NASA

Technical Memorandum 78085

The Solar Reflectance of a Snow Field

(NASA-TM-78085) THE SOLAR REFLECTANCE OF A SNOW FIELD (NASA) 25 p HC A02/MF A01 N78-20575
CSCL 08L
Unclas
G3/43 12569

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JANUARY 1978

National Aeronautics and
Space Administration

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ABSTRACT

The radiative transfer equation has been solved using a modified Schuster–Schwartzschild approximation to obtain an expression for the solar reflectance of a snow field. The parameters in the reflectance formula are the single scattering albedo and the fraction of energy scattered in the backward direction. The single scattering albedo is calculated from the crystal size using a geometrical optics formula and the fraction of energy scattered in the backward direction is calculated from the Mie scattering theory. Numerical results for reflectance are obtained for visible and near infrared radiation for different snow conditions. Good agreement has been found with the whole spectral range. The calculation also shows the observed effect of aging on the snow reflectance.

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INTRODUCTION

Interest in developing techniques for remote measurement of snow parameters (e.g. temperature profile, density grain size, water equivalent and free water content), have significantly increased in recent years. One of the more promising techniques to monitor these parameters is that of microwave radiometry. A recent study by Chang et al., (1976) has used a theoretical scattering model to relate the observed microwave brightness temperature to the physical temperature and snow grain size for snow on glacier ice. By utilizing this remotely determined snow grain size and density information, the solar spectral reflectance from a snow field can be inferred. The knowledge of spectral reflectance is important because it determines the amount of solar energy that will be reflected by a snow field. The present study is an attempt to calculate the solar reflectance of a snow cover in terms of parameters which depend upon the grain size and density.

For incident solar radiation, the energy reflected at the surface constitutes only a small fraction (about 5 percent) of the total energy that is reflected by a snow field. For visible and near infrared radiation, the scattering by individual snow crystals is such that major portion of the radiation gets scattered in the forward direction. The small fraction of energy that is scattered in the backward direction, however, gets enhanced considerably by the cumulative effect of many scatterers. Thus the major portion of the incident solar radiation which gets reflected by a snow cover is due to the volumetric effect of many scatterers.

The appropriate equation which needs to be solved for calculating the cumulative effect of many scatterers is the radiative transfer equation. An analytical solution of the radiative transfer equation when the individual scatterings are in the near forward direction is rather difficult, but a solution can be obtained by numerical methods (Choudhury, 1977). In this paper we will discuss an analytical solution with modified Schuster–Schwartzschild approximation (Sobolev, 1963; Sagan and Pollack, 1967). The expression for spectral reflectance obtained from this analytical solution of the radiative transfer equation can be evaluated in terms of snow parameters. The theory presented here is intended to show the influence of snow parameters on the solar reflectance of snow.

In the following the general expressions and equations needed to study the spectral reflectance of a snow cover are given. It also includes the background information regarding the snow parameters and the solar intensity distribution which are expected to influence the spectral reflectance. The analytic solution of the radiative transfer equation will be discussed and an expression for the spectral reflectance obtained. In addition comparisons are made for the obtained numerical results and the observations.

GENERAL FORMULATION

The solar energy incident on the snow surface can be divided into two components: the collimated intensity, I_c, falling at an angle θ_0 with respect to the normal and the nearly isotropic diffuse intensity I_d which arises due to the atmospheric scattering of the radiation. The flux of the incident radiation is given by

$$F_i = I_c \mu_o + I_d \pi \tag{1}$$

where $\mu_o = \cos\theta_o$.

Since most of the incident radiation is contained in the collimated component, the flux of the radiation reflected by the snow-air interface is approximately given by

$$F_R = r(\theta_o)(I_c \mu_o + I_d \pi) \tag{2}$$

where $r(\theta_o)$ is the Fresnel reflectivity of the surface.

