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TECHNIQUES FOR CORRECTING APPROXIMATE FINITE DIFFERENCE SOLUTIONS

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SUMMARY

A method of correcting finite-difference solutions for the effect of truncation error or the use of an approximate basic equation is presented. Applications to transonic flow problems are described and examples given.

INTRODUCTION

Probably the paramount objective in computational fluid dynamics is to reduce the computing cost while still obtaining a solution of acceptable accuracy. The most direct way of reducing the cost is by algorithm improvement but there are other ways of cost reduction for practical applications. One way is to use a coarse discretization in the numerical procedure which reduces the number of data points to be computed. Another method is to use some approximate form of the governing equations or boundary conditions that leads to an easier and more rapid computation. While generally representing all of the essential features of the complete solution these approximate solutions can fail to accurately capture important finer details. However, if the approximate solutions could be easily corrected in some way to account for these deficiencies and if the correction is universal or even partially universal then computing costs could be reduced without sacrificing accuracy. It is the derivation of such corrections that is discussed in this paper.

The particular problems addressed in the present work are concerned with finite difference calculations of two-dimensional transonic flow problems, an important area of aerodynamics research. A common simplification in transonic flow calculations is the use of the approximate transonic small disturbance equation with thin wing boundary conditions rather than the full potential equation with an exact treatment of the boundary conditions. The error introduced by this approximation and the means of correction are discussed in this paper. A second form of approximation discussed is the use of a coarse finite difference mesh which is then corrected to give a solution typical of a much more refined mesh. Basically the correction method devised for both these cases is an extrapolation from the approximate solution to the "exact" solution. For the transonic flow problems considered, this correction is obtained only for a "similar" or "nearby" solution, for example, the flow around a different airfoil which has all the features of the problem to be corrected and which has much the same pressure distribution. In this way the same correction can be used for all "nearby" problems, in fact a partially universal correction.

Because of the general nature of the pressure distributions in transonic flow calculations in which shock waves of shock capture regions and regions of rapid pressure change occur a modification of the method of strained coordinates developed by Nixon (ref. 1) is used. In this technique the coordinates are strained such that all the rapidly varying parts of the solution are constrained to the same location which improves the range of validity of the corrections.

Results for the pressure distribution around several airfoils with corrections for both mesh size and the use of approximate equations or boundary conditions are presented showing the satisfactory application of the theory. The most important restriction in the choice of both corrections and approximate solutions is that they *must* represent *all* the essential features of the final solution.

BASIC THEORY FOR CORRECTIONS

Consider the mathematical problem defined by

$$L(\phi) = 0 \tag{1}$$

and the boundary condition

$$B(\phi) = F(x_i) \quad \text{on some boundary C} \tag{2}$$

where $L(\ )$ is a differential operator and $B(\ )$ is an operator which may be differential; $F(x_i)$ is a specified function of the coordinates x_i.

It is proposed to solve the system equations (1) and (2) by a finite difference method. If $L_\Delta(\ )$ and $B_\Delta(\ )$ denote difference operators corresponding to $L(\ )$ and $B(\ )$ in some computational mesh characterized by Δ then the difference equations are

$$L_\Delta(\phi_\Delta) = 0 \tag{3}$$

$$B_\Delta(\phi_\Delta) = F(x_i) \tag{4}$$

where ϕ_Δ is the converged solution of the difference equation.

For a given difference scheme

$$\phi = \phi_\Delta + \Delta^n R(\phi_\Delta) \tag{5}$$

where $R(\phi_\Delta)$ depends on the essential character of the solution, that is on ϕ_Δ and its derivatives; the exponent n depends on the order of accuracy of the difference scheme. As $\Delta \rightarrow 0$ the difference solution should approach the exact solution. If it is assumed that Δ is sufficiently small that all the essential features of the solution are represented by ϕ_Δ then $R(\phi_\Delta)$ does not change significantly with decreasing Δ. In this case the difference solution ϕ_Δ will approach the exact solution ϕ monotonically as $\Delta \rightarrow 0$.

Consider now the case where two difference meshes, characterized by Δ_1 and Δ_2, are used where $\Delta_2 < \Delta_1$. Then from equation (5)

$$\phi_{\Delta_2} - \phi_{\Delta_1} = -\left[\Delta_2{}^n R_2\left(\phi_{\Delta_2}\right) - \Delta_1{}^n R_1\left(\phi_{\Delta_1}\right)\right] \tag{6}$$

where R_1 and R_2 are the two error terms in the finite difference approximation. It is assumed that the solution on the Δ_1 mesh is a fairly good approximation to the solution on the fine mesh Δ_2. Then the quantity

$$\left[\Delta_2{}^n R_2\left(\phi_{\Delta_2}\right) - \Delta_1{}^n R_1\left(\phi_{\Delta_1}\right)\right]$$

can be assumed to be small.

A trivial relation connecting ϕ_{Δ_2} and ϕ_{Δ_1} is

$$\phi_{\Delta_2} = \phi_{\Delta_1} + \left(\phi_{\Delta_2} - \phi_{\Delta_1}\right) \tag{7}$$

From equation (6), equation (7) the correction

$$\phi_{\Delta_2} - \phi_{\Delta_1} \sim \underline{0}\left[\Delta_2{}^n R_2\left(\phi_{\Delta_2}\right) - \Delta_1{}^n R_1\left(\phi_{\Delta_1}\right)\right] \tag{8}$$

which is small and as $\Delta_1, \Delta_2 \to 0$ the correction is zero.

