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of Elliptical Contacts for
Materials of Low Elastic Modulus
II - Starved Conjunction**

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and Space Administration

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ELASTOHYDRODYNAMIC LUBRICATION OF ELLIPTICAL CONTACTS

FOR MATERIALS OF LOW ELASTIC MODULUS

II - STARVED CONJUNCTION*

by Bernard J. Hamrock and Duncan Dowson†

Lewis Research Center

SUMMARY

By using the theory and numerical procedure developed by the authors in earlier publications, the influence of lubricant starvation upon minimum film thickness in starved elliptical elastohydrodynamic conjunctions for low-elastic-modulus materials has been investigated. Lubricant starvation was studied simply by moving the inlet boundary closer to the center of the conjunction. The results show that the location of the dimensionless inlet boundary m^* between the fully flooded and starved conditions can be expressed simply as $m^* = 1 + 1.07 [(R_x/b)^2 H_{\min, F}]^{0.16}$, where R_x is the effective radius of curvature in the rolling direction, b is the semiminor axis of the contact ellipse, and $H_{\min, F}$ is the dimensionless minimum film thickness for the fully flooded condition. That is, for a dimensionless inlet distance m less than m^* , starvation occurs; and for $m \geq m^*$, a fully flooded condition exists. Furthermore, it has been possible to express the minimum film thickness for a starved condition as $H_{\min, S} = H_{\min, F} [(m - 1)/(m^* - 1)]^{0.22}$.

Contour plots of the pressure and film thickness in and around the contact are presented for both the fully flooded and starved lubrication conditions. It is evident from the contour plots that the inlet pressure contours become less circular and that the film thickness decreases substantially as the severity of starvation increases.

The results presented in this report, when combined with the findings previously reported, enable the essential features of starved, elliptical, elastohydrodynamic conjunctions for materials of low elastic modulus to be ascertained.

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INTRODUCTION

Only in recent years has the complete numerical solution of the isothermal elastohydrodynamic lubrication (EHL) of elliptical contacts successfully emerged. The analysis requires the simultaneous solution of the elasticity and Reynolds equations. The authors' approach to the theoretical solution has been presented in two previous publications (refs. 1 and 2). The first of these publications (ref. 1) represents an elasticity model in which the conjunction is divided into equal rectangular regions with a uniform pressure applied over each region. The second (ref. 2) gives a complete approach to the solution of the elastohydrodynamic lubrication problem for elliptical contacts.

The most important practical aspect of the EHL elliptical-contact theory (ref. 2) is the determination of the minimum film thickness within the contact. That is, the prediction of a film of adequate thickness is extremely important for the successful operation of machine elements in which these thin, continuous, fluid films occur. In reference 3 the fully flooded results obtained from the theory given in references 1 and 2 are presented. A fully flooded condition is said to exist when the inlet distance of the conjunction ceases to influence in any significant way the minimum film thickness. In reference 3 the influence of the ellipticity parameter and the dimensionless speed, load, and material parameters on minimum film thickness was investigated. Thirty-four cases were used in obtaining the fully flooded minimum-film-thickness formula.

In reference 4 the basic theory developed in references 1 and 2 was used to study the effect of lubricant starvation on the pressure and film thickness within the conjunction. A simple expression for the dimensionless inlet boundary distance was obtained. This inlet boundary distance defines whether a fully flooded or a starved condition exists in the contact. Fifteen cases, in addition to three presented in reference 3, were used to obtain simple expressions for the minimum and central film thicknesses in a starved conjunction.

The work presented in references 1 to 4 is for a material of high elastic modulus (e.g., steel). The work presented in reference 5 and in the present paper is for a material of low elastic modulus (e.g., nitrile rubber). For such materials the distortions are large even with light loads. Another feature of the EHL of low-elastic-modulus materials is the negligible effect of pressure on the viscosity on the lubricating fluid. Engineering applications in which elastohydrodynamic lubrication is important for low-elastic-modulus materials include seals, human joints, tires, and elastomeric-material machine elements.

The problem of fully flooded line contacts has been solved theoretically for low-elastic-modulus materials by Herrebrugh (ref. 6), Dowson and Swales (ref. 7), and Baglin and Archard (ref. 8). The solutions of references 6 and 7 were obtained numerically and are based on simultaneous solutions of the hydrodynamic and elasticity equations; the analytical solution of reference 8 relied on the assumption of a simplified form

for the film shape in the contact region. Biswas and Snidle (ref. 9) used the approach of reference 8 to solve the point-contact situation. Reference 5 presents, to the best of the authors' knowledge, the first complete numerical solution of the problem of fully flooded, isothermal, elastohydrodynamic lubrication of elliptical contacts for low-elastic-modulus materials. That is, no assumptions were made as to pressure or film thickness within the contact, and compressibility and viscous effects were considered.

The present paper uses the basic theory developed in references 1 and 2 in extending the work on materials of low elastic modulus given in reference 5 by studying the effect of lubricant starvation on pressure and film thickness within the conjunction. The approach used here is similar to that used in reference 4 in studying starvation effects for materials of high elastic modulus. That is, the objective of this work is to provide a simple expression for the dimensionless inlet boundary distance. This inlet boundary distance defines whether a fully flooded or a starved condition exists in the conjunction. An additional objective is to develop simple expressions for the minimum film thickness under the starvation condition. Eighteen cases, in addition to three presented in reference 5, were used in obtaining the starvation results. To show more fully what occurs in going from a fully flooded condition to a lubricant starvation condition, contour plots of pressure and film thickness in and around the contact are presented.

SYMBOLS

a	semimajor axis of contact ellipse
b	semiminor axis of contact ellipse
E	modulus of elasticity
E'	$\frac{2}{\frac{1 - \nu_A^2}{E_A} + \frac{1 - \nu_B^2}{E_B}}$
F	normal applied load
G	dimensionless material parameter, $E'/p_{iv, as} \approx \alpha E'$
H _{min}	dimensionless minimum film thickness, h_{min}/R_x
H _{min, F}	dimensionless minimum film thickness for a fully flooded conjunction, $h_{min, F}/R_x$
H _{min, S}	dimensionless minimum film thickness for a starved conjunction, $h_{min, S}/R_x$
h	film thickness

k	ellipticity parameter, a/b
m	dimensionless inlet distance
m^*	dimensionless inlet distance at boundary between fully flooded and starved conditions
P	dimensionless pressure, p/E'
p	pressure
$p_{iv, as}$	asymptotic isoviscous pressure
R	effective radius of curvature
r	radius of curvature
U	dimensionless speed parameter, $u\eta_0/E'R_x$
u	mean surface velocity in x-direction, $\frac{1}{2}(u_A + u_B)$
W	dimensionless load parameter, $F/E'R_x^2$
$\left. \begin{matrix} x & X \\ y & Y \end{matrix} \right\}$	coordinate system defined in paper
α	pressure-viscosity coefficient
η_0	atmospheric viscosity
ν	Poisson's ratio

Subscripts:

A	solid A
B	solid B
F	fully flooded conjunction
H	materials of <u>high</u> elastic modulus
min	minimum
S	starved conjunction
x, y	coordinate system defined in paper

ANALYSIS

Boundary Between Fully Flooded and Starved Conditions

The nodal structure used in obtaining the fully flooded results of reference 5 is shown in figure 1. In this figure the coordinate X is made dimensionless with respect

to the semiminor axis b of the contact ellipse, and the coordinate Y is made dimensionless with respect to the semimajor axis a of the contact ellipse. The ellipticity parameter k is defined as the semimajor axis divided by the semiminor axis of the contact ellipse ($k = a/b$). The coordinates X and Y are made dimensionless such that the Hertzian contact ellipse becomes a Hertzian circle regardless of the ellipticity parameter. This Hertzian contact circle is shown in figure 1 with a radius of unity. The pressure is assumed to be ambient at the edges of the computing area. In figure 1 the dimensionless inlet distance m , which is equal to the dimensionless distance from the center of the contact to the inlet edge of the computing area, is shown.

Lubricant starvation can be studied simply by reducing the dimensionless inlet distance m . A fully flooded condition is said to exist when the dimensionless inlet distance ceases to influence in any significant way the minimum film thickness. When starting from a fully flooded condition as shown in figure 1 and decreasing m , the value at which the minimum film thickness first starts to change is called the fully flooded - starved boundary and is denoted by m^* . Therefore, lubricant starvation was studied by using the basic elastohydrodynamic lubrication theory developed in reference 2, considering solid materials of low elastic modulus, and observing the effect of reducing the dimensionless inlet distance.