The refracted portion of the incident radiation will progress through the medium (snow) getting weaker due to scattering and absorption. It is convenient to separate the refracted portion of the collimated radiation into two components: the unattenuated radiation and the radiation which arises due to scattering. This separation is done because the unattenuated radiation remains angularly concentrated along the direction of refraction but the angular distribution of the scattered radiation depends upon the scattering phase function. If multiple reflection at the boundaries of the medium is negligible then the unattenuated intensity at depth x is given by:

$$I_u(\tau) = \left(1 - r(\theta_o)\right) n^2 \, I_c \, \delta(\phi - \phi_o) \, \delta(\mu - \mu_o) \exp(-\tau/\mu_o') \tag{3}$$

where n is the index of refraction, μ_o' is the angle of refraction:

$$\mu_o' = \left[1 - \frac{1 - \mu_o^2}{n^2}\right]^{1/2}$$

and τ is the optical depth

$$\tau = \int_0^X \gamma_e(x')\, dx' \tag{4}$$

where $\gamma_e(x)$ is the extinction coefficient at depth x. The delta functions in (3) represent the constraint due to the collimated nature of the radiation.

The flux of the unattenuated radiation is given by

$$F_u(\tau) = \left(1 - r(\theta_o)\right) I_c \mu_o \exp(-\tau/\mu_o') \qquad (5)$$

The intensity which arises due to scattering of the incident radiation is to be obtained by solving the radiative transfer equation (Chandrasekhar, 1960):

$$\mu \frac{dI_s(\tau,\mu)}{d\tau} = -I_s(\tau,\mu) + \frac{\omega}{2}\int_{-1}^{1} p(\mu,\mu')\, I_s(\tau,\mu')\, d\mu' + \frac{\omega}{4\pi} n^2 I_c \left(1 - r(\theta_o)\right) p(\mu,\mu_o') \exp(-\tau/\mu_o') \qquad (6)$$

where μ is the angle of radiation, ω is the single scattering albedo and $p(\mu,\mu')$ is the scattering phase function.

The boundary conditions for the radiative transfer equation are as follows:

(1) If the snow-air interface is assumed to be a geometrically smooth surface then the appropriate boundary condition is

$$I_+(\tau = 0, \mu) = r(\mu)\, I_-(\tau = 0, \mu) + \left(1 - r(\mu)\right) n^2 I_d \qquad (7)$$

where I_+ is the radiation going downward into the snow and I_- is the radiation going upward towards the atmosphere. The first term on the right hand side represents the intensity reflected by the snow-air interface and the second term accounts for the refracted component of the diffuse intensity.

(2) The second boundary condition is associated with the snow-soil interface. The general boundary condition will be of the form:

$$I_-(\tau_o, \mu) = r_1(\mu)\; I_+(\tau_o, \mu) \tag{8}$$

where $r_1(\mu)$ is the reflectivity of the snow-soil interface and τ_o is the total optical thickness of snow.

Once the solution of equation (6) is known subject to the above boundary conditions, one can calculate the flux of the radiation emerging from snow as:

$$F_e = 2\pi \int_o^1 I_-(o,\mu)\; \left(1 - r(\mu)\right) \frac{\mu d\mu}{n^2} \tag{9}$$

The albedo of snow is to be calculated using equations (1), (2) and (9) as:

$$A = \frac{F_R + F_e}{F_i} \tag{10}$$

The volumetric effect of scattering which is mainly responsible for large values of albedo is associated with the emergent flux F_e appearing in equation (10) and is defined through the equation (9). The calculation of the emergent flux requires the solution of the radiative transfer equation (6).

APPROXIMATE EXPRESSION FOR SNOW REFLECTANCE

To obtain a closed form analytic solution of the radiative transfer equation (6), the following assumptions are made.

1. The medium (snow) is a homogeneous plane parallel layer, consisting of statistically equal size ice spheres.

2. The Schuster-Schwartzchild or two-stream approximation for the intensity is valid.

3. The angular asymmetry in the scattering process can be described by introducing a parameter which weights the scattered intensities.

