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THE ROLE OF O-TYPE NEUTRAL LINES IN MAGNETIC MERGING DURING SUBSTORMS AND SOLAR FLARES

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ABSTRACT

Observational evidence suggests that magnetospheric substorms may be associated with the formation of a pair of neutral points or lines in the geomagnetic plasma sheet, containing an X-type point (or line) and an O-type one. While magnetic merging theory has concentrated almost entirely on X-type neutral configurations (points, lines or sheets), here the role of O-type configurations is examined, with special attention to three points: (1) How does the X-O configuration extend in three dimensions? To this end, an analytical model of the configuration was derived, useful for visualizing the geometry and for numerical treatment of plasma flows in it; (2) What modifications are needed in the MHD condition $\underset{\sim}{E} = \underset{\sim}{v} \times \underset{\sim}{B}$ near the O-type line, where it tends to make $\underset{\sim}{v}$ grow without limit? By analyzing equations of motion for charged particles near an O-type neutral line and their solutions in limiting cases, it was found that at a certain distance from the neutral line the mean particle motion became decoupled from that of magnetic field lines (which obey the MHD condition). The decoupling distance depended on initial conditions in momentum space, suggesting that the MHD approximation which averages out such conditions may not suffice for describing plasma dynamics near the neutral line. Similar problems arise with merging flows near X-type neutral lines, and although the treatment there is more difficult and requires more approximations, it appears that the same qualitative conclusions apply there as well; and (3) What is the role of O-type neutral lines in particle acceleration? It was found that after inflowing particles are decoupled from the field line motion, they go over to a mode of runaway acceleration along the neutral line. This process is much more efficient along an O-type line than along an X-type line and it is concluded

that if merging occurs at an X-O pair, two particle populations may be expected -- low energy particles accelerated adiabatically by earthward convection past the X-type line, dependent mainly on the total amount of flux which has been merged, and high-energy particles convected towards the O-type line and undergoing there runaway acceleration. The second acceleration process depends critically on the rapidity of merging and is therefore expected to vary considerably from event to event. All this agrees with observations, and similar processes may also be important in solar flares, where a "Y-type neutral point" has been proposed, which actually represents the limiting form of an X-O configuration.

CLASSIFICATION OF NEUTRAL POINTS

The structure of magnetic field lines near a simple neutral point was first investigated by Dungey [1953, 1963] who based his work on the Taylor expansion of the magnetic field $\underset{\sim}{B}$ in the vicinity of some given point P. This expansion begins with

$$\underset{\sim}{B} = \underset{\sim}{B}_0 + \underset{\sim}{r} \cdot \nabla \underset{\sim}{B}_0 + \ldots \tag{1}$$

where $\underset{\sim}{B}_0$ and $\nabla \underset{\sim}{B}_0$ are computed at the point P and the radius vector $\underset{\sim}{r}$ is drawn from that point. If P is a neutral point then $\underset{\sim}{B}_0$ vanishes and along any field lines passing through P the magnetic field must satisfy

$$\underset{\sim}{B} = \lambda_i \underset{\sim}{r} \tag{2}$$

Comparison with (1) shows that λ_i must be a real eigenvalue of the dyadic $\nabla \underset{\sim}{B}_0$ and that $\underset{\sim}{r}$ is aligned with the corresponding eigenvector. Dungey noted that since $\nabla \underset{\sim}{B}_0$ may have either 1 or 3 real eigenvalues, there should exist two main classes of neutral points, having either 1 or 3 field lines passing through them. He named such points O-type and X-type neutral points, respectively, since their expected configurations resembled somewhat the letters O and X (Figures 1).

Dungey's analysis was repeated in a rigorous fashion by Fukao et al. [1975] who showed that the field line structure of an O-type point consists in general of a set of spirals rather than of nested closed curves; however, if there exists a neutral line along which $\underset{\sim}{B} = 0$, the field line structure indeed resembles Figure 1b. Their analysis covered all possible configurations and included the special cases of degenerate eigenvalues and eigenvalues which vanish.

In most studies involving neutral points in space attention has been focused on X-type neutral points or neutral lines, or on their limiting form, neutral sheets. The main reason for this preference was that X-type points can always be expected to occur on the boundaries of different topological regimes of field lines emanating from two or more sources [Sweet, 1958]. A prime example of this is the "open magnetosphere" model [Dungey, 1961] where at least two X-type points are always expected on the boundary between open and closed field lines (Figure 2a). In contrast, no O-type points are required in such a model.

A further reason for the interest in X-type neutral points or lines has been that they form a key ingredient in theories of magnetic merging of field lines originating from different sources and embedded in a plasma in which the MHD approximation

$$\underset{\sim}{E} = - \underset{\sim}{v} \times \underset{\sim}{B} \tag{3}$$

is valid [e.g. Vasyliunas, 1975]. Because such a merging process can transfer magnetic flux from one topological regime to another and overcome the "freezing" of flux to the plasma [e.g. Stern, 1966] it has been widely believed that merging is responsible for solar flares and magnetospheric substorms, where magnetic energy appears to be rapidly converted and where charged particles are accelerated to high energies. Indeed, observations near the midplane of the geomagnetic tail have shown that while at geomagnetically quiet times the north-south magnetic component B_z there is generally directed northward, as expected for disturbed dipole field lines, during substorms it may briefly reverse to become southward [e.g. Hones, 1975, 1977; Nishida and Hones, 1974], suggesting the existence of X-type structures.

O-TYPE STRUCTURES IN THE TAIL

Originally it was believed [e.g. Axford et al., 1965] that the neutral point involved in substorms was the nightside X-type point of the open magnetospheric configuration of Figure 2a. However, more recent observations suggest that this is not the case -- that while the magnetospheric tail extends well beyond the moon's orbit (60 R_E) before becoming pinched off, substorm activity originates much closer to earth, at distances of the order of 10-15 R_E.

Thus, if an X-type neutral point is involved in the mechanism of substorms, it arises in the interior of the tail, in a configuration resembling Figure 2b [Schindler, 1975; Hones, 1977]. As can be seen, this configuration also requires the existence of an O-type neutral point or line.

The purpose of this work is to use analytical models in order to examine three questions related to such O-type neutral structures (points or lines):

(1) What is the 3-dimensional form of the field in Figure 2b?

(2) According to a commonly expressed view, when reconnection occurs in an X-type structure in the geomagnetic tail, a certain amount of magnetic flux originating in the northern lobe (in Figure 2b) flows through the structure, merges with an equal amount of flux from the southern lobe and then the combined field lines convect earthwards. As they con ect,

such field lines become more dipole-like, reducing the stress in the magnetosphere and releasing magnetic energy.

However, if an O-type line exists as in Figure 2b, an equal amount of flux flows into the "bubble" surrounding the O-type line, converges towards the line and finally vanishes there. Does such behavior contradict MHD theory?

(3) Finally, the role of an O-type field configuration in the acceleration of charged particles is examined. As will be seen, there exist reasons to believe that particle acceleration can proceed much more efficiently inside the "bubble" than in the X-type region.

In what follows, these three questions are addressed in the order given above. To simplify the discussion it will be assumed that we are dealing with O-type lines similar to the one shown in Figure 1b. A more general definition of an O-type line which is sometimes used (e.g. Vasyliunas [1975], p. 304, penultimate parag.) allows a small component of $\mathbf{B}$ to persist along the "neutral line" (except at isolated "neutral points" where $\mathbf{B} = 0$): according to Fukao et al., [1975], the field near that line would then have a small radial component, which would transform the nested closed field lines into spirals. Such fields are much more difficult to analyze and we presume that the components thus added are small and that the conclusions reached here are still qualitatively correct.

ANALYTICAL MODEL FOR "BUBBLE IN TAIL"

The only attempt to describe how the interior X and O lines of Figure 2b might be extended to the third dimension was made by Vasyliunas [1976, Figure 5]. Vasyliunas suggested that initially the O-type line and the X-type line in the middle of the plasma sheet were topologically continuous. During a substorm, he proposed, both expanded until they reached the flanks of the tail, at which point the magnetic linkages became shuffled, the X-type line wound up connected to the dayside "Dungey" point (Figure 2a) while the O-type line linked up with the nightside neutral point. In what follows we shall investigate an analytical model of the initial configuration suggested by Vasyliunas, which is practically dictated by the over-all topology: the later developments mentioned above are much more speculative and are not relevant to the present sutdy.

The analytical model is produced by taking a simple 2-dimensional model of the tail field

$$\underset{\sim}{B} = (z/L)\ B_0\ \hat{\underset{\sim}{x}} + B_0\ \hat{\underset{\sim}{z}} \tag{4}$$

(B_0 and L are constant, the x direction is sunward and z = 0 is the midplane of the tail) and superimposing upon it a current ring in the z = 0 plane, flowing counterclockwise when viewed from z > 0. The aim here is pure electromagnetic modeling and no attempt is made to derive a plasma population which supports such a field in a self-consistent manner.

Jackson [1962] has provided formulas for the magnetic field $\underset{\sim}{b}'$ of a circular current filament of radius a, but these do not suit the present purpose, since $\underset{\sim}{b}'$ becomes infinite as one approaches the filament. What is required for a realistic model is a ring carrying a distributed current density varying smoothly enough so that when it is added to the field (4), B_z is reversed near the middle of the ring but $|B_z|$ nowhere becomes unrealistically large.

