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A Life-Cycle Description of Underground Coal Mining

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Assistant Secretary for Energy Technology
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ABSTRACT

This paper describes an initial effort to relate the major technological and economic variables which impact conventional underground coal mining systems, in order to help identify promising areas for advanced mining technology. The point of departure is a series of investment analyses published by the United States Bureau of Mines, which provide both the analytical framework and guidance on a choice of variables.

The result is an algebraic expression for the selling price of clean coal, as a function of labor and capital productivities, required return on investment, average wage rate, equipment availability, initial development cost, recovery factor, tonnage losses due to debris and washing, and similar gross technology descriptors. A preliminary investigation of the structure of this price model reveals a hyperbolic dependence on labor and capital productivities and strong sensitivity to required return on investment, productivity of capital and labor, tonnage lost in beneficiation, wages and salaries, and operating supplies.

Numerical applications of the pricing model are based on a room and pillar mine in 72-inch coal under 800 feet of overburden, producing 1.5 million tons/year of clean coal. Mineral rights are acquired under an option-lease arrangement which requires a minimum annual production payment to the lessor. Construction and initial development extend over three years before capacity production is attained. Although the model formally requires a fixed amount of equipment and personnel throughout the era of capacity production, an extension shows how to accommodate varying annual production levels.

ACKNOWLEDGMENTS

The work reported here represents interim results from the Advanced Coal Extraction Systems Definition Project, a study performed at the Jet Propulsion Laboratory, Pasadena, California for the Fossil Energy Program, United States Department of Energy, via an interagency agreement with the National Aeronautics and Space Administration.

The idea for this paper originated in the summer of 1976 as a result of discussions with William B. Schmidt, Acting Director, Division of Solid Fuel Mining and Preparation, U.S. Department of Energy. Subsequently, the staff of the Process Evaluation Office became collaborators in the work as a result of the strong interest of Carl W. DiBella, Acting Chief, Engineering and Economic Standards Section, U.S. Department of Energy.

The authors would like to take this opportunity to express appreciation for contributions made by several other individuals. Louis H. Berkshire and E. L. Hemingway of the Process Evaluation Office within Engineering, and Economic Standards, DOE, assembled the data on the 1.98 million ton/year shaft mine used in the numerical application of the model. Keith R. Ugone of JPL transformed these data into the format required by the model and performed all of the price calculations and sensitivity analyses. Reviewers of an earlier version of the paper included Michael J. Hudak, Manager of Mines, Fairmont Operations, Consolidation Coal Company; Dr. R. V. Ramani, Professor of Mining Engineering, Pennsylvania State University; Drs. Doan L. Phung and William Gilmer, both of the Institute for Energy Analysis, Oak Ridge; and Drs. James W. Doane and Richard P. O'Toole of the JPL Economics and Policy Analysis Group. The authors are very grateful for the many comments and suggestions made by the reviewers, and wish to absolve them from any errors of fact or conceptualization which remain in the paper. Finally, we wish to thank Elizabeth L. Foster of JPL for the superb job she did preparing this manuscript for publication.

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A LIFE-CYCLE DESCRIPTION OF UNDERGROUND COAL MINING*

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1.0 BACKGROUND

Under an interagency agreement between NASA and the United States Department of Energy, the Jet Propulsion Laboratory has been asked to assist in defining and developing radically new systems for mining deep coal seams. Radically new systems are understood to be those which promise 1) a substantial performance advantage over current technology, or 2) the economic extraction of coal from reserves not presently minable.

In developing or evaluating a new concept, one must consider system performance in four distinct areas:

- Economics: The mine-mouth cost of processed coal;
- Health and Safety: The degree of hazard associated with operating a system;
- Resource Conservation: Possible adverse impacts upon the future exploitation of coal or other natural resources of a mining site;
- Environment: Possible adverse impacts upon the physical environment.

This paper deals solely with those aspects of mine development and operation which can be translated into dollars and cents. Provision is made to include the cost of providing a safe workplace, protecting the physical environment,

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*This document presents interim results from the Advance Coal Extraction Systems Definition Project, JPL, Pasadena, sponsored by the United States Department of Energy, in collaboration with the National Aeronautics and Space Administration.

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or preserving unexploited resources; however, this version of the life-cycle costing model makes no attempt to identify separately these expenditures.

The cost model developed below has a structure which satisfies three criteria:

- It is simple. The model is an algebraic expression, which permits one to see at a glance, the relation between the price of coal and such factors as labor productivity, recovery ratio, wage rate, operating costs, investment in equipment and construction, etc.
- It considers all major determinants of cost generally experienced in underground mining; however, the differential impacts of mining conditions (top, bottom, gas, pitch, overburden, etc.) are not considered.
- Finally, it has a structure which makes it easy to add a cost breakdown in terms of mining system functions or activities.

The objectives set for this modeling effort make it almost unique within the publicly accessible literature: the authors are not aware of any similarly detailed studies which combine technological and economic variables to produce an algebraic model of an underground mining system. A brief survey of past work suggests that existing models fall into three categories:

1) Present value of capital budgeting models oriented toward common descriptions of expenditures and revenues;
2) Simulations of mining operations which typically describe system operations at the level of elemental machine operations; and
3) More integrative algebraic models which attempt some synthesis of ecnomics, machine performance, and mining conditions.

The U.S. Bureau of Mines model mine studies are excellent examples of the present value method.[3,5,6] The following section summarizes the major features of this type of description; however, the reader will find that it differs little from the approach used to evaluate any substantial capital project. Since the present value technique is widely used in the private sector, it is no surprise that others[10,11,13] have used this tool in assessing the economic aspects of mining ventures. Most of these models describe the logic of the present value method, present formulae for describing particular types of cash flows, and then illustrate the method using numerical examples. Few attempt, as does this paper, to express the key relationships algebraically.

In contrast to a present value model, computer simulation offers the possiblity of a very rich description of a mining system. Indeed, the ability of a simulation to capture the salient details of interactions among men, machines, and geology makes it especially attractive to mine planners.[4,7,9] In addition, simulation has been used with encouraging results in the evaluation of novel systems whose architecture is an obvious extension of current mining technology.[8] However, detailed simulation as currently employed appears to be of limited value in either developing or evaluating innovative concepts because of the difficulty of distilling salient system characteristics from numerical output. It is challenging to generalize from a set of simulation runs to the overall performance of the technology modeled; using those results to draw conclusions about radically new technology is an order of magnitude more challenging. In a word, an algebraic model, although it may lack a great deal in realistic detail, seems much better suited for R&D planning than a simulation.

Indeed, an integrative algebraic model — one which ties together hardware performance, mining conditions, and cost — seems ideally suited to establishing R&D priorities and setting goals for development programs. As implied by the strong interest in capital budgeting and simulation models, the bulk of the effort in cost modeling has been oriented to mine planning rather than R&D policy. Thus, it is no surprise to find relatively few algebraic models in the published literature. M. B. Zimmerman[14] recently devised such a model in order to forecast long term coal supply prices for the United States. Zimmerman used empirical data on section production and mining conditions to develop a mine-mouth cost model which explictly involved seam thickness and number of producing sections. An unusual feature of this model is its attempt to relate future production costs to forecasts of depletion — i.e., the need to mine progressively thinner seams as thicker ones become exhausted. Z. Ajdukiewicz,[1] in his study of conventional coal mining as practiced in Eastern Europe, addressed many of the same issues as Zimmerman and used similar econometric tools. However, unlike Zimmerman's work, Ajdukiewicz's results are quite technology specific. One could make similar comments about COMINEC's study of longwall technology and its potential for adaptation to American mining conditions.[2] This work is particularly noteworthy because of its attempt to define optimal system architecture as a function of equipment performance, seam characteristics, and costs.

Each of these studies is valuable for its identification of variables, suggestions about a modeling approach, and guidance about the appropriate level of detail. R&D policy decisions require answers to questions, such as, the following:

- What is a first approximation to the capital and operating cost?
- What aspects of system performance are most crucial to achieving (or bettering) the forecast cost?
- To what extent do these projections allow for additional expenditures to solve problems of environmental protection and worker health and safety — problems often difficult to define until a system undergoes test and demonstration?
- What appears to be the optimal scale of operations?

In the conceptualization or early development stage, the information about a novel system is usually not sufficient to justify computer simulation as a means of answering questions like those above. Since some estimates of system economics are crucial to a development decision, one inevitably falls back upon the capital budgeting framework because it is simple to use and provides meaningful output even with crude data. However, in constructing this type of model, one must be careful to portray the essential links between technological performance and cost. The algebraic models noted above face the same challenge, but choose to begin with a description of the technology, and subsequently derive the necessary cost factors. Since introducing technology into the capital budgeting scheme is a relatively easy thing to do, this analysis starts there, taking the USBM model mine studies as the point of departure.

Development of the model begins in Section 2.0 below with a review of the methodology employed in the Bureau of Mines model mine studies. Section 3.0 contains a brief discussion of model scope and an identfication of the major technological and cost factors to be considered. Derivation of an algebraic expression for the life-cycle cost, or equivalently, the required selling price is presented in Section 4.0. followed in Section 5.0 by an investigation of properties of general interest — price-productivity relationships and price sensitivities. The analysis is concluded in Section 6.0 by comparison with empirical data and numerical illustrations of the selling price and sensitivity calculations for a representative shaft mine. Appendices to the report describe a) an extension of this analysis to accommodate a production capability which varies from year to year; b) calculation of interest during construction; c) detailed cost data for the illustrative mine; and d) input data for the life-cycle cost calculation.

2.0 REVIEW OF BUREAU OF MINES INVESTMENT MODELS

Beginning in 1974, the United States Bureau of Mines issued a series of information circulars which provide detailed estimates of the capital investment and operating costs for underground bituminous coal mines.[3,5,6]

As the name of the series implies, these published studies are meant to assist mine operators in planning a new mine. Consequently, the circulars are organized around familiar cost summaries — investment in construction, equipment, and working capital; breakdown of operating cost; depreciation; consumables; etc. All of the capital outlays are discounted by the appropriate present worth factor, and then summed to yield the present worth of the aggregate mine investment. Assuming constant sales throughout the mine life, the authors compute the required selling price for unwashed coal which will 1) amortize the investment, 2) cover the annual operating cost while making due allowance for the tax impact of depreciation and depletion, and 3) allow for adequate return to debt and equity holders. In short, the information circulars present a comprehensive, easily understood analysis, firmly based on the widely accepted principles of capital budgeting and engineering economics.

These studies consider most of the material financial impacts. However, a mine operator faced with the current market and regulatory environment, would undoubtedly find the following extensions quite helpful:

- Addition of a preparation plant;
- Alternatives to outright purchase of mineral rights, e.g., the currently popular scheme of off-balance sheet financing via option-lease;
- Guidance in estimating the price impact of an initial development which is more costly and time consuming than drift entry;
- Variation in such financial quantities of interest as return on investment, depreciation rate, royalty percentage, the structure of union welfare charges, local taxes, etc.; and
- Variations in section output.

All of these extensions are addressed in the numerical analysis and appendices of this report.

There has been considerable discussion in recent years over the cost impacts of 1) the greatly reduced labor productivity currently experienced by the industry and 2) the continued inflation in the cost of mining equipment and labor. These comments were found to be very stimulating in formulating an investment evaluation scheme which treats the major technical and economic factors in symbolic fashion. This report takes a first step in that direction by translating into algebra key costing rules and related assumptions made in the Bureau's model mine studies. It is hoped that this symbolic treatment of cost will be a guide to answering such questions as the impact of

- Boosting labor productivity,
- Reducing the initial development time,
- Increasing equipment availability, or
- Mechanizing certain functions in conventional coal mining.

3.0 SCOPE AND STRUCTURE OF THE MODEL

As mentioned in the introductory remarks, the primary objective of this paper is the development of an economic tool for evaluating novel extraction concepts. Thus, it must employ terms familiar to the industry and accept data in the form typically available to systems designers. The resource-oriented cost breakdown (plant and equipment, labor, and consumables) adopted by the IC 8600 model mine studies is very well suited to this purpose, even though the intent of the 8600 reports is to provide assistance in detailed project planning.

Because the thrust of this paper is advanced system evaluation rather than project planning, some of the detail required in a mine plan will be omitted in order to focus attention on the primary economic factors which shape system performance and cost. For example, it is expedient to simplify somewhat the treatment of taxes, depreciation, depletion, production royalties, capital structure, etc., to render the algebra tractable. Although some of these simplifications may be too imprecise for planning a mine, they seem quite appropriate for screening new ideas in a preliminary phase of development.

The idea behind the analysis is the estimation of the minimum annual revenue required by an operator. This is accomplished by selecting a "typical" rate of return, then solving for that life-cycle cost which covers all operating expenses, amortizes the capital outlays, and yields the specified return on investment (assuming 100% equity). Thus, the resulting dollar/ton figure is merely an advance estimate of the price an operator must charge to realize the rate of return traditionally required by the industry. A comparison of this price with the projected production cost for current technology permits a first order judgment about the commercial appeal of a new extraction scheme.

In the interest of algebraic simplicity, variables are defined as if they were constant over the life of a mine. For example, the annualized life-cycle cost of coal, labor and capital productivities, annual production, real wages, cost of replacement parts and machines, etc., are all treated as if they were fixed. In fact, this is not as limiting as it may appear. Life-cycle costing commonly employs annualized quantities which reduce year-to-year variations in a schedule of cash flows (for example, annual revenue) to one annualized figure which has a present value identical to the time varying schedule. Thus, one may extend the static character of the model by interpreting selected variables as annualized quantities. Appendix A provides detailed guidance in calculating the cost per ton of coal when annual production quantities vary over the mine life.

The sections which follow give a more detailed picture of model structure by, first, identifying the major variables, and second, listing major assumptions about modeling both technological and financial aspects of system operation.

Variables

ρ_L: Labor productivity. Unless otherwise stated, labor productivity will be based on all of the people at the site. (tons of raw coal/man-shift.)

ρ_E: Productivity of capital, a measure of the capital intensiveness of a mining system. Capital is defined as the present value of initial and deferred investments in construction and equipment. (tons of raw coal/yr/$ invested.)

M_T, M_H: Number of personnel at the site, and those paid on an hourly basis, respectively.

$\overline{w}$: Average wage rate. A subscript may be added to indicate the labor force of interest, e.g., $\overline{w}_T$ symbolizes the weighted average wage for everyone at the site, while $\overline{w}_H$ is the rate for hourly employees only. ($/man-shift.)

p_A: Price of land. The initial outlay for mineral rights, either the single purchase price or the price of an option. ($/acre.)

η: Recovery factor, defined as the fraction of raw coal extracted as a proportion of coal initially in the seam being mined.

v_s: Seam density, considering the combined effects of thickness, pitch, partings, etc. (tons/acre.)

α_R: Dirt and debris mined with the coal, expressed as a fraction of the raw coal tonnage.

α_P: Washing losses, defined as the fraction of raw coal which is lost in washing and other beneficiation processing.

α_A: Anticipated downtime for scheduled maintenance, equipment downtime, and scheduled worker non-productive hours (breaks, transportation, etc.). This effect is expressed as a fractional reduction in capital productivity ρ_E.

α_U: Unanticipated downtime including slowdowns due to unscheduled maintenance, difficult geology, work stoppages, etc. Again, the effect is a reduction in ρ_E.

r: Rate of return. The discount rate which yields an aggregate net present worth of zero when applied to all cash flows resulting from both investment and operations.

p_C: Annualized price of coal at the mine mouth is that constant dollar amount per ton which will cover all costs and produce the specified rate of return to the mine operator. (\$/ton.)

These variables are judged to be the major determinants of the mine-mouth price of coal. Many other variables of lesser rank will be identified in the course of model development. Some of these secondary variables are mentioned in the list of assumptions which follow.

Assumptions

- Annual production tonnage may be a constant over the life of a mine, or alternatively, may vary from year to year. In the latter case, Appendix A shows how to compute the annualized value for required revenue.

