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S-Band Radar Calibration Test of the Lightning Detection and Ranging System (LDAR)

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National Aeronautics and
Space Administration

John F. Kennedy Space Center

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S-BAND RADAR
CALIBRATION TEST OF THE
LIGHTNING DETECTION AND RANGING SYSTEM (LDAR)
RCA 620-5001
Prepared for
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S-BAND RADAR
CALIBRATION TEST OF THE
LIGHTNING DETECTION AND RANGING SYSTEM (LDAR)
RCA 620-5001

June 1, 1978

SUMMARY

A calibration and an accuracy assessment of the Lightning Detection and Ranging System (LDAR) was attempted through a simultaneous track of an S-Band radar transponder and a lightning simulator carried by the NASA-6 airplane on a test flight. Included in the report are the results of the calibration test, error analyses of the radar and the LDAR systems, and recommendations for future calibration tests.

OUTLINE

- I. INTRODUCTION
- II. DESCRIPTION OF CALIBRATION TEST
- III. DETAILED DISCUSSION OF TEST RESULTS
 - A. Systematic Errors
 - 1. Comparison of LDAR with Radar Data
 - Y-Coordinate Differences
 - X-Coordinate Differences
 - Height Differences
 - Ground Range Differences
 - Azimuth Angle Differences
 - Time-of-Arrival Differences
 - Error Vectors
 - B. Random Errors
 - Radar
 - LDAR
- IV. RADAR SYSTEM ERRORS
- V. LDAR SYSTEM ERRORS
- VI. CONCLUSIONS
- VII. RECOMMENDATIONS
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I. INTRODUCTION

A theoretical error analysis of the Lightning Detection and Ranging System (LDAR) was published¹ in March 1977. To supplement the theoretical analysis, experimental measurements of the accuracy of the system at a variety of points in the measurement field were desired. An opportunity for the sought-for experimental verification presented itself in August 1977 when the NASA-6 aircraft became available for a test flight.

The airplane was already equipped with an S-Band radar beacon. By adding a 45 pound lightning simulator package and a 65 inch whip antenna, we were in a position to take simultaneous LDAR and radar position readings in all four quadrants at different altitudes, limited only by the 15,000 feet altitude limit of the NASA-6, and the decreasing level of the lightning simulator signal strength at long distances. Such a calibration flight was conducted on August 24, 1977. The detailed results of the LDAR-Radar calibration flight are presented in this report.

II. DESCRIPTION OF CALIBRATION TEST

The LDAR system, as currently implemented, actually consists of two independent, four-station, hyperbolic time-of-arrival networks of the 120°-Y configuration described in "An Accuracy Analysis of the LDAR System"¹. The central station of the two networks is common, hence we have a total of seven stations. The LDAR System determines position from the differences in the time of arrival at four stations, having a base line length of approximately 10 km. In operation the relative times of arrival are recorded, as well as the computed position in space (X,Y,Z) from which the pulse originates. The configuration and its location at the Kennedy Space Center is shown in Figure 1.

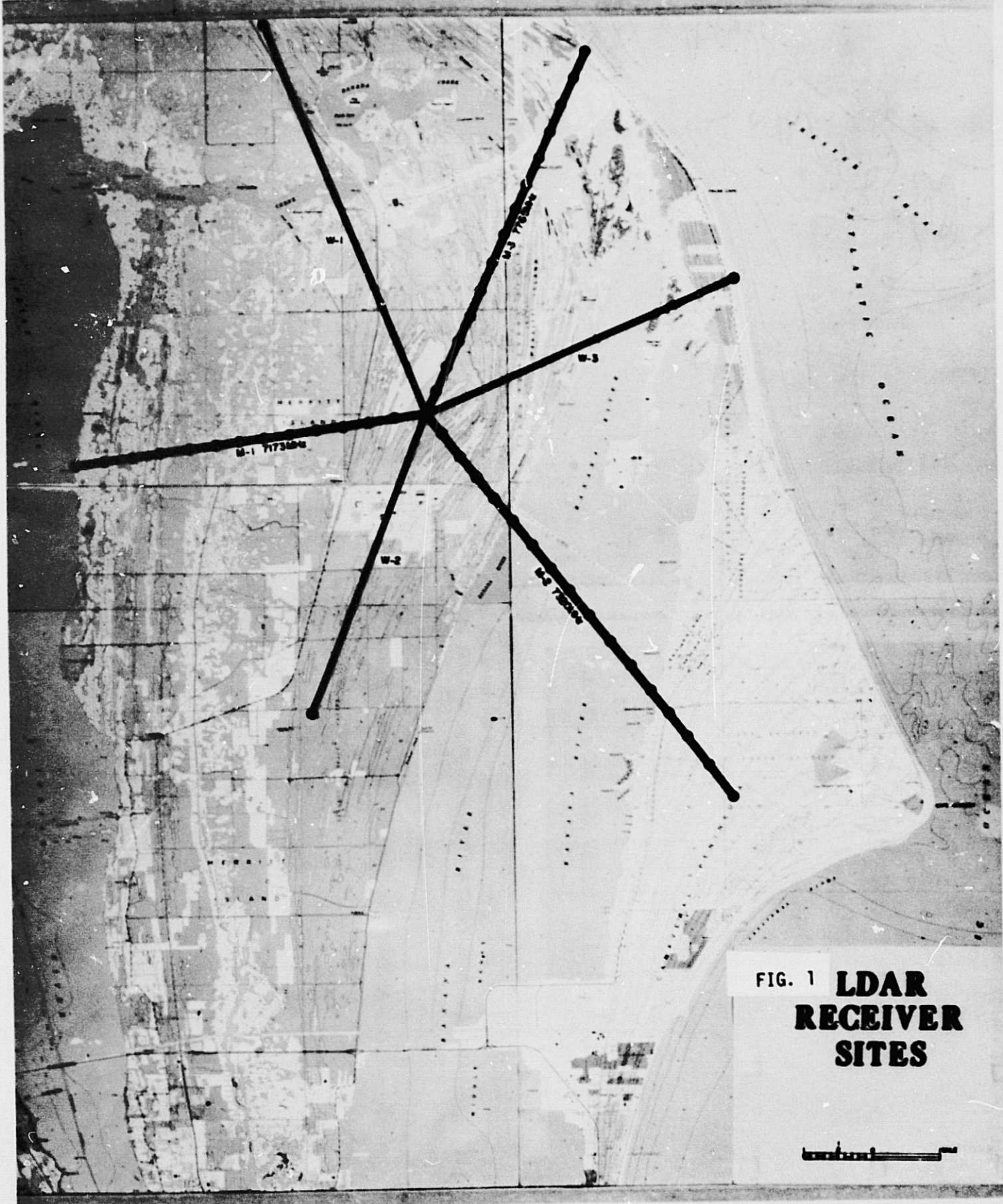


FIG. 1 **LDAR
RECEIVER
SITES**

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The second, independent network of stations is used to provide a check against false data points. In operation, data is not accepted unless the measurements of the two independent sets of stations agree within 10%. Actually, the agreement is much better than that - less than 5% for distances out to two times the baseline length, that is out to 20 km. Within the baseline of the system, that is within 10 km, the agreement is more like 2%.

We have observed that the difference between the two systems is a fairly good measurement of the random error of the measurement.

The calibrations tests disclosed an additional advantage of having two systems spaced by 60 degrees, that is of having two systems spaced symmetrically. The calibration data showed that, in general, the altitude error of System #1 was of sign opposite to the altitude error of System #2. This apparently results from the complementary orientation of the two systems. Advantage of this observation is taken by using the average of the two altitude readings. Since the errors are of opposite sign they tend to cancel, producing a much smaller total error than we would have by using one system alone, or even by using two systems that were not spaced at 60 degrees so as to achieve symmetry.

The Lightning Simulator

Lightning simulation is produced by raising the voltage across a 1/4 inch air gap to breakdown. The lightning simulator package consists of a 30,000 volt commercial high voltage power supply, whose output voltage is controlled by an autotransformer at the 115 volt ac input.

The output of the high voltage supply feeds a 0.003 μ f, 30,000 volt capacitor through a 120,000 ohm, 2 watt carbon resistor. A 1/4 inch air gap is connected across the capacitor, and serves to discharge the capacitor in an impulsive manner when the breakdown potential of the gap is reached.

The 120,000 ohm resistor protects the high voltage supply from the arc's short circuit, and at the same time, permits a slow charge-buildup for the next breakdown. A 65 inch whip antenna was connected to the hot end of the spark gap.

In operation the autotransformer input is increased, increasing the output of the high-voltage supply, until arc-over of the air gap occurred (at approximately 20,000 volts). The input voltage is then increased by an additional 10 percent to achieve stable operation. The repetition frequency of the arc, as well as the energy in the gap can be controlled by adjustment of the autotransformer. The lightning simulator was operated at approximately 2 discharges per second.

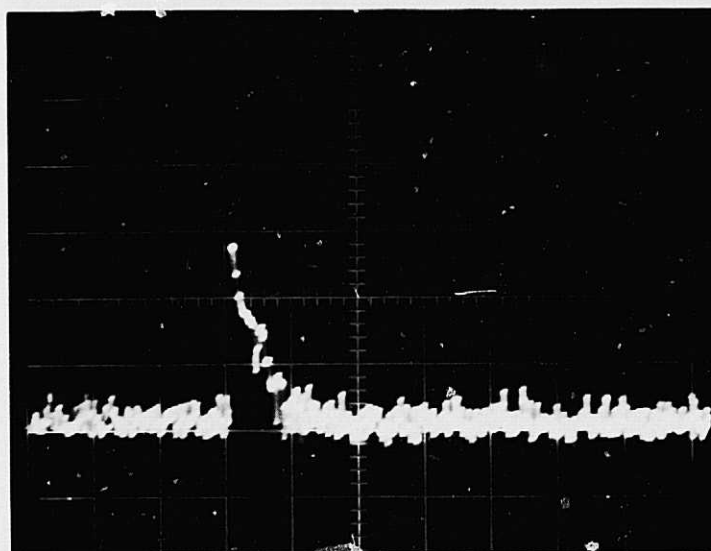
Figure 2 shows a typical received lightning simulator waveform, that is the envelope detected 30-50 MHz input to the LDAR system. This lightning simulator waveform is typical of lightning waveforms received by the system.

The Flight Plan

The scheduled flight plan is shown in Figures 3, 4, 5 and 6. A circular flight about the LDAR System is desired to test the accuracy of the system in all quadrants. A flight across the center of the system, along the azimuth of one of the legs, provides data at varying distance, as well as illuminating the effects of flying over a leg. A climb from 3000 feet altitude to 9000 feet altitude provides data on the effect of changing altitude. Next the original circular path around the LDAR system is repeated, this time at 9000 feet altitude. Finally the plane is scheduled to fly out to Bithlo and back along the Shuttle corridor to provide a check of the accuracy at larger distances, and provide data along the Shuttle corridor.

