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INTRODUCTION

A statistical analysis technique has been developed in this paper to estimate the snow temperature and mean crystal radius of snowfields over Greenland and Antarctica. Recent experimental work (Reference 1) has demonstrated that variations in crystal size and the physical temperature of snow field observations from space give large variations in the microwave brightness temperature. Since the observed brightness temperature for several wavelengths show distinct responses to snow parameters, the Scanning Multichannel Microwave Radiometer (SMMR) measurements on board a spacecraft would appear to be applicable in extracting the snow temperature and mean crystal radius profiles. The SMMR experiment, to be launched on board the Nimbus-G and Seasat-A spacecraft, will make observations in wavelengths of 0.8, 1.4, 1.7, 2.8, and 4.6 cm. The estimated error for the parameters derived from this technique is approximately 1.5 K for temperature and 0.001 cm for mean crystal radius in the presence of 1 K rms noise for each SMMR channel.

MICROWAVE EMISSION FROM SNOWFIELDS

The Mie scattering theory (Reference 2) was used to explain the phenomena of electromagnetic wave propagation within a snowfield. Since snow and ice fields generally consist of closely packed non-spherical particles, two assumptions had to be made before applying the Mie scattering theory. It was first assumed that the shape of snow and ice particles were spherical. Secondly, it was assumed that the particle scatters incoherently and was independent of the distance between scatterers. Under these two assumptions, the extinction and scattering cross section for a snowfield could be calculated by utilizing the Mie scattering theory. These quantities were subsequently used in solving the radiative transfer equation numerically.

In the microwave region, the Rayleigh-Jeans approximation to the plane radiative law applies quite well. Therefore, the intensity of thermal radiation is directly proportional to the brightness temperature. The radiative transfer equation (Reference 3) for a scattering media may be written as

$$\cos\theta \, \frac{dT_B(\theta,Z)}{dZ} + \gamma_{abs}(Z)\,(T_B(\theta,Z) - T(Z)) \tag{1}$$
$$= \frac{\gamma_{sca}(Z)}{2} \int_0^{\pi} T_B(\theta_s, Z)\, F(\theta,\theta_s)\, \sin\theta_s \, d\theta_s - \gamma_{sca}(Z)\, T_B(\theta,Z)$$

where θ is the propagative direction, $T_B(\theta,Z)$ is the brightness temperature at depth Z in the θ direction, T(Z) is the thermodynamic temperature of the absorbing media, $F(\theta,\theta_s)$ is the scattering phase function, $\gamma_{abs}(Z)$ and $\gamma_{sca}(Z)$ are the volume absorption and scattering coefficient at depth Z, respectively.

In Camp Century, Greenland, the approximate crystal size for the top 20 m firn cover was 0.5 mm (Reference 4) which is approximately one-twentieth the size of the shortest SMMR wavelength. The size of the crystal is well within the range of the Rayleigh equation so that the Rayleigh scattering calculations can be used instead of the Mie scattering equation to economize on computer time. The radiative transfer equation is then solved by the invariant imbedding technique (References 5 and 6).

When the snow depth is great, the reflection and emission from the ground surface, below the snowfield, can be neglected and the brightness temperature can be expressed as

$$T_B(\theta,0) = \int_0^{\infty} T(Z)\,\gamma(Z)\, e^{-\int_0^Z \gamma(Z)\sec\theta\, dZ}\, dZ. \tag{2}$$

The factor multiplying T(Z) in the integrand of equation (2) is by definition the temperature weighting function. It represents the effective radiative depth within the medium of the snowfield. Figure 1 shows the calculated temperature weighting function for each of the SMMR channels.

STATISTICAL RETRIEVAL TECHNIQUE

Snow temperature, at discrete levels, can be estimated from SMMR measurements by correlation with the temperature over the weighting function layer. Since the correlation is not exact, the derived temperature at discrete levels from SMMR data will not be as accurate as the average temperature over the weighting function layer.