Fortunately, Jackson also gives a simple approximation $\tilde{b}$ which tends to $\tilde{b}'$ at large distances and also near the origin and near the z-axis, and which fulfills all the above requirements. That field is given (in spherical coordinates r, θ, φ) by the vector potential

$$\tilde{A} = \hat{\varphi}\, b_0\, r \sin\theta\, (a^2 + r^2 + 2ar \sin\theta)^{-3/2} \tag{5}$$

leading to field components

$$b_r = b_0 \cos\theta \frac{(2a^2 + 2r^2 + ar \sin\theta)}{(a^2 + r^2 + 2ar \sin\theta)^{3/2}} \tag{6a}$$

$$b_\theta = -b_0 \sin\theta \frac{(2a^2 - r^2 + ar \sin\theta)}{(a^2 + r^2 + 2ar \sin\theta)^{5/2}} \tag{6b}$$

The above approximation represents a broad current ring producing field lines which enclose the ring $r = 2a$ in the $z = 0$ plane (although at large distances they approximate the field of a filament at $r = a$). In the $z = 0$ plane of the corresponding cylindrical system (i.e. the plane $\theta = \pi/2$) b_z is southward (i.e. negative) whenever $r < 2a$ and northward (i.e. positive) whenever $r > 2a$, but the magnitude $|b_z|$ is small everywhere in that plane except near the origin (Table 1), so that when $\tilde{b}$ is added to the field of equation (4), the reversal region is small, with a diameter of the order of a (while $\tilde{b}$ reverses over a diameter 4a). It is surrounded by true X-type and O-type neutral lines, since both $\tilde{b}$ of (6) and $\tilde{B}$ of (4) have only z components in the plane $z = 0$ and wherever these components cancel the total field vanishes.

Before calculating and plotting the combined field it is useful to transform everything into normalized units -- i.e. scale all field intensities in units of B_0 and all distances in units of the scale length L. The configuration then depends on two dimensionless parameters -- the normalized intensity of the ring current field

$$C_1 = b_0/B_0 \tag{7a}$$

and the normalized radius of the ring

$$C_2 = a/L \tag{7b}$$

The range of these parameters is rather limited. The first parameter C_1 must be negative and less than -0.5, otherwise no reversal of B_z occurs, and it is not likely to fall below -2, since then the southward B_z peaks at 3 B_0, which is about the limit of observed southward fields. Since L in the tail is of the order of 1 R_E, C_2 will approximately equal the diameter of the reversal region in earth radii, which can be taken to be around 1.

Figures 3 show results for $C_1 = -1.5$, $C_2 = 2$. Specifically, they present projections of field lines onto planes of constant y/L (where y is the dawn-dusk coordinate), corresponding to y/L = 0, 0.25, 0.5, 0.55 and 0.75. As can be seen, the neutral lines approach each other as $|y|$ increases and they vanish at a value of y/L slightly exceeding 0.5.

The value of B_y was also computed; B_y/B_0 was found to be rather small, suggesting that to a rather good approximation the field lines do not depart far from planes of constant y.

Topologically the model contains a distinct region, a "bubble" centered on the O-type line, in which field lines are not connected to the main geomagnetic field: its shape resembles somewhat that of a football (American style). Since the detailed analysis of particle dynamics in this region is difficult, it will be handled in the next section for a somewhat simplified geometry, in which the "bubble" is straightened out to a cylinder.

PARTICLE MOTION NEAR AN O-TYPE LINE

As a model for the magnetic field in the vicinity of an O-type neutral line we consider a field

$$B = \nabla\alpha \times \nabla\beta \tag{8}$$

where in cylindrical coordinates (ρ, φ, z) the Euler potentials (α, β) are given by

$$\alpha = -B_0\, \rho^2/2\rho_0 \qquad \rho < \rho_0$$

$$= B_0\, (\rho_0/2 - \rho) \qquad \rho > \rho_0 \tag{9}$$

and

$$\beta = z \tag{10}$$

(in the context of the geomagnetic tail it should be noted that the direction taken here as the z axis is aligned with the solar magnetospheric y axis, so that the system of coordinates used here is not aligned with the one used in the preceding section).

For $\rho > \rho_0$

$$\underset{\sim}{B} = B_0 \hat{\underset{\sim}{\rho}} \tag{11}$$

while for $\rho < \rho_0$ the magnitude of the field decreases in proportion to ρ and vanishes along the z axis.

For simplicity, the calculation below assumes that ρ is always less than ρ_0; the external region $\rho > \rho_0$ could have been included, but this would have only lengthened the calculation without introducing any new qualitative feature.

Assume now that an electric field

$$\underset{\sim}{E} = E_0 \hat{\underset{\sim}{z}} \tag{12}$$

exists in this region and that the bulk velocity of the plasma obeys the MHD condition

$$\underset{\sim}{E} = - \underset{\sim}{v} \times \underset{\sim}{B} \tag{3}$$

Since $\underset{\sim}{E}$ and $\underset{\sim}{B}$ are orthogonal, $\underset{\sim}{v}$ satisfies

$$\underset{\sim}{v} = \underset{\sim}{E} \times \underset{\sim}{B}/B^2 \tag{13}$$

which is the well-known relation for the electric drift velocity. In the given configuration this $\underset{\sim}{v}$ is everywhere directed inwards, towards the neutral line, and its magnitude tends to infinity as $\rho \to 0$ and $B \to 0$. Thus the velocity of magnetic field lines described by (3) converges towards the O-type neutral line and its magnitude tends to infinity just before those lines vanish into oblivion at $\rho = 0$.

If $\underset{\sim}{v}$ is viewed as a field line velocity then such a behavior creates no difficulties, because the motion of magnetic field lines is a mathematical abstraction and is not physically measurable. On the other hand, such behavior would not be appropriate for the bulk velocity of the ambient plasma, which is an observable material velocity.

If the ambient plasma obeys the guiding center approximation at distances of the order ρ_0 or greater, than far away from the neutral line the velocity $\underset{\sim}{v}$ given by (13) will indeed be its bulk velocity, and the plasma flow there will converge inward with a gradually increasing velocity. Near $\rho = 0$, however, the guiding center approximation breaks down and one may expect that the MHD relation (3) is no longer valid. In what follows it will be shown in more detail how exactly this transition occurs, how the motion of the plasma becomes decoupled from that of magnetic field lines and what the consequences may be.

The approach adopted is a slight generalization of the one described by Stern [1975]. Under the conditions described above, the Hamiltonian for a particle of charge q and mass m is

$$H = \frac{1}{2m} \left[p_\rho^2 + p_\varphi^2/\;^2 + (p_z - qc\, \rho^2)^2\right] - q\, E_o\, z \qquad (14)$$

where (p_ρ, p_φ, p_z) are canonical momenta conjugate to (ρ, φ, z) and where

$$c = - B_0/2\, \rho_0$$

Suppose first that $E_0 = 0$ (i.e., no electric field exists). Then p_φ and p_z are both constants of the motion, with values which will be denoted by g and h, and the entire motion reduces to a motion in a one-dimensional potential V

$$H = p^2/2m + V \tag{15}$$

$$V = (1/2m)\ [g^2/\rho^2 + (h - qc\rho^2)^2] \tag{16}$$

The potential V depends only on $u = \rho^2$, is non-negative and (except for the singular case $g = 0$, which must be treated separately) tends towards infinity both as $\rho \to 0$ and as $\rho \to \infty$. The particle is thus trapped in a potential well and the bottom of this well is located at some value ρ_1 of ρ, satisfying

$$\partial V(\rho_1) \partial u = 0 \tag{17}$$

The particle's motion oscillates in that potential well around ρ_1, so that ρ_1 may be viewed as representing a "generalized guiding center" -- and indeed, for low energy particles far away from the z axis, it does tend towards ρ of the guiding center.

Now let the electric field be included, so that H is given by (14). The canonical equations of motion then give

$$p_z = q\ E_0\ (t - t_0) \tag{18a}$$

$$m\ \dot{z} = q[E_0(t - t_0) - c\ \rho^2] \tag{18b}$$

The complete motion cannot be solved, but in principle one could derive an approximate solution using the approach of Sonnerup [1971], as follows. The limit $E_0 = 0$, described by (15) and (16), is one-dimensional and therefore formally soluable: one can thus (in principle) derive for it an adiabatic invariant

$$J = \oint p_\rho \, d\rho = (2m)^{1/2} \oint (W - V(\rho))^{1/2} \, d\rho$$

$$= J(W, g, p_z) \qquad (19a)$$

where W is the constant of energy. If E_0 is no longer zero but remains relatively small, J is still an approximate constant of the motion: inverting (19a) to the form

$$p_z = f(J, W, g) \qquad (19b)$$

and substituting (18a) gives then

$$q \, E_0 \, (t - t_0) = f(J, W, g) \qquad (20)$$

If J and g are assumed to be conserved, the last relation describes (at least implicitly) the variation of the energy W with time.

For particle motion near a neutral sheet, J was derived explicitely in terms of elliptic integrals by Sonnerup [1971] , who then incorporated the effects of a weak electric field in the manner described here. For the present calculation, the analytical form of J is too complicated and we shall contend ourselves with a semi-qualitative treatment, stressing the limiting properties of the motion very far from the line z = 0 and very close to it.

We begin by substituting (18a) into (14). The Hamiltonian then still has formally the form (15), with the potential

$$V(\rho, z, t) = (1/2m) \{ g^2/\rho^2 + q^2 [E_0(t - t_0) - c\rho^2]^2 \} - q \, E_0 \, z \qquad (21)$$

However, z is now no longer a canonical coordinate, since p_z is gone: instead, it is parameter evolving according to (18) (the canonical relations for ρ and Ψ remain the same as before). For any fixed z the dependence of V on ρ is as before a potential well bottoming at ρ_1, representing a "generalized guiding center". If $u = \rho_1^2$ then by (17)

$$\partial V/\partial u = K(u) = - a^2/u^2 + 2\, q^2\, [E_0(t - t_0) - cu] = 0 \qquad (22)$$

The difference now is that due to the dependence on t, ρ_1 gradually shifts, i.e. $\partial \rho_1/\partial t \neq 0$. Differentiating (20)

$$2\, \rho_1 (\partial \rho_1/\partial t)\, (\partial K/\partial u) + K/\partial t = 0 \qquad (23)$$

from which

$$\frac{\partial \rho_1}{\partial t} = \frac{E_0}{2\, \rho_1 c - (2g^2/q^2\, \rho_1{}^5)} \qquad (24)$$

This result is easily interpreted. Far from the z axis, where $_1$ is large, the second term in the denominator is small and may be neglected. By equations (8) - (10), the first term there is simply -B, so the flow in this limit is directed radially inwards with the electric drift velocity E_0/B: the mean motion in this region is thus in accord with (3) and (13).

