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# THE OPTIMIZATION OF SELF-PHASED ARRAYS FOR DIURNAL MOTION TRACKING OF SYNCHRONOUS SATELLITES

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16. Abstract

The diurnal motion of a synchronous satellite necessitates mechanical tracking when a large aperture, high gain antenna is employed at the earth terminal. An alternative solution to this tracking problem is to use a self-phased array consisting of a number of fixed pointed elements, each with moderate directivity. Non-mechanical tracking and adequate directive gain are achieved electronically by phase coherent summing of the element outputs. The element beamwidths provide overlapping area coverage of the satellite motion but introduce a diurnal variation into the array gain.

The optimum element beamwidth and pointing direction of these elements will be obtained under the condition that the array gain is maximized simultaneously with the minimization of the diurnal variation. Optimal design examples will be presented for arrays consisting of two and four elements tracking several models of diurnal satellite motion.

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## I. INTRODUCTION

The design of an earth-space communication link which utilizes a synchronous satellite at centimeter and shorter wavelengths presents the problem of choosing an antenna and tracking method. Although a synchronous satellite is considered to be "stationary" with respect to an observation point on the earth, it experiences East-West perturbations due to earth oblateness and solar radiation and North-South perturbations resulting from solar and lunar gravitational forces. These motions appear to an observer at a fixed point on the earth as a diurnal (periodic over twenty-four hours) variation in the apparent look-angle which projects on a plane normal to the propagation path as an approximate ellipse with eccentricity from zero to one or as any of a number of other closed paths. This "diurnal motion" must be tracked if narrow-beamwidth antennas are chosen for a communication link.

Narrow-beamwidth antennas offer high gain where a large system gain-bandwidth product is desired. However, their large apertures and the necessity of mechanically tracking diurnal motion impose high cost on the antenna and mount equipment. In addition, a failure in the mechanical system or the control electronics disables the communication link entirely. A single element of moderate aperture offers an alternative tracking method. Such an antenna may be fixed pointed if its beamwidth is wider than the satellite motion. However, its gain will be low compared to a large aperture tracking antenna and hence such a system would be susceptible to precipitation fading. An alternative to a single large aperture tracking antenna or a single moderate aperture fixed antenna is to utilize a large number of small aperture elements in a self-phased array. (Figure 1).

A self-phased array antenna provides high gain in a particular direction by phase coherent summing of the outputs of many small aperture elements, each having wide beamwidth. The array may be fixed pointed and non-planar, employing the cohering electronics to track diurnal motion. Although a large array eliminates the need for a tracking mount and large aperture antenna, the associated increase in the number of front ends and element processing circuitry may more than offset any savings. However, if one element fails, the signal-to-noise ratio degrades gracefully instead of catastrophically.

In contrast, this paper examines the characteristics of a self-phased array system consisting of a relatively small number of elements, each employing an antenna of moderately large aperture and narrow beamwidth (Figure 1).

The small array is designed by choosing an elemental beamwidth which approximately covers the expected diurnal satellite motion. Adequate system margin is achieved by adding a sufficient number of such elements. This compromise design simplifies the circuitry required for an array with a large number of elements, eliminates the need for

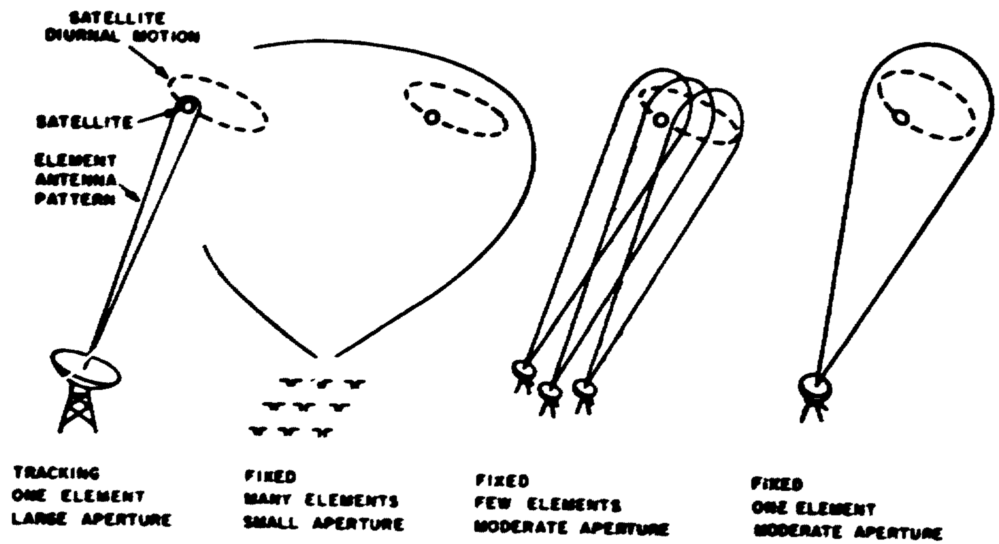


Figure 1. Synchronous satellite tracking methods.

mechanical tracking, and permits graceful degradation in signal-to-noise ratio with the loss of an element. However, because beamwidth is on the order of the diurnal satellite motion, the amplitude of the received signal will be a function of time (with a daily period) and a function of the geometric factors determined by the stabilization and perturbations of the satellite orbit. This paper addresses the question of optimizing the choice of antenna elements and their pointing in order to achieve maximum system gain margin simultaneously with the minimization of the diurnal variation.

## II. GAIN OPTIMIZATION

Array directive gain is defined as

$$G_D = \frac{\text{maximum radiation intensity}}{\text{average radiation intensity}} \tag{1}$$

for the array phase-locked to a source within the angular limits of acquisition of the antenna-receiver system. As the satellite undergoes its diurnal motion, $G_D$ will vary with time as the array electronically tracks the satellite. Consider a typical diurnal gain variation as shown in Figure 2 and define minimum directive gain as

$$G_M = \min_{24\ \text{hrs}} \{G_D(T)\} \quad .$$

Fix the number of antenna elements at N and define the half-power elemental beamwidth in elevation as $\theta_e$ and in azimuth as $\phi_e$. Let the beamwidths be equal and normalize to the largest angular variation of the satellite motion, $\alpha$ (refer to Figure 3):

$$\frac{\theta_e}{\alpha} = \frac{\phi_e}{\alpha} = B \quad . \tag{2}$$

With N fixed, $G_M$ may be plotted as a function of B in order to find the maximum of $G_M$. For the optimization problem, $G_M$ will also be a function of the pointing of each element axis in elevation and azimuth ($\bar{\theta}_i$ and $\bar{\phi}_i$ for the $i^{th}$ element). The maximum of $G_M$ will be called the optimum minimum directive gain, $G_M'$:

$$G_M'(B,N,\bar{\theta}_i,\bar{\phi}_i) = \text{Max}\ G_M(B,N,\bar{\theta}_i,\bar{\phi}_i) \quad .$$

A numerical technique was used to determine B and the $\bar{\theta}_i$ and $\bar{\phi}_i$ for various closed-path models of satellite motion.

