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AN INVESTIGATION OF ROTOR NOISE
GENERATION BY AERODYNAMIC DISTURBANCE

by

C. E. Whitfield

DEPARTMENT OF TRANSPORT TECHNOLOGY

Loughborough University of Technology

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C. E. Whitfield , Ph.D.

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Summary

An open rotor has been considered as a process for converting an unsteady velocity inflow into sound radiation. With the aid of crude assumptions 'aero-acoustic transfer functions' have been defined theoretically for both discrete frequency and broad band noise. An experimental study of the validity of these transfer functions has yielded results which show good agreement at discrete frequencies though slightly less good for broad band noise. Agreement in both cases holds over three or more decades of the relevant parameters.

The experimental work involved has necessitated the development of a rotating hot wire anemometry system. A single hot wire probe has been mounted in the nose-cone of the rotor and used to quantify fluctuations in the airflow onto a single rotor blade for the transfer function results. Further theoretical analysis has revealed that the sound field can be expressed in terms of blade-to-blade correlations in the airflow, and results from two probes rotating simultaneously have been modelled mathematically and inserted in the theory. Preliminary results show encouraging agreement with experimental data.

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NOTATION

A	defined in text
a_0	speed of sound
B	number of blades
B_e	effective bandwidth of filter (defined in text)
b	exponential decay rate (defined in text)
$C_e(\tau)$	envelope of velocity auto-correlation pulse peak amplitudes
c	blade chord
$D(\tau)$	circumferential component of fluctuating blade force
$D(f) \left\{ \begin{array}{l} D(g) \end{array} \right.$	defined in text
$E(f)$	defined in text
F_i	external force per unit volume
F_r	defined in text
f	frequency (Hz) - normally frequency of interest
G	dummy variable
$G_2(f)$	one-sided power spectrum of velocity auto-correlation pulse shape
$\left. \begin{array}{l} g \\ g_1 \\ g_2 \end{array} \right\}$	frequency (Hz) - variable
$H(g)$	frequency response function of filter
h	rotational frequency of force (Hz)
$J_n()$	Bessel Function of the first kind
$K_n()$	modified Bessel Function of the second kind
k	summation index
$L(t)$	unsteady lift on fan blade
$L(g)$	Fourier Transform of unsteady lift
L	summation index
M_i	component of instantaneous convection Mach number in direction $\underline{i}$

M_r	component of instantaneous convection Mach number in direction $\underline{r}$ of the observer
m	summation index
n	summation index
$P(f)$	Fourier Transform of sound pressure
$P_D(f)$	drag integral
$P_T(f)$	thrust integral
$P_p(\epsilon, f)$	time-varying power spectrum of sound pressure
$\overline{P_p(f)}$	time-averaged power spectrum of sound pressure
$\overline{P_{ax}(f)}$	on-axis time-averaged power spectrum of sound pressure
$p(\epsilon)$	fluctuating acoustic pressure
Q	rate of introduction of mass per unit volume
$Q(f)$	defined in text
R	radius of rotating point force
$R_\lambda(\tau)$	cross-correlation between velocities impinging on two blades λ spaces apart
$R_{kl}(\tau)$	cross-correlation between velocities impinging on blades k and l
r	distance between observer and source <u>or</u> variable of integration along fan blade - defined in text
r_i	distance from centre of fan hub to observer
$S(\sigma)$	Sears Lift Function
$S_R(\tau)$	pulse shape function of velocity correlation
$S_R(q)$	Fourier Transform of pulse shape
τ	time (defined in text)
$T(\tau)$	axial component of fluctuating blade force
τ_j	fluctuating stress per unit volume
t	time (defined in text)
U	mean velocity
$\overline{v}$	mean square turbulence velocity
$v'(q)$	amplitude of fluctuating velocity

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$v'_k(q)$	amplitude of fluctuating velocity impinging on blade k
$v'_{kl}(q)$	cross-power spectrum between velocities impinging on blades k and l
$v'_\lambda(q)$	cross-power spectrum between velocities impinging on two blades λ spaces apart
$v'_m(q)$	measurable velocity power spectrum
$v(x, t)$	instantaneous velocity
$\begin{matrix} x \\ y \\ z \end{matrix} \}$	cartesian coordinate system
x_i	observer coordinates
y_i	source coordinates
α	blade incidence
α'	exponential decay rate
$\beta_n(f)$	defined in text
γ	defined in text
$\delta(\)$	Dirac Delta Function
ζ	defined in text
θ	angular position of force
λ	blade-to-blade space
μ	summation index
ν	summation index
ξ	defined in text
$\rho(t)$	fluctuating density
σ	reduced frequency (defined in text)
τ	retarded time <u>or</u> time lag (defined in text)
ϕ	initial angular position of force

. CHAPTER 1

INTRODUCTION

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This thesis describes an experimental and theoretical study of the noise generated by a turbulent airflow entering an open rotor. The objectives of the study are first, to demonstrate by experimental results that a definite relationship exists between turbulence in the airflow and the noise generated; second, to show how a simple theoretical model can predict this relationship with reasonable accuracy; and third, to use the experimental results in the development of a more realistic theory. The position of this study in relation to the available work on rotor noise is outlined below.

1.1 Historical Background

The main feature of the sound radiated by a rotor is that the sound source is rotating. The first demonstration of sound from a rotating source appears to have been performed by Mach (Ref. 1) as an illustration of the Doppler effect. His apparatus was described by Rayleigh (Ref. 2) in his classic book. It was noted that if the observer were situated in the plane of rotation of the source (a whistle) the note heard fluctuated in pitch due to the movement of the source, whereas an observer on the axis of rotation heard a steady note.

The sound from a rotating source seemed destined to remain an interesting curiosity until the advent of the aeroplane. The general interest in aviation at the time of the First World War was paralleled by an upsurge of research into the noise from propellers and the first attempts to formulate a theory describing propeller noise radiation date from this time. Theoreticians were, however, hampered by the lack of good experimental data. An early Aeronautical Research Committee report (Ref. 3) noted, for instance, that propeller noise is at a minimum on the fan axis and that

propellers make considerably less noise in flight than when tested on the ground under nominally identical conditions. In a comprehensive historical review of the study of propeller noise which has been used as the base for this discussion, Lowson (Ref. 4) points out that although these results were obtained mainly by subjective means "reported results of tests correctly define features which were still a matter of controversy fifty years later". There is obviously much to be said for the educated ear. Permanent records of the sound were made at this time on a phonograph machine. Further analysis of the data was possible by measuring the depth of the groove thus created - a laborious process.

The difference in sound radiated by a propeller in flight and the same propeller on a test stand is an important effect. Morfey (Ref. 5) in another comprehensive review paper mentions that it was also observed by Waetzmann (Ref. 6) and Prandtl (Ref. 7) at about the same time. The effect has recently regained some prominence with the advent of the turbo-fan aero engine.

The first attempts to formulate a theory to describe the sound from a propeller were made at the end of the First World War. Lynam and Webb (Ref. 8) modelled the propeller by a continuous ring of stationary sources and a ring of sinks with arbitrary axial displacement. A different approach was tried by Bryan (Ref. 9) who endeavoured to predict the sound field from a point source in circular motion. This was an early example of the retarded time concept. Neither of these papers successfully predicts the amplitude of the sound radiated by a propeller and it was not until some years later when Gutin (Ref. 10) produced his classic paper that a theoretical formulation was found which agreed with the then

available experimental data. Gutin considered that the thrust and torque forces on the rotor could be identified as acoustic dipoles. His analysis predicts the sound field which results when a force of constant absolute strength rotates in a circle. His calculated directional characteristics for the fundamental and second harmonic of blade passing frequency agree with the experimental results of Kemp (Ref. 11) and Paris (Ref. 12), showing a maximum behind the plane of the disc and a non-zero level in the disc plane. This theory was extended by Deming (Ref. 13 and 14) and Gutin (Ref. 15) to include the effects of blade thickness. Later, Garrick and Watkins (Ref. 16) added the effects of forward speed to the original Gutin theory (Ref. 10), but other than this, the steady force model has remained unchanged to the present day and the harmonic noise resulting from the action of steady forces on a rotor is widely referred to as 'Gutin' noise.

Sound from a rotor does not consist solely of discrete frequency radiation. There is a broad band spectrum through which the tones protrude. The first experiments to investigate the sources of this broad band spectrum were performed by Stowell and Deming (Ref. 17) who measured the noise from rotating cylindrical rods. Now, if a cylinder is placed in a uniform flow a wake of vortices is shed from it (the Karman vortex street). The frequency of vortex shedding is governed by the Strouhal number, and the fluctuating force on the cylinder is expressed as Aeolian tone noise. Stowell and Deming found that the broad band sound radiated from the rotating cylinders was in the frequency range predicted by the variation of Strouhal frequency over the rods. Yudin (Ref. 18) did not restrict himself to rods of circular cross-

-section but made a more general study of the noise from rotating shapes. In addition, Yudin produced a theoretical analysis for the sound on similar lines to that of Gutin for tone noise.

Yudin's results demonstrated the increase in radiated sound which results from running a flat plate in its own wake rather than at a slight angle of incidence. In doing this he was repeating an observation made by G. I. Taylor (Ref. 19) twenty years previously. Taylor found that when he waved his toasting fork through the air with the prongs parallel to the plane of motion, the noise generated was greater than that radiated if the prongs were perpendicular to the plane of motion. Kramer (Ref. 20) performed a similar experiment to that of Yudin to demonstrate this effect for aerofoil sections. He rotated a single blade of high aspect ratio and achieved a reduction of three dB in the sound measured on the axis of rotation by running with an angle of attack of one degree in either direction compared with the level obtained at zero angle of attack. Thus, both self-generated effects such as vortex shedding and external causes - free stream turbulence for example - are important contributors to broad band noise radiation.

By the end of the Second World War, foundations had been laid for the understanding of the noise from rotating sources. The Gutin theory for steady forces generated the harmonic radiation and the broad band noise was described in terms of vortex shedding. It was found however by, for example, Hicks and Hubbard (Ref. 21) that while the steady force theory predicted the lower harmonics with reasonable accuracy, the values calculated for the higher orders were systematically underestimated. This was especially so for low tip speeds and many-bladed propellers. One other obvious discrepancy

between the Gutin theory and experiment is that the theory predicts zero harmonic sound on the fan axis whilst there is a definite harmonic content in the measured sound radiated. The lack of agreement between theory and experiment is especially relevant to the study of helicopter rotor noise where the higher harmonics are the greatest contributors to the perceived noise. The use of steady force theory to predict helicopter rotor noise suffers from an inherent handicap. Helicopter rotors operate in such a way as to cause large unsteady loads on the blades and, with hindsight, it appears obvious that these should contribute to the noise. In the 1960s, a great deal of work was done to establish the significance of these unsteady forces. Schlegel et al (Ref. 22) and Loewy and Sutton (Ref. 23) performed computational studies which clearly showed the effect of unsteady blade loads on the sound radiated. Lowson and Ollerhead (Ref. 24) used loading estimates extrapolated from measured data to include the higher loading harmonics and using a point force assumption, were able to predict the discrete frequency noise from a helicopter rotor with reasonable accuracy.

The theoretical method used by Lowson and Ollerhead was a continuation of earlier work by Lowson (Ref. 25). Lowson followed Lighthill (Ref. 26) in obtaining the exact equations for sound generation from the exact equations of aerodynamics and solved these to obtain the far field radiation from a point force in arbitrary motion. Lowson's result can be applied to the steady force example considered by Gutin and the resulting expression is identical with that obtained by Gutin. The extension of this theory to cover fluctuating loads results in the prediction of a non-zero level on the rotor axis and also in agreement with experimental data for the higher

blade passing frequency harmonics. For low speed fans, where the tip Mach number is less than about 0.6, this fluctuating force mechanism is the dominant cause of discrete frequency noise.

The principle behind the discrete frequency sound radiation by a rotor applies equally to broad band radiation. For discrete frequency radiation, the fluctuating forces are harmonically related to the frequency of rotation, but for broad band noise all frequencies radiate. The origin of these random fluctuating forces was originally ascribed to vortex shedding from the trailing edge, but it is now felt that the broad band spectrum is a function of the inflow turbulence.

To investigate the effect of external turbulence as a broad band noise source, Sharland (Ref. 27) measured the noise radiated by a small plate placed in an open jet. He found that the level of sound radiated when the plate was positioned in the laminar core of the jet was considerably less than that radiated when the plate was in the turbulent jet flow. Sharland also estimated the noise radiated. A similar expression using less restrictive assumptions, has been developed by Lawson (for example Ref. 4). Sharland's experiment demonstrated the relative importance of external and self-generated turbulence for the broad band noise. Lawson et al (Ref. 28) investigated the effect of blade tip shape on the sound from a low speed open rotor. They found that by modifying the tip shape it was possible to affect the high frequency broad band spectrum and that there was some Strouhal number dependence. The reduction in sound level observed was consistent with the assumption that separated vortex flow over the blade tips was a source of noise.

In practice it has been found that the distinction between broad band noise and discrete frequency radiation is hard to draw. Leverton

(Ref. 29) observed the differences obtained when testing a rotor under calm conditions and with a slight wind. Under calm conditions no harmonics are visible, but these immediately appear with the wind. There is, however, still a definite background level. Barry and Moore (Ref. 30) tested a low speed ducted fan, and found that at higher frequencies it was hard to distinguish between fluctuations in the rotational harmonics and broad band noise.

This far, the discussion has concentrated on the part played by fluctuating forces in the radiation of sound from rotating blades. Other mechanisms do, however, exist. In the early days of propeller noise research, Deming (Ref. 14) and Gutin (Ref. 15) showed the possible effects of blade thickness. The existence of thickness sources is now generally accepted, and whereas at low speeds and frequencies of radiation they can be ignored, at tip Mach numbers approaching unity, and high frequencies, they must be considered. Hawkings and Lowson (Ref. 31) have studied the effects of thickness sources in some detail.

A further mechanism which has been postulated as a source of radiation from a blade is the aerodynamic stress system around it. Lighthill (Ref. 26) first showed the possibility of fluctuating stresses acting as quadrupole sound sources. Some years later, Ffowcs Williams and Hawkings (Ref. 32) showed that the fluctuating stress system on a blade must also radiate. This source mechanism tends to be ignored in practice on two counts - first, theory based on the fluctuating force (dipole) mechanism is in agreement with available experimental data; and second, estimation of quadrupole source strength is difficult.

1.2 Present Work

This thesis concentrates on fluctuating forces as the source of

sound from a subsonic rotor. The fluctuating force mechanism is now well understood (e.g. Ref. 32) - theory suggests that the acoustic field is caused by fluctuating forces on the rotating blading; the harmonic components of these forces give rise to the discrete frequency constituents of the field and the remaining force components generate the broad band noise.

A major difficulty encountered in the verification of this theory has been estimation of the precise strength of these blade forces. Lowson and Ollerhead (Ref. 24) used loading estimates extrapolated from measured data in their prediction of helicopter main rotor noise, whilst Lowson (Ref. 33) used an entirely theoretical approach in the successful prediction of compressor noise. An alternative method used by some workers is to infer the strength of the forces from the known acoustic data. In this approach, the strength of the fluctuating axial forces is obtained from on-axis sound pressure measurements, thus guaranteeing agreement between theory and experiment. This technique was used by Barry and Moore (Ref. 30) and also by Brown and Ollerhead (Ref. 34).