The Actual Flight

The actual flight was close to the flight plan. The only exception was



← 20 Microseconds →

Fig. 2 LIGHTNING SIMULATOR WAVEFORM

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3000 FT ALTITUDE

10 MI RADIUS

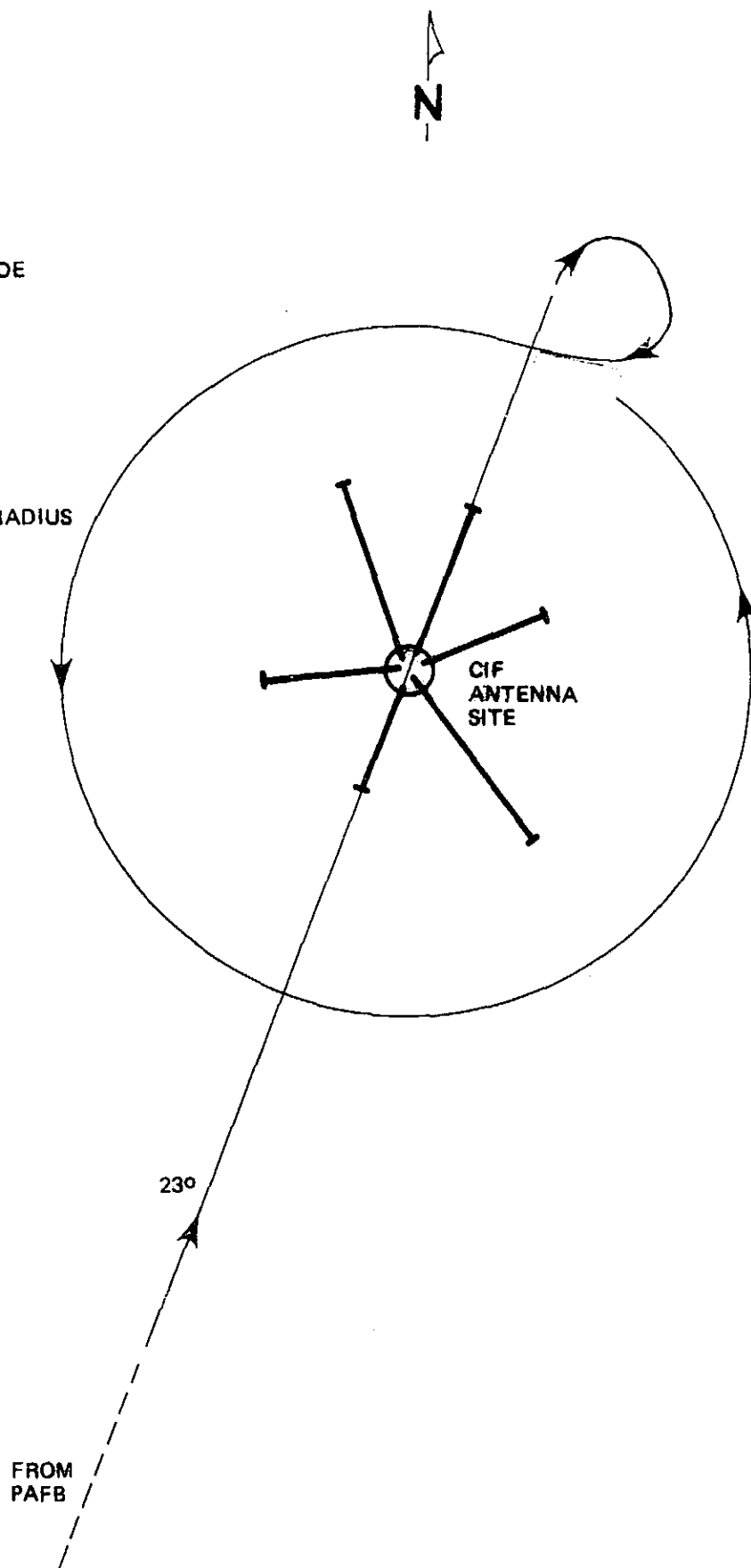


FIGURE 3. FLIGHT PLAN

3000 AND
9000 FT ALTITUDE

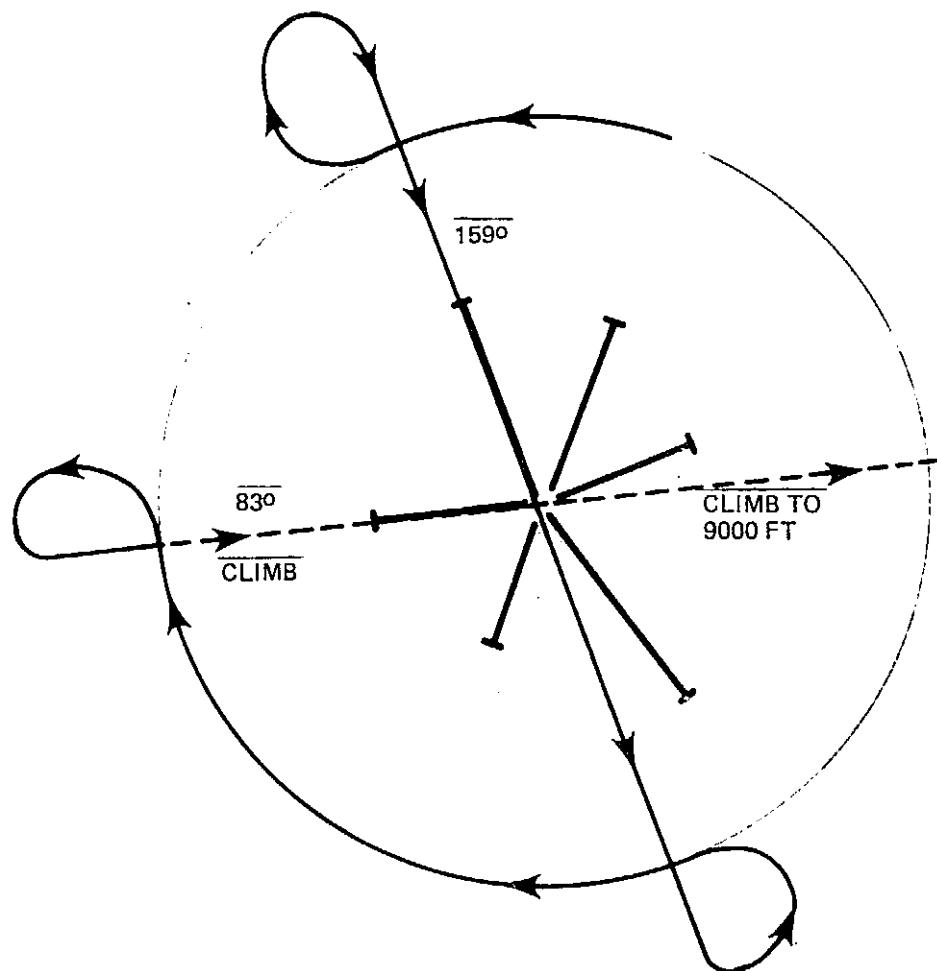


FIGURE 4. FLIGHT PLAN

9000 FT ALTITUDE



10 MI RADIUS

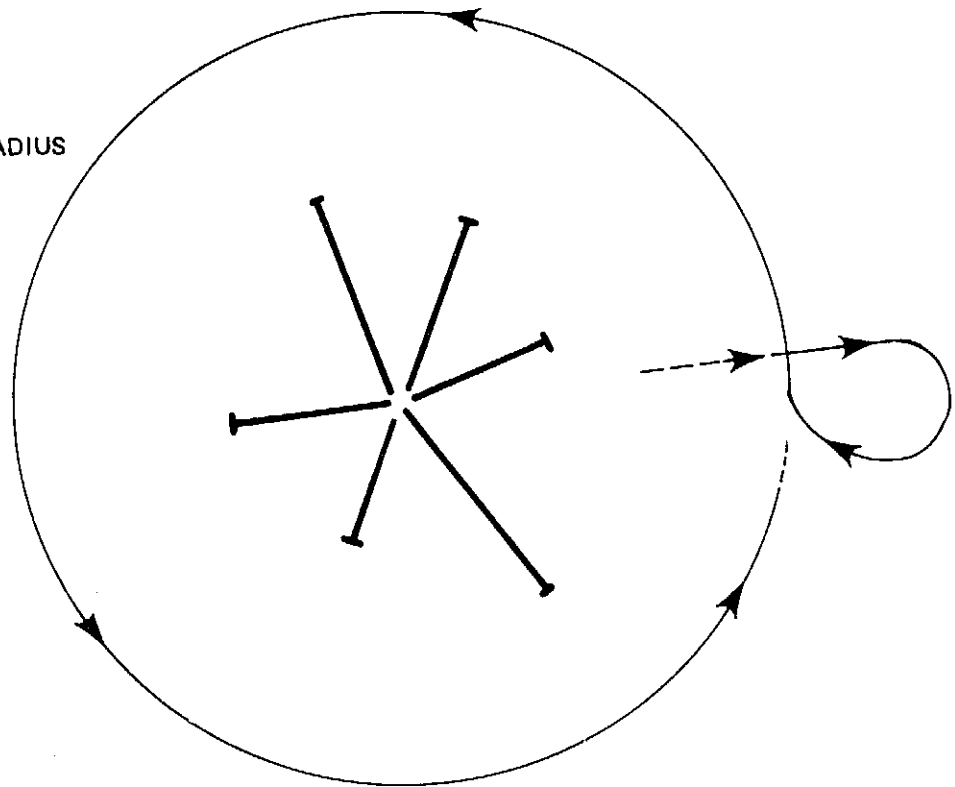


FIGURE 5. FLIGHT PLAN

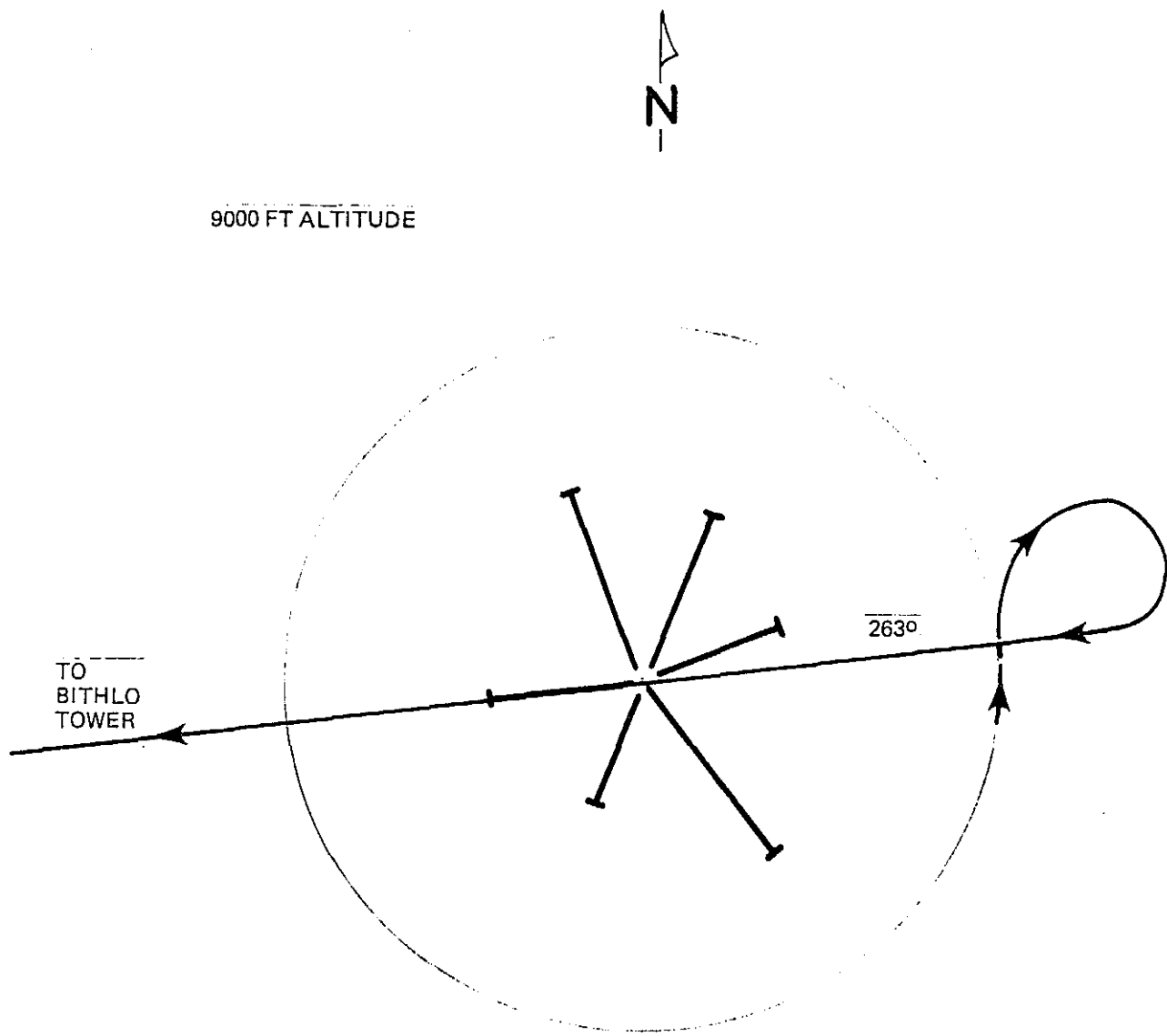


FIGURE 6. FLIGHT PLAN

a modification of the planned altitudes of 3000 and 9000 feet to 3500 and 9500 feet, respectively, to conform to established FAA UFR flight regulations.

Selected LDAR plots of LDAR's track of the plane are shown in Figures 7 to 13. The remainder of the report will concern itself with a detailed comparison of the radar and LDAR track to assess the accuracy of the LDAR System.

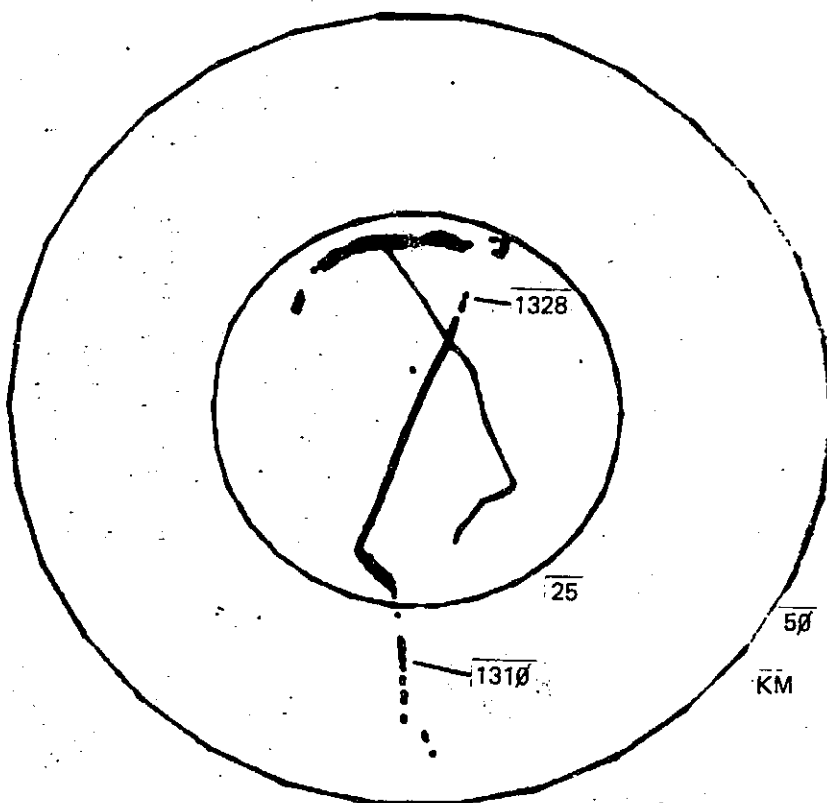
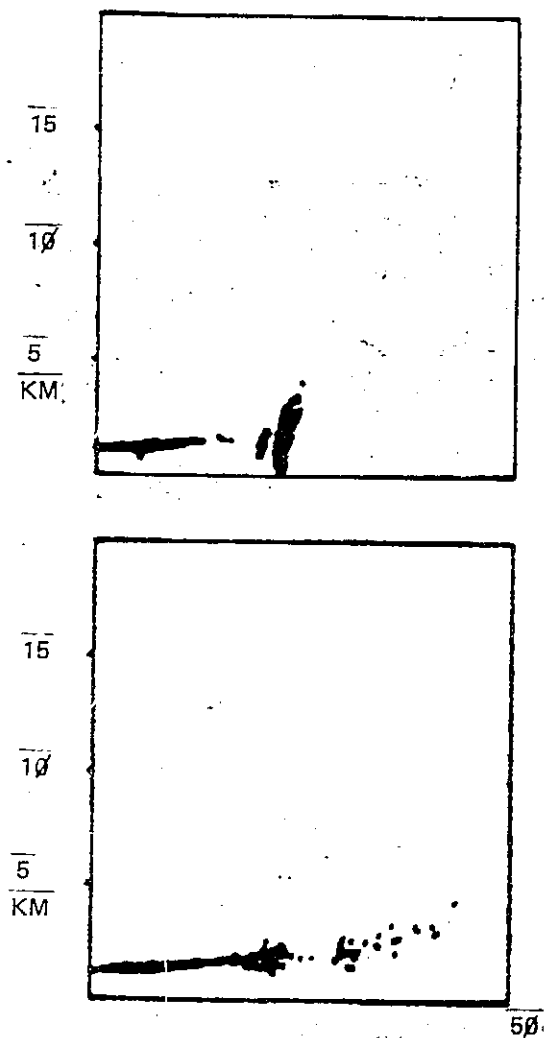
Radar Characteristics

The S-Band radar that was used in the LDAR calibration test is shown in Figure 14, along with its operating characteristics.

Of most interest are the published error characteristics given in Table I. The error characteristics of the radar set the limits to which the LDAR System calibration could be carried out.

TABLE I PUBLISHED S-BAND RADAR ERROR CHARACTERISTICS

<u>With Beacon</u>	<u>Random Error (σ)</u>	<u>Systematic Error</u>
Range	45 ft	100 ft
Azimuth	1.2 to 2 mils	1 mil
Elevation	1.2 to 2 mils	1 mil



REAL TIME	1000	POINTS/PLOT
DAY 236	TIME 1304	:22
	TO 1328	:26

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FIGURE 7. LDAR TRACK OF NASA-6

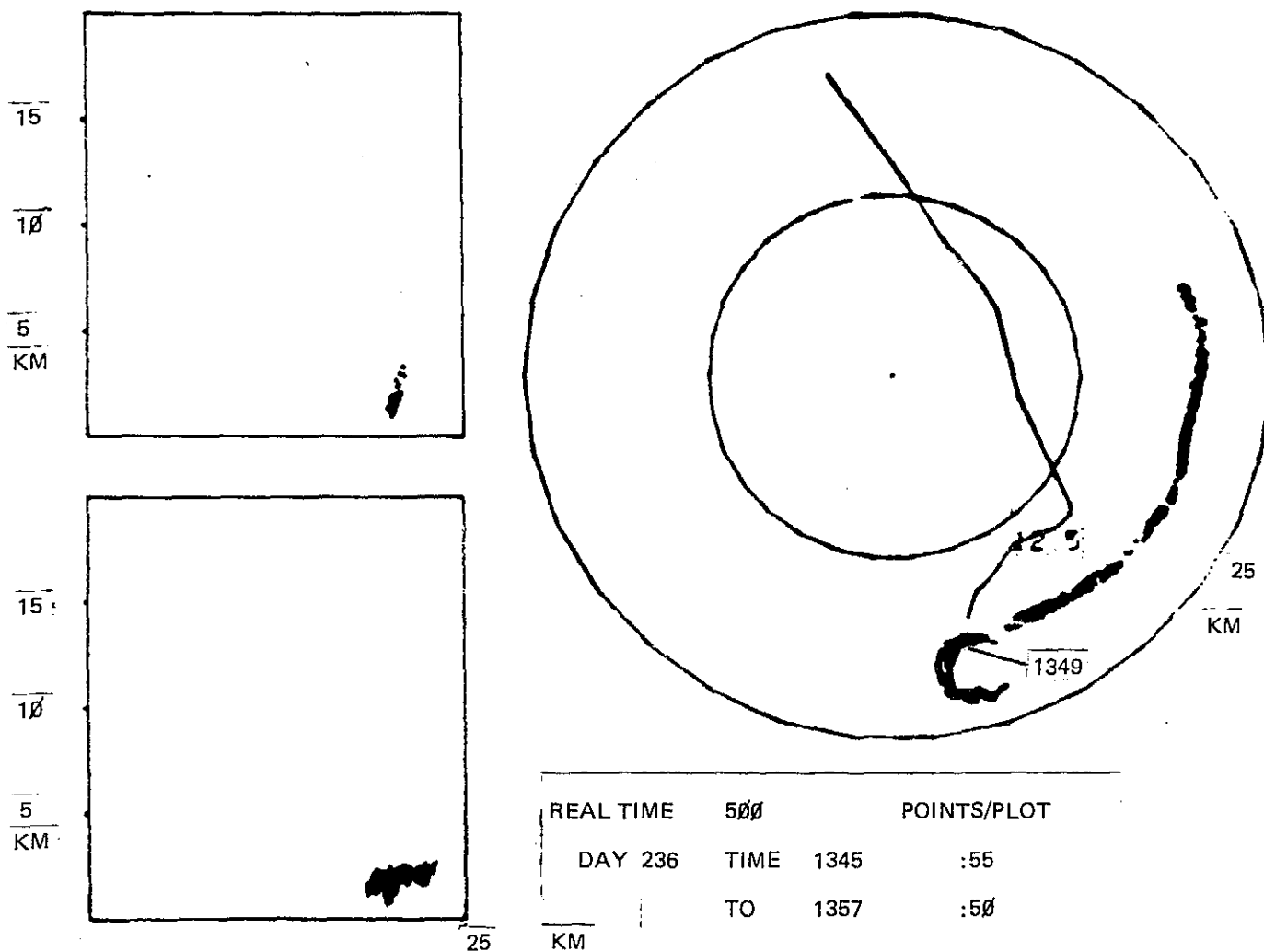
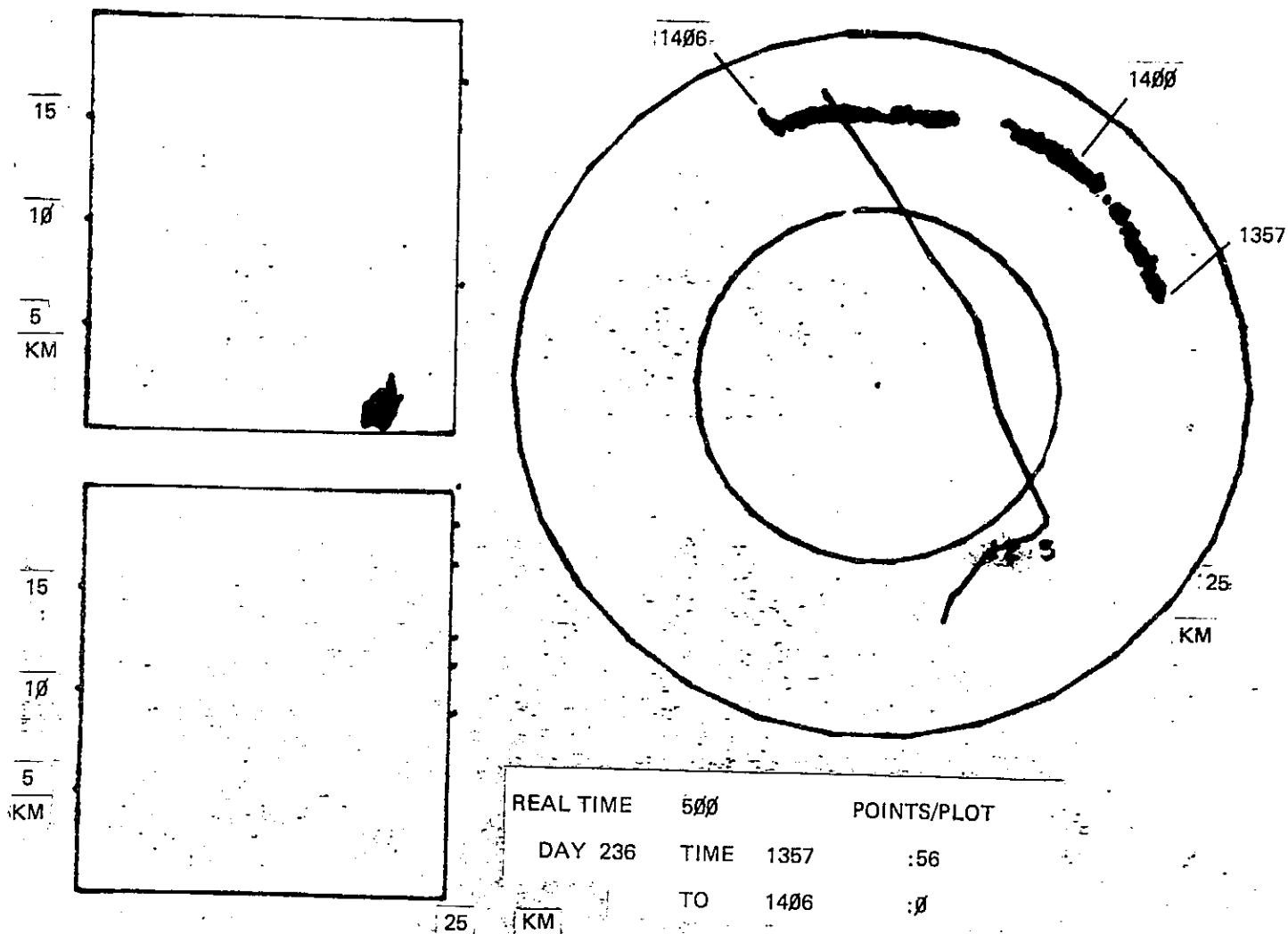


FIGURE 8. LDAR TRACK OF NASA-6



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FIGURE 9. LDAR TRACK OF NASA-6

