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AIRFOIL PROFILE IN A NON-UNIFORM FLOW

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16. Abstract

Airfoil profile in nonuniform flow; theory of airfoil section past two-dimensional non-uniform flow is developed; theory is based on representation of airfoil section by vortex and source distributions and it can be used for calculation of aircraft wings in homogeneous and inhomogeneous flow, as well as for calculation of straight and radial blade and vane-cascades.

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AIRFOIL PROFILE IN A NON-UNIFORM FLOW

Jan Polášek*

We will discuss a method for calculating the flow around a profile in a nonhomogeneous flow field. This method is based on the idea that the profile is replaced by a continuous distribution of vortices and sources along the profile skeleton. The method is suitable for solving the direct and the indirect problem of profile theory. /533**

The results given can be applied for calculating the air foil in an homogeneous and a non-homogeneous flow field, or for calculating straight and radial turbine blades, and guide blades, etc. For practical applications, the results are given in the form of formulas, suitable for numerical calculations. Examples are also given.

SYMBOLS:

c	profile skeleton chord
y_8	profile skeleton ordinate
t	distribution of profile thickness (measured from skeleton)
y_{mt}	ordinate of profile central line
t	symmetric thickness distribution (measured from the central line)
x_p, y_p	coordinates of points on the profile
R_1	curvature radius of leading edge
R_2	curvature radius of trailing edge
ϑ	trigonometric auxiliary variable (identical with the

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** Numbers in margin indicate pagination in original foreign text.

	polar angle in the complex auxiliary plane ζ)
u, v	velocity components
W_K	contour velocity
U_o	basic velocity
Γ	total circulation over profile /618
γ	circulation distribution
q	source distribution
A_n, C_n	coefficients in the expansion of profile thickness
B_n	coefficient in the expansion of skeleton direction factor
$\bar{B}_n$	coefficient of the expansion of the directional factor of central line
g_n	coefficient in the expansion of circulation distribution
q_n	coefficient in the expansion of source distribution
μn, νn	coefficients in the expansion of components of primary velocity

COMPLEX PLANE Z

The same notation is used in the physical plane, but the quantities are considered dimensionless. The lengths are referred to one-half of the chord length (c/2) and the velocities are referred to the basic velocity (U_o).

$\mathfrak{w}$	complex velocity in plane z
u_γ, v_γ	velocity components induced by vortices
u_q, v_q	velocity components induced by sources
u_K, v_K	components of contour velocity

COMPLEX PLANE Z:

X, Y	rectangular coordinates $Z = X + iY$
X_ν, Y_ν	coordinates of points on the profile
$\mathfrak{W}$	complex velocity
U, V	components of primary velocity in plane Z
U_γ, V_γ	velocity components induced by vortices
U_q, V_q	velocity components induced by sources
U_K, V_K	components of contour velocity

COMPLEX PLANE ζ:

ρ, ϑ	Polar coordinates $\zeta = -\rho e^{-i\vartheta}$
ε	distance of point from unit circle ($\varepsilon = \rho - 1$)
$\bar{w}_\zeta$	complex velocity in ζ plane
v_ρ, v_ϑ	radial and azimuthal velocity components

INTRODUCTION

The idea of Birnbaum of using a continuous vortex distribution to replace the flow around a thin profile has been found to be very fruitful. The original paper of Birnbaum [1] was followed by similar studies [2-6], and several applications were considered: From the flow around a thin profile in a homogeneous flow to the flow around a profile in a nonhomogeneous or non-steady flow field. Then this was applied to the solution of guide blades, and blade cascades. Another advance was the consideration of the blade thickness, which can be achieved by a suitable distribution of sources on the profile skeleton. /619

In the calculation of the flow around profiles with a moderate curvature, a further simplification is introduced by decomposing the vortices and sources along the profile chord, instead of along its skeleton. The calculations become cleared with these simplifications, and they can be performed without great mathematical knowledge. The so-called first-order theory gives very good results for the ratio between the shape of an infinite thin profile and the circulation distribution. However, the calculation of the contour velocity and the influence of the finite thickness, especially near the leading edge, lead to certain difficulties and inaccuracies. Various authors have tried to eliminate or reduce this by introducing various correction factors. Deeper mathematical investigations have shown that in the derivation of the relationships between the shape of an infinite thin profile and the circulation distribution in a homogeneous flow field, only the "third order" terms were ignored, because the "second order" terms drop out automatically (as can be seen by comparison with the results of paper [6], for the case of a moderately curved profile in a homogeneous flow field).

In other cases, especially when calculating the velocity distribution along the blade contour and the influence of finite profile thickness, the ignored "second order" terms become noticeable and have a substantial influence, especially near the leading edge.

For these reasons, in this paper we developed a complete "second order" theory for the flow around a profile which has moderate curvature and is not too thick in a nonhomogeneous flow field. The paper is given in two parts: in the first part, we derive the theory and in the second part we give the theoretical results in terms of formulas for numerical calculations. These formulas are then used for the practical solution of several selected problems. First, we will calculate the flow around a symmetric and curved profile in a homogeneous flow field, and then we will consider the flow around the same profile in a grid arrangement.

FIRST PART

THEORETICAL ANALYSIS

2. Infinitesimally-thin profile in a non-homogeneous flow field

The profile under consideration is placed in the complex plane z = x + iy in such a way that one chord lies along the x-axis, and the origin of the coordinate system is at the center. The lengths are considered dimensionless and are referred to one-half of the chord length (c/2).

The profile equation is:

$$\boxed{y = y_s(x), \quad -1 \leq x \leq 1;} \tag{2.1}$$

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where

$$\boxed{y_s(-1) = y_s(1) = 0.} \tag{2.2}$$

Instead of the variable x, we will introduce the trigonometric auxiliary variable ϑ with the equation

$$x = -\cos\vartheta \quad ; \quad 0 \leqq \vartheta \leqq \pi \tag{2.3}$$

For the calculations, it seems appropriate to express the differential quotient of the function $y_s(x)$ in the form of a trigonometric series:

$$\frac{dy_s}{dx} = \sum_{n=0}^{\infty} B_n \cos n\vartheta \,. \tag{2.4}$$

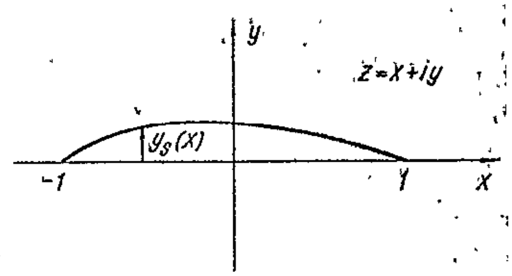

Figure 1

The profile shape is then given by the expression

$$y_s = \frac{1}{2}\left\{B_0(1-\cos\vartheta) + \sum_{n=1}^{\infty}\frac{B_{n-1}-B_{n+1}}{n}(1-\cos n\vartheta)\right\} \tag{2.5}$$

The coefficients B_n must satisfy the following condition because of equation (2.2):

$$B_0 = \sum_{m=1}^{\infty} \frac{B_{2m}}{4m^2-1} \,. \tag{2.6}$$

Also the components of the velocity of the primary inhomogeneous flow field at the profile location, as well as the circulation density (referred to the length unit of the chord), are expressed as a continuous distribution of vortex threads on the profile using trigonometric series:

$$u = 1 + \sum_{n=0}^{\infty} \mu_n \cos n\vartheta, \tag{2.7}$$

$$v = \sum_{n=0}^{\infty} \nu_n \cos n\vartheta, \tag{2.8}$$

$$\gamma = 2\left(g_0 \operatorname{cotg}\frac{\vartheta}{2} + \sum_{n=1}^{\infty} g_n \sin n\vartheta\right).^{1)} \tag{2.9}$$

The velocities and circulation density are considered dimensionless, and are referred to a suitable basic velocity U_0. The coefficients in the expansions (2.4), (2.7), (2.8), and (2.9) are assumed to be small of first order, in the sense that the quantities μ_n, ν_n, g_n and B_n are of first order compared with 1. For large n, they go to zero like the terms of a geometric series with a quotient smaller than 1. The theory is developed as a second order theory in such a way that everywhere the products of two first-order variables are considered as second-order variables, and the products of three first-order variables are ignored. /621

In practice, it is sufficient to approximate the trigonometric expansions (2.4), (2.7), and (2.8) using trigonometric polynomials having a relatively low order.

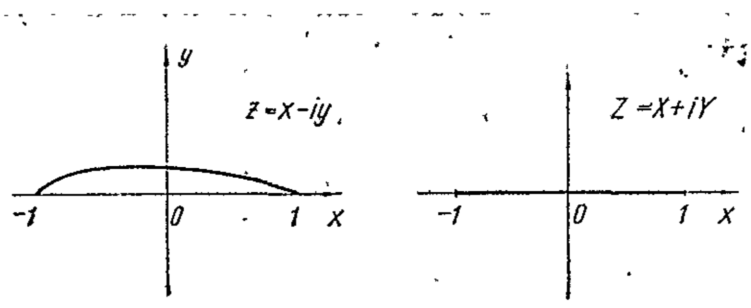

Figure 2

In addition, we will introduce the complex plane Z = X + iY and the complex function:

$$z = Z + if(Z) \tag{2.10}$$

It maps the plane Z onto the plane z in such a way that the distance $-1 \leq X \leq 1$ along the real axis of plane Z is mapped onto a curve y_s in the plane z. In the plane, Z, we introduce for the variable X in the interval $-1 \leq X \leq 1$, Y = 0 a trigonometric auxiliary variable ϑ as

[1] The circulation density on the profile is $\gamma_s = \frac{\gamma}{[1+(dy_s/dx)^2]^{1/2}}$. The quantities γ and dy_s/dx are subjected to the restriction that the difference between γ_s and γ is small to third order, and therefore, we can set $\gamma_s = \gamma$ for our theory.

follows:

$$X = -\cos\vartheta \quad , \quad 0 \leq \vartheta \leq \pi. \tag{2.11}$$

The function f(Z), for example, can be written in the form

$$f(Z) = (1 - Z^2) \sum_{n=0}^{\infty} \beta_n Z^n \tag{2.12}$$

The following relationships can hold between the coefficients β_n and /622
the coefficients B_n in the expansion of the directional factor (2.4), which are linear relationships

$$\begin{aligned} \beta_1 &= B_0 - B_2 + B_4 - \dots, \\ 2(\beta_0 - \beta_2) &= B_1 - 3B_3 + 5B_5 - \dots, \\ 3(\beta_1 - \beta_3) &= 4B_2 - 16B_4 + \dots, \\ &\dots\dots\dots\dots \\ &\dots\dots\dots\dots \end{aligned} \tag{2.13}$$

Since we will perform our analysis in the image plane Z, it is necessary to map the velocity of the primary flow field from the plane z onto plane Z. For the mapping of the primary velocity, we have

$$W = U - iV = (u - iv)\frac{dz}{dZ} \tag{2.14}$$

We have the following relationship for the derivative of the mapping function along $-1 \leq X \leq 1$, $Y = 0$:

$$\frac{dz}{dZ} = 1 + i\frac{df}{dZ} = 1 + i\frac{dy_s}{dx}, \tag{2.15}$$

so that the expressions for the components of mapping of the primary velocity* along the interval $-1 \leq X \leq 1$ along the real axis of the plane Z are given by the following:

$$\begin{aligned} U &= u + v\frac{dy_s}{dx}; \\ V &= v - u\frac{dy_s}{dx}. \end{aligned} \tag{2.16}$$

*The mapping (2.10) maps the flow field in the plane z again onto any flow field in plane Z - which is a mapping of the flow field from plane z. Expressed more simply, this mapping flow field is simply called a flow field in the plane z.

After substituting from (2.4), (2.7), and (2.8), we find:

$$
\begin{aligned}
U &= 1 + \sum_{n=0}^{\infty} \mu_n \cos n\vartheta + \frac{1}{2}\left(\nu_0 B_0 + \sum_{k=0}^{\infty} \nu_k B_k \right) + \frac{1}{2} \sum_{n=1}^{\infty} \left[\sum_{k=0}^{n} \nu_k B_{n-k} + \right. \\
&\qquad \left. + \sum_{k=0}^{\infty} (\nu_k B_{n+k} + \nu_{n+k} B_k) \right] \cos n\vartheta, \\
V &= \sum_{n=0}^{\infty} \nu_n - B_n \cos n\vartheta - \frac{1}{2}\left(\mu_0 B_0 + \sum_{k=0}^{\infty} \mu_k B_k \right) - \frac{1}{2} \sum_{n=1}^{\infty} \left[\sum_{k=0}^{n} \mu_k B_{n-k} + \right. \\
&\qquad \left. + \sum_{k=0}^{\infty} (\mu_k B_{n+k} + \mu_{n+k} B_k) \right] \cos n\vartheta.
\end{aligned}
\tag{2.17}
$$

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The complex velocity in the plane z, which is induced at the profile (2.5) by the distributed vortices (2.9), is given by:

$$
u_\gamma - i v_\gamma = \frac{i}{2\pi} \int_{-1}^{1} \frac{\gamma(x_0)\, dx_0}{z - z_0}
\tag{2.18}
$$

With the mapping (2.12), the velocity (2.18) is transformed to

$$
U_\gamma - i V_\gamma = \left[1 + i \frac{df}{dZ} \right] \frac{i}{2\pi} \int_{-1}^{1} \frac{\gamma(X_0)\, dX_0}{X - X_0 + i\,[Y + f(Z) - f(X_0)]}
\tag{2.19}
$$

Along the segment $-1 \leq X \leq 1$ of the X axis, we have:

$$
U_\gamma - i V_\gamma = \left(1 + i \frac{dy_s}{dx} \right) \frac{i}{2\pi} \int_{-1}^{1} \frac{\gamma(X)_0\, dX_0}{X - X_0 + i(y_s - y_{s0})}
\tag{2.20}
$$

The expression (2.20) is transformed into

$$
U_\gamma - i V_\gamma = \left(1 + i \frac{dy_s}{dx} \right) \frac{i}{2\pi} \int_{-1}^{1} \frac{\gamma(X_0)\, dX_0}{(X - X_0)\left(1 + i \frac{dy_s}{dx} + i \frac{y_s - y_{s0}}{X - X_0} - i \frac{dy_s}{dx} \right)}
\tag{2.21}
$$

and the integrand is expanded according to powers of

$$
\left\{ \left(1 + i \frac{dy_s}{dx} \right)^{-1} \left(\frac{y_s - y_{s0}}{X - X_0} - i \frac{dy_s}{dx} \right) \right\}
$$

If we restrict ourselves to second-order terms in this expression, then we can write

$$
U_\gamma - i V_\gamma = \frac{i}{2\pi} \int_{-1}^{1} \left[1 + i \left(\frac{dy_s}{dx} - \frac{y_s - y_{s0}}{X - X_0} \right) \right] \frac{\gamma_0(X_0)\, dX_0}{X - X_0},
$$

or, after substituting the variable ϑ, according to (2.11), we have:

$$U_\gamma - iV_\gamma = \frac{i}{2\pi}\int_0^\pi \left\{1 + i\left[\sum_{n=0}^\infty B_n \cos n\vartheta - \frac{1}{2}\left(B_0 + \sum_{n=1}^\infty \frac{B_{n-1}-B_{n+1}}{n}\cdot\frac{\cos n\chi - \cos n\vartheta}{\cos\chi - \cos\vartheta}\right)\right]\right\}\cdot \tag{2.22}$$
$$\cdot\left\{2g_0(1+\cos\chi) + \sum_{n=1}^\infty g_n\left[\cos(n-1)\chi - \cos(n+1)\chi\right]\right\}\frac{d\chi}{\cos\chi - \cos\vartheta}.$$

The incomplete integrals on the right side of (2.22) are to be considered in terms of the Cauchy principal value. For example, they were calculated in [6], Appendices I and IV. Therefore, we will only give /624
the final result:

$$\begin{aligned} U_\gamma - iV_\gamma = {} & \pm\left(g_0 \operatorname{cotg}\frac{\vartheta}{2} + \sum_{n=1}^\infty g_n \sin n\vartheta\right) + i\left(g_0 - \sum_{n=1}^\infty g_n \cos n\vartheta\right) + \\ & + \left[\frac{1}{2}B_1\left(g_0 + \frac{1}{2}g_1\right) + \frac{1}{3}B_2\left(g_0 + \frac{1}{2}g_2\right) + \frac{1}{2}B_3\left(g_0 + \frac{1}{4}g_1\right) + \right. \\ & \left. + \frac{7}{15}B_4 g_0 + \frac{1}{2}B_5 g_0 + \dots\right] + \\ & + \cos\vartheta\cdot\left[B_2\left(\frac{4}{3}g_0 + \frac{2}{3}g_1\right) + B_3\left(g_0 + \frac{1}{2}g_2\right) + \right. \\ & \left. + B_4\left(\frac{16}{15}g_0 + \frac{2}{15}g_1\right) + B_5 g_0 + \dots\right] + \\ & + \cos 2\vartheta\cdot\left[B_3\left(\frac{3}{2}g_0 + \frac{3}{4}g_1\right) + \frac{6}{5}B_4 g_0 + \frac{7}{6}B_5 g_0 + \dots\right] + \\ & + \cos 3\vartheta\cdot\left[B_4\left(\frac{8}{5}g_0 + \frac{4}{5}g_1\right) + \frac{4}{3}B_5 g_0 + \dots\right] + \cos 4\vartheta\cdot\frac{5}{3}B_5 g_0 + \dots \end{aligned} \tag{2.23}$$

The + sign applies for the top side of the interval $-1 \leq X \leq 1$, and the - sign for the lower side of this interval. In order not to have to write both signs for the velocity, we will consider the angle ϑ in the interval $[-\pi, \pi]$. The positive value of angle ϑ will refer to the top side, and the negative value will refer to the lower side of the interval (Figure 3).