The method used for obtaining the temperature and mean radius profiles from SMMR measurements are essentially a regression analysis of snowfield temperature and mean crystal radius, and the numerically-computed snow emission. In a statistical sense, it was attempted to minimize the mean-square deviation of the estimated profile from the *a priori* profile.

The statistical retrieval method used in this study is similar to those employed by C. D. Rodgers (Reference 7) and N. E. Gaut (Reference 8). Let us consider an ensemble consisting of sets of snow temperature profiles T(Z) or mean crystal radii r(Z) whose elements give the temperature at discrete depths of the snowfield and the corresponding ten channels of brightness temperature $\phi(T_B)$. It is assumed that the snow temperature T(Z) and mean crystal radius r(Z) are related to the brightness temperature $\phi(T_B)$ by matrix D and D′ respectively

$$T(Z) = D \cdot \phi(T_B) \tag{3}$$

$$r(Z) = D' \cdot \phi(T_B). \tag{3'}$$

The measured data vector $\phi(T_B)$ used for determining the snow temperature is defined as the measured brightness temperature minus the ensemble mean

$$\phi(T_B) = \begin{bmatrix} 1 \\ T_{B_{1V}} - \overline{T}_{B_{1V}} \\ T_{B_{1H}} - \overline{T}_{B_{1H}} \\ ,, \\ ,, \\ ,, \\ ,, \\ T_{B_{5V}} - \overline{T}_{B_{5V}} \\ T_{B_{5H}} - \overline{T}_{B_{5H}} \end{bmatrix} \tag{4}$$

where T_{B_n} is the brightness temperature from SMMR channel n and an overbar indicates its ensemble mean. The inferred snow temperature and mean crystal radius vector were chosen to have the elements corresponding to the depth levels of: (1) surface, (2) 1 meter, (3) 4 meters, (4) 7 meters, and (5) 10 meters of the snowfields which approximately relate to the peak of the weighting functions.

Matrix D was chosen to minimize the expected error between the SMMR determined profile and the actual temperature profile, given by

$$D = C(T,\phi)\, C^{-1}(\phi,\phi) \tag{5}$$

$$D' = C(r,\phi)\, C^{-1}(\phi,\phi) \tag{5'}$$

where C is a correlation matrix, the ij th elements is $C_{ij}(X,Y) = X_i Y_j$ and the C_{ij} can be calculated from snow statistics. When $C(\phi,\phi)$ is singular or nearly singular its inverse is undefined. Under this condition, the matrix inverse calculated for this matrix $C^{-1}(\phi,\phi)$ will introduce considerable errors. The singularity problem was avoided by including a random noise in the simulated brightness temperatures. This method of handling singularities is similar to J. W. Waters et al. (Reference 9). To derive the D or D′ matrix from equation (5) or (5′), it is necessary to have the $C(T,\phi)$ and $C(r,\phi)$ in cross-correlation matrices of the snow temperature and mean crystal size with the measured data vector, and $C^{-1}(\phi,\phi)$ the inverse of the auto correlation matrix of the data vector. Since it does not appear to be practical to collect the necessary ensemble by measurements, the two vectors were obtained from *a priori* measured field data and the radiative transfer calculations. The snow data used were compiled by H. J. Zwally (Reference 10) with measurements taken from several stations in Greenland and Antarctica. In Table 1 the mean annual surface temperature (T_m) and the scattering coefficient per unit length (γ_s) are listed for each station location. The physical temperature profile of the snowfield can be expressed as a function of T_m, Z, and t (time of year).

$$T(Z) = T_m - 15e^{-0.3Z} \cos [0.99(t-84)-(97+20Z)] \qquad (6)$$

The maximum surface temperature occurs at t = 0 and the minimum surface temperature at t = 365/2. By utilizing equation (6) and the scattering coefficient, it is possible to determine the expected brightness temperature emerging from the snowfield for various snow conditions.

