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SPECKLE NOISE IN SATELLITE BASED LIDAR SYSTEMS

by

C. S. Gardner

RRL Publication No. 488

Final Report
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I. INTRODUCTION

In recent years laser radar (lidar) systems have developed into useful tools for remote monitoring of the earth's atmosphere. Satellite-borne lidar systems provide the attractive feature of wide coverage and in some cases may be the only practical means of probing the upper atmosphere. The quality of lidar data depends of course on the system noise characteristics. The receiver output will be contaminated by the usual background and shot noise components and in some cases by speckle noise. In certain situations speckle noise dominates and can seriously limit system performance.

Laser speckle has been studied extensively. Goodman[1] published one of the earliest papers describing speckle effects on optical radar performance and recently an entire issue of the Journal of the Optical Society[2] was devoted to speckle. In this report we develop and summarize the equations which relate the statistics of the speckle noise at the receiver output to the lidar system parameters. Fortunately, we were able to adapt much of the existing theory to the satellite-based lidar problem. Much of the material was obtained from the books Laser Speckle and Related Phenomena edited by J. C. Dainty[3] and Statistical Properties of Scattered Light by Crosignani, Di Porto and Bertolotti[4] and from the November 1976 special issue of JOSA on speckle.

The lidar system model is described in Section II, and in Section III the statistics of the signal and noise at the receiver output are derived. Scattering media effects are discussed in Sections IV and V. Polarization and atmospheric turbulence are considered in Sections VI and VII. And finally, in Section VIII, the major equations are summarized and evaluated for some typical system parameters.

II. SYSTEM MODEL

A typical satellite-based lidar system is illustrated in Figure 1. The transmitting telescope is used to project an image of the optical source onto the scattering medium. In most cases the source would be a narrowband laser. However, since speckle noise is significantly influenced by source temporal coherence, we will assume only that the source coherence bandwidth is a small percentage of the center wavelength. Although a major portion of the propagation path will be in free space, in the lower atmosphere turbulence effects could become important and will be considered. The scattering medium could be a rough surface such as the earth or an ensemble of scattering centers such as aerosols (clouds) and air molecules.

The receiving telescope model is illustrated in Figure 2. The spatial filter following the objective lens is normally a field-of-view (FOV) stop which is used to reject background radiation. After the field stop the optical signal is collimated, passed through an interference filter and then focused onto a photodetector. Finally, the signal from the photodetector is passed through an electrical filter to further limit the system bandwidth.

The problem we are concerned with is the detectability of the scattered signal. The receiver output will be contaminated by the usual background and shot noise components and in some cases by speckle noise. In certain situations speckle noise dominates and can seriously limit system performance. In the following sections we will present the equations which relate the statistics of the photodetector output to the lidar system parameters and the characteristics of the scattering medium.

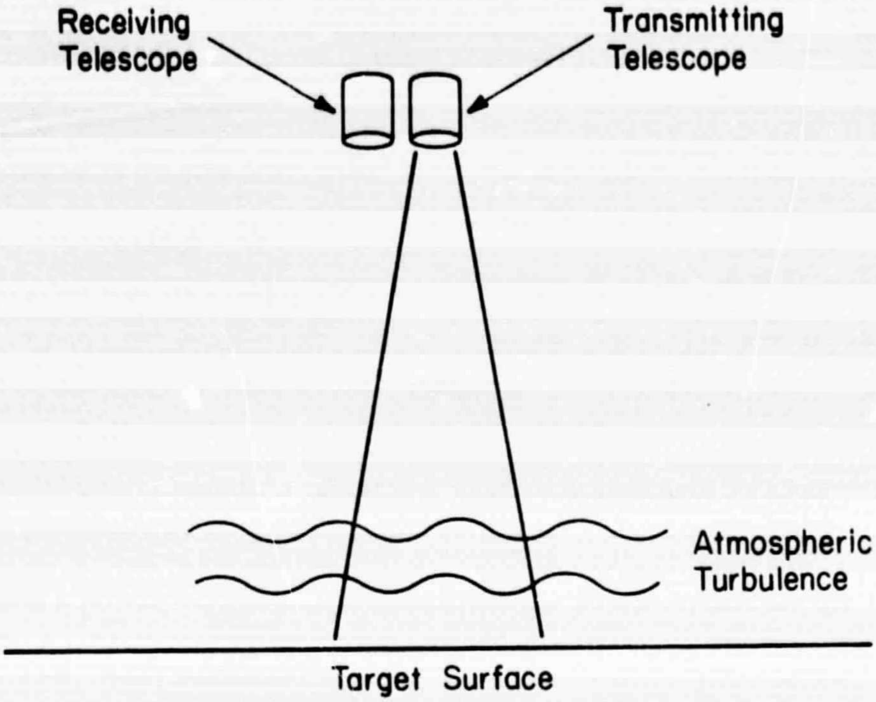

Fig. 1. Lidar Geometry

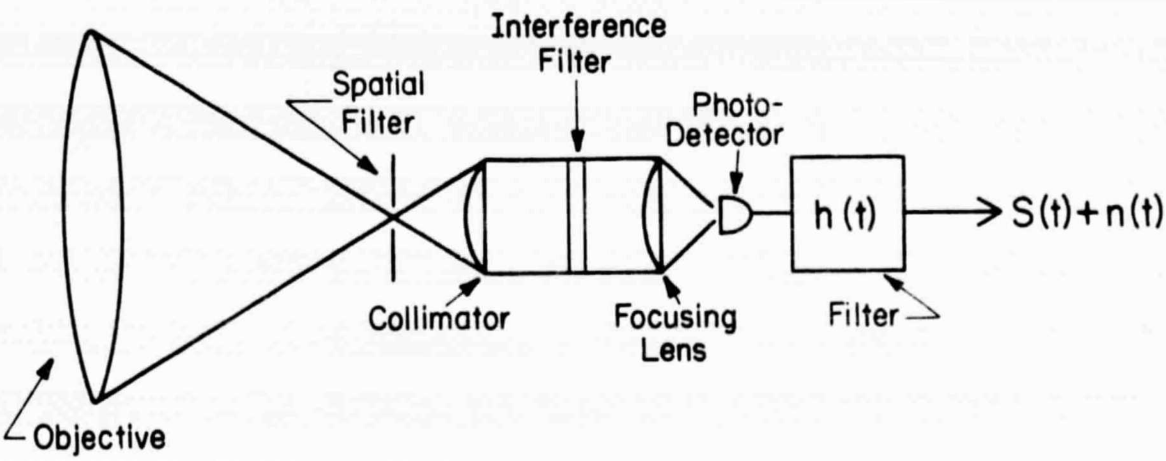

Fig. 2. Receiving System Model

III. STATISTICS OF THE RECEIVED SIGNAL

The signal statistics at the receiver output can be deduced from the statistics of the optical radiation incident on the receiver aperture. Let $u(\underline{r},z,t)$ be the analytic signal representation for a single polarization component of the electrical field. For a quasi-monochromatic field u takes the form

$$u(\underline{r},z,t) = |A(\underline{r},z,t)|e^{i\theta}e^{i\omega_c t} \tag{1}$$

where θ is the phase and ω_c is the center frequency. For the moment we will assume the scattering medium is diffuse so that the received field consists of a large number of randomly phased contributions arising from different scattering centers. If the scatterers are randomly distributed over distances which are large compared to the optical wavelength, the scattered signal will be incoherent. Under these conditions A is Gaussian distributed while the intensity obeys negative exponential statistics[5]

$$I(\underline{r},z,t) = |A|^2 \tag{2}$$

$$p(I) = \frac{1}{\langle I\rangle} \exp(-\frac{I}{\langle I\rangle}) \tag{3}$$

where $\langle I\rangle$ is the average intensity.

The scattering conditions can be relaxed to include the cases where the surface is not rough compared to λ and where the scattered field contains a coherent specular component. These situations are discussed in Section IV.