- Mine closing costs are small, and occur so far in the future that they have negligible impact on annualized price.
- The model assumes a zero escalation rate for all categories of expenditures and revenues.
- The effects of changes in the market price of coal, and the impact of altered labor or capital productivity over the life of a mine are not considered, i.e., this is a static model.
- Tax life of equipment is the same as its economic life — how long it lasts before replacement is necessary.
- Union welfare payments are treated in accord with the 1974 Bituminous Wage Agreement, i.e., a portion of this charge is proportional to tonnage, and a portion, to hours worked.
- General overhead is assumed to be proportional to the combined cost of labor and operating supplies.
- Depletion allowance is specified as 10% of annual sales.
- The combined state and federal income tax rate is assumed to be 50%. Local taxes are modeled as a fixed proportion of annual sales.*
- Insurance costs are calculated as a fixed percentage of initial investment.
- Depreciation is treated as a fixed annual charge with no inflation in the cost of replacing equipment; accelerated depreciation may be handled by computing the equivalent annualized charge.
- Consumption of supplies, water, and power are assumed to be a function only of the annual volume of raw coal produced; in practice expenditures on consumables depend strongly upon seam geometry and mining conditions.

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*This differs somewhat from the treatment of state and local taxes in the USBM model mine studies series, but maintains the same spirit of simplicity.

This concludes the enumeration of major variables and assumptions. As remarked earlier, they retain most of the essential features of the mine investment problem, while simplifying some of the computational details. Although the resulting expression for price may be inadequate for the evaluation of a new mining venture, it appears to provide a reasonable point of departure for the assessment of novel technology.

4.0 DEVELOPMENT OF THE MODEL

As explained in the discussion of model scope and structure, the annualized selling price is based on the revenue required to cover all operating costs, amortize capital outlays, and yield a specified return on aggregate investment. As formulated below, the model is a straightforward present value analysis which uses a predetermined rate of return in discounting the cash flows. Accordingly, the starting point for the analysis is the fundamental requirement that the net present value of all cash flows be zero,* or in symbolic terms,

$$\sum_i PV_i(r)\,(R_i - E_i) = 0 \qquad (1)$$

where R_i and E_i are, respectively, the revenue and expenditures in the i^{th} year, and $PV_i(r)$ is a multiplier which converts these flows to present values appropriate to the year incurred, at the discount rate r. Note that it is the summed present value of revenues minus expenditures which equals zero, not yearly amounts.

Within the framework of this model, it is useful to distinguish capital outlays from operating expenditures. Thus, we define E_i as:

$$E_i \equiv E_{ki} + E_{pi}$$

*Currently, there are a number of financial techniques for evaluating time-phased investments of this sort. The two most commonly used techniques are:

- Internal rate of return, which determines that discount rate which yields a net present value of zero; or
- Net present value, which simply computes the net present value corresponding to an assumed discount rate, presumably the opportunity cost of capital.

Although it may not be apparent from the form of Eq. (1), the model developed below is philosophically similar to the net present value approach because it determines that price which assures a specified rate of return.

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where

E_{ki} = Expenditure on capital equipment in the i^{th} period; and

E_{pi} = Expenditure on operating expense in the i^{th} period.

This permits Eq. (1), the requirement on discounted cash flows, to be written as:

$$\sum_i PV_i(r)\,(R_i - E_{pi} - E_{ki}) = 0$$

or

$$\sum_i PV_i(r)\,E_{ki} = \sum_i PV_i(r)\,(R_i - E_{pi}) \tag{2}$$

Equation (2) is a mathematical statement of the requirement that the present value of all investment outlays be equal to the present value of the net revenue from operations. Thus, if capital investment and operating expenditures are specified, Eq. (2) determines the revenue required to meet a target rate of return.

Revenue Requirement

The annual revenue requirement is calculated by making a set of assumptions which greatly simplify the analysis. Although the key assumptions were presented above in Section 3.0, it is worthwhile pausing to highlight the treatment of inflation and the annual cash flows from operations. The first simplification to be highlighted is the treatment of inflation.

As noted in Section 3.0, the model developed here does not provide for inflation in capital costs or operating costs. This may limit the realism of the results somewhat in those cases where labor, equipment, and operating supplies are escalating at substantially different rates, or where forecast changes in market conditions justify a changing rate of return over the life of a mine. Such considerations, although rather important in planning a new mining venture, are viewed as matters of secondary concern in assessing novel extraction concepts or identifying priority areas for increased R&D. Thus, it was decided not to include the effects of inflation in this model.

The second simplification assumes that annual sales and operating costs are constant and equal to their capacity values during the period which begins with all sections producing, and ends with mine deactivation. At first glance this may appear to be a serious restriction because production does fluctuate throughout the life of a mine, and tails off in a fairly predictable way as operations draw to a close. However, any apparent limitation can be circumvented by interpreting the constant values for production and cost as if they were annualized or "levelized" values which are equivalent in present value impact to a time series that varies from year to year. Appendix A explains the details of calculating an annualized value which is equivalent to the original time series.

Model development continues by defining the origin of time (t_o) as the beginning of the year in which capacity production is first reached. In terms of the summations above, this definition of t_o implies that $i=1$ corresponds to the first year of capacity production. Cash flows which occur prior to capacity production — e.g., flows related to resource assessment, mineral rights acquisition, permits, construction, equipment purchase, initial development, etc., — must accordingly be discounted forward (compounded) to t_o.

Mathematically these assumptions can be written as:

$$R_i = \begin{cases} R \ , & \text{for } 1 \le i \le T \\ R_o , & \text{for } i = 0 \end{cases}$$

$$E_{pi} = \begin{cases} E_p , & \text{for } 1 \le i \le T \\ O \ , & \text{for } i \le 0 \end{cases} \tag{3}$$

Where T is the projected mine life and R_o represents the present values of revenues which occur prior to time period $i=1$. Defining E_{k_o} as the present value at t_o of all expenditures occurring before capacity production, and substituting Eq. (3) into Eq. (2), one obtains

$$\hat{E}_K = \sum_{i=1}^{T} PV_i(r)\, E_{k_i} + \left(E_{k_o} - R_o\right) = (R - E_p) \sum_{i=1}^{T} PV_i(r) \tag{4}$$

where $\hat{E}_K$ symbolizes the present value of all capital expenditures, including equipment replacement. Note that capital outlays prior to capacity production $(E_{k_o} - R_o)$ are computed as net of coal sold. The factor $(R - E_p)$ represents the annual cash flow generated from operations, and is assumed to be constant in light of Eq. (3).

The summation on the right-hand side of Eq. (4) is the reciprocal of the capital recovery factor $\gamma(r, T)$. Thus, Eq. (4) may be rewritten as:

$$\hat{E}_K \; \gamma(r, T) = (R - E_p) \equiv \pi_p \tag{5}$$

where

$$\frac{1}{\gamma(r, T)} \equiv \sum_{i=1}^{T} PV_i(r)$$

and π_p denotes net annual cash flow from operations.

Cash flow has three sources: profit from operations, depreciation, and depletion. In particular,

$$\pi_p \equiv \text{Cash Flow} = \text{Net Profit} + \text{Depletion} + \text{Depreciation}$$

or

$$\text{Net Profit} + \text{Depletion} = \text{Cash Flow} - \text{Depreciation}. \tag{6}$$

The net profit computation provides an alternative expression for (Net Profit + Depletion). Assuming a combined state and federal income tax rate of τ, one may express net profit as

$$\begin{aligned}\text{Net Profit} &= (\text{Annual Sales} - \text{Operating Cost} \\ &\qquad - \text{Depletion} - \text{Income Tax}) \\ &= (1-\tau)(\text{Sales} - \text{Operating Cost} - \text{Depletion}).\end{aligned}$$

After rearranging, one finds

$$\begin{aligned}\text{Net Profit} + \text{Depletion} &= (1-\tau)(\text{Sales} - \text{Operating Cost}) \\ &\qquad + \tau(\text{Depletion}).\end{aligned} \tag{7}$$

If depletion is assumed to be a fixed proportion of annual sales, then Eq. (7) becomes

$$\text{Net Profit} + \text{Depletion} = \left[1-\tau(1-\delta)\right] \text{Sales} - (1-\tau)(\text{Operating Cost}) \tag{8}$$

where δ is depletion allowance as a proportion of sales.

The desired relation is obtained by equating the alternate expressions for (Net Profit + Depletion) given by Eqs. (6) and (8):

$$\text{Cash Flow} - \text{Depreciation} = 1-\tau(1-\delta) \;\; \text{Sales} - (1-\tau)(\text{Operating Cost})$$

so that

$$\text{Sales} = \frac{1}{1-\tau(1-\delta)}\left[(1-\tau)(\text{Operating Cost}) + \text{Cash Flow} - \text{Depreciation}\right] \tag{9}$$

Upon setting the combined state and federal tax rate τ equal to 50%, and the depletion allowance δ to 10%, one obtains the expression for annual sales used in the USBM model mine studies, namely.

$$\text{Sales} = \frac{1}{0.55} \left\{ 0.50\ (\text{Operating Cost}) + \text{Cash Flow} - \text{Depreciation} \right\}. \quad (9a)$$

For convenience of subsequent manipulation, define the following terms:

S: Annual sales

C_p: Annual operating cost

D: Annual depreciation charge.

Then Eq. (9) may be rewritten as:

$$S = \frac{1}{1 - \tau(1 - \delta)} \left\{ (1 - \tau)\, C_p + \pi_p - D \right\}$$

or, after substituting for π_p from Eq. (5),

$$S = \frac{1}{1 - \tau\,(1 - \delta)} \left\{ (1 - \tau)\, C_p + \gamma \hat{E}_K - D \right\} \quad (10)$$

This is the fundamental expression from which flows all of the subsequent analytical development. Examination of Eq. (10) reveals that it can handle a situation where several years are required to achieve capacity production if $\hat{E}_K$ includes production start-up expense. Constant sales and operating costs during the capacity production period remain a constraint on this cost model. However, a method for handling variations in annual production as pointed out above, is described in Appendix A.

Equation (10) can be converted to an equivalent price per ton by dividing annual sales by the annual tonnage capability of the mine. The tonnage expression must provide for converting raw coal into processed coal free of rock, dirt, and other foreign matter. Thus, define the following:

V_C: Annual tonnage of clean or processed coal;

V_R: Annual production capacity of raw coal;

α_R: Fraction of raw coal that is rock, dirt, and other debris;

α_P: Fraction of coal lost in the cleaning process.

One may then express the processed coal tonnage as:

$$V_C = V_R (1 - \alpha_R)(1 - \alpha_P) \tag{11}$$

At first glance, Eq. (11) may seem unduly complex. However, a little reflection on the aggregate characteristics of coal mining technology reveals that α_R and α_P are factors which can be separately influenced by the system designer, and so merit individual attention in describing novel system performance.

In view of Eq. (11), the expression for the selling price (p_C) becomes

$$p_C = \frac{S}{V_C} = \frac{\frac{1}{1 - \tau(1 - \delta)}\left\{(1 - \tau)C_P + \gamma\hat{E}_K - D\right\}}{V_R(1 - \alpha_R)(1 - \alpha_P)} \tag{12}$$

Equation (12) reduces to the expression used in the USBM model mine studies if α_R and α_P are set to zero, $\tau = 0.50$, and $\delta = 0.10$.

The remainder of the analysis is an elaboration of the terms in the numerator — first, capital investment, then depreciation, and finally, operating cost.

Capital Investment

The present value of capital investment, $\hat{E}_K$, may be expressed as the sum of initial investment in plant and equipment, $\hat{K}_{EO}$, deferred investment in equipment, $\hat{K}_{EF}$, the cost of acquiring rights to the land, $\hat{K}_A$, and net expenditures to obtain initial access to the seam and bring all sections up to capacity production, $\hat{K}_D$. Thus, $\hat{E}_K$ may be written as

$$\hat{E}_K = \hat{K}_{EO} + \hat{K}_{EF} + \hat{K}_A + \hat{K}_D \tag{13}$$

where the circumflex superscript denotes present value as of the beginning of year t_o, (equivalent to $i = 1$ in Eq. (3)). An expression for each term on the right-hand side of Eq. (13) is developed below.

The present value of initial and deferred investment in plant and equipment, $\hat{K}_{EO} + \hat{K}_{EF}$, can be described in terms of the capital intensiveness, or capital productivity, ρ_E, of the mining system. Mathematically, ρ_E, is defined as:

$$\rho_E \equiv \frac{V_R}{\hat{K}_{EO} + \hat{K}_{EF}} \qquad (14)$$

where

V_R = Desired annual tonnage of raw coal (tons/year)

ρ_E = Productivity of capital, with capital defined as the present value of all capital investment less land and development costs (tons/year/$).

The productivity of capital is determined by a number of factors, including equipment capability under "ideal conditions"; the need for spares; scheduled downtime for maintenance, travel time to and from the face, lunch, etc., construction delays; unplanned work stoppages due to bad mining conditions, equipment failures, or labor disputes; interest charges during construction and initial development; and the need to replace equipment periodically.

All of these factors may be combined in a fairly simple fashion to assemble an estimate for ρ_E. Begin by defining the following quantities:

K_{EO}: That amount which must be invested under the ideal conditions of n [illegible] certain costs, and no system downtime. K_{EO} is an [illegible]cal cost, exclusive of any interest charges.

α_A, α_U: The fraction of capital productivity lost due to anticipated and unanticipated system downtime, respectively.

λ_{EO}: That fraction of initial investment represented by interest during construction.

β_E: The ratio of aggregate deferred investment to the initial capital outlay, adjusted for productivity losses.

Then one may express the present value of initial and deferred investment as

$$\hat{K}_{EO} + \hat{K}_{EF} = \frac{\left(1+\lambda_{EO}+\beta_E\right)K_{EO}}{\left(1-\alpha_A\right)\left(1-\alpha_U\right)} = \frac{V_R}{\rho_E} \tag{15}$$

Although this is a crude portrayal of a rather complex set of relationships, it does permit a rough estimate of the sensitivity of price to some of the major determinants of capital productivity.

Using Eqs. (14) and (15), K_{EO}, the historical value of initial investment, can be easily related to capital productivity:

$$K_{EO} = \frac{V_R\left(1-\alpha_A\right)\left(1-\alpha_U\right)}{\rho_E\left(1+\lambda_{EO}+\beta_E\right)} \tag{15a}$$

The expression for $\hat{K}_{EO}$ and $\hat{K}_{EF}$ can be further elaborated as follows:

$$\hat{K}_{EO} = \frac{\left(1+\lambda_{EO}\right)K_{EO}}{\left(1-\alpha_A\right)\left(1-\alpha_U\right)} = \frac{\left(1+\lambda_{EO}\right)V_R}{\rho_E\left(1+\lambda_{EO}+\beta_E\right)} \tag{15b}$$

and

$$\hat{K}_{EF} = \frac{\beta_E\,K_{EO}}{\left(1-\alpha_A\right)\left(1-\alpha_U\right)} = \frac{\beta_E\,V_R}{\rho_E\left(1+\lambda_{EO}+\beta_E\right)} \tag{15c}$$

Interest during construction — that is, the impact of discounting on cash flows prior to t_o — can be significant due to extended equipment deliveries, facility construction, and progress payments made on equipment which requires a lengthy time to manufacture. To ascertain the relationship between plant and equipment cost, K_{EO}, and its associated interest during construction, one must consider the timing and amounts of outlays, and the appropriate discount rate. Once the expenditure schedule is determined, or approximated by some simple functional form, the interest fraction λ_{EO} can be calculated directly (see Appendix B for details of this calculation).

The next category of capital investment is the initial expenditure on mineral rights. Begin by defining the following:

p_A: The price of an option or the outright purchase price (\$/acre).

v_s: Area density of the seam, considering the combined effects of thickness, pitch, partings, and other anomalies (tons/acre).

η: Planned recovery factor.

T: Mine life, as defined above (years).

