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Determination of Some Dominant Parameters of the Global Dynamic Sea Surface Topography from GEOS-3 Altimetry

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ABSTRACT

Gradients of the sea surface topography (SST) - i.e., heights of the sea surface in relation to the geoid - are essential for the real-time modelling of ocean dynamics. Ocean current measurements indicate the existence of SST gradients as large as ±0.1 m per 10^2 km. A prerequisite for remote sensing SST from satellites is a geoid model with at least ±6 cm resolution through equivalent wavelengths.

The only potential source of such data is a satellite-determined gravity field model. The internal statistics of the best such model available at present (GEM 9) indicate that favourable signal-to-noise exists for the recovery of the dominant parameters of the quasi-stationary dynamic sea surface topography from GEOS-3 altimetry.

The 1977 altimetry data bank available at Goddard is analyzed for the geometrical shape of the sea surface expressed as surface spherical harmonics after referral to the higher reference model defined by GEM 9. The resulting determination is expressed as quasi-stationary dynamic SST. Solutions are obtained from different sets of long arcs in the GEOS-3 altimeter data bank as well as from sub-sets related to the September 1975 and March 1976 equinoxes assembled with a view to minimizing seasonal effects.

The results obtained are compared with equivalent parameters obtained from the hydrostatic analysis of sporadic temperature, pressure and salinity measurements of the oceans and the known major steady state current systems with comparable wavelengths.

*On Leave of Absence from the University of New South Wales, Sydney, Australia

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The most clearly defined parameter (the zonal harmonic of degree 2) is obtained with an uncertainty of ±6 cm. The preferred numerical value obtained (−43 cm) is smaller than the oceanographic value (−46) largely due to the effect of the correction for the permanent Earth tide. Similar precision is achieved for the zonal harmonic of degree 3. The precision obtained for the fourth degree zonal harmonic reflects more closely the accuracy expected from the level of noise in the orbital solutions, being a factor of 3 inferior to the values quoted above.

Attempts to obtain the harmonics ζ_{s111} and ζ_{s110} were not successful because of the masking effect of the non-geocentricity of the system of reference used. The dominant effect is a southward displacement of 1.5 m along the polar axis.

The results presented in this paper are preliminary. While some further progress of a limited nature may be forthcoming with improvements in the definition of orbits, the most important requirement for significant advances in remote sensing surface ocean dynamics using altimeter data, is the refinement of low degree tesseral harmonics of the satellite-determined gravity field model to 2 parts in 10^9.

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DETERMINATION OF SOME DOMINANT PARAMETERS OF THE GLOBAL DYNAMIC SEA SURFACE TOPOGRAPHY FROM GEOS-3 ALTIMETRY

1. INTRODUCTION

The basic equations of non-tidal motion of the surface layer of the oceans, excluding wave motion, can be represented on an $x_1 x_2$ coordinate system in the local horizon (x_1 oriented east, x_2 oriented north) by (e.g., see Mather 1976 for a review of the derivation)

$$\ddot{x}_i + f\,\epsilon_{ijk}\,\delta_{j3}\,\dot{x}_k = -g\frac{\partial \zeta_s}{\partial x_i} - \frac{1}{\rho_w}\frac{\partial p_a}{\partial x_i} + F_i \qquad (1),$$

obtained when i takes the values 1 and 2, f being the Coriolis parameter given by

$$f = 2\omega \sin\phi \qquad (2),$$

δ_{ij} is the Kronecker delta,

$$\epsilon_{ijk} = \begin{cases} 1 \text{ if subscripts are ordered } 1231231\ldots \\ -1 \text{ if subscripts are ordered } 132132\ldots. \\ 0 \text{ if } i=j;\ i=k,\ j=k \end{cases} \qquad (3),$$

$\ddot{x}_i$, $\dot{x}_i$, F_i being components of accelerations, velocities and frictional forces acting on the surface layer of the oceans in the directions x_i, g is local gravity, p_a the atmospheric pressure on the sea surface, ρ_w the density of sea water and ζ_s the dynamic sea surface topography (SST).

ζ_s is defined as the radial departure of the sea surface from the geoid. As the term "sea surface topography" has been used in the past to refer to the height of the sea surface above any arbitrary reference surface, it has been considered necessary to qualify ζ_s by the use of the additional adjective "dynamic" as in the title of this paper. However, the abbreviation "SST" will always be used in this paper to refer to the quantity ζ_s, as defined above. The height of the sea surface above the adopted reference surfaces will be called the height anomaly, designated by ζ if S is a rotating equipotential ellipsoid of revolution, or ζ' if S were the higher reference model (Mather 1974, p. 90 et seq.).

The currents in the surface layer of the oceans can be as large as 2×10^2 cm s^{-1}. Table 1 summarizes the factors necessary to maintain a current of 10 cm s^{-1} at latitude 45°. Figure 1 is a representation of the dynamic sea surface topography as produced from steric anomalies in the oceans, compiled primarily from (Wyrtki 1975, Reid et al. 1977, and Lisitzin 1974). Such global representations are not instantaneous, being based on sporadic measurements of

Table 1

Magnitudes (Rounded Off) of Factors Which Can Maintain a Surface Layer Current Velocity of 10cm s^{-1} at Latitude 45°

Factor	Required Magnitude
Sea Surface Topography Gradient $\partial\zeta_s/\partial x_\alpha$	0.4arcsec (20cm per 10^2 km)
Atmospheric Pressure Gradient $\partial p_a/\partial x_\alpha$	30mb per 10^2 km
Wind Stress*	40m s^{-1} (83mph)

*F_α computed from the relation

$$F_\alpha = 2 \times 10^{-6} \frac{(W_1^2 + W_2^2)^{1/2}}{\rho_w H} W_\alpha$$

H = Depth of mixed layer (10^2m)

temperature and salinity in the oceans. It is estimated that the contours can be in error by up to ±20cm due to temporal variations. This model is merely a useful guide in the design of techniques to recover the SST from satellite altimetry. It should also be noted that the contours shown contain a zero degree term of +1.14m due to the oceanographic datum which is of no geodetic consequence.

Fast flowing quasi-stationary mid-latitude currents like the Gulf Stream in the western North Atlantic (e.g., NOAA 1975) and the Kuroshio in the western North Pacific (e.g., Cheney 1977) are maintained by steep sea surface topography gradients in excess of 1m per 10^2 km but with relatively short wavelengths. Other more moderate current systems are maintained by the interplay of wind, sea surface topography and frictional forces acting on the surface layer due to continental shelf margin bottom topography. The effect of SST on ocean circulation is a function of latitude (equation 1) being a minimum in equatorial waters.

The GEOS-3 altimeter had a design criterion of ±50cm resolution in the short pulse mode. The comparison of overlapping passes of such altimetry (Mather and Coleman 1977, Table 2) showed that the altimeter appeared to provide a resolution of at least ±20cm on a relative basis, for features with wavelengths greater than 30km, assuming that the discrepancies were not of oceanographic origin. Spectral analysis of pairs of overlapping passes indicated that non-trivial strengths of signal were obtained for several wavelengths in excess of 100km (Mather 1977, p. 25). Further details of investigations of this type using a wider data base, are reported in (Mather et al. 1978b).

These studies indicated that a basis existed for the recovery of limited information about the global SST from the short pulse mode GEOS-3 altimetry, despite the following shortcomings in the data:

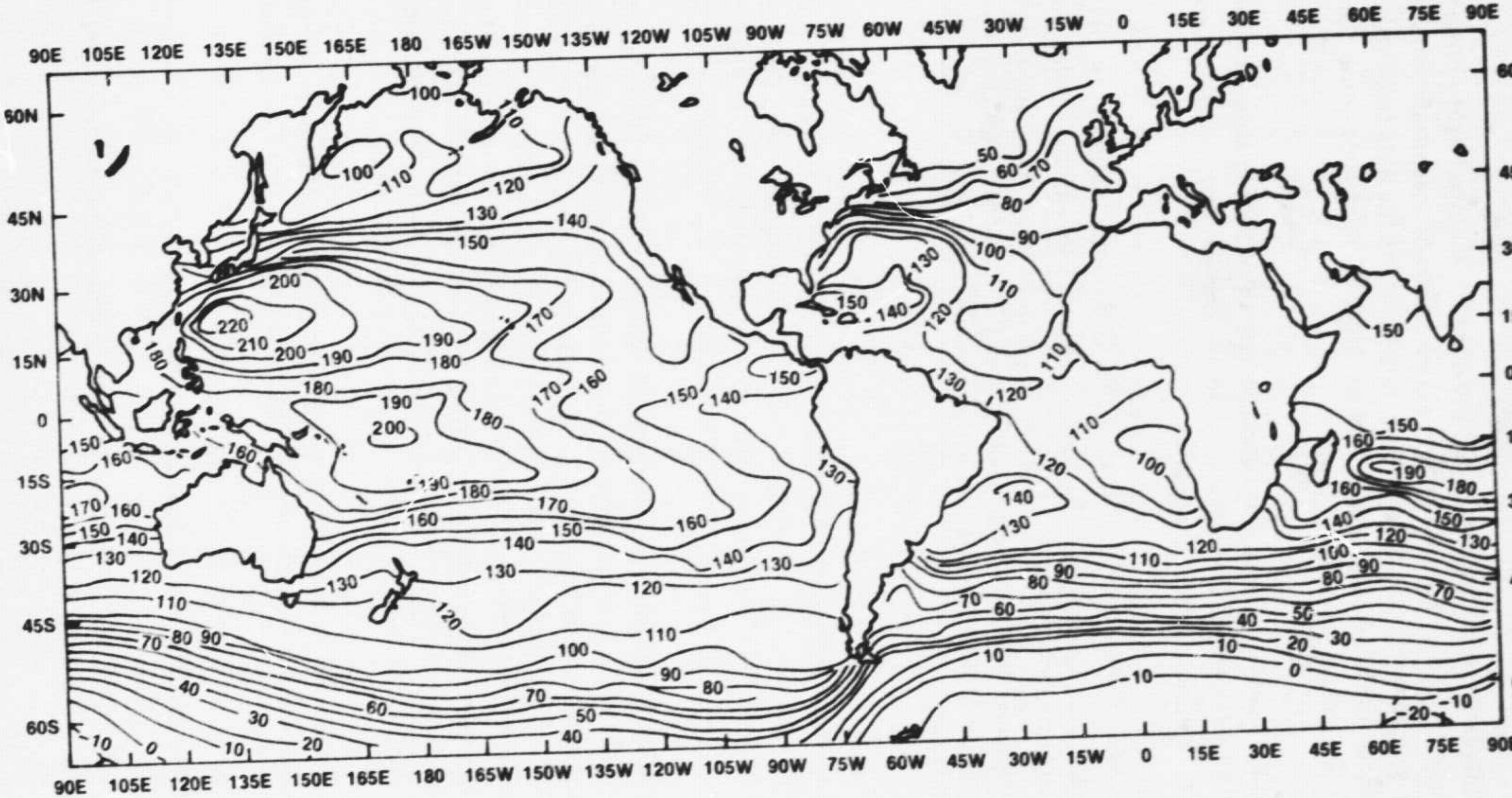

Figure 1. Quasi-stationary Dynamic Sea Surface Topography Relative to 1000 db Surface
Wavelengths > 10^3 km
Contour Interval – 10 cm

(i) The orbits (and hence the values of ζ provided by Wallops Flight Center) were subject to error. These were as much as two orders of magnitude worse than the relative precision of the altimetry data, especially for the data collected in 1975. In 1976, the orbital errors were about one order of magnitude worse than the resolution of the altimeter.

(ii) The GEOS-3 altimetry data was collected as a set of finite passes with lengths up to 9000 km. No attempt was made to obtain near-simultaneous global coverage as envisaged in the case of SEASAT-A.

A study of overlapping passes in the Tasman and Coral Seas (Mather et al. 1977, p. 36) showed that the root mean square (rms) discrepancy between sea surface heights on two passes of length greater than 3000 km dropped on the average to around ±30 cm or less when the radial orbital error was modelled by a bias (b) and a tilt (c). This practice has been widely used by more than one GEOS-3 Principal Investigator when attempting to refine the altimeter data for various studies.

The monthly analyses of GEOS-3 altimetry data in the Sargasso Sea (Mather et al. 1978b) shows that models can be constructed for each monthly data set using the technique of crossover constraints so that the variation of height at a common point in different solutions has an rms of around ±40 cm. Some of this variation is probably due to oceanographic causes.