Table 1

Location and Mean Annual Surface Temperature and Scattering Per Unit Length (Zwally, 1977)

Location	T_m * (K)	$\gamma_s(m^{-1})$
South Pole, Antarctica 90°S	222	0.222 + 0.00863Z
Plateau, Antarctica 79°15′S; 40°30′E	216	0.220 + 0.0275Z
Camp Century, Greenland 77°11′N; 61°10′W	249	0.163 + 0.0647Z
Byrd, Antarctica 79°59′S; 120°01′W	245	0.152 + 0.0968Z
Inge Lehmann, Greenland 77°57′N; 39°11′W	243	0.162 + 0.1178Z
Site 2, Greenland 76°59′N; 56°04′W	249	0.0921 + 0.0212Z
South Ice, Antarctica 81°57′S; 28°50′W	242	0.0422 + 0.0805Z

*Mean annual surface temperature.
Z is snow depth in meters.

To demonstrate the behavior of such an estimation scheme, an ensemble of eighty-four different snow temperature and mean crystal radius profiles were used to generate the combination of snow crystal and temperature profiles from the surface, down to 25 m in depth, in 1 m intervals. In Figures 2 and 3, the ensemble mean profile is shown along with the standard deviation for the ensemble at each level. Also plotted is the residual standard deviation, where the rms noise is 1 K for each SMMR channel. Figures 2 and 3 demonstrate that the variance is reduced, in this case, where the SMMR data is used in retrieval as compared with the *a priori* statistical ensemble.

To test the behavior of the retrieval scheme, Figures 4 and 5 show the results of retrieval by using statistical data. For the Camp Century, Greenland site (77°11'N, 61°10'W), both the mean crystal profile and snow temperature profile are similar to the ensemble mean. Under these conditions, the retrieved parameters fit very well with the known parameters. In the South Pole site (90°S), the snow crystal radius profile differs considerably from the ensemble mean. Hence, the retrieved crystal radius profile does not fit as well as in the previous case. For the temperature profile, the retrieved data still gives good estimates as the variability of the snow temperature is much less than the mean snow crystal size.

CONCLUSION

A statistical retrieval technique has been developed to estimate the snow temperature and mean crystal radius profiles from SMMR data. This technique is used in the study of the snow temperature and mean crystal radius profiles of snowfields over Greenland and Antarctica where the snow truth is difficult to obtain by conventional methods.

The accuracy of retrieval is influenced by the variance of the parameters in the chosen statistical ensemble, and the success of this method depends on having a good estimate for the snow temperature and crystal size correlations with the brightness temperatures for the location in question. The ensemble used consists of data from stations with different snow accumulation characteristics at various times of the year. The detailed structure of the snowfield, such as the ice layers, are not included as further study is required to determine from the geographical locations which method might be reliably used. In addition, the effect of the atmosphere on the retrieved data should be taken into account.

The computational method presented in this paper may also provide a basis for utilizing SMMR data to further develop a new retrieval technique for eventual incorporation into satellite weather and climate data collecting systems.