On the other hand, theories exist which relate the fluctuating force components on a blade to the components of the unsteady velocity field impinging on that blade (see, for example, Ref. 35-37). There is thus a direct link between the aerodynamic and acoustic fields, and the rotor can be regarded as a machine for converting the unsteady aerodynamic inflow into sound radiation. The acoustic field is the result of the interaction between the rotor blades and the unsteady flow which holds for both discrete frequency and broad band noise. It follows therefore that if the unsteady velocity field can be measured, the difficulty of estimating the blade forces can be

circumvented. This is the approach which has been followed in this work.

In Chapter 2 the complete analysis followed is set out from start - the so-called 'Lighthill equation' (Ref. 26) - to finish - the definition of 'transfer functions' between the input velocity field and the output acoustic field for discrete frequency and broad band noise. The approach follows that of Lowson (Ref. 33) using his expression for the far-field pressure from a point force in arbitrary motion (Ref. 25). Fourier Transformation is used in preference to Fourier Series analysis in order to deal with frequencies not harmonically related to the shaft rotational frequency. The step from blade forces to incoming aerodynamics is taken via the work of Sears (Ref. 36) and in so doing two restrictive assumptions are made. First, the amplitude of the velocity fluctuation is constant along the leading edge of the blade; and second, the lift generated is in phase along the blade span. The resultant lift force is thus the maximum that could be obtained. In the transition from the Fourier Transform of the sound pressure to its Power Spectrum it was found that a slight excursion into the realms of non-stationary signal analysis was required. It is believed that this particular treatment has not been used before. Ffowcs Williams and Hawkings (Ref. 32) recognised the problem but avoided it to achieve the same result.

The 'aero-acoustic transfer function' is defined as the ratio of the magnitude of the acoustic output to the magnitude of the aerodynamic input at the same frequency. This definition arose initially through the limitations of the analysis equipment available to process the results and it has since been incorporated into the

theory. No phase effects are included. By making simple assumptions about the input flow it is possible to obtain an absolute value for the transfer function which depends solely upon the number of blades, their geometry, the speed of rotation, the frequency under consideration and a constant function of the experimental set-up for the instance when the observer (i.e. microphone) is on the axis of the fan. To test this theory requires means for measuring aerodynamic input (in rotating co-ordinates) and acoustic output simultaneously and independently. It is also necessary to analyse the data obtained in a suitable fashion. Chapters 3 and 4 describe these procedures.

Chapter 3 describes the reasons for and use of a hot wire anemometer probe which rotates with the blading. This is, in more than one sense of the word, a revolutionary procedure. The only previous attempt to rotate a hot wire probe known to the author, was that of Ufer (Ref. 38) who used a specially designed probe in a constant temperature system to measure the flow behind a blade row. In the present system, a standard 'bought-out' probe is held by a pin vice in the nose cone of the fan. The signal is extracted by means of slip-rings which were expected to affect the performance of the probe cable. After a series of tests it was decided that, surprisingly, the equipment supplied with the 'bought-out box' was the most effective. The only major problem encountered was probe retention. Also described in Chapter 3 are the experimental rig used, the anechoic chamber and the sound pressure measurements taken. The recording system used is mentioned.

In Chapter 4 a comparison is made between the two systems which are available to analyse the results. These are a 'traditional'

* Namely that the fluctuating velocity is constant in magnitude along the blade span and generates a lift force which is in phase along the span.

analogue narrow band filter and a digital system with Fast Fourier Transform (FFT) software. The merits of each system are discussed individually and a comparison is made between them on the grounds of ease of operation, accuracy, speed of operation etc. The digital system can also be used for operations in the time domain, a fact which is made use of in Chapter 5.

Chapter 5 is naturally sub-divided into two sections - first a discussion of the aero-acoustic results obtained, which range from the on-axis microphone and rotating hot wire spectra as such, to a comparison between the theoretical and experimental results obtained for the aero-acoustic transfer function. The transfer function results cover about four decades of the frequency parameters used, and agreement between theory and experiment is very good for discrete frequencies - less good for broad band noise. Reasons for this difference in agreement are put forward.

A point which arises is the difference in the sound generated when the fan is running in a 'clean' flow compared with the noise when the recirculation around the side walls is re-ingested. Before recirculation the agreement between theoretical and experimental aero-acoustic transfer functions breaks down. Examination of the coherence functions between aerodynamic input and acoustic output proves that some form of relationship does exist, though it is obviously not of the form assumed previously. The conclusion drawn is that the inflow (and hence blade response) is not as specified in Chapter 2.

This leads to the second stage of the discussion.

Experimental results are presented which were obtained using two rotating hot wire probes to investigate correlation lengths in

the axial and relative inlet flow directions. These reveal that for this particular fan operating in this particular environment an eddy takes approximately 18 revolutions to pass axially through the fan. It is this well-correlated structure which gives rise to blade passing frequency harmonic tones with the randomness in the relative inlet flow producing the aerodynamic contribution to the overall broad band noise.

The experimental auto- and cross-correlation functions obtained have been modelled mathematically and substituted into the theory of Chapter 2. It becomes clear that the aero-acoustic transfer function depends upon the axial scale of the incoming turbulence, whilst for the acoustic and aerodynamic spectra both axial and relative scales need to be specified.

One particular example is studied and it is seen that the spectra obtained consist of harmonics related to the axial decay rate, superimposed on a broad band spectrum resulting from the blade-to-blade correlation chosen. A comparison between theory and experiment is shown. It appears that this theory can be used with aerodynamic data from model inlets to predict the noise generated by an aircraft in flight as opposed to results obtained from ground testing.

The conclusions that arise from this work appear at the end of Chapter 5 and are:

- 1) The theoretical model developed is based on the fluctuating force (dipole) mechanism of sound generation. The fluctuating lift force on the blades is related to turbulence in the inflow to the fan by unsteady aerofoil theory and an 'aero-acoustic transfer function' is defined as the ratio of the magnitudes of the acoustic

and velocity power spectra at the same frequency. This formulation is simplified by placing the observer on the axis of the fan. The on-axis aero-acoustic transfer function is quantified for both discrete frequency and broad band noise by consideration of the effects of blade-to-blade correlation in the airflow. The two extremes of perfect and zero blade-to-blade correlation are considered for comparison with experimental results - perfect correlation giving the discrete frequency aero-acoustic transfer function, and zero correlation the broad band aero-acoustic transfer function. With these assumptions, the value of the theoretical on-axis aero-acoustic transfer function is dependent solely upon frequency, speed of rotation, blade number and a constant function of the blading. There are no empirical factors involved.

2) To establish experimentally the existence of a definite relationship between the velocity entering a fan and its on-axis acoustic spectrum requires simultaneous and independent measurement of both the acoustic and velocity data. Obtaining the required acoustic information merely involves positioning the microphone as required in a suitable free-field environment (anechoic chamber) but if hot wire anemometry is to be used in the measurement of the turbulent flow entering the fan, the effects of the fan itself on a stationary wire must be considered. This work establishes the use of rotating hot wire anemometry as a viable experimental technique which requires no particular sophistication of instrumentation at speeds up to 1600 rpm (a wire speed of 43 m/s). The restriction on higher speeds is a result of problems in probe retention, not of wire strength. Given a more sophisticated mounting system, it is anticipated that the limiting factor found when increasing speed

would be the rubbing speed of the slip ring brushes on the rings themselves.

3) The experimental results show that there is a definite connection between the velocity input to and the acoustic output from an open rotor. Comparisons between the experimental and theoretical on-axis aero-acoustic transfer functions show agreement to within 10 dB over three decades of the relevant parameters for both discrete frequency and broad band noise. This agreement holds when the incoming turbulence intensity is of the order of 3%. When the turbulence intensity is less than 3% it is found that an experimental aero-acoustic transfer function exists[†] at discrete frequencies only but it is not of the form suggested by the theory. It is felt that some of the restrictive assumptions made in the development of the theory are not applicable here.

4) The idealised conditions of full and zero blade-to-blade correlation do not exist in real life. Experimental auto- and cross-correlation functions obtained from rotating hot wire anemometers show that the inflow appears to consist of long thin eddies which require approximately 18 revolutions to pass through the fan. The form of these correlation functions can be modelled mathematically and inserted into the theory. By this method, velocity and acoustic spectra are predicted as well as the aero-acoustic transfer function. It is found that the aero-acoustic transfer function is a function of the axial decay of the eddies, while the acoustic and velocity spectra depend upon both the axial decay and also the correlation of the turbulence in the direction of the relative velocity.

[†] i.e. there is coherence between the velocity and acoustic signals measured.

5) The theoretical model used takes no account of any span-wise variation in the turbulence field, and hence assumes that the lift generated on a blade is the maximum possible. In practice, however, there is a correlation pattern along the span in addition to those in the axial and V_{∞} directions. It is felt that the investigation of this span-wise effect, first experimentally and then theoretically, would be a profitable line for future work. The extension of the rotating hot wire technique to higher speeds is also to be recommended. This would involve the development of an improved mounting system for the hot wires - using the blades themselves as supports, for example - and some other system for transmitting the data. A system like this could be used to investigate the duct boundary layer in a ducted rotor.

One other line of enquiry is the apparent dependence of the form of the aero-acoustic transfer function on the turbulence intensity in the inflow. This is of importance since aircraft, in general, do not operate in recirculating flow situations.

6) In practice, this work is one further step along the road to the prediction of the sound generated by a rotor. It has confirmed, by experimental means, that the fluctuating force (dipole) source mechanism describes the sound field of a low-speed open rotor with reasonable accuracy for both discrete frequency and broad band noise. It has demonstrated that, given some knowledge of the blade-to-blade correlation in the inflow it is possible to predict the acoustic spectrum of the rotor. Thus, data from model inlet tests can be used in the prediction of fan noise. These results should apply in all instances where low-speed fans are used - air conditioning plants for example.

The development of rotating hot wire anemometry provides a useful experimental tool for the investigation of: velocity impinging on fan blades, duct boundary layers, tip flow behind a rotating fan and other similar situations.

The work as a whole shows that a small-scale, low-speed fan can be used to provide information which may lead to larger, more expensive, test rigs, and also demonstrates the development of theory to fit experimental data.

Some of the work described in this thesis has been presented in Ref. 4, 28 and 39-44.

CHAPTER 2

THEORY

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2.1 Background

The derivation of the far field sound pressure of a point fluctuating force in arbitrary motion is sketched.

The formulation of the general theory of aerodynamic sound is due to Lighthill. The 'Lighthill Equation' can be obtained directly from the aerodynamic equations of continuity and momentum. It can be written, in tensor notation,

$$\frac{\partial^2 \rho}{\partial t^2} - a_0^2 \frac{\partial^2 \rho}{\partial x_i^2} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} - \frac{\partial F_i}{\partial x_i} + \frac{\partial Q}{\partial t} \quad (1)$$

The left hand side of equation (1) is recognised as the equation governing sound propagation in a uniform acoustic medium at rest. The terms on the right hand side represent volume distributions of the possible sources of sound present in the field.

Here

$$T_{ij} = \rho v_i v_j + p \delta_{ij} - a_0^2 \rho \delta_{ij}$$

$$\delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

F_i is the external force per unit volume acting on the fluid and Q is the rate of introduction of mass per unit volume.

The solution to equation (1) for an unbounded fluid can be written

$$\rho - \rho_0 = \frac{1}{4\pi a_0^2} \int_V \left[\frac{G}{r} \right] dv(y) \quad (2)$$

where the right hand side of equation (1) has been written as G and the square brackets $\left[\right]$ imply evaluation of their contents at retarded (or source) time $\tau = t - \frac{r}{a_0}$. Here t is observer

time and $r(\tau)$ the distance from source to observer. y is a dummy variable of integration referring to the source position.

We are concerned with the sound field generated by a moving force - this is represented by replacing G in equation (2) by $-\frac{\partial F_i}{\partial y_i}$ (from equation (1)).

$$p - p_0 = - \frac{1}{4\pi a_0^2} \int \int \left[\frac{1}{r} \frac{\partial F_i}{\partial y_i} \right] dv(y) \quad (3)$$

Lowson (Ref. 25) solved equation (3) to show that the far field sound pressure received from a point fluctuating force in arbitrary motion can be written

$$p(t) = \int \frac{x_i - y_i}{(1-M_r)a_0 r} \frac{\partial}{\partial \tau} \left(\frac{F_i(\tau)}{4\pi r (1-M_r)} \right) \quad (4)$$

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where x_i are the observer coordinates

y_i are the source coordinates

M_r is the component of the instantaneous convection

Mach number in the direction r .

2.2 Application to Rotor Noise

The general expression of equation (4) is used to obtain the far field sound spectrum of a rotor blade in terms of the fluctuating blade forces. All frequencies are included.

The far field acoustic spectrum of an isolated rotor is found to consist of harmonics of the blade passing frequency superimposed on an overall 'broad band' spectrum.

Lowson (Ref. 33) used equation (4) to show that the magnitude of sound in rotor blade passing frequency and its harmonics can be predicted from a knowledge of the periodic fluctuating forces acting on the blading. This was demonstrated by using Fourier Series

techniques to relate the sound harmonic magnitude to the Fourier components of the fluctuating thrust and drag on the blades.

If Fourier Transform rather than Fourier Series techniques are employed, attention is not confined to harmonics of blade passing frequency and this approach has been followed in the present work. By this means the broad band portion of the spectrum can be analysed.

The Fourier Transform of equation (4) above gives the magnitude and phase of sound observed in the far field at all frequencies of the spectrum.

$$P(f) = \int_{-\infty}^{+\infty} \left[\frac{x_i - y_i}{(1-M_r) a_0 r} \frac{\partial}{\partial \tau} \left(\frac{F_i(\tau)}{4\pi r (1-M_r)} \right) \right] \exp \left\{ -2\pi i f t \right\} d\tau ;$$

substituting $\tau = t - \frac{r}{a_0}$ and $d\tau = dt(1-M_r)$ to allow for retarded time effects gives:

$$P(f) = \int_{-\infty}^{+\infty} \frac{(x_i - y_i)}{a_0 r} \frac{\partial}{\partial \tau} \left(\frac{F_i(\tau)}{4\pi r (1-M_r)} \right) \exp \left\{ -2\pi i f \left(\tau + \frac{r}{a_0} \right) \right\} d\tau. \quad (5)$$

Equation (5) can be integrated by parts to give

$$P(f) = \int_{-\infty}^{+\infty} \left(\frac{i f F_i(\tau)}{2 a_0 r} - \frac{F_i(\tau)}{4\pi r (1-M_r)} \left(-\frac{M_r}{r} + \frac{(x_i - y_i) M_r}{r^2} \right) \right) \exp \left\{ -2\pi i f \left(\tau + \frac{r}{a_0} \right) \right\} d\tau, \quad (6)$$

where

$$F_r(\tau) = \frac{F_i(\tau) (x_i - y_i)}{r},$$

the component of the force in the direction of the observer.

The second term in equation (6) only applies in the acoustic near field. The original expression (equation (4)) neglected near field terms and thus the far field result obtained for the

* It was pointed out by the external examiner that equation (4) as written must therefore describe the sound field in its entirety. Dr. Lowson observed that this is the effect of using $\partial/\partial \tau$ rather than $\partial/\partial t$ as in Ref. 25.