A correction for the coarse mesh solution ϕ_{Δ_1} could be obtained from equation (6) if the error terms $R_1(\phi_{\Delta_1})$, $R_2(\phi_{\Delta_2})$ were known — an almost impossible task a priori. However, if some similar or "nearby" solution to ϕ_Δ is known, denoted by $\bar{\phi}_\Delta$, and if it is assumed that

$$\left(\bar{\phi}_{\Delta_2} - \bar{\phi}_{\Delta_1}\right) = \left(\phi_{\Delta_2} - \phi_{\Delta_1}\right) + \underline{0}\left[\epsilon_1\left(\phi_{\Delta_2} - \phi_{\Delta_1}\right)\right] \tag{9}$$

where ϵ_1 is some small parameter then an approximation to equation (7) is

$$\phi_{\Delta_2} = \phi_{\Delta_1} + \left(\bar{\phi}_{\Delta_2} - \bar{\phi}_{\Delta_1}\right) + \underline{0}\left\{\epsilon_1\left[\Delta_2{}^n R_2\left(\phi_{\Delta_2}\right) - \Delta_1{}^n R_1\left(\phi_{\Delta_1}\right)\right]\right\} \tag{10}$$

It is assumed that solutions for the nearby problem are known for both Δ_1 and Δ_2 mesh systems. Since both ϵ_1 and $\{\epsilon_1[\Delta_2{}^n R_2(\phi_{\Delta_2}) - \Delta_1{}^n R_1(\phi_{\Delta_1})]\}$ are small quantities then a good approximate correction is given by

$$\phi_{\Delta_2} = \phi_{\Delta_1} + \left(\bar{\phi}_{\Delta_2} - \bar{\phi}_{\Delta_1}\right) \tag{11}$$

Hence, if the correction for truncation error (from the Δ_1 mesh to the Δ_2 mesh) is known for the nearby problem $\bar{\phi}_\Delta$ by direct comparison then equation (11) will correct the solution ϕ_{Δ_1} to the order of accuracy implied in equation (10).

A second form of correction is when a simplified equation or boundary condition is used instead of equations (1) and (2). Thus, the system solved is

$$\hat{L}(\phi) = 0 \tag{12}$$

with the boundary condition

$$\hat{B}(\phi) = F(x_i) \quad \text{on} \quad \hat{C} \tag{13}$$

where $\hat{L}(\;)$ and $\hat{B}(\;)$ are operators that give approximate forms of the equations (1) and (2); the boundary $\hat{C}$ may be an approximation to the exact boundary C. As before, these equations will be solved by finite differences with the difference operator

$$\hat{L}_\Delta(\hat{\phi}_\Delta) = 0 \tag{14}$$

and boundary conditions

$$\hat{B}_\Delta(\hat{\phi}_\Delta) = F(x_i) \quad \text{on} \quad \hat{C} \tag{15}$$

where $\hat{\phi}_\Delta$ is the converged solution of the discretized problem defined by equations (14) and (15). It is assumed in simplifications of this nature that the solution of the system, equations (14) and (15), is a good approximation to the accurate solution of equations (3) and (4). Thus

$$\hat{\phi}_\Delta = \phi_\Delta + \underline{O}(\varepsilon_2\phi_\Delta) \tag{16}$$

where ε_2 is some small quantity.

As before, a trivial relation between $\hat{\phi}_\Delta$ and ϕ_Δ is given by

$$\phi_\Delta = \hat{\phi}_\Delta + (\phi_\Delta - \hat{\phi}_\Delta) \tag{17}$$

and from equation (16)

$$(\phi_\Delta - \hat{\phi}_\Delta) \sim \underline{O}(\varepsilon_2\phi_\Delta) \tag{18}$$

If a solution to a nearby problem, denoted by $\bar{\phi}_\Delta$, for both the system equations (3) and (4) and the simplified system equations (14) and (15) is known then a correction to $\hat{\phi}_\Delta$ is given by

$$\phi_\Delta = \hat{\phi}_\Delta + \left(\bar{\phi}_\Delta - \hat{\bar{\phi}}_\Delta\right) \tag{19}$$

where $\hat{\bar{\phi}}_\Delta$ is the solution of the nearby problem using the simplified operators $\hat{L}_\Delta(\;)$ and $\hat{B}_\Delta(\;)$.

If

$$\left(\bar{\phi}_{\Delta} - \hat{\bar{\phi}}_{\Delta}\right) = \left(\phi_{\Delta} - \hat{\phi}_{\Delta}\right) + \underline{O}\left[\varepsilon_3\left(\phi_{\Delta} - \hat{\phi}_{\Delta}\right)\right]$$

where ε_3 is a small quantity then the formal accuracy of equation (19) is $O(\varepsilon_2\varepsilon_3)$.

Hence, if the correction for the simplified problem (from $L(\ )$ to $\hat{L}(\ )$, etc.) is known for the nearby problem, then equation (19) will correct the solution $\hat{\phi}_{\Delta}$ to an order of accuracy of $(\varepsilon_2\varepsilon_3)$.