There is no problem in generating orbits such that crossover discrepancies are not much greater than ±3 m. These discrepancies are partly due to force model errors used in orbit integration. They can be reduced to less than ±2 m by introducing crossover constraints, preferably in terms of understood probable errors in the force field models, after elimination of about 10% of the altimetry data affected by excessive biases probably due to time tag errors. Given orbits which have a radial uncertainty of $\pm\ell$ cm, further improvement can be achieved by averaging. The global data bank of sea surface heights (ζ or ζ') can provide information for the recovery of a feature of the SST under the following conditions:

(i) Sufficient GEOS-3 data is available for the average representation of any particular wavelength so that the resolution is $\pm\ell/\sqrt{n}$ cm, where n is the number of samples.

(ii) The amplitude of the feature is greater than $\ell/\sqrt{n}$ cm.

(iii) The error in the geoidal model with wavelengths comparable to that of the feature, are significantly less than $\pm\ell/\sqrt{n}$ cm.

It is not difficult to conjure up a scenario in which features of the SST can be determined with a precision of ±10 cm. For example, it has been observed that the second degree of the SST is significantly larger than other terms in its representation by a surface harmonic series (Mather 1975, p. 67). On the basis of the previous study (Mather et al. 1977), it can be concluded that a basis exists for the recovery of this term from the GEOS-3 data bank, despite its flaws due to inadequate tracking distribution and the irregular manner in which the data were collected in both space and time.

Other circumstances in which the magnitude of ζ_s exceeds the noise level of the altimeter are in the neighborhood of fast-flowing mid-latitude currents like the Gulf Stream. While there is little problem in obtaining ζ or ζ' on a relative basis in such circumstances with the required precision, the same cannot be said for ζ_s due to the lack of knowledge of the geoid profile over such short wavelengths (less than 1000km), as discussed at length in (Mather et al. 1978b, Sec. 9).

The present study therefore concentrates on the definition of parameters of the global quasi-stationary dynamic SST through those wavelengths for which the Earth's gravity field model is known with an acceptable level of precision. Section 2 deals with the data available for the present series of investigations. Section 3 deals with the technique for obtaining a solution from an analysis of long arcs of GEOS-3 altimetry. Section 4 deals with the Equinox Experiment designed to eliminate seasonal effects on the estimation of the quasi-stationary constituent of SST. The reference system plays a significant role in the evaluation of SST from a data set which is not truly global under circumstances where the solution can only be obtained for selected harmonics of the gravity field. This is discussed in greater depth in Section 5. Section 6 deals with the problem of modelling data which is restricted to ocean areas alone, with special reference to the use of the model in generating parameters related to ocean circulation.

Section 7 analyzes the results obtained and Section 8 presents the conclusions drawn from the present study.

2. THE 1977 GEOS-3 ALTIMETRY DATA BANK

The orbits used in reducing GEOS-3 altimeter data were computed at Wallops Flight Center (WFC) using the GEM 7 gravity field model (Wagner et al. 1977) or at the Naval Surface Weapons Center, Dahlgren, Va., using tracking from 16 to 60 Doppler stations and a gravity field model specially tailored to the GEOS-3 spacecraft (Stanley 1978). Some orbits were also computed at Goddard Space Flight Center. The precision of the orbits provided by WFC appears to be better in 1976 than in 1975 (See Table 2). However, it cannot be inferred from these results that the precision of orbit determination is necessarily a function of time (ibid).

In earlier studies (Mather et al. 1977, p. 34; Mather 1977, p. 24), it was shown that the application of corrections for a tilt c and bias b to each of a pair of "overlapping" passes over 3000km lengths, made the median disagreement between passes drop to about ±30cm. This

Table 2

The Analysis of Crossovers (XO) for Twelve Ten Day Arcs used in the Equinox Experiment Wallops Orbits (See Section 4)

Arc	Duration		No. of Altimeter Passes	Crossover Statistics					
				Unadjusted			Constrained*		
	From	To		No. of XO's	Mean Discrepancy (m)	Root Mean Square Residual (±m)	No. of XO's	Mean Discrepancy (m)	Root Mean Square Residual (±m)
1	9. 1.75	9.10.75	65	111	+0.76	4.4	95	+0.35	1.6
2	9.11.75	9.20.75	51	40	+5.05	19.7	28	+0.07	2.5
3	9.21.75	9.30.75	63	98	+3.49	10.6	89	+0.66	1.8
4	10. 1.75	10.10.75	49	48	+2.19	3.6	39	+0.47	1.8
5	10.11.75	10.20.75	61	61	+1.01	2.1	59	+0.47	1.5
6	10.21.75	10.31.75	51	62	+1.20	2.7	55	+0.21	2.0
7	3. 1.76	3.10.76	83	38	-2.08	1.9	35	-1.17	1.9
8	3.11.76	3.20.76	52	53	+0.10	2.4	53	-0.07	1.7
9	3.21.76	3.31.76	46	28	+0.10	2.4	27	-0.35	1.7
10	4. 1.76	4.10.76	34	30	+0.25	1.5	30	-0.01	1.2
11	4.11.76	4.20.76	29	10	-0.62	2.7	10	0.00	1.8
12	4.21.76	4.30.76	40	63	-1.15	2.1	62	-0.34	1.8

*Using the Orbit Error Model Defined in Equation 9 and a ±5m cut off which excluded all passes with average XO discrepancies in excess of this limit.

figure could be improved further if the slight non-coincidence of ground tracks were also allowed for (Mather et al. 1978b, Sec. 5).

In the search for the dominant features of the dynamic SST, the primary objective is determination of the geometry of the sea surface in Earth space as defined in relation to a geocentric coordinate system. Such a model is directly compar... to the geoid in any determination of ζ_s. This would also apply to any satellite determined gravity field used in lieu of the latter in the search for dominant parameters defining the global distribution of ζ_s. This relation is conceptually d· cribed by Figure 2.

In view of the variable quality of the Wallops orbits, it was necessary to attempt an orbit improvement so that the radial orbital errors could be reduced

(a) from values in excess of ± 10m (e.g., Mather et al. 1977, p. 30) to values smaller than ± 2m; and

(b) from values less than ± 2m to the relative precision of ± 30cm implied in the analysis of overlapping passes (Mather 1977, p. 25).

The principal check on the quality of orbit integration is the crossover (XO) discrepancy d. While it is possible to achieve a median value of ± 2m for d from orbit integration (Table 2), the improvement at (b) cannot be obtained from the GEOS-3 tracking data alone. The best tracking data available at the present time is laser data taken from the network of stations shown in Figure 3. The laser ranging precision varies from around ± 10cm for the NASA lasers to around ± 1 m for other systems. The resulting orbit obtained from integration in the form of the instantaneous satellite position $X_{is}(t)$ at time t on a geocentric Earth space coordinate system X_i, with the X_3 axis passing through the CIO pole and the $X_1 X_3$ plane being that of zero longitude (λ). The instantaneous position $X_{iss}(t)$ of the sea surface at the point with geodetic coordinates (ϕ_g,λ) is obtained from the altimeter range h(t) using the relations

$$X_{iss}(t) = X_{is}(t) - \ell_i\, h(t) + o\left\{10^{-8} h\right\} \tag{4}$$

where ℓ_i are defined by the equations

$$\ell_1 = \cos\phi_g \cos\lambda \qquad \ell_2 = \cos\phi_g \sin\lambda \qquad \ell_3 = \sin\phi_g \tag{5}$$

Timing is critical in correlating the orbital ephemeris with the time tag in the altimeter sensor data record as dh/dt can be as large as 20 m s^{-1}. For example, a constant timing bias of θ ms causes sea surface height errors of 2θ cm forcing south-to-north passes to have an error which is equal and opposite to that in a north-to-south pass at the same location. This has serious implications when attempting to enforce crossover constraints.