It should be noted that with the first approximation we are neglecting the presence of all internal inhomogenities like enlarged ice glands and ice lenses which sometimes form during the process of metamorphism. The other two approximations are associated directly with the method of solution. It has been shown that for a optically thick medium, the solution of the radiative transfer equation under the above set of approximations is reasonably accurate (Sagan and Pollack 1967).

The Schuster-Schwartzschild or two-stream approximation is based upon partitioning the scattered radiation into mean intensities along the forward and the backward hemispheres. In single scattering, if β is the fraction of energy scattered in the backward hemisphere with respect to the direction of incidence, then we can replace the radiative transfer equation (6) by the following equations:

$$\frac{dI_+}{d\tau} = - I_+ + \omega(1 - \beta) I_+ + \omega\beta I_- + \omega(1 - \beta) I' \exp(-\tau) \tag{11}$$

and

$$-\frac{dI_-}{d\tau} = -I_- + \omega(1-\beta)I_- + \omega\beta I_+ + \omega\beta I' \exp(-\tau) \qquad (12)$$

where

$$I' = [1 - r(o)]\ (I_c + I_d)$$

and I_+, I_- are respectively the mean intensities along the forward and backward directions with respect to the direction of incident radiation, and r(o) is the Fresnel reflectivity for normal incidence. It should be noted that these equations represent the radiative transfer in a one dimensional medium (Sobolev, 1963).

The radiative transfer equation (11) and (12) contains two parameters, namely, (1) ω, the single scattering albedo and (2) β, the fraction of energy scattered in the backward direction. We will discuss the method of determining these parameters before solving this radiative transfer equation.

For ice particles in the millimeter range and the radiation in the visible or near infrared region, the size of the particle is significantly larger than the wavelength. Under this condition, Mie scattering theory shows the extinction cross-section undergoes resonance oscillations for small changes in the wavelength and size of particle. This resonance effect, however, can be blurred by assuming that there is a statistical variation in the particle size about its mean radius. With this mean size value, one can use the geometrical optics approximation to calculate the single scattering albedo using the equation (Irvine and Pollack, 1968, Sagan and Pollack, 1967).

$$\omega = 1/2 + 1/2 \cdot \exp(-2k_\lambda r) \quad (13)$$

where r is the radius of the particle and k_λ the absorption per unit length of ice at wavelength, λ. In our calculation, we have used k_λ values tabulated by Irvine and Pollack (1968). Following above discussion, instead of using equation (13), it should be noted that one can also use Mie scattering theory to calculate the single scattering albedo.

Consistent with equation (13) the extinction coefficient required in the calculation of optical depth (equation 4) is given by

$$\gamma_e = \frac{3}{2r}\left(\frac{\rho_s}{\rho_i}\right) \quad (14)$$

where ρ_s and ρ_i are respectively the density of snow and the density of ice.

The fraction of energy scattered in the backward direction, β, in a single scattering has been studied by many inve   s (e.g. Bartky, 1968; Sagan and Pollack, 1967). This parameter can be related to the scattering phase function $p(\cos\theta)$ as:

$$2\beta = 1 - \int p(\cos\theta) \cdot \cos\theta \frac{d\Omega}{4\pi} \quad (15)$$

where θ is the scattering angle and $d\Omega$ is an element of solid angle in the co-ordinate system where θ is the polar angle. The normalization condition for the phase function is

$$\int p(\cos\theta) \frac{d\Omega}{4\pi} = 1 .$$

When the phase function is expanded in terms of Legendre polynomials, the parameter β can then be expressed in terms of the coefficient of the first order Legendre polynomial.

For a given particle size one can use Mie scattering theory to calculate the phase function and then use (15) to calculate the value of β. Calculations performed by Irvine and Pollack (1968) show that when the size of the particle is significantly larger than the wavelength of the radiation, the value of β is about 0.075, i.e., in single scattering only 7 percent of the incident energy is scattered in the backward direction. In our calculation, we have used this value of β (0.075) because for visible and near infrared radiations, the scattering by individual ice crystals is indeed in the near-forward direction.

