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THREE-DIMENSIONAL EFFECTS ON PURE TONE FAN NOISE DUE TO INFLOW DISTORTION

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Abstract
For the prediction of fan noise, the two-dimensional method has been used in many cases. However, the accuracy of the two-dimensional method needs to be assessed against three-dimensional effects. Therefore, two-dimensional, quasi three-dimensional and three-dimensional theories for the prediction of pure tone fan noise due to the interaction of inflow distortion with a subsonic annular blade row have been studied with the aid of an unsteady three-dimensional lifting surface theory developed by Namba. The present study also examines the effects of compact and noncompact source distributions on pure tone fan noise in an annular cascade. Numerical results show that the strip theory and quasi three-dimensional theory are reasonably adequate for fan noise prediction. The quasi three-dimensional method is more accurate for acoustic power and modal structure prediction with an acoustic power estimation error of about 2 dB. This accuracy seems to be affected not by the sub-resonance or super-resonance status of the acoustic modes, but by the reduced frequency of the inflow distortion and by the inflow distortion radial distribution. Also, the compact source prediction is different from the noncompact source prediction by as much as 15 dB in upstream radiation cases and by as much as 6 dB in downstream cases, depending on reduced frequency and interblade phases.

Introduction
A major source of fan noise is unsteady flows interacting with blades and vanes. These unsteady flows are inlet distortions such as atmospheric turbulence, crosswind effects, static test installation wakes, fan inlet boundary layers and the ground vortices that are apparent during many static test conditions, all of which can produce a significant levels of noise with a fan or a turbofan engine. Because these inflow conditions can vary during the course of a test program, and can vary significantly between static testing and forward flight of an engine, the evaluation of noise reduction features using static testing and extrapolation to flight of static noise levels is extremely complicated and may prove erroneous.

For flyover noise prediction, source noise error is only a part of the problem since atmospheric propagation effects and installation effects also appear to contribute significantly. However, the study of fan noise sources is an important part of this problem, and knowledge of the importance and strength of possible sources of unsteady flows described above is particularly important in selecting a facility for simulating flight behavior statically. The degree of importance of these unsteady flows will depend on the particular fan stage and unsteady flow structure. It is therefore important to correlate the results of various fan noise calculations with the existing experimental modal data and thereby define the interplay of scales and intensities in determining noise levels and to suggest the most fruitful inflow control strategies. These calculations can also provide information about fan noise modal structure for acoustic suppressor design.

Many theoretical studies of fan noise have been based on two-dimensional linear cascade models that have made clear the fundamental features of the sound generation process and have shown the effect of the various factors that influence sound radiation and noise. However, it has not been made clear over what range of conditions these current two-dimensional theories are able to predict the unsteady blade forces and the sound field for the case of a three-dimensional annular blade row and for complicated inflow-distortions. The two-dimensional problem is easier to handle and to obtain numerical results for; however, comparison of two and three-dimensional results is needed over a wide range of parameters and inflow conditions.

The available theoretical results on unsteady blade forces for a single airfoil in an oblique gust show that the unsteady lift force amplitude decreases as the spanwise wave number of gust increases. We can infer then, that three-dimensional effects will play an important role on fluctuating blade forces in the case of interaction with an inflow distortion of large radial extent. As Tyler and Sofrin pointed out, the interaction field generates many modes with various circumferential and radial mode numbers, and whether these modes propagate or decay depends strongly on the three-dimensional duct cutoff phenomena. Therefore, the role of three-dimensional effects in the prediction of unsteady blade forces and acoustic power and acoustic modal structure has to be clarified.

A theoretical analysis of the unsteady force and the total acoustic power in a three-dimensional annular cascade was carried out by Namba accepting as input various inflow distortions. In order to predict the forward and aft radiated pure tone energy and the modal energy distribution, Special empha-
sis is placed upon the clarification of the accuracy of available two-dimensional theory for the prediction of pure tone fan noise due to the interaction of inflow distortion with a subsonic annular blade row. For this study, three-dimensional calculations, two-dimensional calculations (strip theory) and quasi three-dimensional calculations are carried out and compared. In the quasi three-dimensional calculation, two-dimensional chordwise dipole distributions at several radial positions are resolved into Fourier-Bessel dipole distributions on an annular blade row, and then, with this dipole distribution, the pure tone fan energy and modal structure in a three-dimensional annular duct are calculated. In addition, the effects of compact and noncompact sources in an annular blade row on pure tone fan noise are also studied.