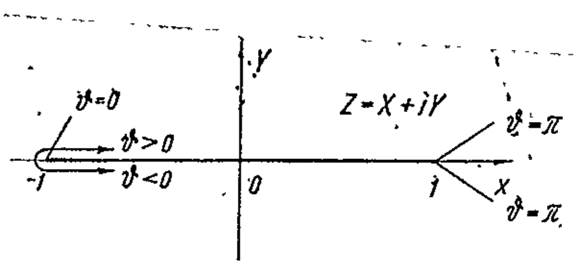

Figure 3

Since the segment $-1 \leq X \leq 1$ lies along the stream line, then the Y-th velocity component must be 0 there, and therefore

$$V_s + V_Y = 0. \tag{2.24}$$

If we substitute expressions (2.17) and (2.23) for V and V in equation (2.24), and if we compare coefficients for equal multiples of the cosine of the angle ϑ then we find a system of equations which gives /625 the relationship between the profile shape and the circulation distribution:

$$\begin{aligned} g_0 &= -B_0 + \nu_0 - \frac{1}{2}\mu_0 B_0 - \frac{1}{2}\sum_{k=0}^{\infty}\mu_k B_k; \\ g_n &= B_n - \nu_n + \frac{1}{2}\sum_{k=0}^{n}\mu_k B_{n-k} + \frac{1}{2}\sum_{k=0}^{\infty}(\mu_k B_{n+k} + \mu_{n+k} B_k);\ n=1,2,\ldots \end{aligned} \tag{2.25}$$

It should be realized that system (2.25) also results from system (2.11) in paper [6], if we set $\omega = 0$ there.

The contour velocity on the profile in the z-plane is obtained from the X-th velocity component along the real axis in the plane Z, when this velocity component is divided by the modulus of the mapping function:

$$w_k = \frac{U_s + U_Y}{[1 + (dy_s/dx)^2]^{1/2}}. \tag{2.26}$$

If we substitute expressions (2.4), (2.17), and (2.13) in equation (2.26), and if we restrict ourselves only to second-order terms, we find:

$$\begin{aligned} w_s = 1 &+ \sum_{n=0}^{\infty}\mu_n \cos n\vartheta + \left[g_0 \operatorname{cotg}\frac{1}{2}\vartheta + \sum_{n=1}^{\infty} g_n \sin n\vartheta\right] + \\ &+ \frac{1}{2}\left(\nu_0 B_0 + \sum_{k=0}^{\infty}\nu_k B_k\right) + \frac{1}{2}\sum_{n=1}^{\infty}\left[\sum_{k=0}^{n}\nu_k B_{n-k} + \sum_{k=0}^{\infty}(\nu_k B_{n+k} + \nu_{n+k} B_k)\right]\cos n\vartheta - \\ &- \frac{1}{4}\left(B_0^2 + \sum_{k=0}^{\infty} B_k^2\right) - \frac{1}{2}\sum_{n=1}^{\infty}\left[\frac{1}{2}\sum_{k=0}^{n} B_k B_{n-k} + \sum_{k=0}^{\infty} B_k B_{n+k}\right]\cos n\vartheta + \\ &+ \left[\frac{1}{2} B_1\left(g_0 + \frac{1}{2} g_1\right) + \frac{1}{3} B_2\left(g_0 + \frac{1}{2} g_2\right) + \frac{1}{2} B_3\left(g_0 + \frac{1}{4} g_1\right) + \right. \\ &\qquad \left. + \frac{7}{15} B_4 g_0 + \frac{1}{2} B_5 g_0 + \ldots\right] + \\ &+ \left[B_2\left(\frac{4}{3} g_0 + \frac{2}{3} g_1\right) + B_3\left(g_0 + \frac{1}{2} g_2\right) + B_4\left(\frac{16}{15} g_0 + \right.\right. \end{aligned} \tag{2.27}$$

(continued)

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$$+\frac{2}{15}g_1\Big)+B_5g_0+\dots\Big]\cos\vartheta+$$
$$+\Big[B_3\Big(\frac{3}{2}g_0+\frac{3}{4}g_1\Big)+\frac{6}{5}B_4g_0+\frac{7}{6}B_5g_0+\dots\Big]\cos 2\vartheta+ \quad (2.27)$$
$$+\Big[B_4\Big(\frac{8}{5}g_0+\frac{4}{5}g_1\Big)+\frac{4}{3}B_5g_0+\dots\Big]\cos 3\vartheta+$$
$$+\Big[\frac{5}{3}B_5g_0+\dots\Big]\cos 4\vartheta+\dots$$

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It should be realized that the expression (2.24) also follows from expression (4.5) in [6], if we set $\omega = 0$.

3. Velocity Field of Sources Distributed on the Skeleton

The finite profile thickness is considered by decomposing the vortices and the sources on the profile skeleton. The density of the continuous distribution of these sources is considered in the form of trigonometric series, just like a circulation distribution:

$$q=[1+(dy_s/dx)^2]^{1/2}q_s=2\Big(q_0\operatorname{cotg}\frac{1}{2}\vartheta+$$
$$+\bar{q}_0\operatorname{tg}\frac{1}{2}\vartheta+\sum_{n=1}^{\infty}q_n\sin n\vartheta\Big),\quad 0\leqq\vartheta\leqq\pi. \quad (3.1)$$

The first term $2q_0\operatorname{cotg}\frac{1}{2}\vartheta$ considers the rounding of the leading edge and the second term $2\bar{q}_0\operatorname{tg}\frac{1}{2}\vartheta$, the rounding of the trailing edge. We have $\bar{q}_0 = 0$ for the profile with a sharp trailing edge. The total yield of all of the sources distributed on the skeleton is given by

$$\int_0^S q_s\,ds=\int_{-1}^{1}q\,dx=2\pi\Big(q_0+\bar{q}_0+\frac{1}{2}q_1\Big). \quad (3.2)$$

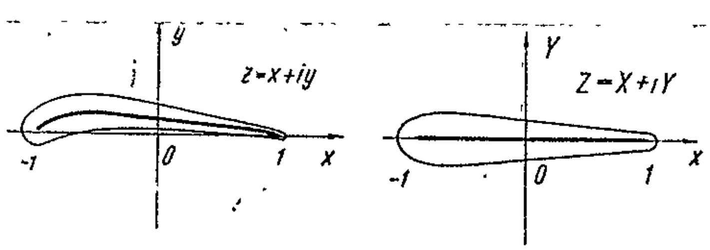

Figure 4

Since the profile is closed, the total yield of the sources must be zero, which results in a condition which the first three coefficients in expansion (3.1) must satisfy:

$$q_0 + \bar{q}_0 + \frac{1}{2} q_1 = 0. \tag{3.3}$$

The complex velocity induced by the vortices on the skeleton is found from the velocity induced by the vortices, and this is done by replacing the coefficients g_n by the coefficients q_n. The entire expression is multiplied by the factor $-i$, (calculation of the term with $\mathrm{tg}\frac{1}{2}\vartheta$, which does not exist in a circulation distribution, is similar to the calculation of the term with $\mathrm{cotg}\frac{1}{2}\vartheta$). After mapping into the Z plane, the velocity on the segment $-1 \le X \le 1$, $Y = 0$ is given by the expression: /627

$$\begin{aligned}
U_q - iV_q = {} & q_0 - \bar{q}_0 - \sum_{n=1}^{\infty} q_n \cos n\vartheta - i\left(q_0 \,\mathrm{cotg}\frac{1}{2}\vartheta + \right. \\
& \left. + \bar{q}_0 \,\mathrm{tg}\frac{1}{2}\vartheta + \sum_{n=1}^{\infty} q_n \sin n\vartheta\right) - \\
& - i\left\{\left[\frac{1}{2}\left(B_1 + B_3 + B_5 + \ldots\right)\left(q_0 + \bar{q}_0 + \frac{1}{2}q_1\right) + \right.\right. \\
& \left. + \frac{1}{3}\left(B_2 + \frac{7}{5}B_4 + \ldots\right)\left(q_0 - \bar{q}_0 + \frac{1}{2}q_2\right) - \frac{1}{8}B_3 q_1 + \ldots\right] + \\
& + \left[\frac{4}{3}\left(B_2 + \frac{4}{5}B_4 + \ldots\right)\left(q_0 + \bar{q}_0 + \frac{1}{2}q_1\right) + \left(B_3 + B_5\right)\left(q_0 - \right.\right. \\
& \left.\left. - \bar{q}_0 + \frac{1}{2}q_2\right) - \frac{2}{5}B_4 q_1 + \ldots\right]\cos\vartheta + \left[\left(\frac{3}{2}B_3 + \right.\right. \\
& \left. + \frac{7}{6}B_5 + \ldots\right)\left(q_0 + \bar{q}_0 + \frac{1}{2}q_1\right) + \\
& \left. + \frac{6}{5}B_4\left(q_0 - \bar{q}_0\right) + \ldots\right]\cos 2\vartheta + \left[\frac{8}{5}B_4\left(q_0 + \bar{q}_0 + \frac{1}{2}q_1\right) + \right. \\
& \left.\left. + \frac{4}{3}B_5\left(q_0 - \bar{q}_0\right) + \ldots\right]\cos 3\vartheta + \left[\frac{5}{3}B_5\left(q_0 + \bar{q}_0\right) + \ldots\right]\cos 4\vartheta + \ldots\right\}.
\end{aligned} \tag{3.4}$$

We would like to add that on the top side of the interval, we have $\vartheta > 0$, and on the bottom side we have $\vartheta < 0$. The skeleton of the profile with a finite thickness becomes the carrier of the singularities (sources and sinks). As a further analysis shows, this is not a pure geometric characteristic of the profile (in contrast to an infinitesimally thin profile, where the skeleton and the profile coincide), but instead it depends on the primary flow field. Instead of equation (2.24) for a profile with a finite thickness, we will consider the expression

$$\boxed{V + V_\gamma + \frac{1}{2}(V_{q+} + V_{q-}) = 0} \qquad (3.5)$$

where V_{q+} and V_{q-} are the values of the velocity V_q on the upper and lower side of the segment $-1 \leq X \leq 1$ of the real axis. Just like equation (2.24), the equation (3.5) is a requirement that the skeleton lies along the stream line. This requirement is only correct for first-order terms. In the case of the second-order terms, deviations have the effect that (3.5) must be replaced by the form (4.25) which is correct for second-order terms. This will be discussed in the following chapter. If we substitute the expansions (3.17), (3.23), and (3.4) in equation (3.5), and if we compare coefficients for the same cosine terms of the angle ϑ, we obtain a system of equations just like in (2.25). This system will be discussed in detail in sections 4 and 7, equations (4.25) and (7.18). The corrections to the second-order terms are also considered there. /628

4. Basic Relationships for Calculating the Flow around Profiles with A Finite Thickness.

Let us consider the profile with a finite thickness. First, we will investigate a profile shape in the image plane Z. Only later on will we transfer to plane z, using equation (2.12).

From expressions (2.23) and (3.4), we can see that the velocities induced by the vortices and the sources are composed of first and second order terms. The first order terms can be looked upon as velocities which are induced in the plane Z by vortices and sources. They are distributed along the real axis along the segment $-1 \leq X \leq 1$, with the following densities

$$\gamma = 2\left(g_0 \operatorname{cotg} \frac{1}{2}\vartheta + \sum_{n=1}^{\infty} g_n \sin n\vartheta\right), \qquad (4.1)$$

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$$q = 2\left(q_0 \operatorname{cotg}\frac{1}{2}\vartheta + \bar{q}_0 \operatorname{tg}\frac{1}{2}\vartheta + \sum_{n=1}^{\infty} q_n \sin n\vartheta\right) \tag{4.2}$$

In the following, the second-order terms are added to the primary velocity field (2.17) in order to simplify the notation. We therefore have the task of estbalishing the relationships between the profile shape and the circulation and source distribution along the profile skeleton. The profile skeleton in the plane Z is identical with the segment $-1 \leq X \leq 1$, $Y = 0$. The velocity of the inhomogeneous flow field into which the profile is placed is given by the expression

$$\tilde{U} - i\tilde{V} = 1 + \sum_{n=0}^{\infty} (\tilde{\mu}_n - i\tilde{\nu}_n) \cos n\vartheta \tag{4.3}$$

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where

$$\begin{aligned}
\tilde{\mu}_0 &= \mu_0 + \frac{1}{2}\left(\nu_0 B_0 + \sum_{k=0}^{\infty} \nu_k B_k\right) + \frac{1}{2}\left(B_1 + B_3 + B_5 + \dots\right)\left(g_0 + \frac{1}{2} g_1\right) + \\
&\quad + \frac{1}{3}\left(B_2 + \frac{7}{5} B_4 + \dots\right)\left(g_0 + \frac{1}{2} g_2\right) - \frac{1}{8} B_3 g_1 + \dots; \\
\tilde{\mu}_1 &= \mu_1 + \frac{1}{2}\left[\sum_{k=0}^{1} \nu_k B_{1-k} + \sum_{k=0}^{\infty}\left(\nu_k B_{k+1} + \nu_{k+1} B_k\right)\right] + \frac{4}{3}\left(B_2 + \frac{4}{5} B_4 + \dots\right)\left(g_0 + \frac{1}{2} g_1\right) + \\
&\quad + \left(B_3 + B_5 + \dots\right)\left(g_0 + \frac{1}{2} g_2\right) - \frac{2}{5} B_4 g_1 + \dots, \\
\tilde{\mu}_2 &= \mu_2 + \frac{1}{2}\left[\sum_{k=0}^{2} \nu_k B_{2-k} + \sum_{k=0}^{\infty}\left(\nu_k B_{k+2} + \nu_{k+2} B_k\right)\right] + \\
&\quad + \left(\frac{3}{2} B_3 + \frac{7}{6} B_5 + \dots\right)\left(g_0 + \frac{1}{2} g_1\right) + \frac{6}{5} B_4 g_0 + \dots \\
&\quad \dots\dots\dots\dots\dots\dots\dots\dots
\end{aligned} \tag{4.4}$$

$$\begin{aligned}
\tilde{\nu}_0 &= \nu_0 - B_0 - \frac{1}{2}\left(\mu_0 B_0 + \sum_{k=0}^{\infty} \mu_k B_k\right) + \frac{1}{2}\left(B_1 + B_3 + B_5 + \dots\right)\left(q_0 + \bar{q}_0 + \frac{1}{2} q_1\right) + \\
&\quad + \frac{1}{3}\left(B_2 + \frac{7}{5} B_4 + \dots\right)\left(q_0 - \bar{q}_0 + \frac{1}{2} q_2\right) - \frac{1}{8} B_3 q_1 + \dots; \\
\tilde{\nu}_1 &= \nu_1 - B_1 - \frac{1}{2}\left[\sum_{k=0}^{1} \mu_k B_{1-k} + \sum_{k=0}^{\infty}\left(\mu_k B_{k+1} + \mu_{k+1} B_k\right)\right] + \frac{4}{3}\left(B_2 + \frac{4}{5} B_4 + \dots\right) \cdot \\
&\quad \cdot \left(q_0 + \bar{q}_0 + \frac{1}{2} q_1\right) + \left(B_3 + B_5 + \dots\right)\left(q_0 - \bar{q}_0 + \frac{1}{2} q_2\right) - \frac{2}{5} B_4 q_1 + \dots; \\
\tilde{\nu}_2 &= \nu_2 - B_2 - \frac{1}{2}\left[\sum_{k=0}^{2} \mu_k B_{2-k} + \sum_{k=0}^{\infty}\left(\mu_k B_{k+2} + \mu_{k+2} B_k\right)\right] + \left(\frac{3}{2} B_3 + \right. \\
&\quad \left. + \frac{7}{6} B_5 + \dots\right)\left(q_0 + \bar{q}_0 + \frac{1}{2} q_1\right) + \frac{6}{5} B_4 (q_0 - \bar{q}_0) + \dots; \\
&\quad \dots\dots\dots\dots\dots\dots\dots\dots
\end{aligned} \tag{4.5}$$

In the following, it is advantageous to introduce the complex plane ζ and the complex function

$$Z = \frac{1}{2}\left(\zeta + \frac{1}{\zeta}\right) \tag{4.6}$$

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This function maps the unit circle in the plane ζ into the interval $-1 \leq X \leq 1$, $Y = 0$ in the plane Z, which is passed through twice.