For a thick snow layer, the general solution of equations (11) and (12) can be obtained by standard methods (see Sobolev, 1963 pp. 33) as

$$I_{+}(\tau) = 1/2\,(1 + \frac{1-\omega}{k})\, C\, e^{-k\tau} - I'\, e^{-\tau} \qquad (16)$$

and

$$I_{-}(\tau) = 1/2\,(1 - \frac{1-\omega}{k})\, C\, e^{-k\tau} \qquad (17)$$

where C is the constant of integration and

$$k^2 = (1-\omega)\,\big(1 - \omega\,(1 - 2\beta)\big)$$

To determine the constant of integration we now impose the boundary condition (7) to obtain

$$C = \frac{2\,I'}{\left((1 + \frac{1-\omega}{k}) - r(o)\,(1 - \frac{1-\omega}{k})\right)} \qquad (18)$$

With the knowledge of complete solution, the reflectance for a one-dimensional system can be calculated as:

$$A = r(o) + I_{-}(\tau = o)\,\left(1 - r(o)\right)\,\frac{1}{(I_c + I_d)} \qquad (19)$$

The explicit expression for reflectance is:

$$A = r(o) + \frac{\left((1 - r(o)\right)^2 \alpha}{- r(o)\,\alpha} \qquad (20)$$

where

$$\alpha = \frac{k - 1 + \omega}{k + 1 - \omega}$$

(Note that for $\omega = 1$ one finds $\alpha = 1$)

The Fresnel reflectivity for normal incidence, $r(o)$, can be calculated from the reflective index of ice, n, using the formula:

$$r(o) = \left| \frac{n - 1}{n + 1} \right|^2 \qquad (21)$$

Since the spectral dependence of n in the visible and the near infrared region is extremely weak (Irvine and Pollack, 1968), we have used a constant 1.30 for n in calculating the $r(o)$.

The expression for reflectance is in close resemblance with that calculated by Dunkle and Bevans (1956) without explicitly making reference to one-dimensional system. The definition of the parameters appearing in equation (20) are, however, quite different. We thus note that although there is a formal resemblance between the expression for reflectance obtained in this paper and that obtained by Dunkle and Bevans (1956), the method of obtaining this expression and the method of calculating the parameters of this expression are quite different in two cases.

The expression for reflectance obtained in this paper is based upon simplifying approximations. We have discussed these approximations and attempted to provide their justification. The ultimate justification of the model is to be found by comparing with the experimental observations. In the next section, we will provide the numerical results and compare them with the observations.

RESULTS AND DISCUSSIONS

By examining equation 20, the single scattering albedo ω is the only varying parameter in calculating the snow reflectance for a given wavelength. Since ω is directly related to the snow crystal radius, it is possible to characterize the snow condition by specifying the radius of the ice crystals.

The ice crystals of fresh fallen snow usually are of complex form and contain sharp corners. The shape of these crystals changes with time depending upon the vapor content and the prevalent temperature. The process of equi-temperature metamorphism leads to the production of fairly uniform and well rounded grains. At the initial stage of this metamorphism the mean radius of the ice crystals is about 0.2 mm. The radius then continues to increase as the process of metamorphism advances. At a fairly advanced stage of this metamorphism, the radius increases to about 1.0 mm. The effect of other metamorphism such as the temperature-gradient metamorphism or the melt-freeze metamorphism, is generally to produce non-spherical and non-uniform ice crystals also the crystals are larger than equi-temperature metamorphism. Thus, although it is not unique, one should be able to use the radius of ice crystal to characterize the stage of metamorphism.

Figure 1 illustrates the changes in the calculated snow reflectance due to the difference in the crystal radius. The chosen radius are the typical snow crystal sizes for different stages of the equi-temperature metamorphism. The overall shape of the curve does not seem to depend upon the radius of the ice crystals but the relative magnitudes of various parts of the curve vary with the radius. The spectral reflectance in the red and near-infrared regions shows maximum sensitivity to the crystal sizes.