Both the option and single purchase price of land can be handled like the initial investment in plant and equipment. Royalties or annual production payments are treated later as a component of operating cost. Accordingly, cost $\hat{K}_A$ maybe expressed as

$$\hat{K}_A = p_A \left(\frac{V_R T}{v_s \eta} \right) (1 + r)^n \tag{16}$$

where n is the time period (in years) between the initial outlay on mineral rights and the time t_o when capacity production begins. The term $(1 + r)^n$ reflects compound interest associated with the land payment. If the land is purchased, $\hat{K}_A$ must be adjusted to account for resale after mining operations cease.

The treatment of investment concludes with expressions for initial development costs. During the development phase, several activities take place. At a very gross level these activities include property assessment; mine design; and construction of field access, general support systems, facilities, and offices. Calculation of development era costs includes expenditures for the tasks listed above, less a credit equal to the revenues generated from coal sold each year prior to capacity production.

Denote the present value of initial development costs as $\hat{K}_D$. Since a mine requires several years to reach capacity production, the expression for $\hat{K}_D$ should explicitly reflect the schedule of annual outlays, $E_{KD}(t)$, the time capacity is achieved, t_o, and the discount rate, r. Using the well known expression for compounding, one may express development cost, net of coal tonnage sold, as

$$\hat{K}_D = \sum_{t=-n_1}^{0} E_{KD}(t)(1+r)^{-t} \tag{17}$$

where n_1 is the year when development era expenditures begin.

Some calculations require the aggregate historical dollar expenditures on initial development, K_D. This quantity is easily expressed as a sum of the $E_{KD}(t)$:

$$K_D \equiv \sum_{t=-n_1}^{0} E_{KD}(t) \tag{17a}$$

It is also useful to define the ratio of these two development costs as

$$d_{INT} = \frac{\hat{K}_D}{K_D} .$$

Equations (15) through (17) are combined to yield an expanded description of capital investment. Recognizing that all terms except $\hat{K}_D$ explicitly involve V_R, one obtains

$$\hat{E}_K = \hat{K}_{EO} + \hat{K}_{EF} + \hat{K}_A + \hat{K}_D$$

$$= V_R \left\{ \frac{1}{\rho_E} + \frac{p_A T}{v_s \eta} (1+r)^n + \frac{\hat{K}_D}{V_R} \right\} \tag{18}$$

Annual Depreciation Charge

The annual depreciation charge, D, depends upon a number of factors, including first costs, economic versus tax lifetimes, salvage values, capitalized repairs, and of course, the form of the depreciation rule. It is difficult to formulate a depreciation expression which reflects all of these factors and is, at the same time, mathematically tractable.* Accordingly, a reasonable course of action is the following: Use data on equipment costs and longevity, plus a depreciation rule to produce an equivalent constant annual charge; then express this annual charge as a fraction of the first cost of all depreciable assets. In symbols, this becomes

$$D = \beta_D \left(\frac{K_{EO}(1 - w_c)}{\left(1 - \alpha_A\right)\left(1 - \alpha_U\right)} + K_D \right) \tag{19}$$

where

$\dfrac{K_{EO}}{(1 - \alpha_A)(1 - \alpha_U)}$: Effective first cost of plant and equipment, considering all work stoppages.

w_c: That fraction of initial capital investment expended on working capital.

K_D: Aggregate historical expenditures on initial mine development, net of any coal sold;

β_D: Ratio between the computed annual charge and total first cost.

Note that K_{EO}, and K_D are historical figures, not present values.

In the case of straight-line depreciation where all depreciable capital outlays occur prior to the year of capacity production, the factor β_D is approximated by the reciprocal of the weighted average lifetime of these assets. If an accelerated rule is used, Eq. (19) requires a side calculation which converts the actual depreciation schedule to an equivalent annual amount D, and the associated ratio β_D (see Appendix A for guidance on performing the annualization calculation). The effect of an accelerated rule,

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*Treatment of depreciation for a new mining venture is much more complex than the situation implied by this summary list of factors. For example, current tax laws treat exploration, development, and production differently with expensing of initial outlays permitted in some cases (see Coal Age, March 1976, pp. 92-94). Consequently, the simplifications inherent in Eq. (19) may lead to a selling price somewhat higher than a more realistic treatment of depreciation.

all other things constant, is to increase β_D since depreciation occurs more quickly than for the straight-line case. Thus, an accelerated rule effectively reduces the price of coal since depreciation is subtracted from yearly costs.

Annual Operating Cost

This model describes operating costs in terms of a commonly used breakdown of resources. Below are listed the major cost categories, plus associated symbols:

C_L: Total labor cost, both hourly and salaried;

C_S: Outlays on operating supplies (e.g., roof bolts, rock dust, replacement parts, etc.), assumed to vary directly with raw coal tonnage for a seam of constant thickness;

C_U: Cost of power and water, assumed to be proportional to raw coal tonnage;

$C_{O/H}$: Payroll overhead (i.e., social security, unemployment compensation, and various fringe benefits), assumed to be 40% of total labor cost in the USBM model mine studies;

C_W: Union welfare payments, which are a function of both raw coal tonnage and the hours worked by those under the union contract (c.f., the 1974 bituminous wage agreement);

C_I: Indirect cost, assumed to be directly proportional to the sum of total labor cost and the cost of operating supplies; in the USBM model mine studies the proportionality factor is 15%;

C_{INS}: Cost of insurance, computed as a percentage of initial capital investment, excluding all interest charges;

C_T: Local taxes, expressed as a percentage of sales, in accord with typical local tax laws;

C_R: Royalty payments, expressed as a percentage of annual sales;

D: Annual depreciation charge;

C_P: Total annual operating cost.

In light of these definitions, annual operating cost may be expressed as:

$$C_P = a_L C_L + a_S C_S + C_U + C_W + C_{INS} + C_T + C_R + D \qquad (20)$$

where

a_L: A constant multiplier which adjusts total labor cost to reflect payroll overhead ($C_{O/H}$) and indirect cost (C_I).

a_S: A constant multiplier which increments the outlay on operating supplies to account for indirect cost (C_I).

The assumptions used in recent USBM model mine studies lead to the following numerical values for a_L and a_S:

$$a_L = 1.55$$

$$a_S = 1.15 \qquad (21)$$

The remainder of this section is devoted to developing an expression for each of the terms in Eq. (20). By construction, each can be related to raw coal tonnage.

Total Labor Cost

The first term in the total cost expression can be written as

$$a_L C_L = a_L V_R \overline{w}_T / \rho_L \qquad (22)$$

where $\overline{w}_T$ is the average wage (\$/man-shift) for all personnel at the site, and V_R and ρ_L are the annual raw coal tonnage and labor productivity, respectively.

Operating Supplies:

As indicated in the list of major cost components above, operating supplies are assumed to vary directly with raw coal tonnage; thus,

$$a_S C_S = a_S c_S V_R \tag{23}$$

where c_S is defined as the per ton cost of supplies.

Cost of Power and Water:

In reality. the utility expense depends both on the scale of operations (i.e., the capacity of the mine) and on the amount of raw coal output. Structurally, this implies that the cost of power and water is the sum of two terms — a fixed cost which depends only on capital investment, and a variable cost which reflects the intensity with which this capacity is utilized. Accordingly, as productivity falls, utility expense falls too, but not as fast. However, in the interests of simplicity, it is assumed that direct proportionality holds between tonnage and cost. Thus,

$$C_U = c_U V_R \tag{24}$$

Union Welfare Expense:

Union welfare charges are based upon both tonnage and working hours, as specified by the 1974 Bituminous Wage Agreement. Exhibit 1 displays the various components of welfare expense.

Exhibit 1

Cost Factors in the 1974 Bituminous Wage Agreement (as of Dec. 6, 1976)

Recipient	Tonnage Charge ($/ton)		Hourly Charge ($/hr)
1950 Pension Trust	0.554		
1950 Benefit Trust	0.190		
1974 Pension Trust	0.076	plus	0.66
1974 Benefit Trust			0.88
TOTALS	0.820	plus	1.54

If the productivity of the unionized workforce is of the order of 10 tons per 8-hour shift, then the tonnage and hourly components of the welfare payment are $8.20 and $12.32 per man, per shift. Since both components are significant, both are included in the welfare cost expression C_W:

$$C_W = c_{WT} V_C + c_{WH} h_S V_R (M_H/M_T)/\rho_L \tag{25}$$

where

- c_{WT}: Welfare payment required per ton of clean coal, as weighed prior to shipment ($/ton);
- V_C: Annual tonnage of clean coal (tons);
- c_{WH}: Welfare payment required per man-hour worked by labor under contract ($/man-hour);
- h_S: Hours worked per shift (hours);
- V_R: Annual tonnage of raw coal produced (tons);
- M_H: Number of shift personnel subject to the union contract — the subscript H denotes hourly workers;
- M_T: Total number of shift personnel at the site;
- ρ_L: Labor productivity, based on all of the personnel at the site (tons/man-shift).

With the help of Eq. (11), clean coal tonnage is easily related to raw coal production. Thus, the above expression becomes

$$C_W = V_R \left\{ (1 - \alpha_R)(1 - \alpha_P) c_{WT} + \left(\frac{M_H}{M_T}\right)\left(\frac{h_s}{\rho_L}\right) c_{WH} \right\} \tag{26}$$

According to Exhibit 1 the two welfare coefficients take the following values as of December 6, 1976

$$c_{WT} = \$0.820/\text{ton}$$

$$c_{WH} = \$1.540/\text{hour}$$

Insurance

Yearly insurance payments are computed as a fraction of the initial capital investment, K_{EO}. Note that interest during construction, land costs, and development expenditures are not relevant to insurance calculations. Thus, the annual insurance cost, C_{INS}, is

$$C_{INS} = c_{INS} \left(\frac{K_{EO}}{(1 - \alpha_A)(1 - \alpha_U)} \right)$$

where c_{INS} is a multiplier which produces the annual premium when applied to the insurance base. Upon expressing K_{EO} in terms of more fundamental quantities, one obtains

$$C_{INS} = c_{INS} V_R \left(\frac{1}{\rho_E (1 + \lambda_{EO} + \beta_E)} \right)$$

Local Taxes

Annual local taxes C_T — for the most part, property taxes — are expressed as a fraction of yearly sales once capacity production begins.* The general form of the relationship is

$$C_T = \sigma S \tag{28}$$

where σ includes all local taxes. In most cases, the local taxes paid by the mining company are effectively the same whether the land is purchased or mineral rights are acquired via an option/lease agreement.

Royalties

As in the case of local taxes, production royalties are expressed as a fixed percentage of sales. Thus, royalty payment, C_R, is written as

$$C_R = \mu S \tag{29}$$

Currently, a typical production royalty is about 5% of gross sales.

Since calculation of the annual depreciation amount is specified by Eq. (19), the above expression for production royalties completes the characterization of the annual operating cost, cash flow, and sales. However, the expression for local taxes and royalty payments when inserted into Eq. (20) and Eq. (10) makes a straightforward calculation of sales awkward. To resolve this, an expanded form of the annual sales requirement is needed. Restating Eq. (10), one obtains

$$S = \frac{1}{1 - \tau(1 - \delta)} \left\{(1 - \tau)\, C_P + \Upsilon \hat{E}_K - D\right\} \tag{10}$$

*Conversations with several county tax offices suggest this treatment of taxes during the era of capacity production.

Define a new variable C_P' as follows:

$$C_P \equiv C_P' + D + C_T + C_R$$

$$\equiv C_P' + D + (\sigma + \mu) S \tag{30}$$

Substituting Eq. (30) into Eq. (10) and rearranging, yields

$$S = \frac{1 - \tau}{1 - \tau\ (1 - \delta) - (1 - \tau)\ (\sigma + \mu)} \left\{ C_P' + \frac{\gamma \hat{E}_K}{1 - \tau} - \frac{\tau D}{1 - \tau} \right\}$$

For notational convenience, let

$$F \equiv \frac{1 - \tau}{1 - \tau\ (1 - \delta) - (1 - \tau)\ (\sigma + \mu)} \tag{30a}$$

so that

$$S = F \left[\left(C_P' - \frac{\tau D}{1 - \tau} \right) + \frac{\gamma \hat{E}_K}{1 - \tau} \right] \tag{31}$$

In its fully expanded form, the sales equation becomes

$$S = \frac{1-\tau}{1 - \tau\ (1 - \delta) - (1 - \tau)\ (\sigma + \mu)} \left[\left(a_L C_L + a_S C_S + C_U + C_W + C_{INS} - \frac{\tau D}{1 - \tau} \right) \right.$$

$$\left. + \frac{\gamma}{1 - \tau} \left(\hat{K}_{EO} + \hat{K}_{EF} + \hat{K}_A + \hat{K}_D \right) \right] \tag{32}$$

This expression includes all of the investment and operating costs typically incurred during production mining. Specific expenditures to assure worker health and safety, conserve natural resources, and protect the physical environment can be included under the appropriate category in Eq. (32) as either an operating or a capital cost.

The following section briefly examines some of the properties of the selling price expression, with primary emphasis on the relationship of price to both labor and capital productivity.

5.0 STRUCTURE AND PROPERTIES OF THE MODEL

This section explores the overall structure of the model and develops price sensitivities for selected variables. Although considerable effort has been expended to keep the model simple, accuracy has compelled the inclusion of rather a lot of detail — so much so, that the model structure is somewhat obscured. Thus, the first order of business is a reformulation which aggregates the detail and reveals key structural features. This is done by treating labor and capital productivities explicitly, and handling the remaining variables as coefficients. The next step will be an exploration of price sensitivity via the familiar technique of differential calculus. Section 6.0 concludes the analysis by presenting an illustrative price calculation and numerical sensitivities for two representative shaft mines — one producing raw coal, the other an identical mine producing washed coal.

Reformulation in Terms of Labor and Capital Productivities

The reformulation begins with Eqs. (12), (30), and (31):

$$P_C = \frac{S}{V_C} = \frac{F\left[\left(C_P' - \frac{\tau D}{1-\tau}\right) + \frac{\gamma \hat{E}_K}{1-\tau}\right]}{V_R\,(1-\alpha_R)\,(1-\alpha_P)} \tag{12a}$$

Next, each term in the expanded form of Eq. (12a) is examined to determine whether it involves labor or capital productivity, or exhibits no explicit tie to either productivity variable. This is done in Exhibit 2.