In the plane ζ, we will introduce polar coordinates ρ and ϑ (Figure 5):

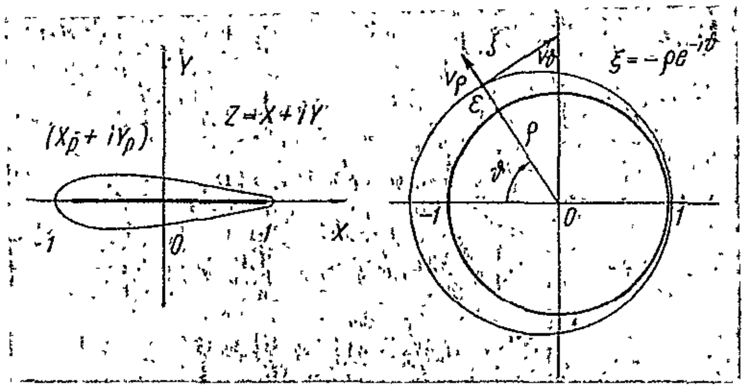

Figure 5

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$$\zeta = -\rho e^{-i\vartheta}, \tag{4.7}$$

if we also set

$$\rho = 1 + \varepsilon, \tag{4.8}$$

where ε is the distance of the point ζ from the unit circle. Then from equation (4.6) we obtain

$$Z = -\frac{1}{2}\left[(1 + \varepsilon) e^{-i\vartheta} + \frac{1}{1 + \varepsilon} e^{i\vartheta}\right] \tag{4.9}$$

or

$$X = -\left(1 + \frac{1}{2}\frac{\varepsilon^2}{1 + \varepsilon}\right)\cos\vartheta, \tag{4.10}$$

$$Y = \varepsilon\left(1 - \frac{1}{2}\frac{\varepsilon}{1 + \varepsilon}\right)\sin\vartheta. \tag{4.11}$$

The velocity field in the plane Z, which is composed of the inhomogeneous primary field (4.3) and the velocity field of the vortices and sources, can be written in a relatively simple form, if we map into the complex plane ζ using the function (4.6):

$$w_\zeta = \frac{1}{2}\left(1 - \frac{1}{\zeta^2}\right)\left\{1 + \sum_{n=0}^{\infty}(-1)^n(\tilde{\mu}_n - i\tilde{\nu}_n)\frac{1}{2}\left(\zeta^n + \frac{1}{\zeta^n}\right) + (q_0 + ig_0)\frac{2}{\zeta+1} + \tilde{q}_0\frac{2}{\zeta-1} - \sum_{n=1}^{\infty}(q_n + ig_n)\frac{(-1)^n}{\zeta^n}\right\} \quad (4.12)$$

The velocity w_ζ is decomposed into a radial part and an azimuth component: /631

$$v_\rho + iv_\vartheta = -w_\zeta e^{-i\vartheta} = -\frac{1}{2}\left(e^{-i\vartheta} - \frac{e^{i\vartheta}}{\rho^2}\right)\left\{1 + \sum_{n=0}^{\infty}(\tilde{\mu}_n - i\tilde{\nu}_n)\frac{1}{2}\left(\rho^n e^{-in\vartheta} + \frac{e^{in\vartheta}}{\rho^n}\right) - (q_0 + ig_0)\frac{2}{\rho e^{-i\vartheta} - 1} - \tilde{q}_0\frac{2}{\rho e^{-i\vartheta} + 1} - \sum_{n=1}^{\infty}(q_n + ig_n)\frac{e^{in\vartheta}}{\rho^n}\right\} \quad (4.13)$$

On the unit circle (that is, for $\rho = 1$), the velocity components are given by the following relationships:

$$v_\rho = \sin\vartheta\left(\sum_{n=0}^{\infty}\tilde{\nu}_n\cos n\vartheta - g_0 + \sum_{n=1}^{\infty} g_n\cos n\vartheta + q_0\operatorname{cotg}\frac{1}{2}\vartheta + \tilde{q}_0\operatorname{tg}\frac{1}{2}\vartheta + \sum_{n=1}^{\infty} q_n\sin n\vartheta\right), \quad (4.14)$$

$$v_\vartheta = \sin\vartheta\left(1 + \sum_{n=0}^{\infty}\tilde{\mu}_n\cos n\vartheta + g_0\operatorname{cotg}\frac{1}{2}\vartheta + \sum_{n=1}^{\infty} g_n\sin n\vartheta + q_0 - \tilde{q}_0 - \sum_{n=1}^{\infty} q_n\cos n\vartheta\right) \quad (4.15)$$

Using this notation, equation (3.5) is equivalent to the system of equations

$$\tilde{\nu}_0 - g_0 = 0, \qquad \tilde{\nu}_n + g_n = 0, \qquad n = 1, 2, \ldots \quad (4.16)$$

so that the radial velocity component on the unit circle is equal to the expression

$$v_\rho = \sin\vartheta\left(q_0\operatorname{cotg}\frac{1}{2}\vartheta + \tilde{q}_0\operatorname{tg}\frac{1}{2}\vartheta + \sum_{n=1}^{\infty} q_n\sin n\vartheta\right) \quad (4.17)$$

which corresponds to sources in the ζ plane, which are distributed along the unit circle with the density

$$\tilde{q} = \frac{1}{2} q\sin\vartheta \quad (4.18)$$

The profile in the plane ζ is given by the closed stream line /632
ABMNA (Figure 6). Its shape is derived from the condition that the total flow passing through the unit circular arc A_1B_1 and the amount of incoming flow through the section A_1A flows through the section BB_1, because the arc AB lies along the stream line and the normal velocity component is 0 on it. Therefore, we have:

$$\int_0^{\vartheta} v_\rho \, d\vartheta + \int_1^{1+\varepsilon_0} v_\vartheta \, d\rho = \int_1^{1+\varepsilon} v_\vartheta \, d\rho \tag{4.19}$$

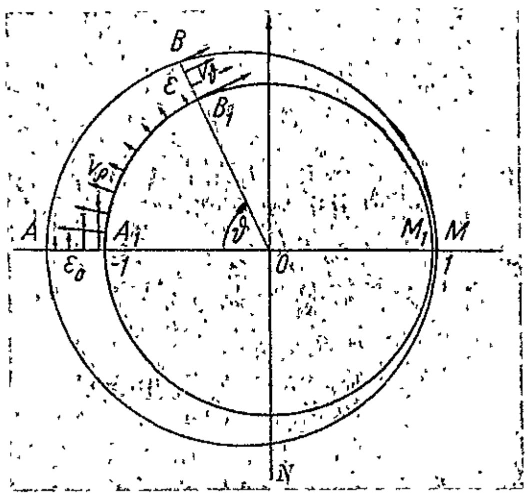

Figure 6

The value of the first integral according to (4.17) is given by:

$$\int_0^{\vartheta} v_\rho \, d\vartheta = \int_0^{\vartheta}\left(q_0 \operatorname{cotg}\frac{1}{2}\vartheta + \bar{q}_0 \operatorname{tg}\frac{1}{2}\vartheta + \sum_{n=1}^{\infty} q_n \sin n\vartheta\right)\sin\vartheta \, d\vartheta =$$
$$= \left(q_0 + \bar{q}_0 + \frac{1}{2}q_1\right)\vartheta + \left(q_0 - \bar{q}_0 + \frac{1}{2}q_2\right)\sin\vartheta - \frac{1}{2}\sum_{n=2}^{\infty}\frac{q_{n-1} - q_{n+1}}{n}\sin n\vartheta \tag{4.20}$$

Because of condition (3.3), the term $\left(q_0 + \bar{q}_0 + \frac{1}{2}q_1\right)\vartheta$ drops out.

In the calculation of the other integrals, in equation (4.19), we will use the expression (4.13). Accordingly, the values of these /633
integrals are given by the imaginary part of the following expression:

$$\int_1^{1+\varepsilon}(v_\rho + i v_\vartheta)\, d\rho = -\frac{1}{2}\int_1^{1+\varepsilon}\Big\{(1 + \tilde{\mu}_0 - i\tilde{\nu}_0)(e^{-i\vartheta} - \rho^{-2}e^{i\vartheta}) + \frac{1}{2}\sum_{n=1}^{\infty}(\tilde{\mu}_n -$$
$$- i\tilde{\nu}_n)[\rho^n e^{-i(n+1)\vartheta} - \rho^{-(n+2)}e^{i(n+1)\vartheta} - \rho^{(n-2)}e^{-i(n-1)\vartheta} + \rho^{-n}e^{i(n-1)\vartheta}] - \tag{4.21}$$

(continued)

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$$
\begin{aligned}
&-2(q_0' + i g_0')(\rho^{-1} + \rho^{-2} e^{i\vartheta}) - 2\bar{q}_0(\rho^{-1} - \rho^{-2} e^{i\vartheta}) - \sum_{n=1}^{\infty}(q_n + i g_n)\cdot \\
&\qquad \cdot(\rho^{-n} e^{i(n-1)\vartheta} - \rho^{-(n+2)} e^{i(n+1)\vartheta})\Big\}\, d\rho = \\
&= -\frac{1}{2}\Big\{(1 + \tilde{\mu}_0 + i\tilde{\nu}_0)[\varepsilon e^{-i\vartheta} + ((1+\varepsilon)^{-1} - 1) e^{i\vartheta}] + \frac{1}{2}(\tilde{\mu}_1 - i\tilde{\nu}_1)\Big[\Big(\varepsilon + \\
&\quad + \frac{1}{2}\varepsilon^2\Big) e^{-i2\vartheta} + \frac{1}{2}((1+\varepsilon)^{-2} - 1) e^{i2\vartheta}\Big] + \frac{1}{2}\sum_{n=2}^{\infty}(\tilde{\mu}_n - i\tilde{\nu}_n)\Big[\frac{1}{n+1}((1 + \\
&\quad + \varepsilon)^{n+1} - 1) e^{-i(n+1)\vartheta} + \frac{1}{n+1}((1+\varepsilon)^{-n-1} - 1) e^{i(n+1)\vartheta} - \frac{1}{n-1}((1+\varepsilon)^{n-1} - \\
&\quad - 1) e^{-i(n-1)\vartheta} - \frac{1}{n-1}((1+\varepsilon)^{-n+1} - 1) e^{i(n-1)\vartheta}\Big] - 2(q_0 + i g_0)[\ln(1+\varepsilon) - \\
&\quad - ((1+\varepsilon)^{-1} - 1) e^{i\vartheta}] - 2\bar{q}_0[\ln(1+\varepsilon) + ((1+\varepsilon)^{-1} - 1) e^{i\vartheta}] - \\
&\quad - (q_1 + i g_1)\Big[\ln(1+\varepsilon) + \frac{1}{2}((1+\varepsilon)^{-2} - 1) e^{i2\vartheta}\Big] - \sum_{n=2}^{\infty}(q_n + i g_n)\cdot \\
&\quad \cdot\Big[-\frac{1}{n-1}((1+\varepsilon)^{-n+1} - 1) e^{i(n-1)\vartheta} + \frac{1}{n+1}((1+\varepsilon)^{-n-1} - 1) e^{i(n+1)\vartheta}\Big]\Big\}
\end{aligned}
\tag{4.21}
$$

Since ε is a small quantity of first order, if we ignore higher than second-order terms, and if we consider equations (4.16), we find:

$$
\begin{aligned}
&\int_1^{1+\varepsilon} v_\vartheta\, d\rho = \varepsilon\Big(1 - \frac{1}{2}\,\frac{\varepsilon}{1+\varepsilon}\Big)\sin\vartheta + \varepsilon\sin\vartheta\Big\{\sum_{n=0}^{\infty}\tilde{\mu}_n \cos n\vartheta + \\
&+ q_0' - \bar{q}_0 + 2(q_0 + \bar{q}_0)\cos\vartheta - \sum_{n=2}^{\infty} q_n \cos n\vartheta + g_0\frac{1+\cos\vartheta}{\sin\vartheta} + \\
&+ \sum_{n=1}^{\infty} g_n \sin n\vartheta.
\end{aligned}
\tag{4.22}
$$

For $\vartheta = 0$, we have:

$$
\int_1^{1+\varepsilon_0} v_\vartheta\, d\rho = 2 g_0 \varepsilon_0 \tag{4.23}
$$

and for $\vartheta = \pi$ we have: /634

$$
\int_1^{1+\varepsilon_\pi} v_\vartheta\, d\rho = 0 \tag{4.24}
$$

For $\vartheta = \pi$, the left side of equation (4.19) is equal to $2g_0\varepsilon_0$ as far as (4.20) and (4.13) are concerned, because according to (4.24) the right side is zero. Since the term $2g_0\varepsilon_0$ is of second order, in our theory we must consider it. Therefore, the equations (4.16) have been given corrections of the following type:

$$\begin{aligned}&\tilde{v}_0 - g_0' + g_0 \varepsilon_0 = 0,\\ &\tilde{v}_1 + g_1 + g_0 \varepsilon_0 = 0,\\ &\tilde{v}_n + g_n = 0, \quad n = 2, 3, \ldots\end{aligned} \tag{4.25}$$

To the integral (4.20), we will add the correction

$$\int_0^{\vartheta} \Delta v_\rho \, d\vartheta = -g_0 \varepsilon_0 \int_0^{\vartheta} (1 + \cos\vartheta) \sin\vartheta \, d\vartheta = -g_0 \varepsilon_0 (1 - \cos\vartheta) - \\ -\frac{1}{4} g_0 \varepsilon_0 (1 - \cos 2\vartheta) \tag{4.26}$$

so that on the left side of equation (4.19) we obtain the following expression for the total current through the arc A_1B_1 and through the section AA_1:

$$\int_1^{\vartheta} v_\rho \, d\vartheta + \int_1^{1+\varepsilon_0} v_\vartheta \, d\rho = \left(q_0 - \bar{q}_0 + \frac{1}{2} g_2\right) \sin\vartheta + \frac{1}{2}\left(q_0 + \bar{q}_0 + \frac{1}{2} q_3\right) \sin 2\vartheta - \\ -\frac{1}{2} \sum_{n=3}^{\infty} \frac{q_{n-1} - q_{n+1}}{n} \sin n\vartheta + g_0 \varepsilon_0 (1 + \cos\vartheta) - \frac{1}{4} g_0 \varepsilon_0 (1 - \cos 2\vartheta). \tag{4.27}$$

By comparing (4.22) and (4.27), we obtain a relationship from which we can determine the profile shape; that is, ε as a function of the angle ϑ. Since we are interested in the profile shape in the Z plane, this relationship is converted from the plane ζ to the plane Z using equations (4.10) and (4.11). Let us call the coordinates of the points on the profile (X_p, Y_p). Then according to (4.10) and (4.11), we have:

$$X_p = -\left(1 + \frac{1}{2} \frac{\varepsilon^2}{1 + \varepsilon}\right) \cos\vartheta, \quad Y_p = \varepsilon \left(1 - \frac{1}{2} \frac{\varepsilon}{1 + \varepsilon}\right) \sin\vartheta. \tag{4.28}$$

After substituting in (4.22) and (4.27), and if we ignore higher order terms above second order, we find /635

$$Y_p \left\{1 + \sum_{n=0}^{\infty} \tilde{\mu}_n \cos n\vartheta + q_0 - \bar{q}_0 + 2(q_0 + \bar{q}_0) \cos\vartheta - \sum_{n=2}^{\infty} q_n \cos n\vartheta + \right. \\ \left. + g_0 \frac{1 + \cos\vartheta}{\sin\vartheta} + \sum_{n=1}^{\infty} g_n \sin n\vartheta \right\} = \left(q_0 - \bar{q}_0 + \frac{1}{2} q_2\right) \sin\vartheta + \frac{1}{2}\left(q_0 + \bar{q}_0 + \right. \\ \left. + \frac{1}{2} q_3\right) \sin 2\vartheta - \frac{1}{2} \sum_{n=3}^{\infty} \frac{q_{n-1} - q_{n+1}}{n} \sin n\vartheta + g_0 \varepsilon_0 (1 + \cos\vartheta) - \\ -\frac{1}{4} g_0 \varepsilon_0 (1 - \cos 2\vartheta), \tag{4.29}$$

$$X_p = -\left[1 + \frac{1}{2} (Y_p / \sin\vartheta)^2\right] \cos\vartheta. \tag{4.30}$$

In order to solve equation (4.29), we will write the function Y_p in the form of a trigonometric series:

$$Y_p = \sum_{n=1}^{\infty} A_n \sin n\vartheta + \sum_{m=1}^{\infty} \left\{ C_{2m} (1 - \cos 2m\vartheta) + C_{2m+1} [\cos \vartheta - - \cos (2m + 1)\vartheta] \right\} \tag{4.31}$$

From this equation, we obtain ε_0 according to (4.11):

$$\varepsilon_0 \left(1 - \frac{1}{2} \frac{\varepsilon_0^2}{1 + \varepsilon_0}\right) = \lim_{\vartheta \to 0} \frac{Y_p}{\sin \vartheta} = \sum_{n=1}^{\infty} n A_n \tag{4.32}$$

and therefore,

$$\varepsilon_0 = \sum_{n=1}^{\infty} n A_n + \frac{1}{2} \left[\sum_{n=1}^{\infty} n A_n\right]^2. \tag{4.33}$$

After substituting expressions (4.31) and (4.33) in (4.29), and after comparison of coefficients of the same sine and cosine terms of the angle ϑ, we obtain two systems of equations:

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$$\begin{aligned}
& q_0 - \bar{q}_0 + \frac{1}{2} q_2 = A_1 + A_1 \left(q_0 - \bar{q}_0 + \tilde{\mu}_0 + \frac{1}{2} q_2 - \frac{1}{2} \tilde{\mu}_2\right) + \\
& + \frac{1}{2} A_2 (2 q_0 + 2\bar{q}_0 + \tilde{\mu}_1 + q_3 - \tilde{\mu}_3) + \\
& + \frac{1}{2} \sum_{n=3}^{\infty} A_n (- q_{n-1} + \tilde{\mu}_{n-1} + q_{n+1} - \tilde{\mu}_{n+1}) + g_0 \left[2 \sum_{m=1}^{\infty} C_{2m} + \sum_{m=1}^{\infty} C_{2m+1}\right] + \\
& + g_1 \left[\sum_{m=1}^{\infty} C_{2m} + \frac{1}{2} C_2\right] + \frac{1}{2} g_2 \left[\sum_{m=1}^{\infty} C_{2m+1} + C_3\right] - \frac{1}{2} \sum_{n=3}^{\infty} g_n [C_{n-1} - C_{n+1}]; \\
& \frac{1}{2} q_0 + \frac{1}{2} \bar{q}_0 + \frac{1}{4} q_3 = A_2 + \frac{1}{2} A_1 (2 q_0 + 2\bar{q}_0 + \tilde{\mu}_1 + q_3 - \tilde{\mu}_3) + \\
& + A_2 \left(q_0 - \bar{q}_0 + \tilde{\mu}_0 + \frac{1}{2} q_4 - \frac{1}{2} \tilde{\mu}_4\right) + \frac{1}{2} A_3 (2 q_0 + 2\bar{q}_0 + \tilde{\mu}_1 + q_5 - \tilde{\mu}_5) + \\
& + \frac{1}{2} \sum_{n=1}^{\infty} A_n (- q_{n-2} + \tilde{\mu}_{n-2} + q_{n+2} - \tilde{\mu}_{n+2}) + \\
& + g_0 \left[C_2 + 2 \sum_{n=3}^{\infty} C_n\right] + g_2 \sum_{m=1}^{\infty} C_{2m} + \frac{1}{2} (g_1 + g_3) \sum_{m=1}^{\infty} C_{2m+1} - \frac{1}{2} g_4 C_2 + \\
& + \frac{1}{2} \sum_{n=3}^{\infty} (g_{n+2} - g_{n-2}) C_n, \\
& - \frac{1}{6} (q_2 - q_4) = A_3 + \frac{1}{2} A_1 (- q_2 + q_4 + \tilde{\mu}_2 - \tilde{\mu}_4) + \\
& + \frac{1}{2} A_2 (2 q_0 + 2\bar{q}_0 + q_5 + \tilde{\mu}_1 - \tilde{\mu}_5) + A_3 \left(q_0 - \bar{q}_0 + \frac{1}{2} q_6 + \tilde{\mu}_0 - \tilde{\mu}_6\right) +
\end{aligned} \tag{4.34}$$

(continued)

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$$
\begin{aligned}
&+\frac{1}{2} A_4 (2q_6 + 2\tilde{q}_0 + q_7 + \tilde{\mu}_1 - \tilde{\mu}_7) + \\
&+\frac{1}{2}\sum_{n=5}^{\infty} A_n (-q_{n-3} + q_{n+3} + \tilde{\mu}_{n-3} - \tilde{\mu}_{n+3}) + g_0\left[C_3 + 2\sum_{n=4}^{\infty} C_n\right] + g_3 \sum_{m=1}^{\infty} C_{2m} + \\
&+\frac{1}{2}(g_2 + g_4)\sum_{m=1}^{\infty} C_{2m+1} - \frac{1}{2}(g_1 + g_5) C_2 - \\
&-\frac{1}{2} g_6 C_3 + \frac{1}{2}\sum_{n=4}^{\infty}(g_{n-3} - g_{n+3}) C_n
\end{aligned}
\qquad (4.34)
$$

$$
\begin{aligned}
&(1 + q_0 - \tilde{q}_0 + \tilde{\mu}_0)\sum_{m=1}^{\infty} C_{2m} + \left(q_0 + \tilde{q}_0 + \frac{1}{2}\tilde{\mu}_1\right)\sum_{m=1}^{\infty} C_{2m+1} + \frac{1}{2}\sum_{n=2}^{\infty}(q_n - \tilde{\mu}_n) C_n + \\
&+ g_0 \sum_{n=1}^{\infty} A_n + \frac{1}{2}\sum_{n=1}^{\infty} A_n g_n = \frac{3}{4} g_0 \sum_{n=1}^{\infty} n A_n ;
\end{aligned}
$$

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$$
\begin{aligned}
&(1 + q_0 - \tilde{q}_0 + \tilde{\mu}_0)\sum_{m=1}^{\infty} C_{2m+1} + (2q_0 + 2\tilde{q}_0 + \tilde{\mu}_1)\left(\sum_{m=1}^{\infty} C_{2m} - \frac{1}{2} C_2\right) - \\
&-\frac{1}{2}(q_2 - \tilde{\mu}_2)\left(\sum_{m=1}^{\infty} C_{2m+1} - C_3\right) + \\
&+\frac{1}{2}\sum_{n=3}^{\infty}(q_n - \tilde{\mu}_n)(C_{n-1} + C_{n+1}) + g_0\left(A_1 + 2\sum_{n=2}^{\infty} A_n\right) + \\
&+\frac{1}{2}\sum_{n=1}^{\infty}(A_n g_{n+1} + A_{n+1} g_n) = g_0 \sum_{n=1}^{\infty} n A_n ,
\end{aligned}
$$

$$
\begin{aligned}
&-(1 + q_0 - \tilde{q}_0 + \tilde{\mu}_0) C_2 + \left(q_0 + \tilde{q}_0 + \frac{1}{2}\tilde{\mu}_1\right)\left(\sum_{m=1}^{\infty} C_{2m+1} - C_3\right) - \\
&-(q_2 - \tilde{\mu}_2)\left(\sum_{m=1}^{\infty} C_{2m} - \frac{1}{2} C_4\right) - \frac{1}{2}(q_3 - \tilde{\mu}_3)\left(\sum_{m=1}^{\infty} C_{2m+1} - C_5\right) + \\
&+\frac{1}{2}\sum_{n=4}^{\infty}(q_n - \tilde{\mu}_n)(C_{n-2} + C_{n+2}) + g_0\left(A_2 + 2\sum_{n=3}^{\infty} A_n\right) - \frac{1}{2} A_1 g_1 + \\
&+\frac{1}{2}\sum_{n=1}^{\infty}(A_n g_{n+2} + A_{n+2} g_n) = \frac{1}{4} g_0 \sum_{n=1}^{\infty} n A_n ,
\end{aligned}
\qquad (4.35)
$$

$$
\begin{aligned}
&-(1 + q_0 - \tilde{q}_0 + \tilde{\mu}_0) C_3 - \left(q_0 + \tilde{q}_0 + \frac{1}{2}\tilde{\mu}_1\right)(C_2 + C_4) - \\
&-\frac{1}{2}(q_2 - \tilde{\mu}_2)\left(\sum_{m=1}^{\infty} C_{2m+1} - C_5\right) - (q_3 - \tilde{\mu}_3)\left(\sum_{m=1}^{\infty} C_{2m} - \frac{1}{2} C_6\right) - \\
&-\frac{1}{2}(q_4 - \tilde{\mu}_4)\left(\sum_{m=1}^{\infty} C_{2m+1} - C_7\right) + \frac{1}{2}\sum_{n=5}^{\infty}(q_n - \tilde{\mu}_n)(C_{n-3} + C_{n+3}) + \\
&+ g_0\left(A_3 + 2\sum_{n=4}^{\infty} A_n\right) - \frac{1}{2}\sum_{n=1}^{2} A_n g_{3-n} + \frac{1}{2}\sum_{n=1}^{\infty}(A_n g_{n+3} + A_{n+3} g_n) = 0 ,
\end{aligned}
$$

. . . .

It is often advantageous to replace the first equations in the system (4.34) by the following equations:

$$2q_0 = \sum_{n=1}^{\infty} nA_n \left\{ 1 + 3q_0 + \bar{q}_0 - \sum_{n=2}^{\infty} q_n + \sum_{n=0}^{\infty} \tilde{\mu}_n \right\}, \tag{4.36}$$

$$2\bar{q}_0 = \sum_{n=1}^{\infty} (-1)^n nA_n \left\{ 1 - q_0 - 3\bar{q}_0 - \sum_{n=2}^{\infty} (-1)^n q_n + \sum_{n=0}^{\infty} (-1)^n \tilde{\mu}_n \right\}. \tag{4.37}$$

These equations are obtained by multiplying each equation of the system (4.34) with its own order number. Then all equations are added /638 once with the same sign and the second time they are added with the alternating symbol.

The first two equations of system (4.35) are linear combinations of remainder equations and therefore will not be considered in the following. It also follows from (4.35) that the coefficients C_n are second-order, so that in systems (4.34) and (4.35), all terms in which coefficients C_n are multiplied by any first-order term can be considered of third order, and can be disregarded. This means that the C_n terms drop out of the system (4.34), and the system (4.35) has a purely diagonal character.

5. Profile Shape in the Vicinity of the Leading Edge and Trailing Edge

As already mentioned in the previous chapter, the Y coordinates of the profile are given by the following expression:

$$Y_p = \sum_{n=1}^{\infty} A_n \sin n\vartheta + \sum_{m=1}^{\infty} C_{2m} [1 - \cos 2m\vartheta] + \\ + \sum_{m=1}^{\infty} C_{2m+1} [\cos \vartheta - \cos (2m+1)\vartheta], \quad -\pi \leqq \vartheta \leqq \pi. \tag{5.1}$$

According to (4.10), we have the following for the X-coordinates of the profile

$$X_p = -\left[1 + \frac{1}{2}(Y_p / \sin \vartheta)^2\right] \cos \vartheta, \tag{5.2}$$

or after substituting equation (5.1) and ignoring terms higher than second order:

$$X_p = -\cos\vartheta\Big\{1 + \frac{1}{2}A_1^2 + A_2^2 + \frac{3}{2}A_3^2 + A_1A_3 + A_1A_5 + 2A_2A_4 + \ldots + 2[A_1A_2 + A_1A_4 + 2A_2A_3 + \ldots]\cos\vartheta + [A_2^2 + 2A_3^2 + 2A_1A_3 + 4A_2A_4 + 2A_1A_5 + \ldots]\cos 2\vartheta + 2[A_1A_4 + A_2A_3 + \ldots]\cos 3\vartheta + [A_3^2 + 2A_2A_4 + 2A_1A_5 + \ldots]\cos 4\vartheta + \ldots\Big\}. \quad (5.3)$$

From expressions (5.1) and (5.2), we then obtain some important information regarding the profile shape in the vicinity of the leading edge and the trailing edge.

For the leading edge, that is, for $\vartheta = 0$, we obtain:

$$Y_p = 0; \; X_p = -\left[1 + \frac{1}{2}\left(\sum_{n=1}^{\infty} nA_n\right)^2\right] \quad (5.4)$$

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or according to (4.36),

$$Y_p = 0; \; X_p \doteq -[1 + 2q_0^2]. \quad (5.5)$$

Without any difficulty, we also obtain the following:

$$\frac{dY_p}{dX_p} = \pm\infty, \quad (5.6)$$

and the curvature radius of the leading edge

$$R_1 = \frac{\left[\sum_{n=1}^{\infty} nA_n\right]^2}{1 + \frac{1}{2}\left[\sum_{n=1}^{\infty} nA_n\right]^2} \doteq 4q_0^2. \quad (5.7)$$

Expression (5.7) can be written in the following way:

$$2q_0 = R_1^{1/2}\left(1 + 3q_0 + q_0 - \sum_{n=2}^{\infty} q_n + \sum_{n=0}^{\infty} p_n\right). \quad (5.8)$$

Finally, we can prove that for

$$X_p = -1, \quad \text{we have} \quad Y_p = \pm 4q_0^2 = \pm R_1. \quad (5.9)$$

From equations (5.5), (5.7), and (5.9), we find that the profile shape can be approximated by a parabola in the vicinity of the leading

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edge:

$$Y_p = \pm 2q_0 2^{1/2} (X_p + 1 + 2q_0^2)^{1/2} = \pm R_1^{1/2} (2X_p + 2 + R_1)^{1/2}. \tag{5.10}$$

Similar conditions also hold for the rounded-off trailing edge, that is, for $\vartheta = \pi$, where $\overline{q}_0 \neq 0$:

$$Y_p = 0; \quad X_p = 1 + \frac{1}{2}\left[\sum_{n=1}^{\infty} (-1)^n n A_n\right]^2 \tag{5.11}$$

or, according to (4.37),

$$Y_p = 0; \quad X_p = 1 + 2\bar{q}_0^2. \tag{5.12}$$

Also, we have

$$\frac{dY_p}{dX_p} = \pm \infty. \tag{5.13}$$

The radius of curvature of the trailing edge is:

$$R_2 = \frac{\left[\sum_{n=1}^{\infty} (-1)^n n A_n\right]^2}{1 + \frac{1}{2}\left[\sum_{n=1}^{\infty} (-1)^n n A_n\right]^2} \doteq 4\bar{q}_0^2 \tag{5.14}$$

or

$$-2\bar{q}_0 = R_2^{1/2}\left[1 - q_0 - 3q_0^3 - \sum_{n=2}^{\infty} (-1)^n q_n + \sum_{n=0}^{\infty} (-1)^n \bar{\mu}_n\right]. \tag{5.15}$$

Also we have: /640

$$X_p = 1, \quad Y_p = \pm 4\bar{q}_0^2 = \pm R_2, \tag{5.16}$$

so that the profile shape in the vicinity of the trailing edge is approximated by a parabola:

$$Y_p = \pm 2\bar{q}_0 (2 + 4\bar{q}_0^2 - 2X_p)^{1/2} = \pm R_2^{1/2} (2 + R_2 - X_p)^{1/2}. \tag{5.17}$$

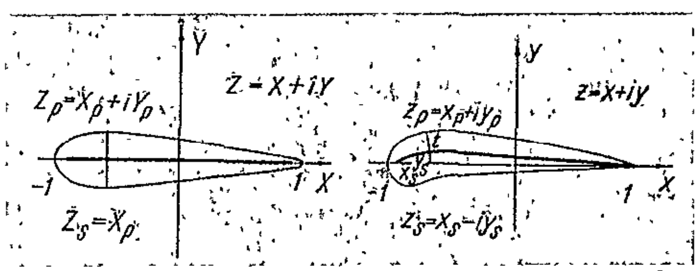

Figure 7

In the case of a sharp trailing edge, that is, for $\vartheta = \pi$ and $\overline{q}_o = 0$, we have:

$$X_p = 1, \; Y_p = 0 \tag{5.18}$$

and

$$\frac{dY_p}{dX_p} = 0. \tag{5.19}$$

Finally, we will make the transition from the image plane Z to the physical plane z (Figure 7):

$$z_p - z_s = Z_p - Z_s + i\,[f(Z_p) - f(Z_s)]. \tag{5.20}$$

The square bracket is replaced by a Taylor series:

$$f(X_p + i\,Y_p) - f(X_p) = i\,Y_p \frac{df}{dZ} + \ldots = i\,Y_p \frac{dy_s}{dx} + \ldots \tag{5.21}$$

and since Y_p as well as dy_s/dx are small, of first order, we can restrict ourselves to the first term of expression (5.21). Equation (5.20) therefore has the form:

$$x_p - x_s + i\,(y_p - y_s) = i\,Y_p \left(1 + i\,\frac{dy_s}{dx}\right). \tag{5.22}$$

In addition to (5.22), according to (2.12), we have:

$$x_s = X_p = -\left[1 + \frac{1}{2}\,(Y_p/\sin\vartheta)^2\right]\cos\vartheta. \tag{5.23}$$

If we decompose (5.22) into a real and an imaginary part, we find:

$$x_p - x_s = -\frac{dy_s}{dx}\,Y_p, \quad y_p - y_s = Y_p, \tag{5.24}$$

which means that the thickness t in the plane z is plotted along the normal to the skeleton, where

$$t = Y_p\left[1 + \left(\frac{dy_s}{dx}\right)^2\right]^{1/2}. \tag{5.25}$$

Since the root value in equation (5.25) differs from 1 only by a second-order quantity, we can write the following for this theory

$$t = Y_p. \tag{5.26}$$

Therefore, it follows that for the profile shape in the vicinity of the leading edge and the trailing edge is the same in the physical plane z as in the image plane Z.