In Figure 2 we show the calculated snow reflectance compared with the experimental values for a nearly fresh snow (O'Brien and Munis, 1975). The results of the calculation are in good agreement with the observations. All prominent spectral structures appearing in the observation are well duplicated in the calculations. The quantitative agreement is also good but should be treated with caution because the experimental results are relative to a standard (white Barium Sulfate) reflector. It is however reassuring that the radius values used in the calculation are in the range expected for a fresh fallen snow.

In Figure 3 we compare the calculated and the observed reflectance of a naturally aged two days old snow (O'Brien and Munis, 1975). There is a good qualitative agreement between them. The radius for the snow crystals which give agreement with the observation are larger than those for the fresh fallen snow (Figure 2). The natural aging process of snow, during the observations, was such that the ambient air temperature was hovering above and below freezing point. It is unlikely that the snow was undergoing the equi-temperature metamorphism. As a result, the ice crystals were probably non-uniform within snow. Furthermore, if the effect of the solar irradiance is interpreted as simply to advance the process of metamorphism of the snow layer in contact with the air, then the radius of the ice crystals in the top layer will be larger than those deep within. Since the absorption coefficient of ice increases with the wavelength, the effective thickness of the snow layer that contributes to the reflectance decreases with the wavelength. Therefore, the reflectance for longer wavelengths should have more influences from the top layer of snow and the reflectance for shorter wavelengths should correspond more to the deeper snow layer. This is a probable cause for the reflectance of the smaller crystals giving better agreement at shorter wavelengths while the reflectance of the larger crystals giving better agreement at longer wavelengths.

Based on calculated results, two potential applications in remote snow reflectance monitoring are possible: (1) the use of multispectral bands to infer the radiation heat flux over a snow covered area; and (2) the use of visible and near-infrared channels to detect melting snow.

The large contrast in snow and soil reflectance makes it possible to monitor quantitatively the amount of solar radiation absorbed by the earth's surface. The degree of heat insulation by snow cover depends on the snow reflectance which is shown to be directly related to the snow crystal size in this paper. Therefore a spaceborne multispectral scanning instrument can provide a method to monitor the heat flux over a snow cover area under the National weather and climate program.

The second potential application, that of detecting melting snow, is based on the calculation that the snow reflectance decreases rapidly with the increase in snow crystal size. The existence of melt water on the snow surface will tend to increase the crystal size, also in the near infrared region the absorption per unit length of water is higher than that of ice. As a result, the reflectance of melting snow is expected to be lower than the dry snow in the near infrared region.

The theory presented here, does show the qualitative and quantitative features of the observed reflectance. However the one dimensional approach can not distinguish the effects of the direct and the diffuse solar radiations. A more complexed analysis is needed to assess the effect of directional irradiation.