Since equation (11) and equation (19) are linear the principle of superposition can be used. Thus, if a solution to the problem defined by equations (12) and (13) is known for some mesh Δ_1 and the solutions for a "nearby" problem are known for the system equations (12) and (13) on the meshes characterized by Δ_1 and Δ_2 and also for the exact formulation, equations (3) and (4), then the solution for the exact problem on the Δ_2 mesh is given approximately by

$$\phi_{\Delta_2} = \hat{\phi}_{\Delta_1} + \left(\hat{\bar{\phi}}_{\Delta_2} - \hat{\bar{\phi}}_{\Delta_1}\right) + \left(\bar{\phi}_{\Delta_1} - \hat{\bar{\phi}}_{\Delta_1}\right) \qquad (20)$$

Hence, once the correction is known for the nearby solution only the coarse mesh solution is required in order to obtain an accurate estimation of the exact solution.

The principles outlined in this section can be applied to other approximations in addition to those considered above.

The usefulness of the correction theory depends on the range of applicability of a given correction, that is, how many problems can be satisfactorily corrected by a given nearby solution. This can only be determined by experiment.

The basis of the present theory depends on the availability of the nearby solutions and that the changes due to the corrections are small. The validity of a perturbation (correction) as outlined in this section requires that the changes due to the correction are smooth. In solutions with discontinuities, for example, transonic flows with shock waves, which is the application considered here, the change due to the correction is not small in the region traversed by the discontinuity. In order to overcome difficulties associated with moving discontinuities the method of strained coordinates developed by Nixon (refs. 1 and 2) is used. This technique is briefly described and extended in the next section.

METHOD OF STRAINED COORDINATES

The method of correcting approximate solutions outlined in the previous section requires that the changes due to the correction are small. In cases containing discontinuities which can alter location when the correction terms are added, the changes are not small in the region traversed by the discontinuity. A means of treating the problem of perturbations in discontinuous transonic flow has been described by Nixon (ref. 1); an outline is given below.

Briefly, the idea is that the problem is reformulated in a strained coordinate system in which the discontinuity remains at the same location throughout the perturbation and hence the difficulties associated with moving discontinuities do not arise explicitly. The required straining is then found as part of the solution. The basic equations in this strained coordinate system are then perturbed about some known solution to give a linear equation for the perturbation quantities similar to those discussed in the previous section. Once the solution of the linear perturbed equation is known, the total perturbed solution in the physical coordinates is then obtained. The major restriction is that the discontinuities must not be lost or generated during the perturbation.

The technique described above was originally developed to treat the discontinuities which can invalidate a perturbation analysis. However, the technique can also be applied to increase the range of application of a valid perturbation analysis. An example from transonic airfoil theory concerns the pressure distribution around an airfoil when shock waves are present; such a pressure distribution is sketched in figure 1. The solid and dashed lines denote two nearby solutions for the pressure distribution. The solution shock waves are captured, that is, the expected discontinuity is smeared over a few mesh spacings and denoted by CD and C'D'. The method of strained coordinates, as given in reference 1, would strain the x-coordinate such that the midpoints of the shock capture regions CD and C'D' coincide. The actual details of the shock capture region are not considered since they are in any event an artificial phenomena.

It can be seen from figure 1 that in the leading edge region the rapid change in the pressure distribution can cause large pressure changes for a small perturbation if the location of the pressure rise shifts slightly in the x-direction. This large effect seriously limits the range of validity of the perturbation analysis since all pressure changes are assumed small. A method of avoiding this difficulty is to strain the coordinates such that representative point on the AB and A'B' curves coincide. This then increases the range of validity in a similar way as the treatment of the shock waves in reference 1.

A further point concerns the treatment of the shock capture regions CD and C'D'. In the earlier applications (refs. 1 and 2) of the theory the same mesh and differential equations were used for computing the pressure distribution in all examples and hence the shock capture characteristics were essentially the same for all cases. For other problems, for example, correction of a coarse mesh solution (truncation error), the shock capture characteristics may differ substantially and it is desirable to correct this behavior. Accordingly, the coordinates are strained such that both the points C,C' and D,D' (the extremities of the shock capture) coincide. As before, the actual flow details in the region CD are considered irrelevant because of the artificial nature of the shock capture.

A more general statement of the above technique is as follows.

1. If a true discontinuity is present, the coordinate straining is such that the location of the discontinuities coincide.

2. If there is a shock capturing type of phenomena, then the straining is such that the extremities of the capture region coincide.

3. If large gradients are present in the solution, then the coordinate straining is chosen so that a representative point in the region of the large gradient coincides.

These conditions constitute perhaps a large number of requirements for the choice of straining. However, a piecewise straining is perfectly feasible provided the end points of the straining (which do not move) lie in regions of the solution for which a small perturbation analysis is valid, for example, in the region BC of figure 1.

APPLICATIONS TO TWO-DIMENSIONAL TRANSONIC FLOW COMPUTATIONS

This section is concerned with the computation of the pressure (or velocity) distribution around airfoils in transonic flow using either the full potential equation or the transonic small disturbance equation.