However, at some critical distance

$$\rho_{1c} = (-g^2/q^2 c)^{1/6} \qquad (25)$$

(note that c is negative) the two demoninator terms become equal, while beyond this distance the second term dominates. The motion is then still inwards (since both terms are negative), but the rate at which ρ_1 decreases slows down to a negligible value.

What happens to the particles? If ρ_1 is viewed as a "generalized guiding center", the motion can be separated into a mean part and an oscillating one, and in particular it is possible to write

$$\rho^2 = \rho_1^2 + (\delta \rho^2)_{osc} \tag{26}$$

For studying the average properties of the motion in the z direction, the oscillating part in (18b) may be neglected, giving after differentiation

$$m\,(\ddot{z})_{aver} = q\,E_0 - 2\,q\,c\;\rho_1\,(\partial \rho_1/\partial t) \tag{27}$$

For large values of ρ_1, by (24), the two terms tend to cancel and to the lowest approximation no appreciable acceleration takes place. A more accurate calculation would show that the particle actually loses some energy, because in the outer regions the magnetic moment μ is approximately conserved (this conservation may be viewed as the limiting case of (19a)). As ρ_1 decreases and the particle moves inwards, it enters regions of progressively weaker $\underset{\sim}{B}$ and if μ is preserved, its energy W also decreases: however, the electric drift velocity, which dominates its motion in these regions, does not depend on W, so that the particle's inward progress continues undiminished.

Once ρ_1 has decreased to the order of ρ_{1c} or less, however, the second term in (27) loses importance and the particle is accelerated almost freely along the z axis. These two regimes are completely analogous to corresponding regions derived by Sonnerup for the vicinity of a neutral sheet.

The apparent conflict with MHD is thus resolved as follows. Magnetic field lines in the O-type configuration flow radially inwards with a steadily increasing velocity given by (13), and they vanish at z = 0 like the mythical snake which swallows its own tail. This velocity however is not physically observable since $\underset{\sim}{B}$ in that region (though not at great distances, where equations (8) - (10) no longer hold) is time independent.

Plasma particles which start out by sharing the inward flow of magnetic field lines decouple from the magnetic field flow at distances of the order of ρ_{1c}, and ρ_{1c} depends, for each particle, on the initial conditions (through the parameter g). Once the decoupling has occurred the particles are removed from the scene by being freely accelerated by $\underset{\sim}{E}$ along the middle of the O-type configuration, which acts as an accelerator tube.

The next section examines the extent to which this kind of "decoupling" occurs in the neighborhood of an X-type neutral line.

X-TYPE NEUTRAL LINES

One of the major problems in merging theory has been that if the MHD condition is assumed to hold in the merging plasma

$$\underset{\sim}{E} = - \underset{\sim}{v} \times \underset{\sim}{B} \tag{3}$$

then some modification must be added to it before it can be applied near $\underset{\sim}{B} = 0$. Such a modification has been frequently introduced by regarding (3) as an approximation to the generalized Ohm's law equation [e.g. Vasyliunas, 1975, eq. 1"; 1976, eq. 2; Rich et al., 1976, appendix] and retaining some other terms from this equation near the neutral point -- for instance, the resistive term (with normal or anomalous resistivity) or inertia terms.

In the preceding section, in connection with merging at an O-type neutral line, a different approach was adopted -- namely, (3) was regarded as an approximation to the mean motion of a charged particle in the plasma. It was then shown that near an O-type neutral line, (3) breaks down and a different regime of motion takes over, one in which particles are steadily accelerated. This breakdown cannot be easily accommodated by MHD theory, because it occurs at different distances for particles which differ by the constant of motion p_φ , and such particles are not distinguished in MHD theory.

It is natural to ask whether such a calculation can be extended to X-type neutral lines. Unfortunately, a new difficulty is encountered here: while the motion near an O-type point in the absence of an electric field can be reduced to one-dimensional form (equations 15-16), the corresponding form for an X-type line remains 2-dimensional [Stern, 1975] and cannot be integrated analytically. The best one can do by analytic methods is to trace the motion of the guiding field line -- rather than that of the guiding center -- as shown below.

If $\tilde{B}$ is given by Euler potentials as in (8) and cartesian coordinates (x, y, z) are used, with the z axis along the neutral line, one finds here

$$\alpha = M^2x^2 - N^2y^2 \qquad (28a)$$

$$\beta = z \qquad (28b)$$

The field lines (Figure 4) are now a set of hyperbolas sharing the asymptotes

$$Mx = \pm Ny \tag{29}$$

which correspond to $\alpha = 0$, and in the absence of electric fields the Hamiltonian is

$$H = (1/2m)\ (p_x^2 + p_y^2) + V \tag{30}$$

$$V = (1/2m)\ [p_z - q(M^2x^2 - N^2y^2)\ ]^2 \tag{31}$$

The "potential" V is non-negative, and since p_z is a constant of the motion, V reaches its absolute minimum value zero along the "guiding field line"

$$\alpha = p_z/q = \alpha_0 \tag{32}$$

At infinity ($x \to \infty$ or $y \to \infty$) V also becomes infinite, while at the origin it reaches a local maximum $p_z^2/2m$. Thus particles for which the (x, y) component W_{xy} of the kinetic energy is sufficiently low, or which are sufficiently distant from the origin, will be confined to a narrow valley centered on $\alpha = \alpha_0$, and by analogy with the preceding section one can state that the particle's generalized guiding center is located somewhere along the bottom of this valley: it is not possible to give the position more precisely, since V is now two-dimensional.

Actually, there exist two hyperbolic valleys along which V vanishes, located symmetrically on opposite sides of the origin. When the particle is far from the origin, that valley in which the particle is not located may be ignored. On the other hand, if the particle comes close to the origin and if W_{xy} is sufficiently large to enable it to cross the pass at the origin (Figure 5), then α_0 can no longer be regarded as the generalized guiding line -- instead, the particle now oscillates irregularly in the region surrounding the origin [Russbridge, 1971, 1977].

Let now an electric field $E_0 \hat{\underset{\sim}{z}}$ be added as before. In analogy with with (15) and (21) we now have

$$H = (1/2m)\ (p_x^2 + p_y^2) + V(x,\ y,\ z,\ t) \tag{33}$$

$$V = (q^2/2m)\ [E_0(t - t_0) - M^2x^2 + N^2y^2]^2 - q\ E_0\ z \tag{34}$$

where z is to be regarded as a parameter satisfying the analog of (18b)

$$m\ \dot{z} = q\ [E_0(t - t_0) - M^2x^2 + N^2y^2] \tag{35}$$

For any constant z, the bottom of the potential "valley" is along the hyperbola

$$m^2x^2 - n^2y^2 = E_0(t - t_0) \tag{36}$$

This can be regarded as the equation of the guiding field line $\alpha = \alpha_0$ which shifts in time towards increasingly larger values of α

$$\alpha_0 = E_0(t - t_0) \tag{37}$$

This again reflects the electric drift velocity of the guiding center, given by (13). This velocity is known to be a valid field line velocity [e.g. Stern, 1966]: even if the position of the guiding center along the field line is not known, if each particle along the line moves with velocity (13), the same field line will continue to contain those particles at all subsequent times. If the energy V_{xy} of the (x, y) motion is low, the particle will oscillate around its guiding field line and its α at any instant can be expressed as

$$\alpha = \alpha_0 + (\delta\alpha)_{osc} \tag{38}$$

Substituting the last 2 equations and (26) in (33) gives

$$(\dot{z})_{aver} = 0 \tag{39a}$$

and hence, in analogy with the result of the preceding section,

$$(\ddot{z})_{aver} = 0 \tag{39b}$$

In other words (as could be expected) the particle moves, on the average, with its electric drift velocity (13) and is not accelerated along z. In addition it may also move along its guiding field line and oscillate across it, and its energy changes slowly in accordance with the conservation of μ, but these details are not resolved by the present approximation.

However, if the particle manages to approach the origin sufficiently close so that it crosses over from one valley to the other, all the preceding breaks down. Its average x and y now oscillate in an irregular manner (investigated numerically by Russbridge [1971, 1977]): it is not clear whether the average of $(M^2x^2 - N^2y^2)$ is zero, but on the other hand, as long as the particle stays near the origin, it remains bounded and it may be argued that when the average time derivative of (35) is computed, its contribution may be neglected. If this is the case then in the central region the particle again undergoes, on the average, a steady acceleration (as in the preceding case), satisfying

$$m\,(\ddot{z})_{aver} = q\,E_0 \tag{40}$$

Thus there exists a strong suggestion that particle motion near an X-type neutral line also contains two limiting regimes -- an adiabatic drift regime far away from the line, in the region where the MHD condition (3) holds, and a runaway acceleration regime near the neutral line itself, where (3) breaks down. The existence of such regimes can be shown explicitly for one particular group of particles, namely those with initial conditions

$$x = 0 \qquad p_x = 0 \tag{41}$$

From Hamilton's equations, if (41) holds at one time, it holds for all other times as well, i.e. the particles are confined to the y axis. The fields sensed by them and the equations describing their motion are then the same as those existing in Sonnerup's null sheet geometry [Sonnerup, 1971], where such limiting regimes can be derived analytically from adiabatic invariance.

There exists, however, one important difference between this case and the O-type line. With the O-type configuration, every one of the particles swept inwards in the adiabatic mode ultimately passes into the second mode and becomes accelerated, unless $\underset{\sim}{E}$ has meanwhile changed.

With the X-type line the present treatment does not indicate what fraction of the particles attains the second mode and becomes accelerated, since all particles guided by the same field line are handled together. Each such field line passes through the origin at $t = t_0$ and at that time some of the particles associated with it will have passed to the second mode. The point to note is that the fraction of such particles cannot be large, because if (13) is regarded as the bulk velocity in the first region, most trajectories of this motion will skirt the vicinity of the origin by a wide margin and only those among them which are headed almost directly towards the origin reach the region of acceleration (individual particle trajectories differ from the trajectories of the bulk motion by the addition of a component $v_{\parallel}$ parallel to magnetic field lines, but this does not affect the conclusions reached here).