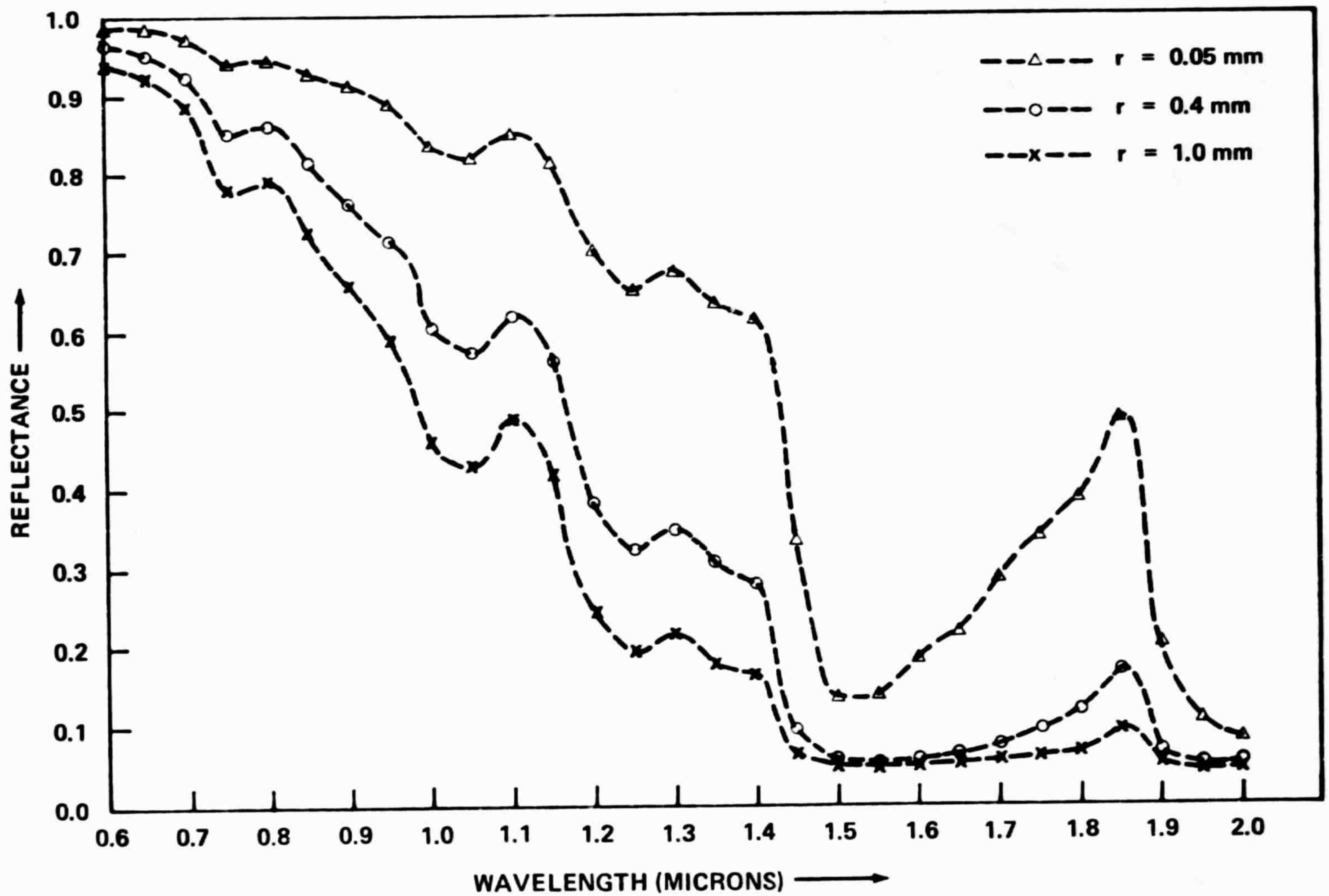

Figure 1. Illustrations of the Effect of Different Snow Crystals on Snow Reflectance