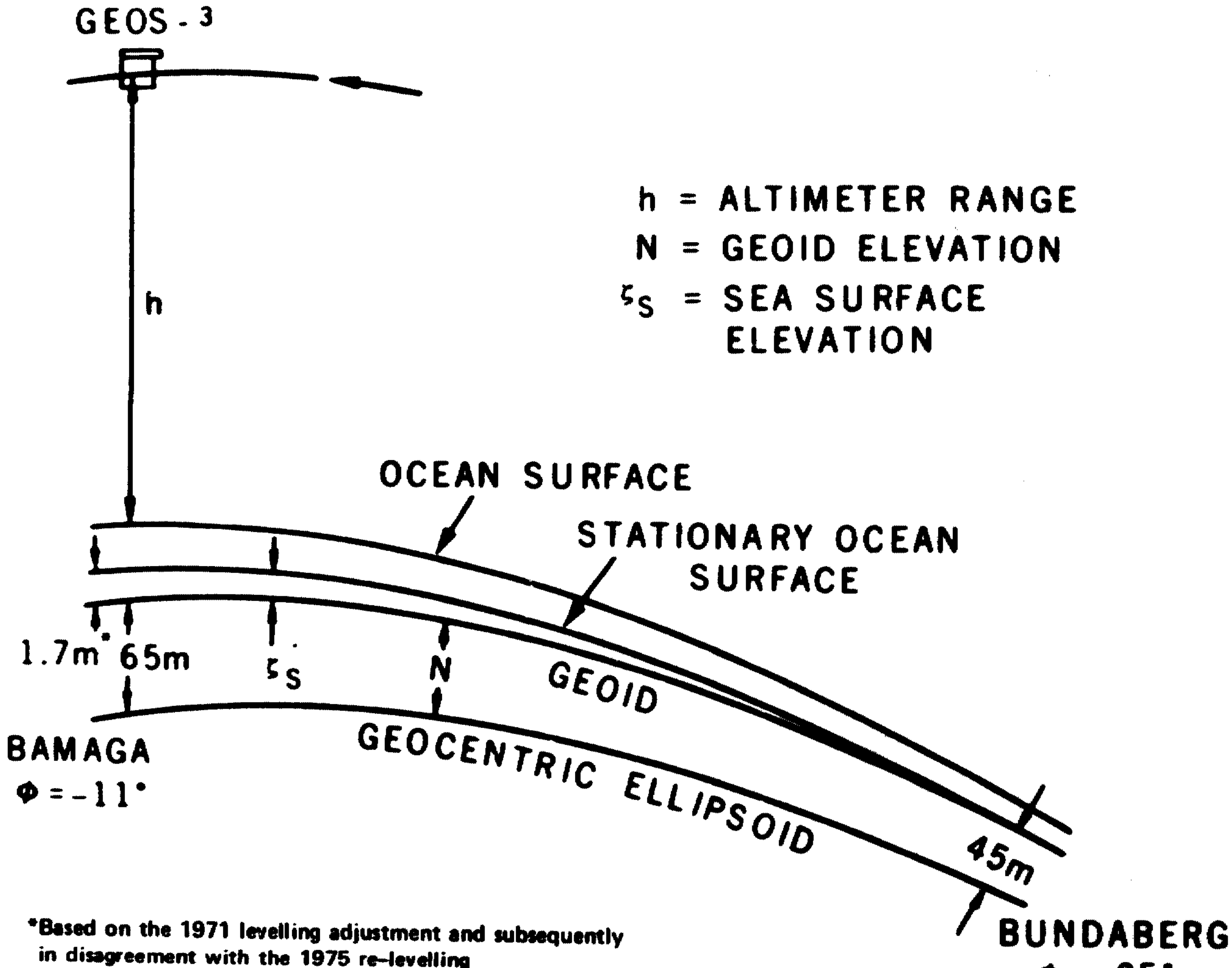

Figure 2. Sea Surface Topography in the Orbital Plane Northeastern Australia

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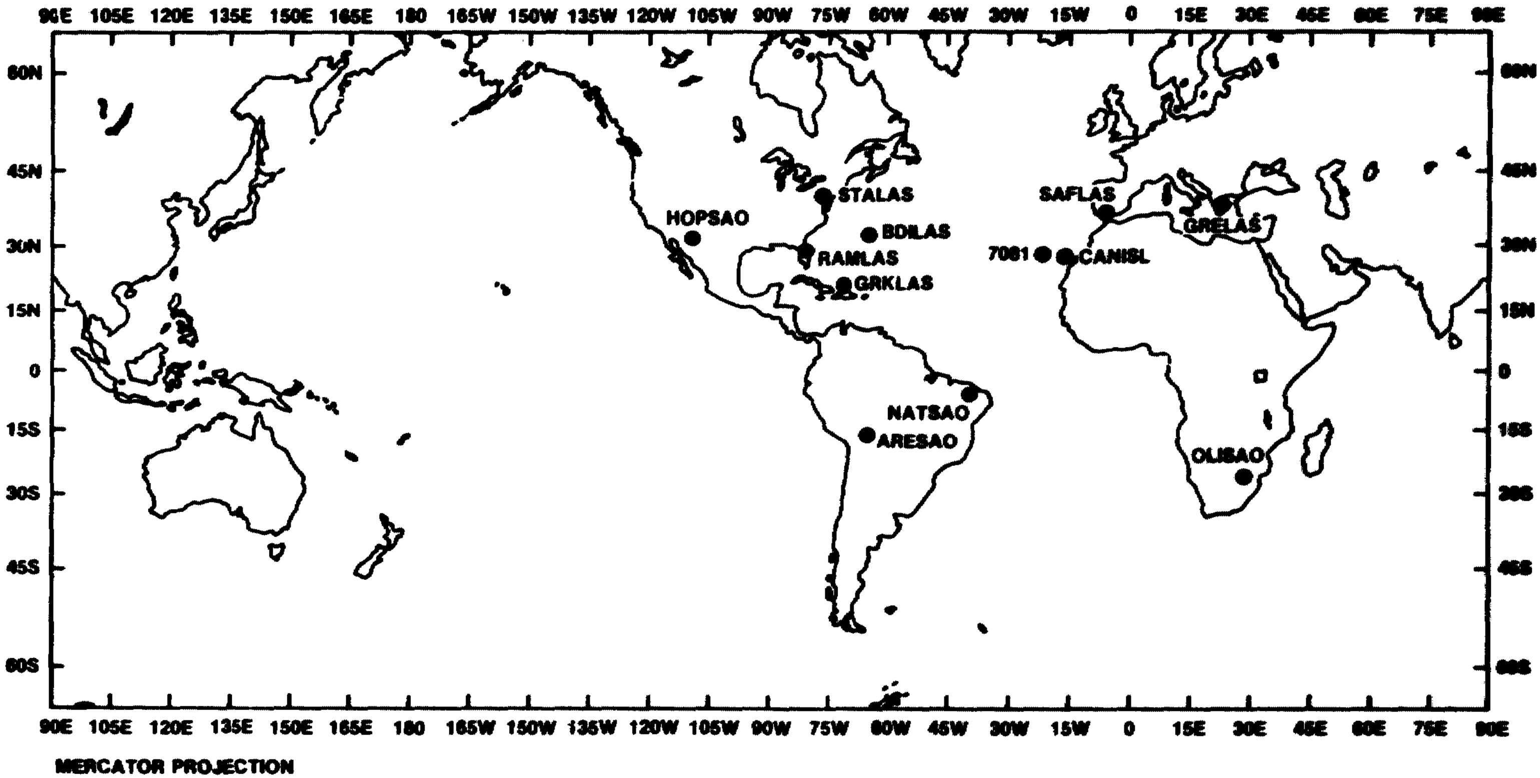

Figure 3. Laser Tracking Station Complement Available for Determining Shape of Sea Surface

Equation 4 assumes that the altimeter is gravitationally stabilized. Orbit integration is usually performed over 5 day arcs using a gravity field model with the quality of Goddard Earth Model (GEM 9) (Lerch et al. 1977). The quality of orbit determination can be partially assessed by looking at the rms residuals for each span of tracking. While these are sometimes not as small as the quality of the tracking would warrant, this is largely attributed to along-track errors. This can be further verified by studying the quality of altimeter crossovers for a 5 day span of data acquisition. A typical case is illustrated in Figure 4.

The geocentric radial distance to the sea surface $R_s(t)$ is obtained from the coordinates $X_{iss}(t)$ using the relation

$$R_s(t) = \left(\sum_{i=1}^{3} \left[X_{iss}(t)\right]^2\right)^{\frac{1}{2}} \tag{6}$$

R_s as evaluated at each altimeter data point during a 5 day arc, can be examined at crossover points for the propagation of the radial component of the orbital error as a function of time, using observation equations of the form

$$R_s(t_2) - R_s(t_1) + \sum_{i=1}^{n} C_i \left[F_i(t_2) - F_i(t_1)\right] = v \tag{7}$$

where C_i are the coefficients required in the solution and F_i are functions of time. Observation equations of this form could, in theory, include the ocean tide. In practice, however, the number of observation equations is rather small (less than 40 for a 5 day arc of GEOS-3 data), precluding this possibility. It is important that crossover constraints be used only to eliminate orbital errors due to unmodelled force field effects. These are estimated as having predominant periods of one half revolution, one revolution, 14 revolutions, a resonance effect with period of approximately 4.70 days and any linear drifts with time.

Twelve ten day arcs of Wallops data selected for the Equinox Experiment (Section 4) were analyzed using crossover constraints per ten day arc using the following model for the orbital error

$$\sum_{i=1}^{n} C_i F_i(t) = C_1 t + C_2 \cos 4\pi \frac{t}{t_o} + C_3 \sin 4\pi \frac{t}{t_o} + C_4 \cos 2\pi \frac{t}{t_o} + C_5 \sin 2\pi \frac{t}{t_o} +$$

$$C_6 \cos \frac{2\pi}{14} \frac{t}{t_o} + C_7 \sin \frac{2\pi}{14} \frac{t}{t_o} + C_8 \cos \frac{2\pi}{66.21} \frac{t}{t_o} + C_9 \cos \frac{2\pi}{66.21} \frac{t}{t_o} \tag{8}$$

where t is the time in days from the start of the 10 day arc and t_o the orbital period in the

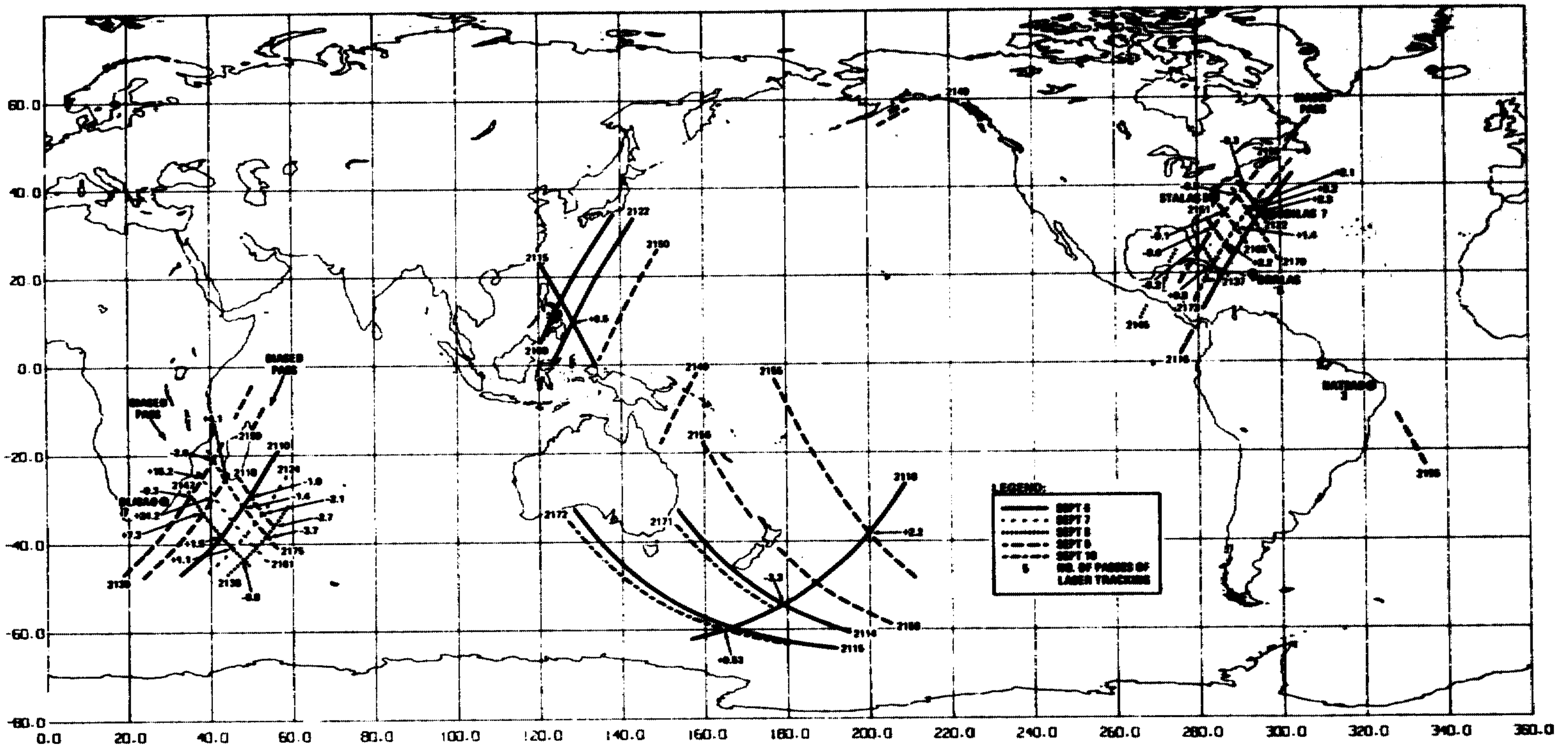

Figure 4. GEOS-3 Altimeter Ground Tracks
September 6-10, 1975

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same units. The results obtained per 10 day arc are set out in Table 2. These results show that the root mean square (rms) residual of crossover discrepancies per 10 day arc averaged ±6.8 m. This figure can be reduced to ±6.1 m by using equation 8 to represent the unmodeled orbital errors (i.e., 18 percent of the power). A study of Table 2 shows that this average crossover discrepancy can be reduced significantly to ±1.8 by rejecting data associated with 9 percent of the XO points on enforcing the criterion that any pass subject to an average XO discrepancy in excess of ±5 m was subject to an unknown source of error, tentatively associated with time tag problems.

Data sets referred to as "constrained" in this context are based on Wallops orbits as amended by XO constraints using equation 8 and the ±5 m cut-off limit explained above. The significance of the coefficients C_i obtained by the use of equation 8 is not apparent at this stage.

It therefore appears doubtful whether there is any means of obtaining profiles of sea surface heights from the orbital ephemeris of GEOS-3 using equation 4 with an absolute radial precision of better than ±1.3 m in global terms at present. However, the internal precision within the pass can be shown to be around ±30 cm from overlapping pass comparisons, as discussed earlier.

Data of this type can play a role in determining parameters of the global dynamic SST under the following conditions:

(a) The error of ±1.3 metres mentioned above and which can be modelled by two parameters (a bias and a tilt) is randomly distributed as a function of position.

(b) The parameter sought should not be much less than ±15 cm. It should have a sufficiently long wavelength to enable its recovery from a global bank of data where satisfactory solution procedures can be devised for recovery of the dynamic SST signal under conditions where the signal to noise ratio approaches 0.1.

If the sea surface topography were modelled using equation 43, the analysis of the data used to construct Figure 1 and summarized in Table 4 (Solution 3) shows that only the coefficients ζ_{s111} and ζ_{s120} satisfy the above criterion. However, if the orbital errors could in some way be brought to below ±60 cm there is some chance that the coefficients ζ_{s110}, ζ_{s130} and ζ_{s140} can also be recovered.

Note that these probabilities are assessed only on the basis of an analysis of the GEOS-3 altimeter data bank and do not take any other factors into consideration. This will be discussed further in Sections 3 and 4.

Table 5

Estimation of the Five Dominant Coefficients of the Quasi-Stationary Dynamic Sea Surface Topography from GEOS-3 Altimetry – Values (cm)

Coefficient			Oceanographic** (from 16, 16) Solution		Long Arc Solutions		Constrained Wallops Orbits			
							Equinox Set		1976 Data Set (till 8.76)	
			Global	Equinox Representation	AGS 658	EGM 19	Unadjusted	Constrained†	Unadjusted	Constrained†
n	m	α	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0	0	1	+114.5	+117.1	–	–	-27	-8	+11	-4
1	0	1	+6.9	+9.6	–	–	-153***	-136***	-165***	-141***
1	1	1	-21.8	-20.6	–	–	+18***	+6***	+30***	-1***
2	0	1	-46.2	-43.6	-45*	-47*	-50*	-35*	-44*	-35*
3	0	1	+6.7	+7.0	+2	+12	+9	-1	+11	-11
4	0	1	-9.5	-9.9	-7	-26	-32	-38	-29	-28
rms variation per 1°x1° sq.			±5	–	±80	–	±172	±149	±184	±161
Average XO Discrepancy rms per 10 day arc			–	–	–	–	±680	±180	±332	±177
No. of 1° blocks			–	–	–	–	13,499	13,275	12,349	12,074
No. of Data Points			–	–	27,674	1757	268,023	232,687	286,197	234,929
Percentage of Data Rejected (XO's)			–	–	–	–	–	9	–	13

*These values should be corrected by +6.1 for the permanent Earth tide (Mather 1978)

**Between 65°S and 65°N

***Includes non-geocentricity of the reference coordinate system

†Using Equation 8 and ±5m average XO limit

3. DETERMINATIONS BASED ON LONG PASS SOLUTIONS

In attempting to reduce the average high level of orbital noise per pass from ±1.3 m to ±30 cm by introducing corrections for tilt c and bias b, the obvious goal is a means of determining the corrections c and b without losing any information about the geometry of the sea surface. The introduction of the corrections c and b per pass effectively removes all information with wavelengths greater than twice the length of the pass unless some special conditions were imposed.