To calculate the signal statistics at the receiver output, it will be necessary to evaluate the mean and covariance function of the intensity fluctuations

$$C_I(\underline{r}_1,t_1,\underline{r}_2,t_2,z) = \langle I(\underline{r}_1,z,t_1)I(\underline{r}_2,z,t_2)\rangle - \langle I(\underline{r}_1,z,t_1)\rangle\langle I(\underline{r}_2,z,t_2)\rangle \quad . \tag{4}$$

Since the field A is a circular complex Gaussian random variable, the covariance function can be expressed in terms of the mutual intensity of the field[5]

$$C_I = |\langle A(\underline{r}_1,z,t_1)A^*(\underline{r}_2,z,t_2)\rangle|^2 = |J_A(\underline{r}_1,t_1,\underline{r}_2,t_2,z)|^2 \quad . \tag{5}$$

The mutual intensity function depends on the characteristics of the scattering medium and the incident optical signal. In Sections IV and V we show that for most cases of interest it can be assumed that A is spatially homogeneous in the transverse direction. The temporal dependence of A arises from a number of factors. Movement of the scattering centers with respect to the receiving telescope caused by Brownian motion, turbulence, wind, or spacecraft motion will introduce temporal fluctuations. Those effects are generally stationary. However, if the optical source is pulsed and the density of scatterers varies with distance (z), the average value of the received intensity will change with time. This effect can be included in our analysis by writing the mutual intensity function in the following form (see Section V)

$$J_A(\underline{r}_1,t_1,\underline{r}_2,t_2,z) = D(t_1,t_2)\tilde{J}_A(\underline{r}_1 - \underline{r}_2,t_1 - t_2) \quad . \tag{6}$$

$D(t_1,t_2)$ is given by

$$D(t_1,t_2) = \int_0^\infty dz P(t_1 - \frac{2z}{c})P(t_2 - \frac{2z}{c})\frac{\rho(z)}{z^2} \tag{7}$$

where P is the incident pulse shape and ρ is the number density of scatterers. If the scattering medium is a rough surface, ρ is a Dirac delta function and

$D(t_1,t_2)$ becomes

$$D(t_1,t_2) = P(t_1 - \frac{2z}{c}) \, P(t_2 - \frac{2z}{c}) \, \frac{\beta_r^2}{z^2} \tag{8}$$

where β_r is the surface reflectivity. Using Equations (6) and (7), the intensity covariance function can be written as

$$C_I = D^2(t_1,t_2) |\tilde{J}_A(\underline{r}_1 - \underline{r}_2, \, t_1 - t_2)|^2 \quad . \tag{9}$$

The average intensity is obtained by simply evaluating the mutual intensity function at the point $\underline{r}_1 = \underline{r}_2$ and $t_1 = t_2$

$$\langle I \rangle = D(t,t) \tilde{J}_A(0,0) \quad . \tag{10}$$

We assume the receiver FOV and interference filter are adjusted to admit all the scattered signal energy which is incident on the telescope objective. The total received signal intensity can therefore be written as

$$I_s(t) = \int d^2\underline{r} W(\underline{r} - \underline{v}t) I(\underline{r},z,t) \, / \int d^2\underline{r} W(\underline{r}) \tag{11}$$

where W is an appropriate real and positive aperture weighting function and $\underline{v}$ is the transverse spacecraft velocity. W is dimensionless. Note for a point detector W is just a Dirac delta function.

Two types of receiver structures will be considered. The analog receiver is applicable to strong signal conditions where h(t) is a low-pass filter (see Figure 2). When the signal is weak, h is an integrator whose output is proportional to the number of detected photons. In both cases the noise at the receiver output will depend on the statistics of I_s.

The mean and variance of the signal at the receiver output can be calculated using Campbell's theorem[6]

$$E[s(t)] = \alpha \langle I_s(t) \rangle * h(t) \tag{12}$$

$$\mathrm{Var}[s(t)] = \alpha \langle I_s(t) \rangle * h^2(t)$$
$$+ \alpha^2 \int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\infty} d\tau_2 C_{I_s}(\tau_1, \tau_2) h(t - \tau_1) h(t - \tau_2) \tag{13}$$

where α is a constant which depends on the photodetector efficiency and C_{I_s} is the temporal covariance function of the received signal.

The variance of S can be regarded as the signal induced noise. The first term in (13) is the quantum or shot noise component arising from the random emission of photoelectrons by the detector. The second term arises from the speckle induced fluctuations in I_s.

The average value of I_s is calculated from (11) using (10)

$$\langle I_s(t) \rangle = D(t,t) \tilde{J}_A(0,0) \quad . \tag{14}$$

The temporal covariance of I_s can be written in terms of $\tilde{J}_A$ and D

$$C_{I_s}(t_1, t_2) = D^2(t_1, t_2) \int d^2\underline{r} |\tilde{J}_A(\underline{r} + \underline{v}\tau, \tau)|^2 R_w(\underline{r}) \tag{15}$$
$$\tau = t_1 - t_2 \quad .$$

R_w is the autocorrelation function associated with the receiver aperture weighting function

$$R_w(\underline{r}) = \int d^2\underline{r} W(\underline{\rho}) W^*(\underline{\rho} + \underline{r}) \quad . \tag{16}$$

The average value of S(t) can now be evaluated by substituting (14) into (12) and using (7)

$$E[S(t)] = \alpha \tilde{J}_A(0,0,z) \int_0^{\infty} dz P^2 \left(t - \frac{2z}{c}\right) * h(t) \frac{\rho(z)}{z^2} \quad . \tag{17}$$

This expression can be further simplified by assuming the transmitter pulse width and receiver bandwidth are chosen so that the small scale structure in $\rho(z)$ can be resolved. In this case $\rho(z)/z^2$ is approximately constant over the important range of integration in (17). By choosing the time reference so that P and h peak at t = 0 we have

$$E[S(t)] = \alpha\tilde{J}_A(0,0,z)\,\frac{\rho(ct/2)}{(ct/2)^2}\int_0^\infty dzP^2\left(t - \frac{2z}{c}\right)*h(t) \quad . \tag{18}$$

Under the same conditions the expression for the variance becomes

$$Var[s(t)] = \alpha\tilde{J}_A(0,0,z)\,\frac{\rho(ct/2)}{(ct/2)^2}\int_0^\infty dzP^2\left(t - \frac{2z}{c}\right)*h^2(t)$$

$$+ \alpha^2\,\frac{c^2}{4}\,\frac{\rho^2(ct/2)}{(ct/2)^4}\int d^2\underline{r}\int_{\infty}^{\infty} d\tau\left|\tilde{J}_A(\underline{r} + \underline{v}\tau,\tau)\right|^2 R_p^2(\tau)R_h(\tau)R_w(\underline{r}) \tag{19}$$

where

$$R_p(\tau) = \int_{-\infty}^{\infty} dtP(t)P(t + \tau)$$

$$R_h(\tau) = \int_{-\infty}^{\infty} d\tau h(t)h(t + \tau) \quad .$$

R_p and R_h are the autocorrelation functions associated with the transmitted pulse shape and impulse response of the receiver filter.

It is instructive to consider the physical implications of (19). If the receiver employs a point detector of infinite bandwidth, both R_w and R_h are Dirac delta functions, in which case the second term in (19) reduces to the square of the average intensity. For a finite aperture and finite bandwidth receiver, R_w and R_h are rather broad functions of $\underline{r}$ and τ with peak values at $\underline{r} = \tau = 0$. Since $\tilde{J}_A$ and R_p are also maximum when their arguments are zero, (19) will be less than the average intensity when a finite aperture and finite bandwidth receiver is used. In this case the receiver spatially and temporally averages the intensity fluctuations. The averaging will depend upon the relationship between the spatial and temporal coherence lengths of the received signal and the receiver aperture size and bandwidth. The averaging will also depend upon the transmitted pulse length through the R_p^2 factor in (19). At any given time t, the signal energy incident on the receiver aperture arises from an illuminated scattering volume whose thickness is determined by the transmitter pulse length. The scattered signals arising from each of the illuminated scattering centers add on an amplitude basis. Consequently, the spatial statistics of the intensity are unaffected by the pulse length (aside from a possible scaling factor). However, since the signals scattered by different centers are uncorrelated, the received intensities at two different times will be partially correlated only if the corresponding scattering volumes have some points in common. Overlapping will occur only if the time difference is smaller than the transmitted pulse length. The intensities will be uncorrelated if the time difference is larger than the pulse length.