The procedure and assumptions introduced in the present study are described as follows. The theoretical model consists of a single three-dimensional annular cascade rotating at constant angular velocity in an annular rigid-wall duct of infinite axial extent. Thus, the duct end reflection, the effect of upstream or downstream blade rows, and the effects of duct area variation are not considered. These effects except for the last one, could be included in the solution procedure of the present study. The undisturbed flow is uniform axially and the relative velocity over the whole blade span is limited to the subsonic range. In order to make the problem tractable, linearized theory is adopted; that is, not only fluctuating quantities due to sound but also those due to the inflow distortion convected with the basic flow, are assumed to be small quantities of the first order in comparison to the mean pressure or the main stream velocity, respectively.

The calculation procedure is as follows. First, the periodic velocity fluctuations at the rotor blades due to the inflow distortion are obtained. Second, in order to cancel the fluctuating component of the velocity normal to the airfoils, acoustic dipoles are assumed to be distributed over the surface of the rotor blade. Fluctuating pressure and velocity fields induced by the rotor blades are expressed through a kernel function in terms of the dipole distribution on a blade. The spanwise distribution of acoustic dipoles is given as a sum of acoustic radial mode components and the kernel function is resolved into the corresponding modal components. Finally, the boundary condition that the fluctuating velocity normal to the airfoil should vanish at each blade surface position is introduced. By solving the resulting integral equation numerically, we obtained the dipole distributions, which are then used to calculate the sound pressure in the far field. Thus, the pure tone acoustic power and modal structure upstream and downstream of the rotor are obtained. The inflow distortion is resolved into its Fourier-Bessel components, and then the effects of each inflow distortion component on the acoustic power is superimposed to obtain the total pure tone acoustic power. It is worth noting that the same acoustic mode can be generated by all the inflow distortion components that have the same circumferential mode number, even though their radial mode numbers are different.

Analytical Model

Fluctuating Pressure Induced by Rotor Blades Row

The theoretical model considered here consists of a single annular blade row with N blades rotating at a constant angular velocity \( \omega_0 \) in an annular rigid-walled duct of infinite axial extent (Fig. 1). The fluid flow is composed of an undisturbed flow with a uniform axial velocity \( \bar{V}_0 \) and small fluctuating flows due to the rotor blades and inflow distortion. The flow is inviscid, of uniform entropy and has no thermal conductivity. The fluctuation induced by the rotor is assumed to be isotropic. The fluctuations of the fluid flow are small compared with the undisturbed flow. It is also assumed that the fluid velocity relative to the blades is subsonic along the whole span and that the blades have no steady load.

A cylindrical polar coordinate system is used with axes \((r, \theta, z)\) fixed to the rotor as shown in Fig. 1. The radial and axial coordinates \( r \) and \( z \) are normalized with respect to the radius \( r_T \) at the blade tip. The dimensionless time \( t \) is correspondingly scaled with respect to \( r_T/\omega_0 \). Then in accordance with the assumptions given above, the continuity, momentum and energy equations are combined to give the following linearized equations \(^{1,12}\) for the fluctuating pressure \( p \)

\[
\left[ \frac{\partial^2}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) \right] p = 0,
\]

where

\[
y^2 - \frac{1}{\sigma_{0}^2} \frac{\partial^2}{\partial \theta^2} = \left( 1 - \sigma_{0}^2 \right) \frac{\partial^2}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) \frac{\partial^2}{\partial \theta^2} - 2 \sigma_{0} \frac{\partial^2}{\partial z \partial \theta} + \frac{\partial^2}{\partial \theta^2} - 2 \sigma_{0} \frac{\partial^2}{\partial \theta \partial z} - 2 \sigma_{0}^2 \frac{\partial^2}{\partial \theta^2}
\]