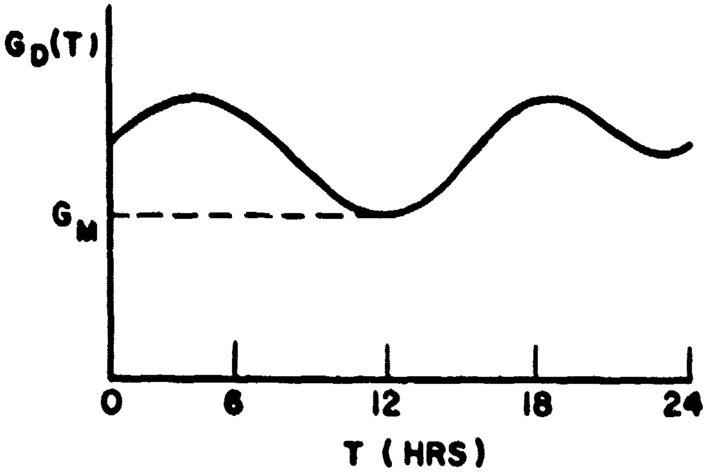


Figure 2. Minimum directive gain $G_M$.

## III. NUMERICAL DESIGN

Diurnal motion may be approximately modelled by an ellipse with a major axis of $\alpha$ degrees and eccentricity E between zero and one, where

$$E = \frac{\text{Minor Axis}}{\text{Major Axis}} \quad . \tag{3}$$

A Gaussian antenna pattern $P_i(\theta,\phi)$ for the $i^{th}$ element, defined as

$$P_i(\phi,\theta) = -3\left[\frac{(\theta-\bar{\theta}_i)^2}{\left(\frac{\alpha B}{2}\right)^2} + \frac{(\phi-\bar{\phi}_i)^2}{\left(\frac{\alpha B}{2}\right)^2}\right] \quad (\text{dB}) \quad , \tag{4}$$

is used as an acceptable representation of a realizable narrow-beam element pattern. The directive gain for a single element may be shown to be, approximately,

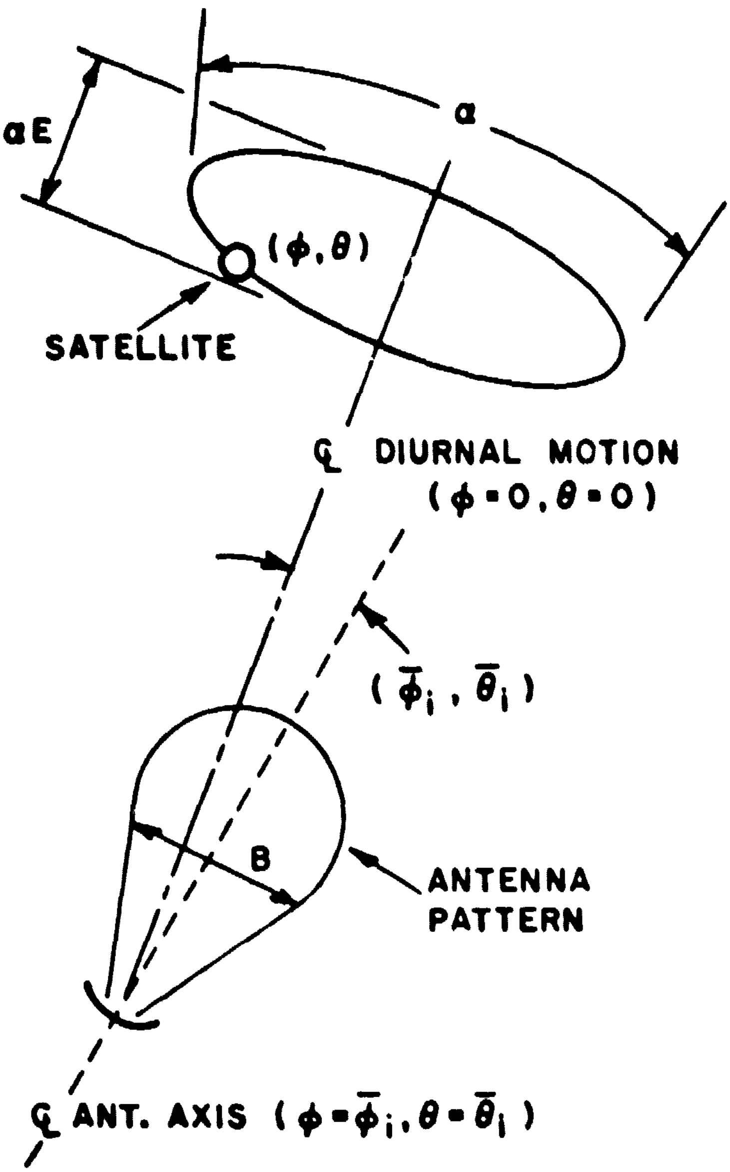


Figure 3. Geometric definitions.

$$G_{D_e} = \frac{41253}{\theta_e \phi_e} \quad [1] , \tag{5}$$

where the half-power beamwidths, $\theta_e$ and $\phi_e$, are given in degrees.

For N fixed and the diurnal motion restricted to an ellipse of eccentricity E and major axis $\alpha$ degrees in the $\phi$ direction, a set of $G_M(B, N, \theta_i, \phi_i)$ was evaluated for a given beamwidth B. The evaluation was performed at five minute time increments along the ellipse in order to find the minimum directive gain $G_M(B,N,\bar{\theta}_i,\bar{\phi}_i)$. The optimum $G_M'$ was obtained as the pointing of all element axes ($\bar{\phi}_i$ and $\bar{\theta}_i$) was varied. The examples which follow show $G_M'$ plotted versus B, which varies from 0.2 to 2.0 in 0.1 unit increments. The same plots also present the corresponding optimum element axis angles $\bar{\theta}_i$ and $\bar{\phi}_i$ as a function of B. Also noted on all plots is $\mathrm{Max}\{G_M'\}$ which is the highest gain realizable for a particular ellipse E and number of elements N.

## IV. CIRCULAR DIURNAL MOTION

Figures 4 and 5 illustrate the optimization of $G_M'$ for two and four antenna elements, respectively, where the diurnal motion is described by a circle (E=1). For N=2 (Figure 4) both elements must be pointed at the center of the circle and have beamwidth $0.85\alpha$ in order to achieve the maximum $G_M'$. Similarly, maximum $G_M'$ for four elements (Figure 5) is achieved with center pointed elements of beamwidth $0.85\alpha$. A tentative conclusion from the numerical model for circular diurnal motion is that maximum optimal directive gain is achieved with center pointed elements of beamwidth $0.85\alpha$, independent of N. The proof is easily shown.

