



## Technical Memorandum 79595

# Nonlinear Landau Damping In The Ionosphere

(NASA-TM-79595) NONLINEAR LANDAU DAMPING IN  
THE IONOSPHERE (NASA) 35 p HC A03/MF A01

N78-29672

CSCL 04A

Inclas

63/46 29035

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**JULY 1978**

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Space Administration

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## INTRODUCTION

The subject of nonlinear wave-particle interactions in the magnetospheric and ionospheric plasmas has attracted considerable attention. For example, such interactions have been discussed in terms of the destruction of adiabatic invariants of trapped particles in the radiation belt [Kennel and Petchek, 1966; Schultz and Lanzerotti, 1974; Williams, 1975], and in terms of the production of a strong braking action against a field aligned flow of electrons [Block, 1975; Papadopoulos, 1977]. Satellite plasma wave observations have indicated the relevance of nonlinear wave-particle interactions to the energetics of the magnetosphere and the ionosphere [Scarf, 1975]. Undoubtedly, there must be nonlinear plasma processes involved in the naturally occurring phenomena. To specify the relevant process, however, is difficult in most cases because of restricted on-board diagnostic capabilities in addition to complicated initial and boundary conditions of the ambient plasma. In order to determine the roles of specific nonlinear processes it will be necessary to conduct active experiments in space where the boundary and initial conditions are chosen so as to simplify the analysis. Although such active controlled space experiments are mainly in the planning stage [Benson, 1977; Fredricks, 1978], important information can be obtained from simple wave transmission experiments performed with ionospheric sounders carried on-board satellites in the topside ionosphere.

The Alouette and ISIS topside sounders stimulate electrostatic waves in addition to the electromagnetic waves which are used to obtain electron density profiles from the satellite altitude down to the altitude of the  $F_2$  peak. The electrostatic waves give rise to plasma resonances. These resonances, which persist for many milliseconds after the 0.1 msec stimulating sounder pulse, are illustrated in Figure 1. Explanations have been offered for all of them except for the resonance observed at the ambient electron cyclotron frequency  $f_H$ . Most have been interpreted by linear theory, but some are due to nonlinear effects (see the



reviews by Muldrew [1972], McAfee [1974], and Benson [1977]). Those where linear explanations have been proposed are the resonances at the electron plasma frequency  $f_p$ , the harmonics of  $f_H$ , the upper hybrid frequency  $f_T = (f_p^2 + f_H^2)^{1/2}$ , and the sequence of resonances designated by  $f_{Qn}$ . The latter resonances occur at frequencies above  $f_T$  and between the harmonics of  $f_H$  corresponding to the maximum frequency bump in the dispersion curves for  $\vec{k} \cdot \vec{B} = 0$  where  $\vec{k}$  is the wave vector and  $\vec{B}$  is the ambient magnetic field. The resonances at  $f_p$ ,  $f_T$ , and  $nf_H$  ( $n = 3$  and  $4$ ) have been interpreted in terms of the reception of oblique echoes of electrostatic waves stimulated by the sounder pulse, the ionospheric reflection being due either to gradients in the ambient electron density  $N_e$  (for  $f_p$  and  $f_T$ ) or in  $\vec{B}$  (for  $nf_H$ ). The resonances observed at  $f_{Qn}$  and the higher order  $nf_H$  resonances have been interpreted either in terms of matching a component of the wave group velocity,  $\vec{V}_g$  to the satellite velocity  $\vec{V}_s$  or, for the very short duration resonances, in terms of waves with  $|\vec{V}_g| \ll |\vec{V}_s|$  that remain near the antenna for only a short time due to the satellite's motion. The  $2f_H$  resonance is not completely understood, but it appears certain to be the result of low velocity waves that can remain in the vicinity ( $\sim 1$  km) of the satellite antenna for several tens of milliseconds.

Linear theory cannot explain the resonance labeled as  $f_{D1}$  in Figure 1. This resonance, first observed and designated as the diffuse resonance by Nelms and Lockwood [1967] and later identified as the first member of a sequence of diffuse resonances by Oya [1970], is the plasma resonance of main interest to the present paper. Linear theory fails because the electrostatic waves that are capable of propagating with small Landau damping quickly leave the vicinity of the sounder antenna and they are not subject to ionospheric reflections as are the electrostatic waves stimulated near resonant frequencies in the medium. Oya [1971a] pointed out that the longest-lasting portion of the  $f_{D1}$  resonance corresponds to the electron cyclotron harmonic wave which has the lowest threshold electron temperature

anisotropy ( $T_{\perp}/T_{\parallel} > 1$  for the Harris instability [Harris 1959], where the perpendicular and parallel symbols refer to directions relative to  $\vec{B}$ . The original anisotropy is due to cyclotron heating resulting from the strong RF pulse. After the turnoff of the RF pulse, however, the anisotropic distribution disappears within a few milliseconds because of the convective loss of the hot electrons away from the satellite region. A nonlinear mechanism involving the 3-wave coupling process was proposed by Oya [1971a] in order to explain  $f_{D1}$  resonant time durations that far exceeded this initial period of sounder-stimulated temperature anisotropy. There are some theoretical and observational problems with this model which will be discussed in the next section. In order to overcome these problems, an alternative nonlinear model (based on the feeding of energy to the  $f_{D1}$  wave via nonlinear Landau damping of the sounder-stimulated  $2f_H$  wave) will be presented in this paper. Nonlinear Landau damping of plasma waves, i.e., the nonlinear wave particle interaction involving the beat of two waves and the thermal particles, has been observed in laboratory experiments [Chang and Porkolab, 1970 and 1972; Gentle and Malein, 1971; Ikezi and Kiwamoto, 1971]. The self-consistent model to be proposed in this paper may be considered to provide, though indirectly, the first support for the existence of this nonlinear process in space.

### Previous Model for the Diffuse Resonance

The work of Oya [1971a] represents the only previous theoretical attempt to explain the long time duration diffuse resonances observed by the Alouette and ISIS topside sounders. His model consists of the following four basic steps.

(1) During the initial 100  $\mu$ sec rf pulse, an anisotropic bi-Maxwellian electron velocity distribution [ $T_{\perp}/T_{\parallel} \gtrsim 5$ ] is produced.

(2) The anisotropic velocity distribution drives the Harris instability to generate the  $f_{Q3}$  and  $f_{D1}$  waves.

(3) A  $2f_H$  wave is generated at the expense of the  $f_{Q3}$  and  $f_{D1}$  waves by a nonlinear wave-wave coupling process.

(4) The  $2f_H$  wave resonantly interacts with electrons to keep  $T_{\perp}/T_{\parallel} \gg 1$  so as to maintain the unstable state that generates the  $f_{D1}$  and  $f_{Q3}$  waves of step (2).

While Oya stressed the importance of the nonlinear wave-wave interaction, energy flow considerations indicate that the  $2f_H$  wave cannot be generated by the process of step (3) above. This statement is based on the Manley-Rowe relations concerning the energy change  $\Delta W$  and angular frequency  $\omega = 2\pi f$  of three waves interacting nonlinearly, i.e.,

$$\pm \frac{\Delta W_1}{\omega_1} = \pm \frac{\Delta W_2}{\omega_2} = \pm \frac{\Delta W_3}{\omega_3} = \text{constant}$$

[Manley and Rowe, 1956] combined with the conservation of energy equation

$$\Delta W_1 + \Delta W_2 + \Delta W_3 = 0$$

which yields the expression

$$\pm \omega_1 \pm \omega_2 \pm \omega_3 = 0.$$

When  $\omega_3 > \omega_2 > \omega_1$ , the choice of signs must be such that  $\omega_1 + \omega_2 - \omega_3 = 0$  which indicates that the Manley-Rowe relations must be written as

$$\frac{\Delta W_1}{\omega_1} = \frac{\Delta W_2}{\omega_2} = -\frac{\Delta W_3}{\omega_3}.$$

Thus, either waves 1 and 2 grow at the expense of wave 3, i.e.,  $\Delta W_3 < 0$ , or wave 3 grows ( $\Delta W_3 > 0$ ) at the expense of waves 1 and 2. Since  $f_{Q3} > 2f_H > f_{D1}$  in the Alouette/ISIS experiment, either the  $f_{Q3}$  wave must grow at the expense of the  $2f_H$  and  $f_{D1}$  waves or vice-versa; the  $2f_H$  wave cannot grow at the expense of the  $f_{Q3}$  and  $f_{D1}$  waves.