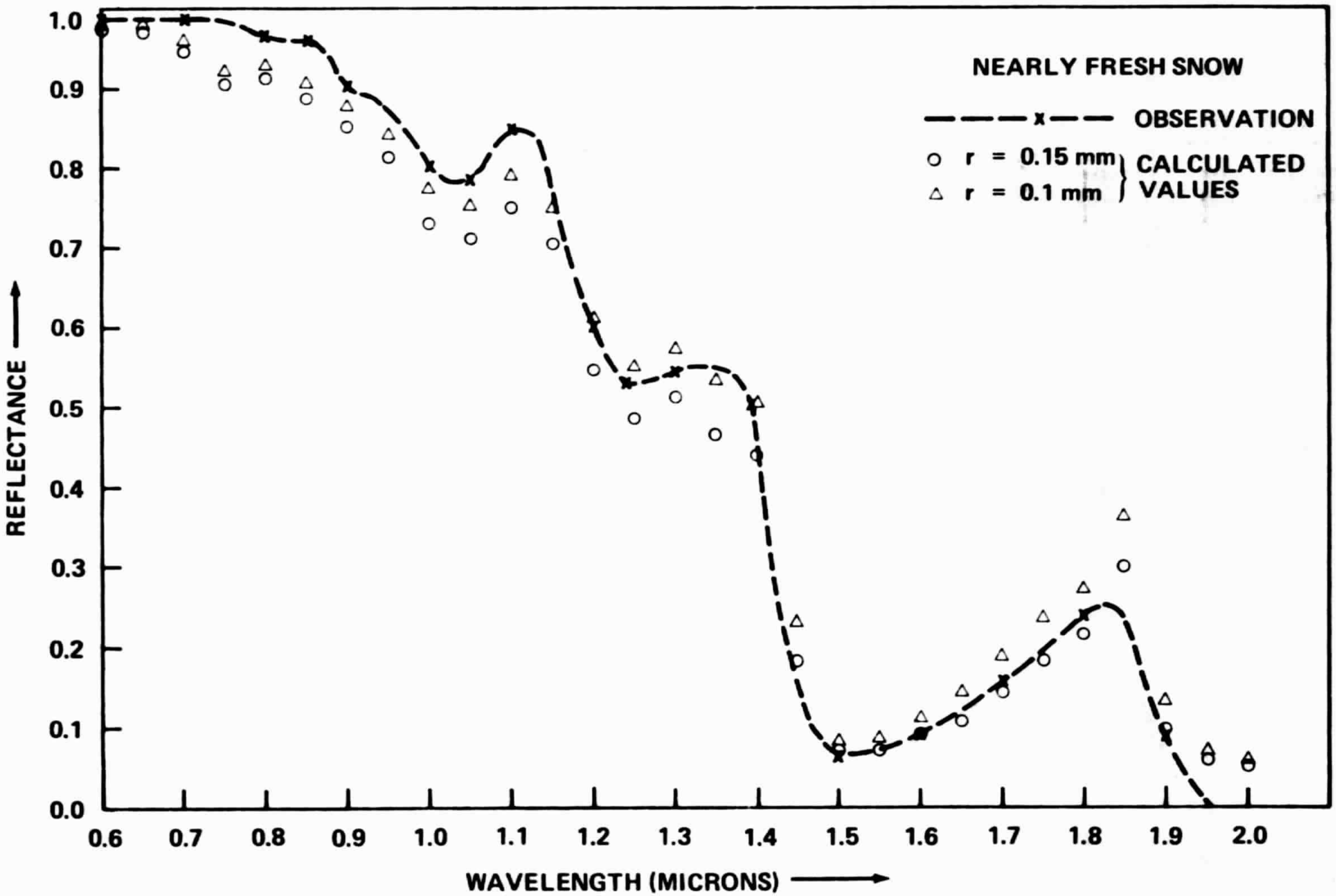

Figure 2. Comparison of Calculated and Observed Reflectance of a Nearly Fresh Snow