Examination of Exhibit 2 reveals several interesting things about model structure. First, every term in the numerator and denominator involves raw coal tonnage V_R, if initial development K_D is characterized in terms of dollars expended per ton of raw coal capacity:

$$k_D \equiv K_D/V_R$$

$$\hat{k}_D \equiv \hat{K}_D/V_R \tag{36a,b}$$

$$p_C = \frac{S}{V_C} = \frac{F\left[\left(C_P' - \frac{\tau D}{1-\tau}\right) + \frac{\gamma \hat{E}_K}{1-\tau}\right]}{V_R(1-e_R)(1-e_P)}$$

Element	Terms Related To: Labor Productivity	Capital Productivity	Other Factors
Denominator:			$V_R(1-e_R)(1-e_P)$
Numerator:			
$F\left(C_P' - \frac{\tau D}{1-\tau}\right) = F a_L C_L$:	$F a_L \bar{w}_T (V_R/\rho_L)$		
$+ F a_S C_S$:			$F a_S c_S V_R$
$+ F C_U$:			$F c_U V_R$
$+ F C_W$:	$F c_{WH} h_S (M_H/M_T)(V_R/\rho_L)$		$+ F c_{WT} V_R (1-e_R)(1-e_P)$
$+ F C_{INS}$:		$F c_{INS} V_R \left(\frac{1}{\rho_E(1+\lambda_{EO}+\beta_E)}\right)$	
$- F\tau D/(1-\tau)$:		$-F\beta_D\left(\frac{\tau}{1-\tau}\right) V_R \left(\frac{1-w_c}{\rho_E(1+\lambda_{EO}+\beta_E)}\right)$	$-F\beta_D\left(\frac{\tau}{1-\tau}\right) V_R \left(\frac{K_D}{V_R}\right)$
$F\gamma \hat{E}_K/(1-\tau) =$		$F\frac{\gamma}{1-\tau} V_R\left(\frac{1}{\rho_E}\right)$	$+\frac{F\gamma V_R}{1-\tau}\left(\frac{\rho_A T}{v_S \eta}(1+r)^n + \frac{\hat{K}_D}{V_R}\right)$

To characterize Local Taxes and Royalties Explicitly in this form

ϵS: $F\epsilon V_R\left[a_L \bar{w}_T/\rho_L + c_{WH} h_S (M_H/M_T)/\rho_L\right] + F\epsilon V_R\left[\frac{1}{\rho_E(1+\lambda_{EO}+\beta_E)}\left(c_{INS} - \frac{\beta_D \tau(1-w_c)}{1-\tau} + \frac{\gamma(1+\lambda_{EO}+\beta_E)}{1-\tau}\right)\right] + F\epsilon V_R\left[a_S c_S + c_U + c_{WT}(1-e_R)(1-e_P) + \frac{\gamma}{1-\tau}\left(\frac{\rho_A T}{v_S \eta}(1+r)^n + \frac{\hat{K}_D}{V_R}\right) - \frac{\beta_D \tau}{1-\tau}\left(\frac{K_D}{V_R}\right)\right]$

μS: $F\mu V_R\left[a_L \bar{w}_T/\rho_L + c_{WH} h_S (M_H/M_T)/\rho_L\right] + F\mu V_R\left[\frac{1}{\rho_E(1+\lambda_{EO}+\beta_E)}\left(c_{INS} - \frac{\beta_D \tau(1-w_c)}{1-\tau} + \frac{\gamma(1+\lambda_{EO}+\beta_E)}{1-\tau}\right)\right] + F\mu V_R\left[a_S c_S + c_U + c_{WT}(1-e_R)(1-e_P) + \frac{\gamma}{1-\tau}\left(\frac{\rho_A T}{v_S \eta}(1+r)^n + \frac{\hat{K}_D}{V_R}\right) - \frac{\beta_D \tau}{1-\tau}\left(\frac{K_D}{V_R}\right)\right]$

Exhibit 2.
Summary of Cost Factors in the Expression for the Price of Coal

In consequence, the resulting expression for the price of coal is formally independent of raw coal tonnage. This implies that no economies of scale are associated with increased coal production, which is not a wholly surprising result given that no scale dependence is reflected in any of the cost components. Although this clearly does not hold for a broad range of mine capacities, the assumption of constant returns to scale is probably a good first approximation for costing a mining system whose capacity is fairly well established. Such is typically the case in any serious evaluation of a novel system. Of course, the best test of such an assumption is comparison with empirical data. This is done below in Section 6.0 with encouraging results.

Another characteristic of interest is the rather simple form assumed by the model when labor and capital productivity are taken as primary independent variables, namely,

$$p_C = \frac{1}{B}\left\{\frac{A_L}{\rho_L} + \frac{A_E}{\rho_E} + A_O\right\} \tag{37}$$

where A_L, A_E, A_O, and B are quantities which involve non-productivity variables exclusively. Exhibit 2 has been organized to assist the identification of these four productivity coefficients.

$$B \equiv (1 - a_R)(1 - a_P)$$

$$A_L \equiv F\left[a_L\overline{w}_T + c_{WH}h_S\,(M_H/M_T)\right]$$

$$A_E = F\left[\left(\frac{1}{1 + \lambda_{EO} + \beta_E}\right)\left(c_{INS} - \frac{\tau\beta_D(1 - w_c)}{1 - \tau}\right) + \frac{\gamma}{1 - \tau}\right]$$

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$$A_O \equiv F\left[a_S c_S + c_U + c_{WT}B - \frac{\tau\beta_D}{1-\tau}\,k_D + \frac{\gamma}{1-\tau}\left(\frac{\rho_A T}{v_S}\,(1+r)^n + \hat{k}_D\right)\right] \qquad (38\ a,b,c,d)$$

and

ρ_L: labor productivity, which is affected by expected make time, set up and idle time, unplanned downtime, amount of coal produced annually, production time per panel, available production hours per year, and haulage capacity.

ρ_E: capital productivity, which is determined by system cost of capital related expenditures (including plant and equipment, capital recovery, tax considerations, and depreciation) and the annual section-hours of production.

F: $\dfrac{1-\tau}{1-\tau(1-\delta)-(1-\tau)(\sigma+\mu)}$, as defined in Eq. (30a).

Equations (37) and (38) provide a reasonably tractable expression for price. In the next section, the properties of this restructured model are explored briefly.

General Properties of the Model

The first property of interest is the behavior of price in terms of the productivity variables. A little manipulation of Eq. (37) reveals 1) that there is a hyperbolic relation between price and each productivity variable, and 2) that there is a hyperbolic relation between the two productivity variables, with price determining the form of the curve

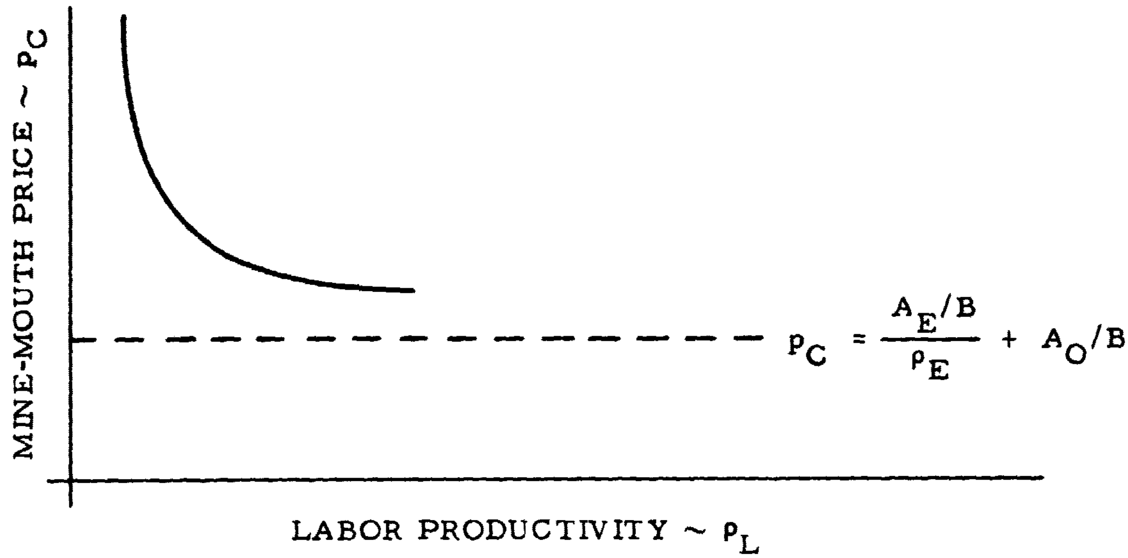
Derivation of the two-variable relationships is the logical point of departure. Consider the relation between p_C and ρ_L. From Eq. (37),

$$p_C\rho_L = \frac{1}{B}\left\{A_L + A_E\,\frac{\rho_L}{\rho_E} + A_O\rho_L\right\}$$

hence

$$\rho_L \left\{ p_C - \left(\frac{A_E/B}{\rho_E} + A_O/B \right) \right\} = \frac{A_L}{B} \tag{39}$$

This is a hyperbola which is asymptotic to the p_C - axis and the line $p_C = ((A_E/B / \rho_E) + A_O/B)$, with A_L/B, determining the sharpness of the "knee" (see the sketch below).



The price asymptote defines the minimum price achievable, given the other assumptions about capital productivity, wage rate, initial development, recovery factor, etc. If this were a general formulation of mining system productivity, the price asymptote could be regarded as a crude measure of the system's potential, assuming that technology and all other factors are fixed. A more meaningful estimate of potential would be the point on the hyperbola corresponding to a very optimistic estimate of labor productivity.

It is important to recognize that this hyperbolic relationship reflects only the reduction in the mine-mouth price of coal associated with increased labor productivity, without consideration of how the reduction is obtained. Typically, increased labor productivity is achieved by investment in more capital intensive equipment. In other words there are generally increased costs associated with an increment in productivity. Conceptually, one finds the range of optimal ρ_L and ρ_E by trading off the price reduction from increased labor productivity, with the increased capital charges which result from additional mechanization. This qualification concerning the relationship between price and productivity applies throughout the analysis.

Now consider the relation between price and capital productivity. Examination of Eq. (37) reveals that this form of the price expression is symmetric in ρ_E and ρ_L. Thus, one may obtain the relation between price and capital productivity by simply reversing the roles of ρ_E and ρ_L in Eq. (39):

$$\rho_E \left\{ p_C - \left(\frac{A_L/B}{\rho_L} + A_O/B \right) \right\} = \frac{A_E}{B} \tag{40}$$

Of course, the graphical form of this expression is the same as the one sketched above for p_C and ρ_L.

The form of the relation among the three variables — ρ_L, ρ_E and p_C — is also easily identified. Begin by eliminating all fractions in Eq. (37)

$$\left(Bp_C - A_O \right) \rho_L \rho_E = A_L \rho_E + A_E \rho_L \tag{41}$$

Now define two new variables,

$$a_L \equiv A_L/(Bp_C - A_O)$$

$$a_E \equiv A_E/(Bp_C - A_O)$$

and make the indicated substitutions in Eq. (41)

$$\rho_L \rho_E = a_L \rho_E + a_E \rho_L \tag{42}$$

Upon adding the quantity $a_E a_L$ to both sides of Eq. (42) and rearranging, one obtains the expression

$$(\rho_L - a_L)(\rho_E - a_E) = a_L a_E \tag{43}$$

which is a hyperbola asymptotic to the lines $\rho_L = a_L$ and $\rho_E = a_E$, with $a_L a_E$ being the hyperbolic constant. Note that the asymptotes change position with changes in price.

Another way to look at the price-productivity relationship is to determine the trade-off ratio between ρ_L and ρ_E at constant p_C. This is easily done by taking the differential of Eq. (37) and setting dp_C to zero. Thus,

$$B dp_C = -\frac{A_L}{\rho_L^2} d\rho_L - \frac{A_E}{\rho_E^2} d\rho_E = 0$$

hence

$$\frac{(d\rho_L/\rho_L)}{(d\rho_E/\rho_E)} = -\left(\frac{\rho_L}{\rho_E}\right)\left(\frac{A_E}{A_L}\right) \tag{44}$$

In words, the ratio of the percentage changes in productivity at constant price varies directly with the ratio of the productivities themselves, and inversely with the ratio of the productivity coefficients.

Price Sensitivity to Small Changes

This brief investigation of model properties concludes with an analysis of the sensitivity of price to small changes in individual variables. The straightforward definition of sensitivity is the partial derivative of price with respect to the variable in question. Partial derivatives for important variables in the price expression are presented in Exhibit 3.

Note that two very important sensitivities are omitted from Exhibit 3 and from the numerical example in the next section. They are the price

Exhibit 3

The Sensitivity of Price to Changes in Individual Variables

Variable Name	Symbol	Price Sensitivity
Labor productivity	ρ_L	$-A_L/\rho_L^2 B$
Capital productivity	ρ_E	$-A_E/\rho_E^2 B$
Debris Fraction in Raw Coal	α_R	$\frac{1}{(1-\alpha_R)^2(1-\alpha_p)}\left[\frac{A_L}{\rho_L}+\frac{A_E}{\rho_E}+A_O\right]-\left(\frac{Fc_{WT}}{(1-\alpha_R)}\right)$
Preparation Losses	α_p	$\frac{1}{(1-\alpha_R)(1-\alpha_p)^2}\left[\frac{A_L}{\rho_L}+\frac{A_E}{\rho_E}+A_O\right]-\left(\frac{Fc_{WT}}{(1-\alpha_p)}\right)$
Average Daily Wage	$\bar{w}_T$	$Fa_L/B\rho_L$
Welfare Payment (per man-hour)	c_{WH}	$F\,h_S\,(M_H/M_T)/B\rho_L$
Hours worked per shift	h_S	$F\,c_{WH}\,(M_H/M_T)/B\rho_L$
Hourly Employee Fraction	(M_H/M_T)	$Fc_{WH}h_S/B\rho_L$
Royalty Fraction of Sales	μ	$\frac{1}{B}\left(\frac{A_L}{\rho_L}+\frac{A_E}{\rho_E}+A_O\right)$
Local Tax Fraction of Sales	σ	$\frac{1}{B}\left(\frac{A_L}{\rho_L}+\frac{A_E}{\rho_E}+A_O\right)$
Cost of Operating Supplies (per ton)	c_S	Fa_S/B
Water and Power Cost (per ton)	c_U	F/B
Insurance Fraction of Capital Cost	c_{INS}	$\frac{F}{B}\left(\frac{1}{\rho_E(1+\lambda_{EO}+\beta_E)}\right)$
Depreciation Factor	β_D	$-\frac{F}{B}\left(\frac{\tau}{1-\tau}\right)\left(\frac{1-w_c}{\rho_E(1+\lambda_{EO}+\beta_E)}+k_D\right)$

Exhibit 3 (contd)

Variable Name	Symbol	Price Sensitivity
Deferred Investment as a Fraction of Total Capital Investment	β_E	$\frac{F\gamma}{1-\tau}\left(\frac{K_{EO}}{(1-\alpha_A)(1-\alpha_U)}\right)\Big/ BV_R$
Interest During Construction Factor	λ_{EO}	$\frac{F\gamma}{1-\tau}\left(\frac{K_{EO}}{(1-\alpha_A)(1-\alpha_U)}\right)\Big/ BV_R$
Anticipated Work Stoppages	α_A	$\frac{F}{(1-\alpha_A)^2(1-\alpha_U)}\left(c_{INS}K_{EO} - \frac{\beta_D\tau}{1-\tau}K_{EO}(1-w_c) + \frac{\gamma}{1-\tau}(1+\lambda_{EO}+\beta_E)K_{EO}\right)\Big/ BV_R$
Unanticipated Work Stoppages	α_U	$-\frac{F}{(1-\alpha_A)(1-\alpha_U)^2}\left(c_{INS}K_{EO} - \frac{\beta_D\tau}{1-\tau}K_{EO}(1-w_c) + \frac{\gamma}{1-\tau}(1+\lambda_{EO}+\beta_E)K_{EO}\right)\Big/ BV_R$
Capital Recovery Factor	γ	$\frac{F}{B}\left(\frac{1}{1-\tau}\right)\left(\frac{1}{\rho_E} + \frac{p_A T}{v_S \eta}(1+r)^n + \hat{k}_D\right)$
Acreage Cost	p_A	$\frac{F}{B}\left(\frac{\gamma}{1-\tau}\right)\left(\frac{T}{v_S \eta}(1+r)^n\right)$
Seam Density	v_S	$\frac{-F}{B}\left(\frac{\gamma}{1-\tau}\right)\left(\frac{p_A T}{(v_S)^2 \eta}(1+r)^n\right)$
Recovery Ratio	η	$\frac{-F}{B}\left(\frac{\gamma}{1-\tau}\right)\left(\frac{p_A T}{v_S \eta^2}(1+r)^n\right)$
Development Cost Per Ton	k_D	$\frac{F}{B}\left(-\frac{\beta_D\tau}{1-\tau} + \frac{\gamma}{1-\tau}(d_{INT})\right)$

sensitivity to mine life and rate of return, both of which are difficult to calculate because of their complex relationship to price.