Fourier Transform of the sound pressure emitted by a point force in arbitrary motion is

$$p(f) = \int_{-\infty}^{+\infty} \frac{if F_r(\tau)}{2a_0 r} \exp \left\{ -2\pi if \left(\tau + \frac{r}{a_0} \right) \right\} d\tau. \quad (7)$$

To make use of this expression in the analysis of a specific problem (for example, fan noise) it is necessary to define both the motion and the force $F_r(\tau)$.

Figure 1 illustrates that, for a point force rotating in a circle, (for example, a point blade force)

$$F_i(\tau) = -T(\tau), \quad -D(\tau) \sin \theta, \quad D(\tau) \cos \theta.$$

$$x_i - y_i = x, \quad y - R \cos \theta, \quad -R \sin \theta.$$

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Hence

$$F_r(\tau) = \frac{(x_i - y_i)}{r} F_i(\tau) = -\frac{x T(\tau)}{r} - \frac{y D(\tau) \sin \theta}{r}$$

and

$$r = |x - y| = (x^2 + y^2 + R^2 - 2Ry \cos \theta)^{1/2}$$

$$\doteq r_1 - \frac{2y \cos \theta}{r_1}$$

where terms of order $\left(\frac{R}{r_1}\right)^2$ and higher have been ignored.

The fluctuating force terms can be defined by their Fourier Transforms.

$$\tau(\tau) = \int_{-\infty}^{+\infty} \tau(y) \exp\{2\pi i y \tau\} dy ,$$

$$\delta(\tau) = \int_{-\infty}^{+\infty} \delta(y) \exp\{2\pi i y \tau\} dy .$$

Then

$$F_r(\tau) = \frac{-x}{r_1} \int_{-\infty}^{+\infty} \tau(y) \exp\{2\pi i y \tau\} dy - \frac{y \sin \theta}{r_1} \int_{-\infty}^{+\infty} \delta(y) \exp\{2\pi i y \tau\} dy .$$

Substitution of these results into equation (7) gives the expression for the Fourier Transform of the sound pressure received from a rotating fluctuating point force as

$$P(f) = \frac{-if}{2a_0 r_1^2} \int_{-\infty}^{+\infty} \left\{ x \int_{-\infty}^{+\infty} \tau(y) \exp\{2\pi i y \tau\} dy + y \sin \theta \int_{-\infty}^{+\infty} \delta(y) \exp\{2\pi i y \tau\} dy \right\} \exp\left\{ -2\pi i f \left(\tau + \frac{r_1}{a_0} \right) \right\} d\tau ,$$

or,

$$P(f) = \frac{-if}{4\pi h a_0 r_1^2} \iint_{-\infty}^{+\infty} \left(x \tau(y) + y \sin \theta \delta(y) \right) \exp\left\{ \frac{iy}{h} (\theta - \phi) \right\} \exp\left\{ \frac{-if}{h} (\theta - \phi) - 2\pi i f \left(\frac{r_1}{a_0} - \frac{2y \cos \theta}{r_1 a_0} \right) \right\} dy d\theta . \quad (8)$$

The Thrust and Drag integrals can be treated separately.

$$P_T(f) = \iint_{-\infty}^{+\infty} \frac{x\tau(g)}{2\pi h} \exp\left\{\frac{i}{h}(\theta - \phi)(g-f) - \frac{2\pi ifr}{a_0} + \frac{2\pi ifRy\cos\theta}{r, a_0}\right\} dg d\theta. \quad (9)$$

Jones (Ref. 45, p.137) gives the result:

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$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} \exp\{-ia \cos x - i\alpha x\} dx = \sum_{n=-\infty}^{+\infty} \exp\left\{-\frac{in\pi}{2}\right\} J_n(a) \delta(\alpha-n),$$

where $J_n(a)$ is a Bessel Function of the first kind and $\delta(\alpha-n)$ is a Dirac Delta Function. This is used in equation (9) to give

$$P_T(f) = \int_{-\infty}^{+\infty} \frac{x\tau(g)}{h} \exp\left\{\frac{i\phi}{h}(f-g) - \frac{2\pi ifr}{a_0}\right\} \sum_{n=-\infty}^{+\infty} \exp\left\{-\frac{in\pi}{2}\right\} J_n\left(-\frac{2\pi fRy}{r, a_0}\right) \delta\left(\frac{f-g}{h} - n\right) dg$$

which can be integrated directly with the result

$$P_T(f) = - \sum_{n=-\infty}^{+\infty} x\tau(f-nh) \exp\left\{-\frac{in\pi}{2} - \frac{2\pi ifr}{a_0} + in\phi\right\} J_n\left(-\frac{2\pi fRy}{r, a_0}\right);$$

or, since

$$J_n(x) = (-1)^n J_n(-x) \quad \text{and} \quad (-1)^n = \exp\{in\pi\},$$

$$P_T(f) = - \sum_{n=-\infty}^{+\infty} x\tau(f-nh) \exp\left\{\frac{in\pi}{2} - \frac{2\pi ifr}{a_0} + in\phi\right\} J_n\left(\frac{2\pi fRy}{r, a_0}\right).$$

In a similar fashion,

$$P_D(f) = \iint_{-\infty}^{+\infty} \frac{y\sin\theta}{2\pi h} \tau(g) \exp\left\{\frac{i}{h}(\theta - \phi)(g-f) - \frac{2\pi ifr}{a_0} + \frac{2\pi ifRy\cos\theta}{r, a_0}\right\} dg d\theta.$$

The integral over θ can be written

$$I_{\theta} = \int_{-\infty}^{+\infty} \sin \theta \exp \left\{ \frac{i\theta}{h} (g-f) + \frac{2\pi i f R y \cos \theta}{r, a_0} \right\} d\theta.$$

This can be integrated by parts to give

$$I_{\theta} = \frac{r, a_0 (g-f)}{2\pi f h R y} \int_{-\infty}^{+\infty} \exp \left\{ \frac{i\theta (g-f)}{h} + \frac{2\pi i f R y \cos \theta}{r, a_0} \right\} d\theta.$$

Using the same result from Jones gives

$$P_D(f) = \int_{-\infty}^{+\infty} \frac{r, a_0 (g-f) D(y)}{2\pi f h^2 R} \exp \left\{ -\frac{i\phi}{h} (g-f) - \frac{2\pi i f r}{a_0} \right\} \\ \sum_{n=-\infty}^{+\infty} \exp \left\{ -\frac{in\pi}{2} \right\} J_n \left(-\frac{2\pi f R y}{r, a_0} \right) \delta \left(\frac{f-g}{h} - n \right) dy.$$

This can be integrated as before to give

$$P_D(f) = \sum_{n=-\infty}^{+\infty} \frac{r, a_0 n D(f-nh)}{2\pi f R} \exp \left\{ in\phi + \frac{in\pi}{2} - \frac{2\pi i f r}{a_0} \right\} J_n \left(\frac{2\pi f R y}{r, a_0} \right).$$

From equation (8),

$$P(f) = \frac{-if}{2a_0 r^2} (P_T(f) + P_D(f)),$$

$$\therefore P(f) = \sum_{n=-\infty}^{+\infty} \left(\frac{2if}{2a_0 r^2} T(f-nh) - \frac{in}{4\pi R r} D(f-nh) \right) \exp \left\{ in \left(\phi + \frac{\pi}{2} \right) - \frac{2\pi i f r}{a_0} \right\} J_n \left(\frac{2\pi f R y}{r, a_0} \right). \quad (10)$$

This equation is, in fact, equation (35) of Ffowes Williams and Hawkins (Ref. 32) expressed in a different form. It resembles equation (11) of Lowson (Ref. 33), but if the frequency f is tied to the rotational frequency (by being set equal to mh , say) equation (10)

above is found to differ from Lowson's result by a factor of $\frac{1}{2}$ in magnitude. This is a direct consequence of allowing f in equation (10) to take all values from $-\infty$ to $+\infty$. Lowson's derivation caters for the single-sided physically measureable spectrum. In this work it was found more convenient to use the double-sided form of the Fourier Transform during the theoretical analysis and then convert to the single-sided spectrum for comparison with experiment.

2.3 The Aero-Acoustic Connection

The relationships between the incoming velocity field and the observed acoustic field are defined for discrete frequency and broad band noise. The first step is to shift the emphasis from blade forces to the input velocity field via unsteady aerofoil theory.

Equation (10) is the Fourier Transform of the far field sound pressure radiated by a rotating point fluctuating force which is represented by its Thrust (axial) and Drag (circumferential) components.

For the case of a fan blade, the force distribution over the blade can be replaced by the total force acting at an effective radius as long as the blade dimensions are small compared with the wavelength of the sound emitted. The forces acting on the air are thus related to the total lift on the blade as follows:

$$T(q) = \int_{\text{span}} L(q) \cos \alpha \, dr \quad ,$$

$$D(q) = \int_{\text{span}} L(q) \sin \alpha \, dr \quad ,$$

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(11)

where α is the angle of incidence of the blade.

Estimation of the precise strength of these blade forces has

been a major difficulty encountered in the verification of this theory. In this analysis a major step is taken by transferring from blade forces to inflow velocity fluctuations which cause these forces.

Sears (Ref. 36) analysed the problem of a wing entering a sinusoidal gust and showed that, if a thin aerofoil is in an unsteady airflow of the form

$$v(x, t) = v'(g) \exp \left\{ 2\pi i g \left(t - \frac{x}{u} \right) \right\},$$

where x is distance downstream of the blade mid-chord point, then the lift per unit span (acting at the quarter-chord point) is given by:

$$L(t) = \pi c \rho u v'(g) S(\sigma) \exp \{ 2\pi i g t \}.$$

Here $\sigma = \frac{\pi c g}{u}$, the reduced frequency, and the Sears Lift Function

$$S(\sigma) = \frac{J_0(\sigma) K_1(i\sigma) + i J_1(\sigma) K_0(i\sigma)}{K_1(i\sigma) + K_0(i\sigma)}$$

where J_0, J_1 are Bessel functions of the first kind and K_0, K_1 are modified Bessel functions of the second kind. This is shown graphically in Figure 2, where the Sears Lift Function is plotted for values from $\sigma = 0$ to $\sigma = \infty$. For a fluctuating inflow containing components at all frequencies, g ,

$$L(g) = \pi c \rho u v'(g) S(\sigma).$$

This expression is substituted into equation (10) via equations (11) to give

$$P(f) = \sum_{n=-\infty}^{\infty} \int_{\text{span}} \left(\frac{2f \cos x}{2a_0 r_1^2} - \frac{n \sin x}{4\pi R_1} \right) i \pi c \rho u v'(f \cdot nh) S(\sigma) \exp \left\{ i n \left(\phi + \frac{\pi}{2} \right) - \frac{2\pi i f r_1}{a_0} \right\} \cdot \int_n \left(\frac{2\pi f R_1}{a_0 r_1} \right) dr. \quad (12)$$

For ease of manipulation a simplified form of notation is now introduced and equation (12) is re-written

$$P(f) = \sum_{n=-\infty}^{+\infty} i \beta_n(f) v'(f-nh) \exp \left\{ i n \left(\phi + \frac{\pi}{2} \right) - i \delta f \right\} J_n(Af) \quad (13)$$

where

$$\beta_n(f) = \pi \rho \int_{s_{\min}} U c S(s) \left(\frac{x f \cos \alpha}{2 a_0 r_1^2} - \frac{n \sin \alpha}{4 \pi R r_1} \right) dr ,$$

$$\delta = \frac{2 \pi r_1}{a_0} \quad \text{and}$$

$$A = \frac{2 \pi R y}{a_0 r_1} .$$

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It can be seen from equation (13) that $v'(f-nh)$ has been assumed constant over the blade span. The justification (or otherwise) of this will be examined later.

For a rotor with B equispaced blades the Fourier Transform of the sound pressure received in the far field then becomes

$$P(f) = \sum_{k=1}^B \sum_{n=-\infty}^{+\infty} i \beta_n(f) v'_k(f-nh) \exp \left\{ i n \left(\frac{\pi}{2} + \frac{2 \pi k}{B} \right) - i \delta f \right\} J_n(Af) .$$

2.4 Power Spectral Analysis

Non-stationary signal analysis techniques are used to obtain the power spectral density of the far field sound pressure in terms of the spectral composition of the incoming airflow.

Of greater use in analysis is the Power Spectral Density of the signal which describes the frequency composition of the data in terms of the spectral density of its mean square value.

In the present work the analytical result is obtained by the use of non-stationary signal analysis theory. This rigorous approach does not appear to have been used before, although the final result

has been obtained by, for example, Ffowes Williams and Hawkings (Ref. 32, equation 39).

In the general case, when the observer is not on the axis of the fan, the signal received from each blade will, by virtue of the rotation of that blade, be Doppler shifted in frequency with a corresponding change in amplitude. The effect of this change in amplitude is that the ensemble mean average of the signal as defined by Bendat and Piersol (Ref. 46) is a function of time and therefore not constant. This time-varying mean implies that the signal cannot be regarded as a stationary random process - the definition of stationarity requires a constant mean value. As a result, computation of the Power Spectral Density requires the use of non-stationary data analysis techniques. In the analysis which follows it is assumed that the velocity field impinging on the blades takes the form of a stationary random process.

The time-scale of the non-stationarity of the process is a function of the rotational frequency and is much shorter than the length of signal used in analysis (in a tape loop for example). The spectrum obtained is thus the 'time-averaged power spectrum' which is derived analytically below.

Following the technique of Bendat and Piersol (Ref. 46 pp. 357-360) a filter is defined whose frequency response function is given by:

$$H(f) = 1 \quad f - \frac{B_e}{2} < f < f + \frac{B_e}{2}$$

= 0 elsewhere

where f is the centre frequency of the filter

B_e is the bandwidth of the filter

The time-varying power spectrum (Ref. 46 p. 360 Eqn. 9.111) is then given by

$$P_p(t, f) = \frac{1}{2B_e} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H^*(g_1) H(g_2) E[P^*(g_1) P(g_2)] \exp\{-2\pi i t (g_1 - g_2)\} dg_1 dg_2. \quad (14)$$

$E[\]$ is the 'expected value' or ensemble average of the quantity in square brackets.

It has already been assumed that the impinging velocity field is stationary and therefore (Ref. 46 p.79)

$$E[P^*(g_1) P(g_2)] = \sum_{k=1}^B \sum_{l=1}^B \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \beta_n^*(g_1) \beta_m(g_2) v'_{kl}(g_1 - nh) \delta(g_1 - g_2 - h(n-m))$$

$$\exp\left\{\frac{i\pi}{2}(m-n) + \frac{2\pi i}{B}(ml - nk) + i\delta(g_1 - g_2)\right\} \int_n(Ag_1) \int_m(Ag_2).$$

$v'_{kl}(g)$ is the cross-power spectrum between the velocity impinging on blade k and that impinging on blade l at the same frequency g.

It is a measure of the power common to both signals.

Substituting into equation (14) and integrating over g_2 gives

$$P_p(t, f) = \frac{1}{2B_e} \int_{-\infty}^{\infty} H^*(g_1) H(g_1 - h(n-m)) \sum_{k=1}^B \sum_{l=1}^B \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \beta_n^*(g_1) \beta_m(g_1 - h(n-m))$$

$$v'_{kl}(g_1 - nh) \exp\left\{\frac{i\pi}{2}(m-n) + \frac{2\pi i}{B}(ml - nk) + i\delta h(n-m)\right\}$$

$$\int_n(Ag_1) \int_m(A(g_1 - h(n-m))) \exp\{2\pi i h t (n-m)\} dg_1. \quad (15)$$

Equation (15) represents the time-varying power spectrum of the signal. This is related to the time-averaged power spectrum by the following (Ref. 46 p.361)

$$\overline{P_p(f)} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} P_p(t, f) dt$$

Thus equation (15) must be averaged over t .