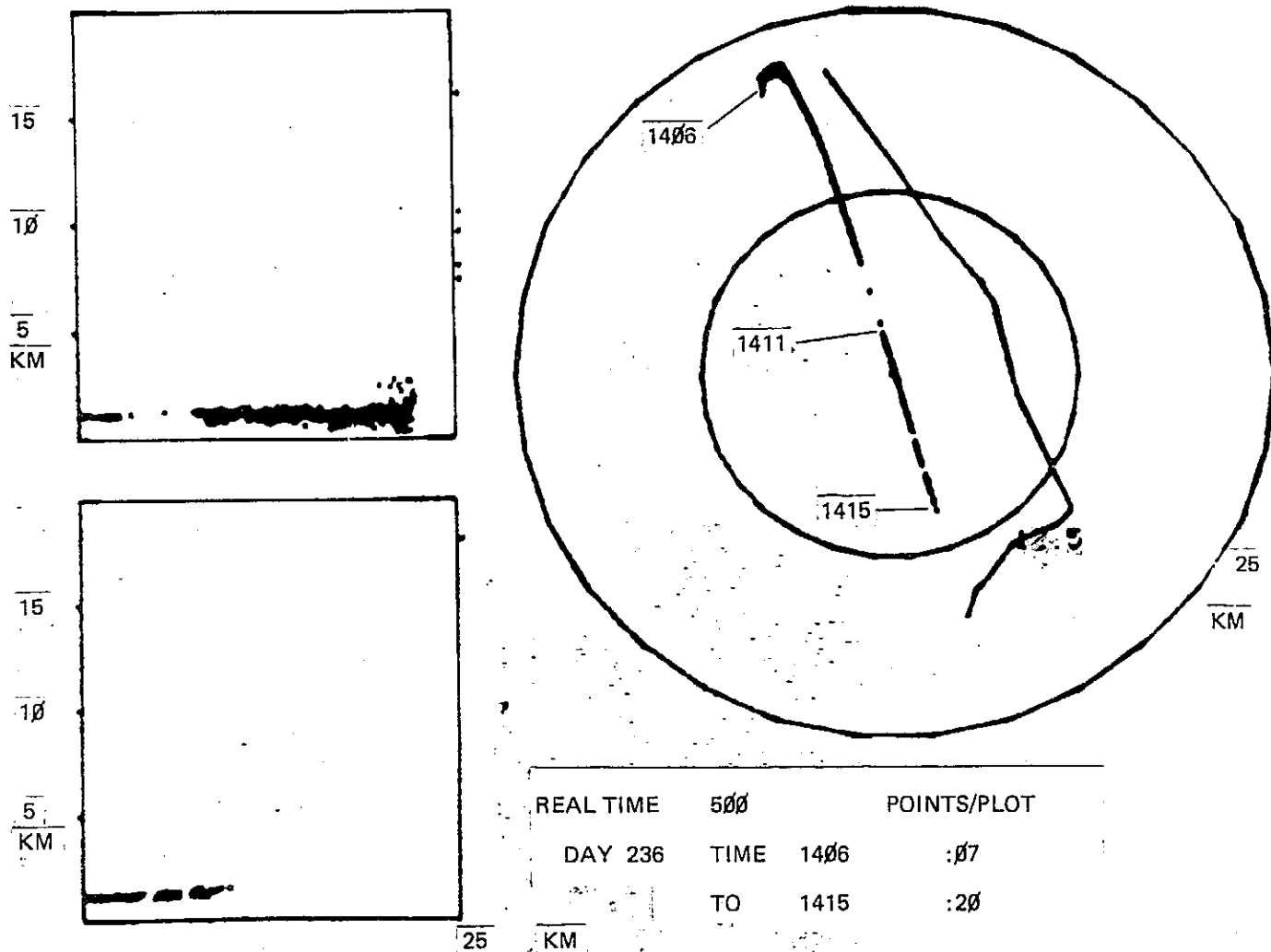
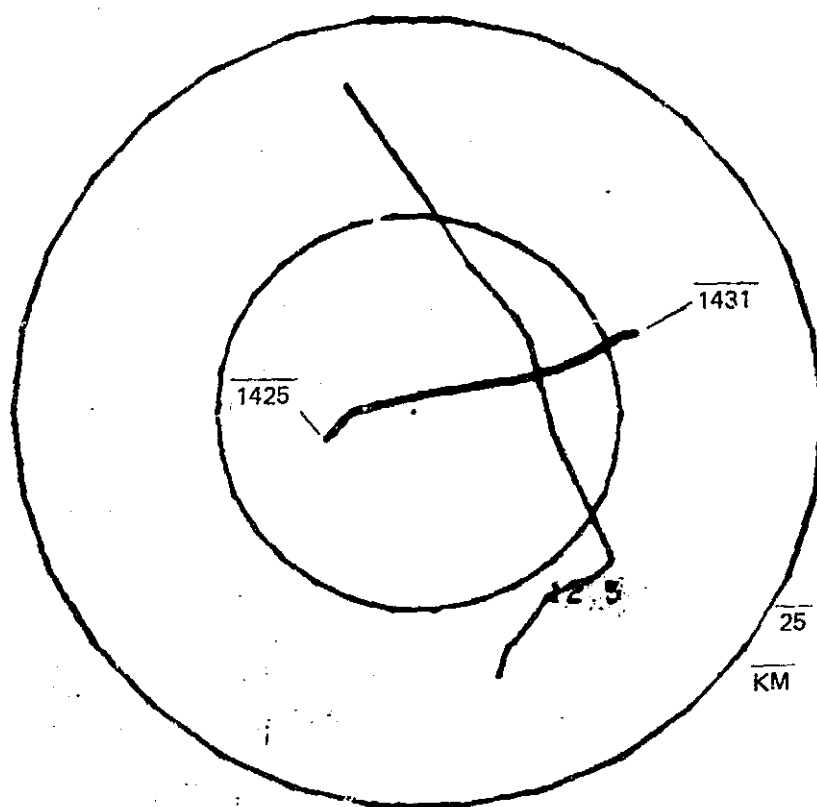
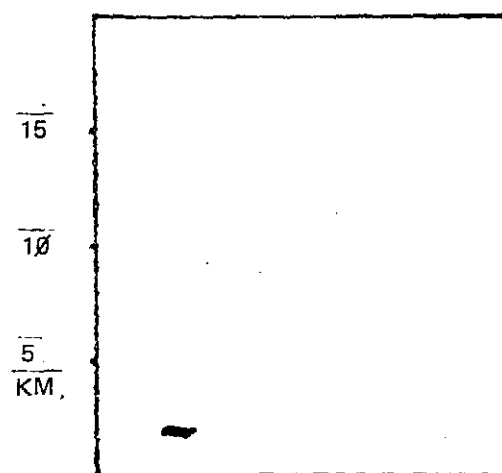
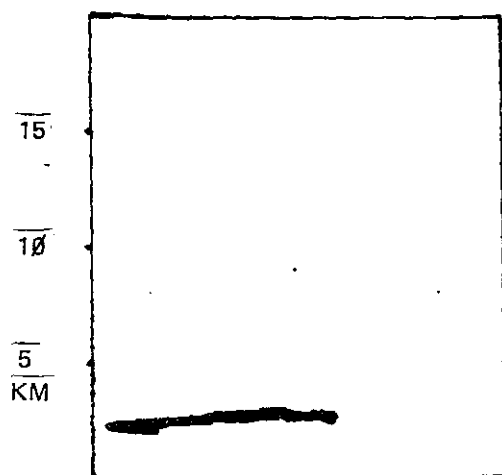


FIGURE 10. LDAR TRACK OF NASA-6



REAL TIME	500	POINTS/PLOT
DAY 236	TIME 1425	:48
	TO 1431	:06

25 KM

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FIGURE 11. LDAR TRACK OF NASA-6

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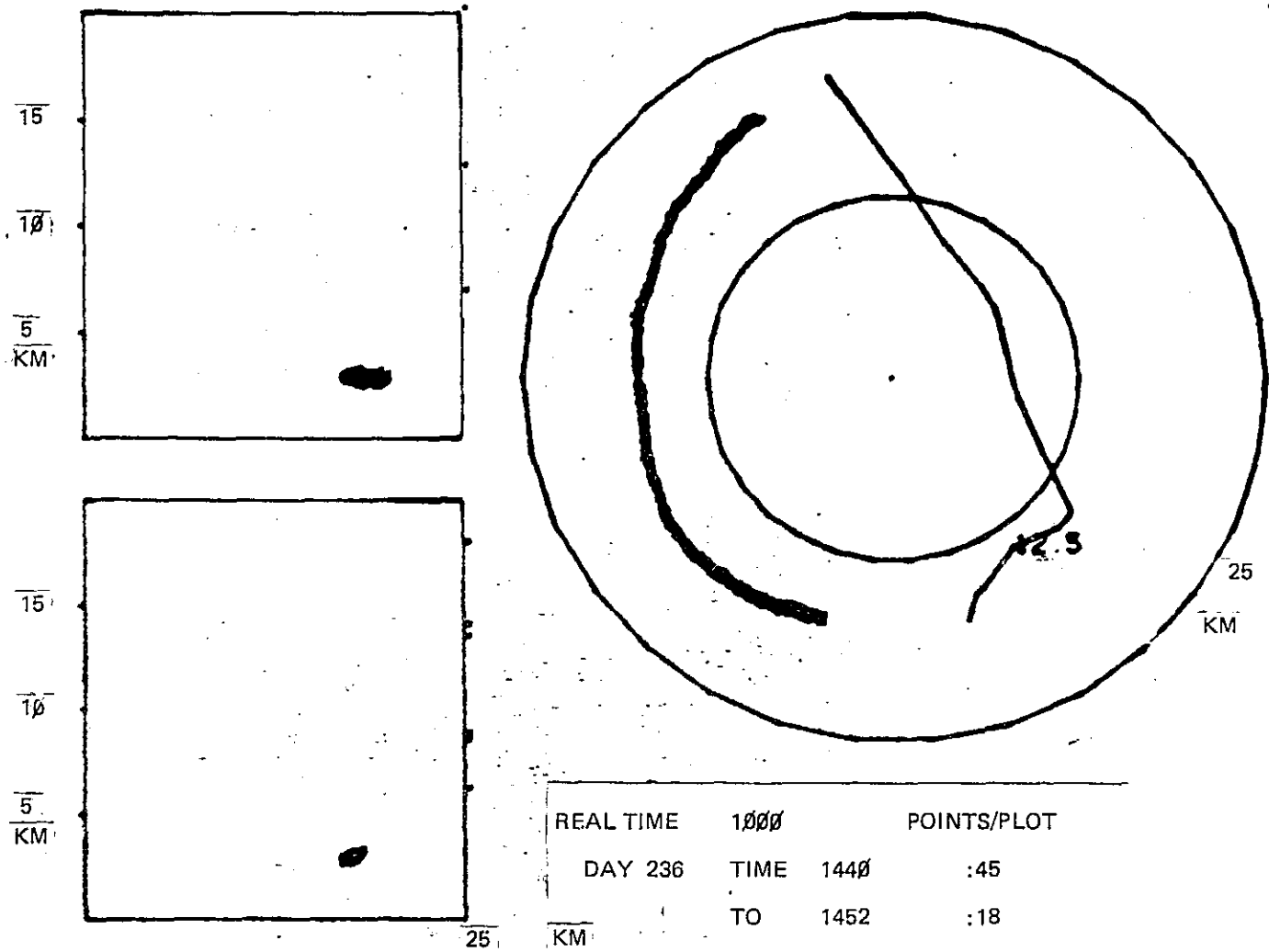
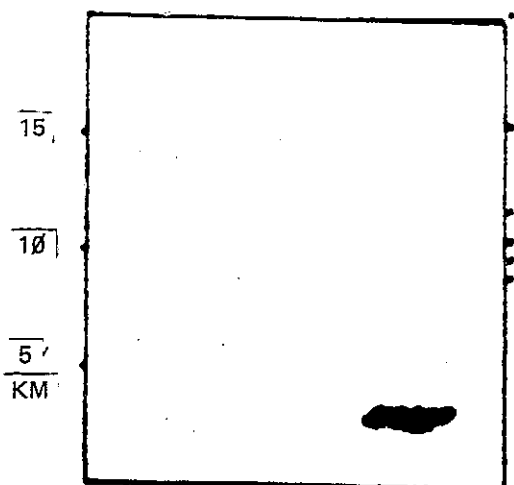
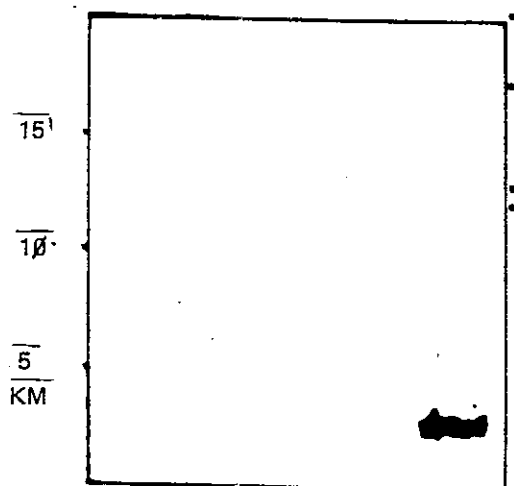
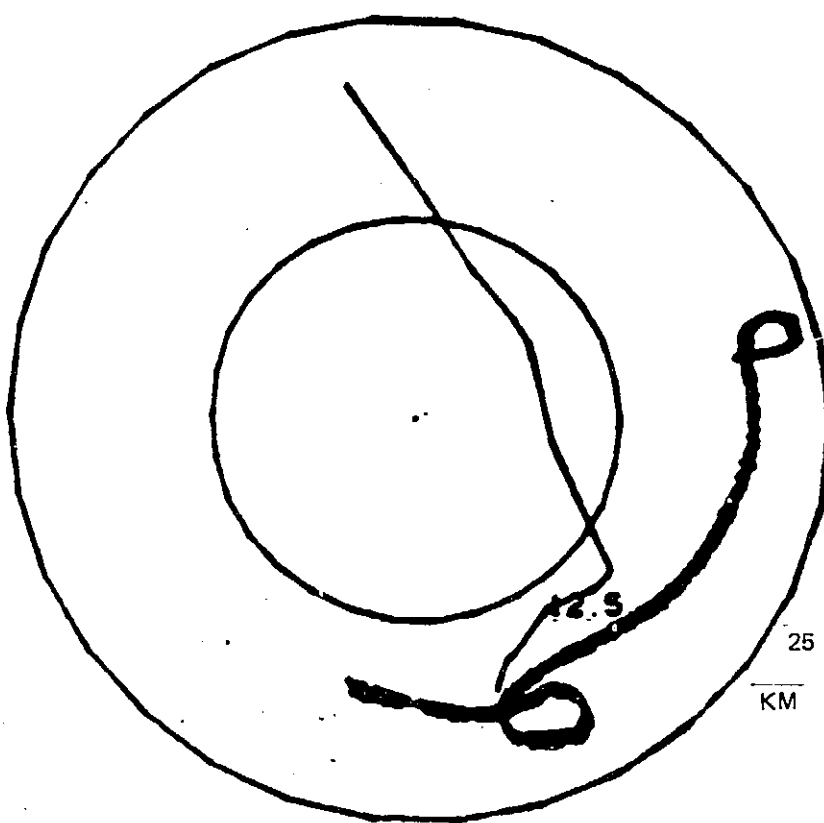


FIGURE 12. LDAR TRACK OF NASA-6.



25



REAL TIME	1000	POINTS/PLOT
DAY 236	TIME 1452	:25
	TO 1506	:05

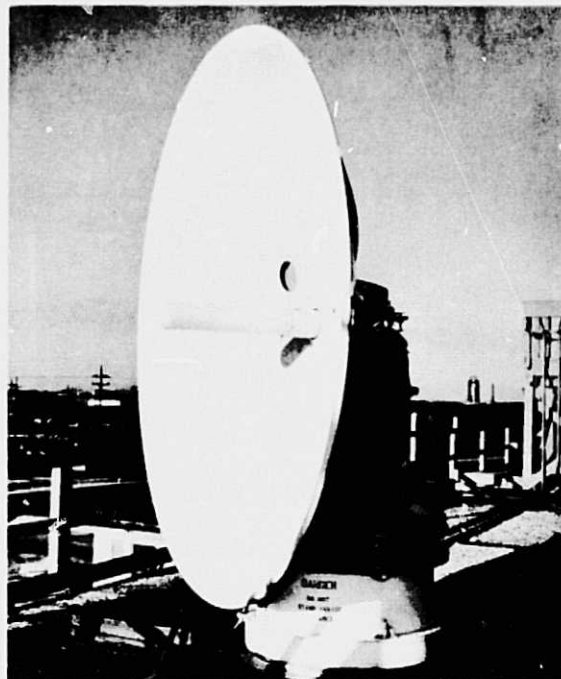
KM

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FIGURE 13. LDAR TRACK OF NASA-6

DESCRIPTION

The Mod II radars at ETR are modified and rehabilitated World War II SCR-584 radars. It is an automatic angle and range tracking radar designed to provide (1) azimuth and elevation angle and range selsyn data, (2) slant range, altitude, ground range, and XY ground position potentiometer data, and (3) boresight camera photographs and digital encoder data. All data are referenced to two calibration target poles, and a target of known range at every station. Mod II provides chiefly a network system to give tracking information for Range Safety and aircraft vectoring.



MOD II RADAR

Tracking with the aid of an airborne S-band beacon, Mod II can provide radar line-of-sight coverage to a 400-nm slant range. The phasing system used for beacon operations permits reliable beacon tracking from two sites simultaneously. This radar can be used to skin track as well as beacon track.

LOCATION

<u>Station</u>	<u>Latitude</u>	<u>Longitude</u>	<u>Elevation (ft)</u>
Radar 1.5	28°30'	80°35'	51.48

PERFORMANCE CHARACTERISTICS

Transmitter

Frequency: 2700 - 2900
Peak power: 250 kw
Pulse width: 0.8 μ sec
Pulse rate: 205 - 1,707 pps

Receiver

Frequency: 2650 - 2950 MHz
Noise figure: 13 db
Bandwidth: 3 MHz

Antenna

Size: 10 ft
Type: parabolic
Gain: 37 db
Scan: conical at 30 cps, 50% or 80% crossover
Power required at receiver antenna: -96 dbm
Polarization: horizontal or vertical
Trackable signal: 12db signal-to-noise

FIG. 14 S-Band Radar

III DETAILED DISCUSSION OF TEST RESULTS

Comparison of LDAR with radar data must recognize the existence of two types of errors in both systems. The first are the bias, or static errors, also known as systematic errors. The second are the random errors, or the unpredictable perturbations, that can only be defined in statistical terms, which are a measure of the scatter of the data, and which can be averaged out.

To determine the systematic errors, one must have available a second, more accurate system, to use as a reference. In this test, the more accurate system is taken to be the S-Band, Mod II Radar whose specifications have been given in Table I. For this system the systematic error is given as 100 feet in range, and 1 milliradian in azimuth and elevation. This will set the limits of accuracy to which we can determine the systematic errors of the LDAR System.

