A Simple Method for Estimating Minimum Autorotative Descent Rate of Single Rotor Helicopters

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ABSTRACT

Flight test results of minimum autorotative descent rate are compared with calculations based on the minimum power required for steady level flight. Empirical correction factors are derived that account for differences in energy dissipation between these two flight conditions. A method is also presented for estimating the minimum power coefficient for level flight for any helicopter for use in the empirical estimation procedure of autorotative descent rate.

NOTATION

\( a \) Blade lift curve slope, 5.73/\text{rad}
\( C_{PMIN} \) Minimum value of \( C_P \) for level flight
\( C_P \) Power coefficient, \( \text{POWER/\( \rho \Omega^3 R^5 \)} \)
\( C_{PO} \) Profile power coefficient
\( C_{POH} \) Hover value of profile power coefficient
\( C_P^I \) Induced power coefficient
\( C_P \) Parasite power coefficient of fuselage and hub

\( C_T \) Thrust coefficient, here \( W/\eta \pi \alpha \omega^2 R^4 \)

\( \beta \) Parasite drag to dynamic pressure ratio, \( D/q, m^2 \)

\( K_I \) Constant to account for nonuniform inflow

\( K_O \) Empirical constant to account for rise in \( C_P \) with \( \mu^3 \)

\( K_{TR} \) Factor used to account for tail rotor power contribution

\( m_0, m_1 \) Correction factors arising from curve fit to experimental flight data

\( P_{AUTO} \) Measured (rate of descent) \( \times (\text{weight}) \), N-m/sec

\( P_{LVL} \) Power for level flight, N-m/sec

\( R \) Rotor radius, m

\( \dot{h} \) Autorotational (power off) rate of descent, m/sec

\( \dot{h}_{LVL} \) Rate of descent based on calculated level flight power required, m/sec

\( V \) Aircraft true airspeed, m/sec

\( W \) Gross weight, N

\( \alpha \) Control axis angle of attack, positive aft, rad

\( \delta \) Mean rotor drag coefficient

\( \lambda \) Inflow ratio, here \( \mu \alpha = C_T/\sqrt{\mu^2 + \lambda^2} \)

\( \mu \) Advance ratio, here \( V/\alpha R \)

\( \rho \) Air density, kg/m\(^3\)

\( \sigma \) Rotor solidity

\( \Omega \) Main rotor angular velocity, assumed same in power off and power on flight, 1/sec
INTRODUCTION

For preliminary design purposes, it is desirable to have convenient and quick methods of reliably estimating helicopter performance. Ideally these methods should yield good approximations to solutions based on more exact models and also provide insight into important physical parameters governing the performance. A convenient and quick way of estimating the autorotative descent rate is based on the energy method, in which the minimum power dissipated in gliding flight is assumed to be equal to that required in level flight. This method, however, consistently overestimates the autorotative descent rate; the power required for gliding flight being less than that for level flight.

In this paper, measured autorotative descent rates from flight test of a number of helicopters are compared with descent rates calculated by the energy method using the minimum power coefficient for level flight, $C_p^{\text{MIN}}$. An equation is given for calculating $C_p^{\text{MIN}}$ and the parameters which govern its value are discussed. Empirical correction factors are derived from the flight measurements to account for the lower energy dissipation in gliding flight so that estimates of autorotative descent rate that agree with flight experience can be obtained using $C_p^{\text{MIN}}$. Finally a sample of a design chart is given that can be used for estimating $C_p^{\text{MIN}}$ for new designs.

ESTIMATION PROCEDURE

Energy Method

The essence of the energy method is recognizing that the rate of change of the potential energy of the helicopter in steady gliding flight is equal to the rate at which energy is dissipated in overcoming rotor and fuselage drag.
Estimates of the rate of energy dissipation, however, are often obtained for a different flight condition: that for level flight. In fact, the power for level flight is often calculated and related directly to rate of descent by the energy equation

\[ \dot{W} = C_{p_{\text{MIN}}} \left(n \rho \sin \theta \right)^3, \quad \text{N-m/sec} \]  

(1)

From the definition of \( C_T \), this can be rearranged to give

\[ \dot{h} = (\pi R) C_{p_{\text{MIN}}} / C_T \quad \text{m/sec} \]  

(2)

In order to see how good this approach is, the measured rates of descent, power off, of several helicopters were plotted vs rates of descent computed with Equation (2). Values of \( C_{p_{\text{MIN}}} \) were obtained from level flight measurements of power required from various Army Aviation Test Activity reports. The results, shown in Figure 1, indicate that the rate of descent so computed is consistently higher than that measured in flight. The source of the discrepancy could be differences in tail rotor power, or differences in aerodynamic drag and main rotor torque from powered to gliding flight; there are insufficient data to isolate the cause. However, if the data are used with correction factors, the method is still useful if one has a good way, from rotor geometry and fuselage drag, of calculating values of level flight \( C_{p_{\text{MIN}}} \) that check well with flight measurements of \( C_{p_{\text{MIN}}} \).