Given the elemental pattern of (4) and directive gain of (5), let N beams be centered on the diurnal motion path ($\bar{\theta}_i=\bar{\phi}_i=0$). The $i^{th}$ element voltage is

$$v_i = 10^{P_i/20} \tag{6}$$

and the array coherent output is

$$P_A = 10 \log_{10}[(v_1+v_2+\cdots+v_N)(v_1+v_2+\cdots+v_N)] \quad . \tag{7}$$

For the circular case, the minimum directive gain is constant at any point in time, so let

$$\theta = 0 \text{ and } \phi = \alpha/2 \tag{8}$$

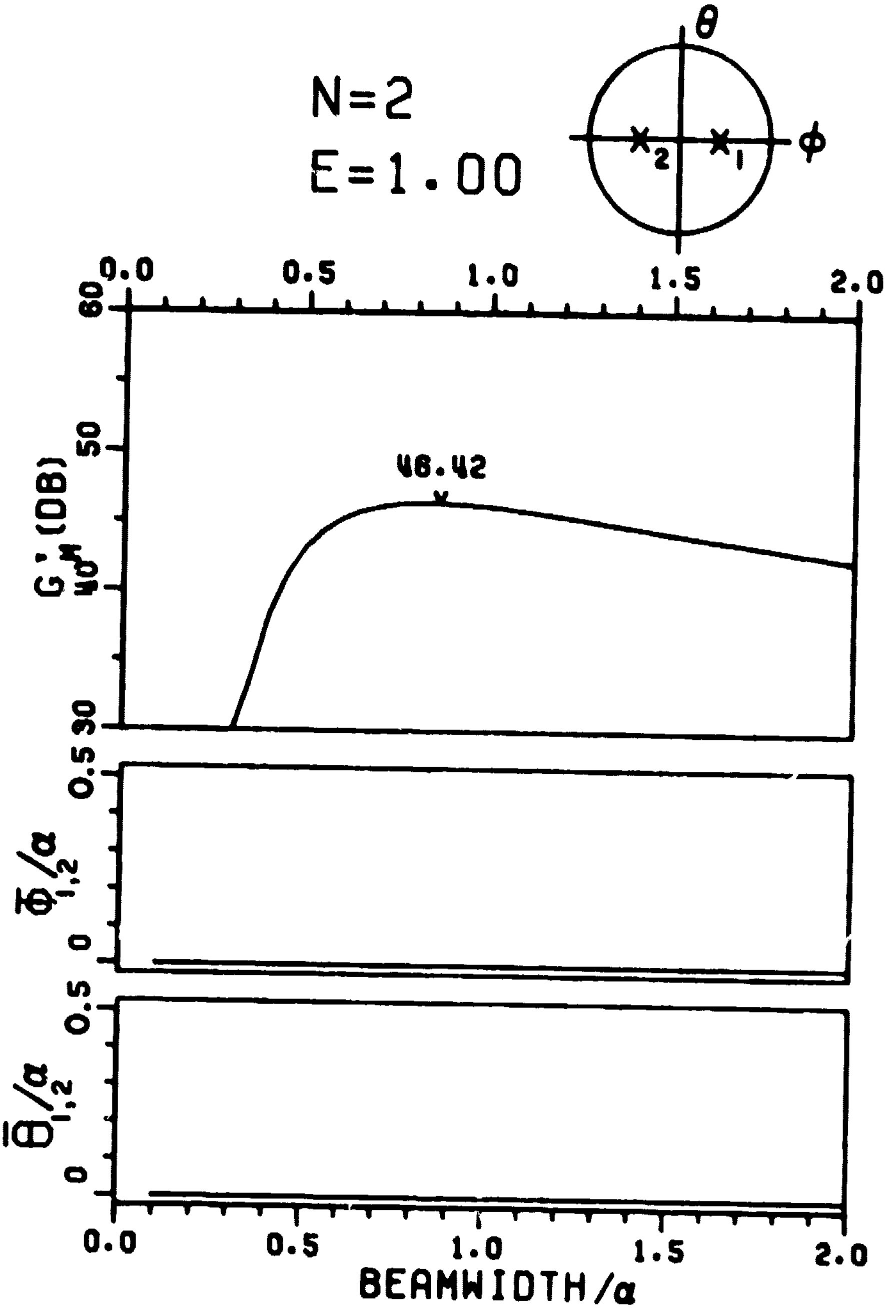


Figure 4. Array optimization for N=2 and E=1.