6. Velocity Distribution along the Profile Contour

When calculating the contour velocity, we will start with the velocity field in the plane ζ: this velocity field is given by (4.12):

$$\mathfrak{w}_\zeta = \frac{1}{2}\left(1 - \frac{1}{\zeta^2}\right)\Big\{1 + \sum_{n=0}^{\infty} (-1)^n (\tilde{\mu}_n - i\tilde{\nu}_n)\frac{1}{2}(\zeta^n + \zeta^{-n}) + \\ + 2(g_0 + i g_0)(\zeta + 1)^{-1} + 2\bar{g}_0(\zeta - 1)^{-1} - \sum_{n=1}^{\infty}(g_n + i g_n)(-\zeta)^{-n}\Big\}. \quad (6.1)$$

From (6.1), it follows that the contour velocity in the plane Z is given by the following if we divide this expression by the derivative of the mapping function (4.6):

$$\frac{dZ}{d\zeta} = \frac{1}{2}\left(1 - \frac{1}{\zeta^2}\right) \quad (6.2)$$

and if we substitute the following for the variable ζ:

$$\zeta = -(1 + \varepsilon)\, e^{-i\vartheta} \quad (6.3)$$

where ε is given by relationships (4.19), (4.28), and (4.29) /642

$$U_K - iV_K = 1 + \sum_{n=0}^{\infty}(\tilde{\mu}_n - i\tilde{\nu}_n)\frac{1}{2}[(1+\varepsilon)^n e^{-in\vartheta} + \\ + (1+\varepsilon)^{-n} e^{in\vartheta}] - (g_0 + i g_0)\cdot \\ \cdot \frac{2}{(1+\varepsilon)e^{-i\vartheta} - 1} - g_0\frac{2}{(1+\varepsilon)e^{-i\vartheta} + 1} - \\ - \sum_{n=1}^{\infty}(g_n + i g_n)(1+\varepsilon)^{-n} e^{in\vartheta}. \quad (6.4)$$

If, in equation (6.4), we substitute $\tilde{\nu}_n$ according to (4.25) and if we ignore terms higher than second order, we find:

$$U_K - iV_K = 1 + \sum_{n=0}^{\infty}\tilde{\mu}_n(\cos n\vartheta - in\varepsilon \sin n\vartheta) - i g_0(1 - \varepsilon_0) + \\ + i g_0 \varepsilon_0 \cos\vartheta - (g_0 + i g_0) \\ \frac{(1+\varepsilon)(\cos\vartheta + i\sin\vartheta) - 1}{(1+\varepsilon)(1+\cos\vartheta) + \frac{1}{2}\varepsilon^2} - g_0\frac{(1+\varepsilon)(\cos\vartheta + i\sin\vartheta) + 1}{(1+\varepsilon)(1+\cos\vartheta) + \frac{1}{2}\varepsilon^2} + \quad (6.5)$$

(continued)

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$$+2(q_0+\bar{q}_0)(1-\varepsilon)(\cos\vartheta+i\sin\vartheta)-\sum_{n=2}^{\infty}\bar{q}_n(1-n\varepsilon)(\cos n\vartheta+i\sin n\vartheta)+$$
$$+\sum_{n=1}^{\infty}g_n(\sin n\vartheta+in\varepsilon\cos n\vartheta) \quad (6.5)$$

By decomposing this expression into a real and imaginary part, we obtain the two components of the contour velocity:

$$U_K=1+\sum_{n=0}^{\infty}\tilde{\mu}_n\cos n\vartheta-$$
$$-g_0\frac{(1+\varepsilon)\cos\vartheta-1}{(1+\varepsilon)(1-\cos\vartheta)+\frac{1}{2}\varepsilon^2}-g_0\frac{(1+\varepsilon)\cos\vartheta+1}{(1+\varepsilon)(1+\cos\vartheta)+\frac{1}{2}\varepsilon^2}+$$
$$+2(q_0+q_0)(1-\varepsilon)\cos\vartheta-\sum_{n=2}^{\infty}q_n(1-n\varepsilon)\cos n\vartheta+ \quad (6.6)$$
$$+g_0\frac{\sin\vartheta}{1-\cos\vartheta+\frac{1}{2}\varepsilon^2}+\sum_{n=1}^{\infty}g_n\sin n\vartheta$$

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$$V_K=g_0\frac{\varepsilon+\frac{1}{2}\varepsilon^2}{(1+\varepsilon)(1-\cos\vartheta)+\frac{1}{2}\varepsilon^2}+q_0\frac{\sin\vartheta}{1-\cos\vartheta+\frac{1}{2}\varepsilon^2}+$$
$$+g_0\frac{\sin\vartheta}{1+\cos\vartheta+\frac{1}{2}\varepsilon^2}-2(q_0+\bar{q}_0)\sin\vartheta+\sum_{n=2}^{\infty}q_n\sin n\vartheta- \quad (6.7)$$
$$-g_0\varepsilon_0(1+\cos\vartheta)+\varepsilon(\tilde{\mu}_1+2\bar{q}_0+2q_0)\sin\vartheta+$$
$$+\varepsilon\sum_{n=2}^{\infty}n(\tilde{\mu}_n-q_n)\sin n\vartheta-\varepsilon\sum_{n=1}^{\infty}n\,g_n\cos n\vartheta.$$

The contour velocity is then

$$W_K=(U_K^2+V_K^2)^{1/2} \quad (6.8)$$

Beyond the surroundings of the leading edge (and for $q_0 \neq 0$ also beyond the rounded-off trailing edge) the contour velocity differs only slightly from 1. Therefore, in this region we can derive a simple expansion for the contour velocity. From expressions (6.6) and (6.7), we obtain the following by ignoring terms higher than second-order:

$$U_K=1+\sum_{n=0}^{\infty}\tilde{\mu}_n\cos n\vartheta+q_0-\bar{q}_0+2(q_0+q_0)\cos\vartheta-\sum_{n=2}^{\infty}q_n\cos n\vartheta+$$
$$+g_0\operatorname{cotg}\frac{1}{2}\vartheta+\sum_{n=1}^{\infty}g_n\sin n\vartheta+\varepsilon\left[-\frac{q_0}{1-\cos\vartheta}+\right. \quad (6.9)$$
$$\left.+\frac{\bar{q}_0}{1+\cos\vartheta}-2(q_0+\bar{q}_0)\cos\vartheta+\sum_{n=2}^{\infty}nq_n\cos n\vartheta\right];$$

(continued)

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$$\Gamma_K = g_0 \operatorname{cotg} \frac{1}{2} \vartheta + \bar{q}_0 \operatorname{tg} \frac{1}{2} \vartheta - 2 (q_0 + \bar{q}_0) \sin \vartheta + \tag{6.9}$$

$$+ \sum_{n=2}^{\infty} g_n \sin n\vartheta - g_0 \varepsilon_0 (1 + \cos \vartheta) +$$

$$+ \varepsilon \left[\sum_{n=1}^{\infty} n (\tilde{\mu}_n - g_n) \sin n\vartheta + \frac{g_0}{1 - \cos \vartheta} - \sum_{n=1}^{\infty} n g_n \cos n\vartheta \right]. \tag{6.10}$$

In the following the letter φ is the angle between the tangent to the profile (Figure 8) and the X-axis; and then we have

$$W_K = U_K \cos \varphi + V_K \sin \varphi. \tag{6.11}$$

Here, tg φ is the directional factor of the profile tangent which can be written as follows to the first approximation according to (4.31) and (4.34): /644

$$\operatorname{tg} \varphi = \frac{dY_p}{dX_p} = \frac{1}{2} \sum_{n=1}^{\infty} n A_n \operatorname{cotg} \frac{1}{2} \vartheta + \frac{1}{2} \sum_{n=1}^{\infty} (-1)^n n A_n \operatorname{tg} \frac{1}{2} \vartheta - \tag{6.12}$$

$$- 2 \sum_{m=1}^{\infty} 2m A_{2m} \sin \vartheta - 2 \sum_{k=2}^{\infty} \left[\sum_{m=0}^{\infty} (2m + k + 1) A_{2m+k+1} \right] \cdot \sin k\vartheta,$$

$$\operatorname{tg} \varphi = \frac{dY_p}{dX_p} = q_0 \operatorname{cotg} \frac{1}{2} \vartheta + \bar{q}_0 \operatorname{tg} \frac{1}{2} \vartheta - 2 (q_0 + \bar{q}_0) \sin \vartheta + \tag{6.13}$$

$$+ \sum_{n=2}^{\infty} q_n \sin n\vartheta.$$

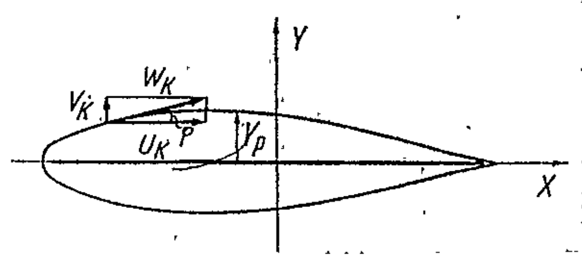

Figure 8

Therefore, $\frac{dY_p}{dX_p}$ can be considered as a first-order variable, and the contour velocity can then be written in the form

$$W_K = U_K \left[1 - \frac{1}{2} \left(\frac{dY_p}{dX_p} \right)^2 \right] + \Gamma_K \frac{dY_p}{dX_p} \tag{6.14}$$

After substitution and if we only consider second-order terms, we find:

$$
\begin{aligned}
W_K = {} & 1 + \sum_{n=0}^{\infty} \bar{g}_n \cos n\vartheta + \bar{q}_0 - q_0 + 2(q_0 + \bar{q}_0)\cos\vartheta - \sum_{n=2}^{\infty} q_n \cos n\vartheta + \\
& + g_0 \operatorname{cotg} \frac{1}{2}\vartheta + \sum_{n=1}^{\infty} g_n \sin n\vartheta + \varepsilon \left[- \frac{q_0}{1-\cos\vartheta} + \frac{\bar{q}_0}{1+\cos\vartheta} - \right. \\
& \left. - 2(q_0 + \bar{q}_0)\cos\vartheta + \sum_{n=2}^{\infty} n q_n \cos n\vartheta \right] + \frac{dY_p}{dX_p} \left[q_0 \operatorname{cotg} \frac{1}{2}\vartheta + \right. \\
& \left. + \bar{q}_0 \operatorname{tg} \frac{1}{2}\vartheta - 2(q_0 + \bar{q}_0)\sin\vartheta + \sum_{n=2}^{\infty} n q_n \sin n\vartheta \right] - \frac{1}{2}\left(\frac{dY_p}{dX_p}\right)^2,
\end{aligned}
\tag{6.15}
$$

where the value of the derivative $\frac{dY_p}{dX_p}$ is given by (6.12) or (6.13) /645
and ε is given by: (according to 4.31):

$$
\varepsilon = \sum_{m=0}^{\infty} A_{2m+1} + 2 \sum_{n=1}^{\infty} \left[\sum_{m=0}^{\infty} A_{2m+n+1} \right] \cos n\vartheta,
\tag{6.16}
$$

or

$$
\begin{aligned}
\varepsilon = {} & \left(q_0 - \bar{q}_0 + \frac{1}{3} q_2 + \frac{1}{15} q_4 + \dots \right) + \left(\bar{q}_0 + q_0 + \frac{1}{4} q_3 + \right. \\
& \left. + \frac{1}{12} q_5 + \dots \right) \cos\vartheta - \left(\frac{1}{3} q_2 - \frac{2}{15} q_4 + \dots \right) \cos 2\vartheta - \\
& - \left(\frac{1}{4} q_3 - \frac{1}{12} q_5 + \dots \right) \cos 3\vartheta - \left(\frac{1}{5} q_4 + \dots \right) \cos 4\vartheta - \\
& - \left(\frac{1}{6} q_5 + \dots \right) \cos 5\vartheta + \dots
\end{aligned}
\tag{6.17}
$$

For the derivative $\frac{dY_p}{dX_p}$ and for ε we will always have two expressions. When solving the even problem, we will use expressions (6.12) and (6.16). The contour velocity is then given by a linear expression in terms of the coefficients g_n and q_n, which is important for calculating the contour velocity for different incident flow directions. When solving the odd problem, we start with the selected circulation distribution (coefficient g_n) and source distribution (coefficient q_n). This selection is specified on the contour by requiring a suitable velocity distribution. Therefore, there must be a way of evaluating this velocity distribution before carrying out the entire calculation. Relationship (6.15) is used for this, in which we substitute expressions (6.13) and (6.17) for dY_ν/dX_p and ε.

As already mentioned, the expression (6.15) for calculating the contour velocity cannot be used in the vicinity of the rounded-off trailing edge. There, we must use formulas (6.6) and (6.8) for obtaining the contour velocity. Directly at the leading edge, that is, for $\vartheta = 0$, we have the following according to (6.6) and (6.7).

$$U_K = 1 + \sum_{n=0}^{\infty} \tilde{\mu}_n - 2q_0 \left(\frac{1}{\varepsilon_0} - 1 + \varepsilon_0\right) + \bar{q}_0 \left(1 - \frac{3}{2}\varepsilon_0\right) - \sum_{n=2}^{\infty} q_n (1 - n\varepsilon_0). \tag{6.18}$$

$$V_K = g_0 \left(\frac{2}{\varepsilon_0} + 1 - 2\varepsilon_0\right) - \varepsilon_0 \sum_{n=1}^{\infty} n g_n \tag{6.19}$$

If in equations (6.18) and (6.19) we substitute according to (4.33) /646
we find

$$\varepsilon_0 = \sum_{n=1}^{\infty} nA_n, \quad \frac{1}{\varepsilon_0} = \frac{1}{\sum\limits_{n=1}^{\infty} nA_n} - \frac{1}{2} + \frac{1}{4}\sum_{n=1}^{\infty} nA_n \tag{6.20}$$

and we have

$$U_K = -\frac{2q_0}{\sum\limits_{n=1}^{\infty} nA_n} + 1 + \sum_{n=0}^{\infty} \tilde{\mu}_n + 3q_0 + \bar{q}_0 - \sum_{n=2}^{\infty} q_n - \sum_{n=1}^{\infty} nA_n \left(\frac{5}{2} q_0 + \frac{3}{2}\bar{q}_0 - \sum_{m=2}^{\infty} m q_m\right), \tag{6.21}$$

$$V_K = g_0 \left(\frac{2}{\sum\limits_{n=1}^{\infty} nA_n} - \frac{3}{2}\sum_{n=1}^{\infty} nA_n\right) - \sum_{n=1}^{\infty} nA_n \sum_{k=1}^{\infty} k g_k. \tag{6.22}$$

If in these expressions, we substitute for ΣnA_n according to equation (4.36), then we have the following:

$$U_K = -2q_0 \left(\frac{5}{2} q_0 + \frac{3}{2}\bar{q}_0 - \sum_{n=2}^{\infty} n q_n\right), \tag{6.23}$$

$$V_K = \frac{g_0}{q_0}\left(1 + 3q_0 + \bar{q}_0 - \sum_{n=2}^{\infty} n q_n + \sum_{n=0}^{\infty} \tilde{\mu}_n\right) - 2q_0 \sum_{n=1}^{\infty} n g_n - 3\, g_0 q_0. \tag{6.24}$$

It should be realized that terms higher than second order were not considered in expression (4.36) for ΣnA_n, and that ΣnA_n itself is a first-order quantity. The ignored higher-order terms have an effect on the reciprocal $1/\Sigma nA_n$ already for the first order terms, and therefore expressions (6.23) and (6.24) are only correct up to first order terms. In the case of a flow without circulation, both components are small of second order in the case of smooth entry ($g_0 = 0$). For a non-smooth entry, U_K is small of second order and V_K is of order 1.

In formula (6.12), which can be considered as a formula for the contour velocity at the leading edge, it is often advantageous to replace ΣnA_n by the radius of curvature of the leading edge, according to (5.7):

$$W_K = V_K = 2g_0 (R_1^{-\frac{1}{2}} - R_1^{\frac{1}{2}}) - R_1^{\frac{1}{2}} \sum_{n=1}^{\infty} n g_n . \tag{6.25}$$

In a similar manner, the following expression gives the velocity along the rounded-off trailing edge (6.26).

$$W_K = V_K = -\bar{g}_0 \left(g_0 - 2 \sum_{n=1}^{\infty} n g_n \right) = R_2^{\frac{1}{2}} \left(\frac{1}{2} g_0 - \sum_{n=1}^{\infty} n g_n \right) \tag{6.26}$$

The X-component of the contour velocity on the rounded trailing edge is small of an order higher than second order, and therefore can be ignored within our theory. It should only be realized that formula (6.26) has a theoretical meaning. This is because in a real flow there is a boundary layer separation in the vicinity of the rounded trailing edge. /647

The formula is derived for the distribution of the contour velocity applied in the image plane Z; therefore, we must derive the relationships for transforming values from the plane Z to the plane z. For the velocity in the z plane, we have

$$w = \frac{1}{\left| \frac{dz}{dZ} \right|} W; \tag{6.27}$$

where $\left| \frac{dz}{dZ} \right|$ is the absolute magnitude of the derivative of the mapping functions (2.10):

$$\frac{dz}{dZ} = 1 + i \frac{df}{dZ} . \tag{6.28}$$

On the abscissa $-1 \leq X \leq 1$, $Y = 0$, we have, according to equation (2.15);

$$\frac{df}{dZ} = \frac{dy_s}{dx} \tag{6.29}$$

and since the profile thickness is small, the derivative df/dZ of the mapping function can be expanded into a Taylor series on the profile:

$$\left.\frac{\mathrm{d}f}{\mathrm{d}Z}\right|_{Z=Z_p} = \frac{\mathrm{d}y_s}{\mathrm{d}x} + \frac{\mathrm{d}}{\mathrm{d}X}\left(\frac{\mathrm{d}y_s}{\mathrm{d}x}\right)(X_p + i\,Y_p - X) + \dots \tag{6.30}$$

If we only restrict ourselves to terms up to second order, then we can omit the difference $X_p - X$ and all other terms not shown of higher order. The derivative $\frac{\mathrm{d}}{\mathrm{d}X}\left(\frac{\mathrm{d}y_s}{\mathrm{d}x}\right)$ can be written as:

$$\frac{\mathrm{d}}{\mathrm{d}X}\left(\frac{\mathrm{d}y_s}{\mathrm{d}x}\right) = \frac{\mathrm{d}}{\mathrm{d}\vartheta}\left(\frac{\mathrm{d}y_s}{\mathrm{d}x}\right)\cdot\frac{\mathrm{d}\vartheta}{\mathrm{d}X} = -\sum_{n=1}^{\infty} n\,B_n\,\frac{\sin n\vartheta}{\sin\vartheta}\,. \tag{6.31}$$

After replacement in equations (6.28) and (6.30), we find:

$$\frac{\mathrm{d}z}{\mathrm{d}Z} = 1 + i\sum_{n=0}^{\infty} B_n \cos n\vartheta + \sum_{n=1}^{\infty} n\,B_n\,\frac{\sin n\vartheta}{\sin\vartheta}\sum_{k=1}^{\infty} A_k \sin k\vartheta \tag{6.32}$$

For an accuracy up to second order, we therefore substitute the following in equation (6.27): /648

$$\begin{aligned}\frac{1}{\left|\frac{\mathrm{d}z}{\mathrm{d}Z}\right|} = {} & 1 - \frac{1}{2}B_0^2 - \frac{1}{4}(B_1^2 + B_2^2 + \dots) - \Big(B_0 B_1 + \frac{1}{2}B_1 B_2 + {} \\ & + \frac{1}{2}B_2 B_3 + \dots\Big)\cos\vartheta - \Big(\frac{1}{4}B_1^2 + B_0 B_2 + \frac{1}{2}B_1 B_3 + \dots\Big)\cos 2\vartheta - {} \\ & - \Big(B_0 B_3 + \frac{1}{2}B_1 B_2 + \frac{1}{2}B_1 B_4 + \dots\Big)\cos 3\vartheta - \Big(\frac{1}{4}B_2^2 + B_0 B_4 + {} \\ & + \frac{1}{2}B_1 B_5 + \dots\Big)\cos 4\vartheta - \Big(B_0 B_5 + \frac{1}{2}B_1 B_4 + {} \\ & + \frac{1}{2}B_2 B_3 + \dots\Big)\cos 5\vartheta - (A_1 B_1 + 2A_2 B_2 + \dots)\sin\vartheta - {} \\ & - (2A_1 B_2 + A_2 B_1 + 3A_2 B_3 + 2A_3 B_2 + \dots)\sin 2\vartheta - (3A_1 B_3 + {} \\ & + 2A_2 B_2 + A_3 B_1 + \dots)\sin 3\vartheta - (4A_1 B_4 + 3A_2 B_3 + 2A_3 B_2 + {} \\ & + A_4 B_1 + \dots)\sin 4\vartheta + \dots\end{aligned} \tag{6.33}$$

The contour velocity is then given by the product of expressions (6.8) or (6.15) and (6.33).