The temporal statistics are also affected by the source coherence, random motion of the scattering centers and spacecraft motion. These effects appear in the $\mathcal{J}_A$ factor in (19). The spatial statistics are determined by the transmitter beam divergence, characteristics of the scattering medium and propagation effects such as turbulence. These factors are discussed in more detail in Sections IV and V.

The relative importance of spatial and temporal coherence length, pulse length, aperture size and receiving bandwidth can be determined by evaluating (18) and (19) for the idealized case where all the functions are Gaussian

$$|\mathcal{J}_A(r,t)|^2 = |\mathcal{J}_A(0,0)|^2 \exp\left(-\frac{r^2}{2\rho_c^2} - \frac{t^2}{2\tau_c^2}\right) \tag{20}$$

$$P(t) = \exp\left(-\frac{t^2}{2\tau_p^2}\right) \tag{21}$$

$$h(t) = \exp(-t^2 B^2) \tag{22}$$

$$W(\underline{r}) = \exp(- \frac{r^2}{R^2}) \tag{23}$$

where

ρ_c = transverse spatial coherence length of signal

τ_c = temporal coherence length of signal

τ_ρ = transmitter pulse length

B = detector bandwidth

R = receiver aperture radius.

Using (20) through (23) in (17), (18) and (19) gives

$$E[S(t)] = \alpha \tilde{J}_A(0,0) \frac{\rho(ct/2)}{(ct/2)^2} \frac{\pi c \tau_p}{2B} \tag{24}$$

$$\mathrm{Var}[S(t)] = \frac{E(S)}{\sqrt{2}} + \frac{E(S)^2}{M_S M_T} \tag{25}$$

where

$$M_S = 1 + \frac{R^2}{\rho_c^2} \tag{26}$$

$$M_T = \left[1 + \frac{1}{B^2 \tau_c^2} + \frac{1}{B^2 \tau_p^2} + \frac{v^2}{B^2 (R^2 + \rho_c^2)}\right]^{1/2} \quad . \tag{27}$$

The equations are valid for scattering by tenuous media such as low density aerosol distributions and air molecules. For rough surface scattering the

$$\frac{\rho(ct/2)}{(ct/2)^2}$$

factor must be replaced by

$$\frac{\beta_r^2}{z^2} \quad .$$

in (24) and τ_p set equal to infinity in the expression for M_T.

The average signal varies with time in proportion to the density of scatterers. The $(ct/2)^{-2}$ factor accounts for decay in signal strength as the distance between the spacecraft and illuminated scattering region increases with time. The first term in the variance expression is the shot noise component. It is proportional to the mean signal. The speckle noise component is proportional to the square of the mean signal and inversely proportional to $M_S M_T$, the effective number of received correlation cells. M_S is the effective number of spatial correlation cells averaged by the receiver aperture while M_T is the effective number of temporal correlation cells.

To reduce speckle noise we would like to make M_S and M_T as large as possible. The parameters R, B and V are generally inflexible. The aperture size R is constrained by weight limitations and the physical construction of the spacecraft. The detector bandwidth B is determined by the required range resolution. And of course the spacecraft velocity is a function of the orbit. Thus the only parameters which may be adjusted are ρ_c, τ_c and τ_p.

The coherence time of the scattered radiation depends on the scattering medium and source coherence. If the scattering medium is a rough surface then τ_c is exactly equal to the source coherence time. If the scattering mechanism is resonant absorption and reradiation, then τ_c is equal to the inverse Doppler line width. For air molecules dominated by Brownian motion, τ_c is on the order of 1 ns. If Rayleigh scattering from air molecules or aerosol and particulate scattering is involved, then $\tau_c = (\tau_d^{-2} + \tau_c^{-2})^{-1/2}$ where τ_{sc} is the source coherence time and $1/\tau_d$ is the Doppler line width. τ_d is on the order of 1 ns for air molecules governed by Brownian motion and

1 ms for aerosols or particles embedded in a turbulent flow (e.g., clouds and smoke plumes). Therefore, except for resonant scattering, τ_c can be controlled by varying the source coherence time.

M_T can also be increased by reducing the source pulse length τ_p. But, there is a practical limit to the reduction. To keep the per pulse signal energy constant, the peak intensity must be increased as τ_p is decreased. However, when resonant scattering is involved, the peak energy density in the scattering medium must be maintained below saturation levels. This problem is most important when the density of scatterers is low.

In the absence of strong turbulence effects ρ_c is a function of the size of the illuminated spot on the scattering medium and the distance from the scatterers to the receiver. If the whole transmitted beam illuminates the s ..tering medium we have (see Section IV)

$$\rho_c \simeq \frac{\lambda}{\theta_T} \tag{28}$$

where λ is the wavelength and θ_T is the transmitter divergence angle. If the transmitter is operating at the diffraction limited divergence, then

$$\theta_T \simeq \frac{\lambda}{R_T} \tag{29}$$

where R_T is the transmitter aperture radius. Usually the transmitter and receiver apertures are nearly equal in size. Thus $M_S \doteq 2$ for diffraction limited operation. The transmitter divergence normally will be much larger than the diffraction limited value to minimize pointing and tracking problems. If the transmitter divergence is increased, the receiver FOV must also be increased so that the receiving telescope sees all of the illuminated scattering volume. Unfortunately, background noise power increases with FOV. If the background noise results from an extended diffuse source, its characteristics will be similar to the scattered signal since both are incoherent and Gaussian.

Uniform background noise sources can be described by their spectral radiance function, $N(f)$, which is defined as the power radiated at frequency f per hertz bandwidth into a unit solid angle per unit of source area.[7] If the source fills the receiver FOV, the effective average noise intensity incident on the receiver aperture is

$$\langle I_n(\underline{r},z,t)\rangle = J_n(0,0) = N(f)B_I\Omega_r \quad . \tag{30}$$

J_n is the noise mutual intensity function, B_I is the bandwidth of the receiver's interference filter and Ω_r is the solid angle FOV of the receiver. Because Ω_r is matched to the transmitter divergence and may change, it is convenient to normalize I_n with respect to a standard FOV such as the diffraction limited value.

$$\langle I_n\rangle = J_{nd\ell}(0,0)\left(\frac{R\theta_T}{\lambda}\right)^2$$

$$J_{nd\ell}(0,0) = N(f)B_I\left(\frac{\lambda}{R}\right)^2 \quad . \tag{31}$$

$J_{nd\ell}$ is just the effective noise intensity for a diffraction limited FOV. The mean and variance of the noise voltage at the receiver output can easily be calculated if we use the form in (20) for the noise intensity correlation.

$$E[n(t)] = E(n_{d\ell})\left(\frac{R\theta_T}{\lambda}\right)^2 = \alpha J_{nd\ell}(0,0)\,\frac{\pi}{B}^{1/2}\left(\frac{R\theta_T}{\lambda}\right)^2 \tag{32}$$

$$\mathrm{Var}[n(t)] = \frac{E(n_{d\ell})}{\sqrt{2}}\left(\frac{R\theta_T}{\lambda}\right)^2 + \frac{E(n_{d\ell})^2}{N_S N_T}\left(\frac{R\theta_T}{\lambda}\right)^4 \tag{33}$$

where

$$N_S = 1 + \left(\frac{R\theta_T}{\lambda}\right)^2 \tag{34}$$

$$N_T = \left[1 + \frac{B_I^2}{B^2} + \frac{v^2}{B^2(R^2 + \lambda^2/\theta_T^2)}\right]^{1/2} \quad . \tag{35}$$

The spatial coherence length of the background noise is determined by the receiver FOV and is equal to λ/θ_T. The temporal coherence length is determined by the interference filter bandwidth and is equal to $1/B_I$.

Background noise contaminates the receiver output by adding a DC component and introducing additional shot noise and speckle noise. Normally the speckle noise component will be small since the interference filter bandwidth is usually much larger than the detector bandwidth. For example, the bandwidth of a 1 Å interference filter centered at 530 nm is nearly 100 GHz compared to detector bandwidths of 1 GHz or less. Thus N_T is on the order of 10^2 or larger.