The longest passes of GEOS-3 altimetry are not greater than 9000 km. Information on the shape of the sea surface which could be lost in tilting and bias correcting such a pass will be of wavelength 18,000 km (i.e., harmonics less than degree 3 in a spherical harmonic representation of the SST). However, any contribution to the sea surface topography which is of degree 2 and symmetrical about the equator will not be lost on introducting corrections for tilt and bias for the following reasons.

Let the ellipsoid of revolution which best fits the geoid have an equatorial radius a and flattening f. Let the difference in flattening between the ellipsoid of revolution which best fits the sea surface and the former be df. If the two ellipsoids have a common tangent at the equator (i.e., they have the same equatorial radius), the changes dR in radial distance R, given by

$$R = a(1 - f\sin^2\phi + o\{f^2\}) \qquad (9),$$

at latitude ϕ between the two ellipsoids is

$$dR = -a\,df\sin^2\phi + o\{af\,df\} \qquad (10).$$

dR is equivalent to ζ_s if both the geoid and the sea surface had only second degree even zonal characteristics. The change δdR in the radial distance between two latitudes ϕ_1 and ϕ_2 is given by

$$\delta dR = -a\,df(\sin^2\phi_2 - \sin^2\phi_1) \qquad (11),$$

ζ_{s120} being directly related to the second degree zonal harmonic in the pseudo-representation of the sea surface as a level surface.

As ζ_{s120} = −46.2 cm (Table 4, Solution 3) the following conclusions can be drawn from a study of Table 3, noting that (e.g., Heiskanen and Moritz, 1967, p. 78)

$$\sqrt{5}\,\zeta_{s120} = -\frac{2a}{3}\,df \qquad (12):$$

(i) No information on the meridian ellipticity of the sea surface or, for that matter, any other characteristic which is symmetrical about the equator (i.e., all even degree zonals in ζ_s) is lost by making corrections for bias and tilt to passes which are nearly symmetrical about the equator.

(ii) The lack of symmetry about the equator can be as large as 1000 km for pa s of length greater than 6000 km without causing more than a 50 percent distortion in the second degree harmonic. This is not considered acceptable for this type of work. However, unless some allowance of this type is made, it is not possible to sample the Indian Ocean region with passes of sufficiently long length.

Table 3

Differential Changes (Unsigned) in Ellipsoidal Height Due to Variation in Flattening over Finite Meridional Arc Lengths (s) (Equation 11)*

Arc Length(s)		Latitude		δ dR (cm)	Maximum δ dR (cm)
Minutes	km	Start	End		
10	4425	60	20.2	96	114
		40	0.2	63	63
		20	-19.8	0	18
15	6650	60	0.4	114	114
		40	-19.7	46	63
		35	-24.7	24	50
		20	-39.7	44	62
20	8850	60	-19.5	97	114
		40	-39.5	1	63
		39.8	-39.8	0	62
		20	-59.5	95	113

*Computed using $df = 2.381 \times 10^{-7}$ (i.e., $\zeta_{s120} = -46.2$cm)

Table 4

Surface Spherical Harmonic Analysis of Oceanographically Determined Quasi-Stationary Dynamic Sea Surface Topography to (5,5) from Data Restricted to Oceans Between Parallels 65°S and 65°N (Units cm)

Coefficients Normalized									
Solution Type		(1) Unconstrained Least Squares Ocean Only		(2) Quadratures Zero on Land		(3) Constrained Least Squares Ocean Only		(4) Quadratures Using Land Bridge*	
n	m								
α		1	2	1	2	1	2	1	2
0	0	110.2		84.3		114.5		113.5	
1	0	5.3		-8.5		6.9		3.4	
1	1	-27.9	-2.7	-34.0	-4.1	-21.8	+2.5	-20.6	+3.5
2	0	-43.7		-39.8		-46.2		-48.7	
2	1	-13.1	-3.4	-8.9	-2.6	-4.0	+4.4	-3.5	+4.6
2	2	+0.8	-4.6	+5.7	+1.0	-0.7	-0.2	-0.1	+0.1
3	0	+18.5		+6.5		+6.7		+1.8	
3	1	-13.2	-11.6	+4.2	-7.5	-4.1	-5.3	-2.0	-3.6
3	2	+1.8	-9.9	+8.5	-13.6	-0.7	-2.4	-0.0	-1.9
3	3	-0.7	-0.4	-3.2	-10.4	-3.0	+1.4	-2.3	+1.7
4	0	2.6		-5.8		-9.5		-11.9	
4	1	-4.4	+2.4	+7.2	+3.6	+2.1	+2.8	+1.9	+2.7
4	2	+2.1	-6.8	+7.8	-0.9	-0.5	+1.2	+2.7	-0.6
4	3	+4.6	-3.3	-2.0	+0.7	+1.2	-0.2	+1.5	-0.4
4	4	-0.7	-2.1	+0.6	-15.3	-1.8	-0.9	-1.9	-0.4
5	0	+14.7		+2.7		+1.0		-3.2	
5	1	-7.6	+3.1	-1.0	+5.5	-3.7	-0.8	-1.0	+0.4
5	2	+1.9	-3.0	+2.5	+7.9	-0.6	+2.4	-1.2	+2.6
5	3	+4.8	-2.5	-4.0	+0.1	+0.6	+0.2	+0.8	+0.1
5	4	+1.7	+0.4	-14.3	+4.9	-0.3	+1.1	-0.4	+1.8
5	5	+0.6	-0.1	-0.8	-6.3	-0.3	-0.8	-0.4	-0.6
Analysis To		8,8				16,16			
Rms fit (±cm)		9				9			

*Zeros assumed for polar regions outside parallels 65°S and 65°N

A basis therefore exists for recovering the dynamic SST from GEOS-3 altimetry from long passes (lengths greater than s) under the following conditions:

(i) There should be sufficient passes of the maximum possible length, symmetrical about the equator, for the recovery of even degree constituents of the sea surface shape.

(ii) Sufficient passes with a tolerance of about 500km from equatorial symmetry should be included to provide an even coverage of the Indian Ocean region.

(iii) As will be seen from the discussion in Section 6, it is desirable to obtain a coverage of the entire oceans to avoid aliasing effects due to incomplete sampling.

Under such circumstances, there are several means of obtaining the necessary set of tilt and bias corrections. The simplest means, as argued above, is by fitting the set of long arcs to the best available geoid model, arc by arc, ensuring that the three conditions set out above are enforced. The following problems arise when using this method:

(a) The resulting sea surface model will not have information on harmonics which are not symmetrical about the equator and with wavelengths greater than twice the length of the pass.

(b) The harmonics of the SST deduced from a global set of such long passes of GEOS-3 altimetry are limited to those terms for which the error in the gravity field model is below at least 1 part in 10^8.

A study of this problem (Mather 1978b, Table 1) shows that, apart from considerations of the quality of the GEOS-3 altimetry data discussed above, the magnitude of the SST coefficients and the estimated errors in the best available gravity field model – GEM 9 – indicate that only the coefficients ζ_{s120}, ζ_{s111}, ζ_{s140}, ζ_{s110}, ζ_{s130}, ζ_{s160} and possibly, ζ_{s121} and ζ_{s221} can be recovered at the present time. Of these, ζ_{s111}, ζ_{s110}, ζ_{s121} and ζ_{s221} cannot be recovered by the application of tilt and bias corrections. It is not clear whether a basis exists for the recovery of ζ_{s130} from such data.

Two types of solutions were obtained using the method of long passes which had orbital errors reduced from the ±1.5 m level to the ±80 cm level by determining corrections b for bias and c for tilt in relation to a gravity field model (GEM). The above three conditions for altimeter passes, although desirable to reduce correlation, are not necessary in this analysis since the bias and tilt parameters can be solved simultaneously with the SST coefficients as in the solution AGS 658 described later. These corrections were determined by setting up observation equations of the form

$$v = h - h_{GEM} - [b + c(t - t_0) + \delta h] \qquad (13),$$

where h is the sea surface height obtained from GEOS-3 altimetry, h_{GEM} the height anomaly at the sea surface computed by the application of Bruns' formula to the GEM model,

(t,t_0) being parameters quantifying the tilt in terms of height increments and δh is the departure of the sea surface from the datum level surface (geoid), being modeled by the relation

$$\delta h = \frac{GM}{R_0 \gamma} \sum_{n=2}^{n'} \left(\frac{a}{R_0}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \Delta C_{\alpha nm} S_{\alpha nm} \qquad (14),$$

where $\Delta C_{\alpha nm}$ are coefficients of the surface spherical harmonic functions $S_{\alpha nm}$ defined by equation 27 and assumed normalized. The coefficients $\Delta C_{\alpha nm}$ obtained from the resulting solution represent the geometrical distortion of the average sea surface from the level surface implicit in the GEM model.

Equation 13, written in matrix form, is

$$v = Ax + Bg - v_0 \qquad (15),$$

where v_0 is the vector of differences $(h - h_{GEM})$, x being the vector of bias and tilt parameters and g the array of coefficients $\Delta C_{\alpha nm}$. The reduced set of normal equations obtained from equation 15 by least squares, can be written in matrix form as

$$\left[B^T B - B^T A (A^T A)^{-1} A^T B\right] g = \left[B^T - B^T A (A^T A)^{-1} A^T\right] v_0 \qquad (16).$$

It can be seen from equation 13 that $A^T A$ for each pass of altimetry, is a 2 x 2 matrix with a simple inverse. Hence, for each pass(p) of altimetry, equation 16 may be written as

$$N_p \, g = R_p \qquad (17)$$

and summed over all passes to give

$$N g = R \qquad (18).$$

If f is the matrix of weight coefficients which allows for variable observational accuracy and if solution constraints are introduced by the relation

$$W g = 0 \qquad (19).$$

the final system of equations in matrix form are

$$(f N + W) g = fR \qquad (19).$$

The elements of the diagonal constraint matrix W (w_{ii} corresponding to the element g_i in the g array which is the coefficient $\Delta C_{\alpha nm}$ in the representation of the SST (see Table 4)), are defined by the relation

$$w_{ii} = \frac{1}{\sigma^2_{nm}} \quad (20),$$

where

$$\sigma^2_{nm} = \frac{1}{2n+1}\left(\frac{\sigma_n}{a}\right)^2 \quad (21),$$

σ^2_n being the degree variances (in cm^2) for the power spectrum of the dynamic SST.

Two sets of solutions were attempted using this technique.

Solution 1 (AGS 658)

Data was selected for this solution to produce the best possible areal coverage. The distribution of data used is shown in Figure 10. 424 passes of GEOS-3 altimeter data, with ground tracks varying in length from 20° to 80°, comprised the data set. The input data were values of the sea surface height computed by Wallops Flight Center. Values of $\Delta C_{\alpha nm}$ were solved for to (5,5) using the following values for σ_n for purposes of constraint:

$$\sigma^2_2 = 1500\,cm^2;\ \sigma^2_3 = 343\,cm^2;\ \sigma^2_4 = 144\,cm^2;\ \sigma^2_5 = 99\,cm^2 \quad (22).$$

For reasons put forward earlier, the coefficients ΔC_{221} and ΔC_{222} needed excessive constraining, as did ΔC_{133}, the respective values of w_{ii} being increased by a factor of 3 over the values given in equation 22. The values used for f in equation 19 was 1 m.

The GEM model used in AGS 658 (i.e., for the generation of h_{GEM}) was GEM 7 (Wagner et al. 1977) while values of the coefficients obtained in Table 5 are based on GEM 9. The degree variances in the uncertainties of the (5,5) model generated in solution AGS 658 in comparison to those for GEM 9 are the following, in cm^2:

Degree	AGS 658	GEM 9
2	66	15
3	30	125
4	22	66
5	12	704

(23).

On comparing the figures at (23) with those at (22), it can be concluded that starting with altimetry data with the level of noise around ±1 m, the technique has the potential to provide significant information on the quasi-stationary dynamic SST, subject to the limitations discussed above. However, the uncertainties in the GEM 9 coefficients, except for those of degree 2 and the zonal terms, restrict the information that can be obtained at the present time.