Since the absorption of wave energy is strong near  $2f_H$  [Oya 1971a] the main significance of  $2f_H$  is in its role as the frequency of the beat wave in Oya's nonlinear wave particle interaction involving the  $f_{Q3}$  and  $f_{D1}$  waves. It is through this Landau damping process that the required temperature anisotropy is maintained in step (4) of his model. Though a temperature anisotropy of  $T_\perp/T_\parallel = 5$  leads to the Harris instability for the  $f_{D1}$  wave [Oya, 1971] in order to have a substantial growth rate  $\gamma$ , i.e.,  $\gamma > 10^{-2} \omega_H$ , an anisotropy of  $T_\perp/T_\parallel \gtrsim 10$  is required [Kiwamoto, 1978]. An extension of the calculations of Kiwamoto [1978] indicate that much larger anisotropies are required for substantial growth of the Harris instability in the  $f_{Q3}$  frequency domain (the maximum  $\gamma$  is only about  $10^{-5} \omega_H$  when  $T_\perp/T_\parallel = 10$ ). Thus there is a basic inconsistency in the Oya model in that the anisotropy required to drive the  $f_{D1}$  wave unstable is maintained by the absorption of energy from the  $2f_H$  wave which, in turn, is the result of nonlinear processes involving the instability driven  $f_{Q3}$  wave. Such a process is in violation of the theory of weak turbulence since it would require the energy density of either the  $2f_H$  wave or the  $f_{Q3}$  wave to be larger than the thermal energy density of the medium.

In addition to the above theoretical arguments, the observational evidence does not support the concept of a nonlinear 3-wave interaction involving the  $f_{Q3}$  wave as an energy source. Figure 2 shows a typical example of the amplitude evolution of the  $f_{Q3}$ ,  $2f_H$ , and  $f_{D1}$  waves observed during a period of sounder mixed-mode operation. In this mode of ISIS 1 operation, the frequency of the stimulating RF pulse is fixed at 0.82 MHz while the receiver frequency is swept over the normal frequency range. Since the plasma perturbation is identical for all values of the receiver frequency, mutual interactions among the three waves, if they exist, should be clearly observed. The  $f_{Q3}$  wave, however, shows a build up of signal strength after the termination of the sounder-pulse whereas the  $f_{D1}$  and  $2f_H$  wave are observed initially with a large amplitude. This behavior is opposite to what would be expected if the  $f_{D1}$  wave grew at the expense of the  $f_{Q3}$  wave in a nonlinear 3-wave interaction. The results presented in Figure 2 are not significantly dependent on the position of the center of the wide frequency spectrum of the transmitted pulse, relative to the resonant frequencies of concern, because similar results are obtained on many consecutive ionograms recorded during changing plasma conditions.

The observational evidence does not even support the possibility of the sounder-stimulated  $f_{Q3}$  wave as the required energy source. The  $f_{Q3}$  wave is sometimes missing or very weak when the  $f_{D1}$  wave is present and the duration of the  $f_{Q3}$  wave is usually less than the duration of the  $f_{D1}$  wave when long duration  $f_{D1}$  waves are observed (see Figure 3). In addition, the results of the detailed frequency scaling presented in Figure 4 indicate that the 3 wave coupling process proposed by Oya is not strictly satisfied since  $f_{D1} + 2f_H > f_{Q3} = (f_{Q3})_{\max}$  where  $(f_{Q3})_{\max}$  is the maximum frequency of the Bernstein mode ( $\vec{k} \cdot \vec{B} = 0$ ) associated with  $f_{Q3}$ . Therefore, the nonlinear model involving the  $f_{Q3}$  wave has many theoretical and observational difficulties to overcome.



The problem of the long lasting  $f_{D1}$  resonance can still be solved in the framework of the theory of weak turbulence, however, if the sounder-stimulated  $2f_H$  wave is used as the energy source. The observations (Figure 3) indicate that the  $2f_H$  wave is always present when the  $f_{D1}$  wave is present and that the duration of the former always exceeds the duration of the latter. The model proposed in this paper uses the sounder-stimulated  $2f_H$  wave as an energy source for sustaining the  $f_{D1}$  wave, and it does not invoke the  $f_{Q3}$  wave. In addition, the resonant time duration is determined by the transit-time for the wave to propagate from the instability generating region back to the antenna. While the importance of this transit-time effect was emphasized by Oya and Benson [1972] and the importance of the sounder-stimulated  $2f_H$  wave was emphasized by Benson [1974], in neither case was the problem of the Landau damping of these waves considered. The latter effect was considered by the work of Kiwamoto [1978] which represents an integral part of the present model.

## PROPOSED THEORY

In the present model for the diffuse resonance it is assumed that the  $f_{D1}$  wave is an unstable electron-cyclotron harmonic wave. This wave is driven by an anisotropic electron velocity distribution created by the high-power sounder pulse. The frequency of the  $f_{D1}$  resonance is determined by the instability. The time duration is attributed to the transit-time of the  $f_{D1}$  wave propagating back to the antenna. A linear theoretical consideration, however, indicates that due to strong Landau damping the instability-produced waves have a shorter lifetime than the observed duration of the  $f_{D1}$  resonance. In order to overcome this dissipation process, a mechanism is required to feed energy to the  $f_{D1}$  wave. A nonlinear Landau damping of the sounder-stimulated  $2f_H$  wave into the  $f_{D1}$  wave is invoked for this purpose.

Instability. An actual experimental condition of fundamental importance to the proposed model is that the transmitted RF Sounder-pulse has a finite frequency width and several harmonic components [Franklin and Maclean, 1969]. When the center frequency of the RF pulse is between  $f_H$  and  $2f_H$ , as in the normal case of observing the  $f_{D1}$  resonance, a substantial amount of power is transmitted near  $2f_H$  [Benson, 1974] which produces an anisotropy by electron cyclotron resonance heating. (The transverse acceleration will be strongest for those electrons which spend the longest time within the RF region; thus, the electrons with small parallel velocity  $v_{||}$  will gain a high perpendicular velocity  $v_{\perp}$ .) Electron cyclotron resonance heating has been proved to be highly efficient in many laboratory experiments [e.g., Dandle et al., 1964]. Oya [1971a] assumed that the heating would result in a bi-Maxwellian velocity distribution which leads to the Harris instability of the  $f_{D1}$  wave observed in space. Fredricks [1971] claimed that a loss cone type (ring) velocity distribution would be responsible for  $f_{D1}$  wave generation.

After the RF pulse is turned off, however, the temperature anisotropy decreases to 1 in a few milliseconds [Oya, 1971a]. A linear analysis shows that the propagation characteristics (the growth rate  $\gamma$  and the angle  $\theta$  between  $\vec{k}$  and  $\vec{B}$ ) change appreciably during the relaxation of the anisotropy [Kiwamoto, 1978]. Most of the wave energy disappears due to strong Landau damping, and the frequency of the dominant wave shifts downward while the propagation angle changes from around  $60^\circ$  to  $80^\circ$ . Even the waves propagating at this larger angle experience strong Landau damping. Thus, without any additional mechanism, all of the  $f_{D1}$  wave energy will damp away in a few milliseconds.

Transit Time. Numerical calculations of the dispersion equation show that the group velocity of the  $f_{D1}$  wave is a few tenths of the electron thermal velocity  $u = (T_e/m)^{1/2}$  but more than 10 times the satellite velocity. This finite  $\vec{V}_g$  causes a convective loss which effectively decreases the  $f_{D1}$  wave energy density near the antenna. For certain values of the propagation angle  $\theta$ , however,  $V_{g\parallel}$  becomes zero so that this component of the convective loss can be neglected. Figure 5 shows  $\theta V_{g\parallel} = 0$  as a function of the wave frequency for various values of the plasma parameter  $\omega_p/\omega_H$  typical in the upper ionosphere. It is seen that a relatively wide range of values for this angle occurs near  $\omega/\omega_H = 1.6$ , which coincides with the frequency range where the  $f_{D1}$  resonance is observed with the longest duration [Oya, 1971a]. The value  $\theta V_{g\parallel} = 0 \simeq 80^\circ$ , corresponding to the minimum frequency satisfying negligible parallel convective loss, also coincides with the propagation angle of the most persistent unstable wave after the relaxation of the instability stimulated by the sounder pulse [Kiwamoto, 1978].