Although signal induced speckle noise will decrease with increasing θ_T, background shot noise and speckle noise will increase. The optimum transmitter divergence can be determined by minimizing the sum Var(S) + Var(n) with respect to θ_T

$$\theta_T\Big|_{\text{optimum}} \simeq \frac{\lambda}{R}\left[\frac{E(S)^2/M_T}{E(n_{d\ell})/\sqrt{2} + E(n_{d\ell})^2/N_T}\right]^{1/4} \quad . \tag{36}$$

If the optimum transmitter divergence is used, the total noise power is

$$\mathrm{Var}(S) + \mathrm{Var}(n) \simeq \frac{E(S)}{\sqrt{2}} + \frac{2E(S)}{M_T^{1/2}}\left[\frac{E(n_{d\ell})}{\sqrt{2}} + \frac{E(n_{d\ell})^2}{N_T}\right]^{1/2}$$

$$\simeq \frac{E(S)}{\sqrt{2}}\left\{1 + \left[\frac{2^{5/2}E(n_{d\ell})}{M_T}\right]^{1/2}\right\} \quad . \tag{37}$$

$E(S)$ and $E(n_{d\ell})$ may be viewed as the average number of signal and noise photons detected in a time interval of length 1/B. Usually $E(n_{d\ell})$ will be very small since it applies to the diffraction limited receiver. In most cases, therefore, the total noise in (37) is approximately equal to the signal shot noise. However, it may not be practical to achieve the optimum value in (37), particularly if the transmitter divergence given by (36) exceeds the angular size of the target.

Although the mean and variance of the signal plus background noise are useful parameters for accessing the lidar system's performance, a complete analysis requires knowledge of the signal and noise probability distributions. The instantaneous intensity evaluated at a single point is exponentially distributed if the scattered signal and noise are both completely incoherent [see Equation (3)] and the photon counting statistics are Bose-Einstein distributed.[4] Because the receiver integrates the intensity over space and time, the intensity statistics will be modified. Goodman[5] has shown that the gamma distribution is a good approximation for the probability density of the integrated speckle patterns

$$p(I) = \frac{\left(\frac{m}{\langle I \rangle}\right)^{m} I^{m-1} \exp\left(-m \frac{I}{\langle I \rangle}\right)}{\Gamma(m)} \qquad (38)$$

$$I > 0$$

where m is equal to $M_S M_T$ for the scattered signal and $N_S N_T$ for the background noise. The photon counting distribution associated with (38) is the negative binomial distribution. If desired, the exact probability density of integrated speckle can be calculated by expanding the scattered field in a Karhunen-Loevre series.[5]

The signal statistics derived in this section are summarized in Section VIII where they are evaluated for some typical system parameters.

IV. SCATTERING BY A ROUGH SURFACE

In the previous section it was shown that the speckle noise power could be calculated from knowledge of the intensity covariance function of the received signal.

$$C_I = \langle I(\underline{r}_1,z,t_1)\ I(\underline{r}_2,z,t_2)\rangle - \langle I(\underline{r}_1,z,t_1)\rangle\langle I(\underline{r}_2,z,t_2)\rangle \quad . \tag{39}$$

If the surface is rough compared to the optical wavelength, the scattered field is a complex circular Gaussian process. The magnitude of the scattered field is Rayleigh distributed and the intensity is exponentially distributed. In this case the intensity correlation function can be written in terms of the mutual intensity of the field.[5]

$$C_I = \left|\langle A(\underline{r}_1,z,t_1)\ A^*(\underline{r}_2,z,t_2)\rangle\right|^2 = \left|J_A(\underline{r}_1,t_1,\underline{r}_2,t_2,z)\right|^2 \quad . \tag{40}$$

If the surface also possesses a specular scattering region which is too small to be resolved by the receiving telescope, then the scattered field will contain a coherent component. In this case the intensity distribution is a modified Rician density.[5]

$$P(I) = \frac{1}{\langle I_r\rangle} \exp\left(-\frac{I + I_s}{\langle I_r\rangle}\right) I_0\left(2\,\frac{\sqrt{II_s}}{\langle I_r\rangle}\right) \tag{41}$$

$$I \geq 0$$

where I_r is the random intensity component and I_s is a coherent specular component. I_0 is a modified Bessel function of the first kind. When the rms fluctuations of the surface roughness are not large compared to the wavelength, the surface can be decomposed into two identically shaped surfaces which are colocated in space, one of which is very rough and the other of which is very smooth but with a reduced reflectivity.[8] In this case the intensity distribution is also a modified Rician density.

The mutual intensity in the observation plane can be written in terms of the mutual intensity in the scattering plane using the Huygens-Fresnel principle. If the observation plane is in the Fraunhofer zone, we have

$$A(\underline{r},z,t) = \frac{1}{\lambda z} \exp\left(-i \frac{\pi}{\lambda z} r^2\right) \int d^2\underline{\rho} A_s(\underline{\rho},t) \exp\left(i \frac{2\pi}{\lambda z} \underline{\rho} \cdot \underline{r}\right) \tag{42}$$

where A_s is the scattered field directly in front of the scattering plane. In most cases the scattered field can be approximated by the formula[5]

$$A_s(\underline{\rho},t) = \beta_r A_i(\underline{\rho} - \underline{v}t,t) \exp\left[i \frac{2\pi}{\lambda} 2h(\underline{\rho})\right] \quad . \tag{43}$$

β_r is the average surface reflectivity, A_i is the complex amplitude of the incident field, $\underline{v}$ is the transverse velocity of the spacecraft and $h(\underline{\rho})$ is the random height distribution of the scattering surface. Equation (43) is essentially the geometric optics approximation for the scattered field. It is reasonably accurate provided the surface slopes are small. The formula implies that the scattered field is reduced in amplitude and phase shifted with respect to the incident field.

The mutual intensity of the scattered field is calculated by assuming the surface height fluctuations are a zero mean homogeneous Gaussian process

$$\begin{aligned} J_{A_s}(\underline{\rho}_1,\underline{\rho}_2,t_1,t_2) = \beta_r^2 J_{A_i}(\underline{\rho}_1 - \underline{v}t_1,\underline{\rho}_2 - \underline{v}t_2,t_1,t_2) \\ \cdot \exp\left\{-\left(\frac{4\pi}{\lambda}\right)^2 [C_h(0) - C_h(\underline{\rho}_1 - \underline{\rho}_2)]\right\} \end{aligned} \tag{44}$$

where J_{A_i} is the mutual intensity of the incident field and $C_h(\underline{\rho})$ is the surface height correlation function. The mutual intensity in the observation plane can now be calculated using (42) and (44)

$$J_A(\underline{r}_1,\underline{r}_2,t_1,t_2,z) = \frac{\beta_r^2}{\lambda^2 z^2} \int\int d^2\underline{\rho}_1\, d^2\underline{\rho}_2\, J_{A_i}(\underline{\rho}_1 - \underline{v}t_1, \underline{\rho}_2 - \underline{v}t_2, t_1, t_2)$$
$$\cdot \exp\left\{-\left(\frac{4\pi}{\lambda}\right)^2 [C_h(0) - C_h(\underline{\rho}_1 - \underline{\rho}_2)]\right\}$$
$$\cdot \exp\left[i\, \frac{2\pi}{\lambda z}\, (\underline{\rho}_1 \cdot \underline{r}_1 - \underline{\rho}_2 \cdot \underline{r}_2)\right] \quad . \tag{45}$$

Note that in (45) we have neglected some multiplicative phase factors. These factors do not affect the intensity correlation function [Eq. (40)] because it depends only on the magnitude of J_A.

