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PROPOSED DESIGN PROCEDURE FOR TRANSMISSION SHAFTING UNDER FATIGUE LOADING

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Abstract

The B106 American National Standards Committee is currently preparing a new standard for the design of transmission shafting. A design procedure, proposed for use in the new standard, for computing the diameter of rotating solid steel shafts under combined cyclic bending and steady torsion is presented. The formula is based on an elliptical variation of endurance strength with torque exhibited by combined stress fatigue data. Fatigue factors are cited to correct specimen bending endurance strength data for use in the shaft formula. A design example illustrates how the method is to be applied.

Introduction

The judicious design of power transmission shafting is not only important from a machine reliability standpoint but from cost and energy conservation standpoints as well. Although the prime design consideration is whether the shaft will provide adequate service life, that is, whether it will resist fatigue failure, it is seldom the only design consideration. The shaft must also be stiff enough between supports to limit deflections of key power transfer elements and sufficiently stiff to avoid vibrational excitation. However, our working knowledge in these other areas is more complete in comparison to our limited knowledge of the fatigue behavior of materials in shafting applications.

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Applying experimentally generated fatigue data to shafting design is certainly not a new approach. However, rarely does the shaft designer have the appropriate fatigue data at his finger tips which matches his application. Although running screening tests on prototype parts is the most prudent approach, very few organizations can afford the cost and time associated with long term endurance testing. Usually the designer can consult a number of design references (1, 2) containing shafting design formulas that give acceptable designs for the majority of applications. However, there is not always consistency from formula to formula. There is often confusion as to which fatigue factors to use and what importance to place on them.

Recognizing the need for a unified, national design standard for power transmission shafting, the ASME organized the American National Standards Committee B106. The Committee's objective is to replace the obsolete code for the Design Transmission Shafting, ASA-B17C which was officially withdrawn in 1954. Its principal shortcoming was that it did not directly consider flexure fatigue as the principal failure mode. At present, the B106 Committee has analyzed several sets of published combined stress fatigue data for alloy steels and has tentatively selected a design method for computing shaft diameters for common loading conditions. To provide additional experimental support for a new shafting design standard, the B106 Committee has proposed a test program to further quantify the effects of combined reversed bending and steady torsional stress on several common shafting steels. It is the purpose of this paper to review the shaft design procedure proposed by the B106 Committee and to illustrate how it might be applied in a typical design application.

Fatigue Failure

Ductile machine elements subjected to repeat fluctuating stresses above their endurance strength but below their yield strength will eventually fail from fatigue. The insidious nature of faitgue is that it occurs without visual warning at operating stresses below plastic deformation. Shafts sized to avoid fatigue will usually be strong enough to avoid elastic failure, unless severe transient or shock overloads occur.

Failure from fatigue is statistical in nature inasmuch as the fatigue life of a particular specimen cannot be precisely predicted but rather the likelihood of failure based on a large population of specimens. For a group of specimens or parts made to the same specification the key fatigue variables would be the effective operating stress, the number of stress cycles and volume of material under stress. Since the effective stresses are usually the highest at points along the surface where discontinuities occur, such as keyways, splines, and fillets, these are the points from which fatigue cracks are most likely to emanate. However, each volume of material under stress carries with it a finite probability of failure. The product of these element probabilities (the "weakest link" criterion) yields the likelihood of failure for the entire part for a given number of loading cycles. This is underlying reason why larger shafts generally have shorter fatigue lives than smaller shafts under the identical stress levels (1, 2).

At present there is no unified statistical failure theory to predict shafting fatigue. However, reasonably accurate life estimates can be derived from general design equations coupled with bench-type fatigue data and material static properties. Fatigue test data is usually obtained in a rotating-beam tester under the conditions of reversed bending. The data generated from these machines are usually plotted in the form of stress-life (S-N) diagrams. On these

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diagrams, the bending stress at which the specimens did not fail, after at least 10^6 cycles for steel, is commonly referred to as the endurance limit. Due to test data scatter, the endurance limit values determine from S-N diagrams usually represent some sort of mean value and must be statistically corrected for higher reliability levels as will be discussed later. It is customary to consider that design stresses less than the endurance limit will produce an "infinite" life design. This is misleading since no part can have a 100 percent probability of survival.