SECOND PART

FORMULAS FOR NUMERICAL CALCULATIONS AND EXAMPLES

7. Formulas for Numerical Calculation

In the previous chapters, we used trigonometric expansions with an infinite number of terms for the variables to include the general case. In practice, it is sufficient to use relatively low-order trigonometric polynomials. In this section, we will discuss formulas for practical applications, in which the first few coefficients of trigonometric expansions are considered in the first order terms (usually, it is not necessary to take more than six terms). In the second-order terms, we will only consider those terms where the index sum is smaller or equal to 4.

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Geometric Variables

The coordinate system is located so that the skeleton chord lies in the x-axis, and the origin of the coordinate system is located at the chord center. The length of the skeleton chord is called c*. Instead of the variable x, we will introduce a trigonometric variable ϑ with the following equation on the skeleton chord.

$$\boxed{x = -\frac{c}{2} \cos \vartheta; \quad 0 \leq \vartheta \leq \pi} \tag{7.1}$$

The skeleton shape is given by the trigonometric expansion of its derivative

$$\boxed{\frac{dy_s}{dx} = \sum_{n=0}^{N} B_n \cos n\vartheta} \tag{7.2}$$

*In this chapter, intended for practical applications, all of the quantities are dimensional. The skeleton chord length is the fundamental length for length measurements, and the velocity U_0 is the fundamental quantity for the velocities. In order to simplify the notation, we will use the same notation for these dimensioned variables as in the complex plane z. Since the theory is not required for performing numerical calculations, there is no danger of confusion.

where

$$B_0 = \sum_{m=1}^{\left[\frac{N}{2}\right]} \frac{B_{2m}}{4m^2 - 1}.$$

From this expansion, we find:

$$y_s = \frac{c}{4}\left\{B_0(1 - \cos\vartheta) + \sum_{n=1}^{N} \frac{B_{n-1} - B_{n+1}}{n}(1 - \cos n\vartheta)\right\}, \tag{7.3}$$

where

$$y_s\left(-\frac{c}{2}\right) = y_s\left(\frac{c}{2}\right) = 0. \tag{7.4}$$

The thickness distribution is plotted along the skeleton normal, and is given by the expansion:

$$t = \frac{c}{2}\left\{\sum_{n=1}^{M} A_n \sin n\vartheta + \sum_{m=1}^{2}\left[C_{2m}(1 - \cos 2m\vartheta) + C_{2m+1}(\cos\vartheta - \cos(2m+1)\vartheta)\right]\right\}, \quad -\pi \leq \vartheta \leq \pi \tag{7.5}$$

The positive values are measured upwards and the negative ones are /650
measured downwards from the skeleton. The base point of the normal to the skeleton has an x-coordinate given by the relationship

$$x = -\frac{c}{2}\cos\vartheta\left[1 + \frac{1}{2}\left(\frac{2t}{c\sin\vartheta}\right)^2\right] \tag{7.6}$$

or

$$x = -\frac{c}{2}\cos\vartheta\left[1 + \frac{1}{2}A_1^2 + A_2^2 + A_1 A_3 + 2A_1A_2\cos\vartheta + (A_2^2 + 2A_1A_3)\cos 2\vartheta\right] \tag{7.7}$$

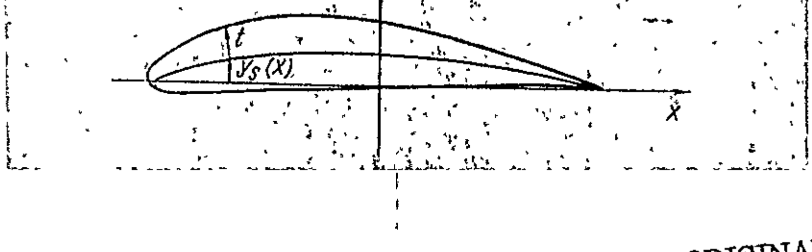

Figure 9

The curvature radius of the leading edge is:

$$R_1 = \frac{c}{2}\left(\sum_{n=1}^{M} nA_n\right)^2. \tag{7.8}$$

The curvature radius of the trailing edge is:

$$R_2 = \frac{c}{2}\left(\sum_{n=1}^{M} (-1)^n nA_n\right)^2. \tag{7.9}$$

The distance between the profile nose from the end of the skeleton onthe leading edge is

$$\Delta_1 = \frac{1}{2} R_1. \tag{7.10}$$

and on the trailing edge,

$$\Delta_2 = \frac{1}{2} R_2. \tag{7.11}$$

Aerodynamic Variables.

The components of the primary velocity field on the point where the skeleton is located are given by the trigonometric polynomials:

$$u = U_0\left(1 + \sum_{n=0}^{N} \mu_n \cos n\vartheta\right), \quad v = U_0 \sum_{n=0}^{N} \nu_n \cos n\vartheta. \tag{7.12}$$

The circulation distribution and source distribution on the skeleton is given by the following trigonometric polynomials: /651

$$\gamma = 2U_0\left(g_0 \operatorname{cotg}\frac{1}{2}\vartheta + \sum_{n=1}^{N} g_n \sin n\vartheta\right), \tag{7.13}$$

$$q = 2U_0\left[q_0 \operatorname{cotg}\frac{1}{2}\vartheta + \bar{q}_0 \operatorname{tg}\frac{1}{2}\vartheta - 2(q_0 + \bar{q}_0)\sin\vartheta + \sum_{n=2}^{M-1} q_n \sin n\vartheta\right]. \tag{7.14}$$

The relationship between the geometric variables and the aerodynamic variables is given by a system of linear inhomogeneous equations between the coefficients of the geometric variables A_n, B_n, and C_n, and the coefficients of the aerodynamic variables g_n, q_n, μ_n, and ν_n. In order to solve this, it is advantageous to decompose the system into three groups of equations and to then solve one group after the other:

The First Group:

$$
\begin{aligned}
&q_0 - \bar{q}_0 + \frac{1}{2} q_2 = A_1 + A_1 \left(q_0 - \bar{q}_0 + \frac{1}{2} q_2 + \mu_0 - \frac{1}{2} \mu_2 \right) + \\
&\qquad + \frac{1}{2} A_2 (2q_0 + 2\bar{q}_0 + \mu_1), \\
&\frac{1}{2} q_0 + \frac{1}{2} \bar{q}_0 + \frac{1}{4} q_3 = A_2 + \frac{1}{2} A_1 (2q_0 + 2\bar{q}_0 + q_3 + \mu_1 - \mu_3) + \\
&\qquad + A_2 (q_0 - \bar{q}_0 + \mu_0) + \frac{1}{2} A_3 (2q_0 + 2\bar{q}_0 + \mu_1), \\
&-\frac{1}{6} (q_2 - q_4) = A_3 - \frac{1}{2} A_1 (q_2 - \mu_2) + \frac{1}{2} (A_2 + A_4) (2q_0 + \\
&\qquad + 2\bar{q}_0 + \mu_1) + A_3 (q_0 - \bar{q}_0 + \mu_0), \\
&-\frac{1}{8} (q_3 - q_5) = A_4 - \frac{1}{2} A_1 (q_3 - \mu_3) - \frac{1}{2} A_2 (q_2 - \mu_2) + \\
&\qquad + \frac{1}{2} A_3 (2q_0 + 2\bar{q}_0 + \mu_1) + A_4 (q_0 - \bar{q}_0 + \mu_0), \\
&-\frac{1}{2n} (q_{n-1} - q_{n+1}) = A_n, \quad n \geqq 5.
\end{aligned}
\tag{7.15}
$$

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It is better to use the following equations, instead of the first of the two equations given above:

$$
2q_0 = \sum_{n=1}^{M} nA_n \left\{ 1 + 3q_0 + \bar{q}_0 - \sum_{n=2}^{4} q_n + \sum_{n=0}^{4} \mu_n \right\}, \tag{7.16}
$$

$$
2\bar{q}_0 = \sum_{n=1}^{M} (-1)^n nA_n \left\{ 1 - q_0 - 3\bar{q}_0 - \sum_{n=2}^{4} (-1)^n q_n + \sum_{n=0}^{4} (-1)^n \mu_n \right\} \tag{7.17}
$$

The Second Group:

$$
\begin{aligned}
&g_0 = -B_0 + \nu_0 + g_0 \sum_{n=1}^{M} nA_n - B_0 \mu_0 - \frac{1}{2} B_1 \mu_1 - \frac{1}{2} B_2 \mu_2 + \\
&\quad + \frac{1}{3} B_2 \left(q_0 - \bar{q}_0 + \frac{1}{2} q_2 \right) + \frac{1}{4} B_3 (q_0 + \bar{q}_0) + \frac{7}{15} B_4 (q_0 - \bar{q}_0); \\
&g_1 = B_1 - \nu_1 - g_0 \sum_{n=1}^{M} nA_n + B_0 \mu_1 + B_1 \left(\mu_0 + \frac{1}{2} \mu_2 \right) + \frac{1}{2} B_2 \mu_1 - \\
&\qquad - B_3 (q_0 - \bar{q}_0) - \frac{4}{5} B_4 (q_0 + \bar{q}_0), \\
&g_2 = B_2 - \nu_2 + B_0 \mu_2 + \frac{1}{2} B_1 (\mu_1 + \mu_3) + B_2 \mu_0 - \frac{6}{5} B_4 (q_0 - \bar{q}_0), \\
&g_3 = B_3 - \nu_3 + B_0 \mu_3 + \frac{1}{2} B_1 \mu_2 + \frac{1}{2} B_2 \mu_1 + B_3 \mu_0; \\
&g_4 = B_4 - \nu_4 + B_0 \mu_4 + \frac{1}{2} B_1 \mu_3 + \frac{1}{2} B_2 \mu_2 + \frac{1}{2} B_3 \mu_1 + B_4 \mu_0; \\
&g_n = B_n - \nu_n, \quad n \geqq 5.
\end{aligned}
\tag{7.18}
$$

The Third Group:

$$C_2 = -\frac{1}{4} g_0 \sum_{n=1}^{M} nA_n + g_0 \left(A_2 + 2 \sum_{n=3}^{M} A_n\right) - \frac{1}{2} g_1 (A_1 - A_3) + \frac{1}{2} g_3 A_1,$$

$$C_3 = g_0 (A_3 + 2A_4) - \frac{1}{2} g_1 A_2 - \frac{1}{2} g_2 A_1, \qquad (7.19)$$

$$C_4 = g_0 A_4 - \frac{1}{2} g_1 A_3 - \frac{1}{2} g_2 A_2 - \frac{1}{2} g_3 A_1.$$

All three groups have the largest terms in the main diagonal, and /653
therefore can be solved using a method of stepwise approximation very easily. In most cases, the first two approximations will be sufficient.

THE EVEN PROBLEM (THE SECOND MAIN PROBLEM) OF PROFILE THEORY

When solving the even problem one determines the circulation distribution and source distribution (that is, the coefficients g_n and q_n) for a profile immersed into a given primary field. From the calculated circulation distribution, one determines the total circulation, the lift, and the velocity distribution along the profile contour.

Since the profile skeleton does not have a purely geometric profile characteristic (it also depends on the primary field), one starts at the profile center line when solving the even problem. This is because most profiles are produced by drawing a suitable symmetric profile along a corresponding center line.

The beginning of the center line is at a distance $\Delta_1 = \frac{1}{2} R_1$ from the leading edge (Figure 10), and its end in the trailing edge, or at a distance $\Delta_2 = \frac{1}{2} R_2$ for a rounded trailing edge. We will determine the coefficients $\bar{B}_n$ for the trigonometric expansion of the slope of the middle line:

$$\frac{dy_{ml}}{dx} = \sum_{n=0}^{N} \bar{B}_n \cos n\vartheta, \qquad (7.20)$$

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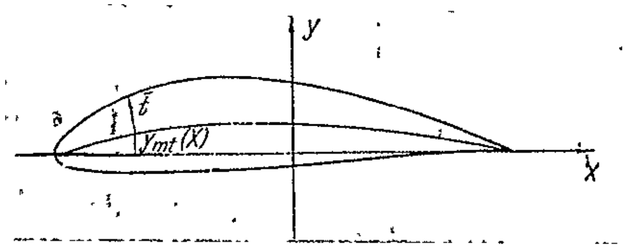

Figure 10

Also, we determined the coefficients A_n of the symmetric thickness distribution:

$$\hat{t} = \frac{c}{2}\sum_{n=1}^{M} A_n \sin n\vartheta, \quad x = -\frac{c}{2}\left[1 + \frac{1}{2}\left(\frac{2\hat{t}}{c\sin\vartheta}\right)^2\right]\cos\vartheta. \qquad (7.21)$$

Considering that the nonsymmetric correction of the thickness distribution (the coefficients C_n in (7.5) are of second order) we can take the following curve when calculating the skeleton: /654

$$y_s = y_{mt} - \frac{c}{2}\left[C_2(1-\cos 2\vartheta) + C_3(\cos\vartheta - \cos 3\vartheta) + C_4(1-\cos 4\vartheta)\right]. \qquad (7.22)$$

The coefficient values which are substituted in system (7.18) are then

$$\begin{aligned} B_0' &= \bar{B}_0 - 2C_3, \\ B_1' &= \bar{B}_1 - 4(C_2 + 2C_4), \\ B_2' &= \bar{B}_2 - 6C_3, \\ B_3' &= \bar{B}_3 - 8C_4, \\ B_n &= \bar{B}_n, \quad n \geq 4; \end{aligned} \qquad (7.23)$$

where the coefficients C_n are given by expressions (7.19). By solving (7.15) and (7.18), we obtain the values of the coefficients g_n and q_n.