For a number of reasons, it is often convenient to modify the above definition of sensitivity as a partial derivative. Foremost among these reasons is the difficulty in comparing rates of change when the variables undergoing change differ in scale by orders of magnitude. For example, it is not very useful to compare a one dollar change in the cost of operating supplies (nominal value, \$2.57/ton) with a dollar change in the price of land option (nominal value, \$50/acre). To circumvent this problem, it is common practice to measure sensitivity in terms of a partial elasticity which calculates the percentage change in price due to a one percent change in a particular variable. This partial elasticity is written as follows:

$$E_{P_C,X} = \left(\frac{\delta P_C}{P_C}\right) \Big/ \left(\frac{\delta x}{x}\right) = \left(\frac{\delta P_C}{\delta x}\right)\left(\frac{x}{P_C}\right) \qquad (45)$$

Thus, to obtain a partial price elasticity for any of the variables appearing in Exhibit 3, one merely multiplies the partial derivative by the ratio of the nominal value of the variable to the nominal price.

6.0 ILLUSTRATIVE APPLICATIONS OF THE MODEL

This section exercises the life-cycle model developed above, using data representative of room and pillar technology. In all, three separate analyses are presented. The first is a straightforward calculation of selling price, done to check model assumptions and structure. The second analysis is a parametric study of the impact of labor productivity upon selling price for productivity values ranging between 5 and 40 tons/man-shift. In practice, it is impossible to vary just one parameter while holding all others constant, i.e., improved productivity is generally achieved at some cost. Nonetheless, the curve resulting from this parameter study fairly accurately mirrors the production cost increase recently experienced by the industry as productivity declined from about 20 tons/man-shift to the current figure of 9 - 10 tons. The third analysis examines the sensitivity of price to small changes in selected technological and financial variables. Like the two other numerical applications, this study of price sensitivity uses the representative room and pillar mine described below. Although only illustrative and suggestive in nature, the sensitivities thus derived lead to some interesting speculations about potentially fruitful directions in mining research and development.

Description of the Representative Mine

The mine chosen for numerical analysis, is a 2 million ton-year shaft mine, using continuous mining equipment to work a 72-inch bituminous seam under 800 feet of overburden. Thus, the mine is representative of conditions encountered in many parts of both the Interior and Appalachian Provinces.

Basic data for this case were prepared in a format similar to the one used in the USBM model mine studies (see Appendix C for details). However, there are several important differences from these studies:

- Shaft entry is assumed here, whereas, drift entry was employed in previous Bureau studies. As a result of the extended period required for construction and initial development, three years was assumed necessary to bring all sections up to capacity production.
- Mineral rights are acquired via the increasingly popular option-lease arrangement, with an initial payment of $50/acre plus a

production royalty of 5% of gross sales. Recent Bureau studies assumed outright purchase at a price of $2500/acre.

- Beneficiation was handled via two sub-cases: 1) a mine producing unwashed or run-of-mine coal, and 2) an operation which provides for a moderate level of washing, resulting in a net tonnage loss of 20% through the preparation plant. Mines in previous Bureau studies produced unwashed coal.
- Like the USBM model mines, this example portrays production as if it were constant from the end of initial development to mine close. However, Appendix A shows how to transform a forecast of varying annual capability to the constant capacity required by the model.

Finally, labor and equipment costs are current as of mid-1977. For convenience of reference, key numerical assumptions are tabulated in Exhibit 4.

Calculation of Selling Price

Calculation of an annualized selling price requires some recasting of the basic costing information tabulated in Appendix C. The resulting inputs to the life-cycle formula are summarized in Exhibit 5, which includes units and algebraic symbols. Note that non-zero values have been specified for all quantities except α_A and α_U, the explicit availability adjustments to capital productivity. Thus, for this example, one must interpret ρ_E as a baseline value of productivity which incorporates a reasonable allowance for downtime without an identification of the source, in accord with the published model mine studies. Specification of numerical values for α_A and α_U awaits performance data on operating sections — data which are not generally available. Of course, assigning zero values to α_A and α_U prevents one from studying the price sensitivity of two very important determinants of production cost.

Exhibit 6 summarizes the calculation of the annualized selling price for both unwashed and washed coal. Although the cost data are purely hypothetical and not specifically related to any working mine, it is encouraging that the selling prices fall within the range of current price quotes F.O.B. mine mouth. Exhibit 7 compares model-derived price with aggregate

Exhibit 4

Summary Description of the Representative Mine

Annual Capability:	1.98 million tons of raw coal, or 1.58 million tons of washed coal.
Seam Thickness:	72 inches.
Overburden:	800 feet.
Coal Density:	80 lbs/ft^3.
Acquisition of Mineral Rights:	Option-lease @ $50/acre, plus 5% of gross sales.
Recovery Factor:	57%.
Extraction Technology:	10 continuous miner sections, each producing 300 tons/shift, three shifts per day, 220 days per year.
Initial Development:	3 shafts, requiring 3 years between beginning of construction and attainment of capacity production.
Preparation:	Assumes storage silos, a railroad loop and a plant employing heavy media jigs, sized for 650 tons/hour; tonnage loss through the plant is 20%.
Rehabilitation of Site:	Not provided for.
Mine Lifetime:	20 years.
Depreciation Method:	Straight-line.
Required Return on Investment:	15%, with 100% equity assumed.

Exhibit 5

Input Data for Computing the Annualized Selling Price of Coal for a 1.98 Million tons/year Shaft Mine in 72" Coal, With and Without Preparation (See Appendix C for detailed costing)

Input Parameter	Symbol	Units	Parameter Value: Without Preparation	Parameter Value: With Preparation
General Descriptors:				
Return on Investment	r	–	0.15	
Mine Life (after capacity production is reached)	T	years	20	
Initial Development Period	n	years	3	
Seam Density	v_S	tons/acre	10,800	
Coal Recovery Factor	η	–	0.57	
Desired Annual Raw Tonnage	V_R	million tons/yr	1.98	
Fraction of Tonnage Lost in Washing	$(1-\sigma_R)(1-\sigma_P)$	–		0.20
Capital Items:				
Initial Investment (gross estimate less interest)	$K_{EO}/(1-\sigma_A)(1-\sigma_U)$	\$	41,330,000	51,483,000
Interest During Construction Fraction	λ_{EO}	–	0.133	0.132
Present Value of Initial Investment	$\hat{K}_{EO}$	\$	46,826,900	58,278,800
Present Value of Deferred Investment	$\hat{K}_{EF}$	\$	9,668,200	9,892,500
Deferred Investment Ratio $(\hat{K}_{EF}/\hat{K}_{EO})$	β_E	–	0.234	0.192
Capital Productivity $V_R/(\hat{K}_{EO}+\hat{K}_{EF})$	ρ_E	ann. ton/\$	0.035	0.029
Lost Productivity Fraction, Anticipated	σ_A	–	0	
Lost Productivity Fraction, Unanticipated	σ_U	–	0	
Present Value of Land Cost (@ \$50/acre for an option)	$\hat{K}_A$	\$	425,400	
Present Value of Net Development Expenditure	$\hat{K}_D$	\$	-1,204,800	
Present Value of Aggregate Capital Investment	$\hat{E}_K$	\$	55,715,700	67,391,900
Working Capital	W_c	\$	5,529,000	5,949,000
Working Capital as a percentage of $K_{EO}/(1-\sigma_A)(1-\sigma_U)$	w_c		0.1338	0.1156
Annual Operating Expense:				
Depreciation Amount	D	\$	3,701,200	4,187,900
Depreciation Fraction	β_D	–	0.107	0.094
Average Wage Per Shift (based on total work force)	$\bar{w}_T$	\$/man-shift	73.08	72.82
Labor Cost Multiplier (to adjust for payroll overhead)	a_L	–	1.55	

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Exhibit 5 (contd)

Input Parameter	Symbol	Units	Parameter Value: Without Preparation		With Preparation
Annual Operating Expense: (contd)					
Total Labor Cost	C_L	$	7,460,000		7,833,500
Labor Productivity (based on raw tonnage)	P_L	tons/man-shift	19,397		18.405
Operating Supplies Multiplier (to adjust for indirect costs)	a_S	—		1.15	
Operating Supplies Cost Per Ton (of raw coal)	c_S	$/ton	2.57		2.83
Total Cost of Operating Supplies	C_S	$	5,081,600		5,611,100
Water and Power Cost Per Ton (of raw coal)	c_U	$/ton	0.628		0.670
Total Water and Power Cost	C_U	$	1,243,400		1,326,700
Union Welfare Expense Per Ton (of coal sold)	c_{WT}	$/ton		0.820	
Union Welfare Expense Per Hour	c_{WH}	$/man-hour		1.540	
Hours Worked Per Shift	h_S	hours		8	
Number of Hourly Workers (3 shifts)	M_H	men	405		424
Total Workforce (3 shifts)	M_T	men	464		489
Total Union Welfare Expense	C_W	$	2,721,300		2,448,100
Insurance Multiplier (applied to insurance base)	c_{INS}	—		0.010	
Insurance Expense	C_{INS}	$	413,300		514,800
Total Operating Costs, Less Local Taxes, Royalties and Depreciation	C_P'	$	21,784,800		22,884,300
Financial Factors:					
Combined Federal and State Income Tax Rate	τ	—		0.5	
Local Tax Rate (as a fraction of gross sales)	σ	—	0.019		0.022
Depletion Allowance (as a fraction of gross sales)	δ	—		0.1	
Production Royalty (as a fraction of gross sales)	μ	—		0.05	
Capital Recovery Factor (to return 15% over 20 years)	γ	—		0.160	
Financial Multiplier	F	—	0.971		0.973

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Exhibit 6

Calculation of Annualized Selling Price
for a 1.98 Million Tons/Year Shaft Mine in 72" Coal
(See Exhibit 5 for inputs to selling price)

Formula for Selling Price:

$$P_C = \frac{F}{\left(1 - \alpha_R\right)\left(1 - \alpha_P\right) V_R} \left\{ C'_P + \frac{\gamma}{1 - \tau} \hat{E}_K - \frac{\tau}{1 - \tau} D \right\}$$

Without Preparation:

$$P_C = \frac{0.97087}{(1)(1)(1,980,000)} \left\{ 21,784,800 + \frac{0.15976}{1 - 0.5} (55,715,700) \right.$$

$$\left. - \frac{0.5}{1 - 0.5} (3,701,200) \right\}$$

$$= \$17.60/\text{ton}$$

With Preparation:

$$P_C = \frac{0.97276}{(1 - 0.20)(1,980,000)} \left\{ 22,884,300 + \frac{0.15976}{1 - 0.5} (67,391,900) \right.$$

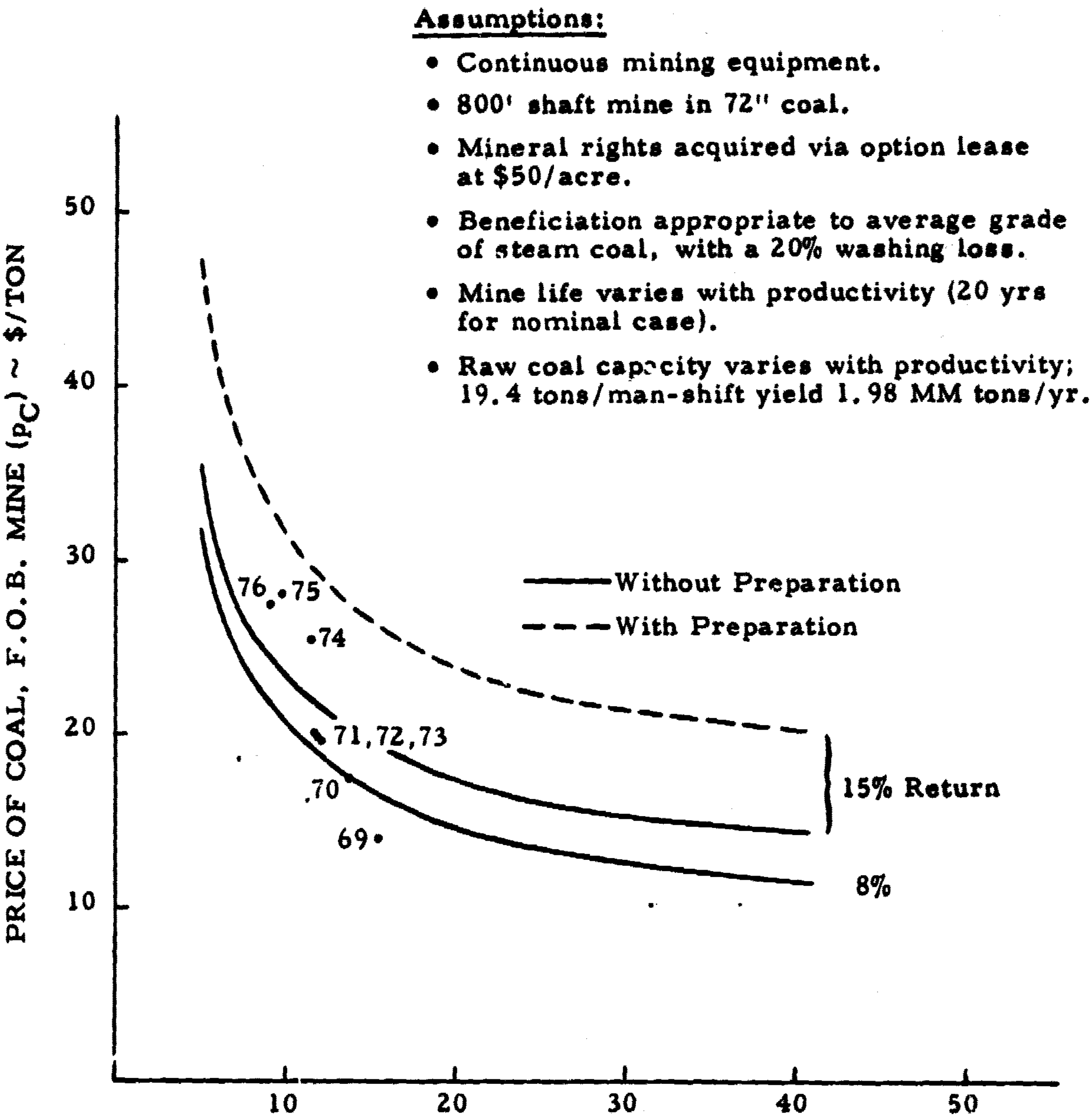
$$\left. - \frac{0.5}{1 - 0.5} (4,187,900) \right\}$$

$$= \$24.71/\text{ton}$$

Exhibit 7

Annualized Price vs. Labor Productivity, with Fixed Equipment and Manning: Model Compared with Production Data (1969 - 1976)

Source of Production Data: U.S. Bureau of Mines annual data on underground production, adjusted for inflation by the implicit price deflator for mining (Survey of Current Business, July 1977)



statistics on underground bituminous production for labor productivities between 5 and 40 tons/man-shift and a return of both 15 and 8 percent. For the higher return, model selling price is shown for both washed and unwashed coal. The life cycle price expression is not intended to explain aggregate industry behavior; nonetheless, it appears to capture fairly well the combined impacts of worsening labor productivity and higher returns experienced by coal operators during the past few years.[12] Exhibit 8 tabulates the annual production data plotted in Exhibit 7, with price adjusted to 1976 constant dollars, using the implicit price deflator for the mining industry.

Sensitivity of Price to Technological and Financial Factors

Exhibit 9 tabulates the price elasticities for small changes in selected technological and financial factors. Small changes mean alterations in the nominal values of one percent or so. As explained above in Section 5.0, a partial price elasticity is defined as the ratio of the percentage change in price to a one percent change in the quantity of interest. Exhibit 9 reveals that price is quite sensitive to the following seven factors:

- Capital Recovery Factor,
- Capital Productivity,
- Labor Productivity,
- Average Wage per Shift,
- Fraction of Raw Tonnage Lost,
- Cost of Operating Supplies, and
- Depreciation Factor.

Each of these factors displays a price sensitivity of 0.10 or more, meaning that a 1.0 percent change in the parameter of interest leads to at least a 0.1 percent change in price.