The major contribution to the integral in (45) occurs for $\underline{\rho}_1$ near $\underline{\rho}_2$. If the receiving telescope is unable to resolve an object in the scattering plane whose scale size is on the order of the spatial coherence length of the scattered field, J_A may be approximated by[5]

$$J_A(\underline{r}_1,\underline{r}_2,t_1,t_2,z) = \frac{\beta_r^2}{\lambda^2 z^2} \int d^2\underline{\rho}\, J_{A_i}[\underline{\rho}, \underline{\rho} - \underline{v}(t_1 - t_2), t_1, t_2]$$
$$\cdot \exp\left[i\, \frac{2\pi}{\lambda z}\, \underline{\rho} \cdot (\underline{r}_1 - \underline{r}_2)\right] \quad . \tag{46}$$

Now, the incident field amplitude A_i is related to the transmitter aperture distribution A_T

$$A_i(\underline{\rho},t) = \frac{1}{\lambda z} \exp\left(-i\, \frac{\pi}{\lambda z}\, \rho^2\right) \int d^2\underline{u} A_T(\underline{u},t) \exp\left(i\, \frac{2\pi}{\lambda z}\, \underline{u} \cdot \underline{\rho}\right) \quad . \tag{47}$$

It is convenient at this point to express A_T explicitly in terms of its temporal and spatial dependence.

$$A_T(\underline{u},t) = P\left(t - \frac{2z}{c}\right) T\left(t - \frac{2z}{c}\right) A_T(\underline{u}) \quad . \tag{48}$$

P is the transmitted pulse shape, T is a random time function which is related to the source temporal coherence, and A_T is the spatial field distribution in the transmitter aperture. Using (48) and (47) we have

$$J_{A_i}(\underline{\rho}_1,\underline{\rho}_2,t_1,t_2) = P\left(t_1 - \frac{2z}{c}\right) P\left(t_2 - \frac{2z}{c}\right) C_T(t_1 - t_2)$$
$$\cdot \frac{1}{\lambda^2 z^2} \exp\left[-i \frac{\pi}{\lambda z}\left(\rho_1^2 - \rho_2^2\right)\right] \iint d^2\underline{u}_1\, d^2\underline{u}_2\, A_T(\underline{u}_1)\, A_T^*(\underline{u}_2)$$
$$\cdot \exp\left[i \frac{2\pi}{\lambda z}(\underline{u}_1 \cdot \underline{\rho}_1 - \underline{u}_2 \cdot \underline{\rho}_2)\right] \tag{49}$$

where C_T is the source temporal coherence function. Substituting (49) into (46) and integrating over $\underline{\rho}$ gives

$$J_A(\underline{r}_1,\underline{r}_2,t_1,t_2,z) = D(t_1,t_2)\, \tilde{J}_A(\underline{r}_1 - \underline{r}_2, t_1 - t_2) \tag{50}$$

$$D(t_1,t_2) = \frac{\beta_r^2 P\left(t_1 - \frac{2z}{c}\right) P\left(t_2 - \frac{2z}{c}\right)}{z^2} \tag{51}$$

$$J_A(\underline{r},\tau) = \frac{C_T(\tau)}{\lambda^4 z^2} \int d^2\underline{u}\, A_T(\underline{u})\, A_T^*(\underline{u} - \underline{r} - \underline{v}\tau) \exp\left[-i \frac{2\pi}{\lambda z}\, \underline{u} \cdot \underline{v}\tau\right] . \tag{52}$$

We will evaluate (52) by assuming the aperture illumination is Gaussian. This is a reasonable assumption if the laser is operating in the fundamental Gaussian mode and if the transmitting telescope does not obscure the central region of the beam. The integral in (52) can be simplified by noting that the magnitude of $\underline{u}$ can never be greater than the transmitter aperture radius R_T. If $R_T vT/\lambda z << 1$, a condition which is usually met, the exponential in (52) can be neglected. Therefore, if we let

$$A_T(\underline{u}) = A_T \exp\left[-\frac{u^2}{R_T^2} - i\frac{\pi u^2}{\lambda f_T}\right] . \tag{53}$$

$\tilde{J}_A$ becomes

$$\tilde{J}_A(\underline{r},\tau) = I_i C_T(\tau) \exp\left[-\frac{|\underline{r} + \underline{v}\tau|^2}{2}\left(\frac{1}{R_T^2} + \frac{\pi^2 R_T^2}{\lambda^2 f_T^2}\right)\right] \tag{54}$$

where f_T is the effective transmitter focal length and I_i is the average incident intensity. It is convenient to write (54) in terms of the effective transmitter divergence angle.

$$\tilde{J}_A(\underline{r},\tau) = I_i C_T(\tau) \exp\left[-\frac{|\underline{r} + \underline{v}\tau|^2}{2} \frac{\theta_T^2}{\lambda^2}\right] \tag{55}$$

$$\theta_T = \left(\frac{\lambda^2}{R_T^2} + \frac{\pi^2 R_T^2}{f_T^2}\right)^{1/2} \quad . \tag{56}$$

If the transmitter beam is collimated, $f_T = \infty$ and θ_T becomes the diffraction limited value λ/R_T. The spatial coherence length is defined as

$$\rho_c = \lambda/\theta_T \quad . \tag{57}$$

Notice that the spacecraft velocity effect is doubled because both the transmitter and receiver are moving. The variance of the received signal is evaluated by integrating $\tilde{J}_A(\underline{r} + \underline{v}T,T)$ over the receiver aperture [see Eqs. (14) and (16)]. The $\underline{v}T$ parameter was introduced by the receiver motion. Transmitter motion introduces the $\underline{v}T$ parameter in Eq. (55). Thus, $\underline{v}$ should be replace by $2\underline{v}$ in the analyses in Section III.

V. SCATTERING BY A RANDOM DISTRIBUTION OF SMALL PARTICLES

Scattering by an ensemble of particles differs from rough surface scattering because the particles are generally in motion, resulting in a continually changing scattering medium. The trajectories of molecules are usually governed by Brownian motion while the motion of material particles such as aerosols are determined by the surrounding medium. The book, Statistical Properties of Scattered Light, by Crosignani, Di Porto and Bertolotti[4], gives an excellent treatment of this subject.

The spatial statistics of the scattered field are very similar to those for rough surface scattering. However, the temporal statistics are considerably different because of the random particle motion. The evaluation of the scattered field is greatly simplified by using the Rayleigh-Gans hypothesis. This hypothesis is similar to the weak scattering assumption often used in turbulence problems. Physically, the Rayleigh-Gans hypothesis implies that the scatterer introduces a small optical perturbation to the surrounding medium and that each point of the scatterer sees the incident field practically unperturbed. Under these conditions the field scattered by the n^{th} particle can be written in the form[4]

$$A_{sn} = A_i(\underline{\rho}_n - \underline{v}t,t,z)\ R_n(t)\ \exp[i\underline{k} \cdot \underline{r}_n(t)] \quad . \tag{58}$$

A_i is the complex amplitude of the incident field, $(\underline{\rho}_n, z)$ are the particle coordinates and $\underline{k}$ is the optical wavenumber vector. $\underline{r}_n$ is the trajectory of a given point on the particle and R_n depends on the rotational motion around $\underline{r}_n$. When the particle is spherical and $\underline{r}_n$ is the trajectory of the center of the sphere, R_n is constant.

If the observation plane is in the Fraunhofer zone, the field contribution from the n^{th} scatterer is given by

$$A_n(\underline{r},z,t) = \frac{A_{sn}}{\lambda z} \exp\left(-i \frac{\pi}{\lambda z} r^2 + i \frac{2\pi}{\lambda z} \underline{\rho}_n \cdot \underline{r}\right) \quad . \tag{59}$$

The total field in the observation plane is calculated by integrating (summing) (59) over all the illuminated particles. Since particles may also be distributed along the direction of propagation (z), the integral must extend over both the longitudinal and transverse dimensions.

$$A(\underline{r},t) = \int_0^\infty dz \int d^2\underline{\rho}_n \frac{A_{sn}}{\lambda z} \exp\left(-i \frac{\pi}{\lambda z} r^2 + i \frac{2\pi}{\lambda z} \underline{\rho}_n \cdot \underline{r}\right) \tag{60}$$

Note that the integral notation in (60) is used for convenience. In reality the total field is calculated by summing over all the illuminated scatterers.