Fatigue Under Combined Stresses

For applications where a simple fluctuating stress of the same kind is acting, for example, a steady bending stress superimposed on a reversed bending stress, a Soderberg failure line connecting the endurance strength with the yield strength provides an acceptable design (1,2). However, most power transmission shafting is subjected to a combination of reversed bending stress (a rotating shaft with constant moment loading) and steady or nearly steady torsional stress. Although a large body of test data has been generated for the simple stress condition, such as pure tensile, flexural or torsional stress, little information has been published for the combined stress condition. This is most likely due, in part, to the additional complexity and cost in making a reliable, high speed combined stress fatigue tester. However, some cyclic bending and static torsional fatigue test data was reported by Kececioglu and Lalli in (3) and Davies in (4). In (3), the endurance limit characteristics of notched AISI 4340 steel specimens was determined for theoretical bending stress concentration factors of 1.42 and 2.34. In (4), 3-percent nickel and nickel-chromium steel specimens were fatigue tested under the same stress combination in a modified Wohler machine. The results from both these experiments appears in fig. 1, where the reversed bending strength for life greater than 10^6 cycles S_b is shown to decrease with an increase in static shear stress S_s. Considering that either fatigue fracture or torsional yielding represents failure, the following elliptical relation reasonably fits the data:

$$\left(\frac{S_b}{S_{re}}\right)^2 + \left(\frac{S_s}{S_{sy}}\right)^2 = 1 \qquad (1)$$

In this equation, S_{re} is the reversed bending endurance strength of the test specimen under bending only and S_{sy} is the torsional yield strength.

The failure relation of eq. (1), is similar to that observed by Gough and Pollard in (5) for rotating-beam specimens loaded under reversed bending in phase with reversed torsion as shown in fig. 2. This data together with that shown in fig. 1 are in reasonable agreement with the distortion energy or von Mises-Hencky failure criterion. This theory predicts static elastic failure when the distortional energy under combined stresses equals or exceeds that in simple tension or bending. There is a great deal of experimental evidence which indicates that of all the failure theories, the distortion-energy theory most accurately predicts yielding of ductile materials under static loading However it is not clear why the distortion-energy theory seems also to hold for some fatigue failures as well.

The distortion-energy elliptical failure relation is not the only one to be proposed for combined cyclic bending and static torsion loading. The tests performed by Ono (6) and Lea and Bodgen (7) suggest that the bending endurance strength of steel is unaffected by the presence of a static torsional stress, even above the torsional yield strength. Based, in part, on this test information, Wellauer (8) recommends that the allowable bending endurance strength and the allowable static torsional stress for gear drive shafts be calculated separately. A comparison between separate stress and combined stress shaft methods is illustrated in fig. 3. From a reliability standpoint, the combined stress relation of eq. (1) will produce a slightly more conservative and thus safer design. However, the differences are not great. For most designs, the difference in shaft diameters will be less than 15 percent. The combined stress fatigue data which the B106 Committee proposes to generate will help clarify this matter.

Shaft Design Formula

For design purposes, allowable strength values must be incorporated into eq. (1) as follows:

$$\left(\frac{S_b}{S_{ea}}\right)^2 + \left(\frac{S_s}{S_{sya}}\right)^2 = 1 \qquad (2)$$

where

S_{ea}	allowable shaft endurance limit, psi = S_e/FS
S_{sya}	allowable shaft torsional yield strength, psi = S_{sya}/FS
S_b	reversed bending stress, psi = $32\ M_b/\pi d^3$
S_s	mean torsional stress, psi = $16\ T_m/\pi d^3$
M_b	reversing bending moment, in-lb
T_m	mean static torque, in-lb
d	shaft diameter, in
FS	factor of safety

Rearranging eq. (2) and noting that for most wrought steels $S_{sy} = S_y/\sqrt{3}$ results in the following formula for computing the diameter of rotating shafts under reversed bending and steady torsional stress (less than torsional yield) with negligible axial loading:

$$d = \left[\frac{32(FS)}{\pi}\sqrt{\left(\frac{M_b}{S_{ea}}\right)^2 + \frac{3}{4}\left(\frac{T_m}{S_{ya}}\right)^2}\right]^{1/3} \qquad (3)$$

Eq. (3) is the basic shaft design equation proposed for the B106 transmission shafting standard. It is also similar to shaft formulas recommended by several design specialists, e.g., (1,8), and identical to that appearing in (2) which was derived theoretically from the distortion-energy failure theory as applied to fatigue loading using the Soderberg criterion.