Taken as an ensemble, these results indicate that the price of coal is most sensitive to changes in capital expenditures. Influencing the capital recovery factor is not within the scope of an R&D program, but impacting capital productivity is. An increase in capital productivity can be achieved by either an increase in production per dollar invested or a decrease in capital outlays per unit of production. Realization of increased capital productivity leads in two directions: 1) an increase in the sustained production rate while

Exhibit 8

Empirical Data for Underground Coal Production

Year	Coal Price (current dollars) F.O.B. Mine*	Implicit Price Deflator for Mining Industry**	Coal Price Adjusted to 1976 Constant Dollars	Productivity (tons/man-shift)	Coal Production (tons $\times 10^6$)
1969	5.62	86.4	14.00	15.61	348
1970	7.40	91.5	17.40	13.76	339
1971	8.87	91.8	20.79	12.03	276
1972	9.70	100.0	20.87	11.91	304
1973	10.84	111.6	20.90	11.66	300
1974	19.86	166.7	25.50	11.31	277
1975	26.28	201.0	27.94	9.54	293
1976	26.56	215.2	26.56	9.10	295

*From U.S. Bureau of Mines, annual statistics on underground coal production.

**From Survey of Current Business (July 1977). Base year 1972 = 100.

Exhibit 9

Price Sensitivities for the Representative Mine

Variable Name	Symbol	Price Elasticity	
		Without Preparation	With Preparation
Capital Recovery Factor	γ	0.497	0.536
Capital Productivity	ρ_E	-0.408	-0.449
Labor Productivity	ρ_L	-0.353	-0.330
Average Wage Per Shift	$\overline{w}_T$	0.322	0.302
Total Loss Fraction (Rock, Dirt, Washing)	α_R, α_P	0	0.242
Operating Supplies Cost Per Ton	c_S	0.163	0.160
Depreciation Factor	β_D	-0.103	-0.104
Deferred Investment as a Fraction of Total Capital Investment	β_E	0.086	0.079
Royalty Fraction of Sales	μ	0.050	0.050
Interest During Construction Factor	λ_{EO}	0.049	0.054
Water and Power Cost Per Ton	c_U	0.035	0.033
Welfare Payment Per Man-Hour	c_{WH}	0.031	0.029
Hours Worked Per Shift	h_S	0.031	0.029
Hourly Employee Fraction	M_H/M_T	0.031	0.029
Local Tax Fraction of Sales	σ	0.019	0.022
Insurance Fraction of Capital Cost	c_{INS}	0.012	0.013
Development Cost Per Ton	k_D	-0.007	-0.007
Acreage Cost	p_A	-0.004	-0.003
Seam Density	v_S	-0.004	-0.003
Recovery Ratio	η	-0.004	0.003

the system is up cutting coal; 2) an increase in system availability or alternatively, a reduction in system downtime; and 3) a reduction in the cost of manufacturing mining equipment. Two of these three directions define traditional areas of activity in mining R&D.

Labor productivity and average wage rate are next in importance in their impact on coal price. Increased labor productivity and decreased wages per shift are desirable, but they are typically achieved through increased capital expenditure (e.g., automation). Since it has been shown that capital productivity (or capital intensiveness) has a larger impact on price than labor productivity for this example, careful capital-labor tradeoffs must be made to effect a price reduction. However, some increase in labor productivity may be achieved simply by reducing equipment downtime via designs which are less susceptible to failure and less demanding of maintenance.

Tonnage losses during the cleaning process are the next most sensitive area, given the nominal assumption of a 20 percent loss from portal to the loading dock. However, preliminary calculations made in the course of preparing the numerical example indicate a strong sensitivity to the particular value chosen for loss. In fact, if losses are assumed to run 5 to 6 percentage points higher, price sensitivity to this factor is comparable to the sensitivity of labor productivity. In any event, thinner seams and other worsening geological conditions forecast for the future suggest a need to remove an increasing amount of rock and dirt from the coal to maintain comparable quality. Reducing the percentage of washing losses would be attractive at first glance, but the increased capital cost of this capability must be evaluated carefully. Current efforts to increase preparation plant capacity to service multiple mines do not address the above problem but rather look for cost reductions per ton via economies of scale.

Operating supplies and depreciation complete the list of factors for which price is quite sensitive. Reduction in operating supplies, although highly desirable, implies a much more cost effective method of roof support, a development not easy to visualize given technology which requires in-seam operation of equipment. Depreciation is similarly viewed as a secondary target of effort, however, more rugged, longer lived equipment — a natural result of improved equipment availability — would have a favorable impact here also. The remaining variables also affect the price of coal to a measurable extent, but have considerably less impact than the seven factors discussed above.

In conclusion, it appears that improvement in labor productivity via automation, or mechanization of equipment is one of several potential targets in a program of mining system research and development. Equally attractive are initiatives to improve capital productivity — i.e., increase equipment availability, enhance throughput, and reduce capital cost per ton.

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APPENDIX A

ANNUALIZED PRICE PER TON OF COAL GIVEN NON-UNIFORM ANNUAL PRODUCTION LEVELS

APPENDIX A

ANNUALIZED PRICE PER TON OF COAL GIVEN NON-UNIFORM ANNUAL PRODUCTION LEVELS*

The annual amount of coal extracted from a mine is not necessarily constant over its useful lifetime. There can be differing yearly production levels both between competing technologies and within a specific mining technique. For a given technology, there may be variations in output once capacity production is reached** for a number of reasons including:

1) Geological conditions which change over time. As the mine continues to operate, the working sections get further from the portal, additional haulage is required, coal seams may thin out or become more difficult to mine, etc.

2) Additional safety procedures required in the mine.

3) Long stoppages for unscheduled maintenance during otherwise available production hours.

4) Lost production due to strikes.

The first two describe the general climate of declining labor productivity and contribute to a reduction in mean coal output over time. A trend line of this phenomenon, over the mine life would reflect a non-increasing output schedule. Points 3) and 4) typify factors causing random fluctuations in annual output. Constant levels of work stoppages are accounted for by α_A and α_U in the cost side of the model and presumably in the expected annual output; however,

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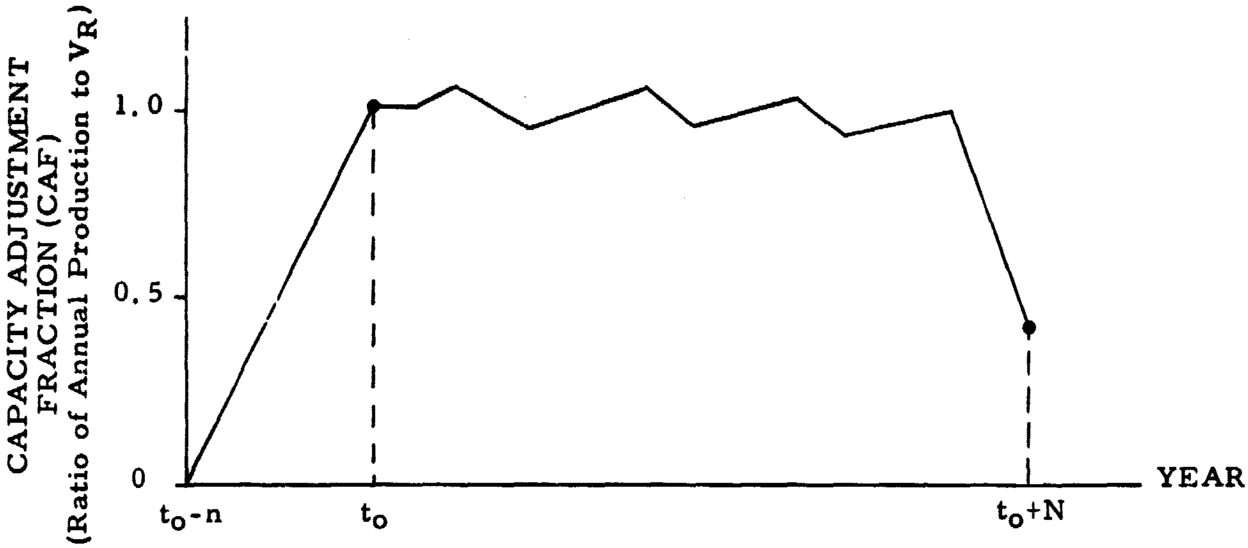
*This appendix is based on an analysis by C. S. Borden, titled, "Generalization of Required Revenues per Unit Output for Non-Uniform Output Flows," Interoffice Memorandum 311.5-5, Jet Propulsion Laboratory, August 25, 1976.

**Development year production amounts are already incorporated in the model as a part of development costs which are net of coal sold during this period.

Note that operating costs based upon annual production (tonnage) must be adjusted yearly to accurately reflect the amount of coal actually extracted. All operating costs can then be summed in a present value sense over the mine lifetime and annualized (by multiplying by the capital recovery factor γ) as are the capital costs. Annualized operating costs then replace the annual operating cost C_P.

there are also yearly variations which occur. Analytically this problem can be handled in a very straightforward manner. The main text defines V_R as the constant amount of coal removed from the mine annually. Suppose now that annual production quantities vary. A ratio of this actual annual production amount to the assumed nominal capacity production level, V_R, is called the capacity adjustment fraction CAF, where CAF > 0. A distribution of hypothetical yearly capacity adjustme ts are displayed in Exhibit A-1.



Time coordinates on the x-axis are:

t_o-n: Year when coal production begins during development phase

t_o: Year when 100% capacity production is achieved

t_o+N: Mine closing.

Exhibit A-1.

This hypothetical shape reflects the assumptions of linear buildup to capacity production, nominal capacity production for a brief period, a varying production around the nominal value for a few years, and then degradation in output to 40% of nominal capacity by year $t_o + N$. Degradation is mostly attributable to reducing the number of sections as mining operations come to an end. Random work stoppages due to machine failures or other reasons would further affect CAF.

To incorporate these effects into the model developed in Section 4.0, the annualized price of coal p_C is defined as:

$$p_C \equiv \frac{S}{V_C} = \frac{S}{V_R\left(1-\alpha_R\right)\left(1-\alpha_P\right)\,CAF} \tag{A-1}$$

where CAF = 1 implies a constant capacity adjustment fraction, as used in the main text. In the case where V_C (and V_R) is constant the present value of required revenues ($p_C \times V_C = S$) over the mine lifetime is computed as:

$$PV(S) = \sum_{i=1}^{N} \frac{p_C \times V_C}{(1+r)^i} \tag{A-2}$$

Year to year changes in output imply a series of different yearly values for CAF. Annual variations in CAF can be handled by recalling that p_C is a constant annualized figure, and rearranging Eq. (A-2) so that

$$PV(S) = p_C \sum_{i=1}^{N} \frac{V_R\left(1-\alpha_R\right)\left(1-\alpha_P\right)\,CAF_i}{(1+r)^i}$$

or

$$p_C = \frac{PV(S)}{V_R\left(1-\alpha_R\right)\left(1-\alpha_P\right)\displaystyle\sum_{i=1}^{N}\frac{CAF_i}{(1+r)^i}} \tag{A-3}$$

If both numerator and denominator are multiplied by the capital recovery factor Y, the numerator becomes the original annualized sales amount, S, and the denominator, the annual coal production times the annualized price per ton adjustment:

$$p_C = \frac{S}{V_R\left(1-\alpha_R\right)\left(1-\alpha_P\right)\gamma \sum_{i=1}^{N} \frac{CAF_i}{(1+r)^i}} \quad \text{(A-4)}$$

Note that the cash flow amount (p_C × actual annual coal production) is the quantity being discounted, not coal output.

Equation (A-4) describes the general case for calculating p_C. In the special case where output is constant and $CAF_i = 1$ for all i, the summation in the demoninator of Eq. (A-4) simplifies to

$$\sum_{i=1}^{N} \frac{1}{(1+r)^i} = \frac{1}{\gamma}$$

and the expression for the annualized price of coal reduces to the fixed capacity form:

$$p_C = \frac{S}{V_R\left(1-\alpha_R\right)\left(1-\alpha_P\right)} = \frac{S}{V_C}$$

APPENDIX B

INTEREST DURING CONSTRUCTION

APPENDIX B

INTEREST DURING CONSTRUCTION

The procedure for calculating interest during construction is presented in this appendix. Compound interest can be a significant amount in the total cost of a mining operation due to expenditures on capital items prior to the year of capacity production. Typical reasons for these early payments are progress payments on equipment, lead time for plant construction, and extended deliveries of support equipment. To determine the interest during construction for these capital expenditures, three inputs are needed: the amount of the outlays, their timing, and the discount rate.

For simplicity, interest during construction is modeled as a multiplier, λ_{EO}, which is applied to historical aggregate investment outlays. Assuming $\alpha_A = \alpha_U = 0$ from Eq. (15b),

$$\hat{K}_{EO} = \left(1 + \lambda_{EO}\right) K_{EO} \tag{B-1}$$

The data required for computing interest during construction are usually presented in the form of a yearly schedule of investment outlays. Thus, the first step in computing λ_{EO} is to convert this schedule of dollar outlays into fractional amounts. Define f_i as the fraction of initial aggregate capital expenditure which occurs in year i of the n-year pre-capacity period, such that

$$f_i \equiv \frac{\text{year i expenditure}}{K_{EO}}$$

where

$$\sum_{i=-n}^{0} f_i = 1$$

Since the dollar amount spent in year i is $f_i \times K_{EO}$, the present value of initial capital expenditure becomes

$$\hat{K}_{EO} = K_{EO} \sum_{i=-n}^{0} \frac{f_i}{(1+r)^i} \tag{B-2}$$

Combining Eq. (B-1) and (B-2) yields

$$\left(1+\lambda_{EO}\right) K_{EO} = K_{EO} \sum_{i=-n}^{0} \frac{f_i}{(1+r)^i}$$

Solving for the interest fraction, one obtains the required result:

$$\lambda_{EO} = \left(\sum_{i=-n}^{0} \frac{f_i}{(1+r)^i} \right) - 1 \tag{B-3}$$

Note that Eq. (B-3) is a general description of interest during construction, which can be used with any expenditure schedule and discount rate. To illustrate this process consider the schedule of expenditures portrayed in Exhibit B-1.

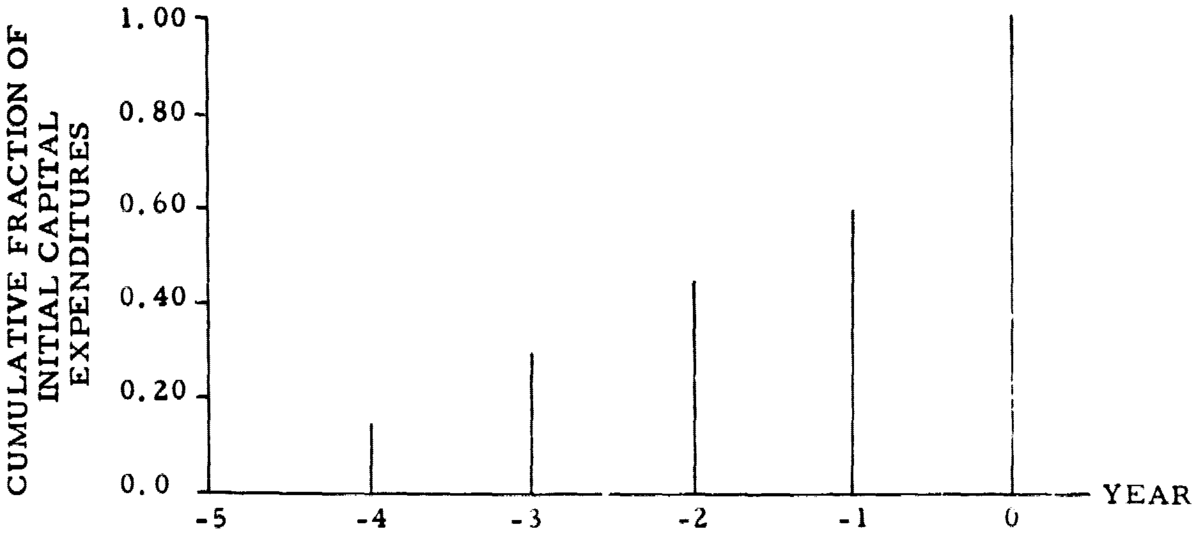


Exhibit B-1.