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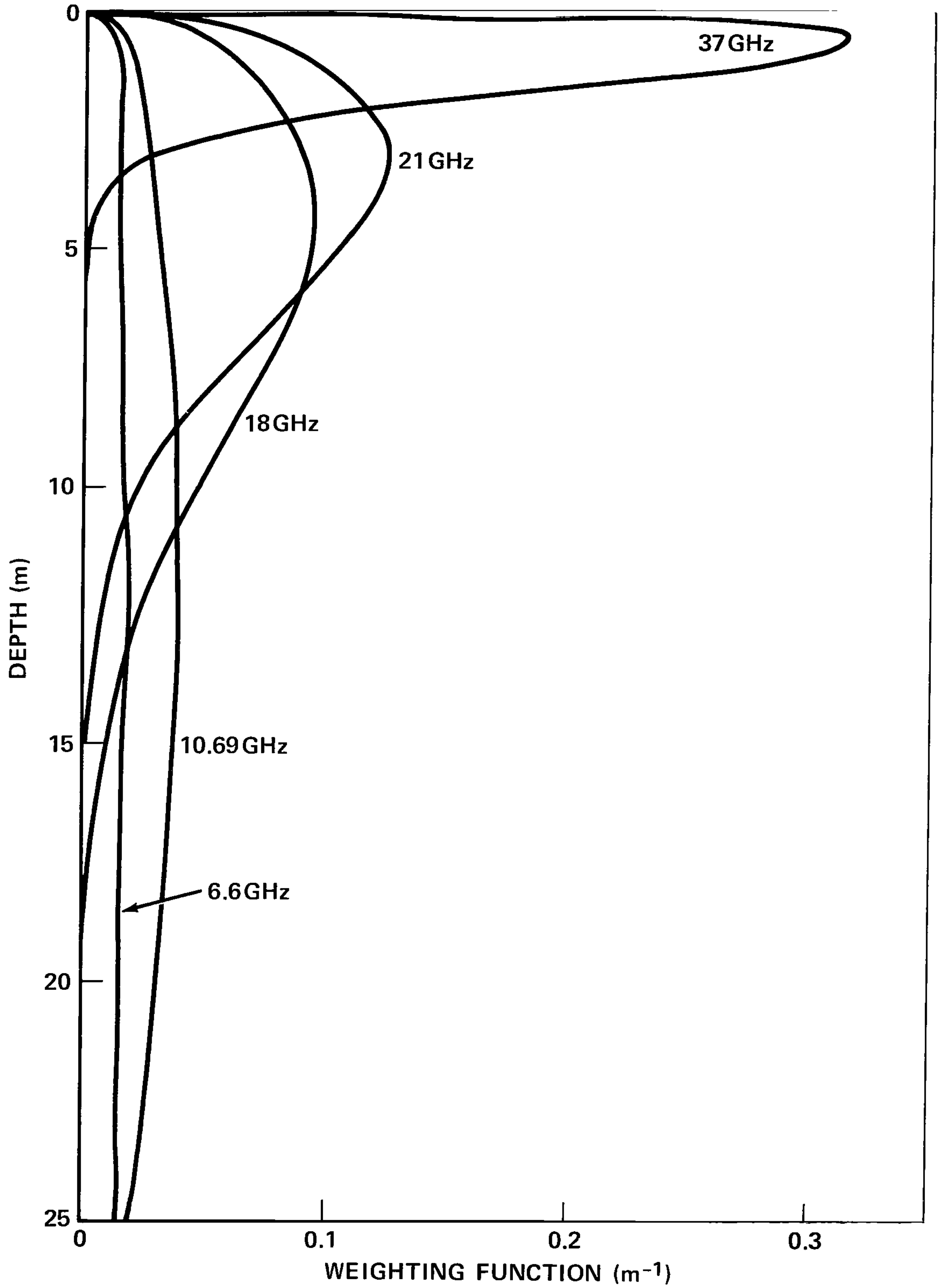

Figure 1. The weighting function for SMMR frequencies.