However,

$$\lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} \exp\{2\pi i h t(n-m)\} dt = 1 \quad \text{if } n-m=0$$

$$= 0 \quad \text{if } n-m \neq 0.$$

Thus, for the time-averaged power spectrum to exist, $n = m$. This is independent of the filter bandwidth B_e .

Furthermore,

$$\int_{-\infty}^{\infty} H^*(q_1) H(q_1) dq_1 = 2B_e$$

and, for

$$B_e \rightarrow 0, \quad q_1 \rightarrow f$$

Thus the power spectral density of the far field acoustic signal becomes

$$\overline{P_p(f)} = \sum_{k=1}^N \sum_{l=1}^N \sum_{n=-\infty}^{+\infty} |p_n(f)|^2 v_{kl}'(f-nh) \left(\int_n(Af) \right)^2 \exp\left\{ \frac{2\pi n i}{B} (L \cdot k) \right\} \quad (16)$$

where all time dependence has vanished from the expression and this time-averaged power spectrum of the original non-stationary signal is, itself, stationary.

If it is assumed that all the blades are identical, the double summation over blade number in equation (16) can be reduced to a single sum over λ , the blade-to-blade spacing.

Equation (16) then becomes:

$$\overline{P_p(f)} = \sum_{\lambda=0}^{B-1} \sum_{n=-\infty}^{\infty} |A_n(f)|^2 v'_\lambda(f-n\lambda) (A_n(f))^2 \exp\left\{ \frac{2\pi n i \lambda}{B} \right\} \quad (17)$$

where $v'_\lambda(g)$ is the cross-power spectrum between the velocities impinging on two blades k and l , λ blade spaces apart. λ is positive for blade l leading blade k .

In order to make use of equation (17) in the prediction of noise from an open rotor, the velocity field $v'_\lambda(g)$ must first be defined. It has been found easiest to regard this in terms of rotor blade-to-blade correlations, bearing in mind that the cross-power spectrum required is the Fourier Transform of the cross-correlation between blades.

$$v'_\lambda(g) = \int_{-\infty}^{\infty} R_\lambda(\tau) \exp\{-2\pi i f \tau\} d\tau.$$

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$R_\lambda(\tau)$ is the cross-correlation between two blades separated by λ spaces at a time lag between the two signals of τ seconds.

2.5 Aero-Acoustic Transfer Functions

Two types of inflow are specified to obtain the relationships between aerodynamic input and acoustic output for both discrete frequency and broad band noise.

There are two extremes of flow regime that can be considered easily - perfect blade-to-blade correlation (which occurs when each blade sees precisely the same flow picture as its predecessor at

that point in space) and zero blade-to-blade correlation - the scale of the turbulence is smaller than the blade spacing and each blade radiates independently. The first of these can be caused by a large static distortion (rig blockage for example), whereas the second is a result of the randomness of the flow.

For full correlation, a distortion can be considered static in both space and time such that (with the same convention on λ)

$$R_{\lambda}(\tau) = R_0\left(\tau + \frac{\lambda}{Bh}\right)$$

and hence

$$v'_{\lambda}(g) = \int_{-\infty}^{+\infty} R_0\left(\tau + \frac{\lambda}{Bh}\right) \exp\{-2\pi i g \tau\} d\tau.$$

This can be integrated to give

$$v'_{\lambda}(g) = v'_0(g) \exp\left\{\frac{2\pi i g \lambda}{Bh}\right\}$$

which is substituted in equation (17):

$$\overline{P_p(f)} = \frac{B^{-1}}{B} \sum_{\lambda=0}^{B-1} \sum_{n=-\infty}^{+\infty} |\beta_n(f)|^2 v'_0((f-n)h) \left(\int_n(Af)\right)^2 \exp\left\{\frac{2\pi i f \lambda}{Bh}\right\}.$$

Now $v'_0(g)$ is a periodic function with fundamental frequency

$\therefore$, the frequency of rotation. Writing

$$f = kh \quad \text{where } k \text{ is an integer, gives}$$

$$\overline{P_p(kh)} = \frac{B^{-1}}{B} \sum_{\lambda=0}^{B-1} \sum_{n=-\infty}^{+\infty} |\beta_n(kh)|^2 v'_0((k-n)h) \left(\int_n(Akh)\right)^2 \exp\left\{\frac{2\pi i k \lambda}{B}\right\}$$

and

$$\sum_{\lambda=0}^{B-1} \exp\left\{\frac{2\pi i R \lambda}{B}\right\} = B \quad k = mB$$

$$= 0 \quad \text{elsewhere.}$$

Therefore, the far field discrete frequency power spectral density obtained from an open rotor is

$$\overline{P_p(mBk)} = \sum_{n=-\infty}^{+\infty} B^2 |\beta_n(mBk)|^2 v_0'((mB-n)h) \left(\int_n(A_n Bk)\right)^2. \quad (18)$$

The other flow regime to be considered is that of zero blade-to-blade correlation.

Here

$$v_\lambda'(g) = 0 \quad \lambda \neq 0$$

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and the summation gives

$$\overline{P_p(f)} = \sum_{n=-\infty}^{+\infty} B |\beta_n(f)|^2 v_0'(f-nh) \left(\int_n(Af)\right)^2$$

which differs from equation (18) by a factor B and represents the far field broad band power spectral density obtained from the rotor.

All that is now required is to sum over the Bessel Functions.

A major simplification is achieved if the observer is placed on the axis of the fan. The argument of the Bessel Function is then zero and the Drag term becomes zero.

Now,

$$\begin{aligned} J_n(0) &= 1 & n=0 \\ &= 0 & n \neq 0. \end{aligned}$$

The summation vanishes leaving

$$\overline{P_{p_a}(mBh)} = B^2 |\beta_o(mBh)|^2 v_o'(mBh)$$

and

$$\overline{P_{p_a}(f)} = B |\beta_o(f)|^2 v_o'(f).$$

In each case the frequency of sound received depends solely upon the same frequency in the velocity spectrum. The relationship between incoming velocity fluctuations to the detector and its on-axis acoustic output can now be expressed fully. Aero-acoustic transfer functions can be defined for the two examples of discrete frequency and broad band noise radiation as

$$\frac{\overline{P_{p_a}(mBh)}}{v_o'(mBh)} = B^2 |\beta_o(mBh)|^2$$

and

$$\frac{\overline{P_{p_a}(f)}}{v_o'(f)} = B |\beta_o(f)|^2$$

From equation (13),

$$\beta_o(g) = \frac{\pi \rho}{2a_o r_i} \int_{s_{min}} U_c S(\sigma) g \cos \alpha \, d\sigma.$$

Lowson (Ref. 33) used the form of the Sears Lift Function given by Kemp (Ref. 47) together with standard asymptotic forms for Bessel functions to obtain, at moderate reduced frequencies,

$$S(\sigma) = \frac{\exp\{i(\sigma - \pi/4)\}}{(2\pi\sigma)^{1/2}}$$

for reduced frequency $\sigma > \pi$.

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Now

$$\sigma = \frac{\pi g c}{U} = \frac{c g \cos \alpha}{2 h r}$$

where r is the blade radius.

Substituting for $S(\sigma)$ gives

$$|\beta_o(g)|^2 = \frac{\pi^3 \rho^2 h^3}{a_o^2 r^2} \left(\int_{\text{span}} \left(\frac{r^3 c}{\cos \alpha} \right)^{1/2} \exp \left\{ i \left(\frac{c g \cos \alpha}{2 h r} - \frac{\pi}{4} \right) \right\} dr \right)^2$$

and the aero-acoustic transfer functions are:

$$\frac{\overline{P_{pa}(mBh)}}{v_o'(mBh)} = \frac{mB^3 \pi^3 \rho^2 h^3}{a_o^2 r^2} \left(\int_{\text{span}} \left(\frac{r^3 c}{\cos \alpha} \right)^{1/2} \exp \left\{ i \left(\frac{cmB \cos \alpha}{2 r} - \frac{\pi}{4} \right) \right\} dr \right)^2 \quad (19)$$

for discrete frequencies only

and

$$\frac{\overline{P_{pa}(f)}}{v_o'(f)} = \frac{B \pi^3 \rho^2 h^3 f}{a_o^2 r^2} \left(\int_{\text{span}} \left(\frac{r^3 c}{\cos \alpha} \right)^{1/2} \exp \left\{ i \left(\frac{c f \cos \alpha}{2 h r} - \frac{\pi}{4} \right) \right\} dr \right)^2 \quad (20)$$

for broad band noise.

If it is assumed arbitrarily that the fluctuating lift force generated on the blade is in phase along the span, equations (19 and 20) simplify to

$$\frac{\overline{P_{\text{Bn}}(mBh)}}{v_o'(mBh)} = \frac{m B^3 \pi^3 \rho^2 h^4}{a_o^2 r_i^2} \left(\int_{\text{span}} \left(\frac{r^3 c}{\cos \alpha} \right)^{1/2} dr \right)^2 \quad (21)$$

for discrete frequencies

and

$$\frac{\overline{P_{\text{Bn}}(f)}}{v_o'(f)} = \frac{B \pi^3 \rho^2 h^3 f}{a_o^2 r_i^2} \left(\int_{\text{span}} \left(\frac{r^3 c}{\cos \alpha} \right)^{1/2} dr \right)^2 \quad (22)$$

for broad band noise.

These equations are the main theoretical results of this Thesis.

It is seen that both transfer functions depend solely upon blade number, rotational speed, frequency and the geometry of the fan blading under consideration. All these quantities are known and, given a means of measuring sound pressure and fluctuating velocity simultaneously and independently, the transfer functions defined here theoretically can be measured experimentally.

2.6 Effect of Blade-to-Blade Correlation

Experimental results are used to extend the theory developed previously.

Examples of the auto- and cross-correlation functions obtained from the rotating hot wire anemometer signal are shown in Figures 27 and 28. These do not correspond to the simple model assumed previously of patches of fully-correlated turbulence which remained so for several fan revolutions, combined with short time-scale turbulence which gave zero blade-to-blade correlation. The measured correlation functions appear to support the 'spaghetti' ideas of Hanson (Ref. 40) with a long 'string' of turbulence drawn into the fan. The auto-correlation function appears to consist of a series of pulses of the same shape $(S_R(\tau))$ whose repetition rate is the

period of rotation of the fan and which are reduced in amplitude at each repetition. This can be represented mathematically as:

$$R_o(\tau) = \sum_{\mu=-\infty}^{+\infty} C_R\left(\frac{\mu}{h}\right) S_R\left(\tau - \frac{\mu}{h}\right) \quad (23)$$

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where $C_R(\tau)$ is the envelope of peak amplitudes.

Similarly, from Figure 28, the cross-correlation function between blades spaced λ apart is given by

$$R_\lambda(\tau) = \sum_{\mu=-\infty}^{+\infty} C_R\left(\frac{\mu}{h} - \frac{\lambda}{Bh}\right) S_R\left(\tau - \frac{\mu}{h} + \frac{\lambda}{Bh}\right).$$

(The same convention on λ applies as previously).

Hence, the velocity cross-power spectrum

$$\begin{aligned} v'_\lambda(g) &= \int_{-\infty}^{+\infty} \sum_{\mu=-\infty}^{+\infty} C_R\left(\frac{\mu}{h} - \frac{\lambda}{Bh}\right) S_R\left(\tau - \frac{\mu}{h} + \frac{\lambda}{Bh}\right) \exp\{-2\pi i g \tau\} d\tau \\ &= \sum_{\mu=-\infty}^{+\infty} C_R\left(\frac{B\mu - \lambda}{Bh}\right) S_R(g) \exp\left\{-2\pi i g \left(\frac{B\mu - \lambda}{Bh}\right)\right\} \end{aligned} \quad (24)$$

$$\text{where } S_R(g) = \int_{-\infty}^{+\infty} S_R(\tau) \exp\{-2\pi i g \tau\} d\tau$$

- the Fourier Transform of the pulse shape.

This expression for the velocity cross-power spectrum can now be used in the previous analysis to predict the power spectrum of the sound pressure.

Equation (17) stated that

$$\overline{P_p(f)} = \frac{B-1}{B} \sum_{\lambda=0}^{B-1} \sum_{n=-\infty}^{+\infty} |f_n(f)|^2 v'_\lambda(f - nh) (f_n(Af))^2 \exp\left\{\frac{2\pi n i \lambda}{B}\right\}.$$

The summation over λ is

$$Q(f) = \sum_{\lambda=0}^{B-1} v'_\lambda (f - nh) \exp \left\{ \frac{2\pi i n \lambda}{B} \right\}.$$

Substituting from equation (24) gives

$$Q(f) = S_2(f - nh) \sum_{\lambda=0}^{B-1} \sum_{\mu=-\infty}^{+\infty} C_2 \left(\frac{B\mu - \lambda}{Bh} \right) \exp \left\{ -2\pi i f \left(\frac{B\mu - \lambda}{Bh} \right) \right\}.$$

It can be seen that not only is there no dependence upon n in the summation, but also, as μ and λ vary, the quantity $B\mu - \lambda$ will take all values between $-\infty$ and $+\infty$ once and once only. The double sum can thus be replaced by a single summation.

i.e.

$$Q(f) = S_2(f - nh) \sum_{\nu=-\infty}^{+\infty} C_2 \left(\frac{\nu}{Bh} \right) \exp \left\{ -\frac{2\pi i f \nu}{Bh} \right\}.$$

Further analysis requires specification of the envelope function and pulse shape.

Consider the case when the envelope function $C_2(\tau)$ is exponential (as suggested by Figures 27 and 28).

$$C_2(\tau) = \exp \{ -\alpha' h |\tau| \}$$

where α' - the exponential decay rate - can be obtained from the experimental results.

$$\frac{Q(f)}{S_R(f-nh)} = \sum_{v=-\infty}^{\infty} \exp \left\{ \frac{-\alpha' |v|}{B} - \frac{2\pi i f v}{Bh} \right\}. \quad (25)$$

This sum was evaluated by Lowson (Ref. 49) as

$$D(f) = \frac{Q(f)}{S_R(f-nh)} = \frac{\sinh(\alpha'/B)}{\cosh(\alpha'/B) - \cos(\xi/B)} \quad \text{where } \xi = \frac{2\pi f}{h}. \quad (26)$$

Which can also be written

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$$D(f) = \frac{1 - \exp\{-2\alpha'/B\}}{1 - 2\cos(\xi/B)\exp\{-\alpha'/B\} + \exp\{-2\alpha'/B\}}.$$

The function $D(f)$ represents the effects of blade-to-blade correlation on the acoustic power spectrum. It is periodic in f with period Bh as expected.

This result can be used to examine the full blade-to-blade correlation and zero blade-to-blade correlation examples considered previously.