The maximum observable time duration of the  $f_{D1}$  resonance will be determined by the convective loss in the perpendicular direction, i.e., by  $r/V_{g\perp}$  where  $r$  is the radius of the hot plasma region and  $V_{g\perp}$  is the component of the  $f_{D1}$  group velocity perpendicular to  $\vec{B}$ . Using typical values of  $V_{g\perp} \simeq u/3$  and  $T_e \simeq 0.3$  eV, gives  $r \simeq 2$  km for the maximum



observed time duration of 25 msec. The required radius of the hot region is therefore  $\lesssim 30$  times the tip-to-tip Alouette/ISIS antenna length of 73 m. For the much more common duration of 11 msec, the required radius becomes 0.9 km or about 10 antenna lengths. In either case the distances must be considered as upper-limits because transit-time effects of the RF field to initiate the hot region remote from the antenna may be significant [Oya and Benson, 1972] but have not been included.

Nonlinear Landau Damping. The time duration of the  $f_{D1}$  resonance will be considerably less than this upper limit of  $r/V_{g1}$  set by perpendicular convection unless there is an additional energy feeding mechanism. A mechanism involving a transfer of energy from the sounder-stimulated  $2f_H$  wave to the instability-generated  $f_{D1}$  wave is consistent with observations because the duration of the  $2f_H$  resonance is always longer than that of the  $f_{D1}$  resonance (see Figure 3). The long duration of the  $2f_H$  resonance indicates that it is a plasma wave which, by itself, interacts with the particles very weakly. Thus the  $2f_H$  wave is assumed to be a Bernstein wave with  $\vec{k}$  almost perpendicular to  $\vec{B}$ . Such a wave cannot be produced by instabilities because an instability implies a strong wave-particle interaction. It can, however, be emitted from the antenna as a broad band component at  $2f_H$  as suggested by Benson [1974]. Thus the energy radiated at  $2f_H$  due to this broad frequency spectrum enters into the present model in 2 ways: the strong RF near field and obliquely propagating electrostatic waves provide the cyclotron heating needed to initiate an anisotropic electron velocity distribution, while the transmitted electrostatic wave energy with  $\theta \simeq 90^\circ$  experiences negligible wave-particle interactions and thus can persist for a long time. The former leads to the plasma wave instability of the present model, whereas the latter is involved in the nonlinear wave-particle interaction.

Essentially the above nonlinear mechanism is the process of nonlinear Landau damping (e.g., Kadomtsev [1965], Sagdeev and Galeev [1969], and Hasegawa [1975]) of energy

from the  $2f_H$  wave to the  $f_{D1}$  wave and the thermal electrons in resonance with the beat oscillation between the two waves according to the resonant condition

$$\omega' - n\omega_H - k_{\parallel}'v_{\parallel} = 0 \quad (n = 0, \pm 1, \pm 2, \dots) \quad (1)$$

where  $v_{\parallel}$  is the parallel component of the thermal electron velocity and the beat oscillation  $\vec{k}' = \vec{k} - \vec{k}'', \omega' = \omega - \omega''$  is produced by the nonlinear interaction between the  $2f_H$  wave  $(\vec{k}, \omega)$  and the  $f_{D1}$  wave  $(\vec{k}'', \omega'')$ .

The wave energy densities  $W_k$  and  $W_{k''}$  of the  $2f_H$  and  $f_{D1}$  waves, respectively, which are normalized by the perpendicular electron thermal energy density as  $W = |E|^2/8\pi N_e T_{\perp}$ , are described by the following equations

$$\frac{\partial W_k}{\partial \tau} = 2\Gamma_k W_k - A_{k,k''} W_{k''} W_k \quad (2)$$

$$\frac{\partial W_{k''}}{\partial \tau} = 2\Gamma_{k''} W_{k''} + A_{k'',k} W_k W_{k''} \quad (3)$$

where  $\tau = t\omega_H$  and  $\Gamma = \gamma/\omega_H$  are the normalized time and wave amplitude growth rates.

Convective terms are not included in the above equations because the effects of convection, which have been considered separately in the previous subsection, are to set an upper limit for the time duration of the  $f_{D1}$  resonance; the goal of the nonlinear interaction described by (2) and (3) is to maintain the  $f_{D1}$  wave during the convective process. The physics in the nonlinear wave-particle interaction is contained in the nonlinear coupling coefficients  $A_{k,k''}$  and  $A_{k'',k}$ .

In order for the  $f_{D1}$  wave to be sustained, the nonlinear energy feeding rate must balance the linear damping rate, i.e., from (3)

$$2\Gamma_{k''} W_{k''} + A_{k'',k} W_k W_{k''} = 0. \quad (4)$$

Then the required minimum normalized energy density of the  $2f_H$  wave is

$$W_{km} = -\frac{2\Gamma_k''}{A_{k,k}''} \quad (5)$$

In (2)  $\Gamma_k = 0$  because the  $2f_H$  wave is assumed to be nearly a Bernstein-mode wave. If  $W_k''$  is too large, the nonlinear damping term  $A_{k,k}''W_k''$  will decrease  $W_k$  quickly below  $W_{km}$ . Accordingly, in order to ensure the long persistence of the  $f_{D1}$  wave,  $W_k''$  must be smaller than the critical value

$$W_{k''c} = \frac{1}{A_{k,k}''\tau_{obs}} \quad (6)$$

where  $\tau_{obs}$  is the observed normalized duration of the  $f_{D1}$  resonance.

Equation (6) determines the upper limit to the  $f_{D1}$  wave normalized energy density. It indicates that in order for the  $f_{D1}$  wave to be observable with a long time duration,  $A_{k,k}''$  should not be too large. On the other hand, for weak turbulence theory to hold,  $W_{km}$  in (5) must be smaller than  $10^{-2}$  [Kadomtsev, 1965]. Because  $A_{k,k}'' \sim A_{k,k}''$  the damping rate  $-\Gamma_k''$  of the  $f_{D1}$  wave must be reasonably small. Accordingly, an important feature required in the present model is that the nonlinear effect should be only moderately efficient so as not to exhaust the  $2f_H$  wave energy in a short time while overcoming the linear damping of the  $f_{D1}$  wave.

## QUANTITATIVE DISCUSSION OF NONLINEAR LANDAU DAMPING

**Formulation.** A general expression for the nonlinear coupling coefficient of nonlinear Landau damping of an electrostatic wave has been obtained by Porkolab and Chang [1972]. Some simplifications to their general expressions are possible in the  $f_{D1}$  resonance problem. In this section, explicit expressions for the nonlinear coupling coefficients  $A_{k,k''}$  and  $A_{k'',k}$  introduced in (2) and (3) will be obtained starting from the general expression.

In a hypothetical case where linear Landau damping is absent, i.e., only nonlinear processes are present, the total photon numbers of the  $2f_H$  and  $f_{D1}$  waves must be conserved in the nonlinear Landau damping process [Sagdeev and Galeev 1969]. From this condition the coupling coefficients  $L_{k'',k}$  and  $L_{k,k''}$ , in the notation of Porkolab and Chang [1972], are related by the expression

$$L_{k'',k} = -L_{k,k''} \quad (7)$$

The coupling coefficients  $A_{k'',k}$  and  $A_{k,k''}$  from (2) and (3) are related to the  $L$ 's as follows:

$$\begin{aligned} A_{k'',k} &= \frac{L_{k'',k}}{\omega_H} \left| \frac{\partial \epsilon}{\partial \omega} \right| N_e T_1 \\ &= -\frac{L_{k,k''}}{\omega_H} \left| \frac{\partial \epsilon}{\partial \omega} \right| N_e T_1 \end{aligned} \quad (8)$$

$$A_{k,k''} = -\frac{L_{k,k''}}{\omega_H} \left| \frac{\partial \epsilon}{\partial \omega''} \right| N_e T_1 \quad (9)$$