The total circulation is:

$$\Gamma = \pi c U_0\left(g_0 + \frac{1}{2}g_1\right). \qquad (7.24)$$

The velocity distribution on the profile contour is given by the following relationship in the region outside of the rounded trailing edge (and also outside of this trailing edge, when the trailing edge is rounded off):

$$
\begin{aligned}
w_K = \frac{U_0}{\left|\frac{dz}{dZ}\right|} \Big\{ & 1 + \sum_{n=0}^{N} \tilde{\mu}_n \cos n\vartheta + q_0 - q_0' + 2(q_0 + q_0')\cos\vartheta - \sum_{n=2}^{M} q_n \cos n\vartheta + \\
& + g_0 \operatorname{cotg}\frac{1}{2}\vartheta + \sum_{n=1}^{4} g_n \sin n\vartheta + \varepsilon\left[\frac{-q_0}{1-\cos\vartheta} + \frac{q_0'}{1+\cos\vartheta} - 2(q_0 + q_0')\cos\vartheta + \right. \\
& \left. + \sum_{n=1}^{4} n q_n \cos n\vartheta\right] + \frac{dY_p}{dX_p}\left[q_0 \operatorname{cotg}\frac{1}{2}\vartheta + q_0' \operatorname{tg}\frac{1}{2}\vartheta - 2(q_0 + q_0')\sin\vartheta + \right. \\
& \left. + \sum_{n=2}^{4} q_n \sin n\vartheta - \frac{1}{2}\frac{dY_p}{dX_p}\right]\Big\},
\end{aligned} \tag{7.25}
$$

$$
\begin{aligned}
\tilde{\mu}_0 = {} & \mu_0 + \nu_0 B_0 + \frac{1}{2}\nu_1 B_1 + \frac{1}{2}\nu_2 B_2 + g_0\left(\frac{1}{2}B_1 + \frac{1}{3}B_2 + \frac{1}{2}B_3 + \frac{7}{15}B_4\right) + \\
& + \frac{1}{4} g_1\left(B_1 + \frac{1}{2}B_3\right) + \frac{1}{6} g_2 B_2;
\end{aligned}
$$

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$$
\begin{aligned}
& \tilde{\mu}_1 = \mu_1 + \nu_0 B_1 + \nu_1\left(B_0 + \frac{1}{2}B_2\right) + \frac{1}{2}\nu_2 B_1 + g_0\left(\frac{4}{3}B_2 + B_3 + \frac{16}{15}B_4\right) + \frac{2}{3}g_1 B_2, \\
& \tilde{\mu}_2 = \mu_2 + \nu_0 B_2 + \frac{1}{2}\nu_1 B_1 + \nu_2 B_0 + \frac{1}{2}\nu_3 B_1 + g_0\left(\frac{3}{2}B_3 + \frac{6}{5}B_4\right) + \frac{3}{4}g_1 B_3; \\
& \tilde{\mu}_3 = \mu_3 + \nu_0 B_3 + \frac{1}{2}\nu_1 B_2 + \frac{1}{2}\nu_2 B_1 + \nu_3 B_0 + \frac{8}{5} g_0 B_4; \\
& \tilde{\mu}_4 = \mu_4 + \nu_0 B_4 + \frac{1}{2}\nu_1 B_3 + \frac{1}{2}\nu_2 B_2 + \frac{1}{2}\nu_3 B_1 + \nu_4 B_0; \\
& \tilde{\mu}_n = \mu_n, \quad n \geq 5;
\end{aligned} \tag{7.26}
$$

$$
\varepsilon = A_1 + A_3 + 2(A_2 + A_4)\cos\vartheta + 2A_3\cos 2\vartheta + 2A_4\cos 3\vartheta; \tag{7.27}
$$

$$
\begin{aligned}
\frac{dY_p}{dX_p} = {} & \left(\frac{R_1}{2c}\right)^{1/2}\operatorname{cotg}\frac{1}{2}\vartheta - \left(\frac{R_2}{2c}\right)^{1/2}\operatorname{tg}\frac{1}{2}\vartheta - 2(2A_2 + 4A_4)\sin\vartheta - 6A_3\sin 2\vartheta - \\
& - 8A_4\sin 3\vartheta;
\end{aligned} \tag{7.28}
$$

$$
\begin{aligned}
\left|\frac{dz}{dZ}\right| = {} & 1 + \frac{1}{2}B_0^2 + \frac{1}{4}(B_1^2 + B_2^2) + \left(B_0 B_1 + \frac{1}{2}B_1 B_2\right)\cos\vartheta + \left(\frac{1}{4}B_1^2 + B_0 B_2 + \right. \\
& \left. + \frac{1}{2}B_1 B_3\right)\cos 2\vartheta + \left(B_0 B_3 + \frac{1}{2}B_1 B_2\right)\cos 3\vartheta + \left(\frac{1}{4}B_2^2 + B_0 B_4 + \right. \\
& \left. + \frac{1}{2}B_1 B_3\right)\cos 4\vartheta + (A_1 B_1 + 2A_2 B_2)\sin\vartheta + (2A_1 B_2 + \\
& + A_2 B_1)\sin 2\vartheta + (3A_1 B_3 + 2A_2 B_2 + A_3 B_1)\sin 3\vartheta.
\end{aligned} \tag{7.29}
$$

In the vicinity of the rounded leading edge and the rounded trailing edge $\left(|\vartheta| < \frac{\pi}{6},\ \frac{5\pi}{6} < |\vartheta| \leq \pi\right)$, where relationship (7.25) already has substantial deviations, it is appropriate to calculate the contour velocity using the formula:

$$w_K = \frac{U_0}{\left|\frac{dz}{dZ}\right|} \cdot (U_K^2 + V_K^2)^{1/2} \tag{7.30}$$

Here, we have /656

$$\begin{aligned} U_K = {} & 1 + \sum_{n=0}^{N} \widetilde{\mu}_n \cos n\vartheta + \bar{q}_0 - \bar{q}_0 + 2(\bar{q}_0 + \bar{q}_0)\cos\vartheta - \sum_{n=2}^{M} q_n \cos n\vartheta + \\ & + \frac{\bar{q}_0 \sin\vartheta}{1 - \cos\vartheta + \frac{1}{2}\varepsilon^2} + \sum_{n=1}^{N} \bar{q}_n \sin n\vartheta + \varepsilon \left[\frac{-\bar{q}_0}{1 - \cos\vartheta + \frac{1}{2}\varepsilon^2} + \right. \\ & \left. + \frac{\bar{q}_0}{1 + \cos\vartheta + \frac{1}{2}\varepsilon^2} - 2(q_0 + \bar{q}_0)\cos\vartheta + \sum_{n=2}^{4} n q_n \cos n\vartheta \right]; \end{aligned} \tag{7.31}$$

$$\begin{aligned} V_K = {} & \frac{q_0 \sin\vartheta}{1 - \cos\vartheta + \frac{1}{2}\varepsilon^2} + \frac{q_0 \sin\vartheta}{1 + \cos\vartheta + \frac{1}{2}\varepsilon^2} - 2(q_0 + \bar{q}_0)\sin\vartheta + \sum_{n=2}^{M} q_n \sin n\vartheta - \\ & - \bar{g}_0 \sum_{n=1}^{N} n A_n (1 + \cos\vartheta) + \varepsilon \left[(\widetilde{\mu}_1 + 2q_0 + 2\bar{q}_0)\sin\vartheta + \right. \\ & \left. + \sum_{n=2}^{4} n(\widetilde{\mu}_n - q_n)\sin n\vartheta + \frac{g_0}{1 - \cos\vartheta + \frac{1}{2}\varepsilon^2} - \sum_{n=1}^{4} n g_n \cos n\vartheta \right]. \end{aligned} \tag{7.32}$$

The velocity along the leading edge is

$$w_K = \frac{U_0}{1 + \frac{1}{2}\left(\sum_{n=0}^{4} B_n\right)^2} \left\{ \left(\frac{R_1}{2c}\right)^{-1/2} g_0 - \left(\frac{2R_1}{c}\right)^{1/2} \sum_{n=1}^{4} n g_n \right\}; \tag{7.33}$$

THE ODD PROBLEM (THE FIRST MAIN PROBLEM) OF PROFILE THEORY

The odd problem in the true sense of the word is to find the profile shape from a specified velocity distribution on the profile contour. Here, we will not solve this problem in this generality. We will introduce a modification so that the shape of the profile placed into the inhomogeneous primary flow field is calculated for a specified skeleton chord, source distribution and vortex distribution on the skeleton.

The expression for the components of the primary velocity at the point of the sought-after profile is found by expanding the components of the primary velocity in the vicinity of the skeleton chord in Taylor series. In this calculation, the first terms of this expansion are considered only. Instead of (7.12), we will write /657

$$u = U_0\left[1 + \sum_{n=0}^{N}\left(\mu_n' + \frac{\partial \mu_n'}{\partial y}\, y_s\right)\cos n\vartheta\right], \qquad (7.34)$$

$$v = U_0\left[\sum_{n=0}^{N}\left(\nu_n' + \frac{\partial \nu_n'}{\partial y}\, y_s\right)\cos n\vartheta\right]. \qquad (7.35)$$

The coefficient values μ_n'; ν_n'; $\frac{\partial \mu_n'}{\partial y}$ and $\frac{\partial \nu_n'}{\partial y}$ are determined from the component values of the primary velocity and their derivatives at the chord points.

The dash notation has been introduced in order to distinguish these coefficients μ_n' and ν_n' from the coefficients μ_n and ν_n of the expansions of the primary velocity components (7.21) calculated directly on the profile. Using (7.3) and considering (7.2), we then have:

$$\begin{aligned}
\mu_0 &= \mu_0' + \frac{c}{8}\frac{\partial \mu_0'}{\partial y}\left(B_1 - \frac{1}{2}B_3\right) + \frac{c}{24}\frac{\partial \mu_1'}{\partial y}B_2 - \frac{c}{16}\frac{\partial \mu_2'}{\partial y}B_1;\\
\mu_1 &= \mu_1' + \frac{c}{6}\frac{\partial \mu_0'}{\partial y}\left(\frac{1}{2}B_2 - \frac{1}{5}B_4\right) + \frac{c}{16}\frac{\partial \mu_1'}{\partial y}B_1 - \frac{c}{16}\frac{\partial \mu_3'}{\partial y}B_1;\\
\mu_2 &= \mu_2' - \frac{c}{8}\frac{\partial \mu_0'}{\partial y}(B_1 - B_3) + \frac{c}{8}\frac{\partial \mu_2}{\partial y}B_1;\\
\mu_3 &= \mu_3' - \frac{c}{12}\frac{\partial \mu_0'}{\partial y}(B_2 - B_4) - \frac{c}{16}\frac{\partial \mu_1'}{\partial y}\left(B_1 - \frac{1}{2}B_3\right) +\\
&\quad + \frac{c}{24}\frac{\partial \mu_2'}{\partial y}B_2 + \frac{c}{8}\frac{\partial \mu_3'}{\partial y}B_1;\\
\mu_4 &= \mu_4' - \frac{c}{16}\frac{\partial \mu_0'}{\partial y}B_3 - \frac{c}{24}\frac{\partial \mu_1'}{\partial y}B_2 - \frac{c}{16}\frac{\partial \mu_2'}{\partial y}B_1,\\
\mu_5 &= \mu_5' - \frac{c}{20}\frac{\partial \mu_0'}{\partial y}B_4 - \frac{c}{32}\frac{\partial \mu_1'}{\partial y}B_3 - \frac{c}{24}\frac{\partial \mu_2'}{\partial y}B_2 - \frac{c}{16}\frac{\partial \mu_3'}{\partial y}B_1;\\
\mu_n &= \mu_n', \quad n \geqq 6.
\end{aligned} \qquad (7.36)$$

$$\begin{aligned}
\nu_0 &= \nu_0' + \frac{c}{8}\frac{\partial \nu_0'}{\partial y}\left(B_1 - \frac{1}{2}B_3\right) + \frac{c}{24}\frac{\partial \nu_1'}{\partial y}B_2 - \frac{c}{16}\frac{\partial \nu_2'}{\partial y}B_1;\\
\nu_1 &= \nu_1' + \frac{c}{6}\frac{\partial \nu_0'}{\partial y}\left(\frac{1}{2}B_2 - \frac{1}{5}B_4\right) + \frac{c}{16}\frac{\partial \nu_1'}{\partial y}B_1 - \frac{c}{16}\frac{\partial \nu_3'}{\partial y}B_1;
\end{aligned} \qquad (7.37)$$

(continued)

$$\begin{aligned}
\nu_2 &= \nu_2' - \frac{c}{8}\frac{\partial \nu_0'}{\partial y}(B_1 - B_3) + \frac{c}{8}\frac{\partial \nu_2'}{\partial y}B_1, \\
\nu_3 &= \nu_3' - \frac{c}{12}\frac{\partial \nu_0'}{\partial y}(B_2 - B_4) - \frac{c}{16}\frac{\partial \nu_1'}{\partial y}\left(B_1 - \frac{1}{2}B_3\right) + \\
&\quad + \frac{c}{24}\frac{\partial \nu_2'}{\partial y}B_2 + \frac{c}{8}\frac{\partial \nu_3'}{\partial y}B_1, \\
\nu_4 &= \nu_4' - \frac{c}{16}\frac{\partial \nu_0'}{\partial y}B_3 - \frac{c}{24}\frac{\partial \nu_1'}{\partial y}B_2 - \frac{c}{16}\frac{\partial \nu_2'}{\partial y}B_1, \\
\nu_5 &= \nu_5' - \frac{c}{20}\frac{\partial \nu_0'}{\partial y}B_4 - \frac{c}{32}\frac{\partial \nu_1'}{\partial y}B_3 - \frac{c}{24}\frac{\partial \nu_2'}{\partial y}B_2 - \frac{c}{16}\frac{\partial \nu_3'}{\partial y}B_1, \\
\nu_n &= \nu_n', \quad n \geqq 6.
\end{aligned} \tag{7.37}$$

Expressions (7.36) and (7.37) are only used where the coefficients μ_n and ν_n appear in the first order terms. In the second order terms, we only write

$$\mu_n = \mu_n', \quad \nu_n = \nu_n'. \tag{7.38}$$

It should also be realized that the coefficients B_n obtained by solving system (7.18), must satisfy condition (7.2). Therefore, everywhere in system (7.18), we set:

$$B_0 = \sum_{m=1}^{\left[\frac{\lambda}{2}\right]} \frac{B_{2m}}{4m^2 - 1} \tag{7.39}$$

It follows from this that the coefficient g_0, when selecting the circulation distribution, is not arbitrary, but instead is a result of solving system (7.18).

After this preparation, we can begin with a solution of systems (7.15), (7.18), (7.19). From the calculated coefficients A_n, B_n, and C_n, we will then find the skeleton form using equation (7.1) and (7.3), and after this the thickness distribution from equations (7.5) and (7.6). The total circulation and velocity distribution on the profile contour are calculated from equations (7.24) - (7.30), similar to the solution of the even problem.

As already mentioned earlier, the velocity distribution on the profile contour is decisive for solving the odd problem. Therefore, we will at least give an approximate expression for the contour velocity, corresponding to the selected vortex and source distribution. This is obtained from the expressions given above, by only considering the first-order terms. In a region outside the rounded trailing edges, /659 the contour velocity is

$$w_K = U_0 \left\{ 1 + \sum_{n=0}^{N} \mu'_n \cos n\vartheta + q_0 - g_0 + 2(q_0 + \bar{g}_0) \cos\vartheta - \sum_{n=2}^{M} q_n \cos n\vartheta + \right. \\ \left. + g_0 \operatorname{cotg} \frac{1}{2}\vartheta + \sum_{n=1}^{N} g_n \sin n\vartheta \right\}. \qquad (7.40)$$

The leading edge velocity is

$$W_K = \frac{g_0}{q_0}. \qquad (7.41)$$

The coefficient g_o is given by

$$g_0 = \nu'_0 - \sum_{m=1}^{\left[\frac{N}{2}\right]} \frac{g_{2m} + \nu'_{2m}}{4m^2 - 1} \qquad (7.42)$$

It is advantageous to estimate the velocity distribution on the profile contour using the formulas (7.40) - (7.42), in every case, before beginning the calculation, so as to exclude those circulation and source distributions which lead to profiles which have unfavorable aerodynamic characteristics. On the other hand, it is possible to use formulas (7.40) - (7.42) for selecting the coefficients g_n and q_n, at least in the first approximation when calculating the profile from the described velocity distribution on its contour.

8. Examples

The usefulness of the theory discussed in this paper will now be illustrated with three practical examples.

8.1 Symmetric profile in a Homogeneous Flow Field

As a first example, we will consider the computation for a modified NACA profile. The profile shape (Figure 11) is given by the coordinate table (Table 1).

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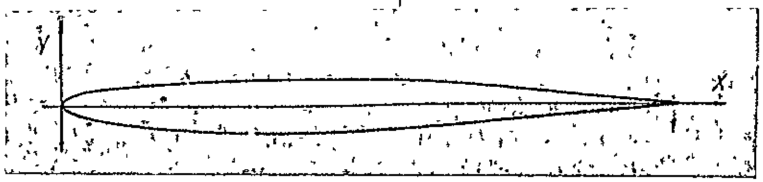

Figure 11

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Table 1

X = nondimensional leading edge distance

Y = nondimensional profile thickness

X	$10^2 \cdot Y$	X	$10^2 \cdot Y$	X	$10^2 \cdot Y$
0	0	0,250	4,040	0,751	2,401
0,007	0,854	0,329	4,278	0,822	1,739
0,030	1,671	0,414	4,309	0,884	1,177
0,067	2,422	0,501	4,108	0,933	0,749
0,117	3,082	0,588	3,683	0,970	0,438
0,177	3,630	0,672	3,084	1,000	0

Using harmonic analysis, we determine the coefficients A_n of the thickness distribution for this profile (see equation 7.21).

$$A_1 = 0{,}0753\,; \quad A_2 = 0{,}0192\,; \quad A_3 = -0{,}0063\,;$$

$$A_4 = 0{,}0002\,; \quad A_5 = 0{,}0008\,; \quad A_6 = -0{,}0002\,;$$

$$\sum_{n=1}^{6} nA_n = 0{,}0987\,; \quad \sum_{n=1}^{6} (-1)^n nA_n = -0{,}0226.$$

Since we have a symmetric profile, all of the coefficients $\bar{B}_n$ (see equation (7.20) are equal to 0. According to equation (7.19), the coefficients C_n are:

$$C_2 = -0{,}017\; g_0 - 0{,}041\; g_1 + 0{,}038\; g_3,$$

$$C_3 = -0{,}006\; g_0 - 0{,}010\; g_1 - 0{,}038\; g_2,$$

$$C_4 = 0{,}003\; g_1 - 0{,}010\; g_2 - 0{,}038\; g_3.$$

The velocities are referred to the x component of the homogeneous primary velocity field so that according to equation (7.12),

$$u = U_\infty, \quad v = U_\infty \operatorname{tg} \alpha_\infty$$

where α_∞ is the angle of attack.