For zero blade-to-blade correlation,

$$\alpha' \rightarrow \infty \quad \text{and} \quad D(f) \rightarrow 1 \quad \text{as anticipated.}$$

For full blade-to-blade correlation the result is not as immediately obvious. However, equation (25) shows that when $\alpha' = 0$,

$$D(f) = \sum_{v=-\infty}^{\infty} \exp \left\{ \frac{-2\pi i f v}{Bh} \right\}$$

and Lighthill, Ref. 50 pp. 67-68, shows that in this instance

$$D(f) = 2\pi B \sum_{s=-\infty}^{+\infty} \delta(s - 2\pi \omega B)$$

or

$$D(f) = B \sum_{s=-\infty}^{+\infty} \delta\left(\frac{f}{h} - B\omega\right) .$$

In other words, the effect of full blade-to-blade correlation is a series of delta functions of amplitude equal to the blade number, occurring at blade passing frequency and its harmonics. This agrees with the result obtained previously.

Equation (26) representing the effect of blade-to-blade correlation can be substituted into equation (17) so that the expression for the acoustic field now becomes

$$\overline{P_p(f)} = \sum_{n=-\infty}^{+\infty} B |\beta_n(f)|^2 (J_n(Af))^2 S_R(f - nh) D(f)$$

where $|\beta_n(f)|^2$ represents the blade response,

$(J_n(Af))^2$ and $(-nh)$ represent the Doppler shift caused by the rotation of the blading,

$S_R(g)$ represents the circumferential spectrum of the turbulence and

$D(f)$ represents the effects of blade-to-blade correlation.

Again, utilising the simplifying effects of placing the observer on the fan axis, gives

$$\overline{P_{p_{ax}}(f)} = B |\beta_0(f)|^2 S_R(f) D(f)$$

and the complete aero-acoustic transfer function as obtained against frequency can be calculated for one set of conditions • i.e. speed and blade number fixed.

The velocity auto-power spectrum is obtained from equation (24) as

$$\begin{aligned} v'_0(g) &= \int_{-\infty}^{+\infty} \sum_{\mu=-\infty}^{+\infty} C_R\left(\frac{\mu}{h}\right) S_R\left(\tau - \frac{\mu}{h}\right) \exp\left\{-2\pi i g \tau\right\} d\tau \\ &= S_R(g) \sum_{\mu=-\infty}^{+\infty} C_R\left(\frac{\mu}{h}\right) \exp\left\{-\frac{2\pi i \mu}{h}\right\}. \end{aligned}$$

We have already assumed $C_R(\tau)$ is exponential

$$C_R\left(\frac{\mu}{h}\right) = \exp\left\{-\kappa' |\mu|\right\}$$

hence

$$E(f) = \frac{v'_0(f)}{S_R(f)} = \frac{\sinh(\kappa')}{\cosh(\kappa') - \cos(\xi)}$$

(where $\xi = \frac{2\pi f}{h}$ as in equation (26))

or

$$E(f) = \frac{1 - \exp\{-2\kappa'\}}{1 - 2\cos(\xi) \exp\{-\kappa'\} + \exp\{-2\kappa'\}}$$

This function $E(f)$ represents the effects of fan rotation on the velocity spectrum. Because only the one summation is involved

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there is no dependence on blade number and $E(f)$ is periodic in f with period h as anticipated.

The on-axis aero-acoustic transfer function was defined previously as the ratio of the magnitudes of the acoustic and aerodynamic power spectra at the same frequency.

Thus

$$\frac{\overline{P_{\text{ac}}(f)}}{v_o'(f)} = \frac{B |\beta_o(f)|^2 D(f)}{E(f)}$$

or, in full,

$$\frac{\overline{P_{\text{ac}}(f)}}{v_o'(f)} = \frac{\frac{B h^3 \pi^3 \rho^2 f}{a_o^2 c^2} \left(\int_0^1 \left(\frac{c r^3}{\sin \alpha} \right)^{1/2} dr \right)^2 \frac{\sinh(\kappa'/B)}{\cosh(\kappa'/B) - \cos(\xi/B)}}{\frac{\sinh(\kappa')}{\cosh(\kappa') - \cos(\xi)}} \quad (27)$$

Which can again be computed directly for a given blade geometry, blade number and speed of rotation.

An interesting point to note here is that the transfer function is not entirely independent of the incoming turbulence. It is a function of blade-to-blade correlation, and hence turbulence, though not of the circumferential scale of the turbulence.

Thus, from a knowledge of the axial decay rate of the turbulence entering a rotor, the on-axis aero-acoustic transfer function can be obtained with some ease.

However, estimation of the acoustic and velocity power spectra requires an assumption of pulse shape. Examination of Figure 29 suggested that $S_p(\tau)$ could be represented by an exponentially decaying

cosine wave.

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i.e.

$$S_2(\tau) = \exp\{-b\hbar|\tau|\} \cos(\eta\pi\hbar\tau)$$

where b , the exponential decay rate and η , a function of the frequency, can be obtained from experimental results.

$S_2(g)$ - the Fourier Transform of $S_2(\tau)$ - is calculated by convolving the Fourier Transform of

$$\exp\{-b\hbar|\tau|\}$$

with the Transform of

$$\cos(\eta\pi\hbar\tau)$$

and is given by Bendat and Piersol (Ref. 46) as

$$S'_2(f) = b\hbar \left(\frac{1}{b^2\hbar^2 + 4\pi^2\left(f + \frac{\eta\hbar}{2}\right)^2} + \frac{1}{b^2\hbar^2 + 4\pi^2\left(f - \frac{\eta\hbar}{2}\right)^2} \right)$$

The physically realisable 'single-sided' power spectrum is

$$G_2(f) = 2S'_2(f)$$

and hence the measurable velocity power spectrum is given in full from equation (24) as

$$v_{om}(f) = \frac{4\bar{v}^2 b\hbar \sinh(\alpha')}{\cosh(\alpha') - \cos(\xi)} \left(\frac{b^2\hbar^2 + 4\pi^2 f^2 + \pi^2 \eta^2 \hbar^2}{(b^2\hbar^2 + 4\pi^2\left(f + \frac{\eta\hbar}{2}\right)^2)(b^2\hbar^2 + 4\pi^2\left(f - \frac{\eta\hbar}{2}\right)^2)} \right) \quad (28)$$

($\bar{v}^2$ - the mean square turbulence velocity - appears because the original auto-correlation was normalised).

$\bar{v}^2$ and $\hbar$ were measured at the time of the experiment and α' , b and η have been obtained from subsequent analysis.

The single-sided acoustic power spectrum becomes in full

$$\overline{P_p(f)} = \sum_{n=-\infty}^{\infty} h^3 \pi^3 \rho^2 \left(\int_{span} \left(\frac{cr^3}{f \cos \alpha} \right)^{1/2} \left(\frac{2f \cos \alpha}{a_0 r_1^2} - \frac{n \sin \alpha}{2\pi R r_1} \right) dr \right)^2 \left(J_n \left(\frac{2\pi R y f}{a_0 r_1} \right) \right)^2$$

$$4 \bar{v}^2 b h \left(\frac{b^2 h^2 + 4\pi^2 (f - nh)^2 + \pi^2 \rho^2 h^2}{(b^2 h^2 + 4\pi^2 (f - nh + \frac{1}{2}h)^2)(b^2 h^2 + 4\pi^2 (f - nh - \frac{1}{2}h)^2)} \right)$$

$$\frac{B \sinh(\alpha'/B)}{\cosh(\alpha'/B) - \cos(\xi/B)} \quad (29)$$

Figures 37 and 38 show examples of the spectra obtained from these formulae for the incoming velocity and the on-axis sound pressure. They are discussed in Chapter 5 together with the results obtained for the aero-acoustic transfer functions - both for discrete frequency and broad band noise.

The object of this Chapter has been to derive aero-acoustic transfer functions which could be compared with experimental data. Later experimental results have extended the original theory. The important results are the aero-acoustic transfer functions:

$$\frac{\overline{P_{p,ax}}(MBh)}{v_0'(MBh)} = \frac{m B^3 \pi^3 \rho^2 h^4}{a_0^2 r_1^2} \left(\int_{span} \left(\frac{r^3 c}{\cos \alpha} \right)^{1/2} dr \right)^2 \quad (21)$$

for discrete frequencies

and

$$\frac{\overline{P_{B,ax}}(f)}{v_0'(f)} = \frac{B \pi^3 \rho^2 h^3 f}{a_0^2 r_1^2} \left(\int_{span} \left(\frac{r^3 c}{\cos \alpha} \right)^{1/2} dr \right)^2 \quad (22)$$

for broad band noise.

For the 1200 rpm, 7-bladed fan considered later, the important results were:

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Aero-acoustic transfer function

$$\frac{\overline{P_R}(f)}{v_o'(f)} = \frac{\frac{B h^3 \pi^3 \rho^2 f}{a_o^2 r_i^2} \left(\int_{\text{span}} \left(\frac{c r^3}{\cos \alpha} \right)^{1/2} dr \right)^2 \frac{\sinh(\kappa'/B)}{\cosh(\kappa'/B) - \cos(\xi/B)}}{\frac{\sinh(\kappa')}{\cosh(\kappa') - \cos(\xi)}} \quad (27)$$

Sound power spectrum

$$\begin{aligned} \overline{P_P}(f) = & \int_{-\infty}^{+\infty} h^3 \pi^3 \rho^2 \left(\int_{\text{span}} \left(\frac{c r^3}{f \cos^3 \alpha} \right)^{1/2} \left(\frac{x f \cos \alpha}{a_o r_i^2} - \frac{n \sin \alpha}{2 \pi r_i} \right) dr \right)^2 \left(\int_{\text{span}} \left(\frac{2 \pi r_i f}{a_o r_i} \right) \right)^2 \\ & 4 \bar{v}^2 b h \left(\frac{b^2 h^2 + 4 \pi^2 (f - nh)^2 + \pi^2 \bar{v}^2 h^2}{(b^2 h^2 + 4 \pi^2 (f - nh + \frac{\bar{v} h}{2})^2) (b^2 h^2 + 4 \pi^2 (f - nh - \frac{\bar{v} h}{2})^2)} \right) \\ & \frac{B \sinh(\kappa'/B)}{\cosh(\kappa'/B) - \cos(\xi/B)} \quad (29) \end{aligned}$$

and Velocity power spectrum

$$v_{om}(f) = \frac{4 \bar{v}^2 b h \sinh(\kappa')}{\cosh(\kappa') - \cos(\xi)} \left(\frac{b^2 h^2 + 4 \pi^2 f^2 + \pi^2 \bar{v}^2 h^2}{(b^2 h^2 + 4 \pi^2 (f + \frac{\bar{v} h}{2})^2) (b^2 h^2 + 4 \pi^2 (f - \frac{\bar{v} h}{2})^2)} \right) \quad (28)$$

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CHAPTER 3

EXPERIMENTS

3.1 The Rig

The primary objective of the experiments was to establish that the aerodynamic input to a fan and its acoustic output are related in a systematic fashion.

The secondary objective was then to determine if this 'aero-acoustic transfer function' was of the form predicted by the theory of Chapter 2. This led to the final, tertiary, objective - the successful prediction of the on-axis rotor noise from the aerodynamic data.

To achieve these objectives it was necessary to design a rig on which aerodynamic and acoustic measurements could be made both independently and simultaneously.

The rig used for the experiments is shown in Figure 3. As can be seen, in essentials it consists of a 0.66m diameter low speed open rotor driven directly by a 7.5 KW D.C. motor. The motor is connected to the drive-shaft through a flexible coupling and the motor and shaft are supported on an angle-iron frame. The fan is slightly over-hung and the frame supports slope away from the fan disc in an attempt to decrease the possibility of blade tip/rig interaction.

The fan used is a commercially available axial flow rotor supplied by Standard and Pochin Bros. Ltd. of Leicester. The hub can take 2, 7 or 14 polypropylene plastic blades which are available with root incidences of 25° , 30° , 35° and 40° . The twist along the span in each case is 20° and full blade details are given in Figure 4. For these experiments the fan was run at speeds of 1000, 1200, 1400 and 1600 rpm in its two- and seven-blade configurations with blades of 30° root incidence. Figures 5-7 show the performance of the seven-bladed fan using these blades.

A more complete presentation of fan performance is given in Ref.28.

The rig is situated in the anechoic chamber of the Loughborough University of Technology Acoustics Facility with the axis of the fan at the mid-height of the chamber. The walls and floor of the chamber are not lined with foam wedges in the customary fashion but with fibre-glass slabs graded to have the greatest density nearest the wall. The fibre-glass is covered with a thin layer of polyurethane foam to prevent erosion by the airflow in the chamber and the net result has been found to give satisfactory absorption above approximately 200 Hz. The construction of the chamber is shown in detail in Figure 8.

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3.2 Instrumentation

The instrumentation used in the experiments falls naturally into two categories, acoustic and aerodynamic. Acoustic measurements were made using a $\frac{1}{2}$ " Brüel and Kjaer microphone type 4133 with F.E.T. preamplifier type 2619 mounted on a boom at a radius of 2.14 metres from the centre of the fan disc and level with it. For this work the microphone was always placed on the fan axis to make use of the simplifications that arise from the absence of Doppler shift caused by the rotating blading. The microphone was powered by a Brüel and Kjaer type 2607 measuring amplifier and the resultant signal was recorded on one track of a Nagra IVS twin track stereo tape recorder.

Measurements made with a stationary hot wire anemometer probe placed .0565m axially upstream of the fan tip produced spectra of the type shown in Figure 9. The peaks occur at harmonics of blade passing frequency and are superimposed on a general background level. It is hard to distinguish between 'self-generated' effects due to the fan and those in the incoming (low velocity) airflow.

Thus the aerodynamic measurements required necessitated the development of a rotating hot wire anemometer system in order to measure the fluctuations in the airflow impinging on one fan blade. Despite initial misgivings this has been achieved by using the latest DISA miniature probe system. A 4mm diameter 235mm long right-angled mounting tube was clamped in the nose cone of the fan and used to carry a DISA type 55POI probe element together with a DISA 55H20 probe support. The probe element was oriented such that the active sensor length was parallel to the blade leading edge as shown in Figure 1C. The hot wire was driven through two channels of sliprings (manufactured by I.D.M. Electronics Ltd.) by either a DISA type 55DOI or a DISA type 55MOI hot wire anemometer unit. Both units were used in the constant temperature mode. The hot wire signal was linearised by a DISA type 55DIO lineariser and amplified by a Brüel and Kjaer type 2606 measuring amplifier. This amplifier was chosen in preference to the type 2607 used for the microphone signal as the 2606 is capable of handling signals with a greater crest factor than can be accommodated by the 2607. It was felt that the hot wire signals might require this capability. The anemometer signal was recorded on the second track of the Nagra IVS tape recorder to give a simultaneous record of the two signals. In order to obtain meaningful results the acoustic and aerodynamic systems were calibrated as follows.

The acoustic measuring and recording systems were calibrated simultaneously using either a Brüel and Kjaer type 4220 pistonphone or a type 4230 acoustic calibrator to produce a known reference level at the microphone. The resultant signal was then recorded. The hot wire system was calibrated in two stages, first by spinning the nose cone/fan disc assembly (without blades) at a series of known

rpm to give a plot of airspeed vs d.c. volts. A typical hot wire calibration curve is shown in Figure 11, whence it can be seen that good linearisation is possible with the rotating probe system. The data acquisition system was then calibrated by recording a 1 KHz 50 mv rms signal on tape at the start of a test series which could be used in conjunction with the d.c. curve to provide an absolute level for the result. It should be mentioned that all the anemometry setting-up procedures that depend on the length and characteristics of the probe cable were performed with the rig running at or near its lowest speed (190 rpm) with a shield over the probe so that (for instance) the resistance and other effects of the sliprings would be taken into account. This method appears to have worked satisfactorily and it has always been possible to obtain a reasonable to good frequency response when setting up the bridge network with a square wave test signal.