Table I shows how changing the dimensionless inlet distance affects the dimensionless film thickness for three groups of dimensionless load and speed parameters. For all the results presented in this paper the dimensionless material parameter G is fixed at 0.4276 and the ellipticity parameter k is fixed at 6. As shown in table I, as the dimensionless inlet distance m decreases and dimensionless minimum film thickness H_{min} decreases.

Table II shows how the three groups of dimensionless speed and load parameters affect the location of the dimensionless inlet boundary m^* . Also given in this table are the corresponding values of the dimensionless minimum film thickness for the fully flooded condition as obtained by interpolation of the numerical values. The dimensionless inlet boundary distance m^* shown in table II was obtained by using the data from table I when the following equation was satisfied:

$$\frac{H_{min, F} - (H_{min})_{m=m^*}}{H_{min, F}} = 0.03 \quad (1)$$

The value of 0.03 is used in equation (1) since it was ascertained that the data in table I could only be obtained to an accuracy of 3 percent.

The general form of the equation that describes how the dimensionless inlet distance at the fully flooded - starved boundary varies is given as

$$m^* = 1 + A \left[\left(\frac{R_x}{b} \right)^2 H_{\min, F} \right]^B \quad (2)$$

The right side of equation (2) is similar to the forms of the equation given in Wolveridge, et al. (ref. 10), Wedeven, et al. (ref. 11), and reference 4. By using the data obtained from table I, the dimensionless inlet boundary distance for materials of low elastic modulus can be written as

$$m^* = 1 + 1.07 \left[\left(\frac{R_x}{b} \right)^2 H_{\min, F} \right]^{0.16} \quad (3)$$

The coefficient of determination r^2 for these results was good at 0.960. The value of r^2 reflects the fit of the data to the resulting equation: 1 being a perfect fit, and zero the worst possible fit. A fully flooded condition exists when $m \geq m^*$, and a starved condition when $m < m^*$. From reference 4 the dimensionless inlet boundary distance for materials of high elastic modulus can be written as

$$\{m^*\}_H = 1 + 3.34 \left[\left(\frac{R_x}{b} \right)^2 H_{\min, F} \right]^{0.56} \quad (4)$$

Comparing equation (3) with equation (4) indicates that the dimensionless inlet boundary for materials of low elastic modulus m^* is less than $\{m^*\}_H$, therefore implying that m^* is closer to the center of the contact.

Film-Thickness Formula

After the limiting location of the inlet boundary for the fully flooded condition is clearly established (eq. (3)), an equation defining the dimensionless minimum film thickness for lubricant starvation conditions can be developed. The relation between the dimensionless minimum film thickness in the starved and fully flooded conditions can be expressed in general form as

$$\frac{H_{\min, S}}{H_{\min, F}} = C \left(\frac{m - 1}{m^* - 1} \right)^D$$

This form of the equation is exactly the same as that used in reference 4, which was determined to be the best form after trying a number of other forms. Table III shows how the ratio of the dimensionless inlet distance parameter to the fully flooded - starved boundary $(m - 1)/(m^* - 1)$ affects the ratio of minimum film thickness in the starved and fully flooded conditions $H_{\min, S}/H_{\min, F}$. A least-squares power curve fit to the 17 pairs of data points

$$\left(\frac{H_{\min, S}}{H_{\min, F}} \right)_i, \left(\frac{m - 1}{m^* - 1} \right)_i \quad \text{where } i = 1, 2, \dots, 17$$

was used in obtaining the dimensionless minimum film thickness for a starved condition for low-elastic-modulus materials

$$H_{\min, S} = H_{\min, F} \left(\frac{m - 1}{m^* - 1} \right)^{0.22} \quad (6)$$

The coefficient of determination r^2 for these results was excellent at 0.990. From reference 4 the dimensionless film thickness for a starved condition for high-elastic-modulus materials was written as

$$\{H_{\min, S}\}_H = \{H_{\min, F}\}_H \left(\frac{m - 1}{m^* - 1} \right)^{0.25} \quad (7)$$

The exponents in equations (6) and (7) are quite similar. Therefore, even though the fully flooded - starved boundary for materials of low elastic modulus was much less than that for materials of high elastic modulus, the starved film thickness equations are quite similar. Figure 2 shows the influence of inlet boundary on minimum-film-thickness ratio for materials of high and low elastic modulus. This figure shows the close agreement between equations (6) and (7).

Therefore, whenever $m < m^*$, where m^* is defined by equation (3), a lubricant starvation condition exists. When this is true, the dimensionless minimum film thickness is expressed by equation (6). If $m \geq m^*$, where m^* is defined by equation (3), a fully flooded condition exists. The dimensionless minimum film thickness for a fully flooded condition $H_{\min, F}$ for materials of low elastic modulus was developed in reference 5 as

$$H_{\min, F} = 7.43(1 - 0.85 e^{-0.31k})U^{0.65}W^{-0.21} \quad (8)$$

where the dimensionless speed parameter

$$U = \frac{\eta_0 u}{E' R_x} \quad (9)$$

the dimensionless load parameter

$$W = \frac{F}{E' R_x^2} \quad (10)$$

and the dimensionless ellipticity parameter

$$k = \frac{a}{b} \quad (11)$$

Simplified Formulas

To make it easier to calculate the ellipticity parameter k , the elliptic integral of the second kind $\mathcal{E}$, and the semiminor axis of the contact ellipse b in equations (3) and (8), the approximate expressions from reference 12 are used and are given here as

$$b = \left[\frac{6\mathcal{E}F}{\pi k E' \left(\frac{1}{R_x} + \frac{1}{R_y} \right)} \right]^{1/3} \quad (12)$$

$$\mathcal{E} = 1 + \frac{3R_x}{5R_y} \quad (13)$$

$$k = 1.03 \left(\frac{R_y}{R_x} \right)^{0.64} \quad (14)$$

where

$$\frac{1}{R_x} = \frac{1}{r_{Ax}} + \frac{1}{r_{Bx}} \quad (15)$$

$$\frac{1}{R_y} = \frac{1}{r_{Ay}} + \frac{1}{r_{By}} \quad (16)$$

The approximate expressions (eqs. (12) to (14)) enable these terms to be easily calculated within 3-percent accuracy without resorting to charts or numerical methods.

CONTOUR PLOTS OF RESULTS

A fuller explanation of what happens in going from a fully flooded to a lubricant starvation condition for materials of low elastic modulus is shown in figures 3 to 6. In figure 3 contour plots of dimensionless pressure ($P = p/E'$) are given for group 3 of table I and for dimensionless inlet distances of 1.967, 1.333, 1.167, and 1.033. In this figure and the remainder of the contour plots to be presented, the symbol $+$ indicates the center of the Hertzian contact. It should be remembered that because of the way the coordinates are made dimensionless, the actual contact ellipse becomes a Hertzian circle regardless of the ellipticity parameter. The Hertzian contact circle is shown in each figure by asterisks. At the top of each figure the contour label and its corresponding value are given. The inlet region is to the left and the exit region is to the right.

The dimensionless pressure contours were kept constant in all parts of figure 3. In figure 3(a), with $m = 1.967$, a fully flooded condition exists. The contours are circular, extending further into the inlet region than into the exit region. As was pointed out in reference 5, the pressure spikes found for high-elastic-modulus materials (ref. 3) were not in evidence in these solutions for low-elastic-modulus materials. The lack of a pressure spike is due to the lack of viscous effects of the fluid in the contact for low-elastic-modulus materials. This in turn is due to considerably less pressure being generated in a contact with low-elastic-modulus materials than in a contact with high-elastic-modulus materials.

In figure 3(b), with $m = 1.333$, a starved condition exists, and the inlet contour is slightly less circular than that in figure 3(a). In figure 3(c), with $m = 1.167$, the starvation is even more severe and the inlet contour is less circular. In figure 3(d), with $m = 1.033$, the starvation is the most severe and the inlet contour is even less circular.