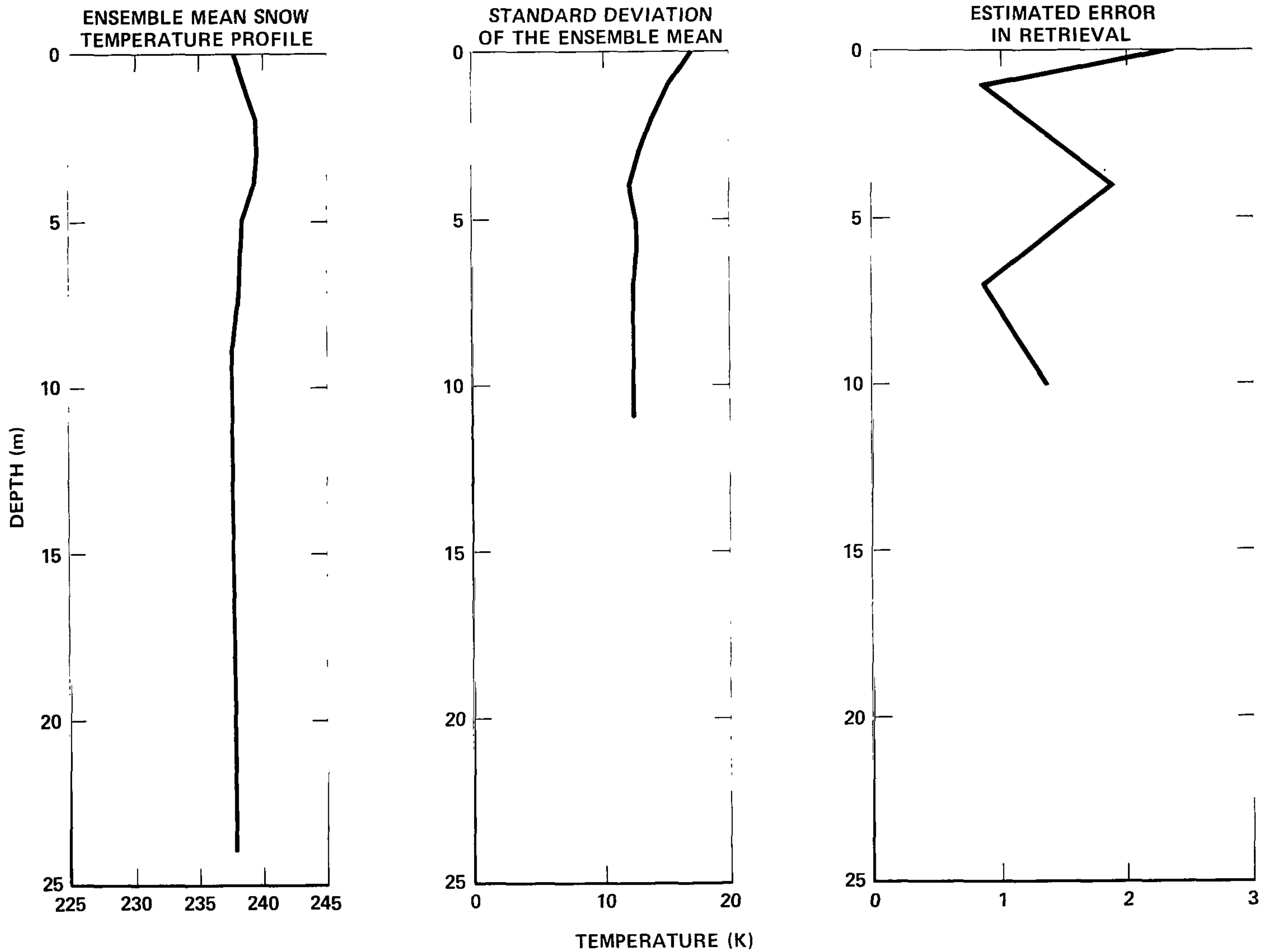

Figure 2. The ensemble mean temperature, standard deviation of snowfields, and the estimated error in temperature retrieval.