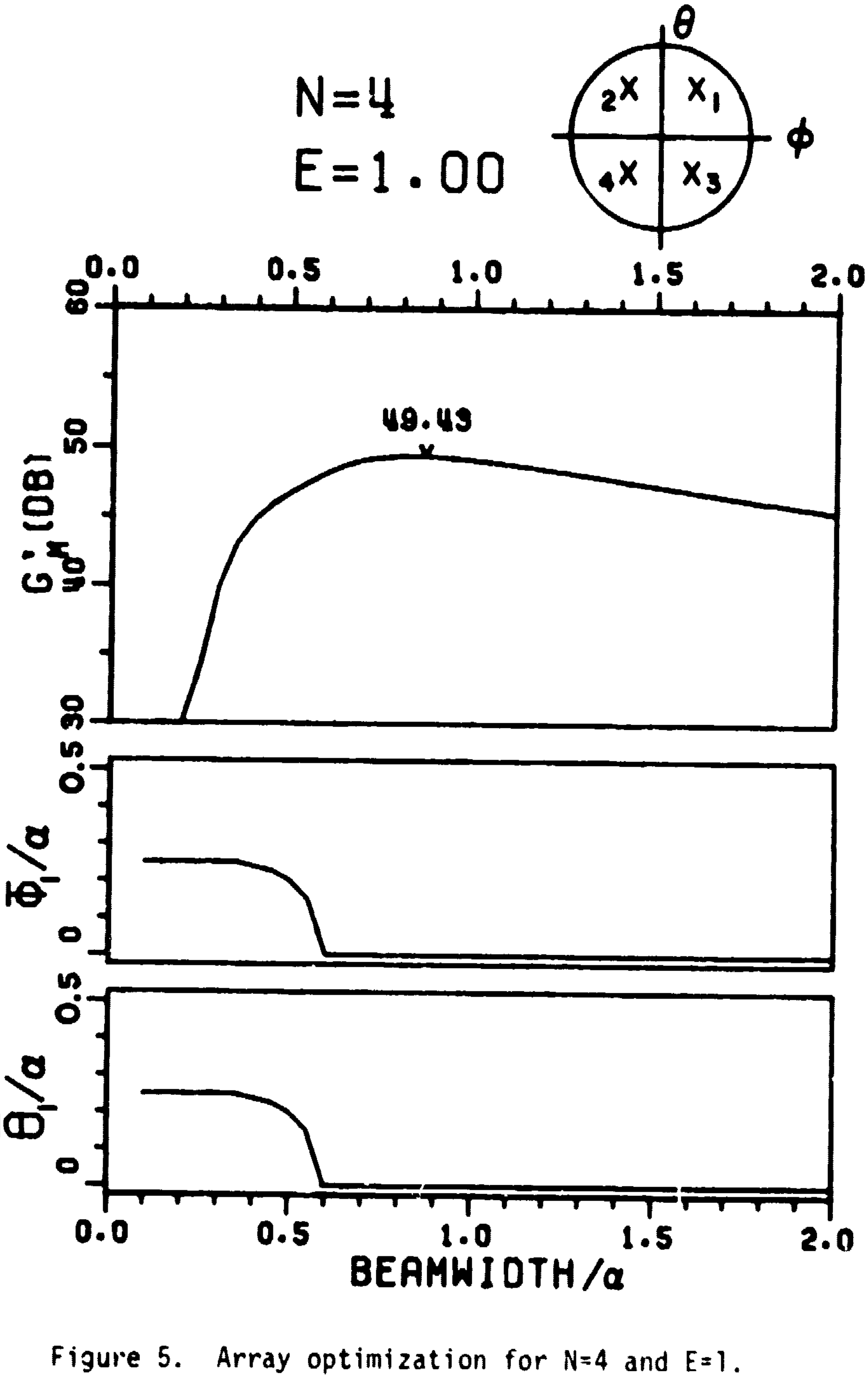


Figure 5. Array optimization for N=4 and E=1.

so that

$$P_A = 10 \log_{10} \left[ \left( \sum_{i=1}^{N} 10^{-3/20B^2} \right) \cdot \left( \sum_{i=1}^{N} 10^{-3/20B^2} \right) \right] \quad . \tag{9}$$

The minimum (constant) directive array gain is then

$$G_M = 10 \log_{10} \left( \frac{41253}{B^2} \right) + 10 \log_{10} \left( N^2 \cdot 10^{-3/10B^2} \right) \quad . \tag{10}$$

Find the maximum $G_M$ and the corresponding B by differentiating (10) with respect to B and equating to zero

$$\frac{dG_M}{dB} = 0 = -10 \log_{10} e \cdot \frac{2}{B} + \frac{6}{B} \quad . \tag{11}$$

$$B \sim 0.83113 \tag{12}$$

and is independent of the number of elements. This value compares quite well with that obtained by numerical techniques for a circular diurnal motion path.

## V. STRAIGHT LINE DIURNAL MOTION

The limiting case of straight line (E=0) optimization is presented in Figures 6 and 7 for N=2 and 4, respectively. The maximum $G_M'$ for the axes remaining center pointed is obtained with $B=0.85\alpha$ in both figures. This breakpoint for the straight line cases is simply a specialized case of the proof for a circle. Equation (8) still describes $\mathrm{Min}\{G_D\}$ because, for the straight line case with center pointing, $G_D$ is always greater than or equal to that value found for a circular path of diameter $\alpha$. Hence, (12) holds for straight line diurnal motion with N elements center pointed.

For the straight line cases with the axis pointing optimized, the maximum $G_M'$ is obtained for the element axes separated and having beamwidths of the order $\alpha/N$. For N=4 (Figure 7), it appears that the four axes are equally spaced along the diurnal path.

To obtain a more quantitative description of these cases, let the axes of the beams be on the satellite path (i.e., $\bar{\theta}_i=0$ and $-\alpha/2 \le \bar{\phi}_i \le \alpha/2$). Restrict N to an even number and assume that the $\bar{\phi}_i$ corresponding to the maximum optimal $G_M'$ are distributed symmetrically about $\phi=0$

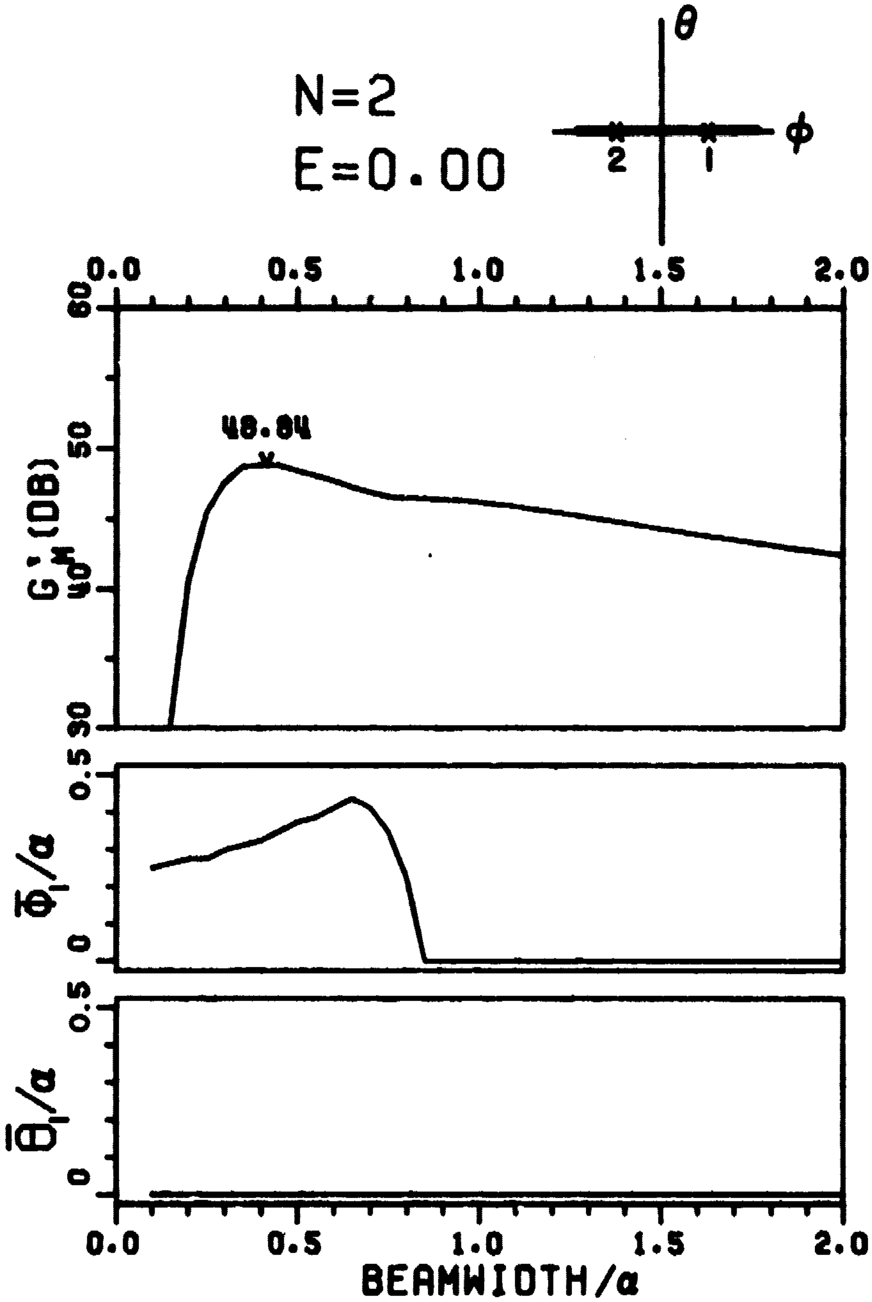


Figure 6. Array optimization for N=2 and E=0.