Random errors do not require a second, more accurate system, for their evaluation, but can be measured for a system, alone, from the scatter of the data. For completeness, the random errors of both the LDAR and radar systems have been determined, and will be found included in this report.

A. Systematic Errors

We will proceed to arrive at an estimate of the systematic errors in the LDAR System by differencing X, Y, and Z measurements, as well as range and azimuth measurements against the radar which, for the purposes of this test, is used as the more accurate system.

1. Comparison of LDAR with Radar Data

Radar data was furnished by the Air Force Eastern Test Range (AFETR) in a Cartesian X, Y, Z coordinate system, centered at the S-Band radar. LDAR normally is presented in meters, about the LDAR System. To make the data more readily comparable, LDAR data was transferred to the origin used by the

radar data and presented in feet in an X, Y, Z system centered at the radar. Much of the data presented in this report will be given in this coordinate system.

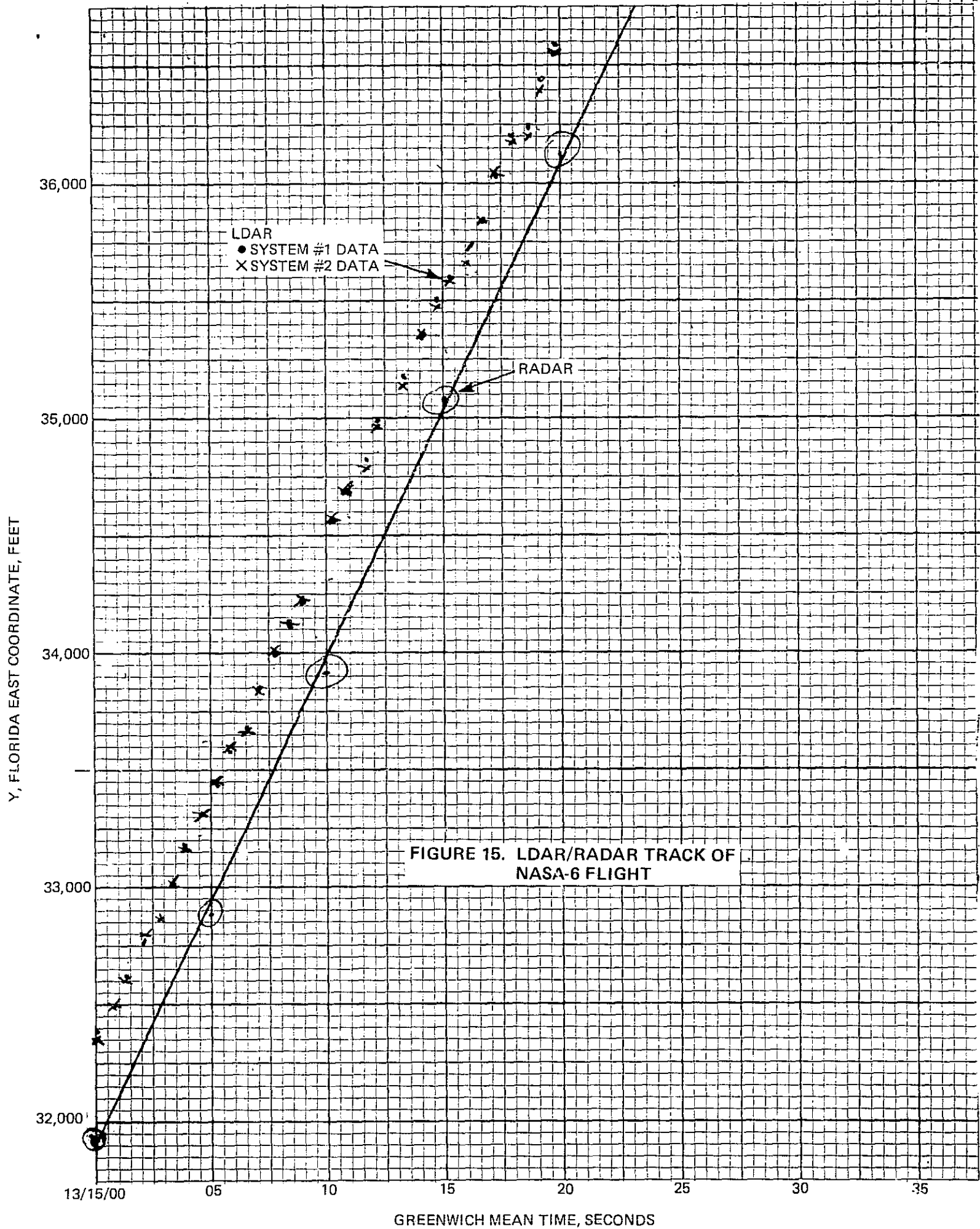
Y-Coordinate Differences

Plots of simultaneous LDAR/radar tracking data are shown in Figure 15 for a straight line portion of the flight (top of Figure 7), and in Figure 16 for a curved portion (upper right portion of Figure 13). Using an X, Y, Z system centered at the radar, the Y position is plotted against Greenwich Mean Time. Both LDAR System #1 and System #2 data are plotted. While radar data is furnished every 0.1 second, only data at 5 second intervals is shown plotted. The random error in the radar data is too small to affect the track shown here. Its magnitude will be discussed later.

The repetition frequency of the LDAR data is determined by the recurrence frequency of the LDAR lightning simulator. This frequency is approximately 1.5 per second, giving a data point spacing of approximately 0.7 seconds. In contrast to the radar, all the LDAR data points are plotted.

In Figure 15 it is apparent the difference between LDAR System #1 and System #2 data is small compared to the difference (500 feet) between LDAR and radar data shown in this plot. Also note that the LDAR data gives a Y coordinate position that is larger than that given by the radar. This was found to be true for all our measurements. The LDAR data always indicated a range that was larger than that indicated by the radar.

Figure 16 is of particular interest in that it not only illustrates the increased distance of the LDAR reading that is noted on all the LDAR radar comparisons, but it also shows a time discrepancy to exist between the two systems. The turn is seen to occur some 1/2 to 1 second earlier in the LDAR



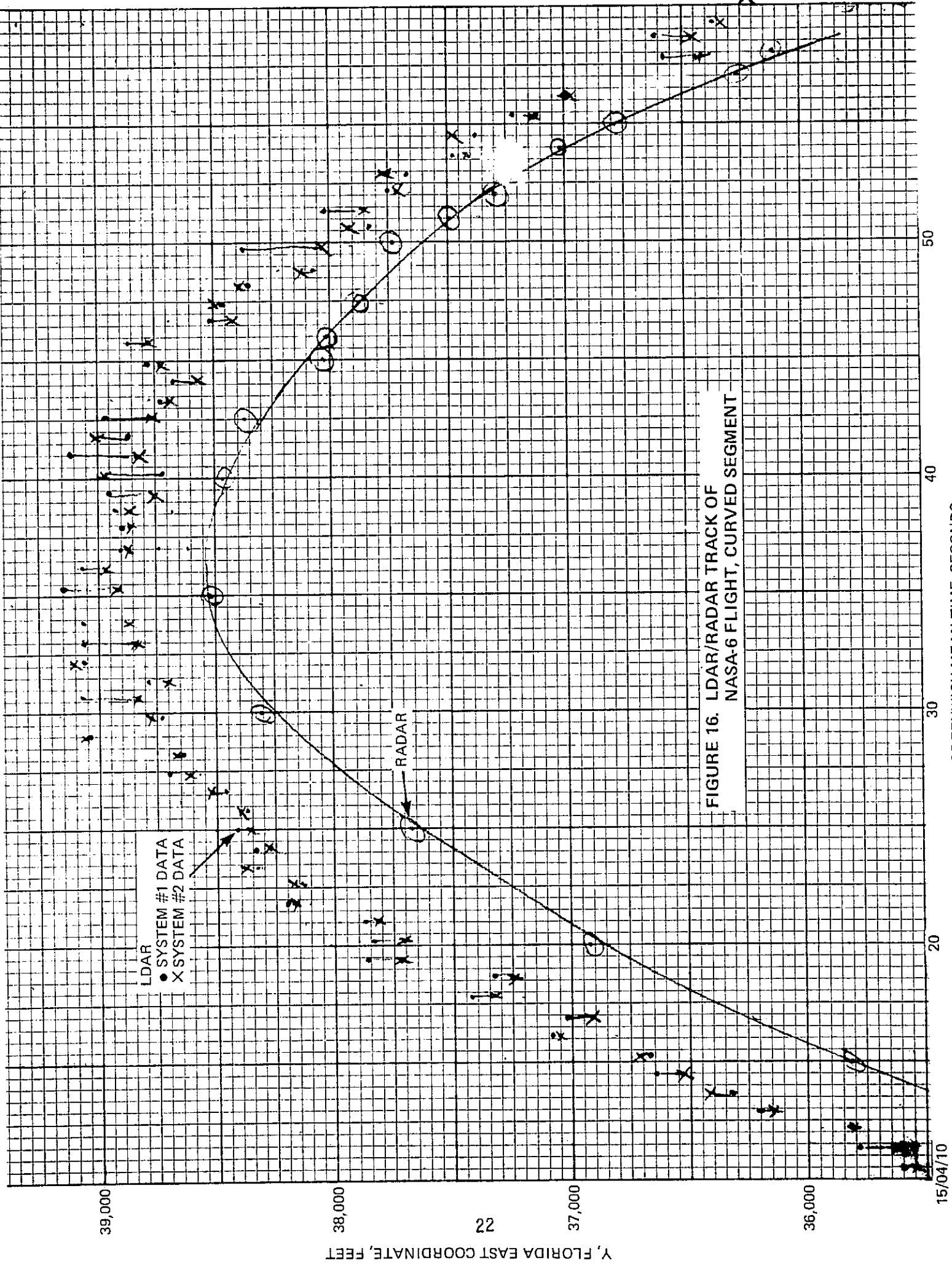


FIGURE 16. LDAR/RADAR TRACK OF
NASA-6 FLIGHT, CURVED SEGMENT

data. Other turns were analyzed with the same result. This discrepancy will be discussed further under Radar Errors.

S-Coordinate Differences

By analysis of a large number of data spans of at least 20 points for both positive and negative values of X , the error plot of Figure 17 was obtained. Plotted here is the difference, $X_R - X_{LDR}$, as a function of the X_{LDR} coordinate about the LDAR origin. Similar plots were obtained for the difference, $Y_R - Y_{LDR}$, as a function of Y_{LDR} .

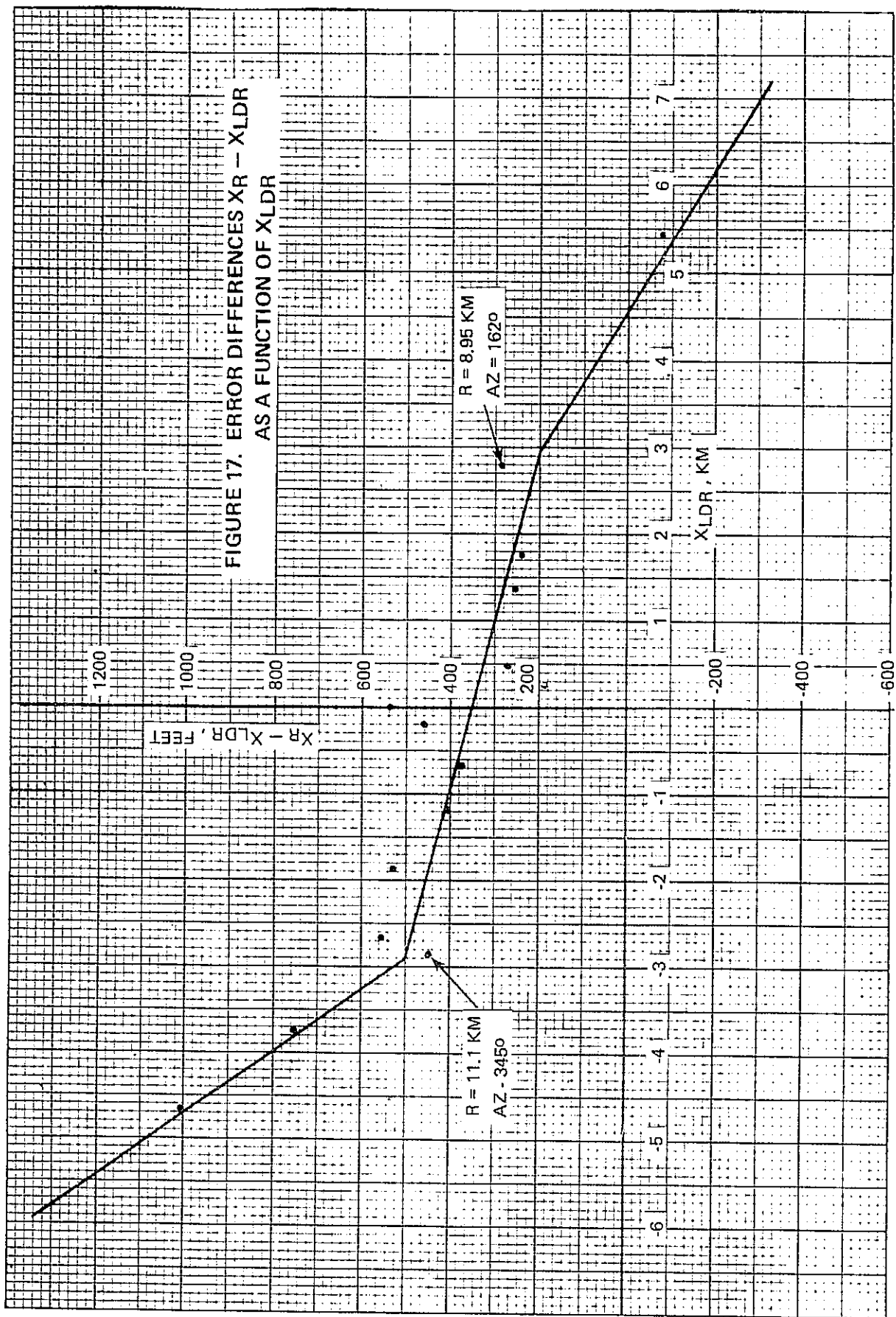
Two types of systematic error are apparent in Figure 17:

- 1) a zero-set error of some 350 feet, indicating that the radar and the LDAR do not agree on the coordinates of the LDAR System origin,
- 2) a scale factor error, which is apparent from the linear behavior of the difference, $X_R - X_{LDR}$, when plotted against distance X_{LDR} . Note that there is a break in the plot, which occurs near the edge of the system baseline, i.e. at 11.1 km /345° and 9.0 km /162°.

When inside the baseline of the system, the scale factor error is 1.6%. When outside the baseline of the system, the scale factor error will vary with distance, in that it depends on how much beyond the break in the curve the data point is located. For example, at 4 km the scale factor error is 3.4%, at 6 km the error is 5%. Note that in every case, the LDAR indicates a longer distance than does the radar.

Height Differences

Height differences, $Z_R - Z_{LDR}$, for a representative number of data points are given in Table II. The magnitude of the differences is distressing. This mean value is +885 meters (2950 feet) and the standard deviation is 120 meters.



In all cases the height indicated by the radar was higher than indicated by the LDAR.

An examination of the problem showed that the large differences are not chargeable to the LDAR system but represent a fault in the radar's height output instrumentation. The matter will be discussed in greater detail under Radar Errors.

TABLE II HEIGHT DIFFERENCES

GROUND RANGE	HEIGHT DIFFERENCES
km	$Z_R - Z_{LDR}$
Meters	
0	880
2	900
4	900
6	880
8	880
10	900
12	870
14	890
16	870
18	900
20	880
22	880
24	880
Mean	885
Standard Deviation	120

Ground Range Differences

Expressing the difference, $R_R - R_{LDR}$, as a percentage gives us the percentage error in indicated ground range R_R . This error is shown plotted in Figure 18 as a function of the ground range R_{LDR} . The points for the plot were taken from all quadrants. Azimuths of the data points are not shown, since the characteristic is independent of the azimuth.

For ranges less than 10 km (that is within the baseline of the system) the error in ground range is small, approximately 1%. Outside the baseline the error rises to some 7% at 60 km, some 6 baselines away.

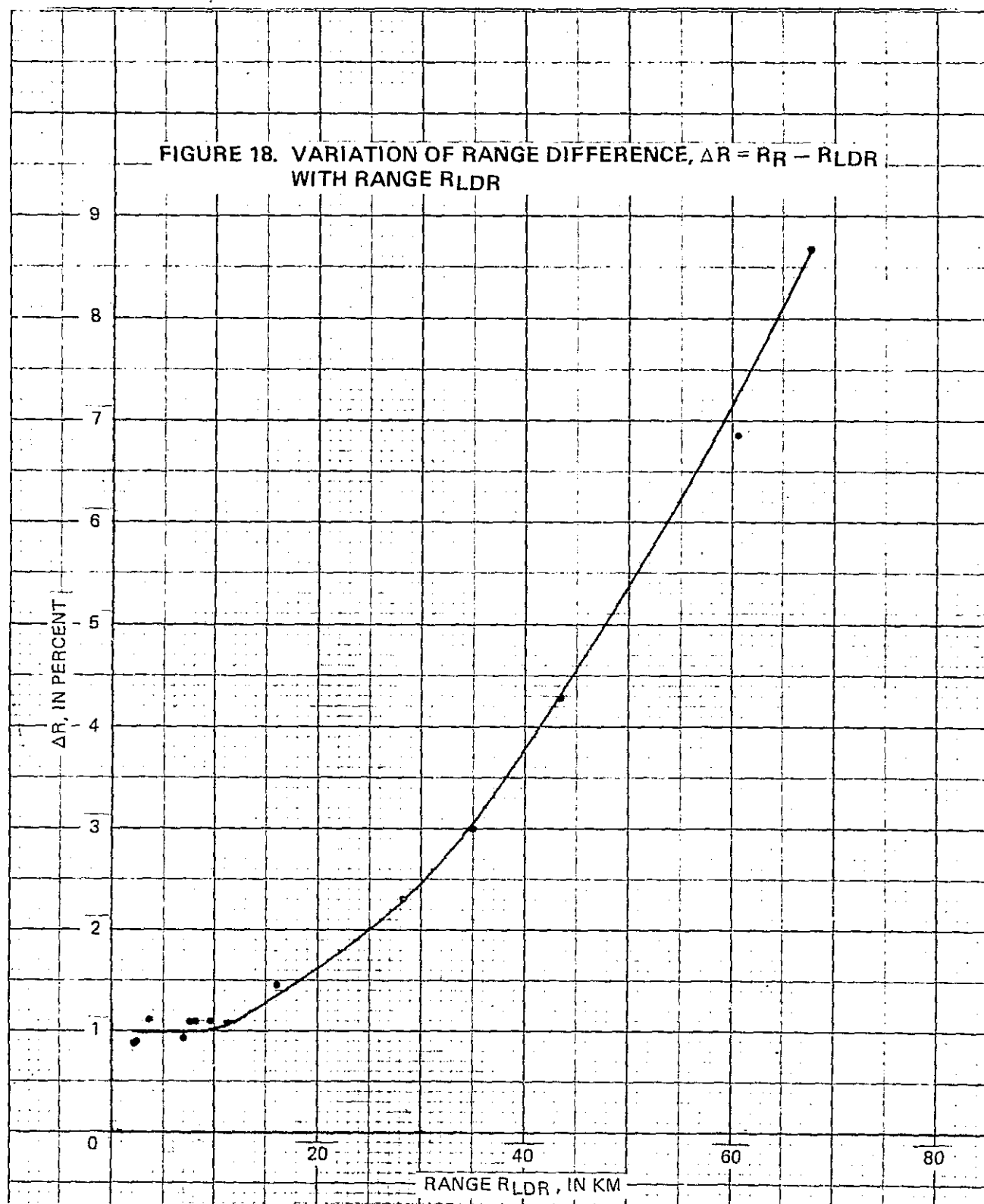
Of considerable significance is the fact that the ground range indicated by LDAR was always larger than the radar ground range, indicating the existence of a systematic error which we will discuss further under Radar Errors.