If $\phi(x, y)$ is the perturbation velocity potential in the Cartesian coordinates (x, y) and ρ is the density then the full potential equation is

$$[\rho(1 + \phi_x)]_x + [\rho\phi_y]_y = 0 \tag{21}$$

where the Cartesian velocity components u, v are given by

$$u = 1 + \phi_x , \quad v = \phi_y \tag{22}$$

and

$$\rho = \left[1 + \left(\frac{\gamma - 1}{2}\right) M_\infty^2 (1 - u^2 - v^2)\right]^{(1/\gamma - 1)} \tag{23}$$

where M_∞ is the free-stream Mach number. Both u and v are scaled with respect to the free-stream velocity; γ is the ratio of specific heats.

The transonic small disturbance equation is

$$(1 - M_\infty^{\;2})\phi_{xx} + \phi_{yy} = (\gamma + 1)M_\infty^{\;q}\phi_x\phi_{xx} \tag{24}$$

where q is an arbitrary parameter.

The tangency boundary condition for equation (21) is

$$\frac{\phi_y(x,y_s)}{1 + \phi_x(x,y_s)} = \frac{v(x,y_s)}{u(x,y_s)} = y_s'(x) \tag{25}$$

where $y = y_s(x)$ specifies the geometry of the airfoil.

The thin airfoil boundary conditions for equation (24) are

$$\phi_y(x, \pm 0) = y_s'(x) \tag{26}$$

where $+0$, -0 denote values on the upper and lower surfaces of the airfoil chord line, respectively.

In the strained coordinate system the x-coordinate is replaced by x' defined as

$$x = x' + \varepsilon x_1(x') \tag{27}$$

where $x_1(x')$ is a straining function and ε is some small parameter. The velocity is then given (ref. 1) by

$$\phi_x(x,y) = \phi_x^{(0)}(x',y)\Big[1 - \varepsilon x_{1_{x'}}(x')\Big] + \varepsilon\phi_{1_{x'}}(x',y) \tag{28}$$

where $\phi_{x'}^{(0)}(x', y)$ is a known base solution. If $\phi_x^{(1)}(x, y)$ denotes a second known solution for a value of the parameter $\bar{\varepsilon}$ then

$$\phi_{1_{x'}}(x',y) = \frac{1}{\bar{\varepsilon}}\left\{\phi_{\bar{x}}^{(1)}(\bar{x}, y) - \phi_{x'}^{(0)}(x',y)\Big[1 - \bar{\varepsilon}x_{1_{x'}}(x')\Big]\right\} \tag{29}$$

where

$$\bar{x} = x' + \bar{\varepsilon}x_1(x') \tag{30}$$

Combination of equations (28) and (29) then leads to

$$\phi_x(x,y) = \phi_{x'}^{(0)}(x',y) + \frac{\varepsilon}{\bar{\varepsilon}}\Big[\phi_{\bar{x}}^{(1)}(\bar{x},y) - \phi_{x'}^{(0)}(x',y)\Big] \tag{31}$$

where the coordinates x', $\bar{x}$ are defined by equations (27) and (30).

For a correction for truncation error ε and $\bar{\varepsilon}$ are identical since it is to be the same change of mesh that is considered. The same result also applies for a correction between the small disturbance and full potential equations. Also as described in section 2, the correction terms are evaluated from a nearby solution, for example, a different airfoil but with a similar velocity distribution. If the solutions to the nearby problems are denoted by an overbar then the corrected value of $u(x, y)$ is given by

$$u(x,y) = u^{(0)}(x',y) + \left[\bar{u}^{(1)}(x,y) - \bar{u}^{(0)}(x',y)\right] \tag{32}$$

where x' is defined by equation (27).

If more than one correction is required then

$$u(x,y) = u^{(0)}(x',y) + \sum_{i=1}^{N}\left[\bar{u}_i^{(1)}(x_i,y) - \bar{u}^{(0)}(x',y)\right] \tag{33}$$

where

$$x_i = x' + \varepsilon_i x_{1_i}(x') \tag{34}$$

and

$$x = x' + \sum_{i=1}^{N} \varepsilon_i x_{1_i}(x') \tag{35}$$

where $\varepsilon_i x_{1_i}(x')$ is the straining function for the ith correction and N is the number of corrections.

Choice of Straining Functions

Two forms of straining functions are given in references 1 and 2. If there is one characteristic point, for example a shock wave, in the solution then a suitable (ref. 1) straining function is

$$\left.\begin{aligned} \varepsilon x_1(x') &= \varepsilon\delta x_A' \frac{x'(1 - x')}{x_A'(1 - x_A')} ; \qquad 0 \leq x' \leq 1 \\ \varepsilon x_1(x') &= 0 , \qquad 1 < x' < 0 \end{aligned}\right\} \tag{36}$$

where x_A' is the location of the characteristic point and $\varepsilon\delta x_A'$ is its movement between the two known solutions.

If there are two characteristic points, x_A', x_B' in the solutions then a suitable (ref. 2) straining is

$$
\left.\begin{aligned}
\epsilon x_1(x') &= \epsilon\delta x'_A \frac{x'(1-x')(x'-x'_B)}{x'_A(1-x'_A)(x'_A-x'_B)} + \epsilon\delta x'_B \frac{x'(1-x')(x'-x'_A)}{x'_B(1-x'_B)(x'_A-x'_B)} \; ; \; 0 \leq x' \leq 1 \\
x_1(x') &= 0 \; , \quad 1 < x' < 0
\end{aligned}\right\} \tag{37}
$$

where $\epsilon\delta x'_A$ and $\epsilon\delta x'_B$ are the movements of x'_A and x'_B, respectively, between the two known solutions.