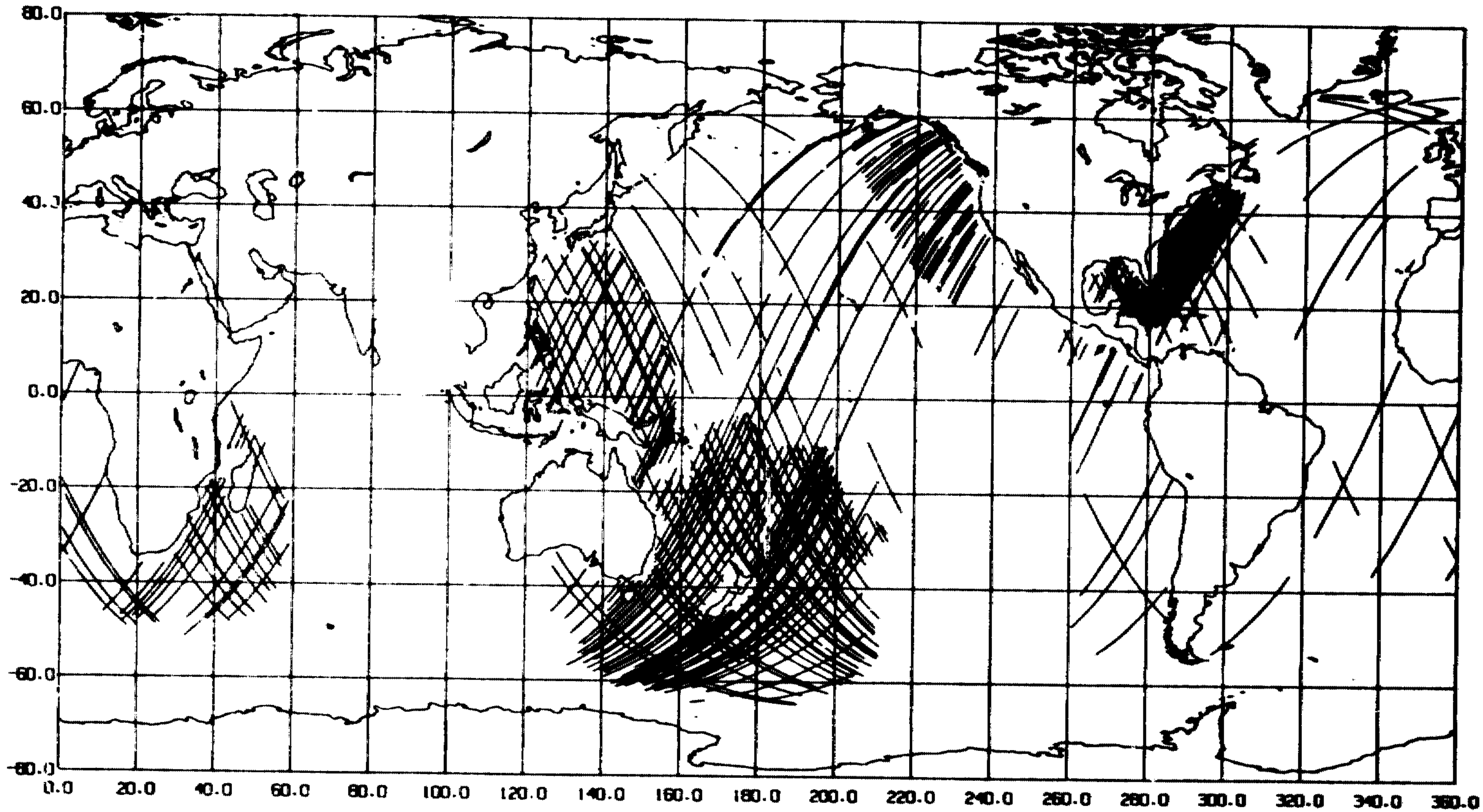

Figure 5. The Distribution of 1975 Data in the Equinox Data Set for 1976.0
September-October, 1975

Table 5

Estimation of the Five Dominant Coefficients of the Quasi-Stationary Dynamic Sea Surface Topography from GEOS-3 Altimetry – Values (cm)

Coefficient			Oceanographic** (from 16,16) Solution		Long Arc Solutions		Constrained Wallops Orbits			
							Equinox Set		1976 Data Set (till 8.76)	
			Global	Equinox Representation	AGS 658	EGM 19	Unadjusted	Constrained†	Unadjusted	Constrained†
n	m	α	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0	0	1	+114.5	+117.1	—	—	-27	-8	+11	-4
1	0	1	+6.9	+9.6	—	—	-153***	-136***	-165***	-141***
1	1	1	-21.8	-20.6	—	—	+18***	+6***	+30***	-1***
2	0	1	-46.2	-43.6	-45*	-47*	-50*	-35*	-44*	-35*
3	0	1	+6.7	+7.0	+2	+12	+9	-1	+11	-11
4	0	1	-9.5	-9.9	-7	-26	-32	-38	-29	-28
rms variation per 1°x1° sq.			±5	—	±80	—	±172	±149	±184	±161
Average X0 Discrepancy rms per 10 day arc			—	—	—	—	±680	±180	±332	±177
No. of 1° blocks			—	—	—	—	13,499	13,275	12,349	12,074
No. of Data Points			—	—	27,674	1757	268,023	232,687	286,197	234,929
Percentage of Data Rejected (X0's)			—	—	—	—	—	9	—	13

*These values should be corrected by +6.1 for the permanent Earth tide (Mather 1978)

**Between 65°S and 65°N

***Includes non-geocentricity of the reference coordinate system

†Using Equation 8 and ±5m average XO limit

Solution 2 (EGM Series)

In this type of solution, an attempt was made to recover primarily ζ_{s120} using the technique described by equations 13 to 21, but with a smaller selection of passes restricted to those which satisfied the more rigid criteria set out following equations 9 to 12. One major problem was obtaining sufficient data to cover the Indian Ocean. In addition, the data sample that satisfied the criteria of symmetry was about 40 times smaller than that used in AGS 658. This substantially increased the level of noise causing solution instability.

Two types of solutions gave satisfactory values of ζ_{s120}. The first type (Solution EGM 19) were passes longer than 4500 km with balance across the equator to within ±500 km. As only 21 passes of data in the 1977 GEOS–3 altimetry data bank satisfied this criterion, it was decided to assemble a second data set (EGM 20) which provided, in addition a coverage of shorter passes (lengths greater than 1500 km) in the Indian Ocean and Atlantic Region. The best results obtained from these two limited data sets were in the case where the higher reference model used was GEM 9 to (30,30) where only zonal coefficients were solved for using a system of constraints defined by equations 19 to 22. The values obtained for ζ_{s120} were −47 cm in the case of EGM 19 and −45 cm for EGM 20. Both these results are in agreement with the value expected from oceanographic considerations, despite the loss of resolution due to the limited data sample which is not well distributed globally (Figure 8). Nevertheless the results substantiate the claim that information on the differential flattening between the sea surface and the geoid is not lost on application of corrections for tilt so long as the pass is symmetrical about the equator.

An effort was made to improve the data coverage by reducing the criterion for equatorial symmetry to 1000 km. The resolution obtained deteriorated further, probably due to the aliasing effect of the additional number of passes asymmetrical about the equator. The result obtained from EGM 19 is shown in Table 5, though the values are not considered reliable enough an estimate of the oceanographic value which is vulnerable to aliasing when evaluated from non-global samples.

This type of solution for ζ_{s120} cannot be recommended due to the poor signal to noise and the paucity of data. However, they were made in an attempt to verify the thesis illustrated in Table 3, that the application of tilt and bias corrections to passes symmetrical about the equator does not result in the loss of the signature of the second degree harmonic of the dynamic SST in the resulting sea surface model. It can be concluded that this has been established.

In conclusion, it can be stated that solution AGS 658 should provide information on the dominant harmonics ζ_{s120}, ζ_{s130} (possibly) and ζ_{s140} of the dynamic SST, having a satisfactory areal distribution (Figure 10) and being confined to long arcs. The results obtained are set out in Table 5. Solutions for harmonics of degree 1 can only be attempted from sea surface models related to geocentric orbits with appropriately controlled levels of noise. As

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In conclusion, it can be stated that solution AGS 658 should provide information on the dominant harmonics ζ_{s120}, ζ_{s130} (possibly) and ζ_{s140} of the dynamic SST, having a satisfactory areal distribution (Figure 10) and being confined to long arcs. The results obtained are set out in Table 5. Solutions for harmonics of degree 1 can only be attempted from sea surface models related to geocentric orbits with appropriately controlled levels of noise. As seen in Section 2, this is possible by applying some constraints when selecting a data base referred to the GEOS-3 orbits, as described in Section 4.

4. THE EQUINOX EXPERIMENT

One objective of satellite altimeter missions is the recovery of data for the synoptic monitoring of ocean circulation. As pointed out in (Mather 1978b, Sec. 4), over 80 percent of the spectrum of SST is contained in about 10 spherical harmonic coefficients through terms with wavelengths longer than 3000 km. Attention is confined to these long wave features defining the global dynamic SST in this study. In the past, the lack of adequate data has forced oceanographers to treat the global dynamic SST as stationary in time. This situation has changed in the past two decades.

The variations in the global SST with time can be classified as follows:

(i) Synoptic variations with periods ranging from 10^1 to 10^2 days, which are probably wind induced.

(ii) Seasonal variations which are largely induced by the transfer of energy between the atmosphere and the surface layer of the oceans as a function of the seasons.

(iii) Long period variations which are beyond the scope of the present study.

The question of seasonal variation has been investigated by Gill and Niiler (1973). A comprehensive analysis of observational data has been undertaken by Wyrtki in the Pacific Ocean (Wyrtki 1975; Wyrtki 1977) and globally by Levitus and Dort (1977).

The figures presented in these studies show that the changes in the SST are a function of season, sea level tending to be at its lowest in northern mid-latitudes during January – February and highest in July to August. The magnitude of the variations is a function of position, varying from approximately 40 cm in the Kuroshio to as little as 4 cm at Guam (Wyrtki 1975, p. 457). The occurrence of such variations is of considerable significance in establishing sampling techniques for the determination of quasi-stationary SST. It is obvious that determinations of the global dynamic SST from data collected during a northern summer will give different results to that collected during a northern winter. The terms affected will probably be the zonal harmonics, three of which contribute to the dominant 80 percent of the spectrum of sea surface topography.

A separate determination of the dominant terms in the global SST was therefore attempted from data which excluded samples recorded during the summer and winter months. This set is called the Equinox Data Set as only the short pulse mode altimetry data collected during the months of September – October 1975 and March – April 1976 were considered in its compilation. It was originally intended to use only the twelve 10 day spans of data which fell within the four month period mentioned. The selected GEOS-3 altimeter data distribution for the September equinox in 1975 is shown in Figure 5 while that for the March 1976 equinox is shown in Figure 6. Each distribution on its own is inadequate for determination of parameters of the global dynamic SST. However the combination of the two (Figure 7), when supplemented by four additional five day spans on either side of the two periods, provided a coverage of 39.8 percent of the 33,902 1° x 1° equi-angular blocks within the parallels 65°S and 65°N which constitute the "global ocean areas" for the definition of the geoid used in this series of studies (Mather et al. 1978a, Sec. 6).

Attempts were made to prepare a set of sea surface heights using the best available orbits so that no information on low degree contributions to the shape of the sea surface were lost as in the case of the techniques described in the previous section. As discussed in Section 2, the less than optimum distribution of tracking stations and limitations in the force field models can be expected to produce errors in the radial component of orbital position and hence, the sea surface heights. The use of crossover constraints provides a means of estimating some of the unmodeled errors in orbit integration, as seems to be the case from a study of the results in Table 2.

The equinox data set was used both in the original and the constrained form, to prepare models of the sea surface as a first stage in the analysis for those long wave components of the dynamic SST which have a favorable signal-to-noise in relation to the errors in the GEM 9 gravity field model. The coefficients sought are set out in Table 5. The results obtained from this analysis are discussed in Section 7. The data derived from the original Wallops data has a higher level of noise to that obtained after the application of the error model at equation 8 in the analysis of crossovers. In both cases the level of noise is significantly larger (a factor of at least 2) than the data sets used in Section 3. Operating under such circumstances is a necessary price which has to be paid in an attempt to retain all information on dynamic SST with wavelengths equivalent to spherical harmonics of degree less than 3 which are not symmetrical about the equator.

Due to this adverse signal-to-noise, sea surface heights obtained directly from orbit integration without the application of arbitrary tilt and bias corrections, require pre-processing into a suitable form prior to analysis. The nature of this pre-processing is described in the next section.

5. THE SYSTEM OF REFERENCE USED IN THE EQUINOX EXPERIMENT

The principal role of a system of reference is the removal of systematic effects in the data which can be eliminated prior to analysis, thereby reducing the signal-to-noise ratio. It is also essential that the modelling procedure used does not inadvertently damp out the signal sought. The model used in reducing the satellite altimetry data is that described as the higher reference model (Mather 1974, pp. 90 et seq.). The use of this system in practice is described in (Mather 1978b, Sec. 4).