Azimuth Angle Differences

Azimuth angle differences, $\theta_R - \theta_{LDR}$, are given in Table III for representative points in the measurement field. Also given are the azimuth angles indicated by LDAR Systems #1 and #2. For each measured quantity, the mean and the standard deviation are given

It is apparent that in their measurement of azimuth, System #1 and System #2 agree very closely. The mean and standard deviation of the differences, $\theta_1 - \theta_2$, are 0.014 ± 0.20 degrees. As pointed out in the accuracy report¹, already referenced, the quantity that the LDAR System measures most precisely is the azimuth.

Examination of the radar-LDAR azimuth differences shows significant values. The average and the standard deviation of $\theta_R - \theta_{LDR}$, are 0.45 ± 0.10 degrees. The difference of 0.45 degrees in the azimuth measurement is indicative of a systematic error, that will be examined in more detail later.



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TABLE III AZIMUTH ANGLE DIFFERENCE

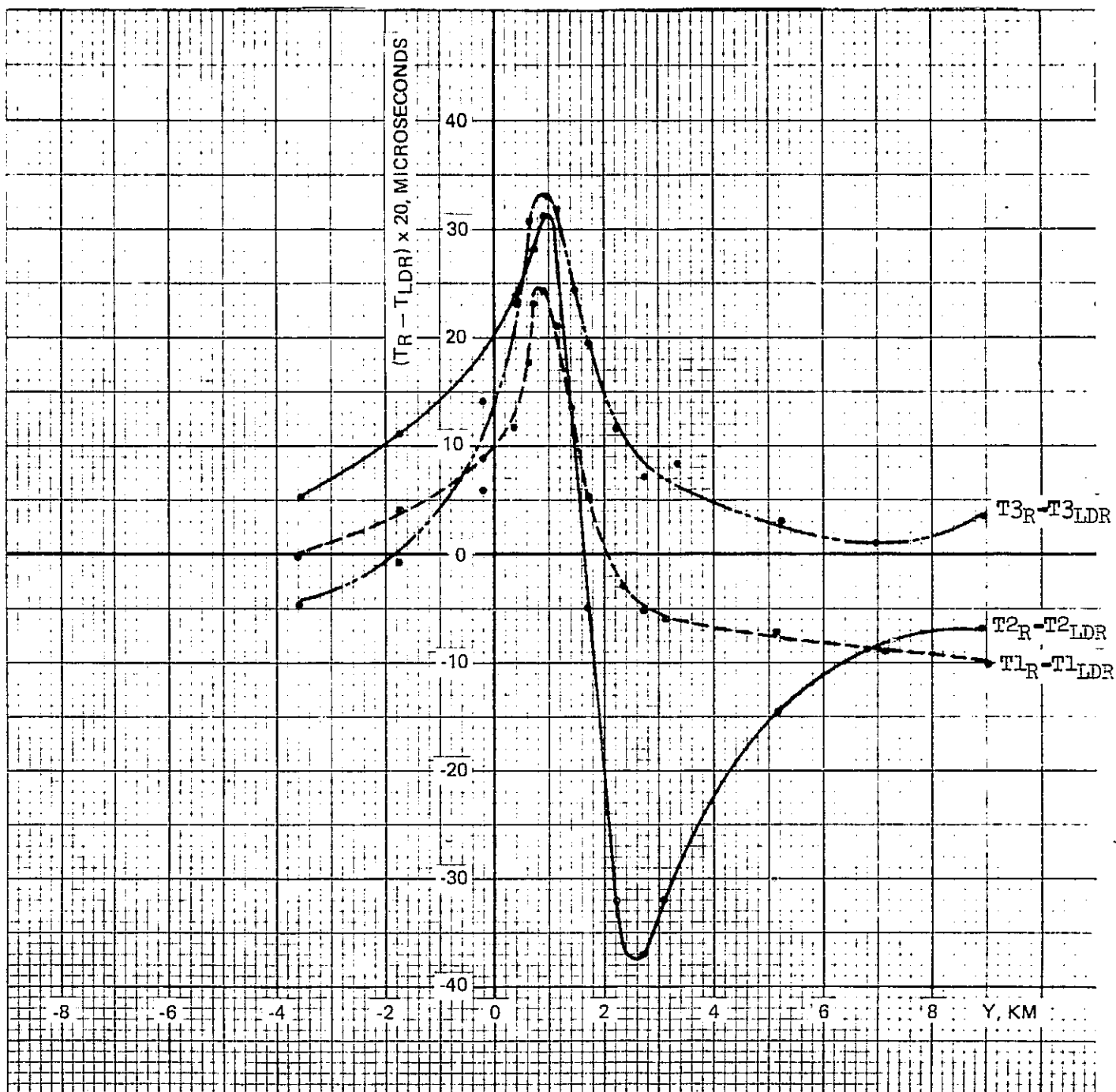
LDAR Position		Difference in Azimuth Radar-LDAR	Difference in Azimuth System #1-#2
Ground Range	Azimuth		
Kilometer	Degrees	Degrees	Degrees
21.2	0.73	0.468	-0.14
18.8	321.6	0.399	0.113
17.2	270.3	0.343	-0.068
20.0	91.0	0.581	0.074
19.6	124.9	0.396	0.02
18.04	232.6	0.639	-0.399
39.8	278.2	0.471	-0.16
21.93	24.4	0.49	0.30
14.50	20.9	0.342	0.126
Mean	14.5	0.45	-0.014
Standard Deviations	--	0.10	0.20

Time-of-Arrival Differences

The LDAR System¹ determines position from the differences in the time of arrival at four stations, having a baseline length of approximately 10 km. In operation the relative times of arrival are recorded, as well as the computed position in space (X, Y, Z) from which the pulse originates.

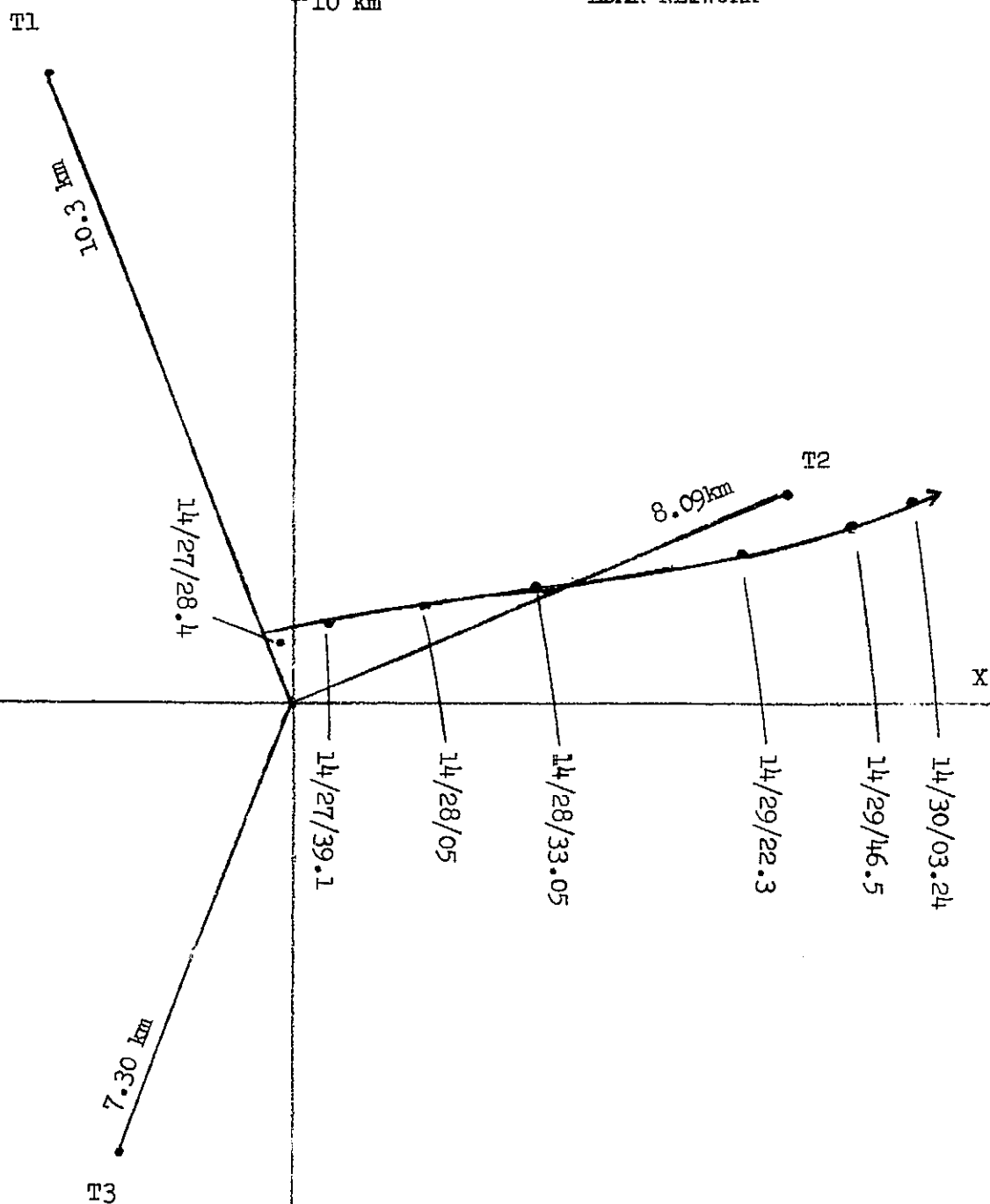
In order to shed light on the reasons for the differences that have been noted in the LDAR and the radar indication of position, computations were carried out as to what the differences in the times of arrival at the LDAR stations would have been for a pulse originating at the position in space indicated by the radar. Since the basic LDAR System has four stations, we have three differences T1, T2, and T3 for the three legs. Signifying the measured LDAR delay times by $T1_{LDR}$, $T2_{LDR}$, and $T3_{LDR}$ and the calculated delays for a point in space given by radar as $T1_R$, $T2_R$, and $T3_R$, we examined the differences $T1_R - T1_{LDR}$, $T2_R - T2_{LDR}$, and $T3_R - T3_{LDR}$ for a systematic difference that would explain the differences in the indicated X, Y, and Z. The initial thought was that the time measurement at the central station had some fixed offset, or that an erroneous delay correction had been used for one of the links that carry the LDAR signal from the outlying site. One would expect such an error to be constant, and would then be able to regress on the data to find a constant offset in all the data. This was not found to be the case.

Insight into one of the problems, however, was provided by plotting the time-of-arrival differences $T1_R - T1_{LDR}$, $T2_R - T2_{LDR}$, and $T3_R - T3_{LDR}$ against distance, as shown in Figure 19. The plane's trajectory relative to the LDAR System is shown in Figure 20. The difference plot is of a most unusual shape, and strongly suggests a systematic error.



ORIGINAL TAPE IS
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FIG. 20 TRAJECTORY OF THE
PLANE RELATIVE TO THE
LDAR NETWORK



Comparison with the plane's trajectory, shown in Figure 20, shows that the differences are a maximum at the ends of the LDAR leg 2 that is being traversed. The peak delay difference shown in the plot is 1.500 microseconds, which is a distressing value when compared with the random error¹ of the delay measurement of 0.025 microseconds. Similar results were obtained for the north-south traverse of the LDAR system.

Computer simulation of various error models disclosed that the type and the magnitude of the error shown is what would be expected if the coordinate axes of the LDAR System differed from the coordinate axes of the radar system by an angular error of approximately 0.4 degrees

Error Vectors

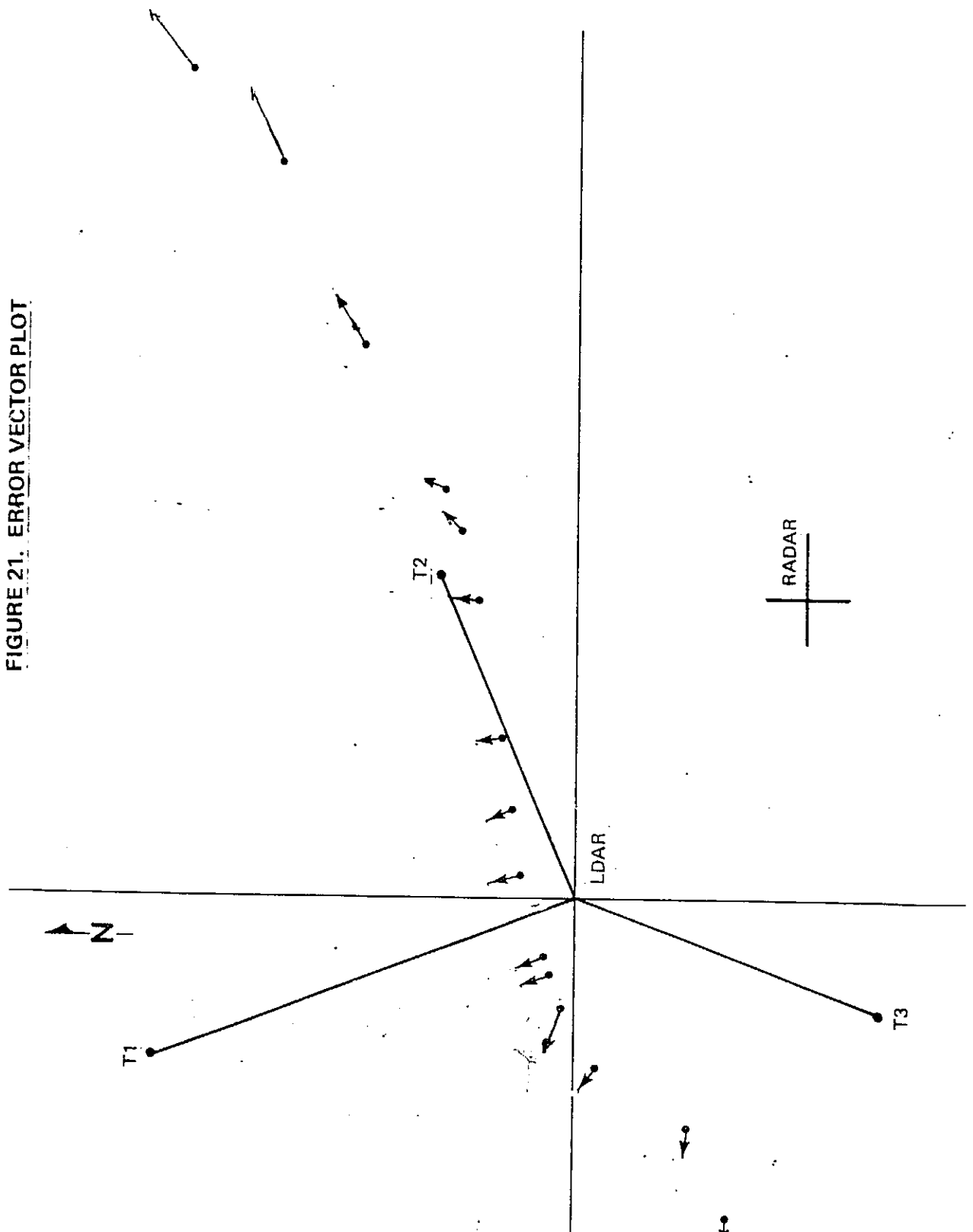
The axes-orientation error, discussed above, was not sufficient by itself to explain the differences noted between the LDAR and radar data. In a continued effort to account for the differences, error vectors $\underline{R} - \underline{R}_{LDR}$ were computed for representative points in the data field. Expressing by $\underline{R}$ the vector from the radar origin to the data point, indicated by the radar and by $\underline{R}_{LDR}$ the vector to the data point indicated by the LDAR we calculated

$$\underline{\Delta R} = \underline{R} - \underline{R}_{LDR}$$

Plotting $\underline{\Delta R}$ to a larger scale than $\underline{R}$ we derive the plot of Figure 21.

Examination of the data of this plot shows, that for the most part, the error vectors can be fitted into a model that has the radar as the origin of the error vectors. This indicated to us that the discrepancy between the radar and the LDAR data was caused by an error in the radar data that depended on the position, distance and orientation of the data point relative to the radar origin. For this reason we turned our attention to a close examination of the radar errors and radar performance. We will discuss the radar errors in detail, after we complete our discussion of the random errors.

FIGURE 21. ERROR VECTOR PLOT



B. Random Errors

In contrast to the systematic errors, which are of low or zero frequency, the random errors are made up of the higher frequency terms. Random errors are statistical in nature, and as such are measured in statistical terms, such as the mean and the standard deviation σ . For a normal distribution, 68% of the data can be expected to fall within plus or minus one sigma of the mean.

Random errors can, in general be averaged out. They are a measure of the scatter of the data, which is in turn dependent on the signal/noise ratio and the magnitude of the quantizing fluctuations.