where  $\epsilon$  is the dielectric function;  $\omega$  and  $\omega''$  are the real part of the angular frequency of the  $2f_H$  and  $f_{D1}$  waves, respectively;  $\frac{\partial \epsilon}{\partial \omega}$  and  $\frac{\partial \epsilon}{\partial \omega''}$  are the frequency derivatives of the Hermitian part of the dielectric function at  $(\vec{k}, \omega)$  and  $(\vec{k}'', \omega'')$ , respectively; and  $N_e$  is the electron

density. The expression for  $L_{k,k''}$  is given in equation (24) of Porkolab and Chang [1972] (hereafter referred to as PC-24) as

$$L_{k,k''} = \frac{4\omega_p^4}{\left| \frac{\partial \epsilon}{\partial \omega} \right| \left| \frac{\partial \epsilon}{\partial \omega''} \right| k'^2 k^2 m N_e} \operatorname{Im} \left\{ \frac{\omega_p^2}{k'^2 \epsilon(k', \omega')} H_{k,k'',k'} H_{k',k,-k''} + T_{k,k''} \right\} \quad (10)$$

where  $m$  is the electron mass,  $\omega' = \omega - \omega''$ , and  $\vec{k}' = \vec{k} - \vec{k}''$ . The elements  $H_{k,k'',k'}$  and  $H_{k',k,-k''}$  are given by PC-19 and 20 as

$$H_{k,k'',k'} = \sum_{s,q} \int_{-\infty}^{+\infty} dv_{\parallel} \int_0^{\infty} dv_{\perp} v_{\perp} \tilde{Y}_{q,s}(v_{\perp}) \left\{ \frac{k_{\perp} [k'_{\perp} Z_{s-q}(k'') + k''_{\perp} Z_q(k')] }{(\omega - s\omega_H - k_{\parallel} v_{\parallel})^2 - \omega_H^2} \right. \\ \left. + \frac{k_{\parallel} [k'_{\parallel} Z_{s-q}(k'') + k''_{\parallel} Z_q(k')] }{(\omega - s\omega_H - k_{\parallel} v_{\parallel})^2} \right\}, \quad (11)$$

and

$$H_{k',k,-k''} = - \sum_{n,p} \int_{-\infty}^{+\infty} dv_{\parallel} \int_0^{\infty} dv_{\perp} v_{\perp} \tilde{Y}_{n,p}(v_{\perp}) \left\{ \frac{k'_{\perp} [k''_{\perp} Z_p(k) - k_{\perp} Z_{p-n}(k'')] }{(\omega' - n\omega_H - k'_{\parallel} v_{\parallel})^2 - \omega_H^2} \right. \\ \left. - \frac{k'_{\parallel} [k''_{\parallel} Z_p(k) - k_{\parallel} Z_{p-n}(k'')] }{(\omega' - n\omega_H - k'_{\parallel} v_{\parallel})^2} \right\}, \quad (12)$$

where

$$\tilde{Y}_{n,p}(v_{\perp}) = J_n \left( \frac{k'_{\perp} v_{\perp}}{\omega_H} \right) J_{p-n} \left( \frac{k''_{\perp} v_{\perp}}{\omega_H} \right) J_p \left( \frac{k_{\perp} v_{\perp}}{\omega_H} \right), \quad (13)$$

and



$$Z_n(k) = \frac{\frac{n\omega_H}{v_\perp} \frac{\partial F_0}{\partial v_\perp} + k_\parallel \frac{\partial F_0}{\partial v_\parallel}}{\omega - n\omega_H - k_\parallel v_\parallel} \equiv \frac{\mathcal{Z}_n(k)}{\omega - n\omega_H - k_\parallel v_\parallel}, \quad (14)$$

and  $s, \ell, n$ , and  $p$  are all possible integers from  $-\infty$  to  $+\infty$ . Here  $J_n$  is the Bessel function of the  $n$ th order, and  $F_0(v_\parallel, v_\perp)$  is the electron velocity distribution function normalized as

$$\int_{-\infty}^{+\infty} dv_\parallel \int_0^\infty dv_\perp v_\perp F_0(v_\parallel, v_\perp) = 1.$$

From PC-22, the last element  $T_{k,k}''$  in (10) is given by

$$T_{k,k}'' = \sum_{n,b,c} \int_{-\infty}^{+\infty} dv_\parallel \int_0^\infty dv_\perp v_\perp W(v_\perp) \vec{k} \cdot \vec{\mathcal{D}} \cdot \vec{k}'', \quad (15)$$

where

$$W(v_\perp) = J_n \left( \frac{k_\perp v_\perp}{\omega_H} \right) J_b \left( \frac{k_\perp v_\perp}{\omega_H} \right) J_{b-c} \left( \frac{k_\perp'' v_\perp}{\omega_H} \right) J_{n-c} \left( \frac{k_\perp'' v_\perp}{\omega_H} \right), \quad (16)$$

and

$$\vec{k} \cdot \vec{\mathcal{D}} \cdot \vec{k}'' = k_\perp k_\perp'' \mathcal{D}_{\perp,\perp} + k_\perp k_\parallel'' \mathcal{D}_{\perp,\parallel} + k_\parallel k_\perp'' \mathcal{D}_{\parallel,\perp} + k_\parallel k_\parallel'' \mathcal{D}_{\parallel,\parallel}. \quad (17)$$

The four elements of the tensor  $\vec{\mathcal{D}}$  are given in PC-23. Calculations indicate that for conditions appropriate to the present problem (see the discussion in the following paragraph), the relevant element is

$$\mathcal{D}_{\perp,\perp} = \sum_{r=1} \frac{1}{2\omega_H} \frac{rk_\perp [k_\perp'' Z_n(k) - k_\perp Z_{n-c}(k'')]}{[(\omega - (b+r)\omega_H - k_\parallel v_\parallel)^2 - \omega_H^2] [\omega' - (c+r)\omega_H - k_\parallel' v_\parallel]}$$

$$= \frac{k_1'' [k_1'' Z_n(k) - k_1 Z_{n-c}(k'')] }{[(\omega - b\omega_H - k_{\parallel} v_{\parallel})^2 - \omega_H^2] [(\omega' - c\omega_H - k_{\parallel}' v_{\parallel})^2 - \omega_H^2]} \quad (18)$$

The above expression for the coupling coefficient can be considerably simplified for the conditions appropriate to the Alouette/ISIS diffuse resonance problem. Since  $\vec{k} \cdot \vec{B} \simeq 0$ ,  $k_{\perp} R \ll 1$  where  $R = a_1/\omega_H$  is the electron cyclotron radius, and the frequency deviations of the  $2f_H$  resonance from the second harmonic of the ambient  $f_H$  value is less than 0.2% [see review by Benson, 1977], the following approximations can be used for the  $2f_H$  wave:

$$|k_{\parallel}/k| \ll 1, \quad (19)$$

$$\omega \simeq 2\omega_H, \quad (20)$$

and since  $|k| \ll |k''|$  and  $\omega \gg k_{\parallel} v_{\parallel}$ , i.e., there are no resonant particles for this near-Bernstein mode wave,

$$k \rightarrow 0 \quad (21)$$

can be used in many of the above equations. Therefore

$$\vec{k}' = -\vec{k}'' \quad (22)$$

Introducing (21) into (13) and (16) yields

$$\tilde{Y}_{n,p}(v_{\perp}) = \begin{cases} 0 & (\text{if } p \neq 0) \\ J_n^2 \left( \frac{k_1'' v_{\perp}}{\omega_H} \right) & (\text{if } p = 0) \end{cases} \quad (13')$$

and

$$W(v_{\perp}) = \begin{cases} J_c^2 \left( \frac{k_{\perp}'' v_{\perp}}{\omega_H} \right) & (\text{if } n = b = 0) \\ 0 & (\text{if } n \neq 0 \text{ or } b \neq 0) \end{cases} \quad (14')$$