The implied expenditure in year i is the cumulative fraction for year i minus the value for year i-1, times K_{EO}. The hypothetical schedule of Exhibit B-1 implies the following series of values for f_i:

$$f_i(-4, -3, -2, -1, 0) = (0.15, 0.15, 0.15, 0.15, 0.40)$$

Once a compound interest rate is specified, λ_{EO} is easily computed from Eq. (B-3). Thus, for an interest rate of 15%,

$$\lambda_{EO} = \frac{0.15}{(1.15)^{-4}} + \frac{0.15}{(1.15)^{-3}} + \frac{0.15}{(1.15)^{-2}} + \frac{0.15}{(1.15)^{-1}} + \frac{0.40}{(1.15)^{0}} - 1$$

$$\lambda_{EO} = 0.26 .$$

APPENDIX C

DETAILED COSTS FOR ILLUSTRATIVE MINE

LIST OF ASSUMPTIONS

1. All equipment, materials, and labor are updated to the third quarter of 1977.

2. Ten continuous miner sections working 3 shifts per day, 220 days per year will produce 1.98 million tons per year, assuming each section averages 300 tons per shift. Two spare mining units are included in the estimate.

3. Three 800-foot shafts are included in the estimate, one for ventilation, one for service, and one for coal removal using a skip hoist. Four 800-foot air shafts are added over the life of the mine.

4. The coal preparation plant is designed for a feed of 650 tons per hour of run-of-mine coal. The 3/8-inch+ is sent to the heavy media and 3/8 × 0 is sent to the deister tables. Tonnage loss through the preparation plant is assumed to be 20%.

5. A three-year period for construction and initial development is assumed, with the mine reaching full capacity at the end of the third year. Raw coal produced during development is presumed salable at $15/ton.

6. Mineral rights are acquired via an option-lease arrangement, with the initial option costing $50/acre and a royalty of 5% of sales paid to the land owner.

7. A mine life of 20 years is assumed, with the operator requiring a 15% return on total invested capital. This figure of 15% is used to compute interest during construction as well.

Table C-1. Capital Investment Summary, 1.98 MM tons/yr Mine.

A. NO PREPARATION:

Item	Quantity	Total Cost
Continuous miner	12	$ 4,248,000
Loading machine	12	1,308,000
Shuttle car	24	2,712,000
Roof bolter	14	1,134,000
Ratio feeder	12	828,000
Auxiliary fan	14	84,000
Mantrip jeep	9	243,000
Mechanic jeep	4	84,000
Personnel jeep	6	114,000
Trickle rock duster	14	91,000
Triple duty rock duster	12	708,000
Supply motor	5	240,000
Supply car	50	250,000
42-inch rope-type mainline belt conveyor	9,000 ft	1,090,000
36-inch rope-type belt conveyor	26,560 ft	2,776,000
Mainline belt power center (300 kV-A)	5	240,000
Section belt power center (150 kV-A)	7	133,000
Section power center (1,000 kV-A)	12	528,000
Section rectifier (200 kW)	12	48,000
Section switch house	12	168,000
Sectionalizing switch house	10	140,000
HV cable (300 MCM AL)	13,100 ft	199,000
PLM coupler	17	24,000
Section cable and coupler	—	142,000
Rectifier for track haulage	2	76,000
Trolley wire	31,300 ft	149,000
Track (60-lb)	31,300 ft	476,000
Fresh water line	31,300 ft	178,000
Pumps and lines	—	57,000
Telephone (page phones)	—	22,000
Conveyor fire protection	—	43,000
Automatic controls and alarms	—	101,000
Scoop tractor	12	570,000
Battery charger	12	48,000
All service mask	36	7,000
Breathing apparatus	24	29,000
Self rescuer	450	27,000
Stretcher set	12	3,000
Safety light	200	13,000
Methanometer	200	117,000
Fire chemical car	8	48,000
Lamp (including accessories)	450	36,000
Dust sampler	35	21,000

Table C-1. Capital Investment Summary, 1.98 MM tons/yr Mine. (contd)

A. NO PREPARATION (contd)

Item	Quantity	Total Cost
Site preparation	—	$ 55,000
Ventilation fan, dual (initial)	—	174,000
Bulk rock dust facility	1	38,000
Substation and distribution	1	123,000
Bathhouse, office, and lamp house	1	529,000
Shop and warehouse	1	354,000
Powder and cap house	1	11,000
Front-end load	1	98,000
Forklift	1	38,000
Bulldozer	1	153,000
Utility truck	1	8,000
Pickup truck	1	6,000
Oil storage	1	27,000
Water tank	1	27,000
Supply yard	1	27,000
Mine drainage treatment plant	—	57,000
Exploration	—	164,000
Landscaping (around physical plant)		16,000
Road and parking lot (assume 1/2-mile road and parking lot)		82,000
Total direct		21,540,000
Field indirect		431,000
Total construction		21,971,000
Engineering		439,000
Overhead and Administration		1,120,000
		23,530,000
Contingency		3,530,000
		27,060,000
Fee		541,000
		27,601,000
Hoists and shafts (initial)		8,200,000
Interest on construction loan[1]		1,790,000
Gross estimate		37,591,000
C; tion-lease on land at $50 per acre		321,600
Net Estimate, No Preparation		$37,912,600

[1] This is an estimate of the interest pa d on a construction loan. However, this item is excluded from the initial investment outlays in the cash flow summary of Table C-9 because it is a part of "interest during construction," a quantity which is computed automatically in the discounting of investment expenditures prior to year 1.

Table C-1. Capital Investment Summary, 1.98 MM tons/yr Mine. (contd)

B. WITH PREPARATION:

Item	Quantity	Total Cost
Net estimate, no preparation		$37,912,600
Plus preparation plant		9,733,000
Net Estimate, With Preparation		$47,645,600

Table C-2. Manning, 1.98 MM tons/yr Mine.

A. NO PREPARATION:

Personnel	Total	Wages[1] Per Day	Cost Per Year
Underground:			
Continuous miner operator	30	$63.08	$ 426,536
Loading machine operator	30	59.84	405,152
Machine operator helper	30	59.84	405,152
Shuttle car operator	60	56.90	771,496
Roof bolter	60	63.08	853,072
Bratticeman	30	54.54	370,172
Utility man	30	56.90	385,748
Mechanic (section)	30	63.08	426,536
	300		4,043,864
Supply motorman	6	55.12	74,800
Beltman	18	54.54	222,103
Trackman	9	54.54	111,052
Wireman	9	54.54	111,052
Mason (precision)	12	56.90	154,299
Pumper	3	54.54	37,017
Utility crew	18	56.90	231,449
Roving mechanic	9	63.08	127,961
Fireboss (union)	3	63.08	42,654
	87		1,112,387
Outside:			
Hoistman	3	55.41	37,591
Lampman	3	52.78	35,856
Front-end loader operator	3	55.41	37,591
Shop mechanic	9	57.18	116,279
	18		227,317
Manpower allowance for sickness and accidents, personnel required for health and safety, and absenteeism (20% of underground and outside labor)			1,076,732
Total hourly personnel	405		6,460,300
Salary:			
Superintendent	1		33,000
General mine foreman	1		22,000
Assistant mine foreman	3		52,800
Section foreman	30		539,000
Maintenance superintendent	1		24,000
General shop foreman	1		16,700
Mine maintenance foreman	3		49,500
Chief mine engineer	1		24,800
Draftsman	1		10,100

A. NO PREPARATION: (contd)

Personnel	Total	Wages[1] Per Day	Cost Per Year
Salary: (contd)			
Survey crew	3		$ 32,700
Safety director	1		21,800
Safety inspector	3		48,000
Dust sampler	3		34,800
Office manager	1		17,400
Timekeeper and bookkeeper	1		12,100
Purchasing supervisor	1		17,400
Warehouseman	4		43,600
Total salaried personnel	59		999,700
Personnel Totals, No Preparation	464		$7,460,000

[1]Figures in this column are for the day shift. Shift differentials for other shifts are reflected in the cost per year.

B. WITH PREPARATION

Personnel	Total	Wages Per Day	Cost Per Year
Total hourly personnel, no preparation	405		$6,460,300
Plus preparation plant hourly	19		237,100
Plus allowance for sickness, accidents absenteeism, etc.			47,400
Total hourly personnel	424		6,744,800
Total salaried personnel, no preparation	59		999,700
Plus preparation plant salaried	6		89,000
Total salaried personnel	65		1,088,700
Personnel Totals, With Preparation	489		$7,833,500

Table C-3. Depreciation Schedule, 1.98-MM tons/yr Mine.

A. NO PREPARATION

Item	Straight-Line Depreciation, Years	Yearly Charge, Dollars
Road and parking lot	20	$ 4,100
Landscaping	20	800
Exploration	20	8,200
Mine drainage treatment plant	10	5,700
Supply yard	10	2,700
Water tank	10	2,700
Oil storage	10	2,700
Pickup truck	5	1,100
Utility truck	5	1,400
Bulldozer	10	15,300
Forklift	10	3,800
Front-end loader	10	8,700
Powder and cap house	10	1,100
Shop and warehouse	20	17,700
Bathhouse, office, and lamp house	20	26,500
Substation	20	6,200
Bulk rock dust facility	10	3,800
Ventilation fan	20	8,700
Site preparation	20	2,800
Coal mine safety equipment	5	89,000
Underground equipment	10	1,910,800
Interim equipment replacement	20	500,000
Subtotal		2,623,800
Hoist and shafts		684,800
Depreciation for field indirect, engineering, overhead and administration, contingency, fee, and interest during development	20	392,600
Total, No Preparation		$3,701,200

Table C-3. Depreciation Schedule, 1.98-MM tons/yr Mine. (contd)

B. WITH PREPARATION

Item	Straight-Line Depreciation, Years	Yearly Charge, Dollars
Total, no preparation		$3,701,200
Plus preparation plant, buildings and equipment	20	486,700
Total, With Preparation		$4,187,900

Table C-4. Power and Water Cost, 1.98 MM tons/yr Mine

A. NO PREPARATION

Number of Units	Operation	Hp per Unit	Hp, Total Load	Hr per Day, Full Load	kW Total Load	Total kWh Requirement
10	Continuous miner	600.0	6,000	15	4,476	67,140
10	Loading machine	160.0	1,600	15	1,194	17,910
20	Shuttle car	135.0	2,700	15	2,014	30,210
10	Roof bolter	50.0	500	18	373	6,714
10	Ratio feeder	125.0	1,250	15	933	13,995
10	Auxiliary fan	30.0	300	18	224	4,032
10	Mantrip jeep	15.0	150	6	112	672
4	Mechanic jeep	15.0	60	15	45	675
6	Personnel jeep	7.5	45	15	34	510
10	Rock duster	30.0	300	12	224	2,688
5	Supply motor	80.0	400	12	298	3,576
3	42-inch conveyor	125.0	375	15	280	4,200
2	36-inch conveyor	100.0	200	15	149	2,235
7	36-inch conveyor	50.0	350	15	461	3,915
1	Ventilation fan		500	24	373	8,952
	Hoist		1,500	15	1,119	16,785
	Extra for pumps, tools, lights, etc.		500	10	373	3,730
	Total, No Preparation					187,939 kWh

Table C-4. Power and Water Cost, 1.98 MM tons/yr Mine (contd)

B. WITH PREPARATION

Number of Units	Operation	Hp per Unit	Hp, Total Load	Hr per Day, Full Load	kW Total Load	Total kWh Requirement
	Total, no preparation					187,939
	Plus preparation plant		1,210	14	902	12,628
Total, With Preparation						200,567 kWh

Note:

Power: $0.03 × 187,939 × 220
= $1,240,400, without preparation

$0.03 × 200,567 × 220
= 1,323,700, with preparation.

Water: 3,000 gal per unit per shift at $0.15 per 1000 gallons
= 3,000 × 30 × 220 × 0.15 ÷ 1,000
= $3,000.

Table C-5. Estimated Annual Production Cost, 1.98 MM tons/yr Mine.

A. NO PREPARATION

	Annual Cost
Direct cost:	
Production:	
Labor	$ 4,712,800
Supervision	899,500
	5,612,300
Maintenance:	
Labor	670,800
Supervision	100,200
	771,000
Manpower allowance	1,076,700
Operating supplies:	
Mining machine parts	1,740,000
Lubrication and hydraulic oil	690,400
Roof bolts and timber	856,200
Rock dust	358,900
Ventilation	524,700
Bits	331,400
Cables	165,700
Miscellaneous	414,300
	5,081,600
Power	1,240,400
Water	3,000
Payroll overhead (40% of payroll)	2,984,000
Union welfare[1]	2,721,300
Indirect cost:	
15% of labor, supervision, and supplies	1,881,200
Fixed cost:	
Taxes and insurance, 3% of mine cost[2]	1,074,000
Depreciation	3,701,200
	4,775,200
Total, No Preparation	$26,146,700

[1]Effective Dec. 6, 1976, under the Bituminous Wage Agreement of 1974.

[2]This is equivalent to a local tax rate of 1.9% applied to annual sales of $34,180,000, plus an annual premium rate of 1% applied to an insurance base of $41,330,000.

Table C-5. Estimated Annual Production Cost, 1.98 MM tons/yr Mine. (contd)

B. WITH PREPARATION

	Annual Cost
Direct cost:	
Production:	
Labor	$ 4,879,900
Supervision	976,300
	5,856,200
Maintenance:	
Labor	740,800
Supervision	112,400
	853,200
Manpower allowance	1,124,100
Operating supplies:	
Mining machine parts	1,740,000
Lubrication and hydraulic oil	690,400
Roof bolts and timber	856,200
Rock dust	358,900
Ventilation	524,700
Bits	331,400
Cables	165,700
Miscellaneous	414,300
Preparation plant	529,500
	5,611,100
Power	1,323,700
Water	3,000
Payroll overhead (40% of payroll)	3,133,400
Union welfare[1]	2,448,100
Indirect cost: 15% of labor, supervision, and supplies	2,016,700
Fixed cost:	
Taxes and insurance, 3% of mine cost[2]	1,366,000
Depreciation	4,187,900
	5,553,900
Total, With Preparation	$27,923,400

[1]Effective Dec. 6, 1976, under the Bituminous Wage Agreement of 1974.

[2]This is equivalent to a local tax rate of 2.2% applied to annual sales of $38,282,400, plus an annual premium rate of 1% applied to an insurance base of $51,483,000.

Table C-6. Operating Cost During 12-month Build-Up to Capacity Production, 1.98 MM tons/yr Mine.[1]

A. NO PREPARATION:

Item	Amount
Total labor and supervision	$ 7,491,600
Operating supplies	2,370,500
Power	437,200
Payroll overhead	2,996,600
Union welfare	2,121,300
Indirect cost	1,479,300
Fixed cost, no preparation	4,775,200
Total first year operating cost	21,671,700
Less credit for coal sold @ $15/ton	22,876,500
Net Development Revenue, No Preparation	$ 1,204,800

[1]First year operating cost covers the period of time required (one calendar year) to place all units in operation within the projected mining plan after initial shafts have been completed. During this period, 1,525,100 tons of raw coal are produced.

B. WITH PREPARATION:

For the mine producing washed coal, it is assumed that the preparation plant is not in operation until the end of this 12-month build-up to capacity production. During this period the coal produced is sold as run-of-mine for whatever price it will bring, assumed to be $15/ton. Thus, "net revenue during development with preparation," is presumed to be the same as for the case of no preparation — $1,204,800.

Table C-7. Insurance Base, 1.98 MM tons/yr Mine.