In figure 4 contour plots of dimensionless film thickness ($H = h/R_x$) are given for group 3 of table I and dimensionless inlet distances m of 1.967, 1.333, 1.167, and 1.033. These film-thickness results correspond to the pressure results in figure 3. The central portion of the film-thickness contours becomes more parallel as starvation is increased (going from figs. 4(a) to (d)). The minimum-film-thickness area moves to the exit region of the contact as starvation is increased. The values of the film-

thickness contours for the most starved condition (fig. 4(d)) are much lower than those for the fully flooded condition (fig. 4(a)).

Figure 5 more clearly describes these film-thickness results. It shows the variation of the dimensionless film thickness on the X-axis for four dimensionless inlet distances. The value of Y is held fixed near the axial center of the contact, and the dimensionless parameters U and W are held constant as given in group 3 of table I. In this figure the central region becomes flatter as starvation becomes more severe. Also, in going from a fully flooded condition to a starved condition the film thickness decreases substantially and the location of the minimum film thickness moves closer to the exit region.

Figure 6 shows the variation of the dimensionless film thickness on the Y-axis for four dimensionless inlet distances. The value of X is held fixed near the axial center of the contact, and the dimensionless parameters U and W are held constant as given in group 3 of table I. In this figure, contrary to what is shown in figure 5 for the fully flooded condition, the central region is flat and the slopes of the curves increase as starvation increases.

CONCLUDING REMARKS

The theory and numerical procedure developed by the authors in earlier publications have been used to investigate the influence of lubricant starvation upon minimum film thickness in starved elliptical elastohydrodynamic conjunctions for materials of low elastic modulus. Lubrication starvation was studied simply by moving the inlet boundary closer to the center of the conjunction. The results show that the location of the dimensionless inlet boundary m^* between the fully flooded and starved conditions can be expressed simply as

$$m^* = 1 + 1.07 \left[\left(\frac{R_x}{b} \right)^2 H_{\min, F} \right]^{0.16}$$

That is, for a dimensionless inlet distance m less than m^* , starvation occurs; and for $m \geq m^*$, a fully flooded condition exists. Furthermore, it has been possible to express the minimum film thickness for a starved condition as

$$H_{\min, S} = H_{\min, F} \left(\frac{m - 1}{m^* - 1} \right)^{0.22}$$

Contour plots of the pressure and film thickness in and around the contact have been presented for both the fully flooded and starved lubrication conditions. It is evident from the contour plots that the inlet pressure contours become less circular and that the film thickness decreases substantially as the severity of starvation increases.

The results presented in this report, when combined with the findings previously reported, enable the essential features of starved, elliptical, elastohydrodynamic conjunctions for materials of low elastic modulus to be ascertained.

Lewis Research Center,
National Aeronautics and Space Administration
Cleveland, Ohio, April 21, 1978,
505-04.

REFERENCES

1. Hamrock, Bernard J.; and Dowson, Duncan: Numerical Evaluation of the Surface Deformation of Elastic Solids Subjected to a Hertzian Contact Stress. NASA TN D-7774, 1974.
2. Hamrock, Bernard J.; and Dowson, Duncan: Isothermal Elastohydrodynamic Lubrication of Point Contacts. Part I - Theoretical Formulation. J. Lubr. Technol., vol. 98, no. 2, Apr. 1976, pp. 223-229.
3. Hamrock, Bernard J.; and Dowson, Duncan: Isothermal Elastohydrodynamic Lubrication of Point Contacts. Part III - Fully Flooded Results. J. Lubr. Technol., vol. 99, no. 2, Apr. 1977, pp. 264-276.
4. Hamrock, B. J.; and Dowson, D.: Isothermal Elastohydrodynamic Lubrication of Point Contacts. Part IV - Starvation Results. J. Lubr. Technol., vol. 99, no. 1, Jan. 1977, pp. 15-23.
5. Hamrock, B. J.; and Dowson, D.: Elastohydrodynamic Lubrication of Elliptical Contacts for Materials of Low Elastic Modulus. Part I - Fully Flooded Conjunction. J. Lubr. Technol., vol. 100, no. 2, Apr. 1978, pp. 236-245.
6. Herrebrugh, K.: Solving the Incompressible and Isothermal Problem in Elastohydrodynamic Lubrication Through an Integral Equation. J. Lubr. Technol., vol. 90, no. 1, Jan. 1968, pp. 262-270.
7. Dowson, D.; and Swales, P. D.: The Development of Elastohydrodynamic Conditions in a Reciprocating Seal. Proceedings of the Fourth International Conference on Fluid Sealing, vol. 2, Paper 1, British Hydromechanics Research Association, 1969, pp. 1-9.

8. Baglin, K. P.; and Archard, J. F.: An Analytic Solution of the Elastohydrodynamic Lubrication of Materials of Low Elastic Modulus. Symposium on Elastohydrodynamic Lubrication, Inst. Mech. Engrs., p. 13.
9. Biswas, S.; and Snidle, R. W.: Elastohydrodynamic Lubrication of Spherical Surfaces of Low Elastic Modulus. J. Lubr. Technol., vol. 98, no. 4, Oct. 1976, pp. 524-529.
10. Wolveridge, P. E.; Baglin, K. P.; and Archard, J. G.: The Starved Lubrication of Cylinders in Line Contact. Proc. Inst. Mech. Eng., (London), Part I, vol. 185, 1971, pp. 1159-1169.
11. Wedeven, L. D.; Evans, D.; and Cameron, A.: Optical Analysis of Ball Bearing Starvation. J. Lub. Technol., vol. 93, no. 3, July 1971, pp. 349-363.
12. Brewe, David E.; and Hamrock, Bernard J.: Simplified Solution for Elliptical-Contact Deformation Between Two Elastic Solids. J. Lubr. Technol., vol. 99, no. 4, Oct. 1977, pp. 485-487.

TABLE I. - EFFECT OF STARVATION ON DIMENSIONLESS
MINIMUM FILM THICKNESS FOR THREE GROUPS OF
DIMENSIONLESS SPEED AND LOAD PARAMETERS

Dimensionless inlet distance, m	Group		
	1	2	3
	Dimensionless load parameter, W		
	0.4405×10^{-3}	0.2202×10^{-3}	0.4405×10^{-3}
	Dimensionless speed parameter, U		
	0.5139×10^{-8}	0.1027×10^{-7}	0.5139×10^{-7}
	Dimensionless minimum film thickness, $H_{\min}$		
1.967	131.8×10^{-6}	241.8×10^{-6}	584.7×10^{-6}
1.833	131.2×10^{-6}	238.6×10^{-6}	572.0×10^{-6}
1.667	129.7×10^{-6}	230.8×10^{-6}	543.1×10^{-6}
1.500	125.6×10^{-6}	217.2×10^{-6}	503.0×10^{-6}
1.333	115.9×10^{-6}	199.3×10^{-6}	444.9×10^{-6}
1.167	98.11×10^{-6}	170.4×10^{-6}	383.5×10^{-6}
1.033	71.80×10^{-6}	120.8×10^{-6}	272.3×10^{-6}

TABLE II. - EFFECT OF DIMENSIONLESS SPEED AND LOAD

PARAMETERS ON DIMENSIONLESS INLET DISTANCE

AT FULLY FLOODED - STARVED BOUNDARY

Group	Dimensionless speed parameter, U	Dimensionless load parameter, W	Dimensionless radius parameter, R_x/b	Dimensionless fully flooded minimum film thickness, $H_{min, F}$	Dimensionless inlet distance at fully flooded - starved boundary, m^*
1	0.5139×10^{-8}	0.4405×10^{-3}	19.41	127.8×10^{-6}	1.661
2	$.1027 \times 10^{-7}$	$.2202 \times 10^{-3}$	24.45	234.5×10^{-6}	1.757
3	$.5139 \times 10^{-7}$	$.4405 \times 10^{-3}$	19.41	567.2×10^{-6}	1.850

TABLE III. - EFFECT OF DIMENSIONLESS INLET

DISTANCE ON DIMENSIONLESS MINIMUM-

FILM-THICKNESS RATIO

Group	Dimensionless inlet distance, m	Ratio of minimum film thickness for starved and flooded conditions, $H_{min, S}/H_{min, F}$	Inlet boundary parameter, $(m - 1)/(m^* - 1)$
1	1.661	1	1
	1.500	.9828	.7564
	1.333	.9069	.5038
	1.167	.7677	.2526
	1.033	.5618	.0499
2	1.757	1	1
	1.667	.9842	.8811
	1.500	.9262	.6605
	1.333	.8499	.4399
	1.167	.7267	.2206
3	1.850	1	1
	1.667	.9575	.7847
	1.500	.8868	.5882
	1.333	.7844	.3918
	1.167	.6761	.1965
	1.033	.4801	.0388