Equation (60) can be simplified by noting that at any given instant the transmitting pulse illuminates a relatively small region of the z axis. Therefore, the phase factor exp $-i \frac{\pi}{\lambda z} r^2$ is essentially constant over the important range of integration. It can be pulled outside the integral signs and dropped since we are only interested in the magnitude of J_A.

$$A(\underline{r},t) = \int_0^\infty dz \int d^2\underline{\rho}_n \frac{A_{sn}}{\lambda z} \exp\left(i \frac{2\pi}{\lambda z} \underline{\rho}_n \cdot \underline{r}\right) \quad . \tag{61}$$

The mutual intensity function for the scattered field in the observation plane can now be calculated using (58) and (61)

$$J_A(\underline{r}_1,\underline{r}_2,t_1,t_2) = \int_0^\infty dz_n \int_0^\infty dz_m \int d^2\underline{\rho}_n \int d^2\underline{\rho}_m \frac{\langle A_{sn} A^*_{sm} \rangle}{\lambda^2 z_n z_m}$$

$$\cdot \exp\left[i \frac{2\pi}{\lambda z_n} \underline{\rho}_n \cdot \underline{r}_1 - i \frac{2\pi}{\lambda z_m} \underline{\rho}_m \cdot \underline{r}_2\right] \tag{62}$$

where

$$<A_{sn}A^*_{sm}> = <A_i(\underline{\rho}_n - \underline{v}t_1,t_1,z_n)\ A^*_i(\underline{\rho}_m - \underline{v}t_2,t_2,z_m)>$$
$$\cdot <R_n(t_i)R^*_m(t_2)\ \exp\{i\underline{k}\cdot[\underline{r}_n(t_1) - r_m(t_2)]\}> \quad . \quad (63)$$

Whenever the rotational motion is decoupled from the translational motion (this is not always the case), (63) becomes

$$<A_{sn}A^*_{sm}> = <A_iA^*_i><R_n(t_1)R^*_m(t_2)><\exp\{i\underline{k}\cdot[\underline{r}_n(t_1) - \underline{r}_m(t_2)]\}> \quad . \quad (64)$$

The ensemble average of the exponential term in (64) is taken over the initial positions and velocities of the particles and over fluctuating quantities of the medium that influence particle motion. Equation (64) can be further simplified if we assume the medium is stationary and homogeneous, which implies that the initial particle positions are uniformly distributed

$$<A_{sn}A^*_{sm}> = <A_iA^*_i><R_n(t_1 - t_2)R^*_m(0)>$$
$$\cdot <\exp\{i\underline{k}\cdot[\underline{r}_n(t_1 - t_2) - \underline{r}_m(0)]\}> \quad . \quad (65)$$

The particle trajectory is given by

$$\underline{r}_n(t) = \underline{r}_{n0} + \int_0^t d\xi\ \underline{v}_n(\xi,\underline{r}_{n0},\underline{v}_{n0}) \quad (66)$$

where

$$\underline{v}_n(t) = \frac{d}{dt}\,\underline{r}_n(t)$$
$$\underline{r}_{n0} = \underline{r}_n(0)$$
$$\underline{v}_{n0} = \underline{v}_n(0) \quad .$$

Using (66), the average of the exponential factor in (65) now becomes

$$\langle \exp\{i\underline{k}\cdot[\underline{r}_n(\tau) - \underline{r}_m(0)]\}\rangle =$$

$$\langle \exp[i\underline{k}\cdot(\underline{r}_{n0} - \underline{r}_{m0})]\exp\left[i\underline{k}\cdot\int_0^\tau d\xi\ \underline{v}_n(\xi,\underline{r}_{n0},\underline{v}_{n0})\right]\rangle \quad . \tag{67}$$

Because of homogeneity the right-hand side of (67) must be independent of the initial positions of the n^{th} and m^{th} particles. This will occur only if the expectation vanishes for $n \neq m$. Under these conditions (62) reduces to

$$J_A(\underline{r}_1,\underline{r}_2,t_1,t_2,z) = \int_0^\infty dz \int d^2\underline{\rho}\ \frac{\rho(z)}{\lambda^2 z^2}\langle A_i(\underline{\rho} - \underline{v}t_1,t_1)\ A_i^*(\underline{\rho} - \underline{v}t_2,t_2)\rangle$$

$$\cdot \langle R_n(t_1 - t_2)\ R_n^*(0)\rangle\langle \exp\{i\underline{k}\cdot[\underline{r}_n(t_1 - t_2) - \underline{r}_{n0}]\}\rangle \tag{68}$$

where $\rho(z)$ is the number density of scatterers. We have assumed that the scattering density is essentially constant throughout the incident beam cross section.

Further evaluation of (67) follows directly from the work on rough surface scattering presented in the previous section. The last two factors in (67) are independent of $\underline{\rho}$ and z and can be removed from the under integral signs. If in addition we use the representation in (47) and (48) for the incident field, (68) becomes

$$J_A(\underline{r}_1,\underline{r}_2,t_1,t_2,z) = D(t_1,t_2)\ \mathcal{J}_A(\underline{r},\tau) \tag{69}$$

$$D(t_1,t_2) = \int_0^\infty dz\ P\left(t_1 - \frac{2z}{c}\right) P\left(t_2 - \frac{2z}{c}\right)\frac{\rho(z)}{z^2} \tag{70}$$

$$\mathcal{J}_A(\underline{r},\tau) = C_T(\tau)\ C_R(\tau)\ C_E(\tau)\ \frac{1}{\lambda^4 z^2}\int d^2\underline{u} A_T(\underline{u})\ A_T^*(\underline{u} - \underline{r} - \underline{v}\tau)$$

$$\cdot \exp\left[-i\ \frac{2\pi}{\lambda z}\ \underline{u}\cdot\underline{v}\tau\right] \quad . \tag{71}$$

where

$$C_R(\tau) = \langle R_n(\tau)\ R_n^*(0)\rangle$$

$$C_E(\tau) = \langle \exp\{i\underline{k}\cdot[\underline{r}_n(\tau) - \underline{r}_{n0}]\}\rangle \quad .$$

From Equations (69) through (71) and (50) through (52) we see that the spatial statistics associated with particle and rough surface scattering are identical. The temporal statistics, however, can be considerably different. For rough surface scattering the temporal statistics are determined entirely by the source, while for particle scattering the temporal statistics are strongly influenced by particle motion.

C_E can be simplified if we assume the particle velocity is a Gaussian random variable. The Gaussian assumption is valid for Brownian motion and for turbulent flow which may be found in clouds and smoke plumes. Using (65), we have

$$C_E(\tau) = \left\langle \exp\left[i\underline{k} \cdot \underline{v}_0\tau + i\underline{k} \cdot \int_0^\tau d\xi \; \Delta\underline{v}(\xi)\right]\right\rangle \tag{72}$$

where $\underline{v}_0$ is the average particle velocity and $\Delta\underline{v}$ is the fluctuating velocity component. The first term in (72) represents the Doppler frequency shift associated with the scattered field. Because we are using a direct detection receiver, the Doppler term will have no effect on the received signal and can be dropped. Thus, C_E may be written in the form

$$\begin{aligned} C_E &= \langle e^x\rangle \\ x &= i\underline{k} \cdot \int_0^\tau d\xi \; \Delta\underline{v}(\xi) \end{aligned} \tag{73}$$

Since x is a zero mean Gaussian random variable, we have

$$\begin{aligned} C_E &= \langle e^x\rangle = \exp\left(-\frac{1}{2}\langle x^2\rangle\right) \\ \langle x^2\rangle &= k^2 \int_0^\tau d\xi_1 \int_0^\tau d\xi_2 \; C_{\Delta v}(\xi_1,\xi_2) \end{aligned} \tag{74}$$

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$$\begin{aligned} C_E &= \langle e^x \rangle = \exp\left(-\frac{1}{2} \langle x^2 \rangle\right) \\ \langle x^2 \rangle &= k^2 \int_0^\tau d\xi_1 \int_0^\tau d\xi_2 \; C_{\Delta v}(\xi_1, \xi_2) \end{aligned} \tag{74}$$

where $C_{\Delta v}$ is the velocity correlation function. If the velocity fluctuations are stationary, $\langle x^2 \rangle$ can be written as

$$\langle x^2 \rangle = -2k^2 \tau \int_0^\tau d\xi \; (1 - \xi/\tau) \; C_{\Delta v}(\xi) \quad . \tag{75}$$

It is interesting to evaluate (75) for the limiting cases where τ is large compared to the velocity correlation time and where τ is small compared to the velocity correlation time. If we let τ_v denote the velocity correlation time, (75) becomes

$$\langle x^2 \rangle = \begin{cases} k^2 \tau^2 \langle \Delta v^2 \rangle & \tau \ll \tau_v \\ 2k^2 \tau \displaystyle\int_0^\infty d\xi \; C_{\Delta v}(\xi) & \tau \gg \tau_v \end{cases} \quad . \tag{76}$$

Notice that the correlation function C_E takes on the Gaussian form whenever $\tau \ll \tau_v$. The velocity correlation time is negligible for Brownian motion and in this case C_E takes on the exponential form which gives rise to the familiar Lorentzian spectrum. For particles suspended in a turbulent flow, the velocity correlation time is usually quite long ($\sim$ms) compared to the observation time so that C_E is Gaussian.