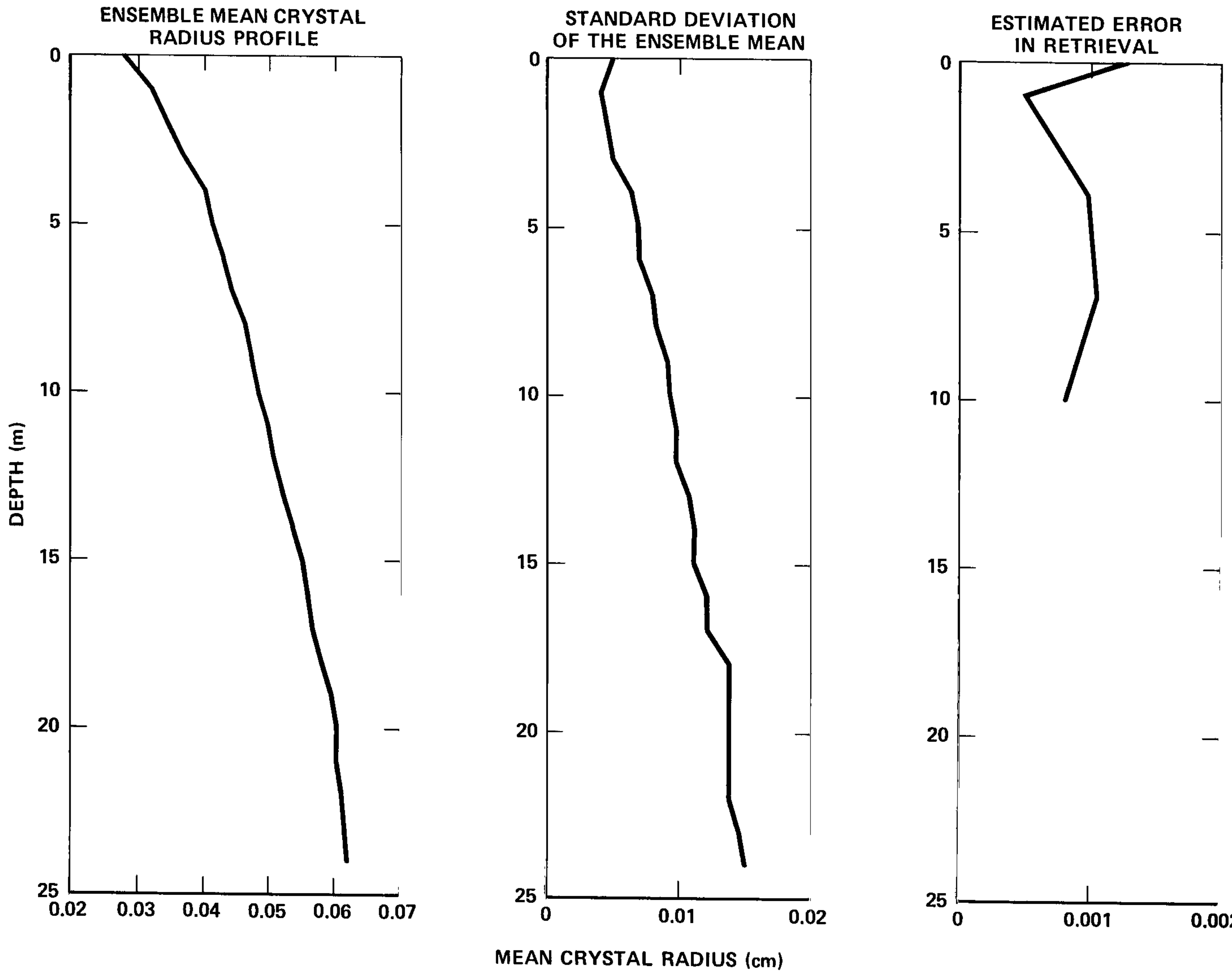

Figure 3. The ensemble mean snow crystal, standard deviation of snowfields, and the estimated error in retrieval.