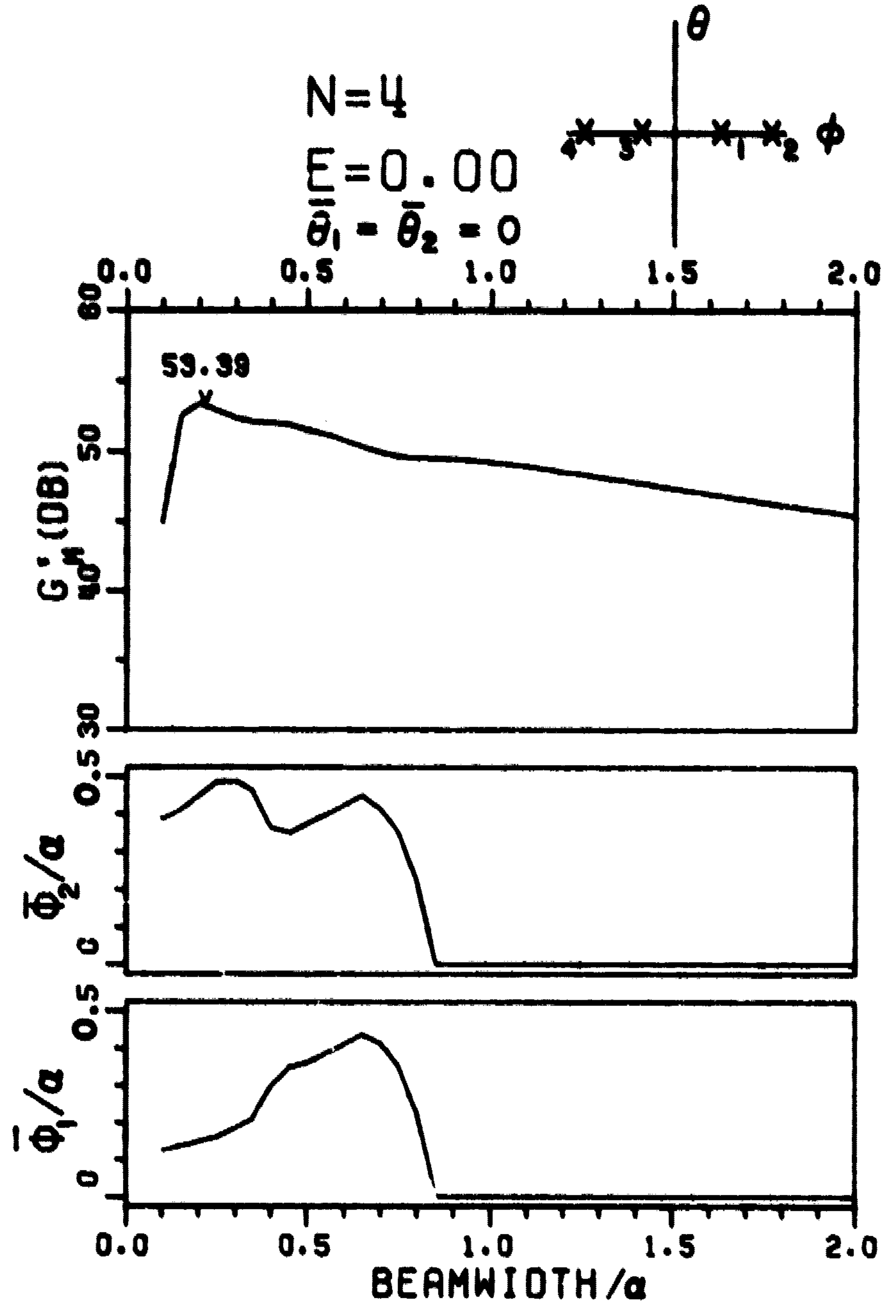


Figure 7. Array optimization for N=4 and E=0.

$$= \phi_{-N/2} = \phi_{N/2} , -\phi_{-N/2+1} = \phi_{N/2-1} , \cdots , -\phi_{-1} = \phi_1 \quad . \tag{13}$$

From the numerical model, assume that the distances between adjacent elements are equal for the optimal case

$$\Delta = \phi_{-N/2+1} - \phi_{-N/2} = \cdots = \phi_1 - \phi_{-1} = \cdots = \phi_{N/2-1} - \phi_{N/2} \quad . \tag{14}$$

A minimum in the diurnal pattern of G will occur at $\phi = 0$ because of symmetry. From the assumption that beamwidth will be of the order $\Delta/N$, the main contribution to G at $\phi = 0$ comes from the coherent sum of the two adjacent beams ($i = \pm 1$). Using (4) through (7),

$$G_M(\phi=0) = 10 \log_{10}\left(\frac{41253}{B^2}\right) + 10 \log_{10}\left[2\cdot 10^{-12/20B^2(\Delta/2)^2} + 2\cdot 10^{-12/20B^2(-\Delta/2)^2}\right] \quad . \tag{15}$$

Differentiate (15) with respect to B and equate to zero to obtain the condition

$$\left(\frac{B}{\Delta}\right)^2 = \frac{3}{20 \log_{10} e} \approx 0.3454 \quad . \tag{16}$$

A second minimum will occur at the endpoint of the diurnal path ($\phi = \pi/2$). Again, assume that the array contribution to $G_M$ at this point is primarily from the adjacent element ($i = N/2$). Using (4) through (7),

$$G_M(\phi=\pi/2) = 10 \log_{10}\left(\frac{41253}{B^2}\right) + 10 \log_{10}\left[10^{-12/10B^2[1/2-(N-1/2)\Delta]^2}\right] \quad . \tag{17}$$

Differentiating (17) with respect to B and using condition (16), one obtains

$$\Delta^2(2N^2-4N+1) + \Delta(4-4N) + 2 = 0 \quad . \tag{18}$$

The smaller root of (18) represents the distance between the axes of two adjacent elements. For example, if N=2, $\Delta$=0.59$\alpha$ and if N=4, $\Delta$=0.27$\alpha$. Figure 6 indicates that maximum $G_M'$ occurs when $2\bar{\phi}=\Delta=.65\alpha$ and Figure 7 gives $2\bar{\phi}_2=\Delta=.30\alpha$. Using (16) and $\Delta$=0.59$\alpha$, one obtains B=0.35$\alpha$ and for $\Delta$=0.27$\alpha$, B=.16$\alpha$. This agrees reasonably well with the numerical model results of 0.4$\alpha$ and 0.2$\alpha$ beamwidths for N=2 and 4, respectively.