Normally the rotational motion of the particles is negligible compared to the translational motion. Either the source coherence function C_T or C_E dominates the temporal coherence of the scattered field. Consequently, if we drop C_R from (71) and use the Gaussian distribution in (53) for A_T, $\tilde{J}_A$ finally simplifies to

$$\tilde{J}_A(\underline{r},\tau) = I_i C_T(\tau) \; C_E(\tau) \; \exp\left[- \frac{|\underline{r} + \underline{v}\tau|^2}{2} \; \frac{\theta_T^2}{\lambda^2} \right] \quad . \tag{77}$$

The analysis in this section must be modified slightly if the scattering mechanism is resonant absorption and reradiation by molecules. For resonant scattering, only the incident energy lying within the molecular absorption band will be scattered. Therefore, the temporal coherence of the scattered field will depend only on the motion of the molecule. The effect can be included in an analysis by simply replacing $I_i C_T$ in (77) by $\tilde{I}_i$, the incident intensity lying in the absorption band. If the total bandwidth of the transmitted signal is narrower than the absorption band, then $\tilde{I}_i$ will be equal to I_i.

VI. POLARIZATION EFFECTS

The analysis in the previous sections considered only one polarization component of the scattered field. In many cases the scattering process will partially depolarize the scattered field so that the total field in the observation plane should be written

$$\underline{A}(\underline{r},z,t) = A_x(\underline{r},z,t)\ \underline{x} + A_y(\underline{r},z,t)\ \underline{y} \tag{78}$$

where $\underline{x}$ and $\underline{y}$ are the unit vectors in the x and y directions. The total intensity is given by

$$\underline{A} \cdot \underline{A}^* = I_x(\underline{r},z,t) + I_y(\underline{r},z,t) \quad . \tag{79}$$

Each polarization component contributes a speckle intensity pattern. Depending on the characteristics of the scattering medium, there can be an arbitrary degree of correlation between I_x and I_y. Goodman[5] has shown that by using a suitable coordinate rotation, the partially polarized field can be decomposed into two uncorrelated polarization components of different intensities. If we denote these two polarization components by I_1 and I_2, their average intensities are given by

$$\begin{aligned} \langle I_1 \rangle &= \frac{1}{2} \langle I_T \rangle (1 + P) \\ \langle I_2 \rangle &= \frac{1}{2} \langle I_T \rangle (1 - P) \end{aligned} \tag{80}$$

where $\langle I_T \rangle$ is the total average intensity and P is the degree of polarization. Because I_1 and I_2 are uncorrelated, the intensity covariance function can be written as

$$C_{I_T} = C_{I_1}(\underline{r}_1,\underline{r}_2,t_1,t_2) + C_{I_2}(\underline{r}_1,\underline{r}_2,t_1,t_2) \quad . \tag{81}$$

The spatial and temporal correlation lengths of I_1 and I_2 are identical and may be calculated using the procedures outlined in Sections IV and V. Thus C_{I_1} and C_{I_2} have the same functional form but differ in magnitude. This implies

$$\frac{C_{I_T}}{\langle I_T\rangle^2} = \frac{C_{I_1}}{\langle I_1\rangle^2} = \frac{C_{I_2}}{\langle I_2\rangle^2} \quad . \tag{82}$$

Since the analysis in the previous sections was based on completely polarized fields, it is convenient to write (81) in terms of the intensity correlation function for complete polarization, i.e., $P = 1$. If we let C_I denote the correlation function for complete polarization, then

$$\frac{C_I}{\langle I\rangle^2} = \frac{C_{I_T}}{\langle I_T\rangle^2} \quad . \tag{83}$$

Using the results in (80) through (83), we find

$$C_{I_T}(\underline{r}_1,\underline{r}_2,t_1,t_2) = \frac{1}{2}\,(1 + P^2)\; C_I(\underline{r}_1,\underline{r}_2,t_1,t_2) \quad . \tag{84}$$

When the field is completely depolarized, $P = 0$ in (84) and the intensity variance is decreased by a factor of two. This is expected because in this case the receiver is actually adding two independent speckle patterns.

The analysis in Section III can now be used to calculate the statistics of the receiver output by simply replacing the intensity covariance function by C_{I_T} and the mean intensity by $\langle I_T\rangle$. The final results show that the mean value of the detected signal is increased to account for the energy in the additional polarization component and the speckle noise power is multiplied by $\frac{1}{2}\,(1 + P^2)$ to account for averaging of the speckle patterns in the two components.

VII. ATMOSPHERIC TURBULENCE

Atmospheric turbulence affects a propagating laser beam by introducing random amplitude and phase fluctuations. This results in a reduction of the spatial and temporal coherence of the beam. Very little work has been reported on the interaction of turbulence and speckle.[9,10] However, some interesting results were derived in the paper by Lee et al.[10] which can be applied to the satellite-based lidar problem.

Lee et al. assumed that phase perturbation of the laser beam is the dominant effect due to atmospheric turbulence. In addition they assumed that the field statistics at the receiver are jointly Gaussian. Utilizing these assumptions, the intensity covariance function of the field in the observation plane was derived for collimated and focused beams. The results are

$$C_I(\underline{\rho}) = \sigma_I^2 \exp\left[-\frac{\rho^2}{2R_T^2} - 4\left(\frac{\rho}{\rho_0}\right)^{5/3}\right] \quad \text{(focused)} \tag{85}$$

and

$$C_I(\underline{\rho}) = \sigma_I^2 \exp\left\{-4\left(\frac{\rho}{\rho_0}\right)^{5/3} - \frac{1}{2}\left[\left(\frac{1}{R_T}\right)^2 + \left(\frac{kR_T}{z}\right)^2\right]\rho^2\right\} \quad \text{(collimated)} , \tag{86}$$

where σ_I^2 is the intensity variance and

$$\rho_0 = (0.546 C_n^2 z k^2)^{-3/5} \quad . \tag{87}$$

ρ_0 is the phase correlation length and C_n^2 is the turbulence structure parameter.

The results in Eqs. (85) through (87) apply to the case where the initial beam profile at the transmitter is Gaussian and the transmitter, receiver and target are all immersed in homogeneous turbulence.

Although this does not apply to the geometry of the satellite-based lidar where only the target may be immersed in turbulence, the results can give some insight into the satellite problem.

For no turbulence $C_n = 0$ and $\rho_0 = \infty$ so that (85) and (86) reduce, as expected, to the cases considered in the previous sections. For strong turbulence, the intensity correlation length is given by either ρ_0 or R_T for the focused case and by either ρ_0, R_T or $\frac{\lambda}{R_T} z$ for the collimated case. In strong turbulence ρ_0 could be small enough so that it determines the speckle scale size.

Physically, the effect can be explained by noting that turbulence reduces the beam coherence as it propagates to the target. Consequently, the beam spreads out, illuminating an area larger than the diffraction limited spot size. As the reflected field propagates back to the receiver, it is again perturbed by the turbulence, further increasing the effective size of the illuminated target. The end result is a reduction in the effective speckle scale size which is inversely proportional to the size of the illuminated target.

Similar effects will occur over the satellite-earth propagation path. ρ_0 will be different from the value given in (87) because the turbulence structure parameter C_n^2 is a function of altitude and the turbulence is concentrated near the target. For high-altitude targets such as clouds, turbulence effects may be minimal since C_n^2 is significant only near the ground.

Turbulence will also affect the temporal statistics by reducing the temporal coherence of the received signal. Because turbulence coherence times are relatively long ($\sim$ ms), these effects are probably negligible compared to the effects of particle motion and transmitter pulse length.