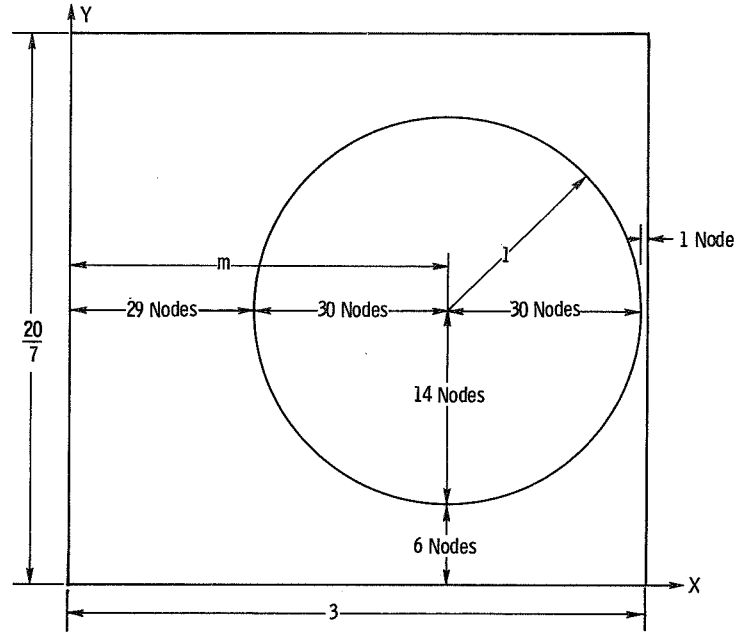


Figure 1. - Nodal structure used for fully flooded numerical calculations.

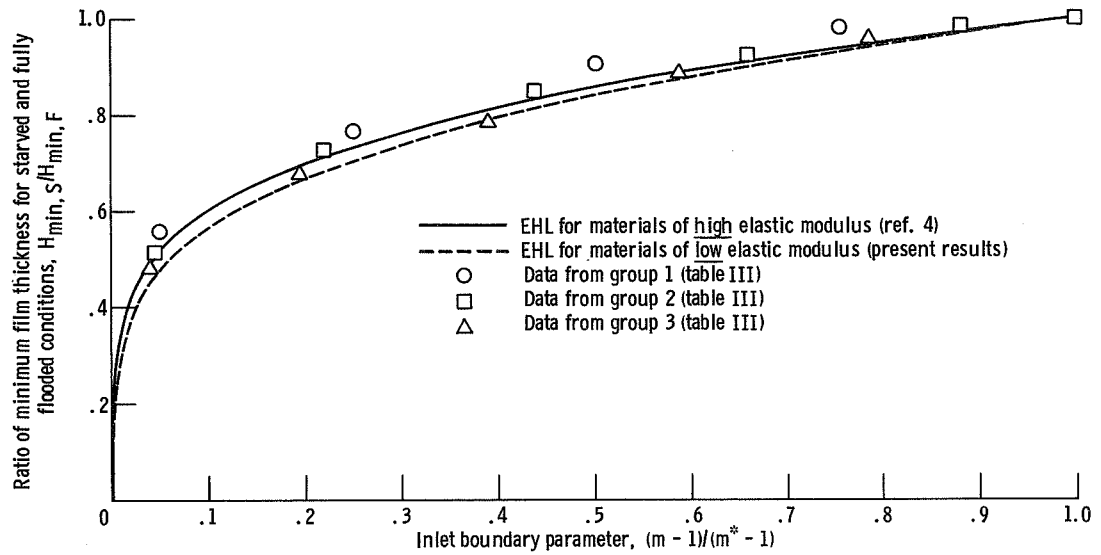
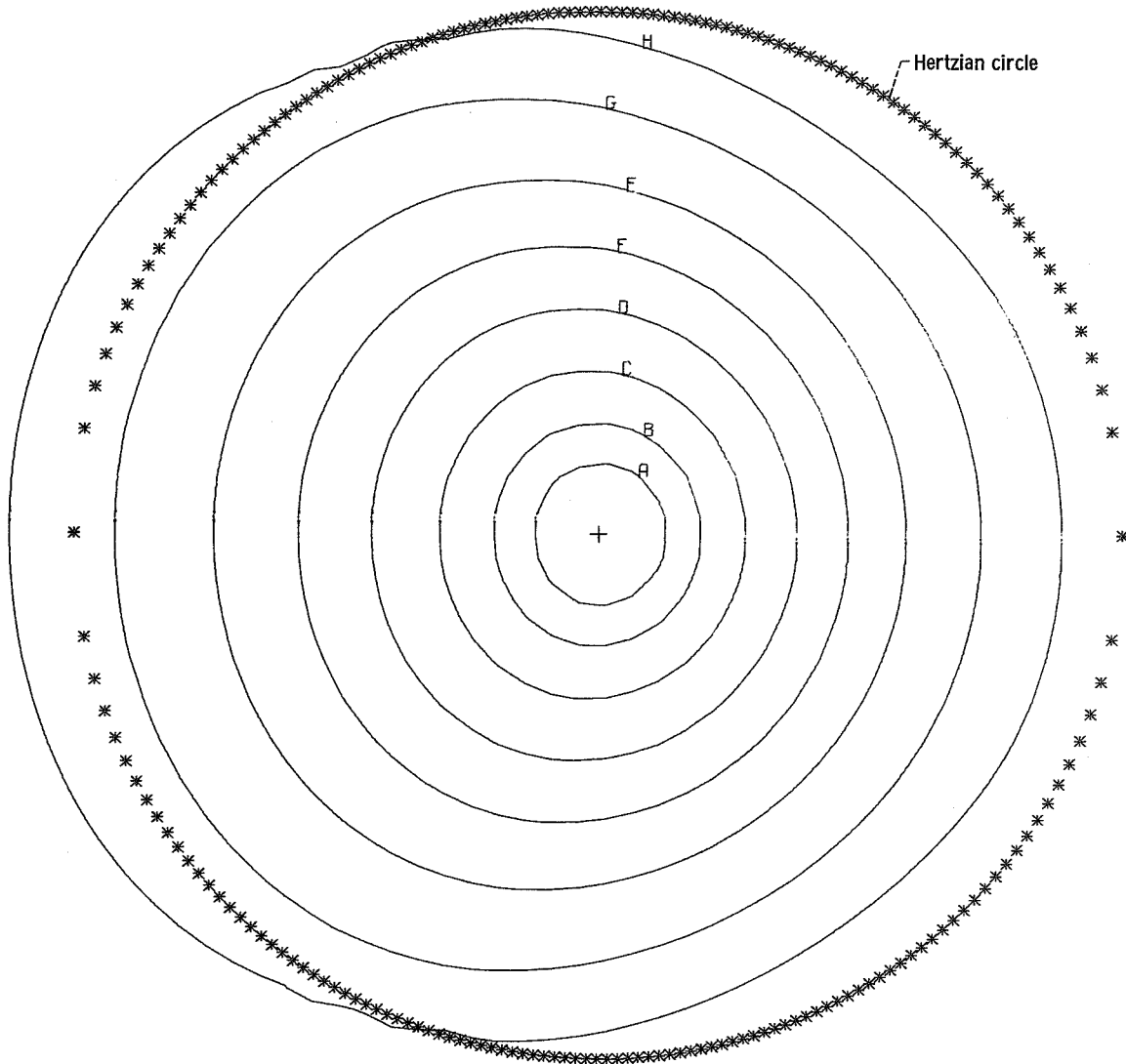


Figure 2. - Influence of inlet boundary on minimum-film-thickness ratio.