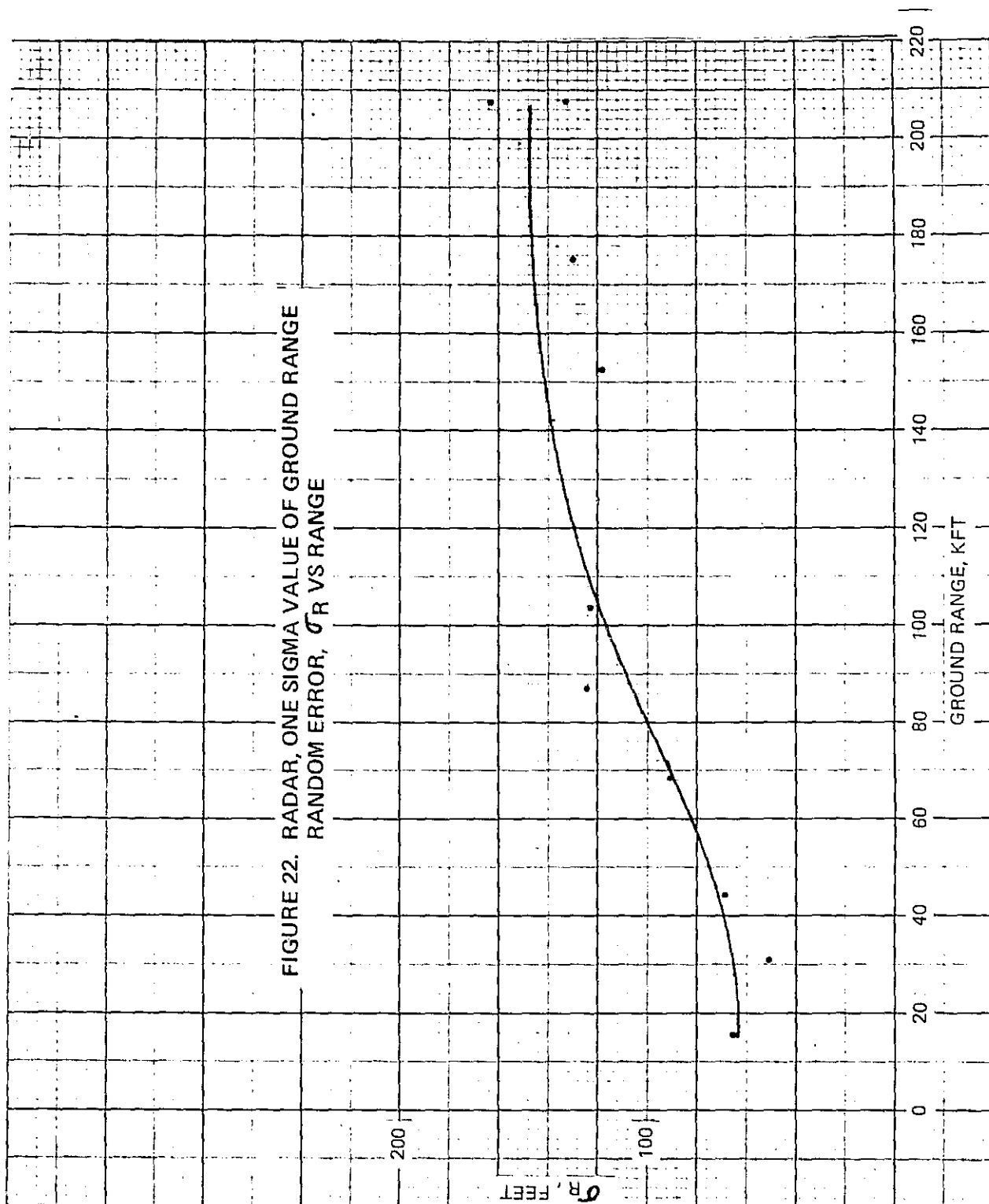
We determined the random errors in the data by examining the statistics of the data fluctuation about the least-squares curve fit to the data. A least-squares polynomial curve fit program was used to determine the best curve fit to the data, and then to determine the standard deviation of the fluctuations of the data about the least-squares curve. Analysis was conducted at numerous portions of the flight data, using 20 point data spans.

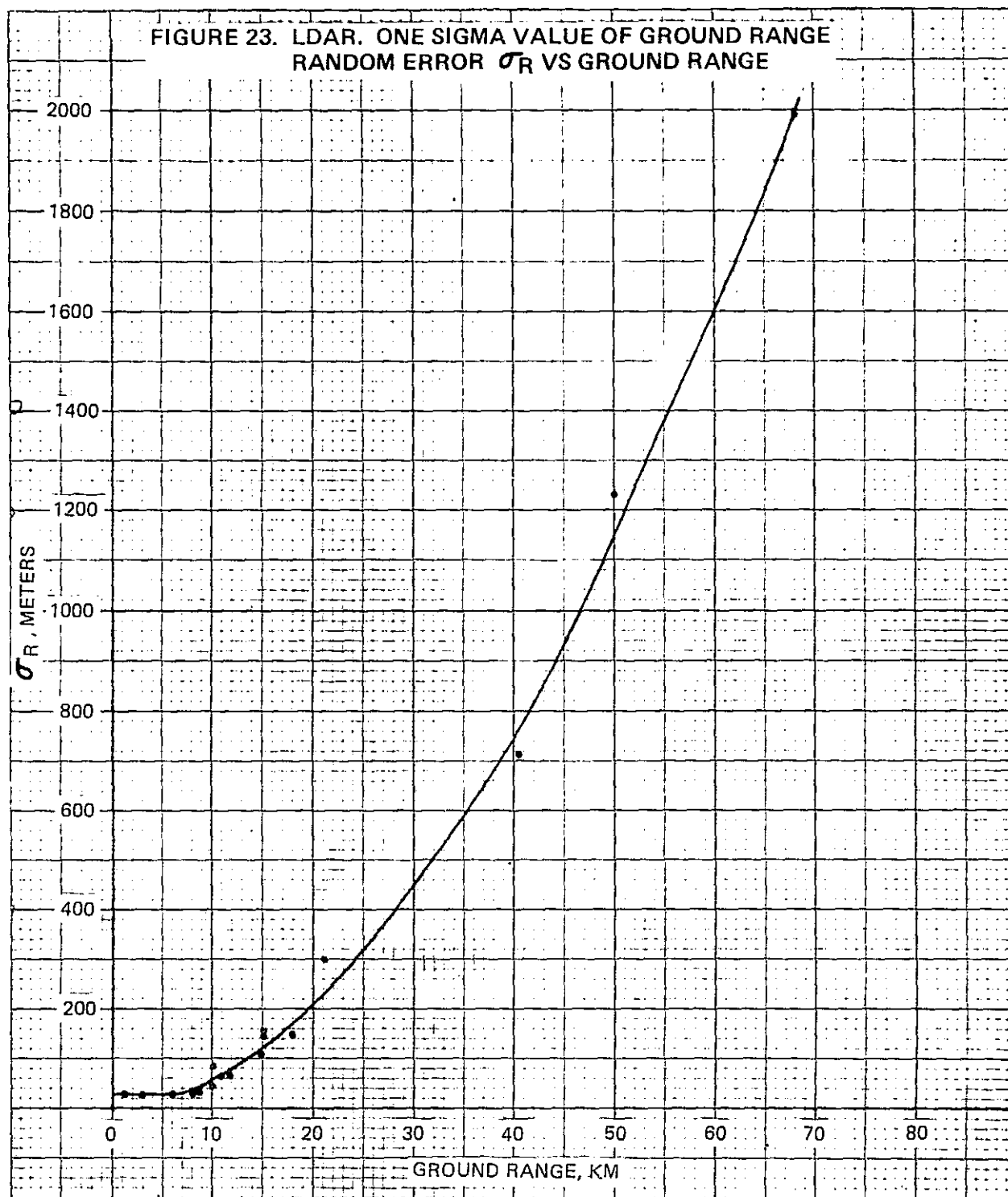
Radar Random Errors

The one sigma values σ_R of the random error of the radar's range measurement are shown in Figure 22 for ranges up to 220,000 feet. For ranges less than 32,000' (10 km), the random error σ_R is approximately 65'. The random error increases slowly with range, up to a value of 140' at 200,000'.

LDAR Random Errors

The random errors in the LDAR data were determined for data segments chosen from 0 to 70 kilometers in ground range. The one sigma values σ_R of the random error in ground range are plotted against ground range in Figure 23. Within the 10 km baseline of the system, the random errors are very low, approximately 80 feet (25m). The random errors increase with range, however. Up to a distance of 20 km or 2 times the baseline length, the increase in random error is moderate, reaching 200 feet (210m) at 20 km. Beyond that distance, the increase can be seen to be very rapid. This is typical of hyperbolic systems¹, and contrasts to the behavior of a radar system as shown in Figure 22.





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IV. RADAR SYSTEM ERRORS

The nature and the magnitude of the LDAR-radar differences necessitated a change in direction from the initial objective of calibrating the LDAR System to a detailed examination of the errors in the radar data.

The observed LDAR-radar differences were reported to A. E. Hoffmann-Heyden, head of the RCA Radar Systems Analysis Group at the Air Force Eastern Test Range. In response, Frank Cockerham of that group was assigned to investigate the accuracy of the 1.5 Mod II S-Band Radar that was used in the LDAR-radar calibration test. Frank Cockerham's investigations have been completed, and the results of his analysis will be included in a report entitled, "Memo for Record, Radar 1.5 Calibration"², which is in typing. We have been supplied with an early draft copy of this report, and have had several informative conversations with Frank Cockerham. The radar errors will be discussed below.

Range Error

The Radar Systems Analysis Group ran calibration tests of the radar against three fixed targets, the Cocoa Water Tower, the Lighthouse, and the CIF Boresite Tower, whose surveyed position was a matter of record. Errors of approximately 1% in the radar's range indication of fixed known targets was found:

Target	Surveyed Range	Surveyed Az	Radar Range Error	
	Yards		Feet	%
Cocoa Water Tower	23,435	225.813	-563	-0.8%
Lighthouse	5,224	139.036	+164	+1.1%
CIF Boresite Tower	9,065	307.750	-104	-0.4%

The Range errors on fixed targets are composed of bias and scale-factor errors, thus accounting for the change in the sign of the error for the Lighthouse. The negative sign in -563 shown for the Water Tower indicates that the radar reading of ground range was 563 feet short of the surveyed range. The azimuth angle is measured clockwise from geodetic north.

In a test to check the linearity of potentiometers, zero offsets, and analog-to-digital conversion from the dc voltage readouts to the digital printout of radar position that was supplied to us, the radar dials were mechanically set to various azimuth positions for a range of 75 kyards at an elevation angle of 45 degrees. This test revealed the existence of quadrant-sensitive errors, as shown in Figure 24

The fact that the error has different values in the four quadrants is significant, in that this is indicative of a zero set bias error in the radar. A similar conclusion was already reached earlier from the comparison with LDAR data.

Azimuth Errors

Calibration tests of the radar against the Cocoa Water Tower, whose surveyed position was a matter of record, revealed an azimuth error of +0.3 degree in the radar's azimuth indication:

Target	Surveyed Azimuth Degrees	Indicated Radar Azimuth Degrees	Error Degrees
Cocoa Water Tower	225.813	226.11	+0.3

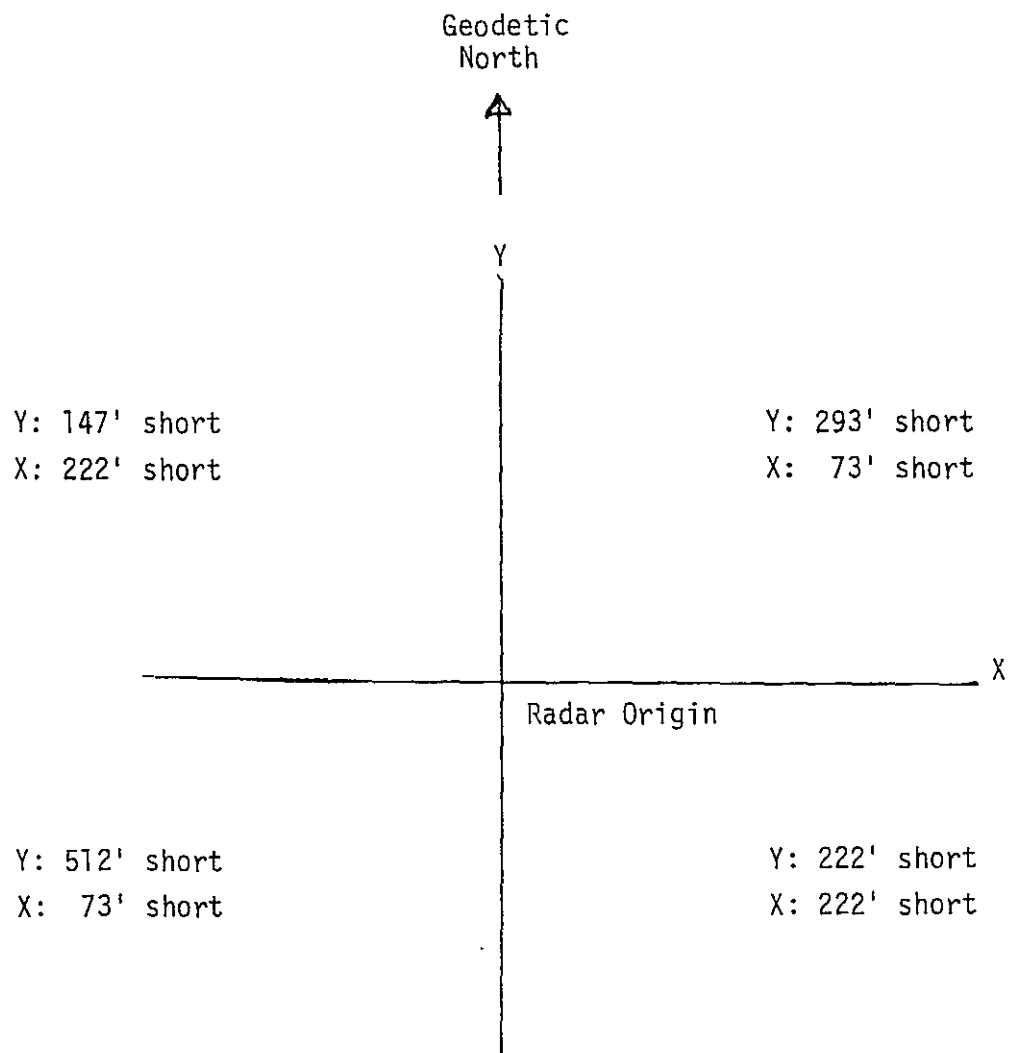


FIG. 24 RADAR, QUADRANT-SENSITIVE ERRORS,
WITH DATA DIALS SET FOR A RANGE OF 75,000 YARDS,
AND AN ELEVATION ANGLE OF 45 DEGREES

The radar error accounts for approximately two thirds of the difference between the LDAR and radar data. The other portion of the error was found to be due to a difference in the azimuthal orientation of the coordinate system used by LDAR and radar, as will be discussed later.

Height Errors

As already been noted in our discussion Height Differences, $Z_R - Z_{LDR}$, and illustrated in Table II, a distressing discrepancy of some 2900 feet (885 meters) was observed in the comparison of LDAR with radar data. Fortunately, an independent indication of height was available from the pilot's altitude log. Very close agreement, generally less than 300 feet, between LDAR's measurement of height and the pilot's log, directed suspicion at the radar's indication of height. Frank Cockerham, of the Radar Systems Analysis Group, after reviewing the data and examining the radar records and the instrumentation that provides the radar indication of height, concluded that the most probable source of the 2900 foot error was a drift in the bias of the 20 year old, tube type, d-c amplifier that furnishes the voltage for the radar height indication.

Transponder Delay Error

The transponder delay is compensated for at the radar. Provided that the correct value of delay is applied at the radar and that the transponder delay does not change, no transponder error is introduced. In order to check on the transponder as a source of error, measurements of transponder delay at various signal strengths were conducted in the laboratory. The measured transponder delays are shown in Table IV.

TABLE IV MEASURED TRANSPONDER DELAYS

Signal Strength	Delay
dB Above Threshold	Microseconds
40	2.8
35	2.7
30	2.7
25	2.6
20	2.7
15	4.0
10	4.1
0	4.2

Except at very low signal strengths, not reached in this test, the transponder delay was 2.7 ± 0.1 microsecond. The transponder delay setting used by the radar operators was the 2 microsecond (328 yards) called for in OD41 for NASA-6. This leaves us with a 0.7 ± 0.1 microsecond, or $344'$ (105 m) $\pm 49'$ slant range error due to the transponder. Its direction is opposite to the 1% range error, already noted. Overhead, the slant range error is essentially a height error, but small compared to the 2900 foot height error already noted. At 10 and 20 km from the LDAR site, the transponder error is essentially a ground range error which amounts to 1.0% and 0.5% respectively.

Timing Errors

As we have already pointed out in the discussion of Figure 16, a timing discrepancy of 1/2 to 1 second between LDAR and radar data was observed in the data analysis of airplane turns. In operation, the LDAR System records timing to within 1 millisecond along with the measured delays T1, T2, and T3, from which position is computed. In contrast, in the operation of the Mod II radar, timing is not sampled along with the

radar data, but is added as the radar data enters the computer, at another location. Robert Rowe of the Data Systems Analysis Group has estimated³ that a possible timing uncertainty of as high as 0.6 seconds could have existed on this test. At a speed of 100 mi/hr, a 0.6 seconds timing error would translate to a 146 foot error in the position of NASA-6. At 10 km this represents an error of 0.4%

Radar Origin Errors

Incorrect information as to the radar location to which the radar data is referenced, will lead to an error in the radar data.

The radar data we received for test T-0370, 24 August 1977, had the following radar origin coordinates printed on the front sheet:

$$1.4 = 28.4927941668^{\circ} \text{ (latitude)}$$

$$-279.4243564159^{\circ} \text{ (longitude)}$$

which has the following equivalent in the Florida East Coordinate System

$$X = 636,310.84 \text{ Feet East}$$

$$Y = 1,512,047.69 \text{ Feet North}$$

The coordinates given above were reviewed by Frank Cockerham of the RCA Radar Systems Analysis Group, and were found to be incorrect. The correct coordinates, according to Frank Cockerham, are:

$$X = 636,306.93 \text{ Feet East}$$

$$Y = 1,511,985.09 \text{ Feet North}$$

Since all our data comparisons were made with the origin supplied with the data, this leaves us with an error in the radar data of ΔX in X, and ΔY in Y of:

$$\text{Error } \Delta X = -4 \text{ Feet}$$

$$\text{Error } \Delta Y = -63 \text{ Feet}$$

Summary of Radar Errors

A summary of the radar errors is given in Table V.

TABLE V SUMMARY OF RADAR ERRORS

		Comments
Range Measurement Error	-0.8 to + 1.1%	See Range Error discussion
Quadrant-Sensitive Bias Error	73 to 145 Feet	See Figure 24
Transponder Delay Error	0.7 Microseconds	(344' long at all ranges; 1% @ 10 km)
Radar Origin Location Error	$\Delta X = -4'$; $\Delta Y = -63'$	(0.2% @ 10 km, N/S)
Azimuth Error	0.17 Degrees	Radar reads 0.17 degrees too small
Timing Error	0.6 Second	(At 100 mi/hr, 146'; at 10 km, 0.4%)
Height Error	+2900 Feet	Radar reads too high

V. LDAR SYSTEM ERRORS

Differences between LDAR and radar data prompted us to re-examine all potential LDAR error sources. In the search, two hitherto¹ unappreciated sources of error were disclosed.