## VI. ADDITIONAL EXAMPLES

Two cases of elliptical model results are presented in Figures 8 and 9 for N=2 and diurnal motion eccentricities E=0.75 and 0.25, respectively. Both indicate that the optimal $G_M'$ for center pointing is still obtained with B=0.85$\alpha$ since, as in the straight line case, $G_D$ is always greater than or equal to that value obtained for a circular path of radius $\alpha/2$ with all elements center pointed. However, maximum optimum gain occurs with the element axes separated and having narrower beamwidths.

Similar results for N=4 are shown in Figures 10 and 11. Here, note that $\bar{\theta}_i$ separation is also required for E=0.75 and B less than 0.45$\alpha$. The behavior is very similar to that of the elliptical cases for N=2. Center pointing of all elements requires B=0.85$\alpha$ and maximum optimum gain is achieved with axis separation and narrower beamwidths.

## VII. SUMMARY AND REMARKS

An elliptical model of the diurnal motion of a synchronous satellite has been presented. A self-phased, fixed pointed array of a small number of antenna elements with parabolic gain patterns was proposed as a means of tracking this motion. An optimization criterion of maximizing array directive gain with the simultaneous minimization of diurnal gain variation was defined and applied to the model. Numerical techniques lead to the conclusion that if all element axes are fixed pointed at the center of the satellite path, the optimization criteria are met with element beamwidths chosen to be 0.83 times the maximum angular extent of the diurnal motion. Higher gain is possible with narrower beamwidths and axis separation, but the design requires that the satellite's diurnal path eccentricity and inclination of the major axis remain fixed.

A summary of the design results for E=0 and 1 is presented in Figure 12. Minimum directive gain is plotted versus $\sqrt{\pi}$ x angular extent of satellite motion in radians x diameter of equivalent total aperture in wavelengths. Using this common definition, a comparison may be made with the cases of a single antenna element using mechanical tracking or a single element fixed pointing at the center of diurnal motion. For a given equivalent total aperture, a larger number of elements results in a directive gain closer to the mechanical tracking case and always

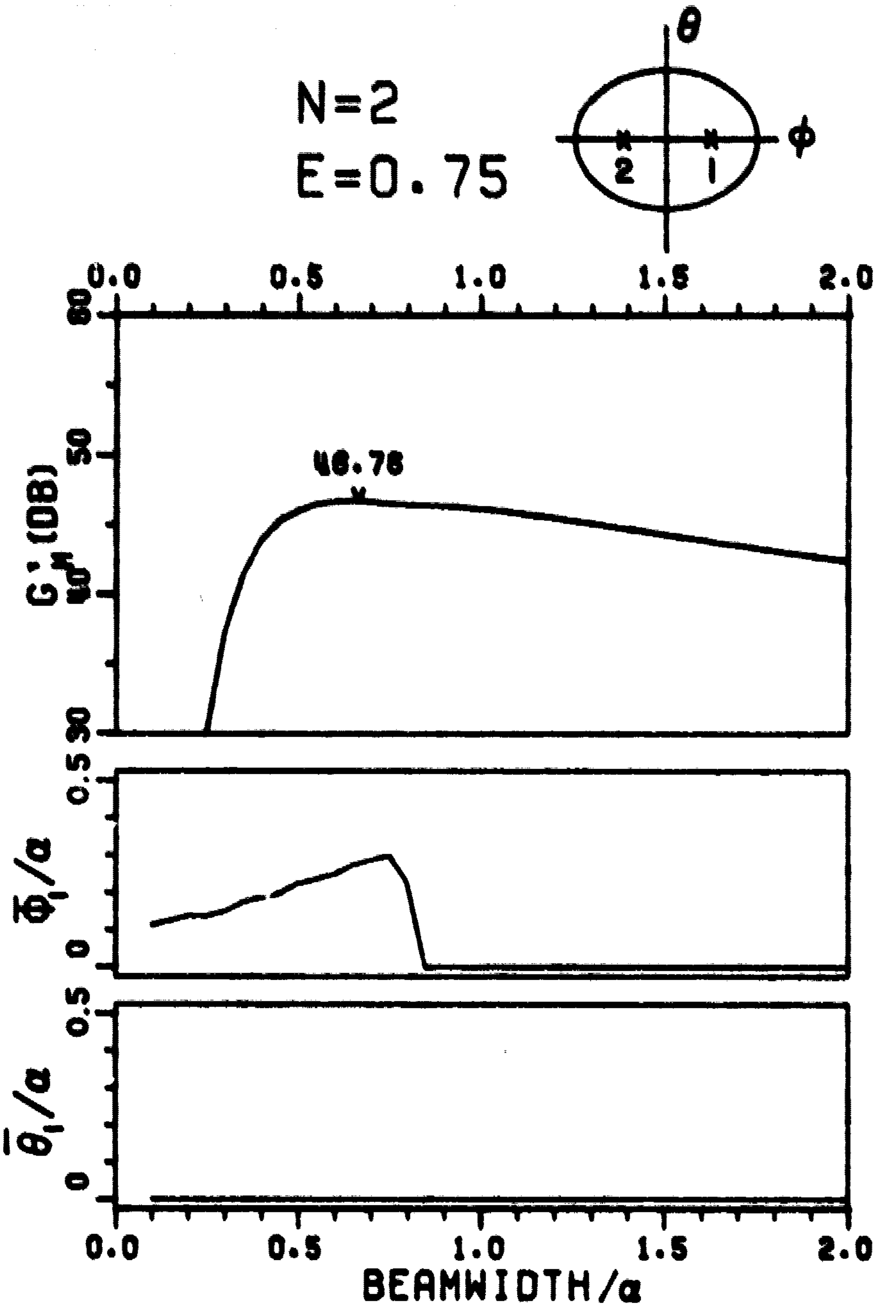


Figure 8. Array optimization for N=2 and E=0.75.