If there are more than two characteristic points then a piecewise version of equations (36) and (37) can be used.

RESULTS

The ideas discussed above are applied to problems in two-dimensional transonic flow. Two types of correction are examined, namely, truncation error and the error induced by the use of the small disturbance equation instead of the full potential equation. In all of the examples the coordinate straining includes the effect of the shock wave, including the shock capture correction discussed in section 3 and the rapid pressure expansion at the leading edge. The choice of the nearby solution for the correction was determined mainly by inspection, that is, whether the pressure distribution in both cases looked similar. In all examples, an interpolation procedure is used to provide data at points other than those used in the computation.

In figure 2 the pressure distribution over the upper surface of an NACA 64A410 airfoil at $M_\infty = 0.74$ and 1.5° angle of attack is shown. The final result is corrected for a mesh of 99×79 from a solution obtained on a mesh of 38×35. The nearby solution is for the same airfoil and Mach number but at 1° angle of attack. All results are computed using the transonic small disturbance theory. It is seen that the large discrepancy between coarse and fine mesh calculations near the leading edge is corrected fairly well by the present scheme as is the shock capture region. The probable reason for the error in the leading edge is that the nearby result does not quite represent the behavior of the problem under consideration.

An important part of the present theory is that the correction need not be computed for the airfoil under consideration but only for an airfoil with a similar form of solution. Correspondingly, some examples were computed using different airfoils for the correction. In figure 3 the pressure distribution around an NACA 0012 airfoil at $M_\infty = 0.84$ at zero angle of attack is shown for a mesh of 99×79 corrected from a computational mesh of 38×35 using the correction obtained for the upper surface of an NACA 64A410 airfoil at $M_\infty = 0.74$ and 1° angle of attack. Although there is some error just ahead of and just behind the shock wave, the main discrepancy in the region of the leading edge has been corrected satisfactorily. A typical correction for the nearby solution is also shown in figure 3.

A second type of correction discussed in this paper is for the use of the transonic small disturbance equation as opposed to the full potential equation. An example of this type of correction is given in figure 4 where the pressure distribution over the upper surface of a Korn airfoil is shown for $M_\infty = 0.78$ and zero angle of attack. This result is obtained by correcting a small disturbance result to correspond to a full potential result. The correction is obtained from computations of both small disturbance and full potential equations for the Korn airfoil at $M_\infty = 0.76$ and zero angle of attack. The agreement of the corrected result with the direct solution is satisfactory except behind the shock wave. In this particular example there are not many points on the airfoil surface behind the shock wave in the small disturbance solution which to a great extent accounts for the lack of resolution.

The final example, shown in figure 5, illustrates the correction between small disturbance and full potential equations. In this case the correction is computed for a different airfoil. In figure 5, the pressure distribution around the upper surface of an NACA 64A410 airfoil at $M_\infty = 0.72$ and zero angle of attack is shown; the correction is obtained from solutions using small disturbance and full potential equations for a 10% biconvex airfoil at zero angle of attack at $M_\infty = 0.8285$. The corrected result agrees satisfactorily with the direct solution.

In all of the examples computed it was essential that the approximate are coarse mesh solutions and the correction solutions must capture all of the characteristic features of the problem under consideration.

CONCLUDING REMARKS

A technique for correcting finite difference solutions for the effect of truncation error and the use of approximate equations or boundary conditions is derived. The correction terms need only be computed from a similar or nearby solution. It is essential that both the correction and the approximate solutions should capture all of the characteristics of the final solution. Since a given correction should be applicable to at least a moderate range of problems the present method should be very useful in practical computations since only the relatively inexpensive approximate computations are required.

The author would like to acknowledge the help of T. L. Holst of Ames Research Center who provided the solutions for the full potential equations.

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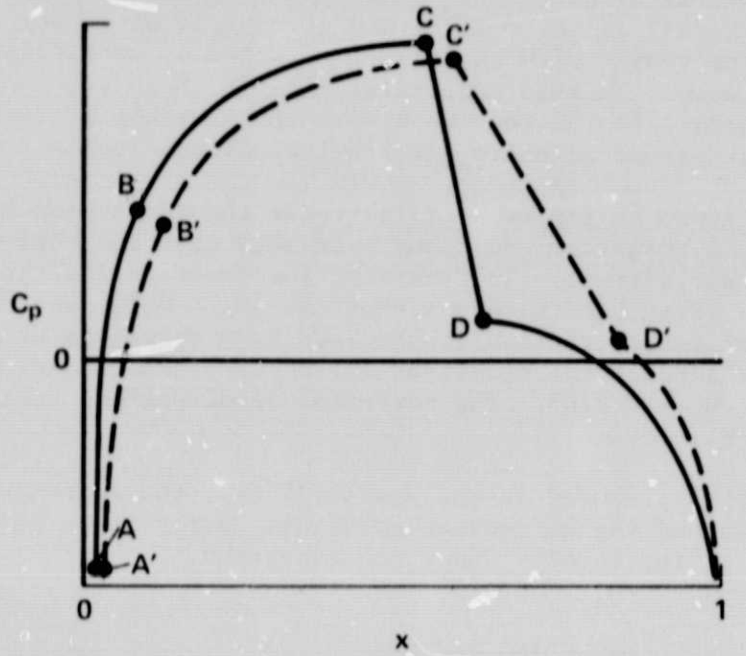

Figure 1.- Sketch of pressure distribution in transonic flow.