Coordinate System Orientation Error

LDAR data is presented in a Cartesian X, Y, Z coordinate system whose origin is at the LDAR central antenna at 23.8 feet above mean sea level, at

$X = 613,593.0$ Feet East

$Y = 1,528,943.48$ Feet North

whose Z axis is vertical and whose Y axis is parallel to the north-south meridian of the Florida East coordinate system.

Radar data is presented in an X, Y, Z Cartesian coordinate system centered at the radar, with Z vertical and the Y axis tangent to the geodetic north-south meridian.

The Florida East coordinate system⁴ is a transverse Mercator coordinate system set up so that its north-south parallels are all aligned with the geodetic north-south meridian at the origin of the Florida East system, i.e. at

Latitude $24^{\circ}20' 00''$

Longitude $81^{\circ} 00' 00''$

which point is assigned the value $X = +500,000$ feet and $Y = 0$ in the Florida East System. As we go east from the origin of the Florida East System, there is a small, but increasing, discrepancy between the north-south meridians of the two systems. At the LDAR site, the difference is 0.18 degrees. Because of this an azimuth (read clockwise from north) in the LDAR system will be 0.18° less than an azimuth read in the geodetic system used by the radar. Adding the coordinate orientation error of 0.18° to the radar system

error of 0.3 discussed in the section, "Radar Errors" accounts for 0.48° of the observed 0.45° azimuth difference between the LDAR and the radar data.

Fixed Lightning Simulator Calibration Errors

A lightning simulator, mounted on the Vertical Assembly Building (VAB) at an azimuth of 354.7 degrees, 5011 meters to the north, is used to calibrate the delays in the LDAR transmission links to the central station, as well as to check the operational status of the LDAR System each morning. Knowledge of the surveyed positions of the seven LDAR sites and the lightning simulator on the VAB permits us to calculate the delays for the lightning simulator signal. An amount of compensating delay is added to each of the six links, just sufficient to make the observed delay for each of the six outlying sites equal to the delay calculated for the surveyed distances. This compensating delay is added to all subsequent time delays measured by the system. Therefore, if the compensating delays are in error, all subsequent measurements will be in error.

Experience has shown that once the initial compensation adjustments are made, further adjustments are not necessary. The primary purpose of the daily calibration check is to check the operational status of the LDAR System.

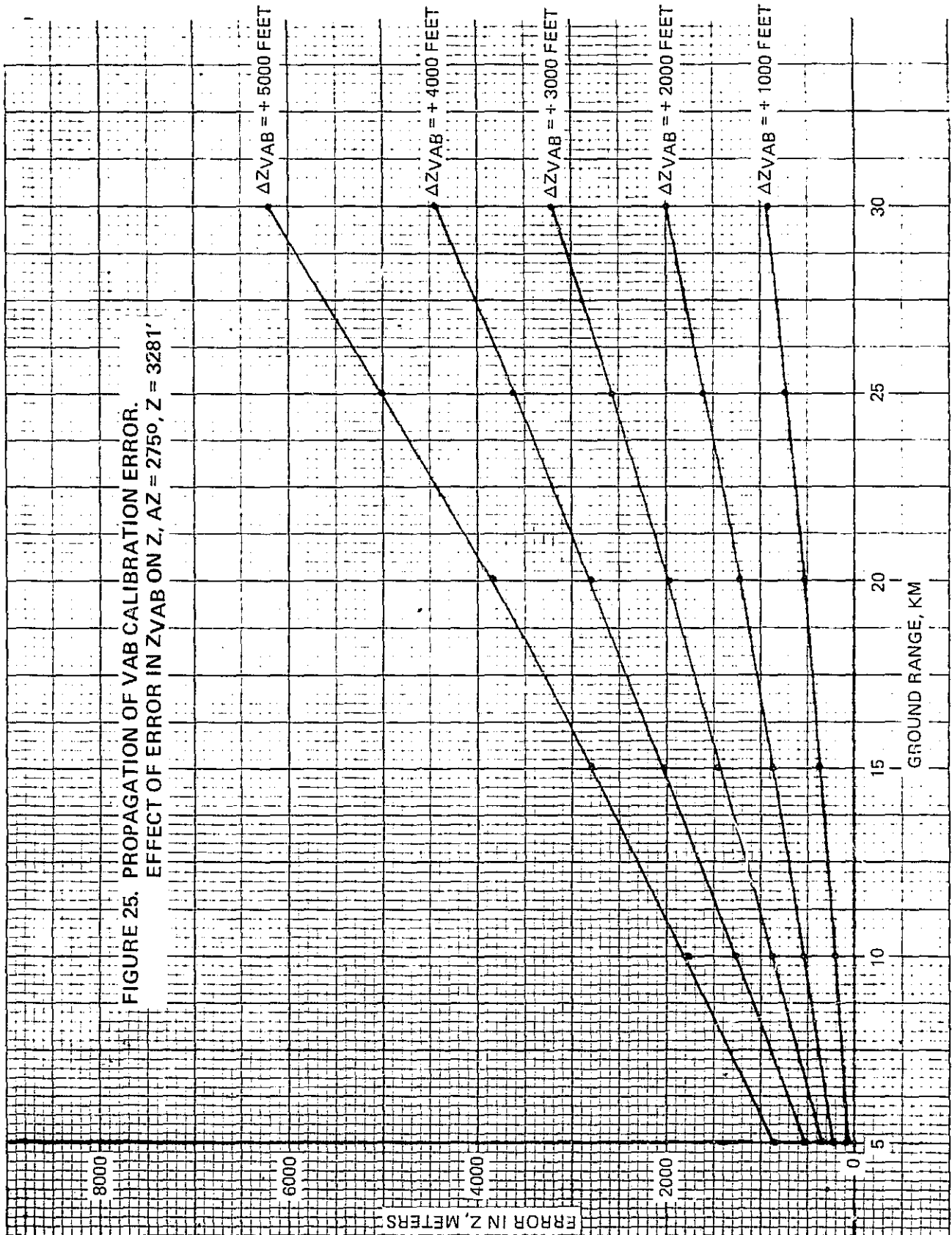
Careful examination of possible LDAR error sources disclosed the fact that small errors in the calibration setting for the VAB lightning simulator can lead to unexpectedly large errors in the LDAR System's measurement of X, Y, and Z. An error in the calibration setting will cause the introduction of a small, but significant, time delay error in all the subsequent six time delays observed by LDAR (three for

System #1 and three for System #2). To illustrate the small size of the calibration time delay errors, consider that an error of 1000' in the calibration setting for the height of the VAB lightning simulator will introduce errors of $0.00515 \mu s$ in T1, a $0.037 \mu s$ in T2, and $0.0237 \mu s$ in T3 in all subsequent time delay measurements of T1, T2, and T3. (Here T1, T2, and T3 are the three time delays of System #1). While these numbers are small in themselves, they become comparatively large at distances comparable to, and especially at distances larger than the baseline, where the time delay increments per unit length become smaller and smaller. In effect the relatively-near position (5 km) of the VAB lightning simulator causes the error to be multiplied at the larger distances, such as 20 km.

Computer simulation of the effects of small errors in the calibration setting of the VAB lightning simulator has been carried out for System #1. The results are shown in Figures 25 to 30.

Figure 25 shows the effects of errors in the system calibration setting for Z_{VAB} , of +1000, +2000, +3000, +4000, and +5000 feet, on subsequent LDAR measurements at data points lying at a 275 degree azimuth, or approximately 90 degrees from the azimuth of the VAB site. While the errors used (+1000, +2000 feet) may at first glance seem to be excessive, they are actually realistic values, as Tables VI and VII will show. The calibration errors were also modeled for measurement errors for data points along a 354 degree azimuth, with similar results.

The importance of careful calibration is shown by the propagation of errors as the range increases. For example for a data point at 10 km distance and 3281 feet height, an initial 2000 foot error in the



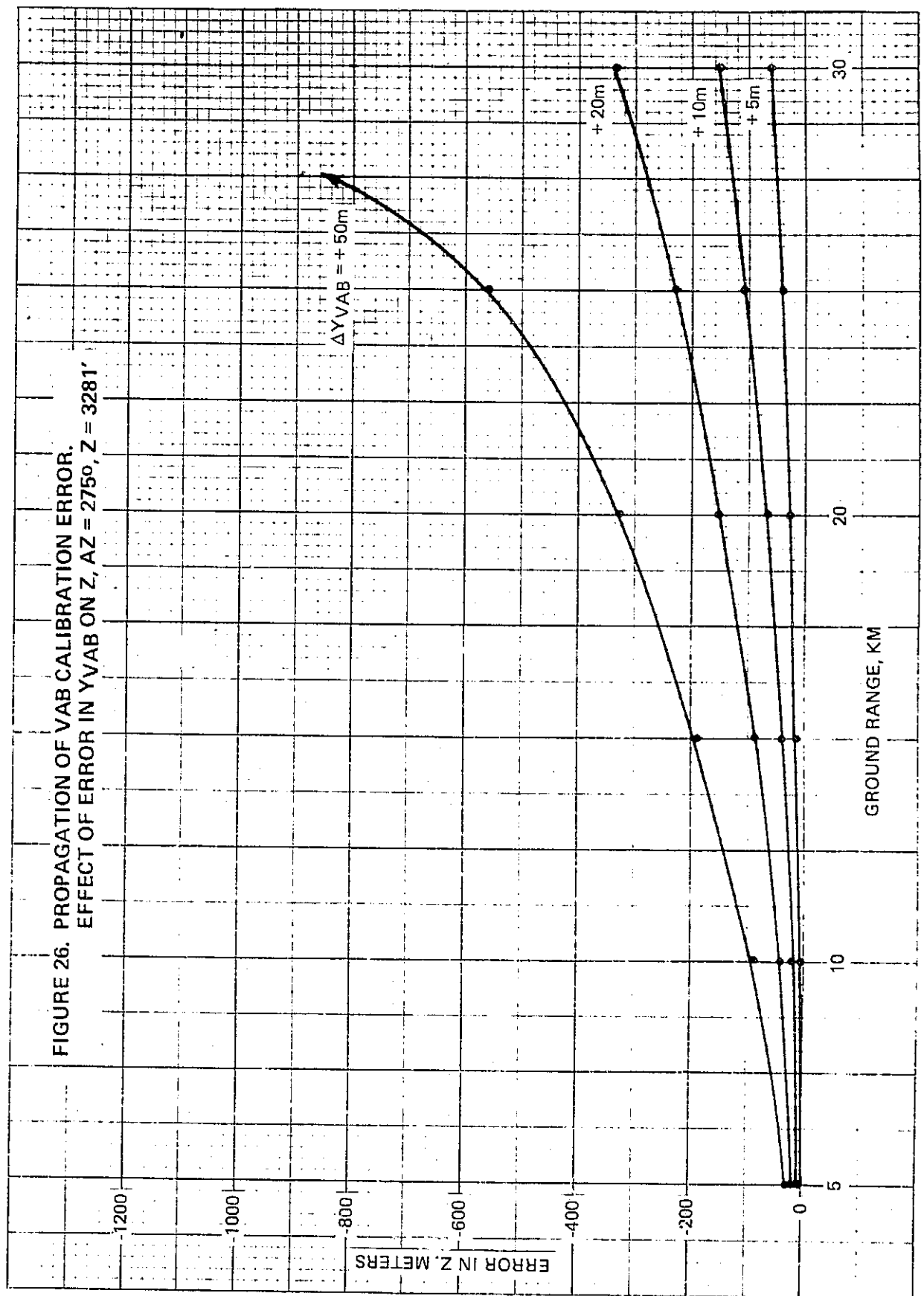
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calibration setting of the height of the VAB lightning simulator, produces a 500 meter error in the height Z measured by LDAR. This error is magnified considerably as the range increases, becoming a 2000 meter height error at a data point 30 km distant.

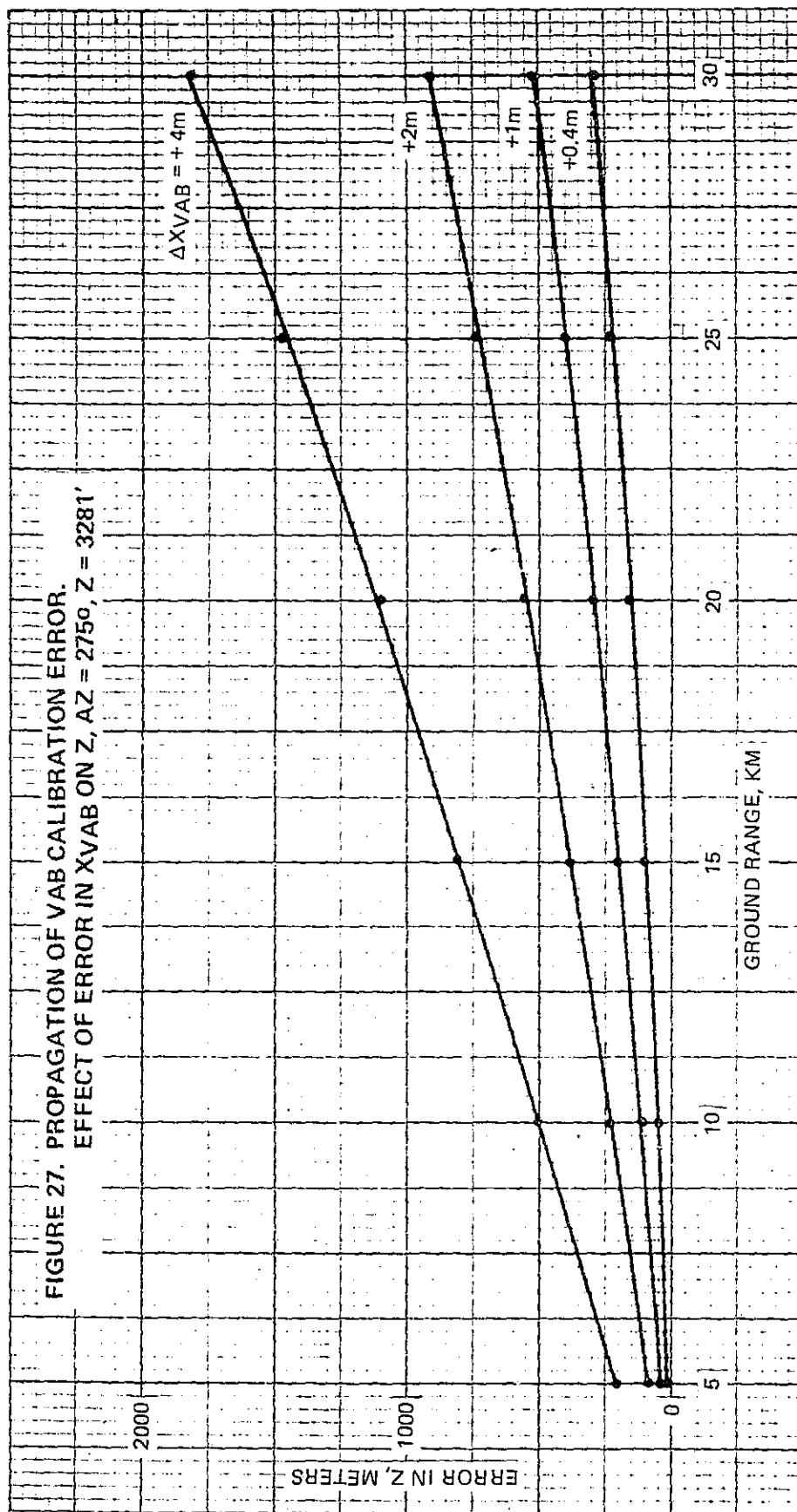
Figure 26 shows the effect of +5, +10, +20 and +50 meter errors in the 5088.9 meter calibration setting of Y_{VAB} . Figure 27 shows the effect of +0.4, +1, +2, and +4 meter errors in the calibration -474 meter setting of X_{VAB} . It is clear that calibration setting errors in both Y_{VAB} and X_{VAB} are reflected into LDAR height measurements, Z . Although substantial, the measurement errors caused by calibration errors in Y_{VAB} and X_{VAB} are not as large as those caused by calibration errors in the height, Z .

Errors in the calibration setting of the height Z_{VAB} are also reflected as errors in the subsequent LDAR measurements of the position coordinates X , and Y . For example, Figure 28 shows the effect of +1000, +3000, +4000, and +5000 foot errors in the calibration setting of the height Z_{VAB} on subsequent measurements of X for points 3281 feet high along a 275 degree azimuth.

Further, errors in the calibration setting of X_{VAB} and Y_{VAB} are reflected in subsequent LDAR measurements of X and Y . Errors in X_{VAB} cause their largest errors at azimuths around 275 degrees in the measurement of X (see Figure 29), and their largest error at azimuths around 354 degrees in the measurement of Y (see Figure 30). The effect of calibration errors in Y_{VAB} is smaller.