Fatigue Modifying Factors

In eq. (3), the reversed bending strength of the shaft to be designed, S_e, is generally different than the endurance limit of rotating-beam specimens, S_{re}, commonly listed in design tables such as in (10). A number of service factors have been identified by Marin (11) which can be used to modify the uncorrected bending endurance limit of test specimen, S_{re}, as follows:

$$S_e = k_a k_b k_c k_d k_e k_f S_{re} \qquad (4)$$

where

- S_e corrected reversed bending endurance limit of the shaft
- S_{re} reversed bending endurance limit of the rotating-beam specimen
- k_a surface finish factor
- k_b size factor
- k_c reliability factor
- k_d temperature factor
- k_e duty cycle factor
- k_f fatigue stress concentration factor
- k_g miscellaneous effects factor

At the time of this writing, the B106 Committee has not yet made a final determination of the values for these factors which would be suitable for a shaft design code. The following discussion is intended to briefly highlight values commonly found for these factors in the open literature and to refer the reader to references where more indepth information can be found.

k_a, surface factor. - Since the shaft surface is the most likely place for fatigue cracks to start, surface condition significantly affects endurance limit as shown in fig. 4, from (1). This figure is based on a compilation of test data from several investigations for a variety of ferrous metals and alloys. The figure shows that the endurance characteristics of higher tensile strength steels are more adversely effected by poorer surface finish.

k_b, size factor. - There is considerable experimental evidence that the bending and torsion fatigue strength of large engineering parts can be significantly less than the small test specimens, 0.30 in. in diameter (10,12). This size effect is attributed the greater volume of material under stress and thus, the greater likelihood of encountering a potential fatigue initiating defect in the material's metallurgical structure.

Although there is a lack of complete quantitative agreement between the many investigations of the influence of size, (10) recommends that a design allowance of 10 to 15 percent lower fatigue strength be given for specimens of up to 2 in. in diameter. For machine parts larger in diameter than this, even a greater reduction in fatigue strength may be required. Accordingly, the size factor, k_b can be selected as follows:

k_b	Shaft diameter, in.
1.0	$d \leq 0.3$
.85	$0.3 < d \leq 2.0$
<.85	$d > 2.0$

k_c, reliability factor. - Even under well controlled test conditions, it is clear that the unavoidable variability in the preparation of test specimens and their metallurgical structure will cause a variability in their measured endurance strengths. Endurance limit data published in standard design references usually represent an average value of endurance for the sample of test specimens. Most designs require a much higher survival rate than 50 percent, that is the probability that at least half of the population will not fail in service. Consequently, endurance limit values must be reduced by some amount to increase reliability. The amount of this reduction is dependent on the failure distribution curve. Several design test, e.g., (1,2), suggest reliability factors based on "Normal" or "Gaussian" failure curves can be used when specific test values are not available. A reliability factor value, $k_c = 0.9$ is generally cited for a 90 percent survival rate, based on an assumed standard deviation of 8 percent of the endurance strength (1,2). This estimated standard deviation is close to the recommended standard deviation of 7 percent reported by Kececioglu and Lalli (3).