This equation is nothing other than the acoustic wave equation in the rotor-fixed coordinate and its solution must satisfy the boundary condition requiring that the radial velocity vanish on the rigid wall surfaces at the hub and tip radii. In the present approximation, a rigid, thin airfoil placed in the periodically fluctuating velocity field is directly equivalent to a sheet of acoustic dipoles, or, in other words, an unsteady lifting surface. Therefore, a rotor blade row can be represented by \( N \) acoustic dipole sheets with the dipole axes normal to the surface. The \( N \) acoustic dipole sheets have a constant interblade phase difference and are circumferentially equally spaced at the locations \( \theta = 2\pi/N, \theta_j = 0, 1, 2, \ldots, N - 1 \). The fluctuating pressure due to the rotor blade row can be expressed in the form

\[
P(r, \theta, z) = \sum_{q=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} p_q \rho(r, \theta, z) \alpha^{q|z},
\]

where
\[ \tilde{P}_{q,p}(r, \theta, z) = -\int_0^1 \int_{C_0/2} \frac{\alpha_{q,p}(r, \xi, \eta)}{\sqrt{1 + \frac{\partial P}{\partial \eta}}} \tilde{P}(r, \theta, z) e^{i(n \theta + l \phi)} d\xi d\eta, \quad (4) \]

\[ \alpha_{q,p}(r, \xi, \eta) = \frac{1}{\Lambda(q)} \left[ \frac{1}{\sqrt{1 + \frac{\partial P}{\partial \eta}}} \left( \frac{\partial^2}{\partial \eta^2} + \frac{\partial^2}{\partial \xi^2} \right) \right] \tilde{P}(r, \theta, z) \quad \text{with} \quad \Lambda(q) = \left( 1 + a^2 \right) \frac{\partial^2}{\partial \eta^2} \left( 1 + \frac{2}{1 + a^2} \right) \quad (5) \]

\[ \tilde{P}(r, \theta, z) = \frac{N}{4\pi^2} \sum_{m=-\infty}^{\infty} \lambda_n \left( k_n, l, r \right) \sum_{l=0}^{\infty} \hat{F}_n \left( \xi, \eta, \right) \delta \left( \left( l - l_0 \right) \frac{2\pi}{\Lambda(q)} \right) \quad (6) \]

\[ \delta \left( \left( l - l_0 \right) \frac{2\pi}{\Lambda(q)} \right) \quad (7) \]

\[ \hat{F}_n \left( \xi, \eta, \right) \quad (8) \]

\[ \hat{F}_n \left( \xi, \eta, \right) = \left( n^2 + l^2 \right) \hat{F}_n \left( \xi, \eta, \right) \quad (9) \]

\[ \hat{F}_n \left( \xi, \eta, \right) = \left( n^2 + l^2 \right) \hat{F}_n \left( \xi, \eta, \right) \quad (10) \]

\[ \hat{F}_n \left( \xi, \eta, \right) = \left( n^2 + l^2 \right) \hat{F}_n \left( \xi, \eta, \right) \quad (11) \]

Equation (4) can be written in the Fourier-Bessel double series form as follows:

\[ \tilde{P}_{q,p}(r, \theta, z) = -\frac{N}{4\pi^2} \sum_{m=-\infty}^{\infty} \lambda_n \left( k_n, l, r \right) \sum_{l=0}^{\infty} \hat{F}_n \left( \xi, \eta, \right) \delta \left( \left( l - l_0 \right) \frac{2\pi}{\Lambda(q)} \right) \quad (12) \]
The fluctuating velocity induced by the rotor blade row can be obtained by integrating the linearized Euler's equation of motion as follows:

\[ q(r, \theta, z) = \frac{1}{\rho_0} \int_{-a/2}^{a/2} \int_{0}^{C/2} e^{-i\omega t} \left[ k_{q,r}(r,z) \right] dz \left[ \int_{0}^{a} \frac{1}{\rho_0} \left( \frac{\partial^2}{\partial x^2} - \frac{1}{r} \frac{\partial}{\partial r} \right) \right] dr. \]  

(14)

where \( \rho_0 \) denotes the fluid density in the undisturbed state and \( q = q_0 \sqrt{1 + \frac{3}{2} \alpha^2} \) is the velocity of the undisturbed fluid relative to the blade. Combining equations (14) and (14) leads to the expression of the fluctuating velocity in terms of the blade force mode coefficients. Then the upwash component \( q(r, \theta, z) \) can be expressed in the following form:

\[ q(r, \theta, z) = -\frac{1}{\rho_0} \int_{0}^{a} \frac{1}{\rho_0} \left( \frac{\partial^2}{\partial x^2} - \frac{1}{r} \frac{\partial}{\partial r} \right) \right] dr. \]  

The upwash kernel function contains parameters of \( K, h, q, \), and \( \ell \). The detailed expression for the kernel function \( K_{q,p} \) is given in Appendix A. As shown, \( K_{q,p} \) is divided into three parts the same as \( \omega_0 \); the propagating part \( K_{p} \), the singular part \( K_{q,p} \), and the regular part \( K_{q,p} \). The singular part \( K_{q,p} \) possesses singularities at the lifting surface.