We will now introduce the angle of attack parameter $K = tg\, \alpha_\infty$ for the rest of the calculation. All of the coefficients μ_n and ν_n in the expansions (7.12) are zero, with the exception of ν_o, for which we have:

$$\nu_0 = K.$$

These values are then substituted in equations (7.15)-(7.18), and after this they are solved for the desired vortex and source distribution coefficients /661

$$g_0 = 1{,}093\ K;\ g_1 = -0{,}040\ K;\ g_2 = 0{,}047\ K;\ g_3 = 0{,}002\ K;$$
$$q_0 = 0{,}0544;\ q_1 = -0{,}0107;\ q_2 = 0{,}0344;\ q_3 = 0{,}0075;$$
$$q_4 = -0{,}0083;\ q_5 = 0{,}0021;\ q_6 = -0{,}0004.$$

According to (7.24), the total circulation is

$$\Gamma/cU_\infty = 3{,}371\ K.$$

The velocity distribution on the profile contour is obtained by substitution in equations (7.25) - (7.33), and is shown in Figure 12 for the angles of attack $\alpha_\infty = 0$ and two degrees. The velocity distribution

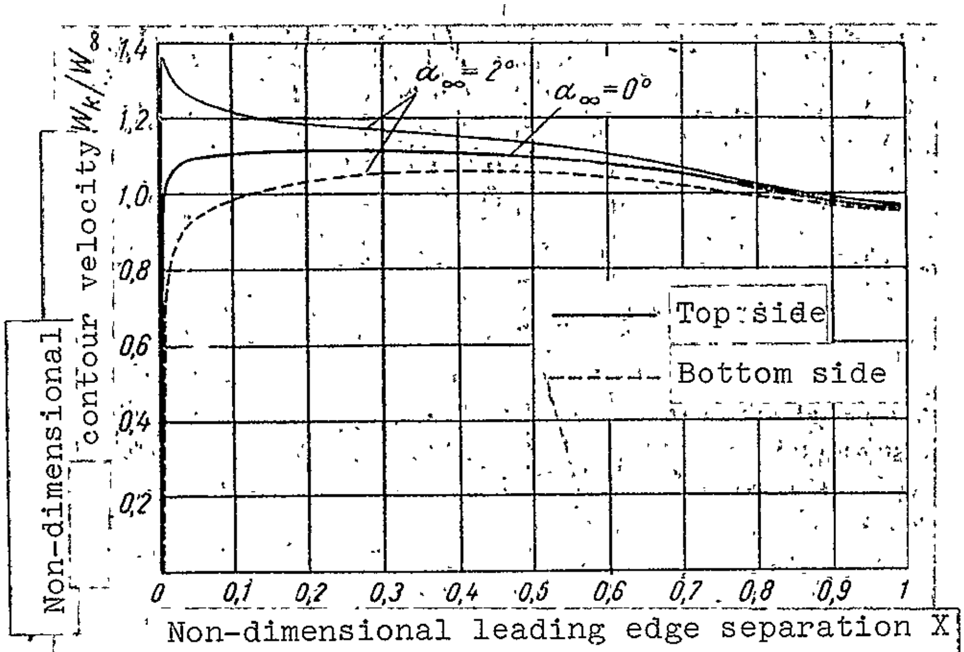

Figure 12

is referred to the total velocity of the primary flow field $W_\infty = U_\infty(1-k^2)^{\frac{1}{2}}$

8.2 Curved Profile in a Homogeneous Flow Field

As a second example, we will now consider a profile with curvature in a flow, which was produced by plotting a symmetric profile mentioned above on a circular arc profile center line. The central angle of the arc shaped center line is $\omega = 29°32'$.

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For circular arc center lines, the coefficients in expansion (7.20) are given by the following expressions:

$$B_1 = \sin\frac{\omega}{2} + \frac{3}{8}\sin^3\frac{\omega}{2} + \frac{15}{16}\sin^5\frac{\omega}{2} + \ldots$$
$$B_3 = \frac{1}{8}\sin^3\frac{\omega}{2} + \frac{15}{128}\sin^5\frac{\omega}{2} + \ldots$$
$$B_5 = \frac{3}{128}\sin^5\frac{\omega}{2} + \ldots$$
$$B_{2n} = 0.$$

In the example given, only two coefficients B_n are different from zero:

$B_1 = 0.2613$ $\qquad$ $B_3 = 0.0022$

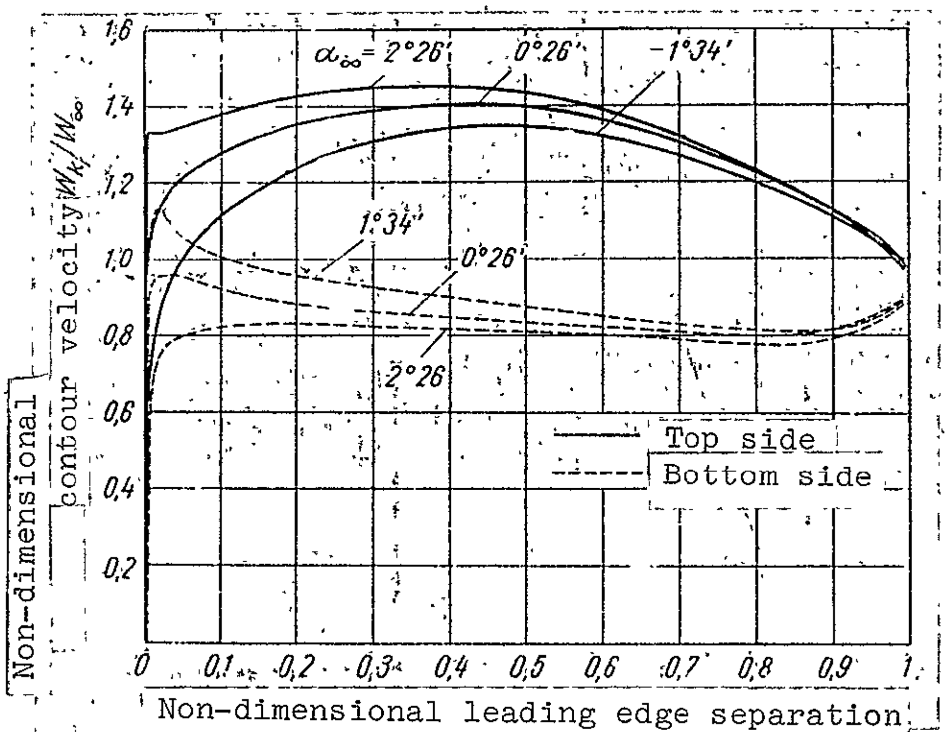

Figure 13

After this, the calculation is completely similar to the previous example, and we will only give the results here.

The total circulation is

$$\frac{\Gamma}{cU_\infty} = 0,4590 + 3,371\ K$$

/6

and the velocity distribution on the profile contour is shown in Figure 13, for $\alpha_\infty = 0°26'$, which corresponds to an entry without shocks. We also show it for $\alpha_\infty = -1°34'$ and $2°26'$.

8.3 Curve Profile in Cascade Configuration

Finally, we will calculate the flow around a profile from a previous example, with a cascade configuration (Figure 14). The division ratio

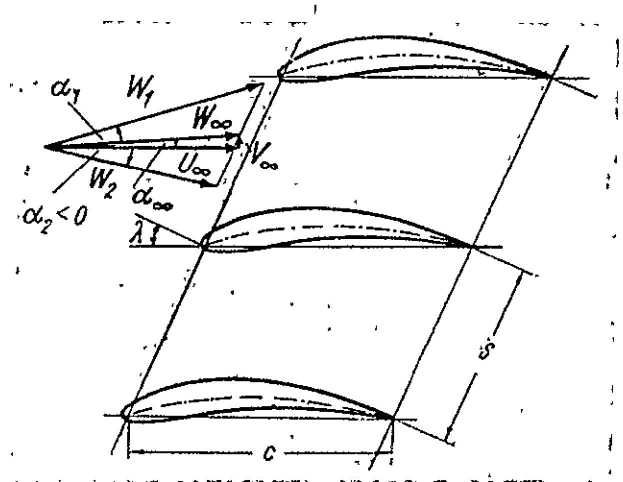

Figure 14

(s/c) and the separation angle (λ) of this cascade are given by

$$s/c = 0.904 \quad \lambda = 34°30'$$

The flow around the profile in a cascade configuration is considered as the flow around an isolated profile in a nonhomogeneous primary flow field [7]. The inhomogeneity of the primary flow (induced velocities) is caused by the other cascade profiles.

The x component ($\bar{U}_\infty$) of the translation velocity (W_∞) is selected as the reference velocity. The translation velocity is defined as the vector average of the incident and departing flow velocities (W_1, W_2). The coefficients μ_n and ν_n in the expansions of equation (7.12) are given by the following expressions (see [8]):

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$$\mu_0 = -0{,}325\, g_0 - 0{,}168\, g_2 + 0{,}006\, g_4 + 0{,}235\, q_0 - 0{,}235\, \bar{q}_0 + 0{,}120\, q_2,$$
$$\mu_1 = 0{,}650\, g_0 + 0{,}368\, g_1 - 0{,}046\, g_3 + 0{,}009\, q_0 - 0{,}009\, \bar{q}_0 - 0{,}007\, q_3,$$
$$\mu_2 = 0{,}085\, g_0 + 0{,}047\, g_2 - 0{,}004\, g_4 + 0{,}009\, q_0 - 0{,}009\, \bar{q}_0 + 0{,}011\, q_2,$$
$$\mu_3 = -0{,}021\, g_0 - 0{,}015\, g_1 + 0{,}003\, g_3 + 0{,}014\, q_0 + 0{,}014\, \bar{q}_0 + 0{,}008\, q_3;$$ /664
$$\mu_4 = -0{,}006\, g_0 - 0{,}002\, g_2 - 0{,}005\, q_0 + 0{,}005\, \bar{q}_0 - 0{,}004\, q_2;$$
$$\mu_5 = 0{,}002\, g_0 + 0{,}001\, g_1,$$
$$\nu_0 = K - 0{,}235\, g_0 - 0{,}120\, g_2 + 0{,}003\, g_4 - 0{,}325\, q_0 + 0{,}325\, \bar{q}_0 - 0{,}168\, q_2,$$
$$\nu_1 = 0{,}470\, g_0 + 0{,}230\, g_1 + 0{,}007\, g_3 - 0{,}085\, q_0 - 0{,}085\, \bar{q}_0 - 0{,}046\, q_3,$$
$$\nu_2 = -0{,}009\, g_0 - 0{,}013\, g_2 + 0{,}008\, g_4 + 0{,}085\, q_0 - 0{,}085\, \bar{q}_0 + 0{,}047\, q_2,$$
$$\nu_3 = -0{,}009\, g_0 + 0{,}002\, g_1 - 0{,}008\, g_3 + 0{,}009\, q_0 + 0{,}009\, \bar{q}_0;$$
$$\nu_4 = 0{,}005\, g_0 + 0{,}004\, g_2 - 0{,}006\, q_0 + 0{,}006\, \bar{q}_0 - 0{,}002\, q_2,$$
$$\nu_5 = 0{,}001\, g_0 - 0{,}001\, g_1.$$

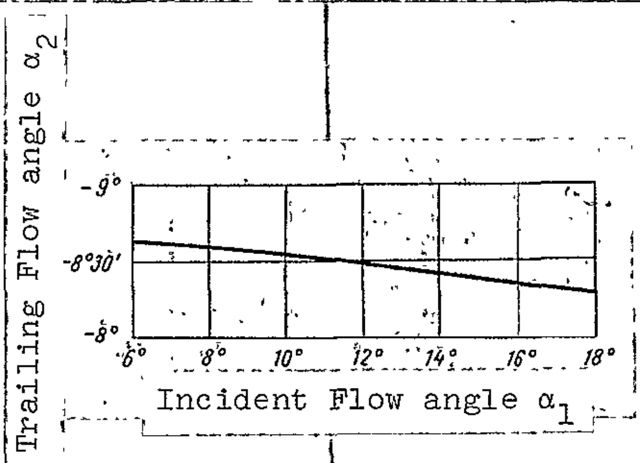

Figure 15

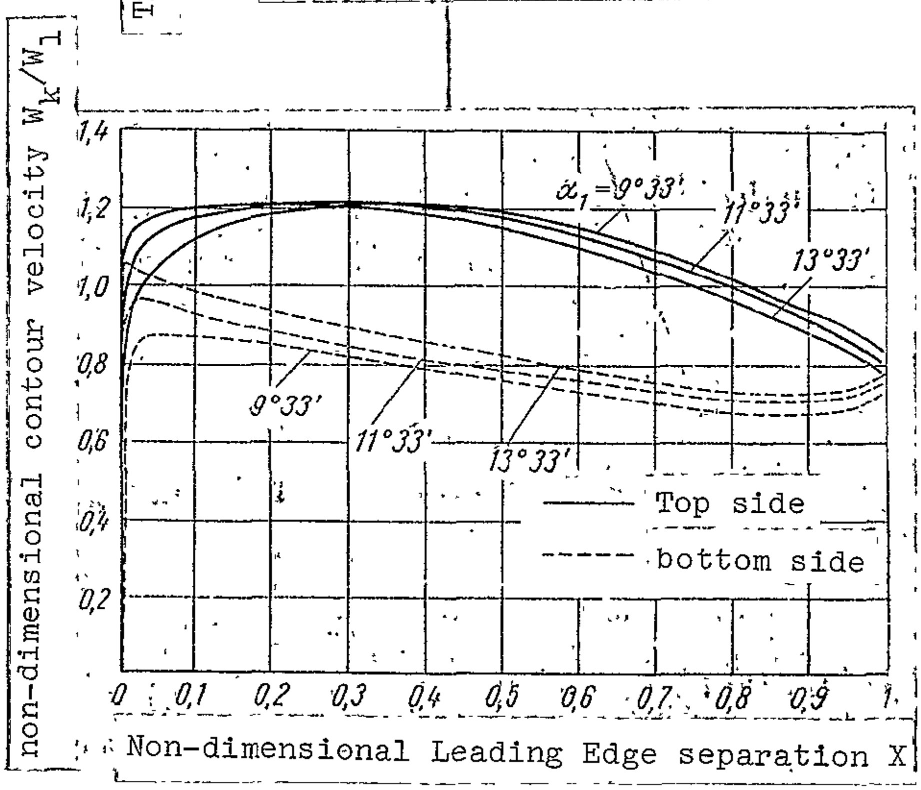

Figure 16

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The calculation is completely similar after this. Of the numerical results, we will only give the departing flow angle α_2 (Figure 15) and the velocity distribution on the profile contour. The velocity distribution is shown in Figure 16 for entry without shocks ($\alpha_1 = 11°33'$) and also for $\alpha_1 = 9°33'$ and $13°33'$. Figure 16 shows the velocity distribution referred to the incident flow velocity.

CONCLUSION

This paper contains a theory of a moderately curved profile in a nonhomogeneous flow field. The theory is built up as a "second order theory". The coefficients in the expansions for the circulation source and thickness distributions, as well as the coefficients in the expansion of the skeleton shape, are considered as small "of first order". All third-order quantities are ignored in the relationships.

The main results of the paper are adapted to the requirements for numerical calculations in the seventh chapter. Without knowing the theory in detail, it can be used. This circumstance is very important, because the derivation of the theory requires a knowledge of functions of a complex variable, and conformal mapping. In a numerical calculation, all that is required is the knowledge of expansions of trigonometric series (harmonic analysis) and the solution of systems of linear inhomogeneous equations. This assumption can be assumed to be satisfied for any technical employee. The systems of linear equations have a very convenient form for numerical calculation. The largest quantities are always located along the main diagonal. This means that we can recommend a step-wise approximation method for the solution, and in this case it also leads to a rapid solution, even for several unknowns.

In practice, we often encounter this type of flow, and this theory was formulated in general terms without direct adaptation to special cases. We can consider the following applications: flow around aerodynamic profiles in a homogeneous flow, flow around profiles near the earth, flow around straight blade cascades, flow around radial blade cascades, flow around guide blades, etc. In all these cases, we can

transform this theory into calculation formulas which will consider the specific properties of the individual applications. This results in a further simplification of the calculations. Of the technical applications, the calculation of the flow around straight and radial blade cascades is probably the most important. Consequently, the results of /666
this paper and the results of papers [7] and [8] were formulated into a calculation procedure for calculating the flow around blade cascades with small thicknesses and moderately curved blades. This has been done for incompressible and compressible subsonic flow, as already discussed in example (8.3).

The formulas for the numerical calculation discussed in chapter 7 can be performed with an electric calculator or a slide rule. They are very well suited for programming of automatic computer installations. The time requirement for a complete calculation of a profile or a blade cascade is reduced to a minimum. In Chapter 8, we discussed examples which have already been calculated on the ZUSE-23 computer.

The results can be generalized to profiles with a high curvature, but with a rather small thickness, as has been shown in paper [6] for the flow around a thin, highly-curved profile in a nonhomogeneous flow field. These generalizations were not included in this paper, because moderately curved profiles are very important in technical applications. Also inclusion of this general case would have made this paper much more cumbersome.

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