Calculation of \( C_{p_{\text{MIN}}} \)

From a simplified expression, \( C_{p_{\text{MIN}}} \) is readily calculated for the level flight power required. The power, expressed as the sum of main-rotor-induced power, profile power, fuselage drag, and a tail rotor power contribution is

\[ C_p = K_{TR} (C_p^o + C_p^i + C_p^f) \]  

(3)
The main rotor profile power $C_{P_0}$ can be expressed as the hover value multiplied by a function of $\mu$. It can be lumped with the fuselage parasite drag by approximating the profile power rise with speed as a $\mu^3$ function.

Reference 1 shows that the increase in profile power with speed can be represented by the factor

$$\frac{C_{P_0}}{C_{P_{OH}}} = 1 + 3\mu^2 + \frac{3}{8} \mu^4$$

(4)

In the region of interest ($\mu = \mu/C_{P_{MIN}}$), this variation is replaced with the factor as follows

$$1 + K_0\mu^3 = 1 + 3\mu^2 + \frac{3}{8} \mu^4$$

(5)

Thus

$$K_0 = \frac{3}{\mu} + \frac{3}{8} \mu$$

(6)

For example, $K_0 \approx 25$ at $\mu = 0.12$.

The induced contribution $C_{P_1}$ is approximated by using the uniform inflow model at higher $\mu$ where $\mu >> \lambda$.

The fuselage drag is treated as a constant. The factor $K_{TR}$ is used to multiply the whole power expression to account for tail rotor requirements.

The expression for power then becomes

$$C_{P} = K_{TR} \left[ C_{P_{OH}} \left(1 + K_0\mu^3\right) + \frac{K_1C_T^2}{2\mu} + \frac{f_{e}\mu^3}{2\pi R^2} \right]$$

(7)

The approximation that thrust = weight is used throughout this note.

The hover profile drag of the blades $C_{P_{OH}}$ can be calculated from

$$C_{P_{OH}} = \frac{\delta d}{8}$$

(8)

$$\delta = 0.009 + 0.3 \left(\frac{6C_T}{\sigma A}\right)^2$$

(9)
Having expressed the power equation in this way, the equation can be easily differentiated with respect to $u$ and its minimum found. The result is

$$C_{p_{MIN}} = K_{TR} \left[ C_{p_{OH}} + 1.144 C_{l}^{3/4} \left( C_{p_{OH}} K_{O} + \frac{f_{e}}{2\pi R^{2}} \right) \right]^{1/4}$$ (10)

$$\nu C_{p_{MIN}} = \left\{ \frac{1.13C_{T}^{2}}{6 \left[ C_{p_{OH}} K_{O} + (f_{e}/2\pi R^{2}) \right]} \right\}^{1/4}$$ (11)

The numerical constants arise from the value 1.13 used for $K_{I}$ and the process of differentiation. The factor 1.13 accounts for the inflow being triangular (with $r/R$) rather than rectangular (uniform). In order to use expression (10) some independent estimate of $f_{e}$ is required. It can be obtained for each helicopter by measuring slopes of $C_{p}$ vs $u^3$ at high speed and correcting for the $C_{p_{OH}} K_{O}$ term.

This can be seen by differentiating (7) with respect to $u$ and ignoring the induced term:

$$\frac{3C_{p}}{3(u^3)} = \frac{f_{e}}{2\pi R^{2}} + C_{p_{OH}} K_{O}$$ (12)

Thus,

$$f_{e} = 2\pi R^{2} \left[ \frac{3C_{p}}{3(u^3)} \right]_{measured} - C_{p_{OH}} K_{O}$$ (13)

A sample flight test result of $C_{p}$ plotted versus $u^3$ is shown in Figure 2 for an AH-1G helicopter. As a matter of interest, the $f_{e}$ results extracted for several other helicopters are shown in Table 1.