The upwash component \( q(r, \theta, z) \) can be expressed as the product:

\[ q(r, \theta, z) = \frac{\omega_q \omega_n}{\epsilon_n} \int_{0}^{a} \frac{1}{\rho_0} \left( \frac{\partial^2}{\partial x^2} - \frac{1}{r} \frac{\partial}{\partial r} \right) \right] dr. \]  

(15)

Inflow Distortion

In this paper, the inflow distortion is assumed to have only an axial velocity component (see Fig. 2). If a Fourier-Bessel analysis of the arbitrary shapes of inflow distortion is carried out, the following axial component of external fluctuating velocity is obtained.

\[ q_{e,a}(r, \theta, z, \phi) = \frac{\omega_q \omega_n}{\epsilon_n} \int_{0}^{a} \frac{1}{\rho_0} \left( \frac{\partial^2}{\partial x^2} - \frac{1}{r} \frac{\partial}{\partial r} \right) \right] dr. \]  

(16-1)

where \( q \) and \( p \) are respectively the circumferential mode number and radial mode number. \( \epsilon_n \) denotes a small quantity which is the ratio of the external fluctuating velocity to the basic flow velocity. Since the upwash component of external fluctuating velocity is given by

\[ q_{e,a} = \frac{-q_{e,a}}{\omega_t} \sqrt{1 + \frac{3}{2} \alpha^2} \]  

(17)

then, one can express the upwash component of the external fluctuating velocity on a blade surface \( \theta = \alpha_0 \) by

\[ [q_{e,a}]_{\theta = \alpha_0} = -\frac{q_{e,a}}{\omega_t} \sqrt{1 + \frac{3}{2} \alpha^2} \]  

(18)

One easily sees that the reduced frequency \( \omega_q \) and the interblade phase angle \( 2\alpha/N \) of inflow distortion sensed by the rotor blades are given by

\[ \omega_q = q_{e,a} \omega_t, \quad \text{and} \quad \sigma = \alpha_0 - \alpha \]  

(19)

where the integer \( \alpha \) is chosen so that \( \alpha = N \alpha_0 \). Then the Fourier coefficient \( B_{q,p} \) can be determined from the axial distortion velocity \( w_{e,a} \) by the relation:

\[ B_{q,p} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{2\pi} \int_{0}^{a} \frac{1}{\omega_t} \frac{1}{\rho_0} \left( \frac{\partial^2}{\partial x^2} - \frac{1}{r} \frac{\partial}{\partial r} \right) \right] dr. \]  

(20)

It is convenient to suppose that the distortion velocity can be expressed as the product:

\[ \frac{\omega_{e,a} \omega_n}{\epsilon_n} = \frac{f_1(r)}{\epsilon_n} \cdot \theta_1(\phi) \]  

(21)

then

\[ B_{q,p} = s_q b_p \]  

(22)

where

\[ s_q = \frac{1}{2\pi} \int_{0}^{2\pi} \frac{1}{\omega_t} \frac{1}{\rho_0} \left( \frac{\partial^2}{\partial x^2} - \frac{1}{r} \frac{\partial}{\partial r} \right) \right] dr. \]  

(23)

and

\[ b_p = \int_{0}^{a} \frac{1}{\omega_t} \frac{1}{\rho_0} \left( \frac{\partial^2}{\partial x^2} - \frac{1}{r} \frac{\partial}{\partial r} \right) \right] dr. \]  

(24)
When the inflow distortion can be represented by N Gaussian profiles (for \( N = 1, 2, \ldots \)) in the circumferential direction,
\[
\psi_1 = (r - 1)\pi/N, \quad (r = 1, 2, \ldots, N)
\]
\[
\psi_2 = (2r - 1)\pi/2N
\]
\( N \) is the number of chordwise acoustic dipole points and is also the chordwise boundary points on a blade. \( G_{a,k}(\xi, q_1, p) \) are represented by
\[
\tilde{G}_{a,k}(\xi, q_1, p) = \frac{G_{o,k}(\xi, q_1, p)}{\left(\frac{G}{2}\sin \theta_1\right)}
\]
The quantity \( \tilde{G}_{a,k}(\xi, q_1, p) \) is calculated from equation (28) by a collocation method. The solution is assumed to satisfy a Kutta condition at the trailing edge i.e., \( G_{a,k}(\xi, q_1, p) = 0 \).