Using (13) and assumption (19), (11) is reduced to

$$\begin{aligned} H_{k,k'',k'} = & \frac{k_{\perp}}{\omega^2 - \omega_H^2} \sum_{\epsilon} \int_{-\infty}^{+\infty} dv_{\parallel} \int_0^{\infty} dv_{\perp} v_{\perp} \left\{ k_{\perp}' J_{-\epsilon}^2 \left( \frac{k_{\perp}' v_{\perp}}{\omega_H} \right) Z_{-\epsilon}(k'') \right. \\ & \left. - k_{\perp}' J_{\epsilon}^2 \left( \frac{k_{\perp}' v_{\perp}}{\omega_H} \right) Z_{\epsilon}(k') \right\} \\ & + \frac{k_{\parallel}}{\omega^2} \sum_{\epsilon} \int_{-\infty}^{+\infty} dv_{\parallel} \int_0^{\infty} dv_{\perp} v_{\perp} \left\{ k_{\parallel}' J_{-\epsilon}^2 \left( \frac{k_{\perp}' v_{\perp}}{\omega_H} \right) Z_{-\epsilon}(k'') \right. \\ & \left. - k_{\parallel}' J_{\epsilon}^2 \left( \frac{k_{\perp}' v_{\perp}}{\omega_H} \right) Z_{\epsilon}(k') \right\}. \end{aligned}$$

Using the expression from PC-20 and 21 for the dielectric function

$$\epsilon(\vec{k}, \omega) = 1 + \frac{\omega_p^2}{k^2} \sum_{n=-\infty}^{+\infty} \int_{-\infty}^{+\infty} dv_{\parallel} \int_0^{\infty} dv_{\perp} v_{\perp} J_n^2 \left( \frac{k_{\perp} v_{\perp}}{\omega_H} \right) Z_n(k), \quad (23)$$

the element  $H_{k,k'',k'}$  can be rewritten as

$$H_{k,k'',k'} = - \frac{k''^2}{\omega_p^2} \left( \frac{k_{\perp} k_{\perp}'}{\omega^2 - \omega_H^2} + \frac{k_{\parallel} k_{\parallel}'}{\omega^2} \right) \epsilon(\vec{k}', \omega') \quad (24)$$

because the  $f_{D1}$  wave must satisfy the dispersion relation, i.e.,

$$\epsilon(\vec{k}'', \omega'') = 0. \quad (25)$$



Since  $H_{\mathbf{k}, \mathbf{k}'', \mathbf{k}'} / \epsilon(\vec{\mathbf{k}}', \omega')$  in (24) is real, only the imaginary part of  $H_{\mathbf{k}', \mathbf{k}, -\mathbf{k}''}$  is necessary for (10). Taking only terms in (12) leading to residues of first order poles satisfying the resonance condition (1) and using PC-25 together with (13'), (20), and (22) yields

$$\begin{aligned} \text{Im } H_{\mathbf{k}', \mathbf{k}, -\mathbf{k}''} = & \frac{\pi k'_1 k_1}{2\omega_H^2} \sum_n \int_{-\infty}^{\infty} dv_{\parallel} \int_0^{\infty} dv_1 v_1 J_n^2 \left( \frac{k'_1 v_1}{\omega_H} \right) \\ & \times \left\{ \delta(\omega' - (n+1)\omega_H - k'_{\parallel} v_{\parallel}) \cdot \left[ \frac{k'_1}{2k_1} \mathcal{B}_0(k) - \mathcal{B}_{-n}(k'') \right] \right. \\ & \left. - \delta(\omega' - (n-1)\omega_H - k'_{\parallel} v_{\parallel}) \left[ \frac{k'_1}{2k_1} \mathcal{B}_0(k) - \frac{1}{3} \mathcal{B}_{-n}(k'') \right] \right\}. \quad (26) \end{aligned}$$

Of the four terms in (17), the contribution of the first term is dominant; the contribution from the remaining terms are smaller by a factor of  $k_{\parallel}/k$  [see condition (19)]. Using the assumptions (19) to (22), the imaginary part of  $\vec{\mathbf{k}} \cdot \vec{\mathcal{D}} \cdot \vec{\mathbf{k}}''$  in (17) is derived as

$$\begin{aligned} \text{Im } \vec{\mathbf{k}} \cdot \vec{\mathcal{D}} \cdot \vec{\mathbf{k}}'' = & \frac{\pi(k_1 k'_1)^2}{6\omega_H^4} \left\{ \delta(\omega' - (c+1)\omega_H - k'_{\parallel} v_{\parallel}) \cdot \left[ \frac{k'_1}{2k_1} \mathcal{B}_0(k) - \mathcal{B}_{-c}(k'') \right] \right. \\ & \times \left[ 1 - \frac{3}{2} \frac{k'_{\parallel} k_1}{k'_1 k_{\parallel}} \left\{ \frac{\omega'}{\omega_H} - (c+1) - \frac{k'_{\parallel}}{4k_{\parallel}} \left( \frac{k_1 a_1}{\omega_H} \right)^2 \left( \frac{\omega_H^2}{\omega_p^2} - \frac{1}{3} \right)^{-1} \right\}^{-1} \right] \\ & \left. - \delta(\omega' - (c-1)\omega_H - k'_{\parallel} v_{\parallel}) \cdot \left[ \frac{k'_1}{2k_1} \mathcal{B}_0(k) - \frac{1}{3} \mathcal{B}_{-c}(k'') \right] \right\}. \quad (27) \end{aligned}$$

Here, the term including the finite wavelength effect  $(k_1 R)^2$  was derived from a slight difference between  $\omega$  and  $2\omega_H$

$$\omega - 2\omega_H \simeq -\frac{\omega_H}{4} \left( \frac{k_1 a_1}{\omega_H} \right)^2 \left/ \left( \frac{1}{3} - \frac{\omega_H^2}{\omega_p^2} \right) \right. \quad (28)$$

Introducing (24), (26), and (27) into (10), yields the following expression for the nonlinear coupling coefficient  $A_{k'',k}$  in (8):

$$A_{k'',k} = \pi \left( \frac{\omega_p}{\omega_H} \right)^4 \left( \frac{k_1 k_1''}{k k''} \right)^2 \frac{k_1' k_1}{k_1' k_1} \frac{a_1^2}{\omega_H \left| \frac{\partial \epsilon}{\partial \omega''} \right|} \\ \times \sum_{\ell=-\infty}^{+\infty} \int_{-\infty}^{+\infty} dv_{\parallel} \int_0^{\infty} dv_1 v_1 \delta(\omega' - (\ell + 1)\omega_H - k_{\parallel}' v_{\parallel}) J_{\ell}^2 \left( \frac{k_1'' v_1}{\omega_H} \right) \\ \times \left[ \frac{k_1''}{2k_1} \mathcal{F}_0(k) - \mathcal{F}_{-\ell}(k'') \right] \cdot \left[ \frac{\omega'}{\omega_H} - (\ell + 1) - \frac{k_{\parallel}'}{4k_{\parallel}} \left( \frac{k_1 a_1}{\omega_H} \right)^2 \left( \frac{\omega_H^2}{\omega_p^2} - \frac{1}{3} \right)^{-1} \right]^{-1} \quad (29)$$

The functions  $\mathcal{F}_0(k)$  and  $\mathcal{F}_{-\ell}(k'')$ , as defined in (14), contain the information about the electron velocity distribution, the non-resonant part of which has been assumed to be bi-Maxwellian, i.e.,

$$F_0(v_{\parallel}, v_1) = \frac{1}{(2\pi)^{3/2}} \frac{1}{a_{\parallel} a_1^2} \exp \left[ -\frac{1}{2} \left( \frac{v_{\parallel}^2}{a_{\parallel}^2} + \frac{v_1^2}{a_1^2} \right) \right] \quad (30)$$

in the dispersion relation (28) for the  $2f_H$  wave. If the same distribution is assumed for the resonant part, (29) reduces to

$$A_{k'',k} = \frac{2\sqrt{\pi}}{3} \left( \frac{\omega_p}{\omega_H} \right)^4 \frac{1}{\omega_H \left| \frac{\partial \epsilon}{\partial \omega''} \right|} \left( \frac{k_1''}{k''} \right)^2 \sum_{\ell=-\infty}^{+\infty} e^{-\xi} I_{\ell}(\xi) Q_{\ell}, \quad (31)$$

where

$$Q_\ell = 6 \left( \frac{1}{3} - \frac{\omega_H^2}{\omega_p^2} \right) \frac{\omega_H}{k_\perp a_\perp} \frac{\omega_H}{k'_\perp a_\perp} \left[ \left( \frac{a_\perp}{a_\parallel} \right)^2 x_{\ell+1} + \ell \frac{\omega_H}{\sqrt{2} a_\parallel k'_\parallel} \right] e^{-x_{\ell+1}^2}, \quad (32)$$

$$x_\ell = \frac{\omega' - \ell \omega_H}{\sqrt{2} a_\parallel k'_\parallel},$$

and

$$\xi = \left( \frac{k'_\perp a_\perp}{\omega_H} \right)^2.$$

The other coefficient  $A_{k,k''}$  in (9) is obtained by replacing  $\left| \frac{\partial \epsilon}{\partial \omega} \right|$  in (31) by  $\left| \frac{\partial \epsilon}{\partial \omega} \right|$ .