The measured and estimated values of $C_{p_{MIN}}$ for several helicopters are shown in Figure 3; the values shown result from using values of $K_{TR} = 1.10$ and $K_{O} = 24.5$. The calculated values show good agreement with measured flight values.
Empirical Correction Factors

With the knowledge of the value of \( C_{p\text{MIN}} \), the energy method can be used to estimate the autorotative rate of descent. However, as observed from the data of Figure 1, direct use of \( C_{p\text{MIN}} \) in the energy equation always results in descent rates that exceed those observed. Two methods were explored of correcting the calculated values of rate of descent to bring them into agreement with measured rates of descent. One was setting the \( K_{TR} \) term equal to unity which was not very satisfactory and implied a particular knowledge of the source of the discrepancies between test and calculated values.

The other was simply expressing the discrepancy between measured values and the calculated values as a constant error plus one linearly dependent on calculated rate of descent — in effect acknowledging the true difference between \( C_{p\text{MIN}} \) for level flight and \( C_p \) for minimum autorotation descent rate. This was done by putting a least squares fit through the data shown in Fig. 1. The result is expressed by

\[
h_{\text{EST}} = m_1 \frac{C_{p\text{MIN}}}{C_T} + m_0 \quad \text{m/sec} \quad (14)
\]

where \( m_0 \) was found to be 2.30 m/sec and \( m_1 \) was found to be 0.66.

Method of Estimating Rate of Descent

As an aid to rapid computation and to provide insight into the parameters which influence autorotative rate of descent, Equation (10) can be solved for fixed values of solidity and \( f_e \) and presented as the ratio \( C_p/C_T \). This was done for \( \sigma = 0.04 \) and \( f_e = 0.005, 0.020 \) and 0.060 in Figure 4.

For any new design, the rate of descent can be computed in a straightforward manner using Figure 4 and Equation (14) as follows:
1. Obtain a reasonable estimate of \( f_e \) by using Table 1 and compute \( f_e/\pi R^2 \) for the hypothetical design of interest. See also Reference 2.

2. Compute \( C_T \) from the helicopter weight, rotor speed and air density.

3. Interpolate between constant values of \( f_e/\pi R^2 \) in Figure 4 to obtain the value of \( C_{P_{\text{MIN}}}/C_T \) at the design values of \( f_e/\pi R^2 \) and \( C_T \). Additional charts for \( \sigma \neq 0.040 \) can be readily constructed.

4. Compute rate of descent multiplying \( C_{P_{\text{MIN}}}/C_T \) by \((m_1 OR)\) and adding \( m_0 \), as indicated by Equation (14).

The range of \( C_T \) shown in the charts covers low disc loading, high tip speed, low altitude conditions to low tip speed, high disc loading, and high altitude conditions. For any given set of rotor parameters, there is an optimum \( C_T \) for minimum rate of descent (a minimum of \( C_{P_{\text{MIN}}}/C_T \)).

CONCLUDING REMARKS

Flight test data show that the power absorbed by the main and tail rotor is consistently lower in gliding flight than in level flight, suggesting that if appropriate correction factors are applied, the minimum autorotation descent rate of a new design can be estimated from the power required in level flight.

Equations for the speed for minimum power and for \( C_{P_{\text{MIN}}} \) were derived from simplified, classical equations for level flight power required. The rate of descent is shown to depend mainly on solidity, thrust coefficient, and the ratio of parasite drag area to rotor disc area.

The value of correction factors which may be applied to the equation for level flight power required in order to estimate minimum autorotational rate of descent, are obtained and a graph for estimating rate of descent for new designs is presented.
The utility of using the graph and correction factors is to provide some insight into the parameters affecting autorotation performance and to increase the accuracy and convenience of making such estimates during the preliminary design process.

REFERENCES

Table 1. Values of $f_e$ Obtained from Slopes of Power Curves at High $\mu$ for Several Helicopters.

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<th>$f_e$</th>
<th>Gross weight</th>
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FIGURE CAPTIONS

Figure 1  Comparison of measured autorotation minimum descent rate with that based on $C_p_{MIN}$ from level flight.

Figure 2  $C_p$ vs $\mu^3$ for the AH-1G helicopter.

Figure 3  Comparison of measured and calculated minimum level flight power coefficient.

Figure 4  Ratio of minimum power coefficient to thrust coefficient vs thrust coefficient.
Figure 1  Comparison of measured autorotation minimum descent rate with that based on $C_{p_{\text{min}}}$ from level flight.
Figure 2 $C_p$ vs $\mu^3$ for the AH-1G helicopter.
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Figure 4 Ratio of minimum power coefficient to thrust coefficient vs thrust coefficient.
18. Abstract

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