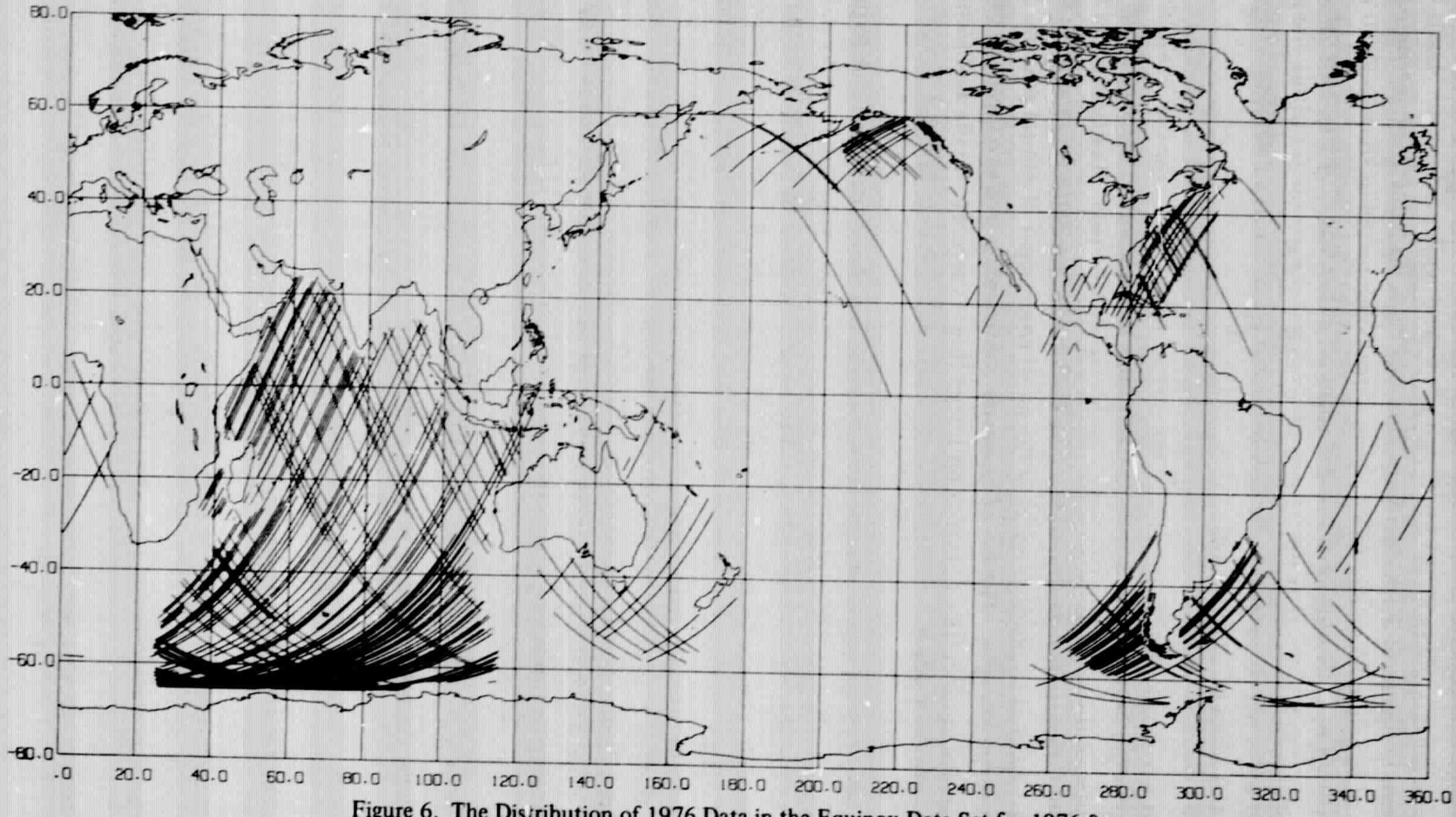

Figure 6. The Distribution of 1976 Data in the Equinox Data Set for 1976.0
March-April, 1976

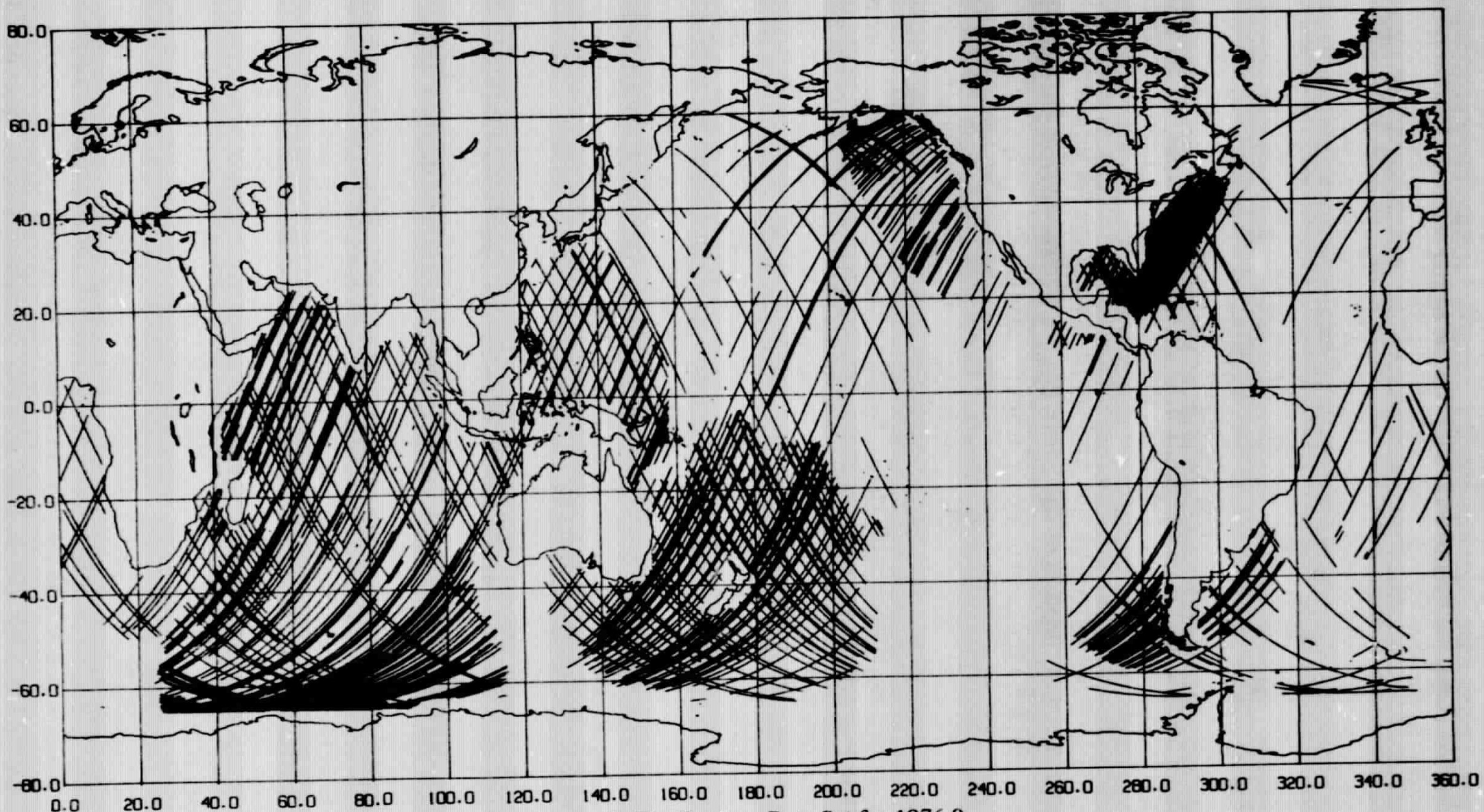

Figure 7. The Equinox Data Set for 1976.0

In the fi..t stage, the height anomaly ζ' on the higher reference system is computed using the relation

$$\zeta' = \zeta - \zeta_m \tag{24},$$

where

$$\zeta_m = \frac{GM}{\gamma R_o} \sum_{n=2}^{n'} \left(\frac{a}{R_o}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} C'_{\alpha nm} S_{\alpha nm} \tag{25},$$

where (R_o, ϕ, λ) are the geocentric spherical coordinates of the sea surface, G is the gravitational constant and M the mass of the Earth, $C'_{\alpha nm}$ are obtained from the satellite determined harmonic coefficients $C_{\alpha nm}$ to degree n′ from the relation

$$C'_{\alpha nm} = C_{\alpha nm} + dC_{\alpha nm} \tag{26}.$$

As described in (Mather et al. 1978a, Sec. 6.1), the corrections $dC_{\alpha nm}$ are primarily due to the differential effect of the atmosphere on downward continuation of the geopotential. These corrections turn out to be negligible for harmonics less than degree 6. The exceptions are the coefficients C'_{120}, C'_{140} and C'_{160}. The use of the height anomaly ζ in relation to a reference ellipsoid (equatorial radius a, flattening f) in equation 24, requires that $C'_{120} = 0$ if the value of f is consistent with the coefficient C_{120}. If this were the case, on forcing the above reference ellipsoid to be part of the higher reference model, the corrections dC_{140} and dC_{160} take specific values which are functions of f (ibid, Appendix). Full details of the computation of ζ_m are given in the reference quoted. The terms $S_{\alpha nm}$ in equation 25 are surface spherical harmonic functions, given by

$$S_{1nm} = P_{nm}(\sin\phi)\cos m\lambda; \quad S_{2nm} = P_{nm}(\sin\phi)\sin m\lambda \tag{27},$$

$P_{nm}(\sin\phi)$ being the normalized associated Legendre function of degree n and order m.

The basic relation satisfied by the height anomaly ζ' at the surface of the Earth is (Mather 1975, p. 72)

$$T = (W_o - U_o) + \gamma\zeta' - \gamma\zeta_s \tag{28},$$

γ being normal gravity.

The difference $(W_o - U_o)$ is established from the GEOS-3 altimetry as described in (Mather et al. 1978a, Sec. 6) and is a known quantity, T, for all practical purposes is the disturbing potential which will be zero for all harmonics less than n′ if the coefficients $C_{\alpha nm}$ are also free from error, the atmospheric potential being neglected in equation 28 as its differential effect has been shown to have no significant influence on relations of low degree at the surface of the Earth not involving upward continuation (ibid, p. 17).

A solution for low degree harmonic coefficients ($n \leq n'$) in ζ_s can only proceed on the basis that the only other term in equation 28 containing such information, is the global data bank of ζ' which, in the present investigation, is only available at sea between parallels 65°S and 65°N.

The input data in the Equinox Data Set is in the form of 13,499 1° x 1° area mean values of ζ' (Table 5), with a rms variability of ±2.7m. The mean variability within a 1° x 1° square from raw Wallops orbits is ±1.7m. As the harmonics being sought in the present study are of degree less than 7, it is preferable to analyze the data in the form of 5° x 5° area means as the signal-to-noise is improved.

Simplified observation equations of the form

$$v = k - \sum_{n=1}^{n'} \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \zeta_{s\alpha nm} S_{\alpha nm} \tag{29}$$

where $\zeta_{s\alpha nm}$ are the surface spherical harmonic coefficients in the global representation of ζ_s, k being defined by either

$$k = \frac{1}{\gamma} (W_o - U_o) + \zeta' \tag{30}$$

or

$$k = \frac{1}{\gamma} \left[W_o - \frac{GM}{R_o} \sum_{n=0}^{n'} \left(\frac{a}{R_o}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} C_{\alpha nm} S_{\alpha nm} - \frac{1}{2} R_o^2 \cos^2 \phi \, \omega^2 \right] \tag{31}$$

U_o being the potential of the rotating equipotential ellipsoid defined by the parameters GM, a, f and ω, where ω is the angular velocity of rotation of the Earth by the relation (e.g., see Mather 1971, p. 83)

$$U_o = \frac{GM}{a} \frac{\alpha}{\sin \alpha} + \frac{1}{3} a^2 \omega^2 \tag{32}$$

where

$$\alpha = \cos^{-1} (1 - f) \tag{33}$$

The satellite altimetry data is used to define the geocentric spherical coordinates (R_o, ϕ, λ) of the sea surface when using equation 31 to define k.

The reference system used for the Equinox experiment was defined by the following set of constants:

$$
\begin{aligned}
c &= 2.997\,924\,58 \times 10^{10} \text{ cm s}^{-1} \\
GM &= 3.986\,004\,7 \times 10^{20} \text{ cm}^3 \text{ s}^{-2} \\
C_{120} &= 1.082\,627\,5 \times 10^{-3} \\
\omega &= 7.292\,115\,15 \times 10^{-5} \text{ rad s}^{-1}
\end{aligned} \qquad (34).
$$

The dependent constants have the following values:

$$f^{-1} = 298.257\,3 \qquad (35).$$

The potential of the geoid as obtained from the analysis of the equinox data (Mather et al. 1978a, Sec. 6) using the constrained set from the Wallops orbits (Table 5),

$$W_o = 6{,}263{,}682.76 \pm 0.18 \text{ kGal m} \qquad (36),$$

giving

$$(W_o - U_o) = +0.08 \text{ kGal m} \qquad (37),$$

for an ellipsoid of equatorial radius 6,378,140.00m.