3.3 The Experiments

The experimental work has followed two natural lines of approach - aero-acoustic and aerodynamic. The objectives of the aero-acoustic work were first, to establish the existence of a transfer function between aerodynamic input to and acoustic output from the fan, and second, if such a function were found to exist, whether it was of the form predicted by the theory. The aerodynamic experiments arose from further consideration of the theory and are discussed later.

To achieve the aero-acoustic objectives a series of tests was made at fan speeds of 1000, 1200, 1400 and 1600 rpm (i.e. tip-speeds of 34.6, 41.5, 48.4 and 55.3 m/s) with the fan in both its two- and seven-bladed configurations. For these tests the hot wire was mounted a radius of 0.256 m (80% radius) - this being the greatest radius

possible using standard DISA mounting tubes - as it was felt that the tip flow would exert the most influence on the noise. The sensor was oriented parallel to the leading edge of one blade and situated approximately half a chord-length upstream along the chord line, and the speed was restricted to the maximum of 1600 rpm from considerations of wire strength and mounting problems - at higher speeds there was a tendency for probes to bury themselves in the walls of the chamber. In all cases the microphone was mounted with the diaphragm perpendicular to the fan axis at a distance of 2.14 metres from the centre of the hub.

It was found that, owing to the enclosed nature of the anechoic chamber, a recirculating flow situation occurred in which the air, having passed through the fan, was pushed round the walls and re-ingested. The onset of recirculation was accompanied by an audible change in the character of the fan noise which became noticeably harsher and increased in amplitude. Each test therefore could produce two sets of data - one "before" and one "with" recirculation - thus providing two measurements of the aero-acoustic transfer function which should be identical. It was felt that if the transfer functions between aerodynamic input and acoustic output were found to be the same under these very different input conditions and also to agree with the values obtained by theoretical arguments this would be a convincing proof of the relationship deduced between input aerodynamics and output acoustics. The test series was repeated in order to obtain information on the repeatability of the results.

Following the establishment of rotating hot wire anemometry as a viable experimental technique it was decided to extend the measuring system from a single rotating probe to two probes in

order to obtain more insight into the nature of the flow. The initial investigation was aimed at confirming the Sears (Ref. 36) model of uniformly convected sinusoidal gusts entering the fan to cause the fluctuating lift forces on the blades. To achieve this, the probes were positioned along the chord line upstream of one blade at the same radius (0.256m) as before. The upstream probe was 0.138m (approximately two chord lengths) from the blade leading edge and the second probe was positioned as previously. The setting up and calibration of the probes was performed as before and two DISA 55D10 linearisers were used to ensure that both systems had the same velocity vs. volts calibration. A typical example of this is shown in Figure 12. The recording/analysis system was calibrated with 1KHz 50mv rms signals from the Brüel and Kjaer 2607 amplifier for one probe system and the 2606 amplifier for the other.

In another test series the two probes have been used to investigate blade-to-blade correlations - again at the same radius (0.256m) as before and with both probes in the same position relative to different blades. The fan was run at 1200 rpm (corresponding to a tip speed of 41.5 m/s) and the probes were positioned on adjacent blades, then separated by one and by two blades. These positions (on the seven-bladed fan) cover the full range of blade-to-blade spacings as either probe can be considered to be leading.

A longer probe mounting tube was manufactured to enable investigation of the flow in the tip region of the blades. Two series of tests were run with this probe. First, using just the tip probe and on-axis microphone a few transfer function recordings were made in the same way as before to check on differences between results obtained at a radius of 0.256m and those at the tip. Second, one probe was placed in its original position with the second probe at

the tip of the same blade - the calibrations were set up to be identical as before - and tests were run (with seven blades again) at 1000 and 1200 rpm corresponding to tip speeds of 34.6 and 41.5 m/s respectively. The objective of these tests was to discover the degree of spanwise correlations along the blade.

Table 1 lists all the experiments which were performed and the results obtained in these experiments were analysed as described in the following Chapter.

CHAPTER 4

ANALYSIS

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This Chapter describes the means available for analysing the data obtained in the experiments of Chapter 3. The respective merits of the systems are compared.

Two methods - one analogue, the other digital - have been available for analysing the data. For analogue analysis the recorded signal is split into short tape loops (of two seconds duration) and the resultant output filtered by a narrow band (3.16Hz) filter (Brüel and Kjaer type 2020) which is swept through the frequency range 20-20,000Hz by a Brüel and Kjaer type 1024 Beat Frequency Oscillator. A permanent record of the output is made by a Brüel and Kjaer 2305 Level Recorder. Spectra such as Figure 13 are the only form of output available with this equipment.

The second, digital, method involves a Hewlett Packard 5451A Fourier Analyser. The heart of this system is a Hewlett Packard 2100A mini-computer with associated Fourier software. The signal to be analysed is input via analogue-to-digital converters and can either be processed directly or stored on digital magnetic tape for attention later. (All input signals are low pass filtered to prevent aliasing). The Fourier software package is addressed via a keyboard - Fourier transformation, change of coordinates from rectangular to polar, the use of a Hanning window etc. being performed at the push of a button-and it is possible to string a series of operations together to form a keyboard program. A permanent record of the output is obtained from a Computer Instrumentation Ltd. digital graph plotter - this output can be in the form of spectra, transfer functions, coherence functions, correlograms or time-averaged data. A comparison between the two systems follows.

The majority of results were required as aero-acoustic transfer functions in the form defined in Chapter 2. To produce these on

the Brüel and Kjaer (analogue) equipment is a time consuming task involving the analysis of the two signals separately followed by extracting the individual levels at the required frequencies and subtracting them (as the scales are logarithmic) to give the transfer function amplitude at each frequency of interest. The short tape loops used ensure that the same signal is analysed at all frequencies and that the aerodynamic and acoustic signals (i.e. the cause and effect) correspond. Restricting the length of the loop to two seconds also enables the 'before' and 'with' recirculation flow regimes to be investigated independently. However, the narrow bandwidth and small timescale of the analysis do not make for great statistical accuracy. The main advantage of this system lies in its ease of calibration and operation. The levels are set from the recorded calibration signals and the system can then be left to its own devices.

Production of results on the Hewlett Packard system requires a different technique. Transfer functions can be produced automatically from two simultaneous inputs either using the same tape loops as before or from data digitised previously and stored on digital magnetic tape. A keyboard program is written to perform the operation which, in this case, will produce not transfer functions alone but power spectra of the input and output signals, their cross-power spectrum and coherence function. (The coherence function is defined as

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$$\frac{|P_{xy}|^2}{P_{xx} \cdot P_{yy}}$$

where $|P_{xy}|$ is the magnitude of the cross-power spectrum between two signals $x(t)$ and $y(t)$,

P_{xx} is the power spectrum of $x(t)$

P_{yy} is the power spectrum of $y(t)$.

It is a measure of the degree of relationship between two signals.

This program is set in motion and the results are plotted out.

Best statistical accuracy is obtained by averaging the results from a large number of independent samples and hence, from this viewpoint, the use of pre-digitised data is to be recommended.

However, using the same tape loops as are used in the analogue analysis does provide a check between the two systems.

It would appear then that the Hewlett Packard equipment which produces transfer functions directly should be a distinct improvement on the Brüel and Kjaer system which does not.

Unfortunately, however, the state of the computer system at Loughborough whilst these results were being analysed made it difficult to obtain a clear-cut decision on this. It was mentioned above that the major attraction of the analogue system lay in the ease with which results can be interpreted quantitatively. Likewise, the main detraction from the merits of the digital system has been found in the area of quantitative interpretation of results. When results are required with a logarithmic amplitude scale the Hewlett Packard produces dBs relative to its own reference level. This reference level must be related to the required reference level via the calibration signal and it has been found that the technique available is both inconvenient and unsatisfactory. All in all, it is felt that the Hewlett Packard system is ideal for obtaining quick qualitative results in the frequency domain but needs further development before quantitative results can be trusted implicitly.

The above discussion concentrates solely on frequency aspects of the two systems - indeed the Brüel and Kjaer system has no other aspect. The Hewlett Packard system however can also be used in the time domain to produce the autocorrelation of a signal or cross correlation between signals. Again, care is needed in the interpretation of these results. To eliminate 'wrap around' error portions of the data must be discarded, leaving valid answers in only the central half of the data block. Also, if correlation functions (i.e. normalised results) are required these are easily obtained for auto-correlations, but obtained with more difficulty for cross-correlations.

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CHAPTER 5

DISCUSSION

The results produced from the experiments of Chapter 3 by the analysis techniques of Chapter 4 can be divided into three categories: acoustic, aerodynamic and aero-acoustic where the latter group overlaps the others to a certain extent. All the acoustic data are in the form of on-axis spectra at different fan speeds and blade numbers and under the two entry flow conditions of 'before' and 'with' recirculation. The effects of changing these parameters are shown in Figures 13 to 15. As would be expected, the 'basic' spectrum consists of an overall broad band spectrum with discrete tones at blade passing frequency and its harmonics. The frequencies of these tones change with fan speed and blade number (Figures 13 and 14).

The comparison between 'before' and 'with' recirculation spectra shown in Figure 15 is of considerably greater interest. Here the 'before' recirculation spectrum shows sharp peaks (whose width is defined solely by the filter shape) at blade passing frequency and harmonics thereof, together with smaller peaks at harmonics of rotational frequency not related by blade number. These are superimposed on a low level broad band spectrum. The 'with' recirculation spectrum has tones at harmonics of blade passing frequency which are considerably broader than before and of greater amplitude. The broad band background level has risen above the rotational frequency harmonic tones observed previously. These changes in the acoustic spectrum correspond to an audible change in the noise of the fan and also to a visible change in the signal received from the rotating hot wire which was monitored on an oscilloscope during the tests. These particular results were obtained using the seven-bladed fan. The two-bladed results follow the same overall pattern but the time taken for the recirculation to build up is longer in this case

as the mass flow through the fan is less.

Corresponding aerodynamic spectra obtained from the rotating hot wire anemometer signal are shown in Figures 16 to 19. Here the basic spectrum again has a background broad band level with tones superimposed on it - at harmonics of rotational frequency this time. There is no difference between Figure 16 (seven blades) and Figure 17 (two blades) in that no one particular harmonic of rotational frequency dominates as in the acoustic examples. This indicates that the probe is indeed seeing the airflow impinging on one blade rather than the potential field of the rotor as is the case with a stationary probe (Figure 9). The effect of speed change is shown in Figure 18 and is, as would be expected, to change the fundamental frequency of the harmonics. The other features remain the same.

The 'before' and 'with' recirculation spectra shown in Figure 19 exhibit similar differences to those observed in Figure 15 for the acoustic signal. 'Before' recirculation the harmonic peaks are sharp and clearly defined, the background level is low. 'With' recirculation the background level rises, the peaks broaden and increase in amplitude and it becomes harder to distinguish between tones and broad band information at the higher frequencies. The recirculating flow would thus appear to contain a broad spectrum of eddies of various sizes - some well correlated (to increase the discrete frequency tones) others not (to raise the broad band level).

Typical results of the repeatability tests at discrete frequencies are shown in Figure 20 for the microphone and Figure 21 for the equivalent hot wire signals. In each figure the level has been plotted against blade passing frequency harmonic number for both 'before' and 'with' recirculation data. As can be seen, agreement between the two sets of experimental values is of the order of 3 dB

and is better 'with' than 'before' recalculation. It was considered that this agreement was close enough to justify the use of results from one set of data only for each point on the transfer function plots.

It will be noticed that whereas the sound pressure level spectra are scaled in dBs re $2 \times 10^{-5} \text{ N/m}^2$ (the threshold of hearing) in the customary fashion, the velocity spectra use dBs re 1 m/s which is taken as a convenient reference level. The actual levels are obtained from a combination of the original system calibration curve (d.c. volts vs m/s) and the analysis of the 50 mv rms 1 KHz signal recorded at the same time as the data. The combination of the sound pressure level and velocity scales results in aero-acoustic transfer functions given in dBs re $2 \times 10^{-5} \frac{\text{N/m}^2}{\text{m/s}}$ as shown in Figures 22 to 24.

Equations (21) and (22) of Chapter 2 defined the on-axis aero-acoustic transfer function as

$$\frac{\overline{P_{ax}}(\text{mBh})}{V_o'(\text{mBh})} = \frac{m \delta \pi^3 \rho^2 h^4}{a_o^2 r_o^2} \left(\int_{span} \left(\frac{r^3 c}{\cos \alpha} \right)^{1/2} dr \right)^2 \quad (21)$$

for discrete frequencies and

$$\frac{\overline{P_{ax}}(f)}{V_o'(f)} = \frac{B \pi^3 \rho^2 h^4 f}{a_o^2 r_o^2} \left(\int_{span} \left(\frac{r^3 c}{\cos \alpha} \right)^{1/2} dr \right)^2 \quad (22)$$

for broad band noise.

In Figures 22 and 23 the results predicted by these formulae are compared with experimental results obtained from the experimental and analysis techniques of Chapters 3 and 4.

These Figures contain the main results of this work.

The theoretical values are absolute, depending solely upon

frequency, blade number, speed of rotation and blading geometry to produce the lines shown. In both these figures each experimental point represents the value of the transfer function obtained in one analysis of one run at a given speed at one blade passing frequency harmonic (discrete frequency transfer function) or one frequency (broad band transfer function). Blade passing frequency harmonics beyond the tenth have been neglected as it is difficult to isolate them from the background level (Figures 13 to 15) and the broad band frequencies were chosen to be in the same portion of the spectrum. The results themselves show agreement between theory and experiment over three decades of the relevant parameter for both discrete frequency and broad band data.

The establishment of this agreement between theory and experiment was the prime objective of this work.

Closer examination of Figure 22 reveals that whilst the agreement between experiment and theory is, on the whole, very good, there is some 'banding' of the experimental results. Individual runs, especially for the two-bladed results, have a steeper slope than predicted. The explanation for this is not known. The experimental points in Figure 23 are, with the exception of one run, consistently lower than the theory predicts. There is also much greater scatter than was seen in Figure 22. This scatter can be attributed in part to the difficulties associated with defining the broad band level in spectra such as Figures 13 to 19.

Figure 24 compares the theoretical result with a least squares fit drawn through discrete frequency data points at 1600 rpm both 'before' and 'with' recirculation. It is apparent that whereas, as maintained above, 'with' recirculation the agreement between the two is good in both magnitude and trend, 'before' recirculation there

is some agreement on magnitude but the theory does not predict the trend of the results. This is unfortunate in that the transfer function should be independent of input - merely a constant relationship between input and output regardless of conditions. This 'before' recirculation result casts doubts on the reasoning behind the transfer function theory and a check on its credibility is needed.

One check is provided by consideration of the coherence function obtained between the hot wire (input) signal and the microphone (output) signal.

The coherence function can take values in the range zero to unity depending on the degree of correlation between the signals - a coherence function of zero means the signals are totally unrelated, a coherence function of unity indicates that the output signal is a function of the input signal only and a value less than unity implies that one or more of the following situations exists.