Dimensionless
pressure,
 $P = p/E'$

A	0.0123
B	.0121
C	.0117
D	.0110
E	.0100
F	.0085
G	.0060
H	.0030

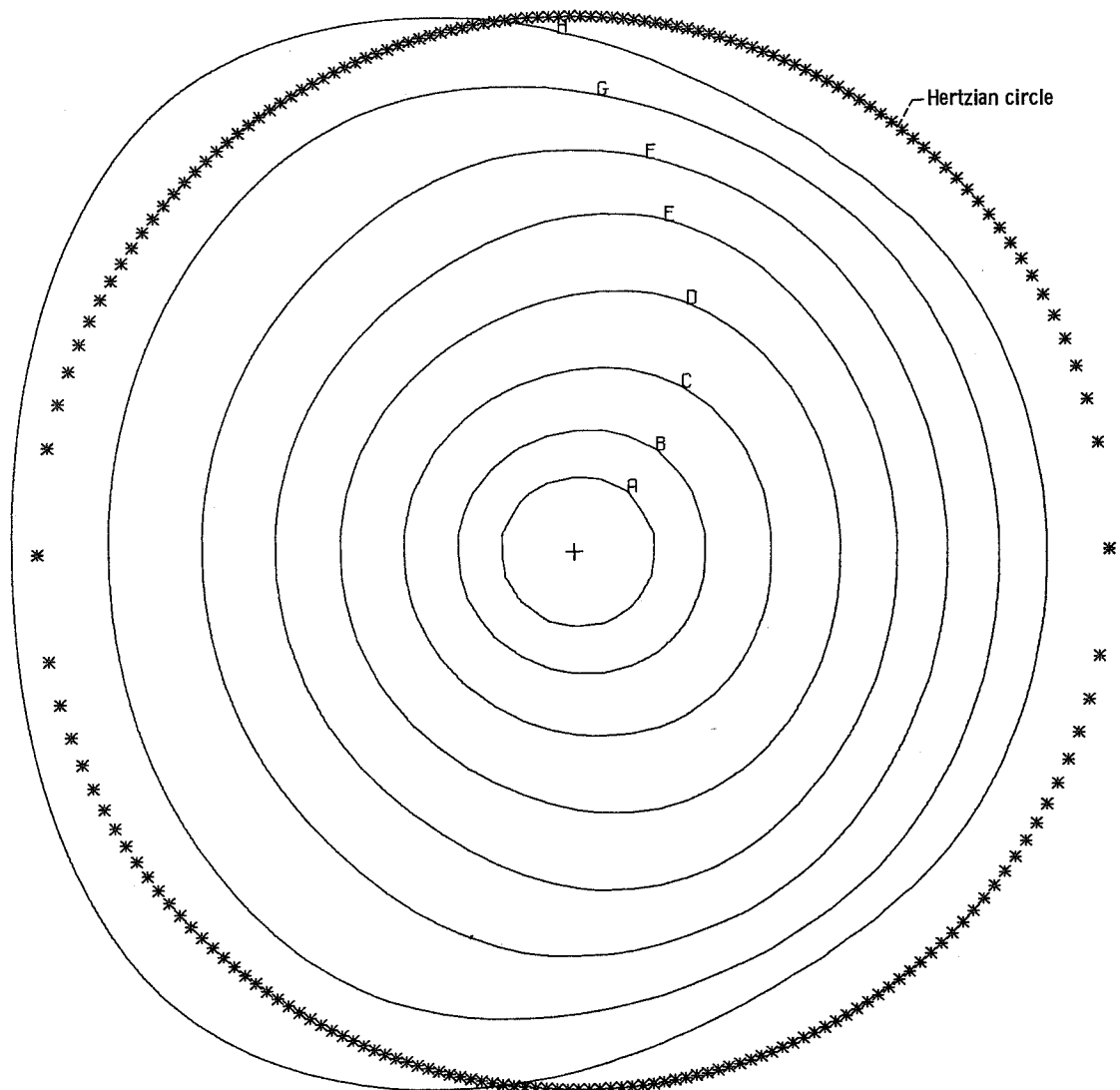


(a) $m = 1.967$.

Figure 3. - Contour plots of dimensionless pressure for dimensionless inlet distances m of 1.967, 1.333, 1.167, and 1.033 and for group 3 of table I.

Dimensionless
pressure,
 $P = p/E'$

A	0.0123
B	.0121
C	.0117
D	.0110
E	.0100
F	.0085
G	.0060
H	.0030

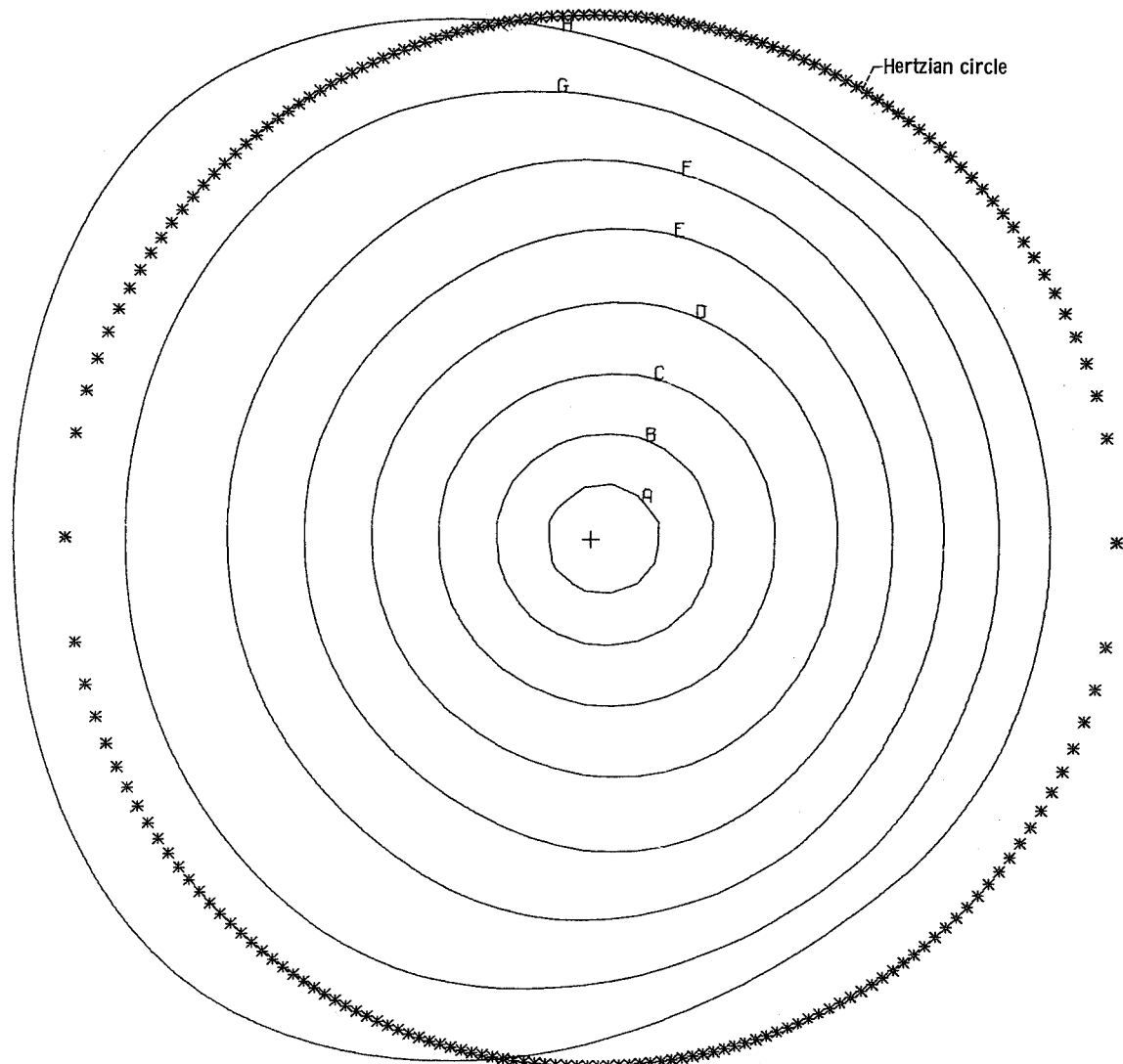


(b) $m = 1.333$.

Figure 3. - Continued.