As an alternate to the normal distribution, the Weibull distribution (12) should be investigated. It is very effective in representing rolling-contact fatigue for bearings and gears and should fit shafting fatigue data more closely than either the normal or log-normal distributions.

k_d, temperature factor. - Operating temperatures higher than about 300° F or lower than about -50° F can have a significant effect on the fatigue limit of steels (2). According to the data presented in (2), at low temperatures (to -200° F) carbon and alloy steel both possess significantly greater bending endurance strength. As the temperature is increased to approximately 700° F, carbon steels actually show a small improvement in endurance strength relative to room temperature values while the endurance strength of alloy steel (AISI-4340) slightly decrease (2). At elevated temperatures, above 800° F, the fatigue resistance of both types of steels drops sharply as the effects of creep and loss of material strength properties become more pronounced.

k_e, duty cycle factor. - Shafts are seldom exposed to constant loading in service. Start-stop cycles, transient overloads, vibrational or shock loading and changes in the load spectrum of the equipment driven by the shaft must be considered by the design. The principal question is how much endurance strength is left in the shaft material which has already been exposed to cyclic stress for given number of stress cycles.

Because fatigue is a cumulative stress cycle phenomena, occasional stop-start cycles and transient overloads totaling a relatively few stress cycles would be expected to have relatively little effect on fatigue life. A number of experimental investigations reported in (12) indicate that repeated application of stresses below the fatigue limit, that is understressing, may actually improve the material's endurance limit. Thus, for applications where the cyclic stresses vary in magnitude, but none exceed the endurance limit S_{re} of the material, $k_e = 1$ would provide a conservation design. However, shafts subjected to stresses greater than S_{re} (that is overstressing) for a significant number of stress cycles would adversely affect the material's endurance properties (12). At present, the available data is too inconsistant to quantify the duty cycle factor k_e for the effects of overstressing. Reference (1) discusses a potentially useful design method, which currently lacks sufficient supportive test data, to graphically adjust the endurance limit on an S-N diagram. For overstressing, some designers, e.g., (2), advocate a Miner's rule or linear cumulative damage theory approach. However, there is some experimental evidence (12) which indicates that the theory generally gives slightly overoptimistic results for steels when high stresses are applied first in the loading sequence.

k_f, fatigue stress concentration factor. - Experience has shown that a shaft fatigue failure almost always occurs at a notch, hole, keyway, shoulder or other discontinuity where the effective stresses have been amplified. The effect of a stress concentration on the endurance limit of the shaft is represented by the fatigue stress concentration factor k_f, where

$$k_f = \frac{\text{endurance limit of the notched specimen}}{\text{endurance limit of a specimen free of notches}} = \frac{1}{K_f} \qquad (5)$$

and where K_f = fatigue-strength reduction factor.

Experimental data (12) indicate that low strength steels are significantly less sensitive in fatigue to notches than high strength steels. The notch sensitivity, q, of material can be used to relate fatigue strength reduction factor K_f to the theoretical (static) stress concentration factor K_t as follows:

$$K_f = 1 + q(K_t - 1) \qquad (6)$$

The appropriate theoretical stress concentration factor, K_t to be used in eq. (6) is the value for bending. This is because the fatigue stress concentration factor, k_f is used to modify the specimen's bending endurance limit, S_{re}. Corroborating this approach is the data shown in fig. 1 from (3) which was generated with two different notch geometeries (K_t = 1.42 and 2.34 in bending) and yet follows the same failure line as given in eq. (1). Values for K_t and q can be found in several design references, such as (1, 2, 10, 12).

k_g, miscellaneous factors. - There are numerous material processing and service factors which are known to influence the endurance characteristics of the shaft but have not yet been fully quantified. These factors include, heat treatment processes such as carburizing, nitriding, flame-hardening, etc., which increase surface strength. Cold working processes, such as shot peening, rolling and drawing usually generate beneficial residual compressive stresses. Vacuum-processing of the steel melt would provide cleaner metallurgical structure with less defects and improved fatigue resistance. Stress corrosion and fretting corrosion, plating, and welding generally have an adverse affect on endurance. There are only some of the factors which should be considered when the application warrants it. A more thorough discussion of these and other miscellaneous fatigue factors can be found from several metal fatigue references such as (10, 12).

Shaft Design Example

The spindle drive shaft shown in fig. 5 is to be machined from AISI-C1045 steel, cold drawn to a Brinell hardness of 217. The spindle carrier a steady torque of 1000 in-lbs and rotates at 6 000 rpm under the loads shown. Operating temperatures are expected not to exceed 150° F and the operating environment will be noncorrosive. The shaft is to be designed for "infinite" life (greater than 10^6 cycles) for a survival rate of 90 percent.