Either a) Extraneous noise is present in the measurements
or b) The system relating $x(t)$ and $y(t)$ is not linear
or c) $y(t)$ is an output due to an input $x(t)$ as well as
 to other inputs.

Figure 25 shows examples of the coherence functions obtained with the fan in its two-bladed configuration for both 'before' and 'with' recirculation flow. 'Before' recirculation there is strong coherence between the signals at all harmonics of rotational frequency, not just blade passing frequency and its harmonics as was deduced theoretically. Between harmonics the coherence function drops to zero - there is thus no relationship between aerodynamic input and acoustic output for the broad band portion of the 'before' recirculation signals. Obviously, however, there is a transfer function between the two at harmonics of rotational frequency. Equally

obviously (from Figure 24) it is not of the form derived theoretically.

If reduced frequency effects are discarded arbitrarily in the theoretical analysis it is found that, for discrete frequencies, the transfer function can be expressed as:

$$\frac{\overline{P_{ax}}(m\beta h)}{v'(m\beta h)} \propto m^2 \beta^4 h^2$$

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Figure 26 shows the result of plotting the 'before' recirculation data against this parameter with a line of gradient 3dB/octave fitted through the points. It would appear that this is a possible form of the transfer function in this instance though the reasons why this should be so are as yet unclear. The coherence at all harmonics of rotational frequency is also unexplained. No 'before' recirculation broad band transfer functions have been considered as there is zero coherence between the signals at these frequencies.

The coherence function obtained 'with' recirculation shown in Figure 25 prompts further discussion of the agreement between experiment and theory shown in Figures 22 and 23 for discrete frequency and broad band transfer functions respectively. In the development of the theory, it was assumed that the amplitude of the fluctuating velocity and the resultant phase of the fluctuating

lift force are constant along the blade span. There is no physical reason why this should occur - the velocity field may be such that the resultant lift forces combine to cancel rather than sum. The theoretical lines of Figures 22 and 23 thus mark the upper limit of the aero-acoustic transfer function. Figures 22 and 25 taken in conjunction suggest that the flow field causing the discrete

frequency tones comes close to that assumed - the total on-axis sound radiated at these frequencies is coherent with the velocity measured at one radial station and the transfer functions obtained from these measurements agree with the theoretical line. This is consistent with the idea of large, well-correlated eddies passing through the fan. The on-axis broad band sound field however is only partially coherent with the velocity at the measuring station, thus implying that other noise sources are present. It is suggested that the flow field consists of small un-correlated eddies, each of which contributes to the noise, but whose combined lift output is less than the in-phase lift assumed in the theory. This mechanism explains both Figures 23 and 25.

The effects of blade-to-blade correlation in the airflow have been studied in detail for the two- and seven-bladed fans running at 1200 rpm.

Figures 27 and 28 show examples of the experimental auto- and cross-correlation functions obtained from the incoming velocity signals. They appear to support the 'spaghetti' argument of Hanson (Ref. 48) with long, thin eddies taking some time (about 18 revolutions) to pass through the fan. The auto-correlation function (Figure 27) appears to consist of a series of pulses, each of the same shape. The repetition rate of the pulses is the period of rotation of the fan and they are reduced in amplitude as the time lag (τ) increases. Figure 28 is similar with the pulse train displaced from the origin by an amount corresponding to the blade separation.

These correlation functions were expressed mathematically in Chapter 2 as:

$$R_x(\tau) = \sum_{\mu=-\infty}^{+\infty} C_R \left(\frac{\mu}{h} - \frac{1}{Bh} \right) S_R \left(\tau - \frac{\mu}{h} + \frac{1}{Bh} \right)$$

where $C_2(\tau)$ is the envelope of pulse peak amplitudes and $S_2(\tau)$ is the basic pulse shape. If it is assumed that the pulse amplitudes decay exponentially,

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$$C_2(\tau) = \exp\{-\kappa'h/|\tau|\}$$

and the value of κ' can be found from the experimental results.

Figure 27 shows an exponential decay curve with

$$\kappa' = 0.225.$$

This value was the mean from a number of correlations. The values ranged from $0.225 + 13\%$ to $0.225 - 6\%$.

Figure 28 shows that when the blade spacing λ is greater than zero (i.e. for a cross-correlation function), the envelope of amplitudes is still centred on the origin despite the offset of the pulse train.

Figure 29 shows an 'expanded' view of the auto-correlation function covering one revolution of the fan. Also shown is a mathematical representation of the pulse shape factor of the form

$$S_2(\tau) = \exp\{-b\tau/|\tau|\} \cos \gamma\pi h\tau.$$

Values of $b = 2.9$ and $\gamma = 4.16$ were obtained to give the best fit with the available data.

The formulation of the blade-to-blade velocity cross-correlation function is thus

$$R_\lambda(\tau) = \sum_{\mu=-\infty}^{+\infty} \exp\left\{-\kappa'h\left|\frac{\mu}{h} - \frac{\lambda}{8h}\right| - b\left|\tau - \frac{\mu}{h} + \frac{\lambda}{8h}\right|\right\} \cos\left(\gamma\pi h\left(\tau - \frac{\mu}{h} + \frac{\lambda}{8h}\right)\right),$$

which has been programmed and the results are shown in Figures 30 and 31 for $\lambda = 0$ (the auto-correlation function) and $\lambda = 4$. These simulated correlation functions can be compared with those obtained experimentally shown in Figures 27 and 28, when it is seen that they perform as expected.

One effect of the expressions chosen for the envelope and pulse shape functions is that a small d.c. value is found to exist. This takes the form for the auto-correlation :

$$R_{\lambda}(\tau) = \frac{2bh \sinh(\alpha')}{(\cosh(\alpha') - 1)(b^2 h^2 + q^2 \tau^2 h^2)}$$

which for the values of b , h , q and α' used here = 0.014.

This is small enough to be neglected for these correlation plots.

Some interpretation of these correlations is required in order to understand what is happening in the flow field. This is best achieved by attempting to visualise first, what the probe 'sees' as it rotates and second, what the correlation function actually means.

Figure 32 shows the triangle of velocities that exists for the rotor. The resultant velocity is the sum of axial and circumferential components. Now, for an auto-correlation, the flow is sampled at discrete time intervals (which, as the probe is rotating, correspond to discrete points in space) and compared with the initial value obtained at time delay $\tau = 0$. After a delay $\tau = 1/h$ sec., (where h Hz is the rotational frequency) the probe is in its original position. Therefore, points on the auto-correlation starting at

$\tau = 0$ and separated by a delay $\tau = 1/h$ sec., describe the axial auto-correlation of the incoming velocity. These points are described by $C_2(\tau)$, the envelope of peak amplitudes.

Definition of the 'pulse shape function' $S_2(\tau)$ requires further

thought as it involves both axial and circumferential components of velocity. However, it can be visualised as being composed of a series of points obtained from the cross-correlations between a static reference probe and one which can be placed in a number of positions around the duct. For each position of this moveable probe the value of the cross-correlation at a time delay corresponding to the angular separation of the probes and the relevant speed of rotation gives one point on the 'pulse shape function'.

In practice, of course, it is impossible to obtain meaningful results immediately upstream of a rotor using stationary hot wire anemometers - this is why a rotating system was devised for the experiments described herein. However, if a series of tests were performed as described by Hanson (Ref. 46) in which the rotor was removed and the intake under consideration 'sucked' it should be possible to obtain information on both axial decay rate $C_2(\tau)$ and pulse shape $S_2(\tau)$ in the form required for the theoretical prediction of the noise that would be generated by a given rotor in a specific intake.

The full equation for the aero-acoustic transfer function obtained when the envelope of peak amplitudes is an exponential decay is given as equation (27) of Chapter 2 and reproduced here.

$$\frac{\overline{P_{\text{Rm}}(f)}}{v_0'(f)} = \frac{\frac{B h^3 \pi^3 \rho^2 f}{a_0^2 r_i^2} \left(\int_{r_{\text{min}}} \left(\frac{c r^3}{\cos \alpha} \right)^{1/2} dr \right)^2 \frac{\sinh(\alpha'/B)}{\cosh(\alpha'/B) - \cos(\xi/B)}}{\frac{\sinh(\alpha')}{\cosh(\alpha') - \cos(\xi)}}$$

Only the axial scale of the turbulence is involved in this transfer function - the pulse shape function occurs in both the numerator and

denominator and hence cancels. Results obtained from this equation using the value of α' found previously are compared with experimental data in Figures 33 and 34 for the seven- and two-bladed fans running at 1200 rpm.

A point to note here is that the transfer function consists of broad peaks at harmonics of blade passing frequency with $(B-2)$ peaks in between (where B is the blade number). In fact, the troughs which occur are a natural result of the division of a function containing peaks at blade passing frequency harmonics by a function with peaks at all rotational frequency harmonics. The troughs are at these harmonics. This effect can be seen in the experimental results shown in Figures 35 and 36 (both 'with' recirculation). On a logarithmic frequency scale the peaks rise at 3dB per octave and are below the level obtained with the 'full correlation' model. The broad band agreement between the theoretical and experimental results is closer than was obtained with the 'zero correlation' model - this is as would be expected since an attempt has been made to model the actual flow conditions as opposed to an ideal state.

More than one representation of the pulse shape function was possible. It is thus necessary to check the velocity spectrum obtained from this formulation against the experimental results before attempting to predict the acoustic spectrum.

Equation (28) of Chapter 2 gives the measureable, single-sided auto-power spectrum of the turbulence impinging on one blade as:

$$V_m(f) = \frac{4\sqrt{2}bh \sinh(\alpha')}{\cosh(\alpha') - \cos(\xi)} \left(\frac{b^2h^2 + 4\pi^2f^2 + \pi^2\eta^2h^2}{(b^2h^2 + 4\pi^2(f + \frac{\eta h}{2}))^2 (b^2h^2 + 4\pi^2(f - \frac{\eta h}{2}))^2} \right)$$

The mean square value $\bar{v}^2$ was obtained at the time of the experiments and found to be $0.81(\text{m/s})^2$ at a rotational speed of 1200 rpm.

The spectrum obtained from this formulation using the empirical values for b , α' and ℓ obtained previously is shown in Figure 37.

The qualitative points to note from this figure are that it comprises tones at harmonics of rotational frequency superimposed on an overall background shape - this is in agreement with the experimental data. However, it can be seen that these tones are the result of the once-per-revolution (i.e. axial) correlation, and the broad band background is the effect of the particular pulse shape chosen. A 'hump' in the broad band spectrum occurs at 41.6Hz, which is equal to $\frac{44}{2}$, the frequency of the cosine wave specified in the pulse shape function $S_2(\tau)$. Examination of Figure 16 which shows the experimental velocity spectrum obtained at this speed reveals a similar trend in the broad band spectrum although the higher harmonics of rotational frequency tend to become swamped by the broad band level. This effect has not been simulated in the theoretical spectrum where the harmonics are all at the same distance from the background.

Consideration of equation (26) of Chapter 2 (reproduced above) reveals that the pulse shape function (or relative velocity correlation) governs the broad band spectrum whilst the envelope function (axial correlation) governs the harmonic levels.

The form chosen for the pulse shape function can thus influence the result obtained to a large extent. If, for example, a band-limited random noise type of auto-correlation had been used (which was equally feasible) the resultant broad-band spectrum would have been flat over the bandwidth specified with all the harmonics superimposed to give the same value whatever the harmonic number. Care must be exercised

in the choice of shape function because the net result is a power spectrum - the amplitude is always positive and the phase zero.

Both the example used (the exponentially decaying cosine wave) and the bandwidth-limited random noise shape discussed above satisfy this criterion.

Quantitatively, it can be seen that while the overall spectrum shape is very good, the values predicted appear to be almost constantly 4dB low for the discrete frequency points. This would suggest that the value of $\bar{v}^2$ used to convert the auto-correlation function was too low. The value used was 0.81 (m/s)^2 which leads to a turbulence intensity of 2.8%. The rms velocity was obtained from measurements made at an early stage of the experiments - before the technique was fully refined.

Equation (29) of Chapter 2 gives the general formula for the single-sided acoustic power spectrum. If the simplifying effects of placing the observer on the far axis are utilised, the following result is obtained.

$$\overline{P_{Ra}}(f) = \frac{h^2 \pi^2 \rho^2 f}{q^2 r^2} \left(\int_{-\infty}^{\infty} \left(\frac{cr^2}{\cos \kappa} \right)^{1/2} dr \right)^2$$

$$4 \bar{v}^2 b h \left(\frac{b^2 h^2 + 4 \pi^2 f^2 + \pi^2 q^2 h^2}{(b^2 h^2 + 4 \pi^2 (f + \frac{qh}{2})^2) (b^2 h^2 + 4 \pi^2 (f - \frac{qh}{2})^2)} \right)$$

$$\frac{B \sinh(\kappa'/B)}{\cosh(\kappa'/B) - \cos(\xi/B)}$$

This equation has been programmed and the result is shown in Figure 38.

As before, the effect of the blade-to-blade correlation is exhibited in the peaks at harmonics of blade passing frequency, while the broad band noise characteristics are a result of the pulse shape chosen.

It can be seen that whereas the harmonic levels produced theoretically are in good agreement with their experimental counterparts, the broad band level predicted is too low. This agrees with the coherence

functions of Figure 25 which suggested that the broad band noise was a function of aerodynamic input and other sources (e.g. blade vibration). The bump at 41.6 Hz occurs as before as a result of the pulse shape chosen.

It should be emphasised that these acoustic and aerodynamic spectra have been built up from a simple model of the incoming turbulence auto-correlation. No attempt has been made to model the turbulence as such. However, it has been shown that variations in the axial decay rate of the incoming turbulence will affect the harmonic level solely, whilst the aerodynamic contribution to the broad band spectrum is a function of the distribution of the turbulence in the relative velocity.

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C O N C L U S I O N S

The objectives of this study of the noise generated by a turbulent airflow entering an open rotor were: first, to demonstrate by experimental results that a definite relationship exists between turbulence in the airflow and the noise generated; second, to show how a simple theoretical model can predict this relationship with reasonable accuracy; and third, to use the experimental results in the development of a more realistic theory. The following conclusions can be drawn from this work.

1) The theoretical model developed is based on the fluctuating force (dipole) mechanism of sound generation. The fluctuating lift force on the blades is related to turbulence in the inflow to the fan by unsteady aerofoil theory and an 'aero-acoustic transfer function' is defined as the ratio of the magnitudes of the acoustic and velocity power spectra at the same frequency. This formulation is simplified by placing the observer on the axis of the fan. The on-axis aero-acoustic transfer function is quantified for both discrete frequency and broad band noise by consideration of the effects of blade-to-blade correlation in the airflow. The two extremes of perfect and zero blade-to-blade correlation are considered for comparison with experimental results - perfect correlation giving the discrete frequency aero-acoustic transfer function, and zero correlation the broad band aero-acoustic transfer function. With these assumptions, the value of the theoretical on-axis aero-acoustic transfer function is dependent solely upon frequency, speed of rotation, blade number and a constant function of the blading. There are no empirical factors involved.