Furthermore, the duration of sustained acceleration in the vicinity of an X-type neutral line may be brief for most particles undergoing it, because the region in which it occurs has 4 "valleys" leading away from it. Numerical tracking of trajectories [Russbridge, 1971, 1977] suggests that sooner or later particles enter these valleys -- and although adiabatic mirroring will generally drive them back out again, their electric drift may meanwhile move them away from the region in which (40) holds.

The relevant conclusion from all this is that if during substorms two magnetic neutral lines of different types are created as shown in Figure 2b, close enough to each other to share the same electric field, then far more particles will undergo runaway acceleration near the O-type line, which draws them in and keeps them well contained, than near the X-type line.

SUBSTORMS AND FLARES

What happens during a magnetic substorm? Observations seen to indicate [Fairfield and Ness, 1970; Caan et al., 1973] that stretched tail lines snap back to a more dipolar shape, decreasing the amount of stretched magnetic flux threading the cross-section of the near-earth tail. The exact mechanism and cause of this leap are still a matter of controversy, but it is widely believed that it is initiated by the formation of an interior X-type neutral point or line as shown in Figure 2b, a view which will be adopted in what follows.

The decrease in the stretched magnetic flux induces an e.m.f. in the circuit consisting of the plasma sheet and the magnetopause [Stern, 1977, sect. 6b] and this inductive e.m.f. is responsible for the enhanced electric field $\underset{\sim}{E}$ generally assumed to exist along the neutral lines from dawn to dusk. It should be stressed that $\underset{\sim}{E}$ is a global phenomenon, due to the decreasing magnetic flux in the tail, and not a local consequence of conditions in the merging region: its energy is derived from the magnetic energy of the decreasing tail flux and it is available in its own turn to accelerate and energize tail particles.

Two simultaneous modes of acceleration may be expected to result from $\underset{\sim}{E}$. First, there would exist an enhanced convection of particles streaming earthwards around the X-line: most of them will stay out of the immediate vicinity of the X-line, although their guiding field lines will pass through it. The energization of these particles will be moderate and it can be estimated from adiabatic conservation laws as being of the order of the ratio between the final and initial field intensities sensed by the convected particles. This process may be the one responsible for substorm particles in the ring current and the aurora, although the properties of such particles may undergo further modification in the inner magnetosphere by local electric fields with $E_{\parallel} \neq 0$.

Secondly, there exists a high energy component ($\sim$ 0.5 Mev) detected in a number of substorms [Roelof et al., 1976, Keath et al., 1976; Hones et al., 1976; Baker and Stone, 1977] and even observed outside the magnetosphere. It is proposed here that such particles are accelerated along the O-type line of a configuration such as the one of Figure 2b. This line probably occupies a "bubble" extending only part of the way across the tail and particles emerging from it become attached to magnetospheric field lines, which is the mode of motion in which the energetic particles are generally observed.

In this connection it should be noted that there exist many similarities between substorms and solar flares [e.g. De Feiter, 1975] and that the radiation from flares suggests that there, too, the accelerated particles can be divided into two populations occupying different energy ranges [Bai and Ramaty, 1976]. The conventional explanation of this is based on a two-stage acceleration process -- initially particles are accelerated to the lower level and a certain fraction of them are accelerated again. If the present model is applicable, however, i.e. if flares occur at X-O pairs of neutral lines within bundles of stretched flux emanating from sunspot regions -- these could be two distinct populations, accelerated in distant ways. In this connection it may be pointed out that the "Y type neutral line" proposed by Sturrock [1973] closely resembles the limiting configuration of an X-O pair when both of its components lie very close together.

CONCLUSION

The theoretical picture of magnetic merging and particle acceleration in substorms presented above is incomplete in many details. Not only can particle trajectories near neutral lines be handled analytically only in some limiting cases and in an idealized geometry, there also exists only a poor understanding of what precedes the substorm -- of how the magnetosphere manages to establish a configuration of plasmas and magnetic fields which can rapidly relax and release energy as soon as a pair of neutral points is established, but not before.

We also lack information about the duration of the primary energy release in substorms (or flares), which is related to the rate at which magnetic flux jumps back. Stern [1977, sect. 6b] estimated that if the change associated with a typical substorm occurred over 30 minutes, the induced e.m.f. would be 40,000 volts: if it takes only one minute, over 10^6 volts are generated and the energetic particles observed in the 0.5 Mev range are readily produced along the O-type line.

In practice, the rate at which the flux changes may well depend on circumstances and vary from one event to the next one. If the present picture is correct, such variations will affect only slightly the energy transfer to plasma convected through the X-type line, for unless the depth of penetration changes for the convected plasma, the same energy is provided to the same number of particles -- only the time required for the process differs. On the other hand, this variation will make a great difference in acceleration effects along the O-type line. Substorms -- and perhaps also solar flares -- which lack a high energy component might well differ from those which possess it only in the rate at which they evolve, which might be slightly slower.

To summarize:

(1) A model has been constructed for the magnetic field configuration of a combination of X-type and O-type neutral lines in the middle of the geomagentic tail. The main feature is a "bubble" centered on the O-type line, the field lines of which are not connected to the main geomagnetic field.

(2) An analytical model of the motion of charged particles in the vicinity of an O-type neutral line, with an electric field aligned parallel to it, has been constructed and analyzed. Using the concept of a generalized guiding center, it is shown that on the average particles move steadily inwards towards the O-type line. At large distances their motion obeys the MHD relation, but as they pass within some critical radius the MHD approximation breaks down and particles tend to become freely accelerated along the O-type line.

Although the motion of charged particles cannot be analyzed as well in the neighborhood of an X-type neutral line, it seems that similar regimes exist there as well. There is a difference, however, in that only very few particles can here reach the region of free acceleration, and confinement in this region is also less thorough than it is near an O-type line. The conclusion is that if particles are accelerated across the plasma sheet along a neutral line, this is far more likely to be the O-type line than the X-type line. It is also concluded that since nonadiabatic behavior is responsible for the breakdown of the MHD approximation near an O-type neutral line (and probably near an X-type line as well), treatment of merging of collision-free plasma near such lines should not be based on the MHD approximation, even if eq. (3) is supplemented by resistive or inertial terms.

(3) If a substorm is powered by the inductive e.m.f. generated when the tail's magnetic field returns to a more dipole-like configuration by flowing through a pair of X-type and O-type neutral lines (Figure 2b), then two distinct mechanisms of praticle acceleration may take place. On the earthward side of the X-type line particles are moderately energized by convection: their final energy depends mainly on the depth to which they penetrate into the inner magnetosphere, while the total energy given to them depends mainly on the amount of reconnected flux. This process is not critically affected by the rate of merging.

In addition, however, particles are also convected towards the O-type line and are accelerated there. Such particles can reach high energies, provided the rate of merging is sufficiently rapid. It is conjectured that while the first mechanism injects particles into the ring current and aurora, the second one is responsible for the fluxes of high energy particles recently observed during substorms by Imp spacecraft. Similar processes, occurring on stretched field lines above sunspots, may explain the existence of two populations of energetic particles in solar flares, commonly ascribed to a two-stage acceleration process.

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FIGURE CAPTIONS

Figure 1a -- Structure of magnetic field lines near an X-type neutral point.

Figure 1b -- Structure of magnetic field lines near an O-type neutral point. This drawing illustrates a special case, corresponding to an O-type neutral line along which $\tilde{B} = 0$.

Figure 2a -- Schematic view of the "open magnetosphere" proposed by Dungey.

Figure 2b -- Magnetic merging at a pair of neutral points formed in the interior of the tail, as proposed by some theories of magnetic substorms. One of these points has an X-type configuration, the other is of the O-type variety.

Figure 3a -- Magnetic field lines in the plane y = 0 of a three-dimensional model of the X-O configuration, as described in the text. Distances are scaled in units of a, the characteristic dimension of the current loop, and the field structure for $z < 0$ is a mirror image of the one portrayed here.

Figure 3b -- Similar lines for y = 0.25 (a/L). The small component B_y is ignored in this graph and in the three that follow it, so that the lines drawn actually describe two-dimensional projections of the magnetic field.

Figure 3c -- Similar to 3b, but for y = 0.5 (a/L). The two neutral points are now rather close to each other.

Figure 3d -- Similar to 3b, but for y = 0.55 (a/L). The lowest field intensity (at x = z = 0) is down to 0.0209 B_0, but no neutral points remain anywhere.

Figure 3e -- Similar to 3b, but for $y = 0.75\ (a/L)$. The shape of the field lines is only moderately affected by the proximity of the current ring, but the field intensity at the origin is depressed to $0.3181\ B_0$.

Figure 4 -- Schematic view of field lines in the x-y plane near a neutral line. Each line here can also be viewed as a line of constant potential , with V defined by equation (31). The two dashed lines are the bottoms of two "valleys" along which $V = 0$, for some particular value of p_z: the choice of a different value will shift the valleys to a different pair of equipotential lines.

Figure 5 -- The variation of the potential V along the line $y = 0$ in Figure 4. Here V is given in units of $p_z^2/2m$ (its value at the origin) while x is in units of $(p_z/qM^2)^{1/2}$. Particles in this potential may be trapped on one side of the origin or may cross from side to side: these modes correspond to two modes of motion distinguished in the work of Sonnerup [1971] and also represent two regimes of motion expected to exist in the vicinity of the X-type neutral line.

TABLE CAPTION

Table 1 -- The values of B/b_0 for the magnetic field of equations (6), at various values of the radial distance r/a in the plane z = 0. The scaling of distances is the same as in Figures 3; the field intensities along z = 0 in Figure 3a equal $1 - 1.5(B/b_0)$, if one identifies x/a there with r/a in the table. The value $B/b_0 = -1/256$ at r/a = 3 is the most negative one encountered.