The material properties of cold drawn, AISI-C1045 steel are given in (1) as

$$S_y = 90 \text{ ksi} \qquad S_u = 103 \text{ ksi}$$

When test data is not available for the endurance strength of the material, it is generally recommended (1, 2, 10, 12) that the endurance limit of polished steel specimens with tensile strengths less than 200 000 psi can be taken as 50 percent of the tensile strength, S_u. Thus the uncorrected endurance limit can be estimated as:

$$S_{re} = 0.5\ S_u = 51.5 \text{ ksi}$$

From fig. 4, for a machined shaft with S_u = 103 ksi,

$$k_a = 0.73$$

Estimating the shaft diameter to be less than 2 in. but greater than 0.3 in., k_b = 0.85. The design calls for a 90 percent survival rate, so k_c = 0.9.

The temperature will not be elevated, so k_d = 1.

The torque loading is applied steadily, so k_e = 1.

Finally the critical point along the shaft has been identified at a shoulder of 1/8 in. fillet radius. (See fig. 5). Tentatively selecting an estimated shaft diameter of 0.75 in., the theoretical bending stress concentration factors for a shaft fillet is K_t = 1.5 and notch sensitivity factor, q = 0.87 for steel with S_u = 103 ksi and a fillet radius = 0.125 in. from (2).

From eqs. (5) and (6) we can calculate the fatigue stress concentration factor.

$$k_f = \frac{1}{1 + 0.87\ (1.5 - 1)} = 0.70$$

Because of the noncorrosive environment and no unusual operating conditions, set $k_g = 1$.

We can now determine the corrected endurance strength by means of eq. (4)

$$S_e = k_a k_b k_c k_d k_e k_f k_g S_{re}$$

$$= (0.73)(0.85)(0.9)(1)(1)(0.70)(1)(51.6)$$

$$= 20.1 \text{ ksi}$$

The bending moment M_b at the shoulder is 108 in-lb as shown from fig. 5(b) and the torque T_M is given as 1000 in-lb. In conventional design applications, the margin of safety should be at least 100 percent for a safe design, so the Factor of Safety, FS, can be set equal to 2.

With eq. (3) and the above design variables, the required shaft diameter d is

$$d = \left[\frac{32(2)}{\pi}\sqrt{\left(\frac{108}{20\ 100}\right)^2 + \frac{3}{4}\left(\frac{1000}{90\ 000}\right)^2}\right]^{1/3}$$

$$= 0.61 \text{ in.}$$

This diameter is somewhat smaller than the first estimate of 0.75 in which was used to select K_t, so a new value of K_t can be selected based on $d = 0.61$ in. and the computation repeated.

Having determined the required shaft diameter to withstand fatigue loading, a calculation should be made to determine if this diameter is also sufficiently large to prevent elastic failure under the severest loading conditions. After determining that the shaft is sufficiently strong, the next step would be to calculate shaft deflections, particularly the shaft slope under the bearings and to check for critical speeds.

Concluding Remarks

A simple design formula for computing the diameter of rotating solid steel shafts under cyclic bending and steady torque has been presented. It considers the flexure fatigue characteristics of the shaft material and makes allowances for application factors which might reduce the endurance strength values from those cited in design tests for polished rotating-beam specimens. The design formula was predicated on an elliptical combined stress failure relation developed from fatigue test data published by two independent investigators. The design formula can also be theoretically derived from the distortion energy or von Mises-Henckey failure criterion. Based on the above, the proposed method seems to be a reasonable basis for a national standard shaft selection procedure. However the approach presented is far from being comprehensive. The effects of complex stresses on fatigue strengths of metals is not well understood. More experimental data is needed to increase confidence in the proposed method and to fill in gaps in our understanding of factors which influence fatigue strength.

In recognition of the work still needed to be done, the B106 Shafting Standards Committee has established a test program to investigate the effects of cyclic bending and the steady torsion on the fatigue characteristics of several common industrial shafting steels. The effects of mill condition, hardness and bending-stress concentration will also be examined as outlined in the test matrix appearing in table I from (13).