**Fringe Tone Acoustic Power**

The pure tone fan noise is composed of a finite number of propagating pressure modes which depend on the parameters \( \nu, \lambda, q, w, M_a \) and \( h \). Using equation (2), we can obtain the expression for sound pressure
\[
P_a(\Xi, w, h) = \sum_{n} \sum_{i=1}^{N} \sum_{k} \tilde{P}_a(n, i, q) \cdot e^{i(n\omega t - k\Xi w)}
\]
where the suffixes + and - denote the sound waves propagating forward (downstream) and backward (upstream) respectively, and \( \tilde{P}_a(n, i, q) \) denotes the nondimensional pressure amplitude in the \( n, l \) acoustic mode, which is given in the noncompact source case by
\[
\tilde{P}_a(n, l, q) = \frac{N}{4\pi^2} \sum_{k=0}^{L-1} \left[ \frac{\nu_{nm}\lambda_{nm}}{\lambda_{nm}} \right]^{1/2} \cdot \frac{a_{k}^{n}}{C_{n,l,k}}
\]
while in the case of a compact source
\[
\tilde{P}_a(n, l, q) = \frac{N}{4\pi^2} \sum_{k=0}^{L-1} \left[ \frac{\nu_{nm}\lambda_{nm}}{\lambda_{nm}} \right]^{1/2} \cdot \frac{a_{k}^{n}}{C_{n,l,k}}
\]

Further details are given by
\[
\psi_1 = -(\pi/2)\cos \psi_1
\]
\[
\psi_2 = -(\pi/2)\cos \psi_2
\]
where
\[ E_{\xi}^{1}(n,l,q) = \frac{e^{i(n\theta_2 + \xi_1)}}{2} E_{\xi}^{1}(n,l,q) \]
\[ = |E_{\xi}^{1}(n,l,q)| \beta_{n}^{2}(n\omega_{A} + \omega) |P_{n}^{(1)}| (\alpha_{n} + n\omega_{A} + \omega)^{2} \]
\[ E_{\xi}^{1}(n,l,q) \] is the modal component of the dimensionless acoustic power.

Two-Dimensionel Theory (Strip Theory) and Quasi Three-Dimensional Theory

A two-dimensional cascade calculation is obtained as the limit of large cylinder radii. In the notation of this paper, this limit corresponds to \( h \to 1 \) and \( N \to \infty \) simultaneously, while keeping
\[ 2r/(N_{c} \sqrt{k_{n}^{2} + 1}) = E \] constant at each radius of the annular blade row and \( q / N = \text{constant} \), where \( E \) is the pitch-chord ratio and \( q / N \) is the ratio of inflow distortion circumferential wave length to cascade pitch.

In the quasi three-dimensional method, first, the chordwise unsteady force distributions (namely, acoustic dipole chordwise distribution) are calculated at \( L \) numbers of radial positions from hub to tip using the two-dimensional cascade calculation. Second, those dipole distributions on a blade are resolved into Fourier-Bessel form with equation (11) and then the pure tone fan noise and modal structure in three-dimensional annular cascade are computed from equations (39) and (40).

Numerical Results

The numerical calculations are carried out in this paper for two different rotor configurations. Their geometric parameters are given in Table I. The fan denoted case No. 1 has a high tip-speed and a large number of blades, while the No. 2 case fan has low tip-speed and a small blade number. Under those conditions, the pitch-chord ratio \( S \), the stagger angle \( \gamma \), the ratio of the relative flow-velocity to the axial flow velocity \( q / \omega_{A} \) vary along the span as shown in Fig. 3. Reduced frequencies of blade force on the blade due to an inflow distortion in cases No. 1 and No. 2 are calculated as \( K_{f} = 0.136 q \) and \( K_{f} = 0.272 q \), respectively. The pure tone acoustic power has been computed for different combinations of the circumferential mode number \( q \) and the radial distribution of the inflow distortion. In order to clarify the three-dimensional character and the accuracy of available two-dimensional theory, the results are compared with those of the strip theory method and the quasi three-dimensional method.