Dimensionless
pressure,
 $P = p/E'$

A	0.0123
B	.0121
C	.0117
D	.0110
E	.0100
F	.0085
G	.0060
H	.0030

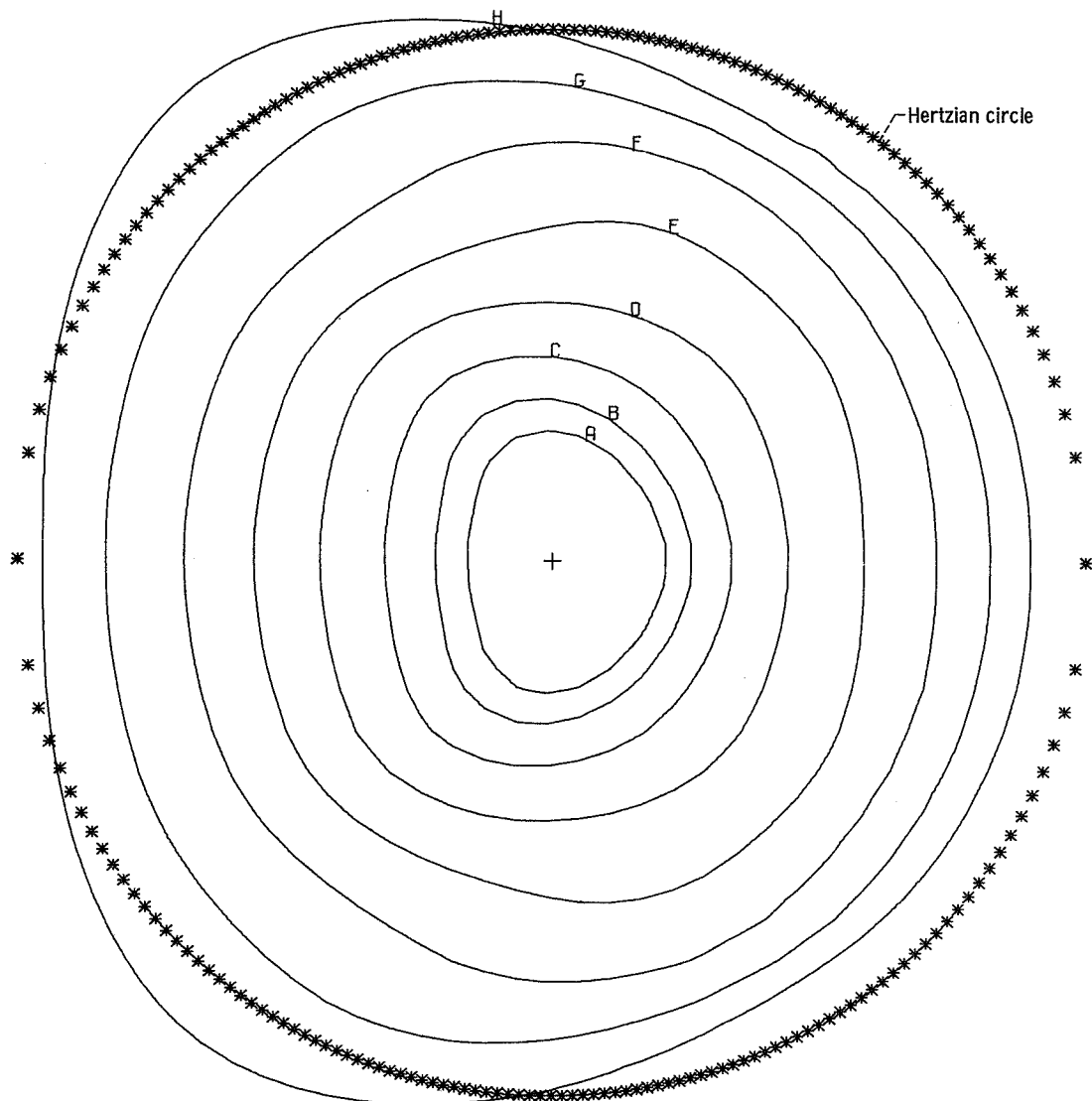


(c) $m = 1.167$.

Figure 3. - Continued.

Dimensionless
pressure,
 $P = p/E'$

A	0.0123
B	.0121
C	.0117
D	.0110
E	.0100
F	.0085
G	.0060
H	.0030

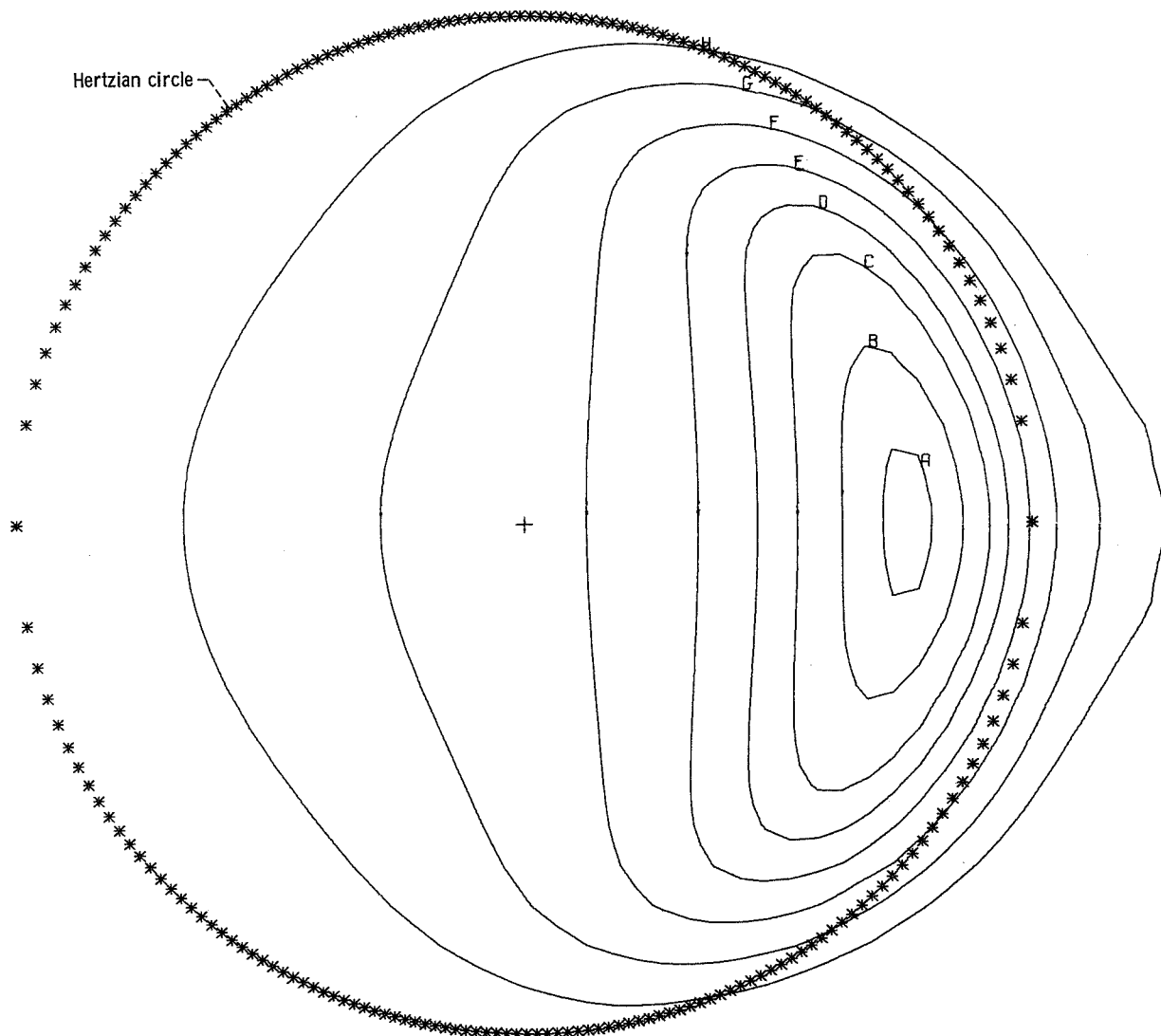


(d) $m = 1.033$.

Figure 3. - Concluded.