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TABLE I. - TEST SPECIMEN MATRIX FOR B106 COMMITTEE'S COMBINED STRESS FATIGUE TESTS

(FROM REF. (13))

Case*	Material	Mill condition	Tensile strength, psi		Brinell hardness number	Theoretical bending stress concentration factor	Cutoff limit, 10^6 cycles
			Yield	Ultimate			
I	AISI-1018	Hot-rolled	43 000	65 000	143	1.00	6
II	AISI-1045	Hot-rolled	59 000	98 000	212	1.00	6
III	AISI-1045	Hot-rolled	59 000	98 000	212	2.00	6
IV	AISI-4140	Cold-drawn	90 000	102 000	223	1.00	6
V	AISI-4140	Cold-drawn	90 000	102 000	223	2.00	6
VI	AISI-4140	Cold-drawn at 1000° F	131 000	153 000	302	1.00	20.4

*125 specimens are required for each case.

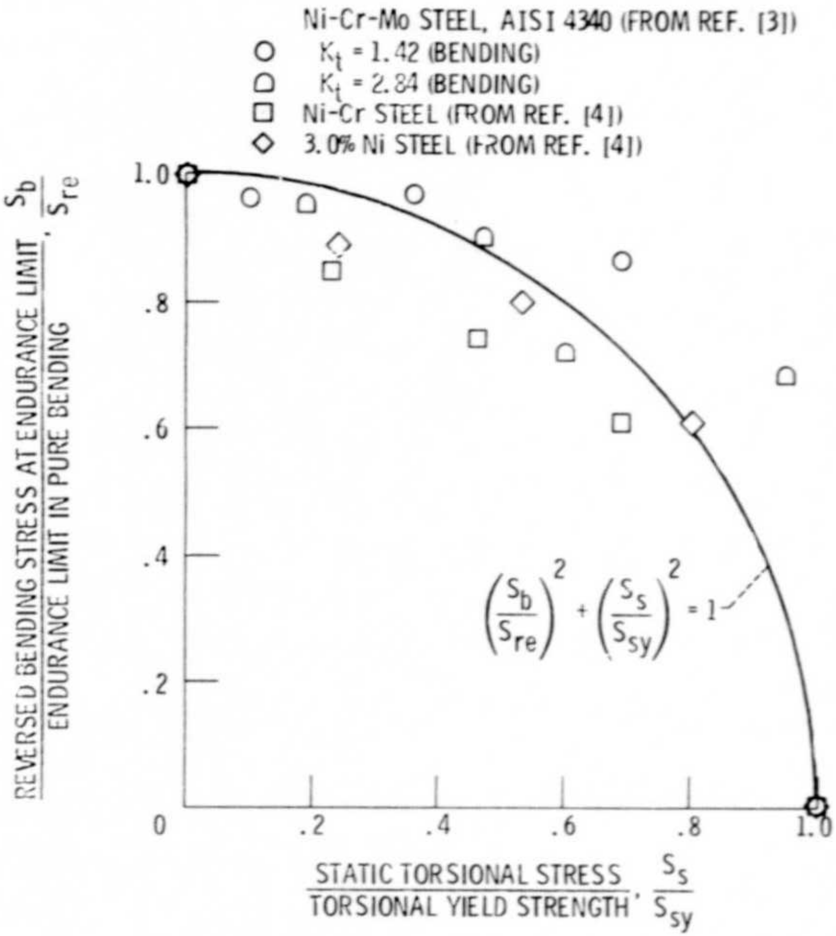

Figure 1. - Combined stress fatigue test data for reversed bending in combination with static torsion.