TABLE 1

r/a	B/b_0	r/a	B/b_0
0	2	0.8	0.114
0.1	1.30	1.0	0.0625
0.2	0.87	1.5	0.0128
0.3	0.60	2.0	0
0.4	0.42	2.5	-0.0033
0.5	0.30	3.0	-0.0039

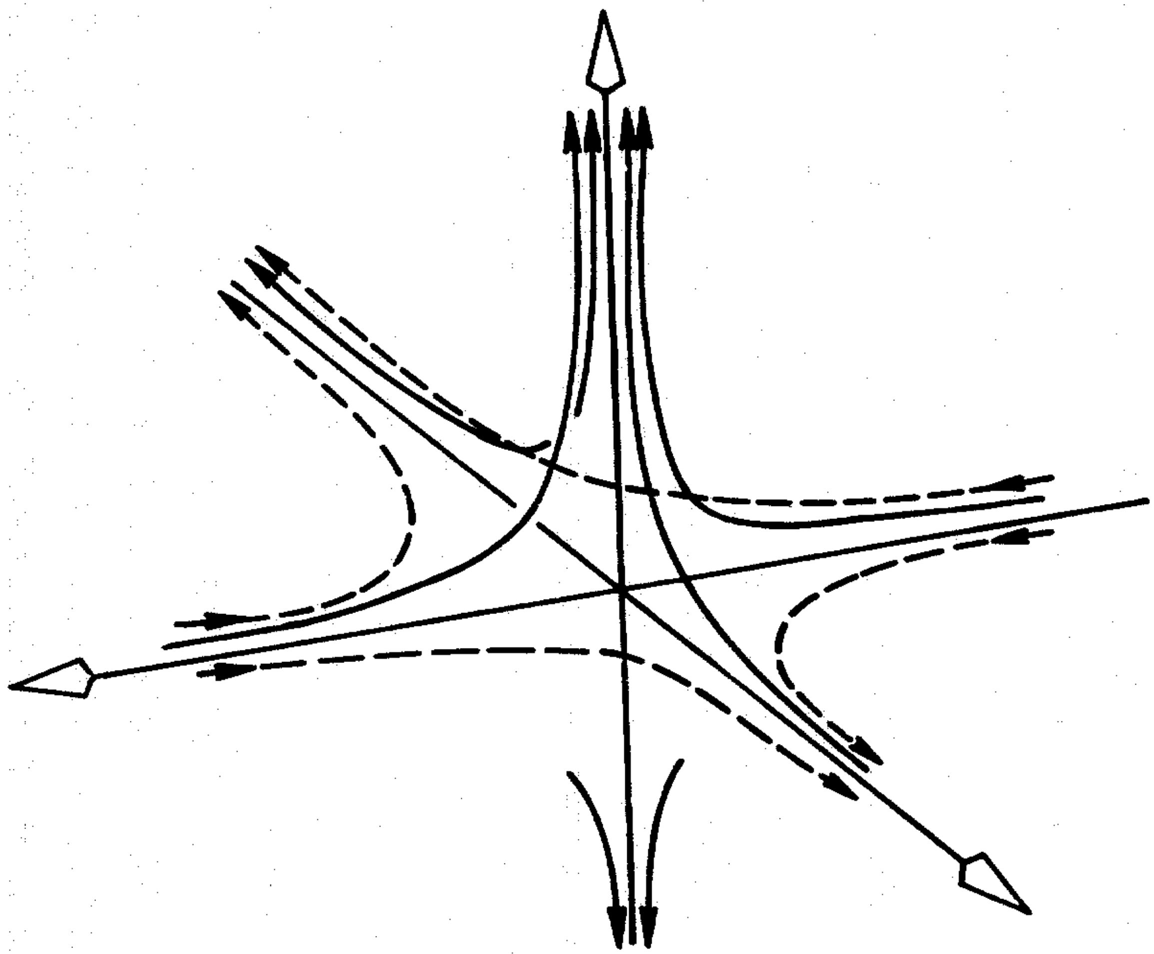


Figure 1a