The finite series approximation of \( L = 7 \) and \( N = 6 \) were used for the calculations in this paper. In accordance with the increase of circumferential wave number \( q \), the number of propagating modes \((n,l)\) increases, and therefore the accuracy of the calculation of the \( P_{n}^{(1)} \) term in equation (15) seems to decrease. In the three-dimensional calculation, the degree of accuracy seems to be greater than in the two-dimensional calculation, because the three-dimensional calculation has three-dimensional effects in the radial direction. So
the calculation for a large number of propagating acoustic modes will require an increase in \( N \).

**Acoustic Power Variation with Circumferential Mode Number of Inflow Distortion**

In Fig. 4, the dimensionless total acoustic powers from the 3-D calculation of the upstream and downstream propagating sound waves are plotted against the circumferential mode number \( q \) of the inflow distortion. The coefficient \( A_q \) in equation (16-1) was held constant over all values of \( q \) and \( N \). In this section, we discuss the numerical results for the noncompact source case. The dimensionless acoustic power at \( q = 10 \) agrees with the Namba’s results. However, acoustic power levels at other values of \( q \) are different from Namba’s results, because in Namba’s calculation the reduced frequency is assumed as constant without regard to the variation of \( q \). In the upper part of Fig. 4, the variation of the generated acoustic modes with the inflow distortion mode number \( q \) is shown. In the notation \( v_{11} \) corresponds to the harmonic number of the pure tone and \( v \) corresponds to the radial mode number of the acoustic mode. Therefore the circumferential mode number of the acoustic mode \( v \) is calculated with the equation \( v = v_{11} N - q \) where \( N \) is the blade number and \( q \) is the circumferential mode number of the inflow distortion.

Figure 5 shows that the number of acoustic modes increases with an increase in \( q \). The acoustic powers of both upstream and downstream propagating sound waves are largest at about \( q = 30 \). This region of large acoustic power corresponds to a large number of low harmonic acoustic modes. In Fig. 4, it is seen that the total acoustic power of downstream propagating waves is higher than that of upstream propagating waves in the whole range of \( q \).

Figure 5 shows the variation of fundamental pure tone component with circumferential wave number \( q \) of inflow distortion for case No. 1. For values of \( q \) less than 20, the fundamental tone acoustic power level of downstream propagating waves is higher than that of upstream propagating waves, but for values of greater than 20, this tendency is reversed. Total acoustic power and the acoustic power of the fundamental pure tone for case No. 2 are shown in Figs. 6 and 7. The total acoustic power radiated downstream is higher than that radiated upstream, while for the fundamental pure tone the downstream acoustic power is sometimes lower and sometimes higher than the upstream acoustic power.

**Compact and Noncompact Sources**

In Figs. 4 to 7, comparisons are made between the compact source prediction based on distributed sources only along a line in the spanwise direction (compact source) and a rigorous prediction based on distributed sources along both the airfoil chord and span (noncompact) for the upstream and downstream propagating sound waves. The divergence of the compact and noncompact source predictions in both the upstream and downstream propagation cases is considerable, especially in the lower reduced frequency range for small values of \( q \). This tendency is similar to that observed by Kajitani in a two-dimensional cascade. The maximum acoustic power difference is 15 dB in the upstream case and 6 dB in the downstream case. The degree of difference decreases as the reduced frequency of the blade forces increases.

At the higher reduced frequency, the unsteady forces on the blade are concentrated near the leading edge and so the phase velocities of sources in the chordwise direction is small. In addition, in the downstream direction, the wave length of sound is stretched due to fluid flow and therefore the ratio of wave length of sound to the chord length, a retarded time effect, is smaller than in the upstream case. Therefore, the difference in acoustic power between noncompact and compact sources is less noticeable in downstream case (see Fig. 5).

In the case of the fundamental tone (Fig. 5), the downstream acoustic power predicted by the compact source model is higher by a maximum of 7 dB than that of the noncompact source model and they approach one another in the higher source reduced frequency range. In the upstream case, the acoustic power level of the compact source prediction is considerably higher than that of the noncompact source prediction in the low reduced frequency range, but in the higher reduced frequency range, the acoustic power of the latter is higher than that of the former by several dB.

Figures 6 and 7 for case No. 2 also show that the acoustic power difference between compact and noncompact sources is smaller than in case No. 1, but still appreciable.