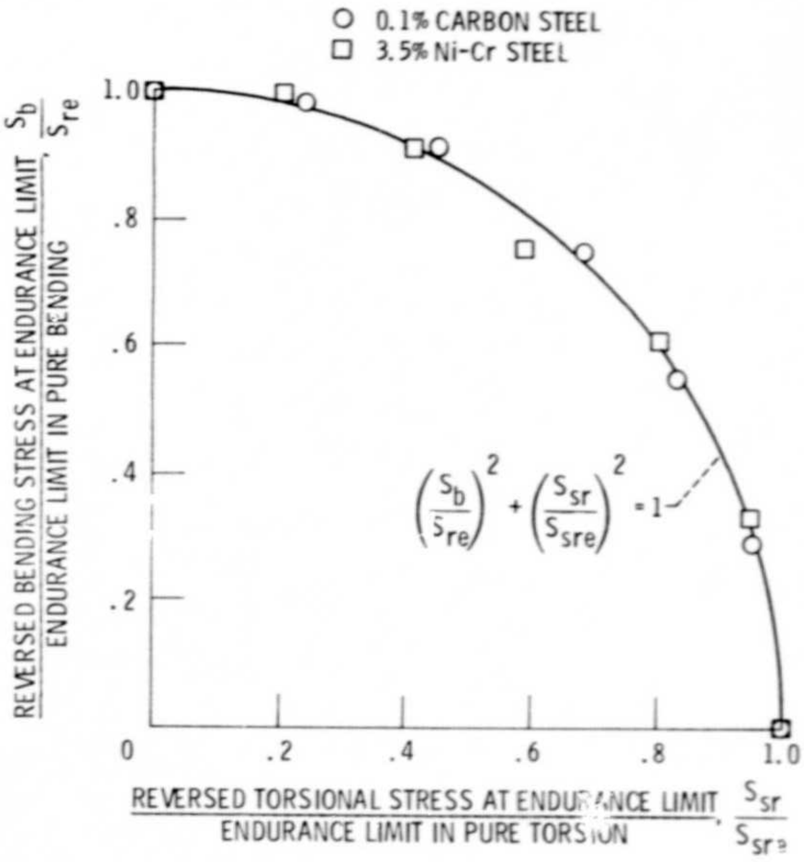

Figure 2. - Combined stress fatigue test data for reversed bending in combination with reversed torsion (from ref. [5]).

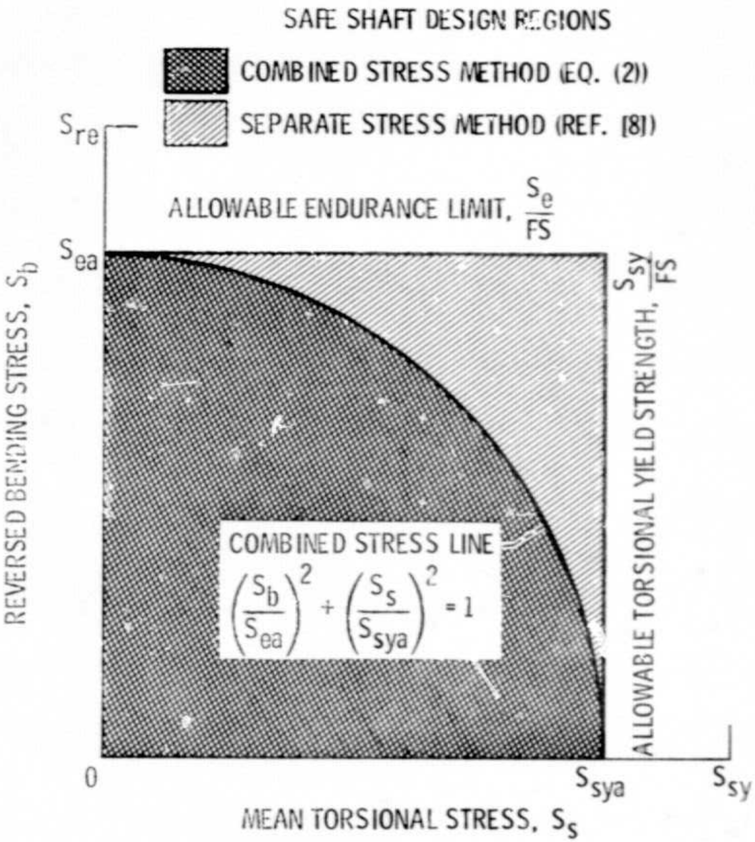

Figure 3. - Comparison of combined stress and separate stress design methods.