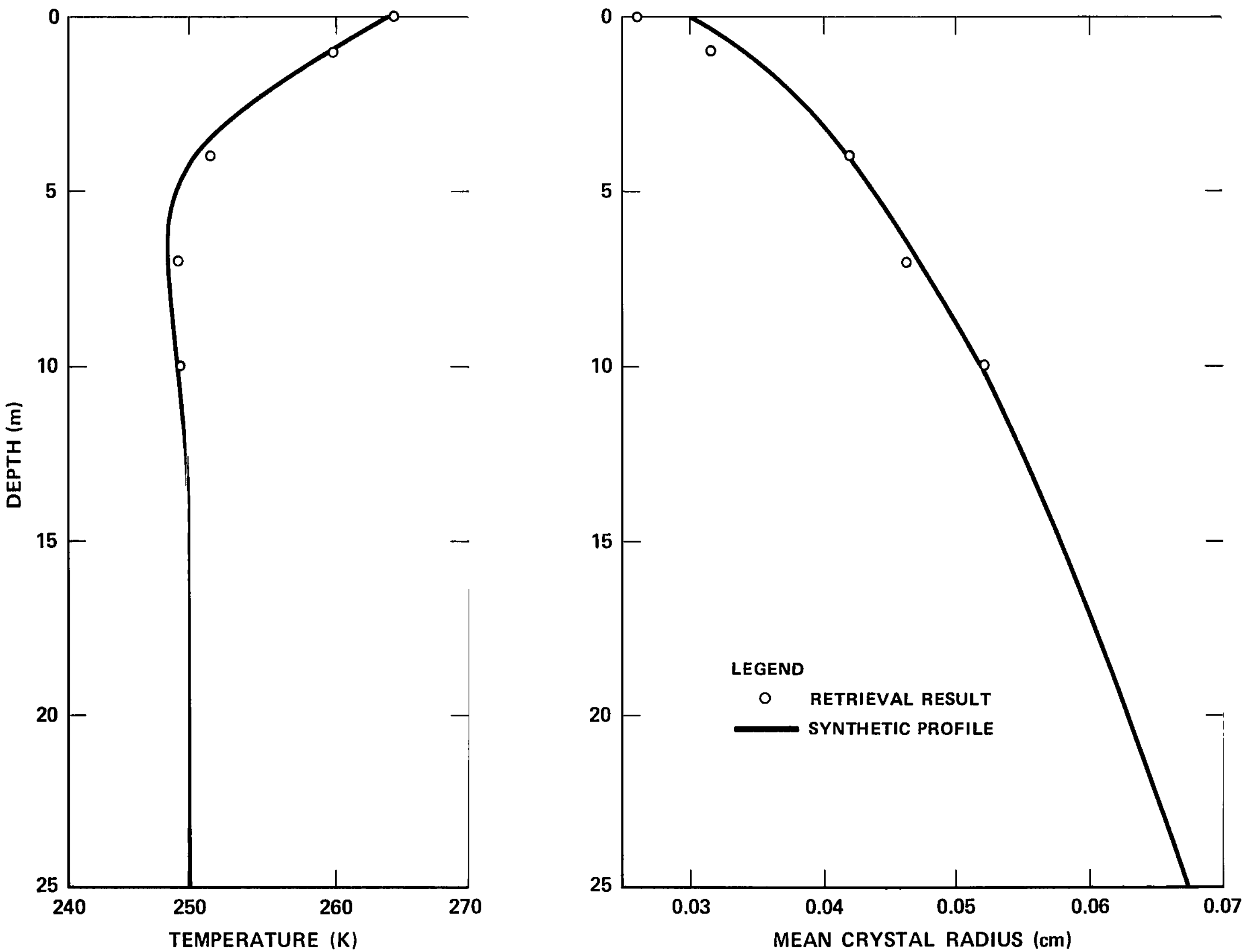

Figure 4. Retrieval data from Camp Century, Greenland (summer profile).