**Numerical results.** Although (31) is valid for a bi-Maxwellian velocity distribution, the long term behavior of the  $f_{D1}$  wave should be considered in an isotropic velocity distribution, i.e.,  $a_\parallel = a_\perp = a$ . The normalized wave number  $kR$  of the  $2f_H$  wave is taken as 0.02; a lower limit that assures the validity of the electrostatic approximation [Oya, 1971b]. The dispersion relation ( $k''R$ ,  $\omega''/\omega_H$ ) of the  $f_{D1}$  wave is determined from (23) if two parameters  $\omega_p/\omega_H$  and  $\theta$  are specified, so that the nonlinear coupling coefficient  $A_{k'',k}$  in (31) is also determined.

Numerical values of this coupling coefficient, and the required minimum energy density of the  $2f_H$  wave  $W_{km}$  from (5), are plotted parametrically in  $\theta$  in Figure 6 corresponding to different values of  $\omega_p/\omega_H$ . The magnitude of the coupling coefficient increases as the frequency approaches  $f_H$  or  $2f_H$  or as  $\theta$  decreases. This behavior results from the decrease in the resonance velocity  $2\pi(2f_H - f_{D1} - nf_H)/k' \cos \theta$ , defined by (1), which increases the number of electrons resonantly interacting with the beat oscillation  $(\vec{k}', \omega')$ . Since the damping rate of the  $f_{D1}$  wave increases in a similar way in response to a decrease in the resonant velocity [see Figure 1b of Kiwamoto, 1978],  $W_{km}$  stays almost constant in

the upper frequency range and then increases with descending frequency in the lower frequency range – the most rapid increase coinciding with the minimum in  $A_{k'',k}$ . Figure 6 also indicates that for a given frequency  $W_{km}$  decreases slightly while  $A_{k'',k}$  increases with increasing  $\omega_p/\omega_H$ . This tendency suggests that the duration of the  $f_{D1}$  resonance may increase with increasing  $\omega_p/\omega_H$  provided that the  $f_{D1}$  wave energy density is not so large that it quickly saps the  $2f_H$  wave, i.e.,  $W_k'' < W_k''_c$  in (6), and the transit time  $r/V_g$  is sufficiently long. The latter condition, however, depends on the heated volume which is an unknown function of  $\omega_p$ ,  $\omega_H$ , the relative geometry between the antenna orientation and  $\vec{V}_g$  and  $\vec{B}$ , the antenna length and the transmitted RF power.

The shaded region in the horizontal axis indicates the frequency range where long-lasting diffuse resonances are observed. The low value of  $W_{km}$  within this frequency region, and at higher frequencies, favors the nonlinear coupling of energy from the  $2f_H$  wave to the  $f_{D1}$  wave. The minimum of  $A_{k,k''} [\sim A_{k'',k}]$  near the shaded region increases the upper limit  $W_k''_c$  in (6) of the allowable energy density of the  $f_{D1}$  wave, consistent with maintaining sufficient  $2f_H$  wave energy to keep the nonlinear process active. This optimization of the nonlinear process increases the lifetime of the  $f_{D1}$  waves, which are preferentially generated in the shaded frequency range by instabilities corresponding to the minimum required anisotropy [Oya, 1971a; Kiwamoto, 1978].

In closing this section, consider the following theoretical and observational checks on the present model. Typical values of  $W_{km}$  in (5) are  $10^{-4}$  (see Figure 6). These values are well within the allowed value of  $10^{-2}$  in weak turbulence theory. Although there is no way experimentally to determine the value of  $W_k$ , a value from  $10^{-4}$  to  $10^{-3}$  seems to be quite reasonable from the observational point of view [ $W_k = 10^{-4}$  corresponds to  $E_k = 0.19$  V/m if  $N_e T = 10^4$  eV cm $^{-3}$ ]. The typical maximum duration of the  $f_{D1}$  resonance is 20 msec  $\approx 6 \times 10^4 \omega_H^{-1}$ . Because  $A_{k,k''} \sim A_{k'',k} \sim 1$ , the corresponding limitation to the  $f_{D1}$  wave

energy is, from (6),  $W_k'' \lesssim 10^{-5}$ . The corresponding potential fluctuation amplitude is

$$\phi_k'' = E_k''/k'' \approx aE_k''/\omega_H < 10^{-2} \text{ V.}$$

Since  $\phi_k'' \propto W_k^{1/2} \propto A_{k,k}^{-1/2}$  from (6), and the extreme limits of  $A_{k,k}''$  in Figure 6 are contained between  $10^{-4}$  and  $10^4$ , the maximum potential fluctuation allowed for the  $f_{D1}$  wave ranges from  $10^{-4}$  to 1V. Because the background noise level including the galactic noise is below  $10^{-5}$  V in the frequency range of interest [Franklin and Maclean 1969], the required range of the  $f_{D1}$  wave amplitude is well within the detectability of the receiver.



## SUMMARY

Active experiments in space can be used to investigate fundamental processes in plasma wave instabilities and nonlinear interactions. The Alouette/ISIS topside sounders represent first generation experiments of this type in the area of wave injection. Among the various plasma resonance phenomena excited by these sounders, is the long duration diffuse resonance which cannot be explained in terms of the reception of low group velocity waves stimulated by the sounder-pulse because it corresponds to large group velocity waves. This resonance is driven by an anisotropic electron velocity distribution which is created in a large volume ( $\sim 1$  to  $2$  km) around the satellite by the high-power sounder pulse. After the decay of the original anisotropy, the waves will be subject to strong Landau damping and will not be able to return to the sounder antenna unless a feeding mechanism is operational to maintain the waves. The low group-velocity sounder-stimulated  $2f_H$  wave remains in the region where the diffuse resonance wave is stimulated and it is available as an energy source via the process of nonlinear Landau damping. Calculations of the nonlinear coupling coefficient indicates that the nonlinear damping of energy from the  $2f_H$  wave to the thermal electrons and the diffuse resonance wave is strong enough to sustain this wave yet weak enough so as not to extinguish the  $2f_H$  wave. The required energy density of the latter is about  $10^{-4}$  of the thermal energy density which is consistent with the theory of weak turbulence. The diffuse resonance can be maintained in this manner during the transit time of the wave from its generation region back to the sounder antenna. The convection loss of wave energy parallel to  $\vec{B}$  is negligible during this process because the component of wave group velocity parallel to  $\vec{B}$  is zero for the most persistent diffuse resonance wave component after the relaxation of the sounder-stimulated temperature anisotropy. Thus the duration of the  $f_{D1}$  resonance is determined by the transit time  $r/V_g$ —provided it does not exceed the nonlinear life-time  $(A_{k \parallel k} W_k)^{-1}$ .

• The above model is consistent with the long duration ( $\sim 20$  msec) diffuse resonances observed on topside ionograms and indicates that nonlinear Landau damping, which has been observed in laboratory plasmas, can be an important process in space.

#### Acknowledgments

The Alouette 2 data used in Figures 1, 3, and 4 were provided by the National Space Science Data Center at the Goddard Space Flight Center; the ISIS 1 mixed-mode operation data used in Figure 2 were provided by the Communications Research Centre, Ottawa (we are grateful to D. B. Muldrew for his assistance in obtaining the latter). One of us (Y. K.) is grateful for stimulating discussions with Dr. A. Hasegawa.