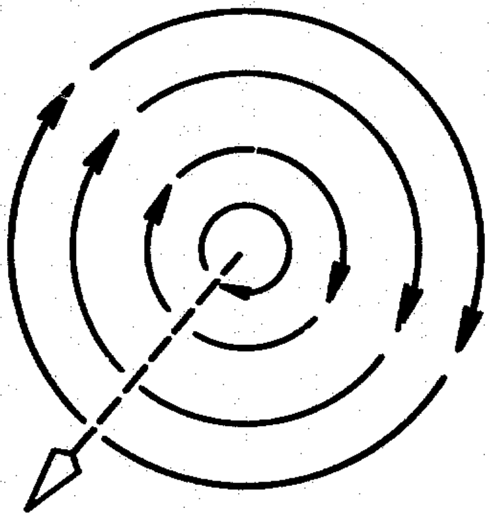


Figure 1b

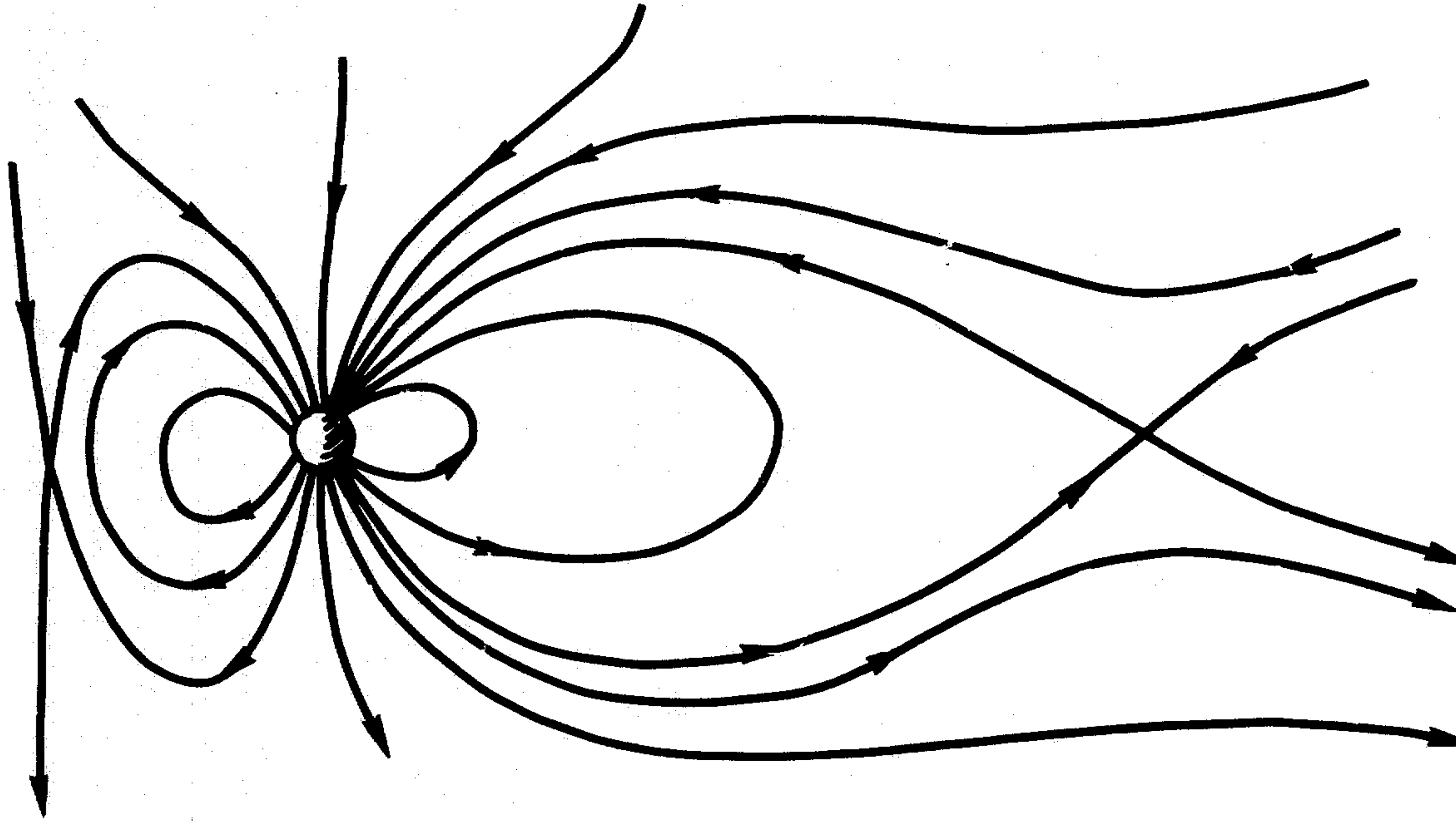


Figure 2a

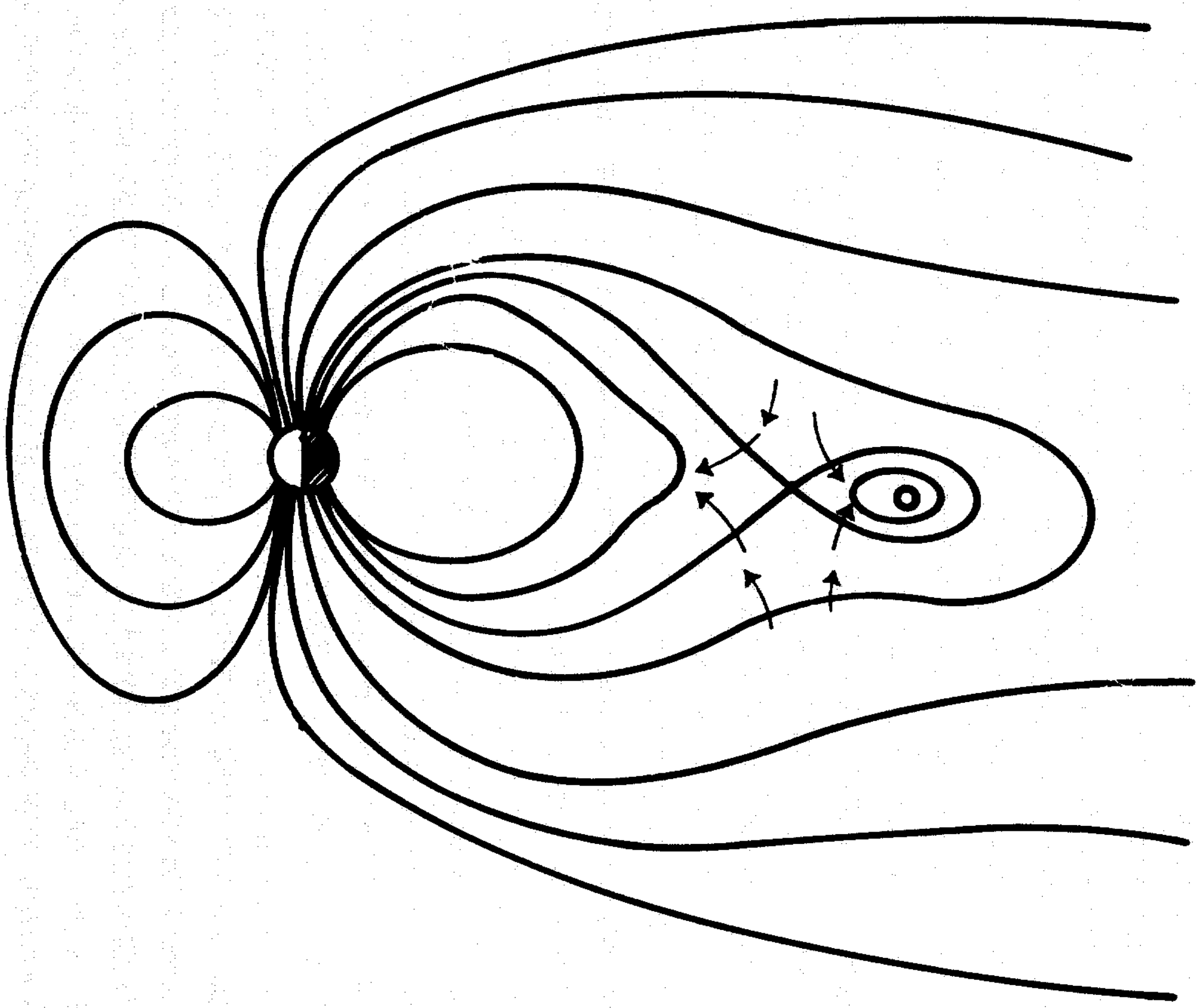
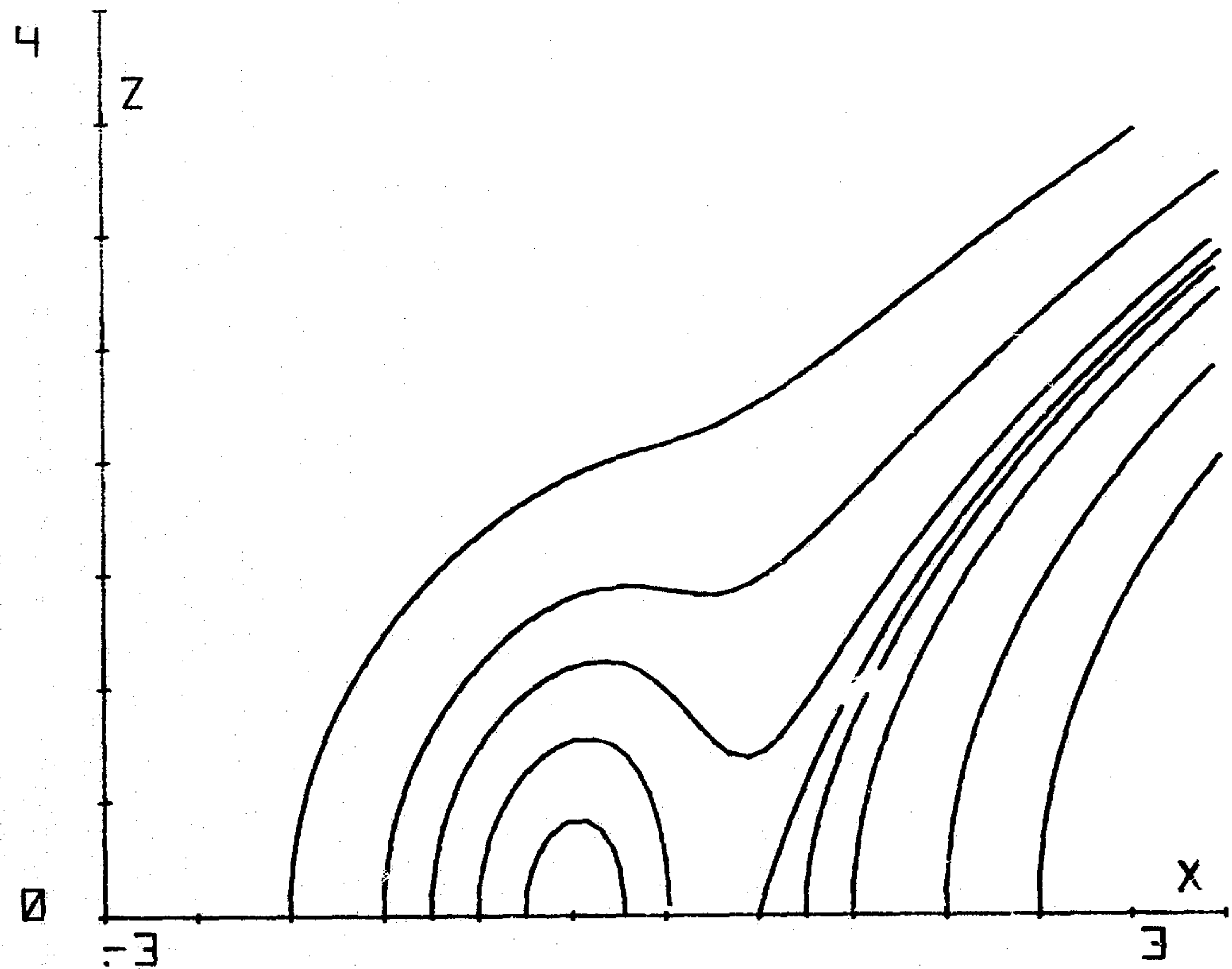