VIII. SUMMARY

In the previous sections the statistics of the signal and noise at the output of a satellite-based lidar receiver were evaluated. These statistics are summarized in Tables I and II for the case where all the system functions are Gaussian. The mean signal, E(s), may be regarded as the average number of signal photons detected in a time interval of width 1/B when the receiver FOV is equal to the transmitter divergence angle θ_T. Similarly, $E(n_{d\ell})$ is the average number of background noise photons detected in a time interval of width 1/B when the receiver FOV is equal to the diffraction limited value of λ/R. The signal variance, Var(s), and background noise variance, Var(n), consist of a shot noise component and a speckle noise. The speckle noise power is a function of the lidar system parameters and the characteristics of the scattering medium.

A useful parameter for measuring the relative quality of lidar data is the signal-to-noise power ratio (SNR) which is defined as

$$\mathrm{SNR} = \frac{E(s)^2}{\mathrm{Var}(s) + \mathrm{Var}(n)} \quad . \tag{88}$$

Under strong signal conditions the signal speckle noise dominates and the SNR becomes

$$\mathrm{SNR} \simeq \frac{2M_s M_T}{(1 + P^2)} \quad . \tag{89}$$

It is instructive to evaluate (89) for some typical lidar parameters. As an example we will use the system specification for the proposed cloud climatology experiment (Advanced Applications Flight Experiment, GSFC).

TABLE I.

$$E[s(t) + n(t)] = E(s) + E(n_{d\ell})\left(\frac{R\theta_T}{\lambda}\right)^2$$

$$\mathrm{Var}\,[s(t) + n(t)] = \frac{E(s)}{\sqrt{2}} + \frac{1}{2}\,(1 + P^2)\,\frac{E(s)^2}{M_s M_T}$$

$$+ \frac{E(n_{d\ell})}{\sqrt{2}}\left(\frac{R\theta_T}{\lambda}\right)^2 + \frac{E(n_{d\ell})^2}{N_s N_T}\left(\frac{R\theta_T}{\lambda}\right)^4$$

$s(t)$ - signal voltage at receiver output

$n(t)$ - background noise voltage at receiver output

$n_{d\ell}$ - equivalent noise voltage for diffraction limited FOV

R - receiver aperture radius

θ_T - transmitter divergence angle and receiver FOV

λ - optical wavelength

P - degree of signal polarization, $0 \leq P \leq 1$

M_s - effective number of spatial correlation cells seen by receiver aperture (signal)

M_T - effective number of temporal correlation cells seen by receiver electrical filter (signal)

N_s - effective number of spatial correlation cells seen by receiver aperture (background noise)

N_T - effective number of temporal correlation cells seen by receiver electrical filter (background noise)

TABLE II.

Scattering Mechanism	M_s	M_T	τ_c	N_s	N_T
Rough Surface	$1 + \frac{R^2\theta_T^2}{\lambda^2}$	$\left[1 + \frac{1}{B^2\tau_c^2} + \frac{4v^2}{B^2(R^2 + \lambda^2/\theta_T^2)}\right]^{\frac{1}{2}}$	τ_{sc}	$1 + \frac{R^2\theta_T^2}{\lambda^2}$	$\left[1 + \frac{B_I^2}{B^2} + \frac{v^2}{B^2(R^2 + \lambda^2/\theta_T^2)}\right]^{\frac{1}{2}}$
Molecules, Resonant	.	$\left[1 + \frac{1}{B^2\tau_c^2} + \frac{1}{B^2\tau_p^2} + \frac{4v^2}{B^2(R^2 + \lambda^2/\theta_T^2)}\right]^{\frac{1}{2}}$	τ_d	.	.
Molecules, Rayleigh	.	.	$\left(\tau_{sc}^{-2} + \tau_d^{-2}\right)^{-\frac{1}{2}}$	.	.
Particulates & Aerosols	.	.	.	.	.

τ_{sc} - source coherence time

τ_d - inverse Doppler linewidth of scattered signal

τ_p - transmitted pulse width

B - receiver electrical filter bandwidth

B_I - receiver interference filter bandwidth

v - transverse velocity of spacecraft

Laser	Receiver
λ - 1.06 μm, 0.532 μm	R - 9 cm
τ_p - 5 ns (FWHM)	B - 100 MHz
τ_{sc} - ∞	V - ∿ 10 km/sec
θ_T - 0.5 mrad (Half-angle 10 dB points)	

Using the above values in (89) we obtain

$$\mathrm{SNR} \simeq \begin{cases} 3.6 \times 10^3 & \lambda = 1.06\ \mu\mathrm{m} \\ 1.45 \times 10^4 & \lambda = 0.532\ \mu\mathrm{m} \end{cases} .$$

These relatively high SNRs are a consequence of large transmitter divergence angle. They illustrate that speckle noise can be maintained at acceptable levels by careful choice of the system parameters.

Although the equations in Tables I and II were derived for the specific case where all system functions are Gaussian, the general expression for the signal variance can be written in a similar form

$$\mathrm{Var}(\mathbf{s}) = \delta E(\mathbf{s}) + \frac{(1 + P)}{2M_S M_T} E^2(\mathbf{s}) \tag{90}$$

where

$$\delta = \frac{R_h(0)}{\int_{-\infty}^{\infty} d\tau\, h(\tau)} \tag{91}$$

$$\frac{1}{M_S} = \frac{\int_{-\infty}^{\infty} d^2\underline{r}\, |\tilde{J}_S(\underline{r})|^2\, R_W(\underline{r})}{[\,|J_S(0)|\int_{-\infty}^{\infty} d^2\underline{r}\, W(\underline{r})]^2} \tag{92}$$

$$\frac{1}{M_T} = \frac{\int_{-\infty}^{\infty} d\tau |\mathcal{J}_t(\tau)|^2 R_p^2(\tau) R_h(\tau)}{[|\mathcal{J}_T(0)| \int_{-\infty}^{\infty} d\tau P^2(\tau) \int_{-\infty}^{\infty} d\tau h(\tau)]^2} \quad . \tag{93}$$

In equations (90) through (93) we have neglected the minor effects caused by the satellite motion and have used the fact that the mutual intensity function is separable

$$\tilde{\mathcal{J}}_A(\underline{r},\tau) = \tilde{\mathcal{J}}_s(\underline{r}) J_T(\tau) \quad . \tag{94}$$

When the receiving aperture diameter is small compared to the spatial coherence length, M_S is approximately one. When the receiving aperture diameter is large compared to P_c, M_s can be approximated by the formula

$$\frac{1}{M_s} \simeq \frac{R_W(0)}{[\int d^2\underline{r} W(\underline{r})]^2} \int d^2\underline{r} \frac{|\tilde{\mathcal{J}}_s(\underline{r})|^2}{|\tilde{\mathcal{J}}_s(0)|^2} \quad . \tag{95}$$

In this case M_s is just the ratio of the receiver aperture area to the effective area of the spatial component of the mutual intensity function. M_T is also approximately one when the bandwidth of h is small compared to the temporal coherence bandwidth of the received signal. When the bandwidth of h is large compared to τ_c, M_T can be approximated by the formula

$$\frac{1}{M_T} \simeq \frac{R_h(0)}{[\int_{-\infty}^{\infty} d\tau h(\tau)]^2} \; \frac{\int_{-\infty}^{\infty} d\tau |\mathcal{J}_T(\tau)|^2 R_p^2(\tau)}{|\mathcal{J}_T(0)|^2 R_p^2(0)} \tag{95}$$

The effective temporal coherence function of the signal is $\mathcal{J}_T R_p$. Thus, M_T is just the ratio of the effective area of the receiver impulse response to the effective area of the temporal coherence function.

As an example, consider the more realistic situation of a photon counting receiver where the aperture is annular with an obscuration ratio of γ and the counting interval is T seconds in duration. Then if $\mathcal{J}_s$, $\mathcal{J}_T$

and P are Gaussian, we have

$$M_s \simeq \frac{\pi R^2 (1 - \gamma^2)}{2\pi\rho_c^2} = \frac{1}{2} (1 - \gamma^2) \frac{R^2}{\rho_c^2} \tag{97}$$

$$M_T \simeq \frac{\left(\frac{T^2}{\tau_p^2} + \frac{T^2}{\tau_c^2}\right)^{1/2}}{\sqrt{2\pi}} \quad . \tag{98}$$

Exact expressions for M_s and M_T have been derived for the more realistic cases such as annular receiving apertures and higher order laser modes. These results will be included in a subsequent report.

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