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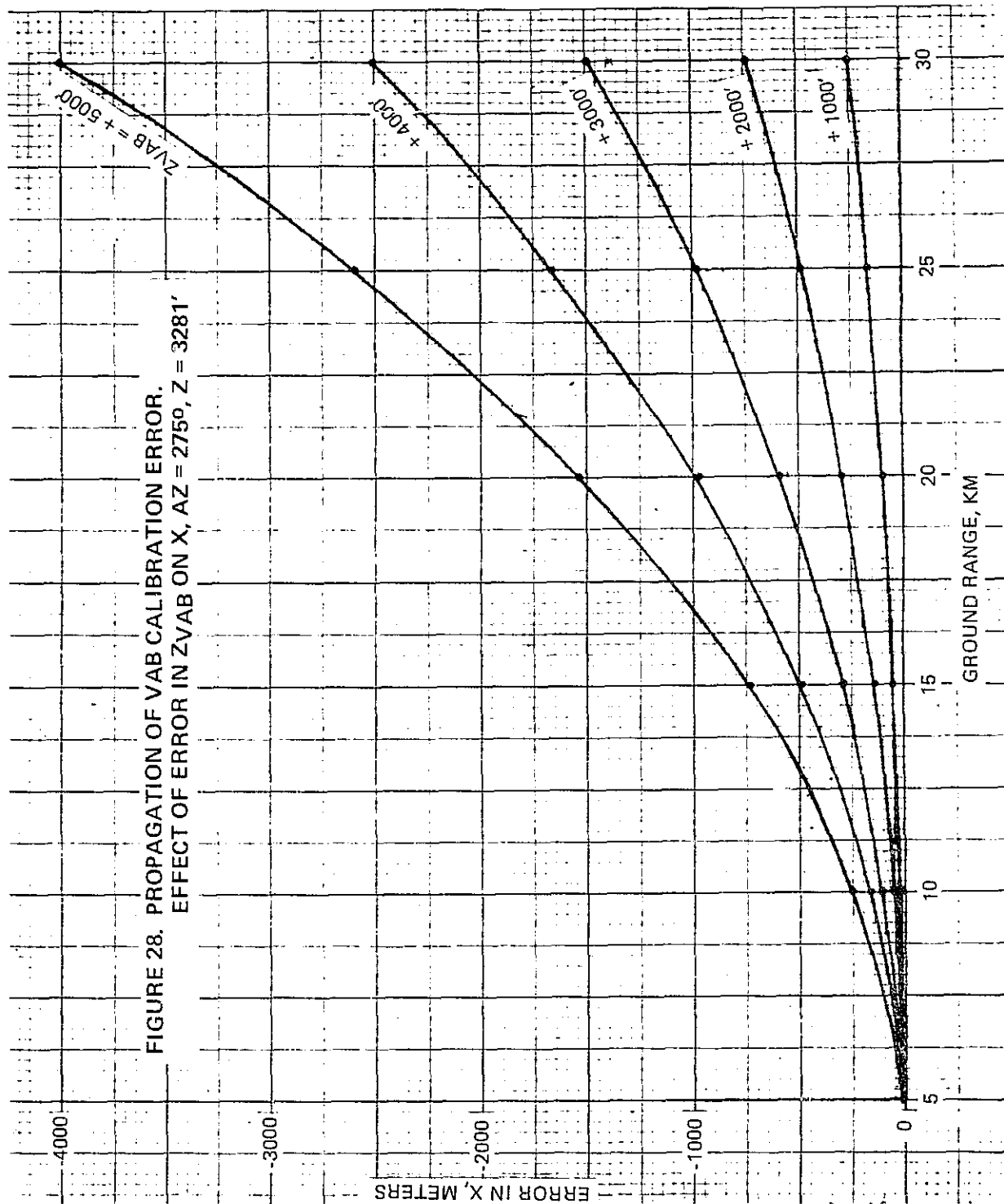


FIGURE 28. PROPAGATION OF VAB CALIBRATION ERROR.
EFFECT OF ERROR IN Z/VAB ON X, AZ = 275°, Z = 3281'

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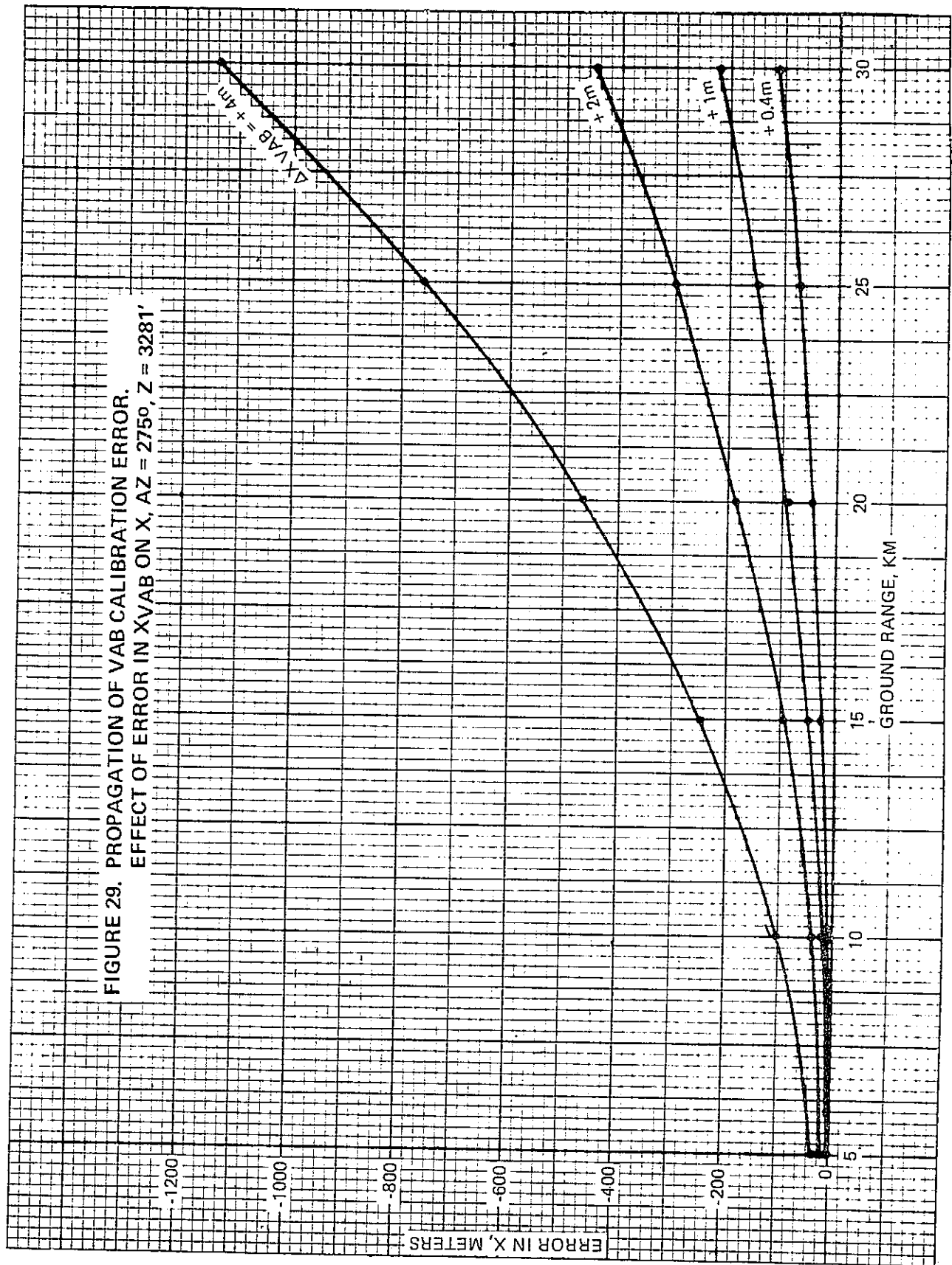
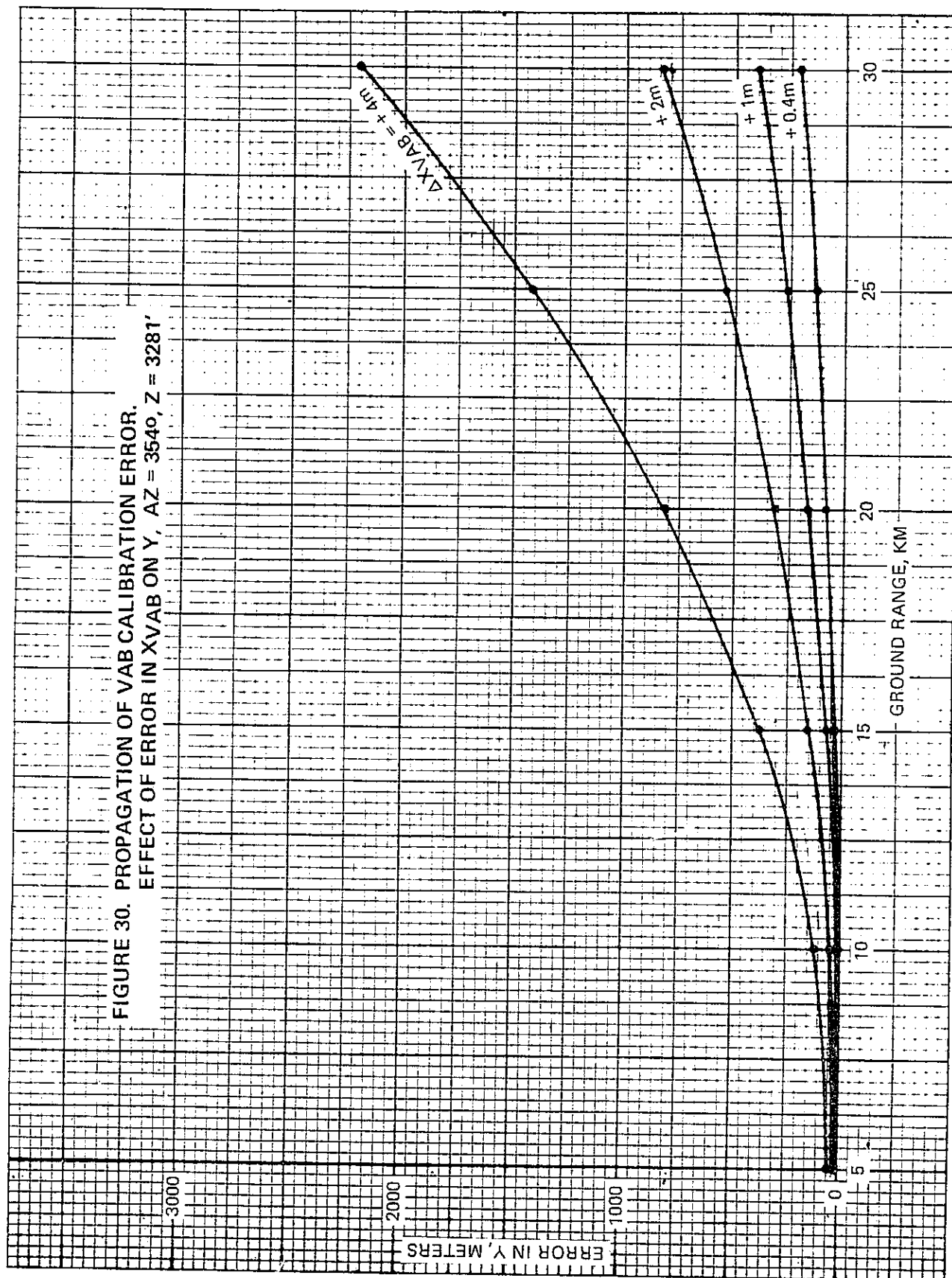


FIGURE 29. PROPAGATION OF VAB CALIBRATION ERROR.
EFFECT OF ERROR IN X/VAB ON X, AZ = 275°, Z = 3281'



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Calibration Procedure - Problems, Past Performance, and Recommendations

The LDAR System calibration is carried out by activating the lightning simulator on the VAB, and introducing a small amount of compensation delay into each of the six data links, just sufficient to make the observed delay for each of the six outlying sites equal to the delay calculated for the surveyed distances. If this is not done correctly, errors will result in LDAR's reading of the

$X_{VAB} = -474.08$ meters $Y_{VAB} = 5055.88$ meters $Z_{VAB} = 163$ meters

position of the VAB lightning simulator. These errors will affect all subsequent readings of the LDAR System.

Practical difficulty is encountered in the calibration setting, particularly in the height Z_{VAB} , because of the extreme sensitivity¹ of the LDAR System in its measurement of Z at low altitudes. Here system noise (at low signal/noise ratios) and quantizing fluctuations cause fluctuations in the output data.

Table VI shows LDAR's reading of the position of the VAB lightning simulator on the morning of the calibration flight. The extreme sensitivity of the height reading Z_{VAB} is apparent. The fluctuations in Z are many times the fluctuations in either X_{VAB} or Y_{VAB} . This data shows the necessity of collecting a sufficient number of calibration data points, as well as the need for the computer to average during collection, so that an average reading will be available to compare with the known position of the VAB lightning calibrator.

The means shown in Table VI indicate calibration errors ΔX_{VAB} , ΔY_{VAB} and ΔZ_{VAB} of

$$\Delta X_{VAB} = -5.8 \text{ meters (-19 feet)}$$

$$\Delta Y_{VAB} = +25.4 \text{ meters (+83 feet)}$$

$$\Delta Z_{VAB} = +638 \text{ meters (+2094 feet)}$$

TABLE VI LDAR SYSTEM CALIBRATION DATA, VAB LIGHTNING SIMULATOR

24 AUGUST 1977

X_{VAB}	Y_{VAB}	Z_{VAB}
Meters	Meters	Feet
-483.5	5116.5	2366
-478.6	5124.7	3428
-478.5	5117.1	2963
-473.7	5110.4	3005
-490.2	5113.2	2860
-478.6	5117.1	2963
-478.6	5117.1	2963
-478.6	5117.1	2963
-478.6	5117.1	2963
-483.7	5116.5	2366
-478.6	5124.7	3428
-478.6	5117.1	2963
-478.6	5117.1	2963
-471.9	5120.5	2488
-478.6	5102.8	1718
-478.6	5117.1	2963
-490.2	5105.8	2293
-490.2	5105.8	2293
-478.6	5117.1	2963
-483.5	5109.2	1643
-478.6	5117.1	2963
-483.5	5116.5	2366
-490.2	5105.8	2293
-483.5	5109.2	1643
-471.9	5120.5	2488
-473.7	5103.1	2470
-478.6	5117.1	2963
-467.0	5106.8	1880
Mean $\pm$ Std. Dev.	-479.9 $\pm$ 5.7 m	5114.2 $\pm$ 5.9 m
		2629 $\pm$ 1629 Ft.
Surveyed Position	-474.1 m	5088.88 m
		535 Ft *
Error	-5.8 m	+25.4 m
		2094 Ft.

* relative to the LDAR System, which is 23.8 feet above mean sea level.

To present some perspective, Table VII presents a listing of the readily-available VAB simulator calibration readings for the preceding 18 months. Each item in the table here represents the average of a number of readings, as indicated by the "Sample Size" in the extreme right column. Three points are worth making. First the errors in the VAB calibration setting on the day of the calibration flight are not atypical. Second, in all the data sets shown, the fluctuations in Z_{VAB} are many times the fluctuations for either X_{VAB} or Y_{VAB} . Third on many occasions a calibration reading for Z_{VAB} was accepted that was several orders of magnitude too large. This was done because the X_{VAB} and Y_{VAB} values were very close. As we have shown, however, X_{VAB} and Y_{VAB} being close is not good enough. Errors in the calibration setting for Z_{VAB} propagate into system measurements of the position X , Y , and Z of the data point.

Clearly future calibration settings of the VAB lightning simulator cannot continue to ignore the value of the height Z_{VAB} as has been done in the past. Calibrations must be conducted so that not only the X_{VAB} and Y_{VAB} values of the VAB calibrator, but also the Z_{VAB} values are within acceptable limits. Based on the error models that we have presented, we recommend that the following calibration error limits be adhered to

$$\begin{aligned}\Delta X_{VAB} &\leq 1 \text{ meter} \\ \Delta Y_{VAB} &\leq 10 \text{ meters} \\ \Delta Z_{VAB} &\leq 300 \text{ meters}\end{aligned}$$

TABLE VII A SUMMARY OF PREVIOUS LDAR SYSTEM CALIBRATION DATA, VAB LIGHTNING SIMULATOR

Date	Mean $\pm$ Standard Deviation			Error			Sample Size
	X _{VAB} Meter	Y _{VAB} Meter	Z _{VAB} Meter	Δ X _{VAB} Meter	Δ Y _{VAB} Meter	Δ Z _{VAB} Feet	
4-06-76	-475.1 $\pm$ 7.6	5090.3 $\pm$ 4.2	695 $\pm$ 79	- 1.0	+ 1.4	1745	17
4-13-76	-471.2 $\pm$ 6	509 .6 $\pm$ 3.8	156,806 $\pm$ 162	+ 2.9	+ 7.7	513,920	12
6-11-76	-473.8 $\pm$ 4.1	5086 $\pm$ 3.96	250,062 $\pm$ 166,652	+ 0.3	- 2.9	819,883	10
8-04-76	-472.6 $\pm$ 6.5	5092.6 $\pm$ 4	270.3 $\pm$ 50	+ 1.5	+ 3.7	351	9
8-13-76	-474.3 $\pm$ 10	5091.8 $\pm$ 7	479 $\pm$ 208	- 0.2	2.9	1035	32
10-19-76	-483.7	5088.5 $\pm$ 4.3	246,990 $\pm$ 283,587	- 9.6	-0.4	809,799	12
10-19-76	-478.4 $\pm$ 5	5091.2 $\pm$ 5	128,664 $\pm$ 110,604	- 4.3	2.3	421,591	12
11-01-76	---	---	541 $\pm$ 215	--	--	1240	32
Mean of the Means	-475.6 m	5091.0 m	98,063 m	-1.5 m	+ 2.2 m	+321,195 Ft.	17
Standard Deviation of the Means	4.2 m	3.3 m	111,961 m	4.2 m	3.3 m	367,327 Ft.	10
Surveyed Position	-474.08m	5088.88m	163.1m [*]				
Error	- 1.5m	+ 2.2m	+ 97,900m				

^{*} relative to the LDAR System, which is 7.3 meters above mean sea level.

VII. RECOMMENDATIONS

1. The LDAR calibration test should be repeated using a C-Band Missile Precision Instrumentation Radar (MIPIR) instead of a Mod II S-Band radar. Not only is the MIPIR a more accurate radar, but, more importantly, its performance is kept up to specification level.
2. If possible, the new Precision Laser Tracking System PLTS should be included in the calibration test. Since PLTS is undergoing operational checkout, the only additional effort involved would be the mounting of a laser reflector on NASA-6.
3. Extra care should be exercised in calibrating the LDAR System against the VAB lightning simulator. The importance of a high degree of precision in this calibration to the accuracy of the LDAR System has been demonstrated in this report. More than the usual amount of calibration data should be taken so that a meaningful statistical sample for the determination of the mean and the random fluctuation of this measurement will be available. The computer program should be modified to read out the mean of at least 50 calibration readings. Delay compensation should be predicated on the mean of at least 50 readings, and not on just a dozen or so data points.
4. Daily calibration data for the VAB lightning simulator should be taken and kept together with the LDAR data gathered for that day.
5. For system calibration, a more distant lightning simulator should be used. A higher-powered lightning simulator, at a distance at least twice the present 5 km, is needed.

VI. CONCLUSIONS

1. The analysis of the tracking data showed that the LDAR System was as, or even more, accurate than the radar that was used to calibrate the LDAR System.
2. The Mod II, S-Band radar did not meet its performance specifications, in fact suffered from a number of serious errors.
3. Within the 10 kilometer baseline of the system, the range error of the LDAR is estimated to be of the order of 1%, and at 10,000 feet the height error of the LDAR is estimated to be approximately 300 feet.
4. The accuracy of the LDAR system decreases outside the baseline. At 20 kilometers the range error is estimated to be 2%, and at 60 kilometers the range error is estimated to be 7%.
5. The accuracy assessment arrived at in this report must be considered approximate because of the poor performance of the Mod II, S-Band radar and its failure to meet specifications. A more confident assessment of LDAR accuracy must await a second calibration test against a precision tracking system, such as the C-Band Missile Precision Instrumentation (MIPIR) Radar.

VIII. REFERENCES

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