Dimensionless
film thickness,
 $H = h/R_x$

A	0.00059
B	.00060
C	.00062
D	.00064
E	.00067
F	.00072
G	.00080
H	.00092

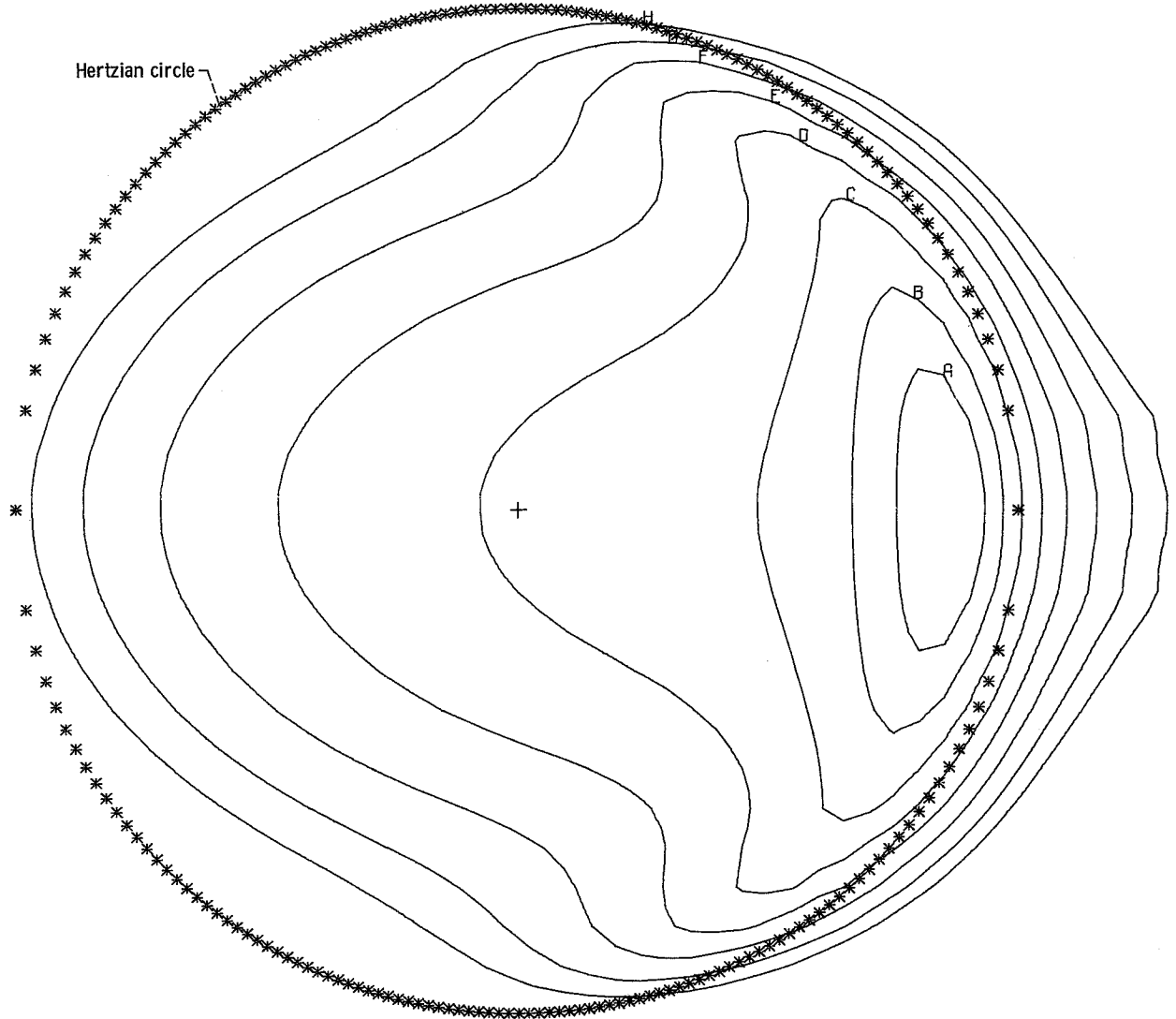


(a) $m = 1.967$.

Figure 4. - Contour plots of dimensionless film thickness for dimensionless inlet distances m of 1.967, 1.333, 1.167, and 1.033 and for group 3 of table I.

Dimensionless
film thickness,
 $H = h/R_x$

A	0.00040
B	.00048
C	.00051
D	.00055
E	.00060
F	.00066
G	.00073
H	.00080

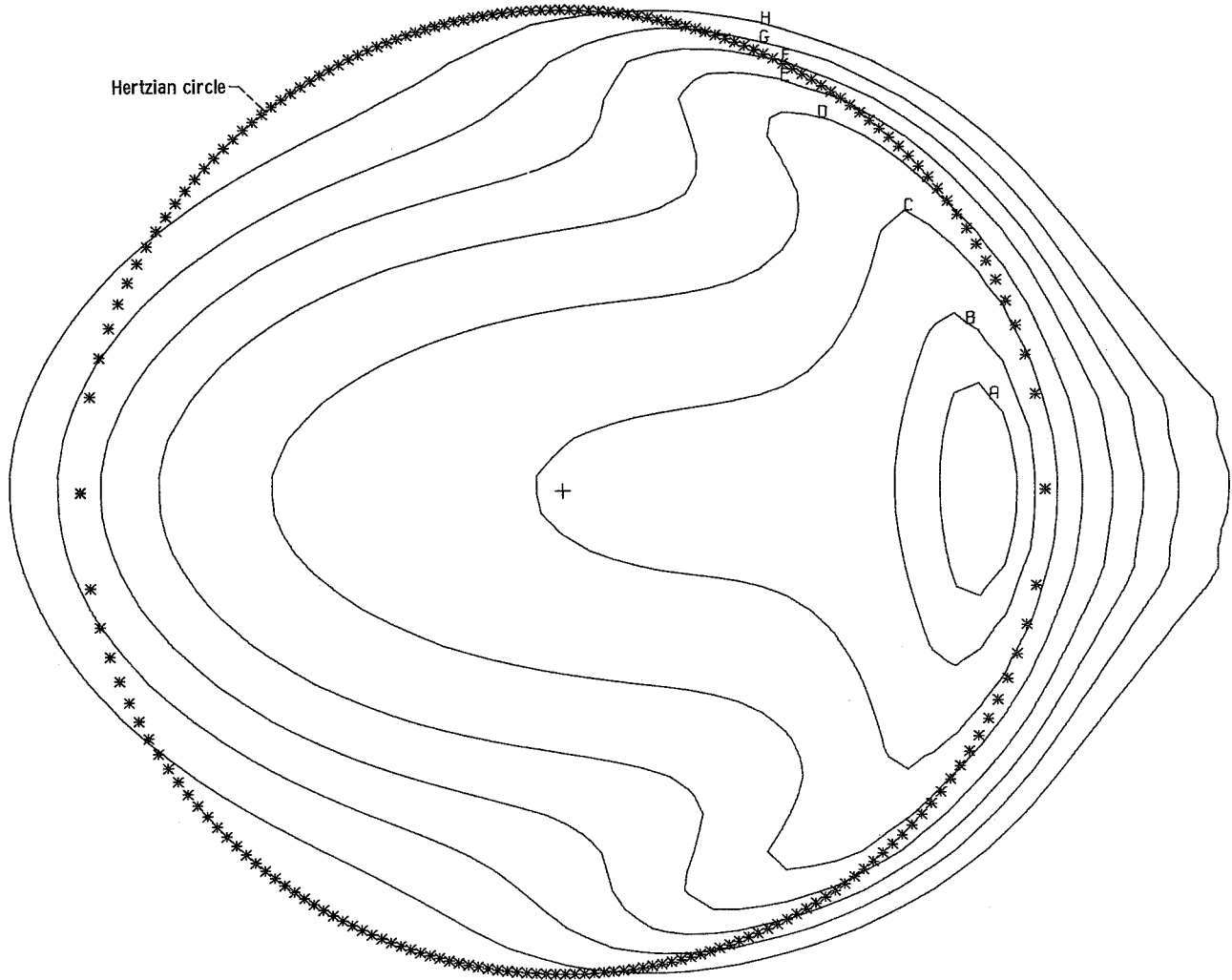


(b) $m = 1.333$.

Figure 4. - Continued.

