$ $0.183058 \mathrm{~S} 6 \mathrm{D}+010.4457897 \mathrm{RD}+000.18269468 \mathrm{O}+010.205139210+01$ $0.211837780+01 \quad 0.443572200+000.189249040+010.202052320+01$ $0.237106830+010.442626160+000.19707 A 170+010.201005950+01$ $0.258789200+010.441873590+000.194708450+010.200247680+01$ $0.273977710+010.441457830+000.196629260+010.199580340+01$ $0.283381 \mathrm{~A} 2 \mathrm{D}+01 \quad 0.44127054 \mathrm{D}+000.197 \mathrm{R} 39950+010.199096770+01$ $0.289093480+010.441191240+000.198580440+010.198897900+01$ $0.292851090+010.441155470+000.199068990+010.198809590+01$ $0.295596550+010.441138260+000.199476780+010.198767510+01$ $0.297825990+010.441170140+000.199716770+010.198747770+01$ $0.29983373 D+010.441127500+000.199978340+010.19874138 D+01$ $0.700000000+010.441128330+000.200000000+010.19874139 D+01$ $0.300634960+010.441127880+000.200108790+010.198742290+01$ $0.3016781 \mathrm{BD}+010.441129070+000.200218640+010.19874518 \mathrm{OD}+01$ $0.302546200+010.441131120+000.200331720+010.198750150+01$ $0.303456520+010.44: 134160+000.250450290+010.198757520+01$ $0.304428950+010.441138390+000.200576940+010.198767810+01$ $0.305487160+010.441144120+000.200714730+010.198781800+01$ $0.306660740+010.44115184 D+000.700867490+010.198800700+01$ $0.30798810 D+01 \quad 0.441162260+000.201040190+010.19882628 \mathrm{D}+01$ $0.30952071 D+010.441176450+000.201 ? 394 R D+010.198861290+01$ $0.31: 32964 D+010.441196100+000.201474510+010.198910010+01$ $0.313515740+010.441 ? 23 R 80+000.20175 R 200+010.198979340+01$ $0.316226100+010.441264140+000.202109720+010.199080720+01$ $0.31968108 \mathrm{D}+010.441324240+000.202555720+010.199233800+01$ $0.32421926 D+010.441416820+000.203139670+010.199473490+01$ $0.370371500+010.44156396 \mathrm{D}+000.20392581 \mathrm{D}+010.199863690+01$ $0.338976490+010.441805730+000.205012370+010.200181500+01$ $0.351310310+010.442207600+000.206536070+010.200577270+01$ $0.369157620+010.442860450+000.208649490+010.201254690+01$ $0.394082450+010.4437 A 5740+000.211369270+010.302305710+01$ $0.42467020 D+010.446497210+000.21900479 D+010.206191750+01$ $0.4552 R 986 D+010.44985366 \mathrm{H}+000.22 ? 17650 \mathrm{D}+01020987158 \mathrm{D}+01$ $0.49256027 \mathrm{D}+01 \mathrm{~N} .455755740+000.270693720+010.213769340+01$
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－0．3926844299259D＋01 0．0 $-0.7535984896560 \mathrm{D}-15$ $0.3535504329608 D+010.10000000000000+02-0.35354933063380+0$ $0.34878685000000-140.10000000000000+02-0.50000000000000+01$ $-0.2777291873984 D+01 \quad 0.0$ －0． 2776268061225 D $-0.2776268061 \geq 25 \mathrm{D}+0$ $-0.3926844299259 D+01 \quad 0.0$ －0．7535984896560D－15 $-0.2444408154633 \mathrm{D}-140.20000000000000+020.27620769925030-14$ $-0.47659400347250-300.20000000000000+020.47597299122530-15$ 0.415650870397 DD－14 0．5294261593354D－14 0．40505R8？10252D－14
 $-0.24444081546330-140.20000000000000+070.27620769925030-14$ $-0.47659400347250-30 \quad 0.20000000000000+020.4759729912253 \mathrm{D}-15$ 0． $41565087039720-140.52942615933540-140.40505882102520-14$ 0． 40 R543629R47？D－14 0．5797798つつ $24210-14-0.4267269790584 D-17$ $0.3487868500000 D-14-0.1000000000000 D+02-0.5000000000000 \mathrm{D}+01$ $-0.3535504329608 D+01-0.10000000000000+0 ?-0.3535493306338 D+01$ $-0.3926844299259 \mathrm{D}+010.0$