The geoid so defined is for epoch 1976.0. It should be noted that W_o is based on a 39.8 percent representation of the ocean areas between 65°S and 65°N, with data distribution as shown in Figure 7. The calibration adopted for the GEOS-3 altimeter is that determined pseudo-geometrically by Martin and Butler (1977). This procedure, while affecting the value of W_o does not influence the dynamic SST parameters determined from the GEOS-3 data bank.

6. MODELLING THE DYNAMIC SEA SURFACE TOPOGRAPHY

Special problems are involved in modelling data which does not continuously cover the surface of the Earth. Oceanic phenomena cover only about 70 percent of the Earth's surface. The data for GEOS-3 only provides data for those parts of the oceans which lie between 65°S and 65°N. On using the 1° x 1° global elevation data bank as a mask for the oceans, this defines the ocean area in terms of 33,902 such squares between the parallels defined above.

Extractable information on the quasi-stationary dynamic sea surface topography is contained in the GEOS-3 altimetry data bank subject to the following qualifications:

(i) The features sought should preferably have a magnitude in excess of at least three times the noise level of equivalent wavelengths in the gravity field.

(ii) Sufficient altimetry data is available to filter out the effect of the tides.

(iii) Adequate data is available to average out any seasonal variations in the SST. If this were not the case, the resolution obtained will only be equivalent to the magnitude of such variations (currently estimated at ±20 cm).

The obvious model for representing the long wave features of the SST is a surface spherical harmonic series

$$\zeta_s = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \zeta_{s\alpha nm} S_{\alpha nm} + o\{r\zeta_s\} \quad (38),$$

where $\zeta_{s\alpha nm}$ are coefficients of degree n and order m, while $S_{\alpha nm}$ are defined by equation 27. The values obtained for the coefficients $\zeta_{s\alpha nm}$ are dependent on the method of solution. The harmonic coefficients can be obtained by two different methods if the data is uniformly distributed over a sphere.

(i) By least squares, using observation equations of the form

$$AX - K = V \quad (39),$$

from equation 29. Values of $\zeta_{s\alpha nm}$ are obtained by minimizing

$$\Phi = V^T W V \quad (40),$$

where W is the matrix of weight coefficients which in this set of computations is $\cos\phi$, where ϕ is the latitude of the equi-angular square in which the data was sampled.

The solution is then

$$X = (A^T W A)^{-1} A^T W K \quad (41),$$

The array X being composed of the coefficients $\zeta_{s\alpha nm}$.

(ii) Alternately, given a distribution of ζ_s, the coefficients $\zeta_{s\alpha nm}$ can be obtained using the relation

$$\zeta_{s\alpha nm} = \frac{1}{4\pi} \iint \zeta_s S_{\alpha nm} \, d\sigma \quad (42),$$

assuming the coefficients to be normalized, $d\sigma$ being the element of surface area on unit sphere.

The error introduced by the spherical assumption is less than ±0.5 cm as the SST does not have a magnitude in excess of ±2 m.

The question of modelling the long wave features of the SST has been discussed before (Mather 1975; Mather 1978a, Sec. 6). Two distinct possibilities are open:

(a) Solve for $\zeta_{s\alpha nm}$ using equations 28 to 31, sampling ζ_s in ocean areas only.

(b) Solve for $\zeta_{s\alpha nm}$ using equation 29 but replacing ζ_s by

$$\zeta_s = k_o(\phi,\lambda) \sum_{n=1}^{\infty} \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \zeta_{s\alpha nm} S_{\alpha nm} \qquad (43),$$

where

$k_o(\phi,\lambda)$ is the ocean function defined by

$$k_o(\phi,\lambda) = \begin{cases} 0 \text{ if } (\phi,\lambda) \text{ on land} \\ 1 \text{ if } (\phi,\lambda) \text{ is oceanic} \end{cases} \qquad (44).$$

Table 4 sets out the coefficients obtained by analyzing the quasi-stationary dynamic SST represented in Figure 1 by four different methods. Solution 1 gives the results obtained by using equation 29 in ocean areas only. Solution 2 gives values of $\zeta_{s\alpha nm}$ obtained by using equations 42, 43 and 44. The results for solution 2 change negligibly between equivalent least squares solutions and values obtained by quadratures. The results in solution 1 for a determination to (8,8) vary considerably when the analysis is carried out by least squares to differing degrees, due to a high level of correlation between the solved coefficients as a result of the incomplete representation of data on the sphere.

A third type of solution was obtained by constraining the coefficients $\zeta_{s\alpha nm}$ by imposing the condition

$$\Phi = V^T W V + X^T W_c X = \text{Minimum} \qquad (45)$$

instead of equation 40 when obtaining a solution. The constraints were imposed through the array W_c, using coefficients of the form

$$W_{c_{nn}} = \left(\frac{10^{-5} R}{n^2}\right)^2 \times 10^3 \qquad (46),$$

for answers in m, where n is the degree of the harmonic and R the mean radius of the Earth.

Solutions obtained in this manner changed only slightly as a function of the degree to which the data was analyzed. A solution of this type is shown as solution 3 of Table 4.

The constrained solution in ocean areas so obtained was checked by interpolation of values in land areas along a parallel using a cubic polynomial in longitude difference $d\lambda$ from the eastern sea/land boundary, of the form

$$\zeta_s = \sum_{i=0}^{4} a_i \, (d\lambda)^i \tag{47}$$

The coefficients a_i are estimated by least squares fitting to between 4 and 8 data points in oceans along the same parallel on either side of the land mass which needs to be bridged.

The results so obtained from both least squares and quadratures are listed as solution 4 in Table 4.

The results in Table 4 emphasize the care needed in selecting a system for modelling the SST. No two models in Table 4 are exactly equivalent, though the models in the last two sets of columns differ only in that zero has been assumed for values of ζ_s outside the parallels 65°N and 65°S in one evaluation by quadratures.

The coefficients in unconstrained solutions are heavily correlated and change dramatically with the maximum degree to which analysis is carried out. The quadratures solutions using zero on land are not directly comparable with harmonic solutions obtained from sets of observation equations formed in ocean areas only. Furthermore, the use of such models in current modelling (Equation 1) in the form of a horizontal derivatives, become unstable close to coastlines, as do the representations of ζ_s, due to forcing the harmonic model to take zero values on land.

The variation in the coefficients for solutions of types 3 and 4 indicate the strong influence of areal representation in solutions. GEOS-3 altimeter data does not provide values of sea surface heights in all oceanic areas between 65°S and 65°N, as can be seen from the distribution of data for the equinox experiment (Figure 7).

It is therefore considered essential that this factor be taken into account when assessing the relative quality of solutions obtained from different incomplete data banks. The dominant surface spherical harmonic coefficients for the quasi-stationary SST used in the present study to represent the fully sampled region between the two sixty-fifth parallels in ocean areas is given in Column 1 of Table 5. Column 2 of this table gives the values obtained had the analysis been done using data sampled only in the areas where altimetry data were available for the Equinox experiment. Both sets of values are obtained from constrained least squares solutions to (16,16).

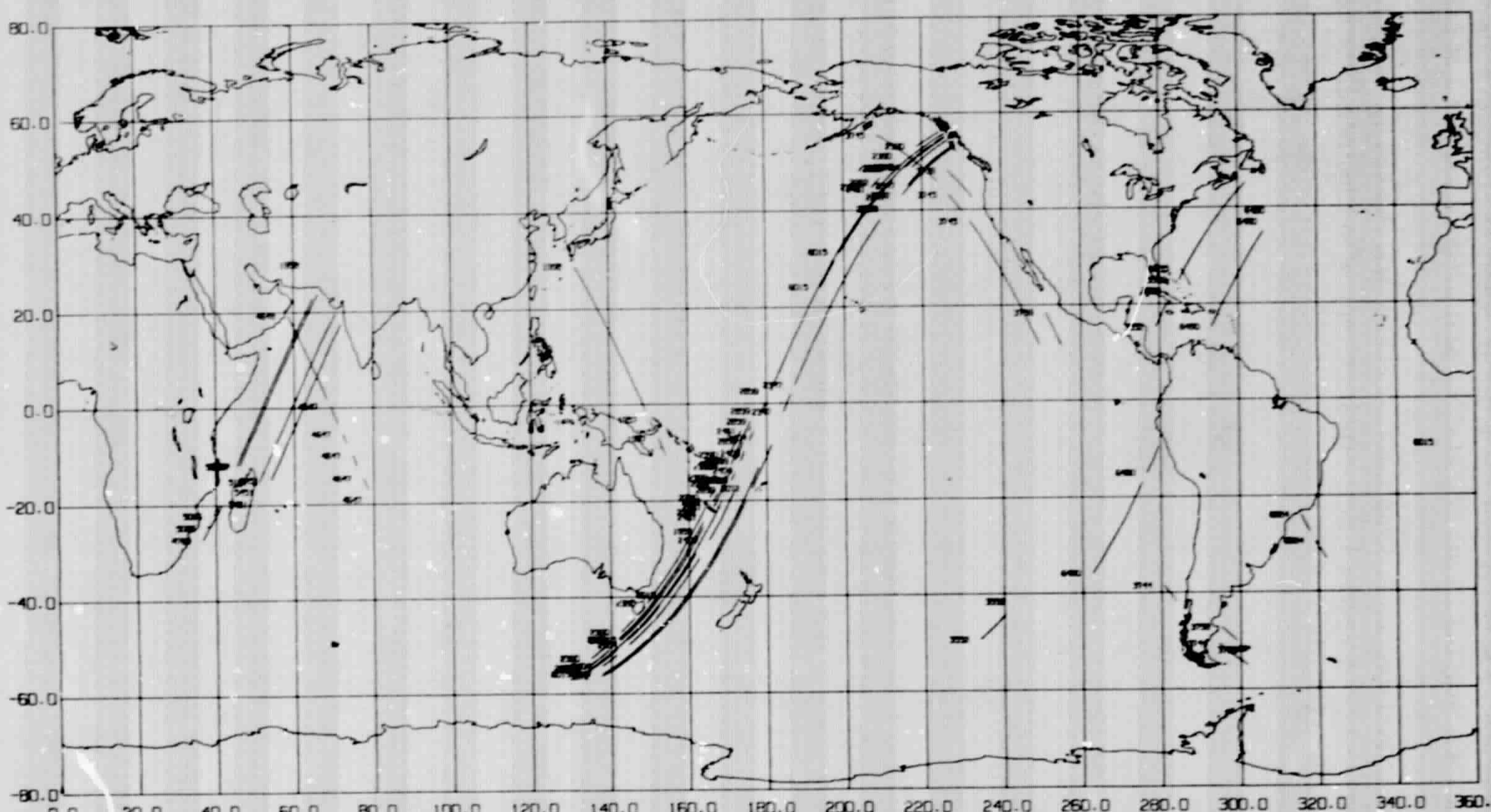

Figure 8. Distribution of GEOS-3 Altimetry in the EGM 19 Data Set

7. THE ANALYSIS OF THE RESULTS

Table 5 presents in summary, the results obtained to date in determinations of the dominant parameters of the dynamic sea surface topography (SST). The results show that correctly filtered GEOS-3 altimetry data can provide heights of the sea surface in relation to a geocentric coordinate system with a sufficiently large signal to noise to permit the recovery of the parameters referred to above. The level of noise in the constrained data sets obtained from the Wallops orbits using equation 8 on 10 day arcs, is estimated from crossover (XO) discrepancies at ±1.3 m. A second indicator is the variability of values in a 1° x 1° square. The rms geoid variation across such a square varies between ±0.3 m to ±0.8 m. The value of ±1.5 m obtained for data in the equinox data set seems to confirm the estimate of ±1.3 m as the level of noise in the data comprising the Equinox Data Set.

It has been a rule of thumb to assume that it is possible to derive long wave signals whose amplitudes are ten times smaller than the noise level provided the background noise does not contain errors of equivalent wavelength and amplitude. The analysis of XO's to model short period orbital error as described in Section 2, reduced the XO discrepancies on average by ±3 m. The resulting constrained solution produces a value of ζ_{s120} which is significantly smaller than the oceanographic value. This casts a shadow over any ad hoc modelling of terms in the enforcement of XO constraints.

In the present set of solutions, a limit of ±1 m were set on the magnitude of the coefficients C_i in equation 8, using equation 45. This may be too high. Therefore, the constrained set of solutions in the present study may not necessarily be the best solutions.