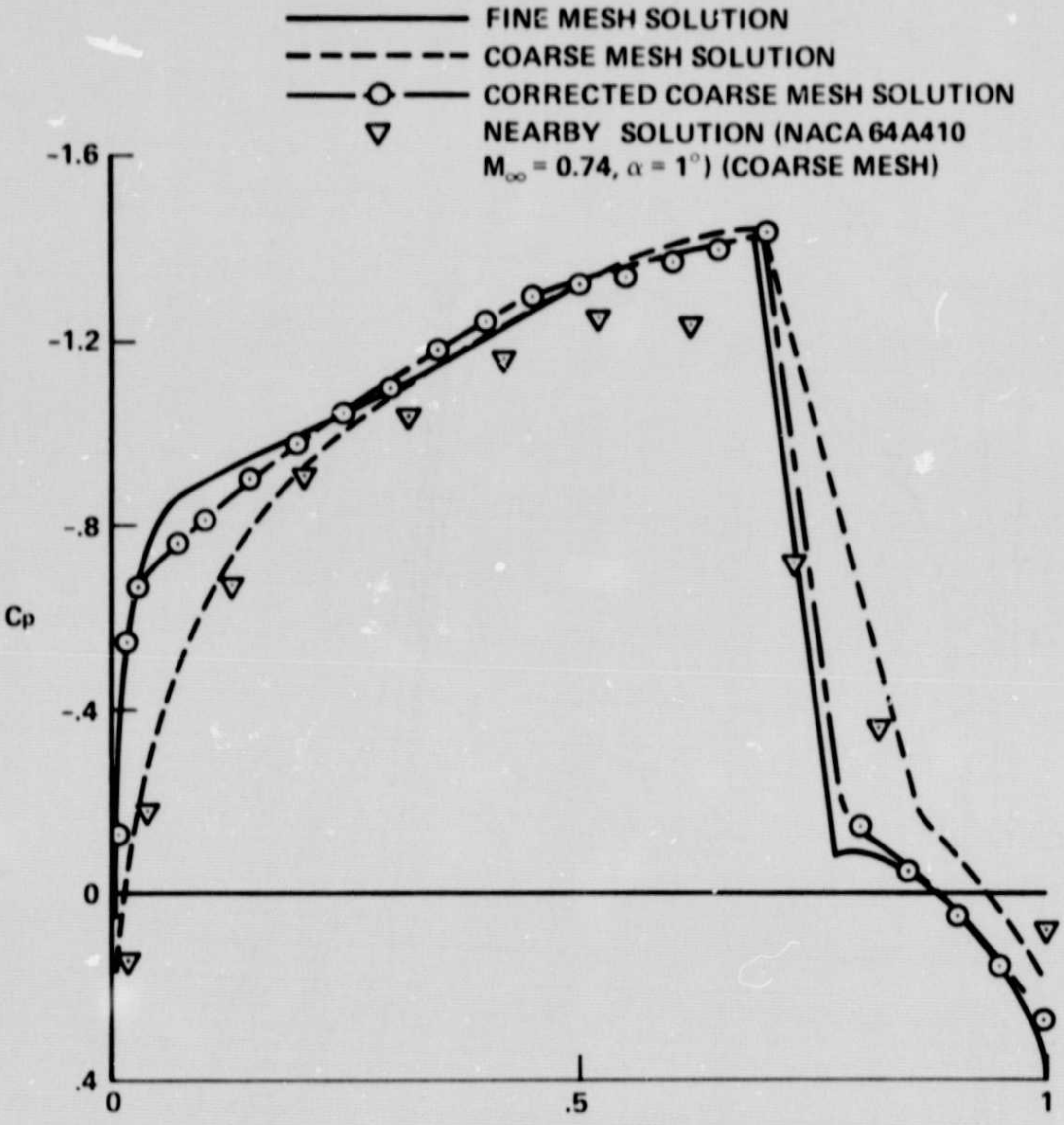

Figure 2.- Pressure distribution around the upper surface of a NACA 64A410 airfoil; $M_\infty = 0.74$, $\alpha = 1.5°$.