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$0.222743660+000.7 ? 9545150+000.148425150+010.48264780 \mathrm{D}+00$ $0.337259 \mathrm{AED}+00 \quad 0.73038799 \mathrm{D}+000.149526430+010.473889530+00$ $0.456149090+000.731571780+000.151111800+010.485010310+00$ $0.581207470+000.733105930+000.153240180+010.455987880+00$ $0.71448115 \mathrm{D}+000.73499 \mathrm{AR} 4 \mathrm{D}+000.15599752 \mathrm{D}+010.446829700+00$ 0 . $\mathrm{R} 5 \mathrm{~B} 484200+000.73775162 \mathrm{D}+000.159505960+010.437586170+00$ $0.101658200+010.739 \mathrm{~A} 4772 \mathrm{D}+000.163942450+010.42837573 \mathrm{D}+00$ 0.11936 R35D $+010.74271866 \mathrm{D}+000.16954804 \mathrm{D}+010.419424610+00$ $0.13755249 \mathrm{D}+010.745663 \mathrm{RRD}+000.176583560+010.411140710+00$ $0.16305256 D+010.74829812 D+000.185401070+010.404260340+00$ $0.19136709 D+010.74990019 D+000.196549960+010.400240620+00$ $0.226048530+010.749137920+000.210353620+010.402141104 \div 00$ $0.25 R 03$ P5 2D +010.745978 R 0 D $+000.22253931 \mathrm{D}+010.410295740+00$ $0.281316 R 2 D+010.7426104 R D+000.230679640+010.41974273 D+00$ $0.30002567 \mathrm{D}+010.7395684410+000.236557560+010.429304170+00$ 0.31612 R65D $+010.73693229 \mathrm{D}+000.24100676 \mathrm{D}+010.438803270+00$ $0.33057179 D+01 \quad 0.73469664 D+000.244451520+010.448164700+00$ $0.343917930+010.73283671 D+000.24713789 D+010.457403050+00$ $0.35648456 \mathrm{D}+0110.73134039 \mathrm{D}+000.24919990 \mathrm{D}+010.46652160 \mathrm{D}+00$ $0.36848098 D+01 \quad 0.73020020 D+000.250718960+010.475524320+00$ $0.380065190+010.779410710+000.251746610+010.484421400+00$ $0.391371620+010.728968570+000.252314010+010.493231900+00$ $0.400000000+010.728 A 64120+000.257446800+010.500000000+00$ $0.411127600+010.729037010+000.252226550+010.508723100+00$ $0.422318160+01 \quad 0.72954628 D+000.25157146 \mathrm{D}+010.51742204 \mathrm{D}+00$ $0.433747390+010.7303 A 83 A D+000.75047112 \mathrm{D}+010.52616640 \mathrm{D}+00$ $0.44558461 D+010.731566960+000.248 R 92 R 0 D+010.535004020+00$ $0.45800643 D+010.733089340+000.246781340+010.54395969 D+00$ $0.471211050+010.734961960+000.244055750+010.553029140+00$ $0.4854379 R D+010.7371 R 4140+000.240601 ? \mathrm{RD}+010.56216699 \mathrm{O}+00$ $0.501001460+010.7397380 R D+000.236253310+010.57126402 D+00$ $0.51835147 D+010.742557890+000.77078963 D+010.580102600+00$ 0.53 RO0500D+01 0.745456100+00 0.2?3976100 +01 0.58829903D+00 $0.56071916 \mathrm{D}+010.748084000+000.21549723 \mathrm{D}+010.59519520 \mathrm{O}+00$ $0.58786865 \mathrm{D}+010.749807250+000.20484475 \mathrm{D}+010.59952803 \mathrm{D}+00$ $0.6214902 R D+010.749408720+000.19144197 D+010.59853594 D+00$ $0.65534543 D+010.74632060 D+000.1784493 R D+010.59061304 D+00$ $0.679861670+010.74 ? R 79600+000.169804970+010.590929740+00$ $0.699177 R 4 \mathrm{D}+010.77970977 \mathrm{D}+000.167693450+010.57116466 \mathrm{D}+00$ $0.715627170+010.737014070+000.159172390+010.561501590+00$ $0.770789810+010.774779740+000.155610520+010.552006460+00$ $0.7437 R 917 D+010.77 ? R 549 R D+000.152 R R 5840+010.54265942 D+00$ $0.756467160+010.771344150+000.150 R 03210+010.533456650+00$ $0.7685 .379 ? 0+010.770197360+000.149775360+010.57 .4395938+00$ $0.7 R 0162170+010.729407000+000.148746670+010.51546958 D+00$ $0.79148397 D+010.72896740 D+000.14768>580+010.506681730+00$ 0. A0n000000.01 $0.7789659 R D+000.147553270+010.500000000+00$