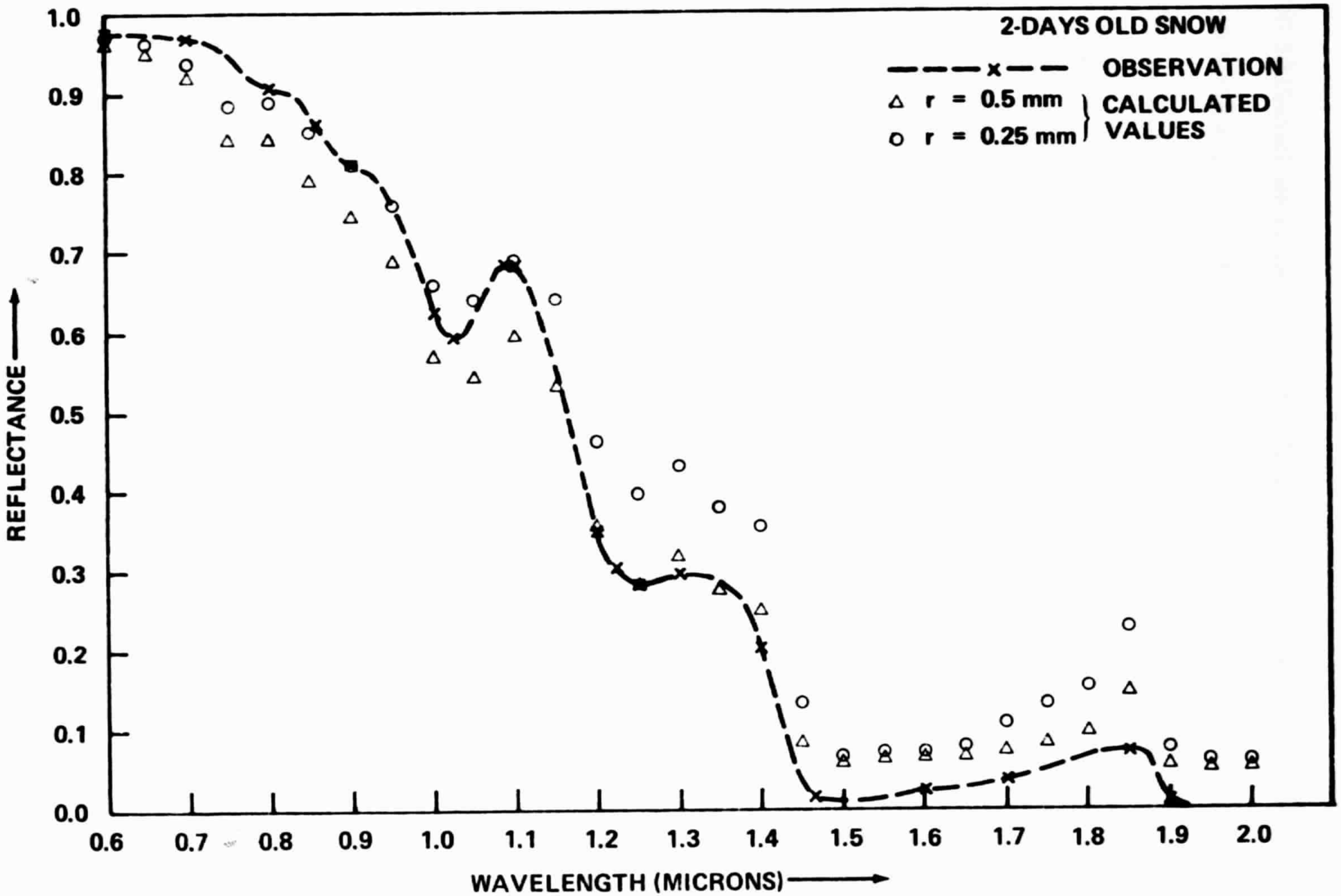

Figure 3. Comparison of Calculated and Observed Reflectance of a Naturally Aged Snow

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FIGURE CAPTIONS

Figure 1. Illustrations of the Effect of Different Snow Crystals on Snow Reflectance.

Figure 2. Comparison of Calculated and Observed Reflectance of a Nearly Fresh Snow.

Figure 3. Comparison of Calculated and Observed Reflectance of a Naturally Aged Snow.

BIBLIOGRAPHIC DATA SHEET

1. Report No. TM 78085	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle THE SOLAR REFLECTANCE OF A SNOW FIELD		5. Report Date January 1978
		6. Performing Organization Code 910
7. Author(s) B. J. Choudhury, Computer Sciences Corp. and A. T. C. Chang, NASA/GSFC		8. Performing Organization Report No.
9. Performing Organization Name and Address Goddard Space Flight Center Greenbelt, Maryland 20771		10. Work Unit No.
		11. Contract or Grant No.
		13. Type of Report and Period Covered
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		
		14. Sponsoring Agency Code
15. Supplementary Notes		

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17. Key Words (Selected by Author(s)) Snow reflectance, Mie scattering theory		18. Distribution Statement Unclassified – Unlimited Cat. 46	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages	22. Price*

*For sale by the National Technical Information Service, Springfield, Virginia 22151.

GSFC 25-44 (10/77)