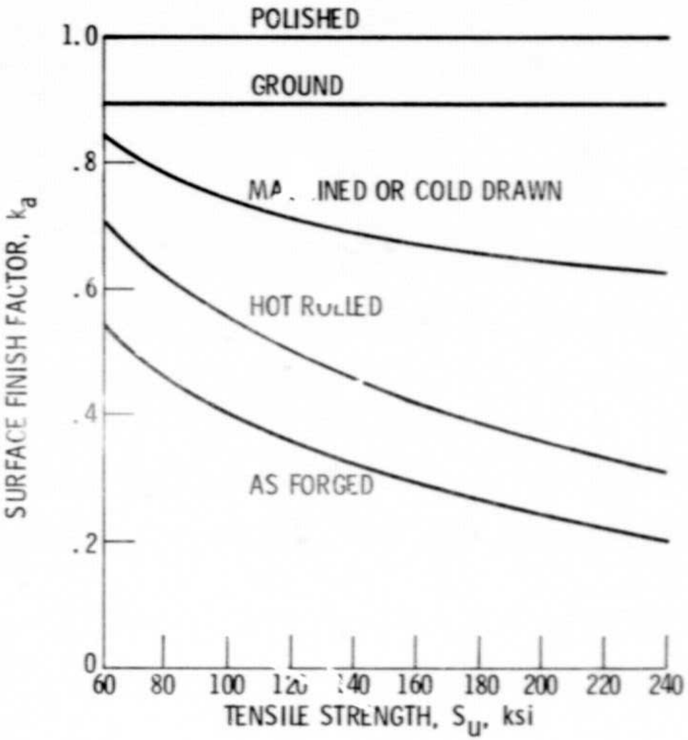

Figure 4. - Modifying factors for surface finish (from ref. [1]).

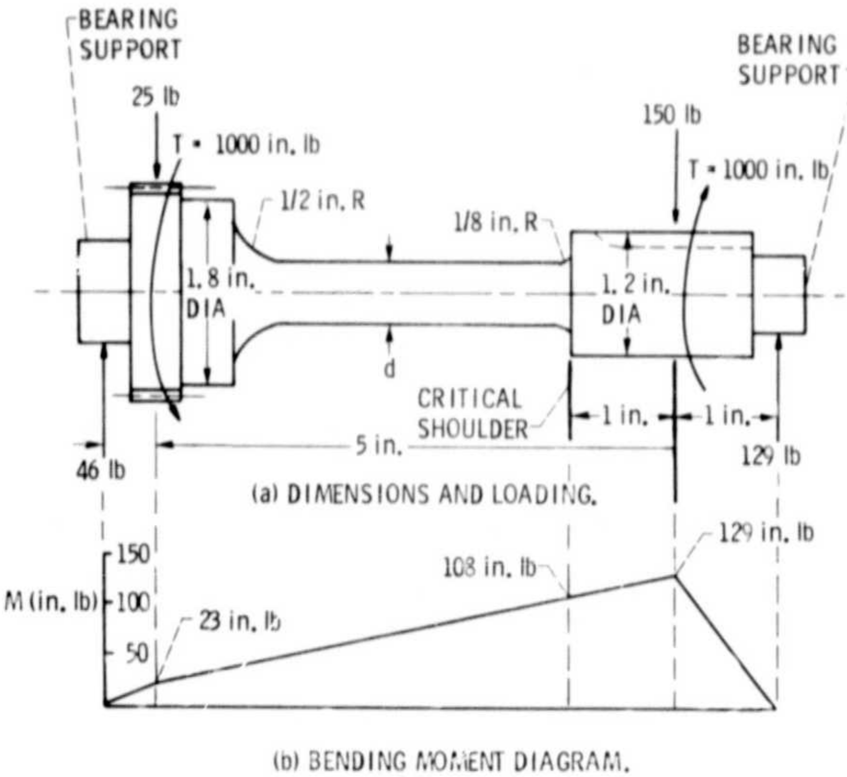

Figure 5. - Spindle drive shaft. Machined from AISI-C1045 steel (cold drawn).