Figure 2b



Fig_re 3a

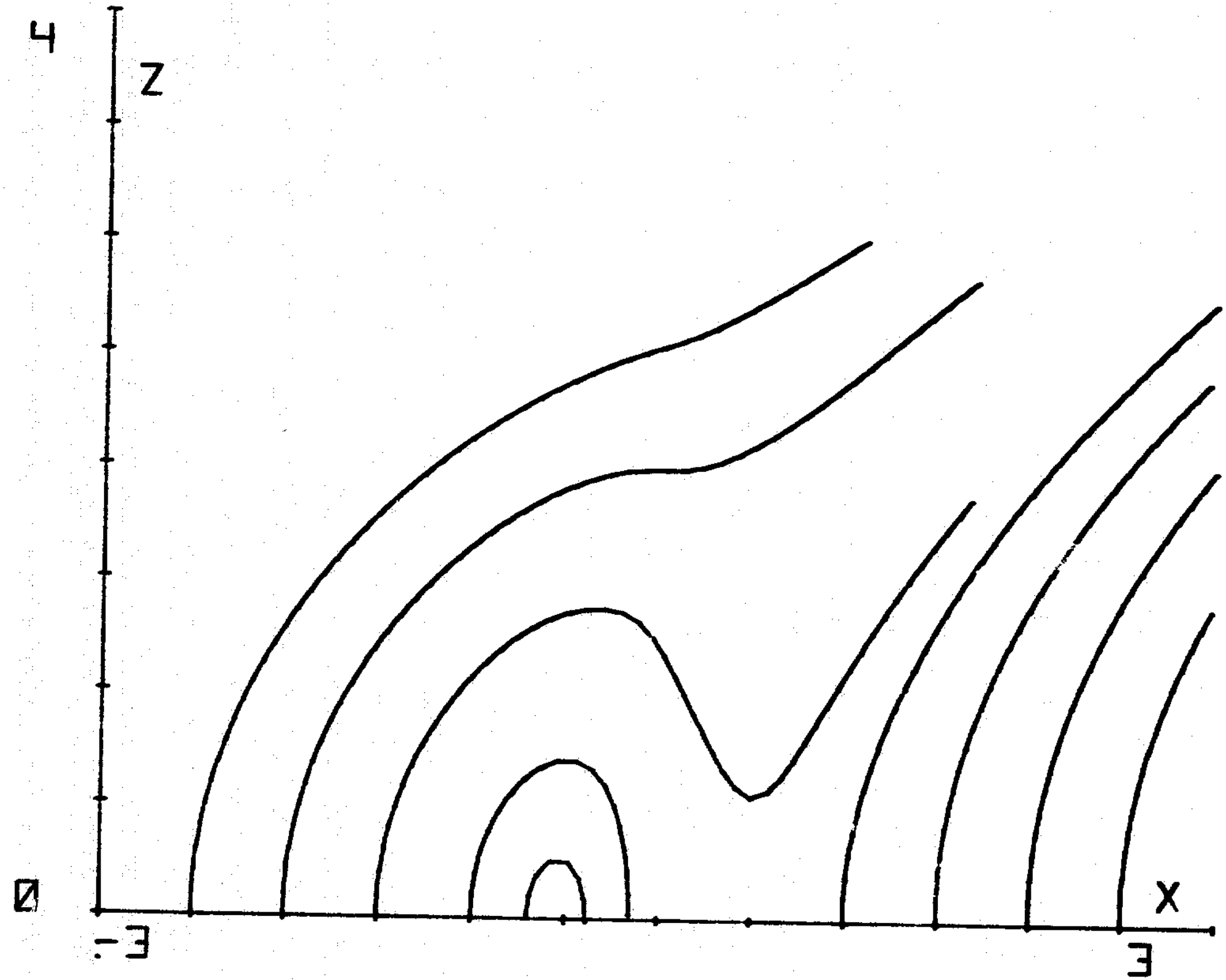


Figure 3b

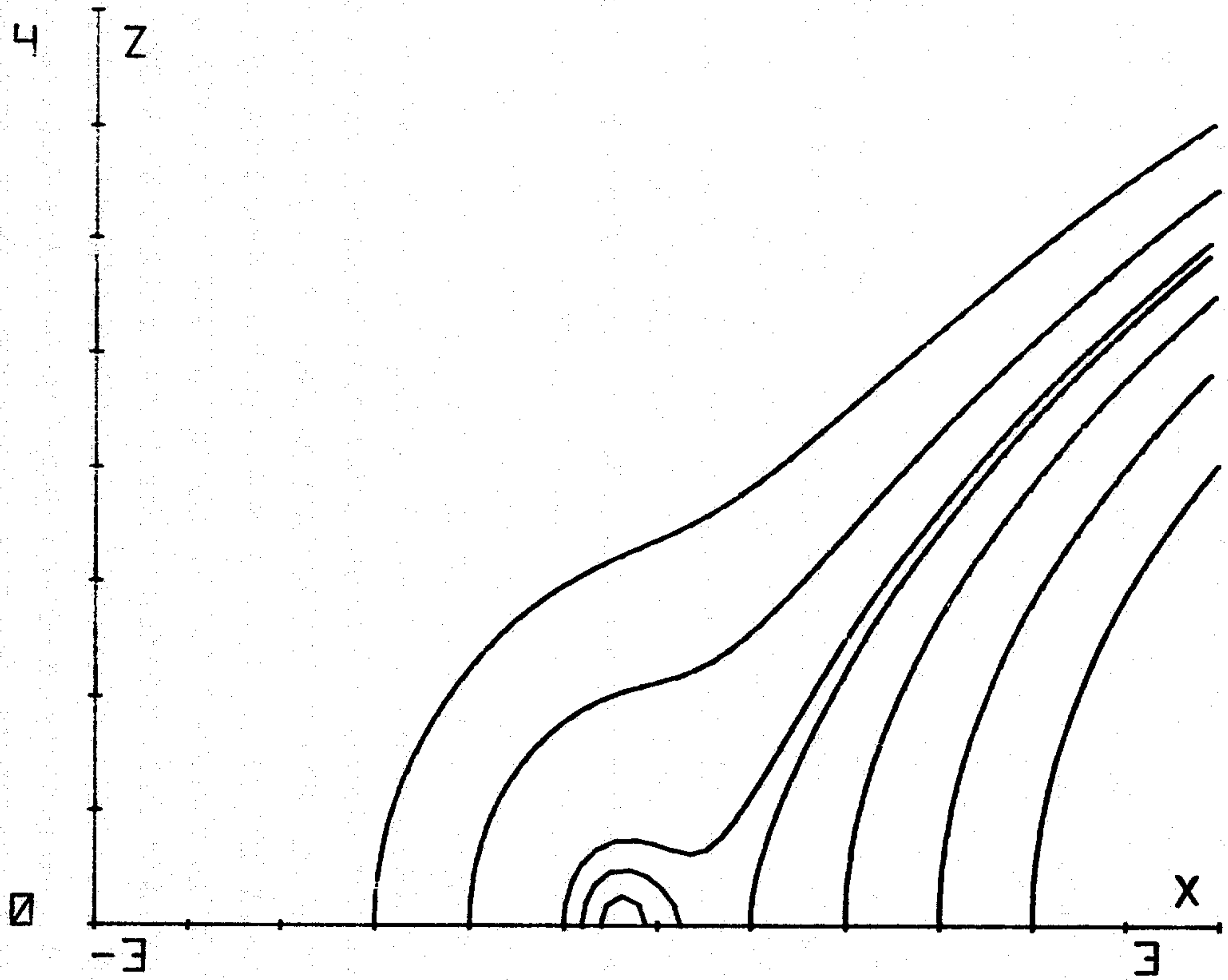


Figure 3c

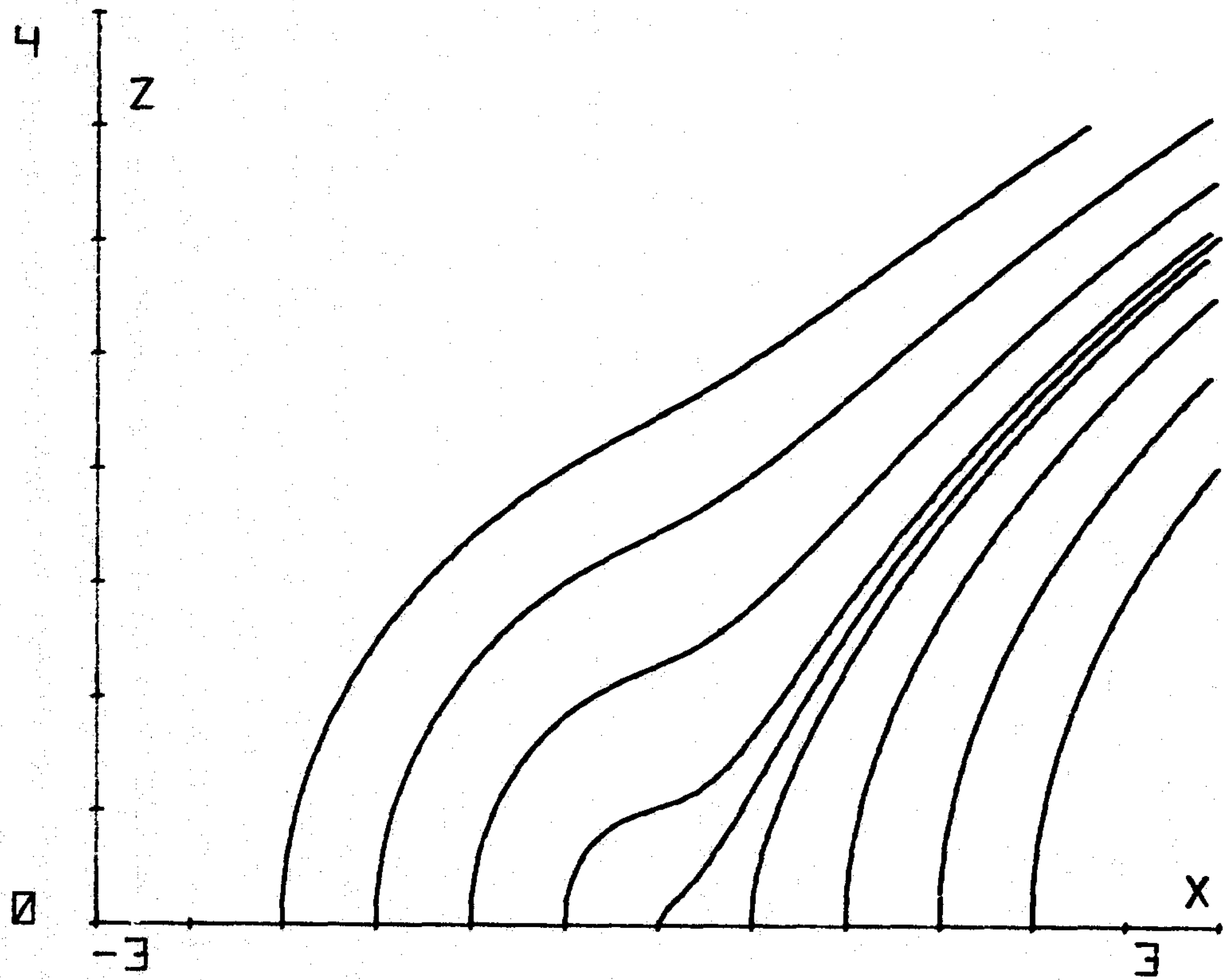


Figure 3d

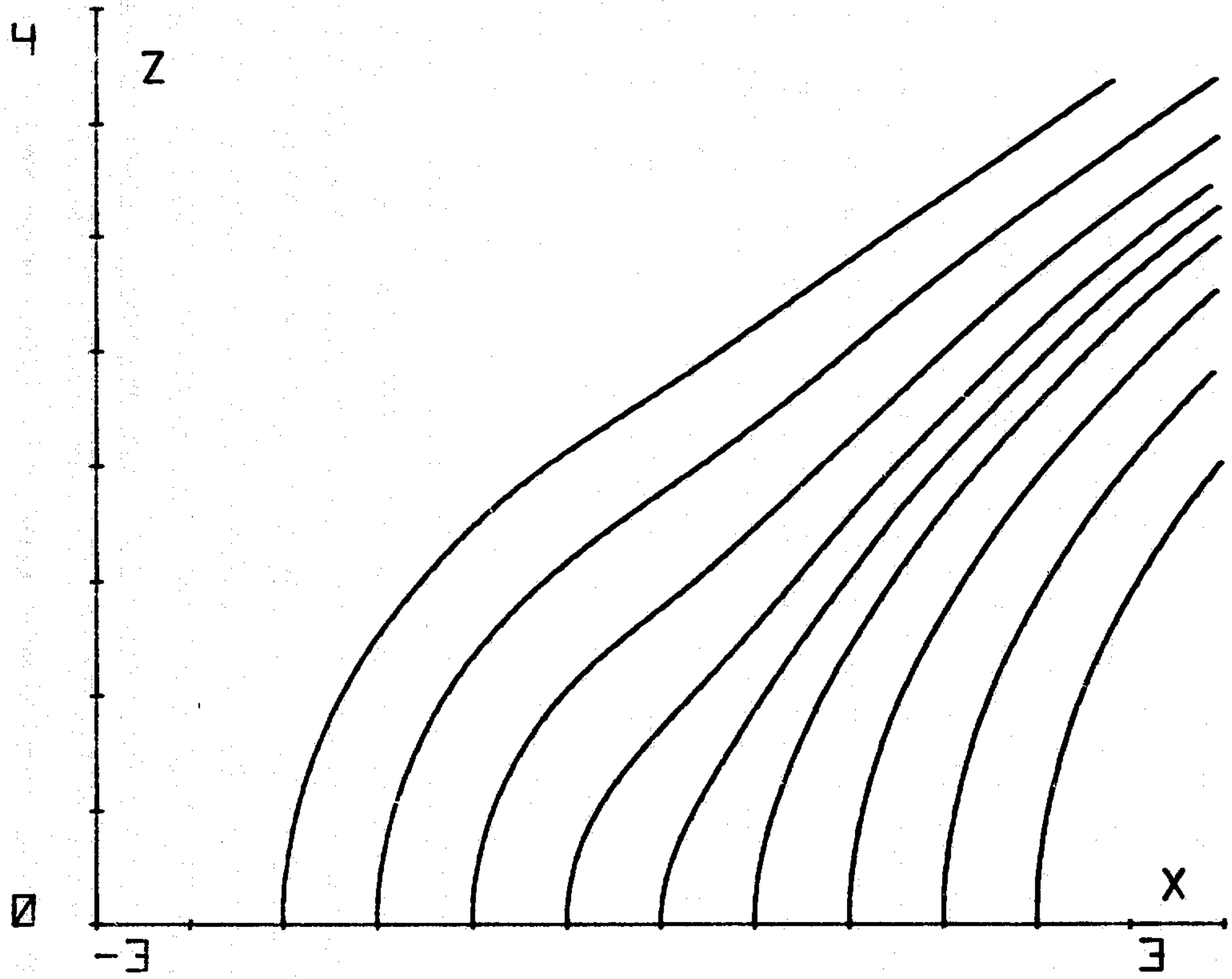


Figure 3e

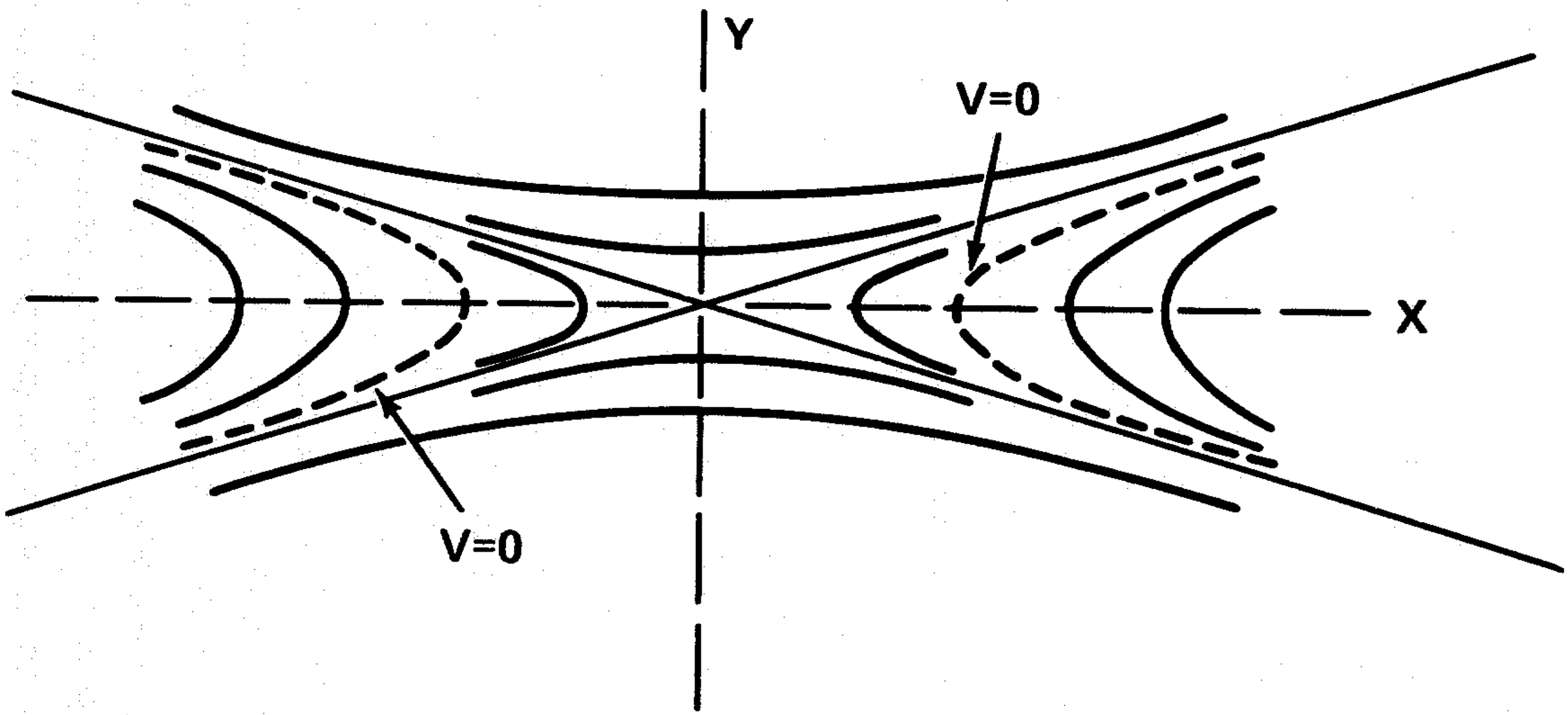


Figure 4

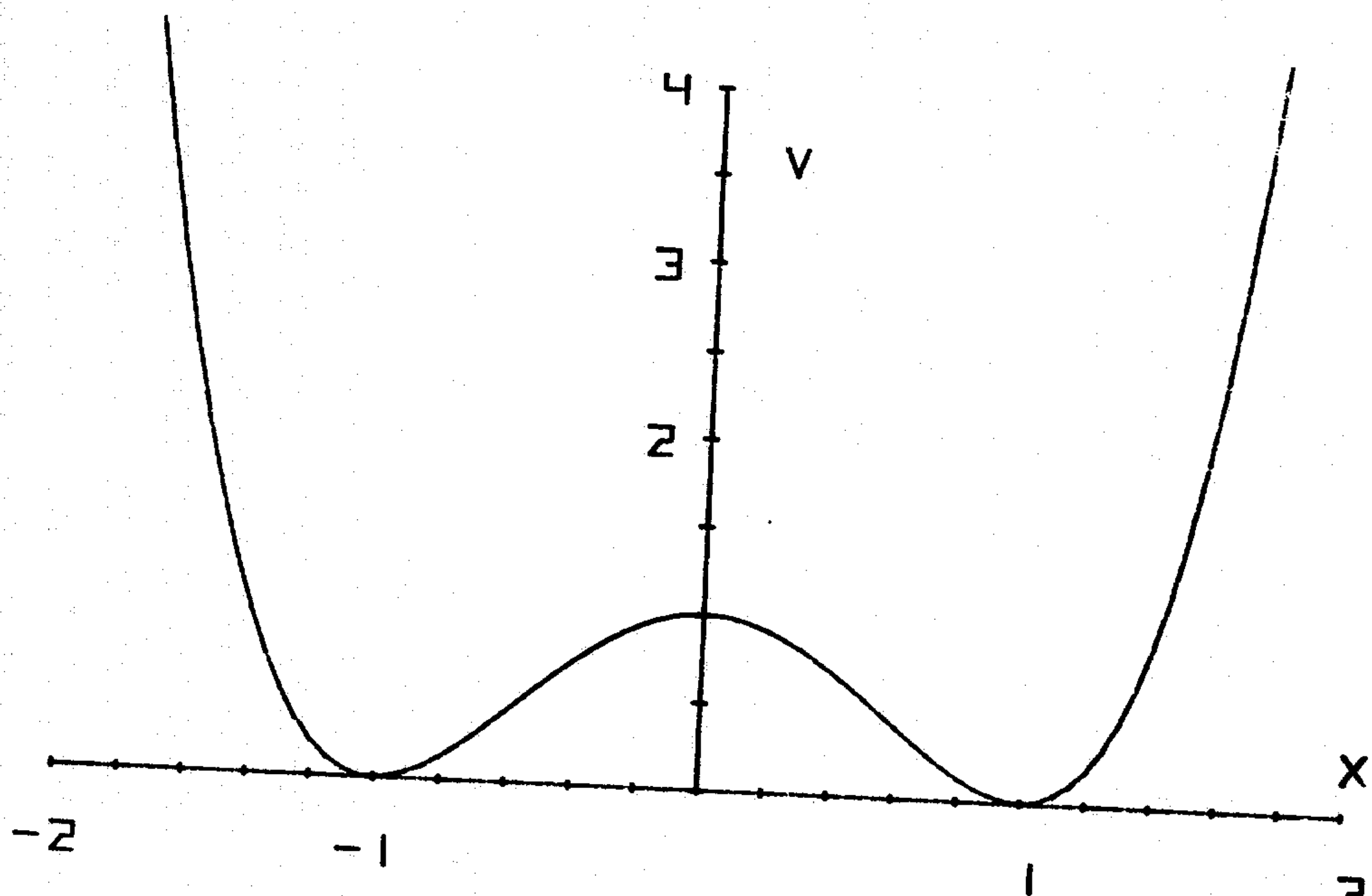


Figure 5

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16. Abstract

A model is derived for the 3-dimensional structure of the magnetic field near an isolated X-O pair of neutral points, such as may arise in the earth's magnetospheric tail during substorms, and possibly in solar flares. In a simplified 2-dimensional model of the neighborhood of the O-type neutral line of this configuration, the particle motion associated with magnetic merging is analyzed. It is shown that runaway acceleration of particles takes place efficiently near the O-type neutral line, where the MHD approximation is expected to break down. Extension to X-type neutral lines and applications to substorms and flares are also developed.

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