The analysis of XO's for the entire 1977 GEOS-3 altimeter data set in 10 day spans indicated that the orbits in 1976 were significantly better than those in 1975. It was therefore decided to compile a <u>1976 Data Set</u> based on data acquired between January and August 1976 (Figure 9). The same two types of solutions as obtained for the Equinox Data Set were repeated and the results listed in the last two columns of Table 5. Again, the constrained solution gives a relatively low value of ζ_{s120}, confirming the need for a review of the procedure for modelling orbital error in enforcing XO constraints.

The Second Degree Harmonic

In view of the reservations about the constrained solutions, the authors are inclined towards a value of −47 cm for ζ_{s120}. However, this value is only a preliminary estimate. It should also be noted that the value is obtained by referring the true sea surface to a <u>static</u> Earth model which is independent of the Earth tide. The latter has a permanent component which can be estimated as contributing to the shape of the sea surface. A value of ζ_{s120} comparable with the oceanographic values is obtained by correcting the above value by +6 cm (Mather 1978c).

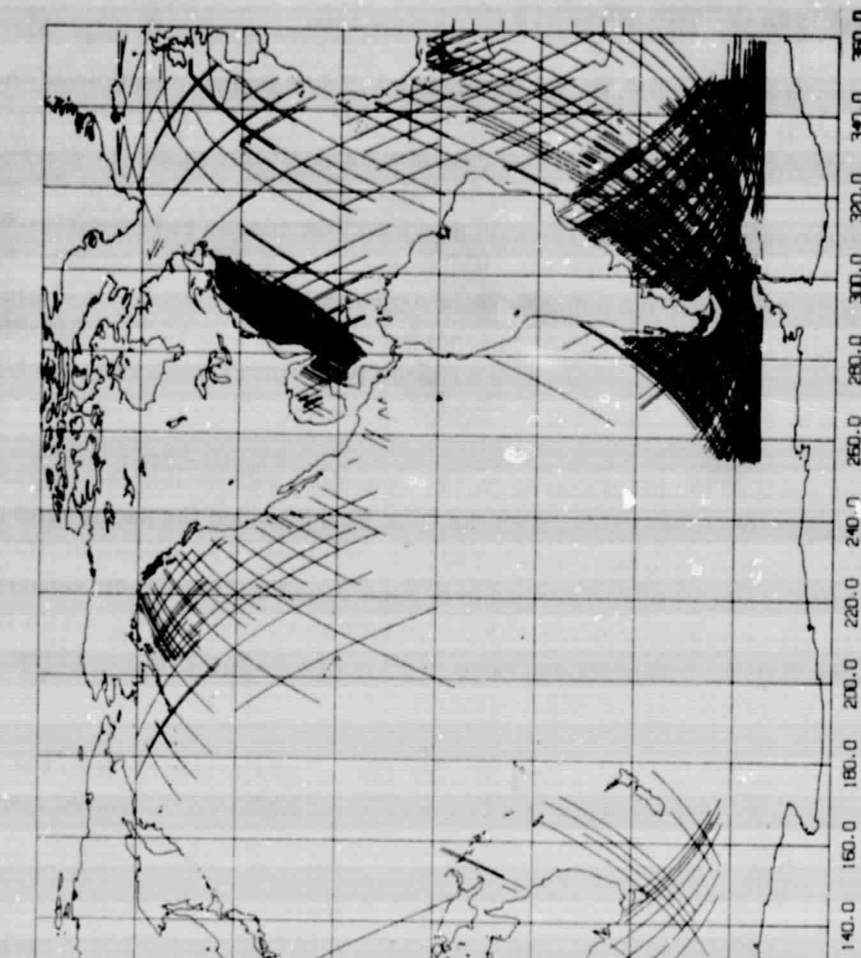

Figure 9. Distribution of GEOS-3 Altimetry in the 1976 Data Set

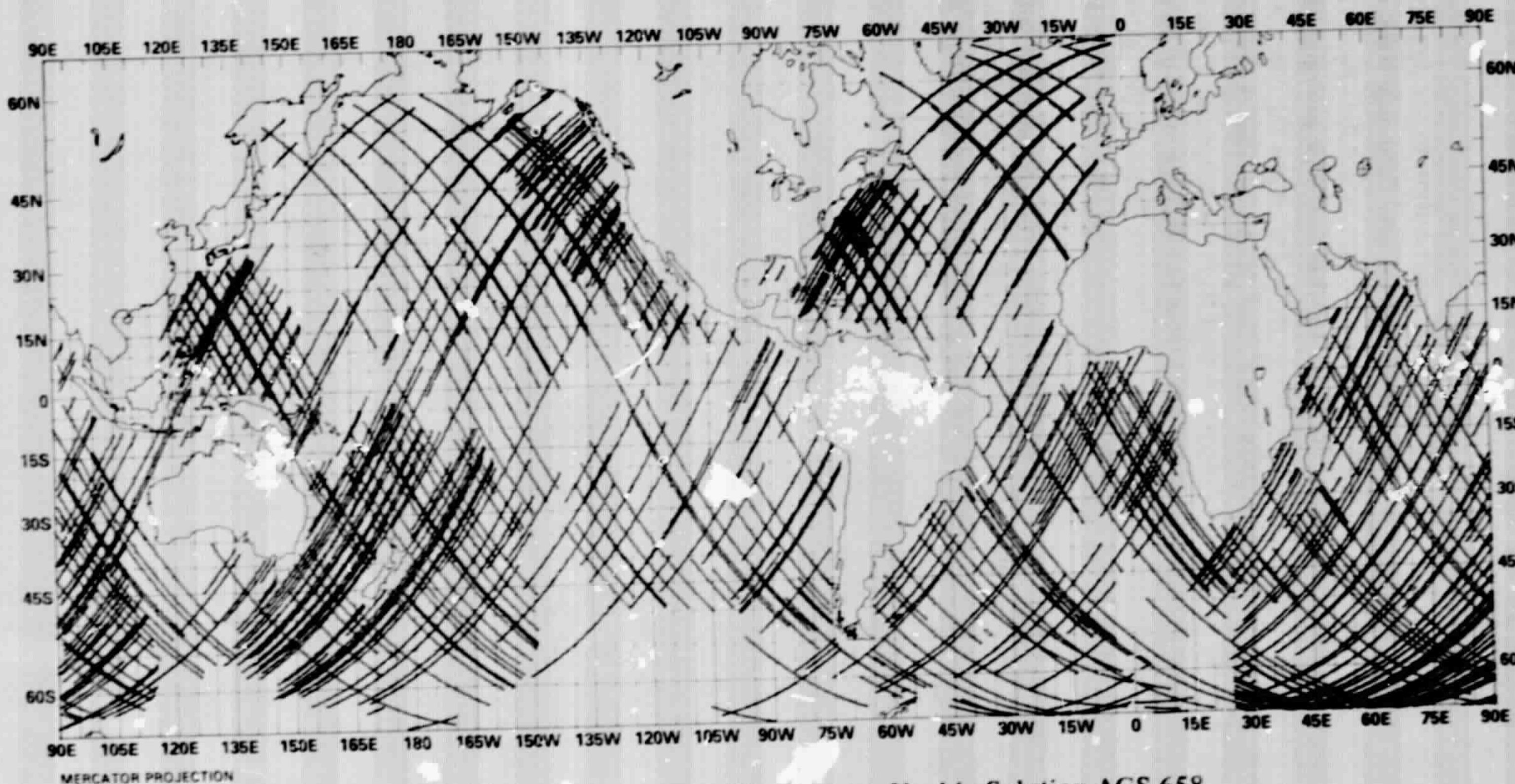

Figure 10. Distribution of GEOS-3 Altimetry Used in Solution AGS 658

The First Degree Harmonic

It had been hoped to recover the second largest contribution to the dynamic SST (ζ_{s111} – Table 5) which has a value −21 cm. Being a first degree harmonic term, it can only be obtained from the analysis of the orbit related sea surface height data bank (i.e., either the Equinox Set or the 1976 Data Set). Solutions for this harmonic can only be obtained if the GEOS-3 orbits refer to the geocenter without error. This is not the case. As the first degree harmonic of ζ_s has a degree variance of $515\,cm^2$, it follows that the center of the ellipsoid of best fit to the sea surface does not depart from the geocenter by more than 25 cm. The degree variance of the value obtained from the Wallops orbits in all cases exceeds $18500\,cm^2$. The only inference that can be drawn is that the origin of the system of reference used in integrating the orbits is displaced from the geocenter by not more than ±1.5 m.

The largest contribution, by far comes from the first degree zonal harmonic (Table 5), which indicates a southward displacement of 1.49 ±0.15 m. As the sources from which orbital information were obtained varied, no specific conclusions are drawn from these numbers at the present time.

The Zonal Harmonics of Degree 3 and 4

The magnitudes of ζ_{s130} and ζ_{s140} are smaller than one-tenth the estimated systems noise in the case of the orbit related SST. The signal to noise approaches the 0.1 level in the case of the long arc solutions AGS 658 and EGM 19. As pointed out earlier, it is debatable whether the long arc solutions can provide estimates of ζ_{s130}. Disregarding values from the suspect constrained solutions, the average result from Table 5 is

$$\zeta_{s130} = +7 \pm 5\,\text{cm} \qquad (48).$$

The standard deviation is significantly smaller than expected from the input data noise levels and is in very good agreement with the oceanographically determined value. If the standard deviations are to be taken at face value, the seasonal effect on ζ_{s130} should be absent in the result from the equinox experiment. The difference of +2 cm is consistent with the 1976 data set representing data during a southern summer. The irregular distribution of data between the hemispheres (Figure 9) cautions against any drawing of conclusions from this result.

The value of ζ_{s140} is not so well defined. The average value obtained without enforcing crossover constraints is

$$\zeta_{s140} = -23 \pm 18\,\text{cm} \qquad (49).$$

The standard deviation is much more in keeping with that estimated from the noise of the input data. The values obtained from orbital and long arc solutions differ somewhat. The reason for this difference is not clear at this stage.

8. CONCLUSIONS

The following conclusions can be reached:

(i) A basis exists for determining three out of the five dominant parameters of the quasi-stationary dynamic SST from GEOS–3 altimetry despite the fact that the data acquisition patterns did not have global oceanographic objectives in view.

(ii) The present series of solutions are confined to the basic orbits generated for Wallops Flight Center, as modified in ways prescribed above. The results obtained are preliminary and subject to revision as improved orbits become available. On the basis of the discussion given above, the best values for the dominant harmonics other than those of degree one, in the SST are the following

$$\begin{aligned} &\zeta_{s120} \text{ (Second Degree Zonal)} = -43 \pm 6\,\text{cm} \quad (-46) \\ &\zeta_{s130} \text{ (Third Degree Zonal)} = +7 \pm 5\,\text{cm} \quad (+7) \\ &\zeta_{s140} \text{ (Fourth Degree Zonal)} = -23 \pm 18\,\text{cm} \quad (-10) \end{aligned} \tag{50}$$

the values in brackets being the oceanographic estimates. The value for ζ_{s120} has taken into account the contribution of the permanent Earth tide and the -2 cm effect introduced by the irregular data sampling (Table 5, Columns 1 and 2).

(iii) It is hoped to improve on these results by a revision of the constraining procedure used in preparing Table 5 (i.e., equation 8) in conjunction with improved laser supported orbits.

(iv) An improved gravity field model for the low degree tesseral terms aiming for a resolution of 3 parts in 10^9 is an important pre-requisite to further progress in this area.

(v) It cannot be too strongly emphasized that an evenly spaced altimeter data coverage of the world's oceans is indispensable for recovering the long wave features of the quasi-stationary sea surface topography with confidence. On the basis of the present study, this could be achieved if the radial component of orbital precision can be improved by one order of magnitude.

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16. Abstract

The 1977 GEOS-3 altimetry data bank at Goddard is analysed for the geometrtical shape of the sea surface as expressed as surface spherical harmonics after referral to the higher reference model defined by GEM 9. The resulting determination is expressed as quasi-stationary dynamic sea surface topography(SST). The results obtained are compared with equivalent parameters obtained from surface measurements of temperature, pressure and salinity. The most clearly defined parameter is the zonal harmonic of degree 2 which is determined to be -43±6 cm. This is smaller than the oceanographic value of -46 cm. Values with slightly larger uncertainties are obtained for the 3rd and 4th degree zonal terms. These values are in reasonable agreement with oceanographic values.

Attempts to obtain the first degree harmonic in the SST were not successful due to the masking effect of the non-geocentricity of the system of reference used. The dominant effect is a southward displacement of 1.5 m along the polar axis. The results are preliminary and subject to improvement with better ephemerides for GEOS-3.

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