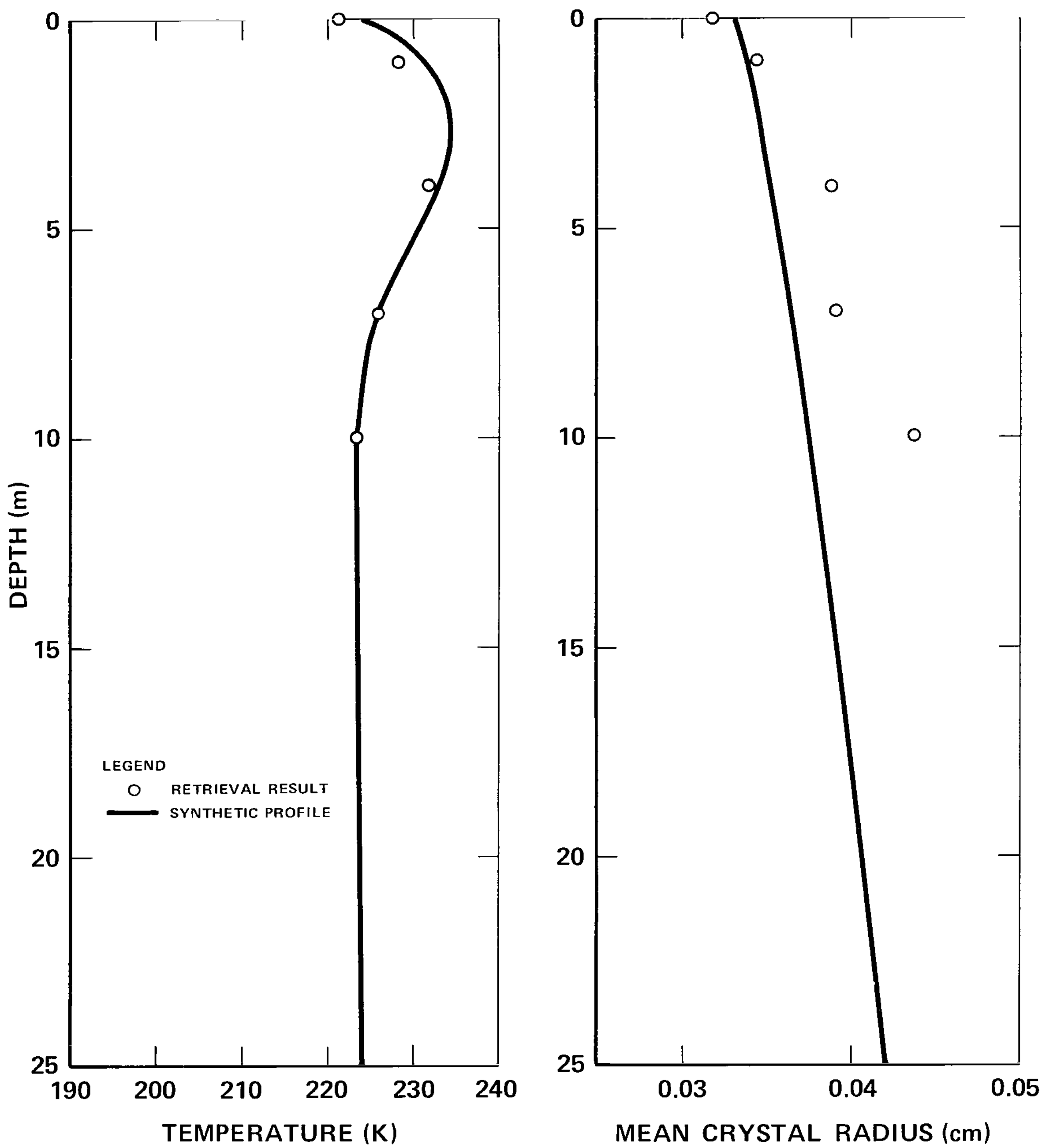

Figure 5. Retrieval data from South Pole (winter profile).

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A statistical retrieval method has been developed in this paper to determine the snowfield temperature profile and mean crystal size by using the scanning multifrequency microwave radiometer on board a spacecraft. The estimated errors for retrieval are approximately 1.5 K for temperature and 0.001 cm for crystal radius in the presence of 1 K rms noise for each SMMR channel.</td></tr>
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