**Acoustic Power Variation Due to Inflow Distortion Radial Distribution**

Figures 8(a) and (b) show the acoustic power variation of the fundamental tone for upstream and downstream propagation. The calculations were for case No. 1 using the 3-D model. In these figures, acoustic powers for the cases of inflow distortion radial distribution are shown. The cases are for inflow distortions with radial extents of 8, 25, 52, 67, and 100 percent of the span from starting from the blade tip (see sketch in Fig. 8). These figures show that at low values of \( q \) all of the upstream acoustic power is generated by the outer 25 percent of blade inflow distortion interaction. At higher values of \( q \), the 25 percent of blade inflow distortion interaction still accounts for over half of the acoustic power. For the downstream case, a similar behavior was observed that involved the 42 percent of blade inflow distortion interaction.

**Modal Structure**

Figures 9 and 10 show the variation of radial mode level for the fundamental pure tone noise for various values of \( q \) for both cases No. 1 and No. 2. It is seen that the acoustic power in the first radial mode is generally higher than that in the second radial mode.

In the case of upstream propagation, for case No. 1 (Fig. 9(a)) the acoustic power of the first radial mode is higher than second radial, mode by more than 7 dB for \( q \) less than 40. In case No. 2, (Fig. 10(a)) the second radial mode level is close
to the first radial mode at \( q = 12 \) and 16, however, for all other values of \( q \), the former is lower by about 5 dB than the latter.

In the case of downstream propagation (Fig. 9(b)), case No. 1 shows that the second radial mode level is lower by more than 3 dB than the first radial mode level. Other radial more levels are lower yet than second radial mode level. In case No. 2 (Fig. 10(b)) the second radial mode level is lower than the first radial component level except in the range of high reduced frequency. Therefore, in all cases, the prediction for both upstream and downstream cases, it seems that only the first radial mode might be enough for acoustic liner design if suppression requirements are modest.

Accuracy of Two-Dimensional Method for Pure Tone Fan Noise Prediction

Comparisons are made of the acoustic powers calculated from the three-dimensional method (3-D), the two-dimensional method (2-D) and the quasi three-dimensional method (quasi 3-D) in order to clarify the three-dimensional effects and the accuracy of the two-dimensional method for fan noise prediction. Figure 11 is for case No. 1 and Fig. 12 is for case No. 2.

Numerical results for both upstream and downstream total acoustic power for case No. 1 (Figs. 11(a) and (b)) show that the acoustic power difference between the two-dimensional and three-dimensional calculation is less than 3 dB. The quasi three-dimensional calculation results based upon the two-dimensional unsteady force are as close to the 3-D results as are the 2-D results.

Results for the fundamental pure tone (Figs. 11(c) and (d)) show that the level differences between 2-D (also quasi 3-D) and 3-D are smaller than for the total acoustic power. This is true for both the upstream and downstream waves.

For case No. 2, Fig. 12(a) shows the upstream total acoustic power difference between 3-D and 2-D to be about 5 dB at some values of \( q \), but the acoustic power difference between 3-D and quasi 3-D is less than 4 dB. For the downstream wave (Fig. 12(b)), the total acoustic power difference is generally 1 dB or less in the region between \( q = 8 \) and \( q = 24 \). Figures 12(c) and (d) show similar close agreements for the fundamental tone.

The survey based on the above data indicates that the quasi 3-D method may be used for pure tone fan noise prediction with an acoustic power estimation error generally less than 42 dB.

Comparison of Modal Structure Between 3-D and Quasi 3-D Methods

Figure 13 shows the comparison between modal structure of the fundamental pure tone based on 3-D and that based on quasi 3-D methods. Figures 13(a) and (b) are for case No. 1 and show the upstream and downstream cases, respectively, while, Figs. 13(c) and (d) are for case No. 2. In both cases, No. 1 and No. 2, the agreement of acoustic powers of corresponding modal levels between 3-D and quasi 3-D methods is good. In both rotor cases, the acoustic power difference of corresponding acoustic modes is less than 3 dB, and in the region of low reduced frequency, the power difference approaches zero. In accordance with these numerical results, the quasi 3-D method also seems to be adequate for acoustic modal structure prediction in the three-dimensional annular fan -- inflow distortion interaction problem.

Concluding Remarks

Comparisons are made among the predictions based on three-dimensional theory, strip theory and quasi three-dimensional theory, in order to clarify the accuracy of available two-dimensional theory for the prediction of pure tone fan noise due to the interaction of inflow distortion with a subsonic annular blade row. The theories were derived with the aid of an unsteady three-dimensional lifting surface theory developed by Namba. Both compact and noncompact sources were considered. Several numerical calculations were carried out and the following results were obtained.