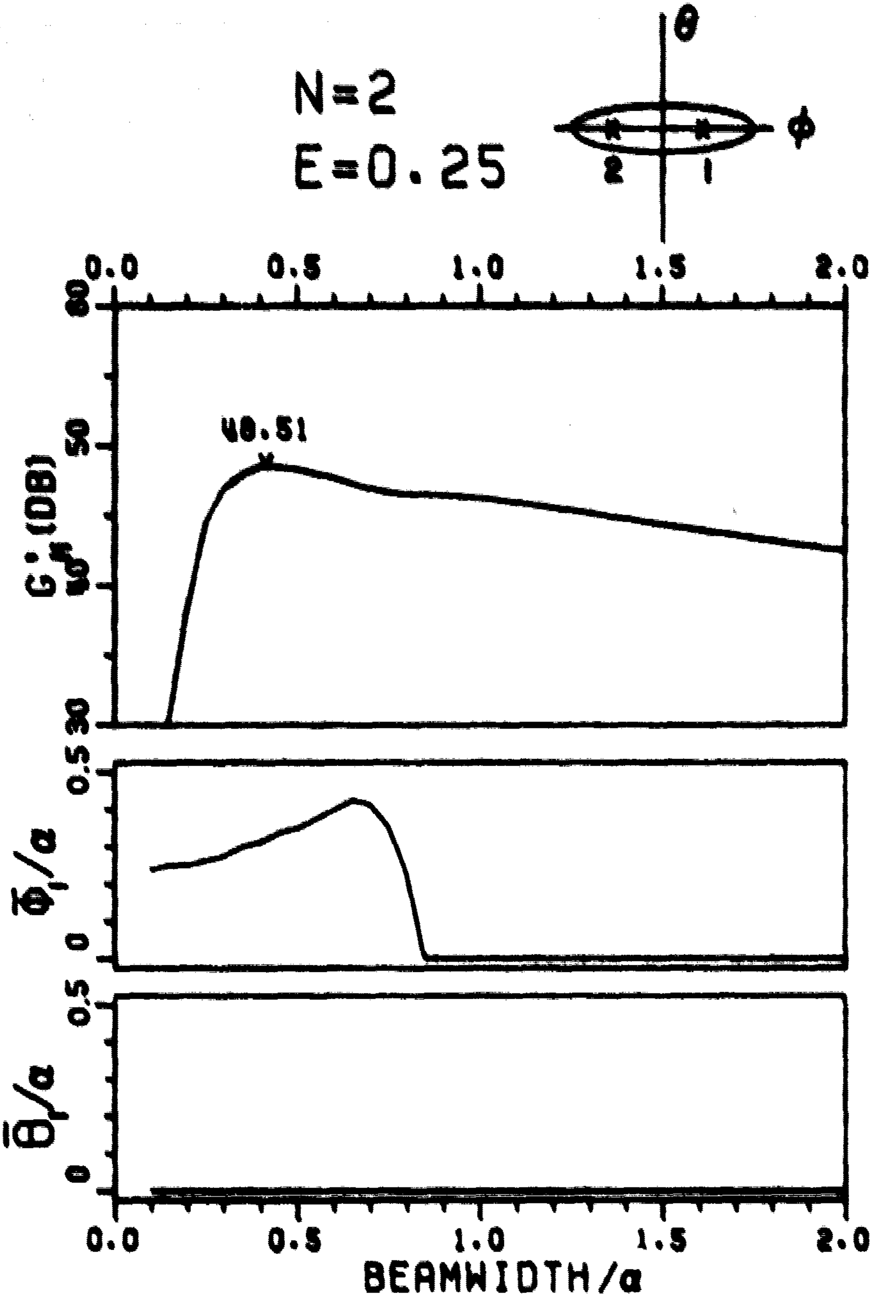


Figure 9. Array optimization for N=2 and E=0.25.

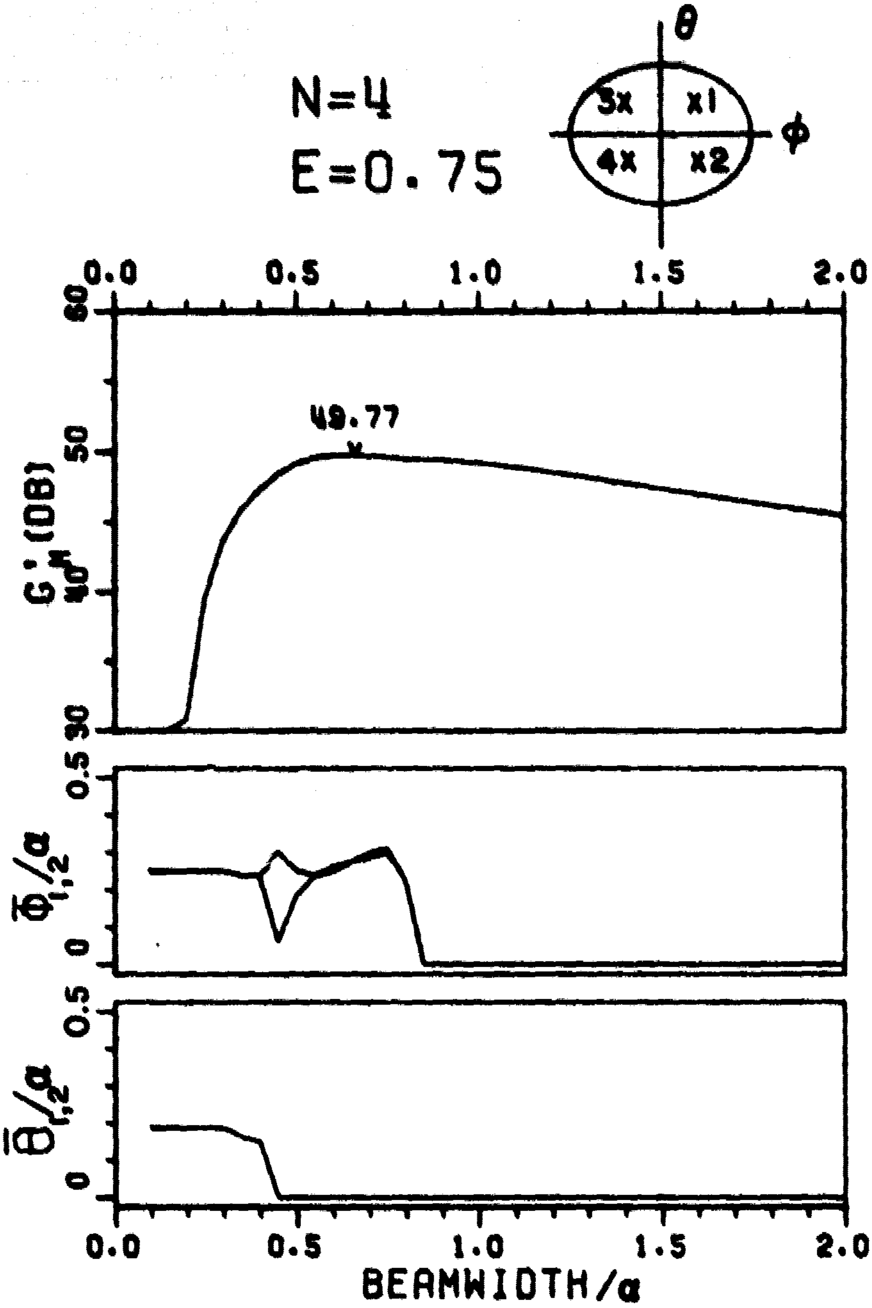


Figure 10. Array optimization for N=4 and E=0.75.