A. NO PREPARATION:

Net estimate for capital investment		$37,912,600
Less:		
Option on land	$ 321,600	
Interest during development	1,790,000	
		2,111,600
		35,801,000
Plus:		
Working Capital		5,529,000
Insurance Base, No Preparation		$41,330,000

B. WITH PREPARATION:

Net estimate for capital investment (No Preparation)		$37,912,600
Less:		
Option on land	$ 321,600	
Interest during development	1,790,000	
		2,111,600
		35,801,000
Plus:		
Working Capital	5,949,000	
Preparation Plant	9,733,000	
		15,682,000
Insurance Base, With Preparation		$51,483,000

Table C-8. Working Capital, 1.98 MM tons/yr Mine.

A. NO PREPARATION

Direct labor	3 months	$1,865,000
Operating supplies	do	1,270,400
Payroll overhead	do	746,000
Indirect cost	4 months	627,100
Fixed cost	0.5 percent of insurance base	179,000
Spare parts		757,300
Miscellaneous		84,200
Total Working Capital, No Preparation		$5,529,000

B. WITH PREPARATION

Direct labor	3 months	$1,958,400
Operating supplies	do	1,402,800
Payroll overhead	do	783,400
Indirect cost	4 months	672,200
Fixed cost	0.5 percent of insurance base	227,700
Spare parts		814,000
Miscellaneous		90,500
Total Working Capital, With Preparation		$5,949,000

Table C-9. Initial Investment Summary, 1.98 MM tons/yr Mine.

A. NO PREPARATION

Net estimate	$37,912,600
Plus working capital	5,529,000
	43,441,600
Less interest during development	1,790,000
Total Initial Investment, No Preparation	$41,651,600

Allocation by year:

Year	Multiplier		Amount
-2	1/6		$ 6,941,900
-1	1/2		20,825,800
0	1/3		13,883,900
		Total	$41,651,600

B. WITH PREPARATION

Net estimate	$47,645,600
Plus working capital	5,949,000
	53,594,600
Less interest during development	1,790,000
Total Initial Investment, With Preparation	$51,804,600

Allocation by year:

Year	Multiplier		Amount
-2	1/6		$ 8,634,100
-1	1/2		25,902,300
0	1/3		17,268,200
		Total	$51,804,600

Table C-10. Cash Flow Summary, 1.98 MM tons/yr Mine
(Costs are shown in thousands of dollars.)

A. NO PREPARATION

Year	Capital Investment	Other Expenditures	Present Value Factor @ 15%	Present Value of Capital Items @ 15%
- 2	6,942		1.3225	9,181
- 1	20,826		1.1500	23,950
0	13,884	(1,205)[1]	1.0000	12,679
1	500	27,923	0.8696	435
2	500		0.7561	378
3	500		0.6575	329
4	1,874		0.[illegible]	1,071
5	959		0.4[illegible]	477
6	500		0.4323	216
7	500		0.3759	188
8	1,874		0.3269	613
9	500		0.2843	142
10	20,543		0.2472	5,078
11	500		0.2149	107
12	1,874		0.1869	350
13	500		0.1625	81
14	500		0.1413	71
15	959		0.1229	118
16	1,874		0.1069	200
17	500		0.0929	46
18	500		0.0808	40
19	500	27,923	0.0703	35
20	500	20,618[2]	0.0611	(307)
				55,478

[1] Revenue during first year of operation.

[2] Includes the effect of all cash inflows in the last year of operation; equipment is assumed to have zero salvage value; working capital is presumed convertible into cash at its historical cost; no reclamation or other decommissioning costs are considered.

Table C-10. Cash Flow Summary, 1.98 MM tons/yr Mine. (contd)
(Costs are shown in thousands of dollars.)

B. WITH PREPARATION

Year	Capital Investment	Other Expenditures	Present Value Factor @ 15%	Present Value of Capital Items @ 15%
- 2	8,634		1.3225	11,418
- 1	25,903		1.1500	29,788
0	17,268	(1,205)[1]	1.0000	16,063
1	500	28,278	0.8696	435
2	500		0.7561	378
3	500		0.6575	329
4	1,874		0.5718	1,071
5	959		0.4972	477
6	500		0.4323	216
7	500		0.3759	188
8	1,874		0.3269	613
9	500		0.2843	142
10	20,543		0.2472	5,328
11	500		0.2149	107
12	1,874		0.1869	350
13	500		0.1625	81
14	500		0.1413	71
15	959		0.1229	118
16	1,874		0.1069	200
17	500		0.0929	46
18	500		0.0808	40
19	500	28,278	0.0703	35
20	500	22,329	0.0611	(333)
				67,161

[1]Revenue during first year of operation.

[2]Includes the effect of all cash inflows in the last year of operation; equipment is assumed to have zero salvage value; working capital is presumed convertible into cash at its historical cost; no reclamation or other decommissioning costs are considered.

APPENDIX D

INPUT DATA FOR LIFE CYCLE COST CALCULATION

APPENDIX D

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Appendix C contains data on manning, equipment, and cost for a representative shaft mine using continuous mining equipment. Appendix D recasts these data, when appropriate, into the format required by the life cycle model. Thus, these calculations provide illustrative operational definitions of all the key variables which appear in the model. The organization of Appendix D follows very closely the sequence of topics covered during the development of the model, with the first portion of the appendix treating capital items, and the second portion, annual operating expense.

<u>Capital Items:</u>

<u>Historical value of initial investment:</u> $K_{EO} \Big/ \left[(1-\alpha_A)(1-\alpha_U)\right]$

$\equiv$ (Gross Estimate) - (Interest during Construction) + (Working Capital)

Upon selecting the required values from Tables C-1 and C-8, one obtains the following:

$$K_{EO} \Big/ \left[(1-\alpha_A)(1-\alpha_U)\right] =$$

$$= 37,591,000 - 1,790,000 + 5,529,000 = \underline{\underline{41,330,000}} \text{ (No Prep)}$$
$$= 47,324,000 - 1,790,000 + 5,949,000 = \underline{\underline{51,483,000}} \text{ (With Prep)}$$

<u>Present value of initial investment:</u> $\hat{K}_{EO}$

$$\equiv (1+\lambda_{EO})\, K_{EO} \Big/ \left[(1-\alpha_A)(1-\alpha_U)\right]$$

Thus, the first step is to compute λ_{EO}, the factor which produces interest during construction (in the present value sense) when applied to K_{EO}. According to Appendix B,

$$\lambda_{EO} \equiv \left\{ \sum_{i=-\infty}^{0} f_i/(1+r)^i \right\} - 1$$

where

f_i: fraction of the initial expenditures occurring in the i^{th} year prior to capacity production

r: specified rate of return.

The required cash flow data are found in Table C-10.

$$\lambda_{EO} = \frac{1}{40,446,800}\left\{\frac{6,941,900}{(1.15)^2} + \frac{20,825,800}{1.15} + \frac{12,679,100}{1}\right\} - 1$$

$$= \underline{\underline{0.13183}} \text{ (No Prep)}$$

$$= \frac{1}{50,599,800}\left\{\frac{8,634,100}{(1.15)^2} + \frac{25,902,300}{1.15} + \frac{16,063,400}{1}\right\} - 1$$

$$= \underline{\underline{0.13183}} \text{ (With Prep)}$$

This permits the immediate calculation of $\hat{K}_{EO}$:

$$\hat{K}_{EO} = (1 + \lambda_{EO})\left\{K_{EO}\Big/\left[(1-\alpha_A)(1-\alpha_U)\right]\right\}$$

$$= 1.13183\ (41,330,000) = \underline{\underline{46,826,900}} \text{ (No Prep)}$$

$$= 1.13183\ (51,483,000) = \underline{\underline{58,278,800}} \text{ (With Prep).}$$

<u>Capital Productivity:</u> ρ_E

$$\equiv V_R \Big/ (\hat{K}_{EO} + \hat{K}_{EF})$$

Since the raw coal tonnage, V_R, and the present value of initial investment $\hat{K}_{EO}$ are known, it remains to compute $\hat{K}_{EF}$, the present value of deferred investment. The revelant figures are obtained by applying the present value factor to the deferred capital expenditures for each year from 1 through 20 of Table C-10, then summing. Note that the flow during year 20 includes the effect of liquidating working capital at its historical value. Mining equipment is presumed to have zero salvage value. Thus,

$$\hat{K}_{EF} = (10,006,000) - 5,529,000/(1.15)^{20} = \underline{\underline{9,668,200}} \text{ (No Prep)}$$

$$= (10,256,000) - 5,949,000/(1.15)^{20} = \underline{\underline{9,892,500}} \text{ (With Prep)}$$

And subsequently,

$$\rho_E = V_R \Big/ (\hat{K}_{EO} + \hat{K}_{EF})$$

$$= 1,980,000/(46,826,900 + 9,668,200) = \underline{\underline{0.03505}} \text{ (No Prep)}$$

$$= 1,980,000/(58,278,800 + 9,892,500) = \underline{\underline{0.02904}} \text{ (With Prep)}$$

For purposes of price sensitivity studies, deferred investment is assumed to be a fixed fraction of initial investment, with each figure being interpreted in its present value sense. In particular

$$\hat{K}_{EF} \equiv \beta_E \left\{ \hat{K}_{EO} \Big/ \left[(1 - \alpha_A)(1 - \alpha_U) \right] \right\}$$

or

$$\beta_E \equiv \hat{K}_{EF} \left[(1 - \alpha_A)(1 - \alpha_U) \right] \Big/ \hat{K}_{EO}$$

$$= 9,668,200/41,330,000 = \underline{\underline{0.23393}} \text{ (No Prep)}$$

$$= 9,892,500/51,483,000 = \underline{\underline{0.19215}} \text{ (With Prep)}$$

<u>Present value of mineral rights:</u> K_A

$$\equiv p_A \left(\frac{V_R T}{v_s \eta} \right) (1 + r)^n$$

an expression involving mine annual capability, V_R, mine life, T, seam density, v_s, planned recovery factor, η, and the specified discount rate, r. Although mineral rights are typically acquired before significant expenditures on equipment or construction (i.e., prior to year -2) for simplicity this cash flow is lumped together with the other year -2 flows. Thus, n is set equal to 2, and one obtains

$$\hat{K}_A = 50 \frac{1,980,000\ (20)}{10,800\ (0.57)} (1.15)^2 = \underline{\underline{425,400}} \text{ (Both Cases)}$$

Note that the above expression for $\hat{K}_A$ applies to all initial expenditures on mineral rights, whether acquired via outright purchase or option-lease.

Present value of net development expenditure: $\hat{K}_D$

$\hat{K}_D$ is computed by adjusting the present value of development expenditures for the present value of coal sold during the build-up to capacity production. Assuming that initial development occurs during the 12-month period of year zero, one is able to compute $\hat{K}_D$ from the information in Table C-6:

$$\hat{K}_D = \left[21,671,700 - 15\,(1,525,100)\right](1.15)^0 = \underline{\underline{-1,204,800}} \text{ (Both Cases).}$$

Present value of aggregate capital investment: $\hat{E}_K$

$$\equiv \hat{K}_{EO} + \hat{K}_{EF} + \hat{K}_A + \hat{K}_D$$

Using the results of the calculations above, one obtains

$$\hat{E}_K = 46,826,900 + 9,668,200 + 425,400 - 1,204,800 = \underline{\underline{55,715,700}} \text{ (No Prep)}$$

$$= 58,278,800 + 9,892,500 + 425,400 - 1,204,800 = \underline{\underline{67,391,900}} \text{ (With Prep)}$$

Working capital as a fraction of initial investment: w_c

$$\equiv W_c(1-\alpha_A)(1-\alpha_U) / K_{EO}$$

Again, this is a quantity whose principal use is in sensitivity analysis. Data from Table C-8 combine with the above results for initial investment to yield,

$$w_c = 5,529,000/41,330,000 = \underline{\underline{0.13378}} \text{ (No Prep)}$$

$$= 5,949,000/51,483,000 = \underline{\underline{0.11555}} \text{ (With Prep)}$$

Annual Operating Expense and Related Variables:

This section begins by grouping operating expense into two components — depreciation, and all other expense — and then computes values for quantities used in the sensitivity analysis.

Depreciation fraction: β_D

$$\equiv D \Big/ \left\{ K_{EO}(1 - w_c) \Big/ \left[(1 - \alpha_A)(1 - \alpha_U)\right] + K_D \right\}$$

Combining the values for depreciation from Table C-3 with previous results for initial capital investment, working capital (which is not depreciable) and net development expenditures, one obtains

$$\beta_D = 3,701,200 \Big/ \left\{ 41,330,000(1 - 0.1338) - 1,204,800 \right\} = \underline{\underline{0.10699}} \quad \text{No Prep)}$$

$$= 4,187,900 \Big/ \left\{ 51,483,000(1 - 0.1156) - 1,204,800 \right\} = \underline{\underline{0.09448}} \quad \text{(With Prep)}$$

Operating expense less depreciation: C_p'

$$\equiv \text{(Labor Related)} + \text{(Supplies Related)} + \text{(Power and Water)}$$
$$+ \text{(Union Welfare)} + \text{(Insurance and Other Fixed Costs)}$$

$$= a_L C_L + a_S C_S + C_U + C_w + C_{INS}$$

Data for all components except union welfare are readily assembled from previous calculations or costing rules of thumb:

Quantity	No Prep	With Prep	Source/Comments
a_L	1.55	1.55	See p. 26 of text
C_L	7,460,000	7,833,500	Table C-2
a_S	1.15	1.15	See p. 26 of text
C_S	5,081,600	5,611,100	Table C-5
C_U	1,243,400	1,326,700	Table C-5
C_{INS}	413,300	514,830	1% of insurance base; Table C-7

Union welfare expense requires a separate calculation. Equation (25) of the text is easily recast into the following form:

$$C_W = c_{WT} V_R (1 - \alpha_R)(1 - \alpha_P) + c_{WH} M_H h_S \text{ (days of operation)}$$

Using the December 1976 values for c_{WT} and c_{WH} (p. 28), the hourly worker count from Table C-2, and a 20 percent washing loss, one obtains

$$C_W = 0.82(1,980,000)(1) + 1.54(405)(8)(220) = \underline{\underline{2,721,300}} \text{ (No Prep)}$$

$$= 0.82(1,980,000)(0.8) + 1.54(424)(8)(220) = \underline{\underline{2,448,100}} \text{ (With Prep)}$$

assuming three 8-hour shifts and 220 days of operation per year. Now it is a simple matter to compute C'_p.

$$C'_p = 1.55(7,460,000) + 1.15(5,081,600) + 0.628(1,980,000)$$

$$+ 2,721,312 + 413,300$$

$$= \underline{\underline{21,784,840}} \text{ (No Prep)}$$

$$= 1.55(7,833,500) + 1.15(5,611,100) + 0.670(1,980,000)$$

$$+ 2,448,089 + 514,800$$

$$= \underline{\underline{22,884,290}} \text{ (With Prep)}$$

The remainder of this appendix is devoted to the calculation of quantities used in sensitivity analysis.

<u>Average wage per shift:</u> $\overline{w}_T$

$$\equiv C_L \Big/ \left[M_T \text{(days of operation)} \right]$$

Assuming 220 days of operation, and using the labor counts and labor cost totals from Table C-2, one obtains

$$\overline{w}_T = 7,460,000/464(220) = \underline{\underline{73.08}} \text{ (No Prep)}$$

$$= 7,833,500/489(220) = \underline{\underline{72.82}} \text{ (With Prep)}$$

<u>Unit cost of operating supplies:</u> c_S

$$\equiv C_S / V_R$$

$$= 5,081,600/1,980,000 = 2.57 \text{ (No Prep)}$$

$$= 5,611,100/1,980,000 = 2.83 \text{ (With Prep)}$$

<u>Unit cost of power and water:</u> c_U

$$\equiv (\text{power} + \text{water}) / V_R$$

Referring to Table C-5, one computes

$$c_U = (1,240,400 + 3,000) / 1,980,000 = \underline{\underline{0.628}} \text{ (No Prep)}$$

$$= (1,323,700 + 3,000) / 1,980,000 = \underline{\underline{0.670}} \text{ (With Prep)}$$

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