1. The compact source prediction in a three-dimensional annular blade row can overestimate the upstream radiation by 15 dB and the downstream radiation by 6 dB. The degree of overestimation depends on blade row characteristics and reduced frequency and interblade phase due to inflow distortion. This result implies that noncompact source effects must be considered.

2. The upstream radiation seems to depend heavily on the tip region of blade inflow distortion interaction for both upstream and downstream cases. The extent of spanwise contribution to the sound was greater in the downstream case than in the upstream case.

3. The numerical results show that for acoustic liner design the consideration of only the first radial acoustic mode is enough when only modest suppression is needed.

4. Strip theory and quasi three-dimensional theory based on the two-dimensional calculation of unsteady forces are reasonably adequate for fan noise prediction. The quasi three-dimensional method is more accurate for acoustic power and for modal structure prediction, with an acoustic power estimation error of 42 dB or less.

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Appendix - Kernel Function \( K_T \)

The kernel function \( K_T \) is composed of the three parts \( K_T(\vec{p}), K_T(\vec{q}), \) and \( K_T(\vec{r}) \), which are given as follows:


TABLE I. - FAN PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor blade number, N</td>
<td>40</td>
<td>15</td>
</tr>
<tr>
<td>Axial chord length, dimensionless, C</td>
<td>0.03458</td>
<td>0.25836</td>
</tr>
<tr>
<td>Rotor speed/axial flow speed, ( w_r / )</td>
<td>2.474</td>
<td>0.627</td>
</tr>
<tr>
<td>Rotor relative Mach number, ( M_r )</td>
<td>0.934</td>
<td>0.865</td>
</tr>
<tr>
<td>Axial flow Mach number, ( M_a )</td>
<td>0.35</td>
<td>0.596</td>
</tr>
<tr>
<td>Hub/tip ratio, ( h )</td>
<td>0.4</td>
<td>0.46</td>
</tr>
</tbody>
</table>

Figure 1. - Analytical model and cylindrical coordinate system.

Figure 2. - The velocity distribution for inflow distortion model at a constant radius plane.
Figure 3. - Radial variations of stagger angle $\gamma$, pitch-chord ratio $S$ and velocity ratio $Q/W_a$ of the rotor.
Figure 4. - Variation of total acoustic power with inflow distortion circumferential mode number, case no. 1. (Uniform radial distribution of inflow distortion.)
Figure 5. Variation of fundamental pure tone noise with inflow distortion circumferential mode number, case no. 1. (Uniform radial distribution of inflow distortion.)
Figure 6. Variation of total acoustic power with inflow distortion circumferential mode number \( q \), case no. 2. (Uniform radial distribution of inflow distortion.)
Figure 7. Variation of fundamental pure tone noise with inflow distortion circumferential mode number $q$, case no. 2 (uniform radial distribution of inflow distortion).
Figure 8. - Variation of fundamental pure tone noise with inflow distortion radial distribution.
Figure 9 - Variation of radial mode levels of fundamental pure tone noise with inflow distortion circumferential mode number. (Uniform radial distribution of inflow distortion.)
Figure 10. - Variation of radial mode levels of fundamental pure tone noise with inflow distortion circumferential wave number. (Uniform radial distribution of inflow distortion.)
Figure 11. - Comparison of acoustic powers among 3-D method, 2-D method, and quasi 3-D method - case 1.
NONCOMPACT

(c) UPSTREAM PROPAGATION; FUNDAMENTAL PURE TONE.

(d) DOWNSTREAM PROPAGATION; FUNDAMENTAL PURE TONE.

Figure 11. - Concluded.
Figure 12. Comparison of acoustic powers among 3-D method, 2-D method, and quasi 3-D method - case 2.
Figure 12. - Concluded.

(c) UPSTREAM PROPAGATION; FUNDAMENTAL PURE TONE.

(d) DOWNSTREAM PROPAGATION; FUNDAMENTAL PURE TONE.
Figure 13. - Comparison of modal structure of fundamental pure tone noise between 3-D method and quasi 3-D method.

(a) UPSTREAM CASE, NO. 1.

(b) DOWNSTREAM CASE, NO. 1.

h = 0.4, N = 40, M_0 = 0.35
M_f = 0.934, C_a = 0.05498
Figure 13. - Concluded.