Dimensionless
film thickness,
 $H = h/R_x$

A	0.00040
B	.00042
C	.00046
D	.00051
E	.00057
F	.00063
G	.00070
H	.00080

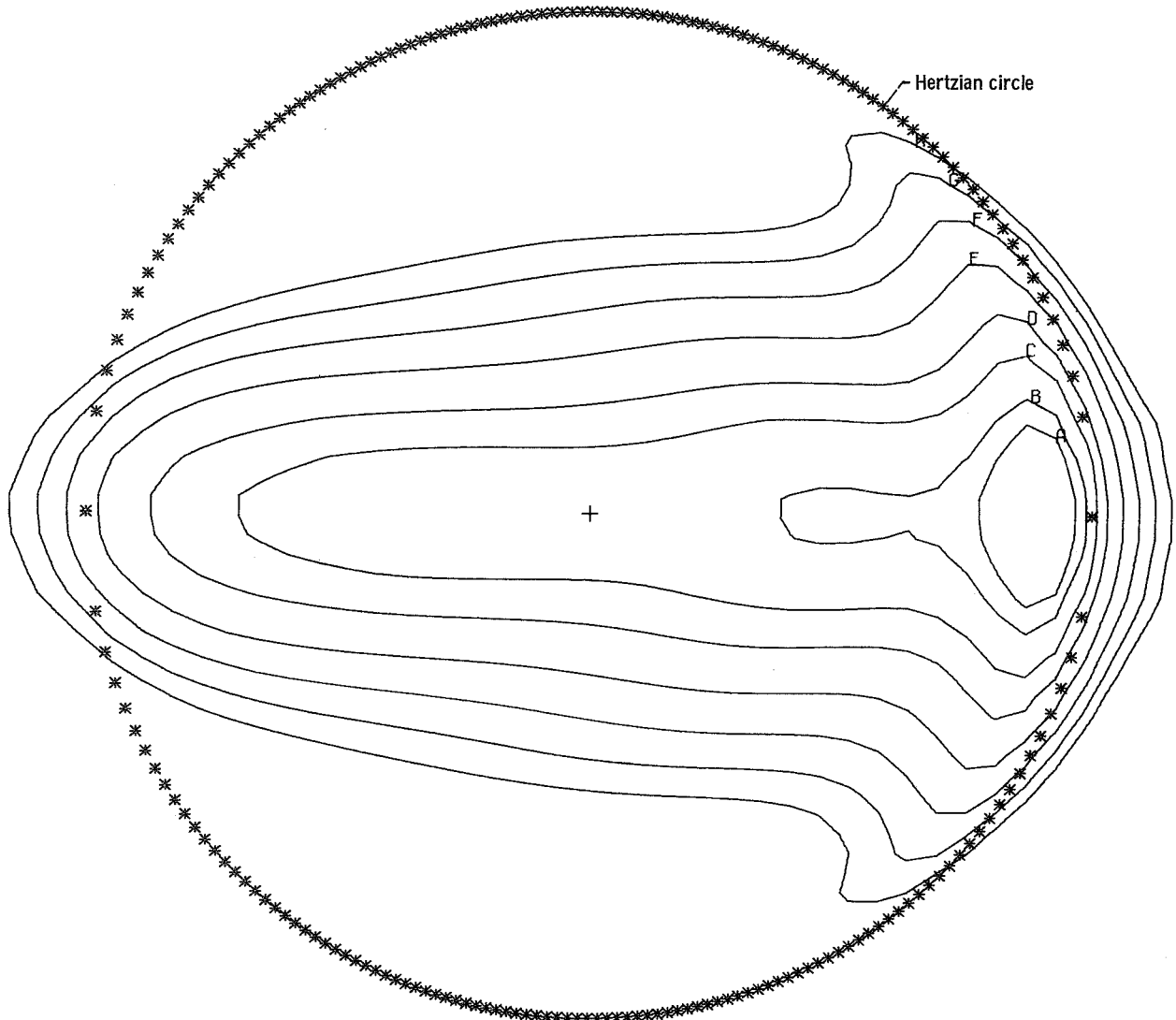


(c) $m = 1.167$.

Figure 4. - Continued.

Dimensionless
film thickness,
 $H = h/R_x$

A	0.00029
B	.00030
C	.00032
D	.00034
E	.00037
F	.00040
G	.00043
H	.00046



(d) $m = 1.033$.

Figure 4. - Concluded.

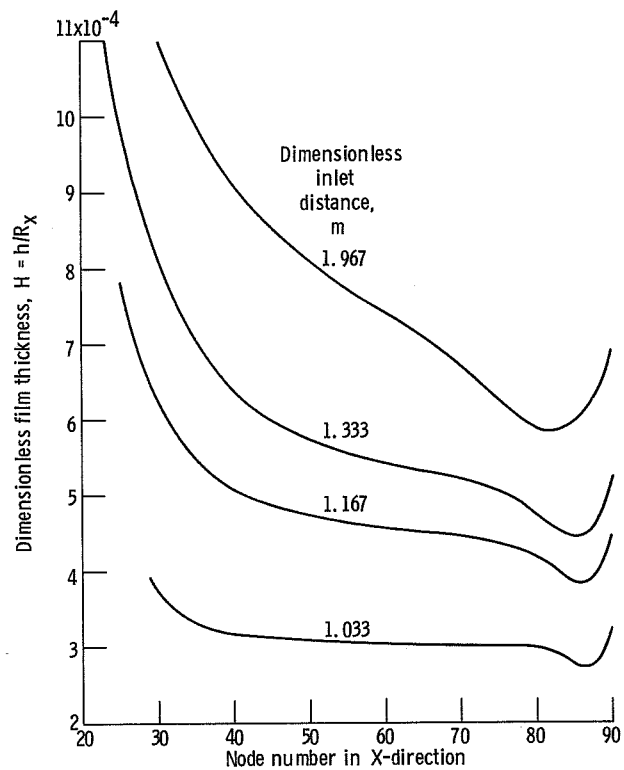


Figure 5. - Variation of dimensionless film thickness on X-axis for four dimensionless inlet distances. The value of Y is held fixed near axial center of contact and the dimensionless parameters U and W are held constant as given in group 3 of table I.

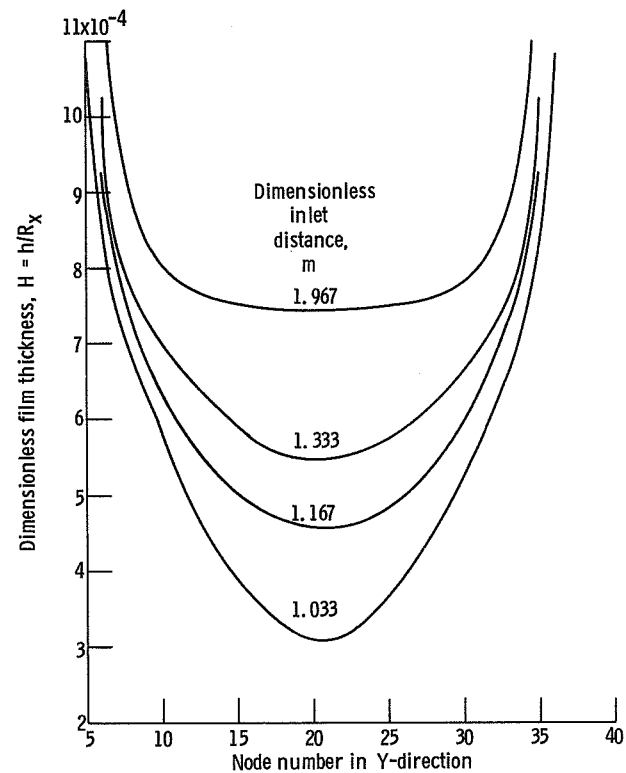


Figure 6. - Variation of dimensionless film thickness on Y-axis for four dimensionless inlet distances. The value of X is held fixed near the axial center of contact and the dimensionless parameters U and W are held constant as given in group 3 of table I.

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16. Abstract The theory and numerical procedures developed by the authors in earlier publications were used to investigate the influence of lubricant starvation on minimum film thickness for low-elastic-modulus materials. Lubricant starvation was studied simply by moving the inlet boundary closer to the contact center. The results show that the dimensionless inlet distance at the boundary between the fully flooded and starved conditions m^* can be expressed simply as $m^* = 1 + 1.07 [(R_x/b)^2 H_{\min, F}]^{0.16}$, where R_x is the effective radius of curvature in the rolling direction, b is the semiminor axis of the contact ellipse, and $H_{\min, F}$ is the dimensionless minimum film thickness for the fully flooded condition. That is, for a dimensionless inlet distance m less than m^*, starvation occurs; and for $m \geq m^*$, a fully flooded condition exists. Furthermore, it has been possible to express the dimensionless minimum film thickness for a starved condition as $H_{\min, S} = H_{\min, F} [(m - 1)/(m^* - 1)]^{0.22}$. Contour plots of the pressure and film thickness in and around the contact are shown for the fully flooded and starved lubrication conditions. These contour plots show that the inlet pressure contours become less circular and that the film thickness decreases substantially as the severity of starvation increases.					
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