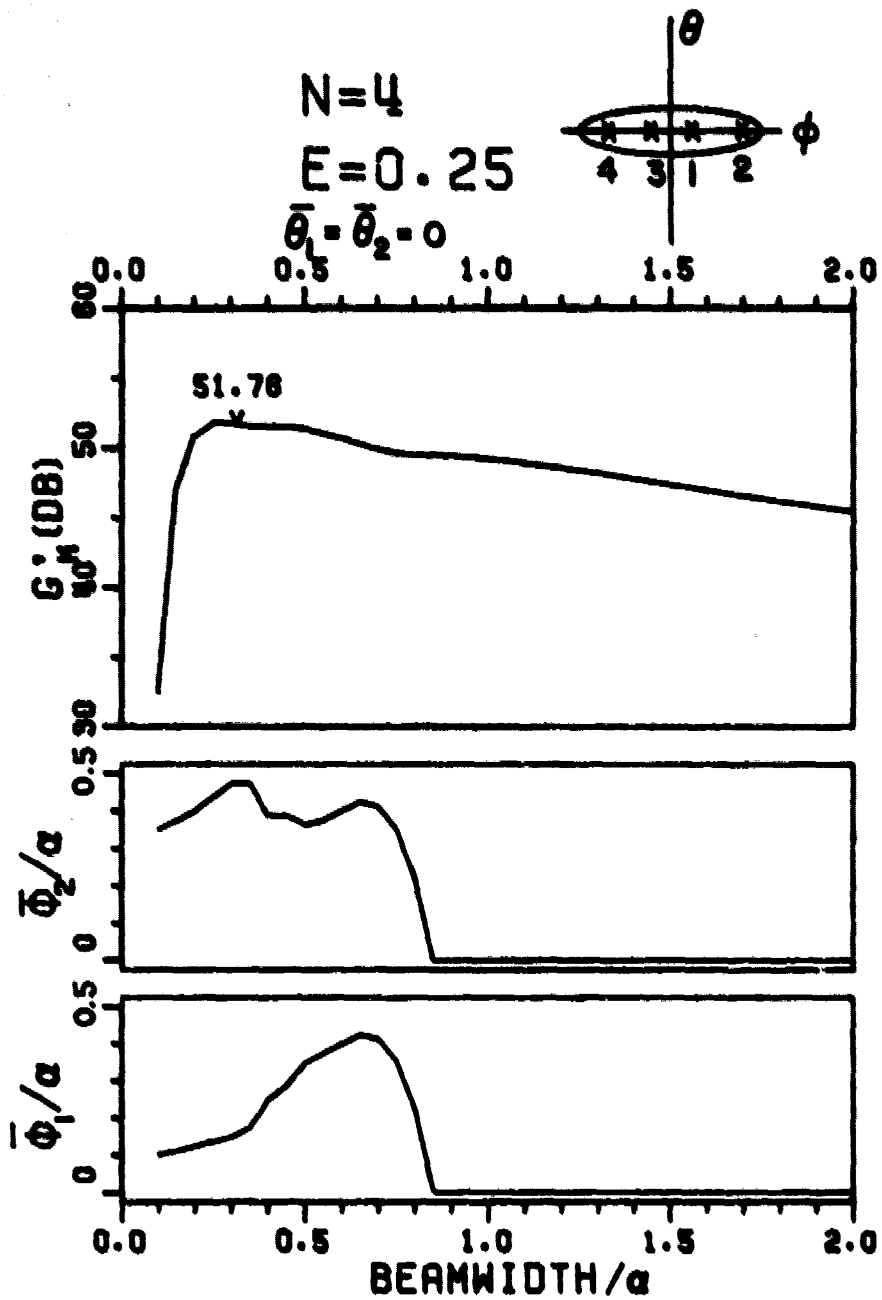


Figure 11. Array optimization for N=4 and E=0.25.

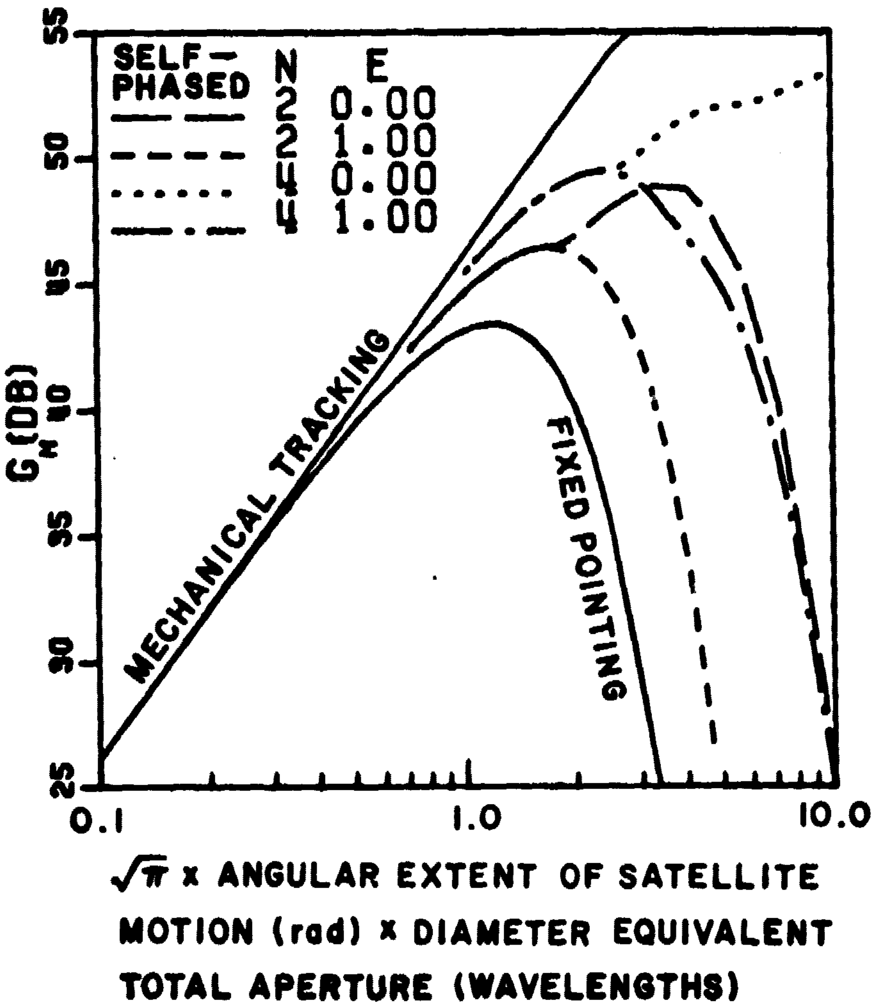


Figure 12. Comparison of mechanical tracking, fixed pointing, and small self-phased arrays.