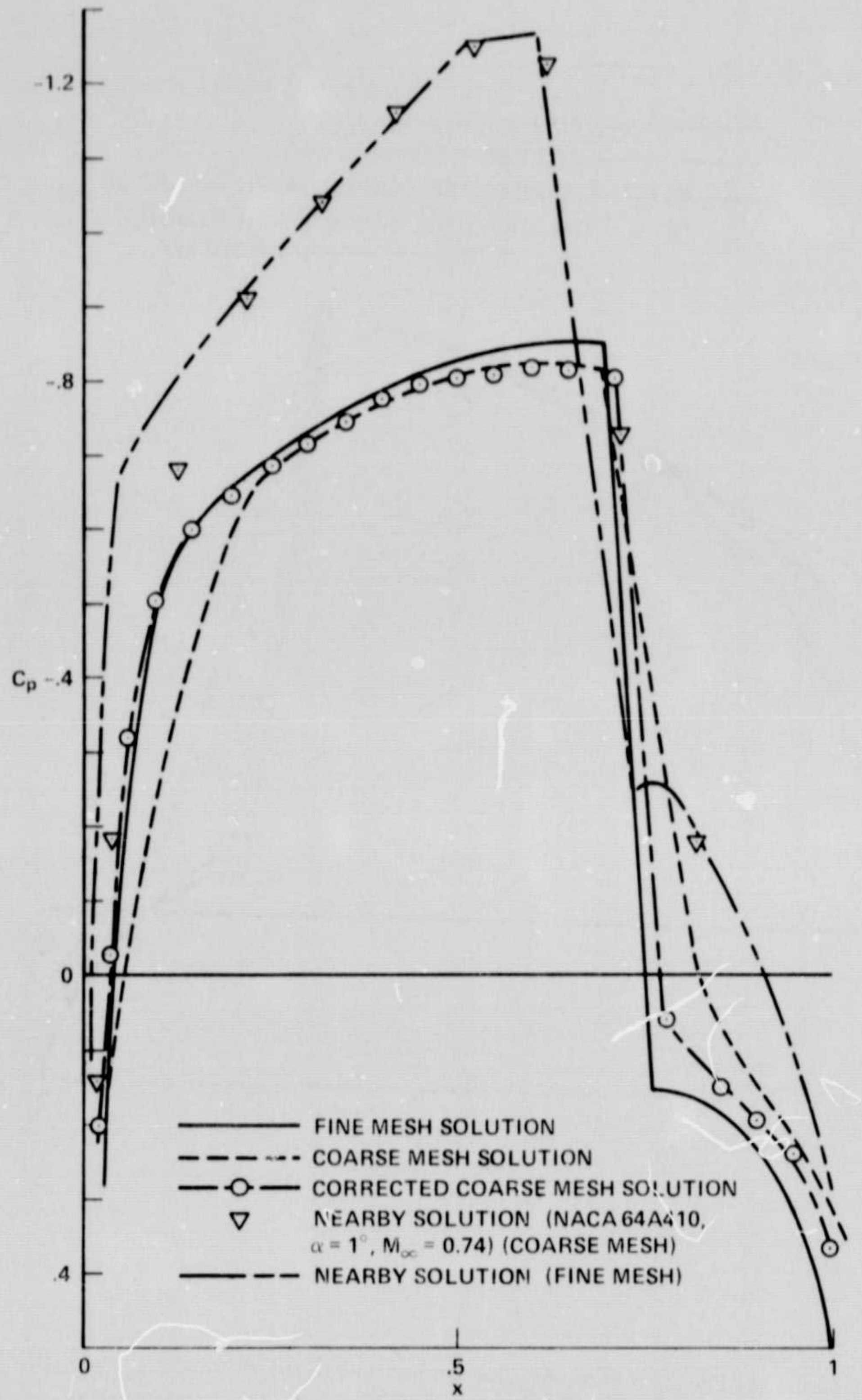

Figure 3.- Pressure distribution around a NACA 0012 airfoil; $M_\infty = 0.84$, $\alpha = 0°$.

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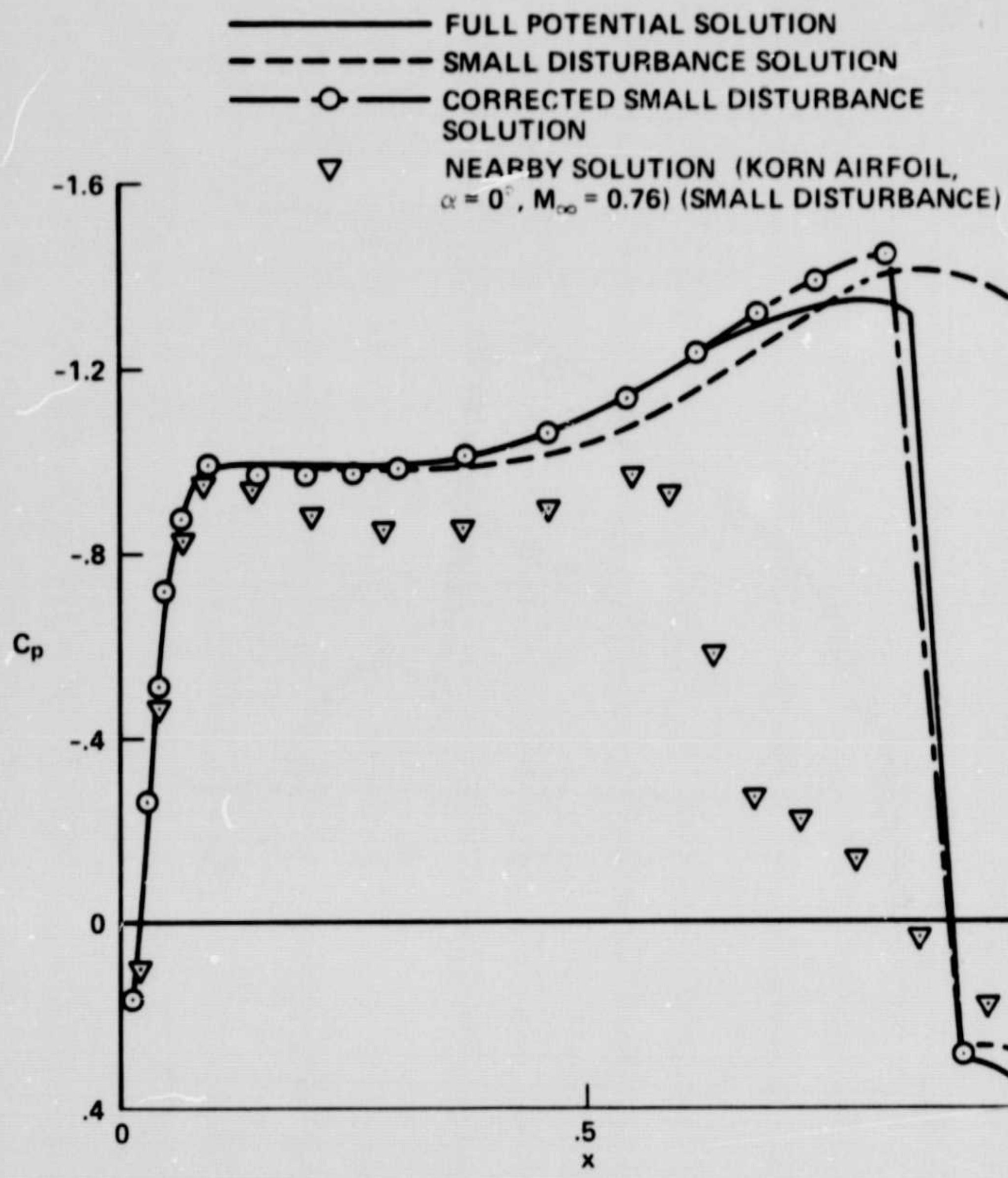