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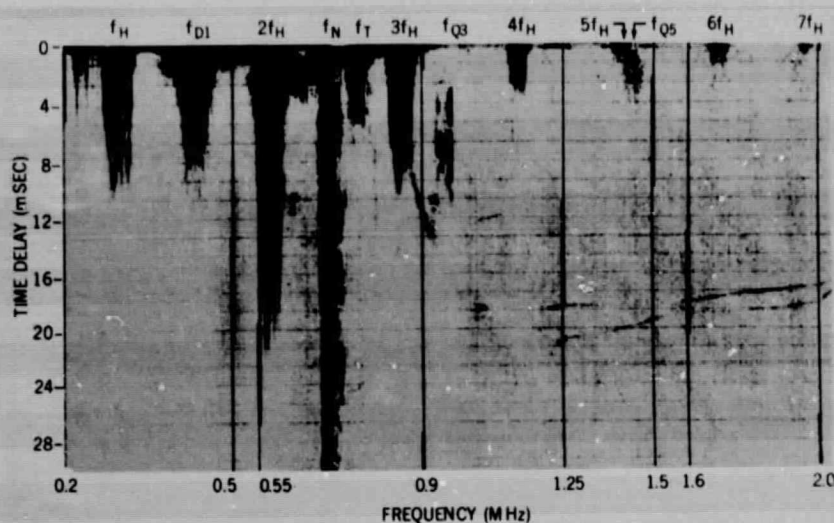


Figure 1. A portion of an Alouette 2 ionogram illustrating resonances. It was recorded at 1214:22 UT on 4 May 1967. The duration of the stimulating transmitter pulse is confined to the top 0.1 msec of the ionogram; more than 400 such pulses (each separated in frequency by 4 1/3 kHz) were transmitted in the portion of the ionogram reproduced. The heavy vertical traces are resonances which are stimulated by these pulses, they are identified at the top of the ionogram by the frequency at which they occur (the plasma resonance is identified by the common ionospheric notation of  $f_N$  rather than the common plasma physics notation of  $f_p$  used in the text). The weaker non-vertical traces are due to the ionospheric reflection of electromagnetic waves radiated by the sounder.

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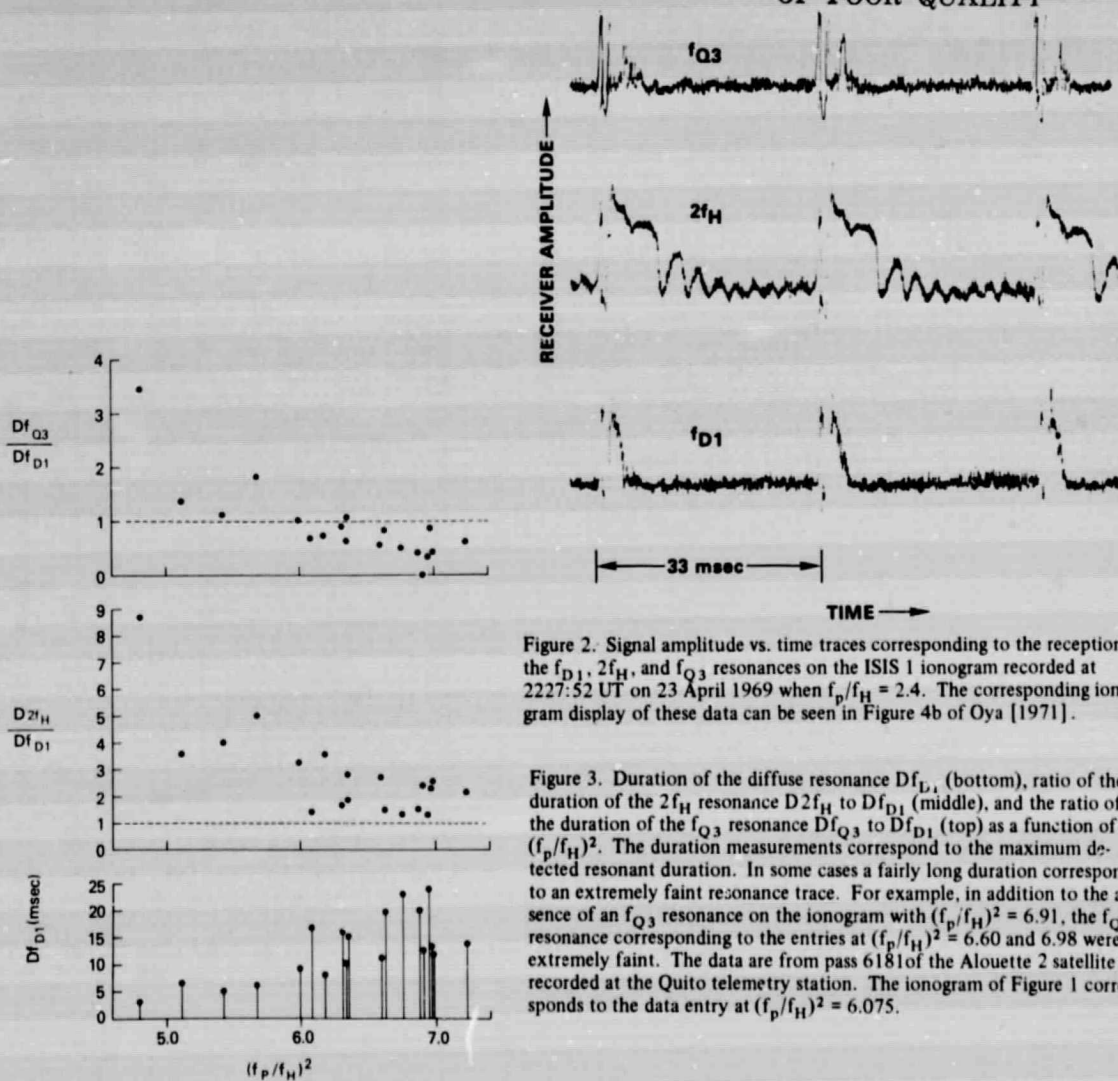


Figure 2. Signal amplitude vs. time traces corresponding to the reception of the  $f_{D1}$ ,  $2f_H$ , and  $f_{Q3}$  resonances on the ISIS 1 ionogram recorded at 2227:52 UT on 23 April 1969 when  $f_p/f_H = 2.4$ . The corresponding ionogram display of these data can be seen in Figure 4b of Oya [1971].

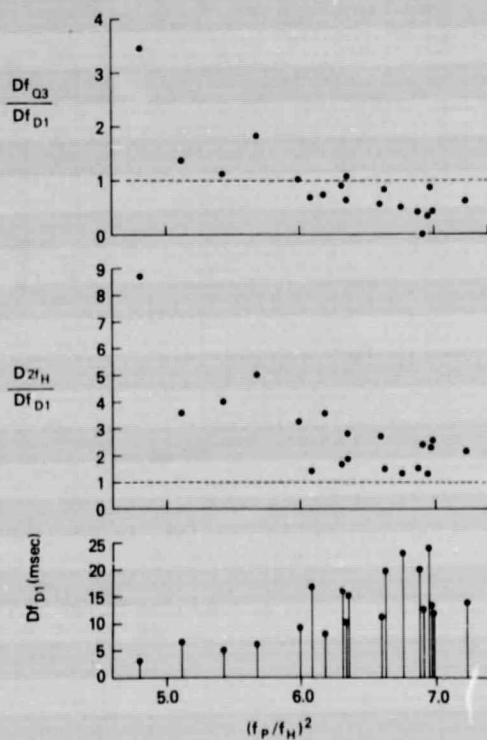


Figure 3. Duration of the diffuse resonance  $Df_{D1}$  (bottom), ratio of the duration of the  $2f_H$  resonance  $D2f_H$  to  $Df_{D1}$  (middle), and the ratio of the duration of the  $f_{Q3}$  resonance  $Df_{Q3}$  to  $Df_{D1}$  (top) as a function of  $(f_p/f_H)^2$ . The duration measurements correspond to the maximum detected resonant duration. In some cases a fairly long duration corresponded to an extremely faint resonance trace. For example, in addition to the absence of an  $f_{Q3}$  resonance on the ionogram with  $(f_p/f_H)^2 = 6.91$ , the  $f_{Q3}$  resonance corresponding to the entries at  $(f_p/f_H)^2 = 6.60$  and  $6.98$  were extremely faint. The data are from pass 6181 of the Alouette 2 satellite as recorded at the Quito telemetry station. The ionogram of Figure 1 corresponds to the data entry at  $(f_p/f_H)^2 = 6.075$ .