Figure 4.- Pressure distribution around the upper surface of a Korn airfoil. $M_\infty = 0.78$, $\alpha = 0°$.

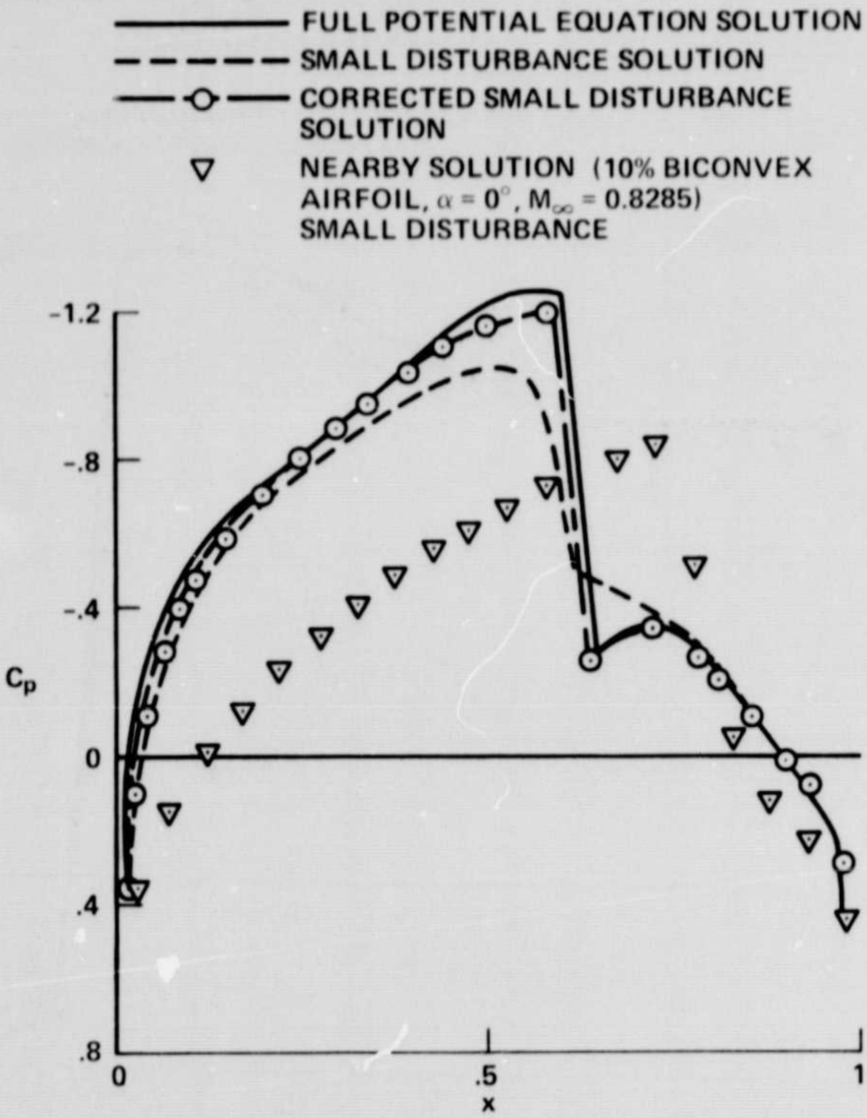

Figure 5.- Pressure distribution around the upper surface of a NACA 64A410 airfoil; $M_\infty = 0.72$, $\alpha = 0°$.