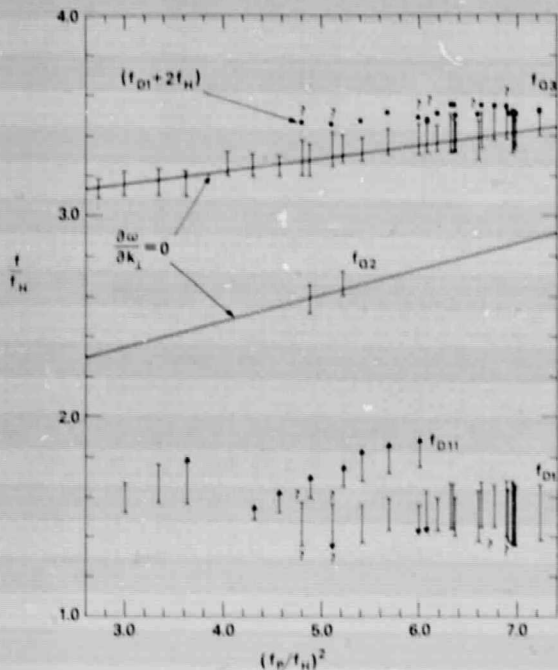


Figure 4. A comparison of the bandwidths of the  $f_{D1}$  and  $f_{Q3}$  resonances for the Alouette 2 data of Figure 3. The question marks and arrow heads indicate uncertainties usually resulting from obscuring features on the ionogram. The points corresponding to  $f_{D1} + 2f_H$  were obtained by adding  $2f_H$  to the center of the observed  $f_{D1}$  bandwidth. The  $\partial\omega/\partial k_\perp = 0$  curves correspond to the maximum frequency bump in the  $\theta = 90^\circ$  dispersion curves. Also presented are the observed frequency spread of the weaker subsidiary diffuse resonance  $f_{D11}$  [Oya, 1970] and the  $F_{Q2}$  resonance which is another member of the sequence of electrostatic resonances identified by Warren and Hagg [1968].

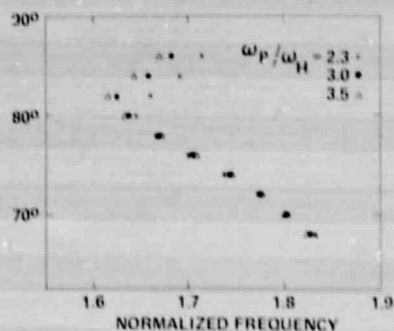


Figure 5. Dependence of the propagation angle  $\theta$  corresponding to  $V_{k1} = 0$  on the normalized frequency  $f/f_H$  for various values of  $\omega_p/\omega_H$ . The results correspond to an isotropic Maxwellian electron velocity distribution.

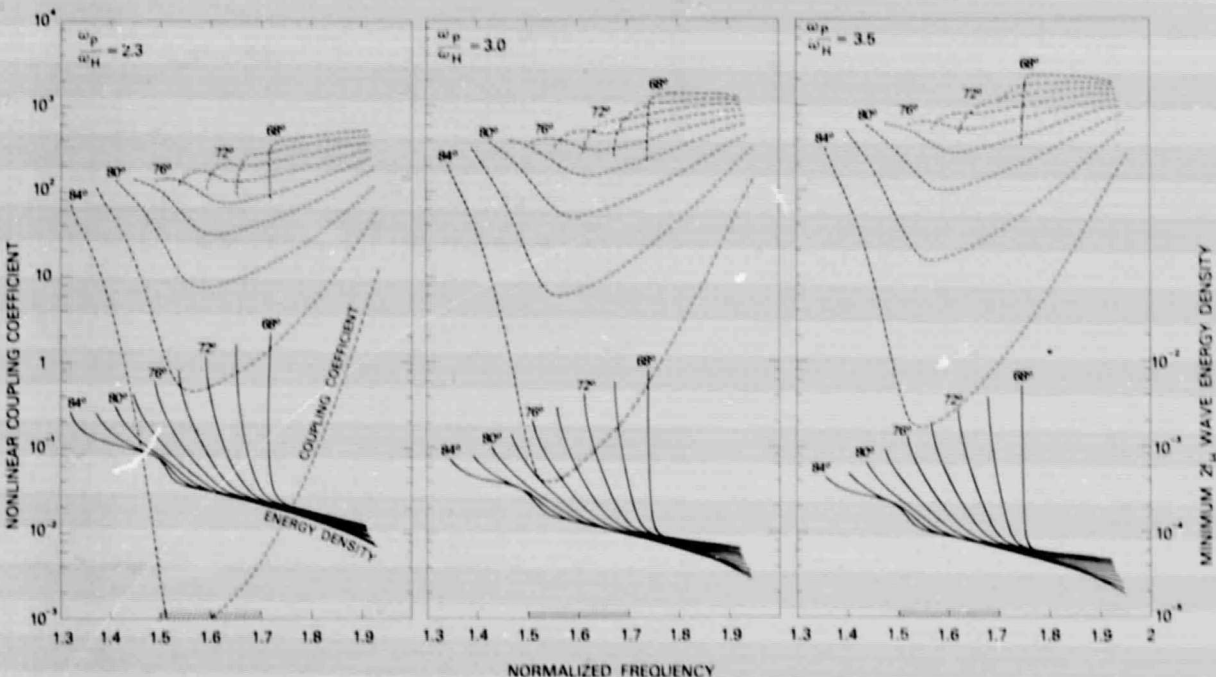


Figure 6. Numerical values of the nonlinear coupling coefficient  $A_{k^*,k}$  from (31) and the minimum required energy density of the  $2f_H$  wave  $W_{k,m}$  from (5) for 3 different values of  $\omega_p/\omega_H$ . In each case the results are presented for values of the propagation angle  $\theta$  ranging from  $68^\circ$  to  $84^\circ$ . All calculations correspond to an isotropic Maxwellian electron velocity distribution.



1. Report No. TM 79595	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle Nonlinear Landau Damping in the Ionosphere		5. Report Date July 1978	
		6. Performing Organization Code 621	
7. Author(s) Yasuhito Kiwamoto and Robert F. Benson		8. Performing Organization Report No.	
9. Performing Organization Name and Address NASA/Goddard Space Flight Center Laboratory for Planetary Atmospheres Greenbelt, Maryland 20771		10. Work Unit No.	
		11. Contract or Grant No.	
12. Sponsoring Agency Name and Address NASA/Goddard Space Flight Center Laboratory for Planetary Atmospheres Greenbelt, Maryland 20771		13. Type of Report and Period Covered  Technical Memorandum	
		14. Sponsoring Agency Code	
15. Supplementary Notes  Submitted for publication in <u>Journal of Geophysical Research</u>			
16. Abstract A model is presented to explain the non-resonant waves which give rise to the diffuse resonance observed near $3/2 f_H$ by the Alouette and ISIS topside sounders, where $f_H$ is the ambient electron cyclotron frequency. These waves are the result of plasma wave instabilities driven by anisotropic electron velocity distributions initiated by the high-power (400 W) short duration (100 $\mu$ sec) sounder pulse. In a strictly linear analysis, these instability-driven waves will decay due to Landau damping on a time scale much shorter than the observed time duration of the diffuse resonance. Calculations of the non-linear wave-particle coupling coefficients, however, indicate that the diffuse resonance wave can be maintained by the non-linear Landau damping of the sounder-stimulated $2f_H$ wave. This $2f_H$ wave is always observed with a time duration longer than the diffuse resonance wave, which indicates that it is available as an energy source for the proposed non-linear mechanism. The time duration of the diffuse resonance is determined by the transit time of the instability-generated and nonlinearly-maintained diffuse resonance wave from the remote short-lived hot region back to the antenna. The model is consistent with the Alouette/ISIS observations, and clearly demonstrates the existence of nonlinear wave-particle interactions in the ionosphere. Such interactions have been observed in the laboratory and have long been speculated as important in the energetics of the earth's magnetosphere and ionosphere.			
17. Key Words (Selected by Author(s)) nonlinear Landau damping, wave particle interactions, ISIS, plasma resonances		18. Distribution Statement	
19. Security Classif. (of this report)  U	20. Security Classif. (of this page)  U	21. No. of Pages  30	22. Price*