2) To establish experimentally the existence of a definite relationship between the velocity entering a fan and its on-axis

acoustic spectrum requires simultaneous and independent measurement of both the acoustic and velocity data. Obtaining the required acoustic information merely involves positioning the microphone as required in a suitable free-field environment (anechoic chamber) but if hot wire anemometry is to be used in the measurement of the turbulent flow entering the fan, the effects of the fan itself on a stationary wire must be considered. This work establishes the use of rotating hot wire anemometry as a viable experimental technique which requires no particular sophistication of instrumentation at speeds up to 1600 rpm (a wire speed of 43 m/s). The restriction on higher speeds is a result of problems in probe retention, not of wire strength. Given a more sophisticated mounting system, it is anticipated that the limiting factor found when increasing speed would be the rubbing speed of the slip ring brushes on the rings themselves.

3) The experimental results show that there is a definite connection between the velocity input to and the acoustic output from an open rotor. Comparisons between the experimental and theoretical on-axis aero-acoustic transfer functions show agreement to within 10 dB over three decades of the relevant parameters for both discrete frequency and broad band noise. This agreement holds when the incoming turbulence intensity is of the order of 3%. When the turbulence intensity is less than 3% it is found that an experimental aero-acoustic transfer function exists at discrete frequencies only but it is not of the form suggested by the theory. It is felt that some of the restrictive assumptions made in the development of the theory are not applicable here.

4) The idealised conditions of full and zero blade-to-blade correlation do not exist in real life. Experimental auto- and cross-correlation functions obtained from rotating hot wire anemometers show that the inflow appears to consist of long thin eddies which require approximately 18 revolutions to pass through the fan. The form of these correlation functions can be modelled mathematically and inserted into the theory. By this method, velocity and acoustic spectra are predicted as well as the aero-acoustic transfer function. It is found that the aero-acoustic transfer function is a function of the axial decay of the eddies, while the acoustic and velocity spectra depend upon both the axial decay and also the correlation of the turbulence in the direction of the relative velocity.

5) The theoretical model used takes no account of any span-wise variation in the turbulence field, and hence assumes that the lift generated on a blade is the maximum possible. In practice, however, there is a correlation pattern along the span in addition to those in the axial and V_{∞} directions. It is felt that the investigation of this span-wise effect, first experimentally and then theoretically, would be a profitable line for future work. The extension of the rotating hot wire technique to higher speeds is also to be recommended. This would involve the development of an improved mounting system for the hot wires - using the blades themselves as supports, for example - and some other system for transmitting the data. A system like this could be used to investigate the duct boundary layer in a ducted rotor.

One other line of enquiry is the apparent dependence of the

form of the aero-acoustic transfer function on the turbulence intensity in the inflow. This is of importance since aircraft, in general, do not operate in recirculating flow situations.

6) In practice, this work is one further step along the road to the prediction of the sound generated by a rotor. It has confirmed, by experimental means, that the fluctuating force (dipole) source mechanism describes the sound field of a low-speed open rotor with reasonable accuracy for both discrete frequency and broad band noise. It has demonstrated that, given some knowledge of the blade-to-blade correlation in the inflow it is possible to predict the acoustic spectrum of the rotor. Thus, data from model inlet tests can be used in the prediction of fan noise. These results should apply in all instances where low-speed fans are used - air conditioning plants for example.

The development of rotating hot wire anemometry provides a useful experimental tool for the investigation of: velocity impinging on fan blades, duct boundary layers, tip flow behind a rotating fan and other similar situations.

The work as a whole shows that a small-scale, low-speed fan can be used to provide information which may lead to larger, more expensive, test rigs, and also demonstrates the development of theory to fit experimental data.

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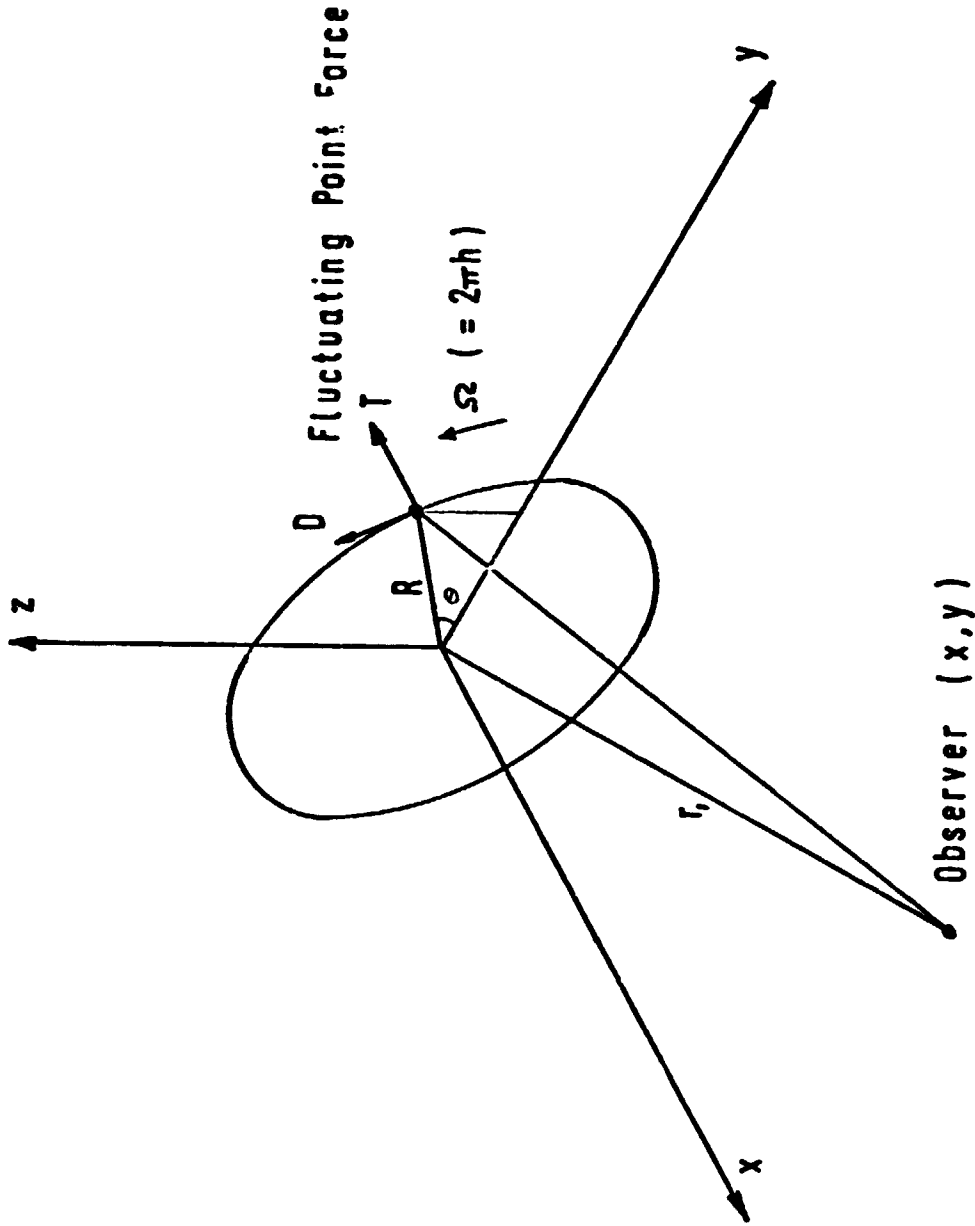
TABLE I - THE EXPERIMENTS

<u>DESCRIPTION OF TEST</u>	<u>FAN SPEED</u>	<u>BLADE NO.</u>	<u>HOW ANALYSED</u>
Stationary hot wire in front of fan	1000 1200 1500	7	Spectra.
Investigation of slip ring noise	1200	7	Spectra. (Found to be satisfactory).
Check on frequency response of hot wire system	1000 1200 1500	0	Transfer Function Analysis. (Found to be satisfactory).
Rotating hot wire at .256m radius. Micro- phone on axis.	1000 1200 1400 1600	7, 2	Spectra, aero-acoustic transfer functions.
Rotating hot wire at blade tip. Microphone on axis.	1000 1200 1400 1600	7	Spectra, aero-acoustic transfer functions. (Results agreed with previous experiments).
Two wires on chord line .256m radius.	1000 1200 1400 1600	7	Auto- and cross-correlations, also wave-form reductions to average out steady effects. (Found to be negligible).
Two wires .025m upstream of adjacent blades ($\lambda:1$),	1000 1200 1400 1600	7	Auto- and cross-correlations.

continued...../

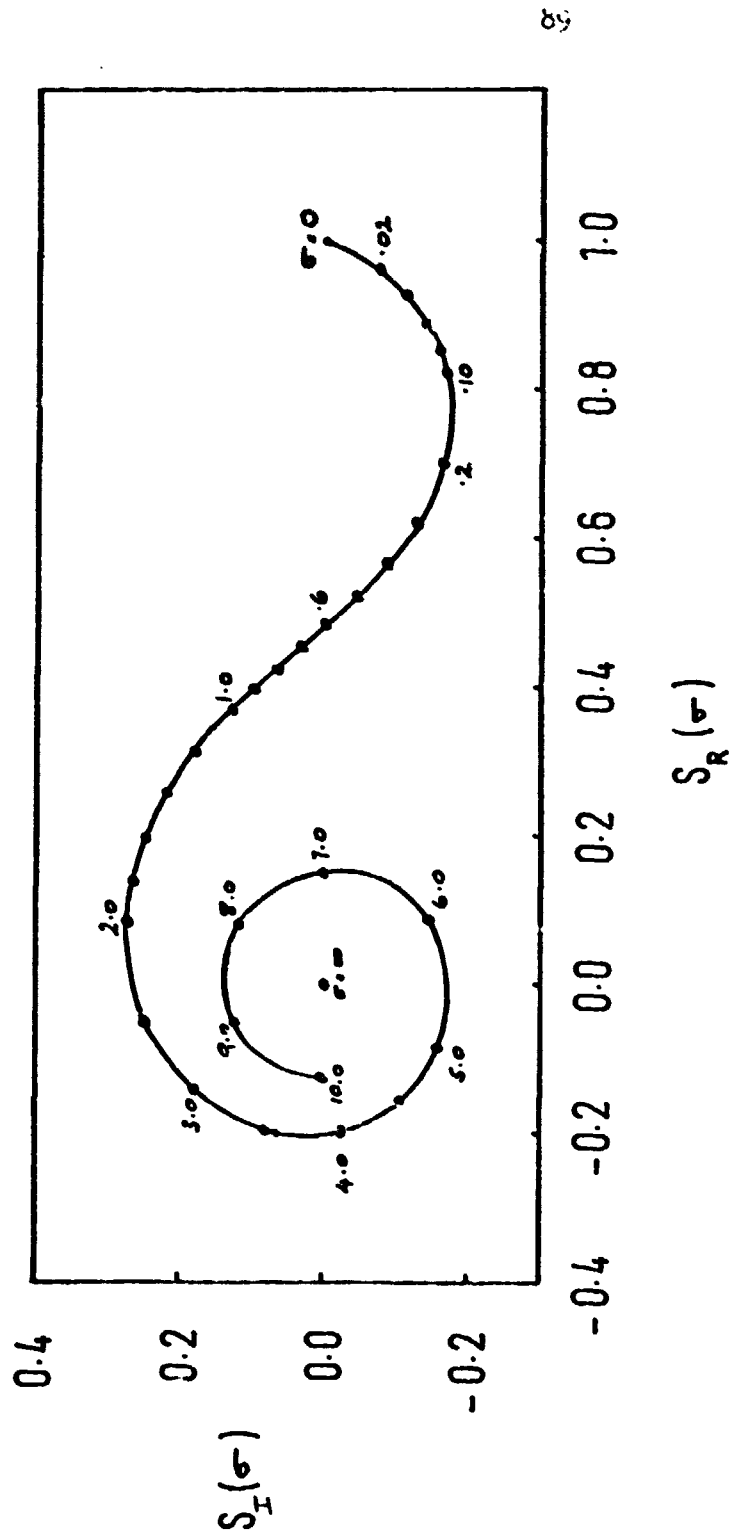
<u>DESCRIPTION OF TEST</u>	<u>FAN SPEED</u>	<u>BLADE NO.</u>	<u>HOW ANALYSED</u>
Two wires .025m	1200	7	Auto- and cross-correlations
upstream of	1200		
separate blades.	1200		
(λ. 1-6)			
Two wires on one blade	1200	7	Cross-correlations.
one at tip, one at	1000		(Results inconclusive).
.256m radius.			

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Rotating Point Force

Figure 1



89

The Sears Lift Function (from Ref.47)

Figure 2

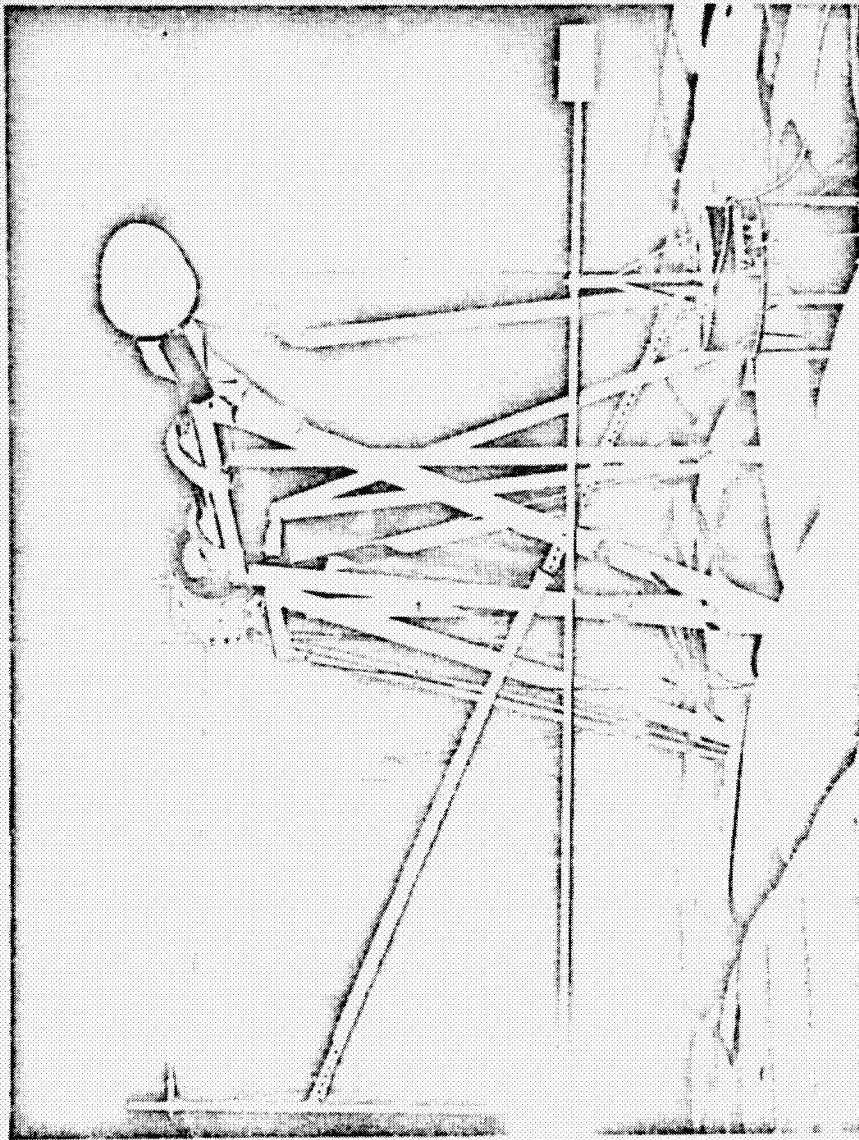
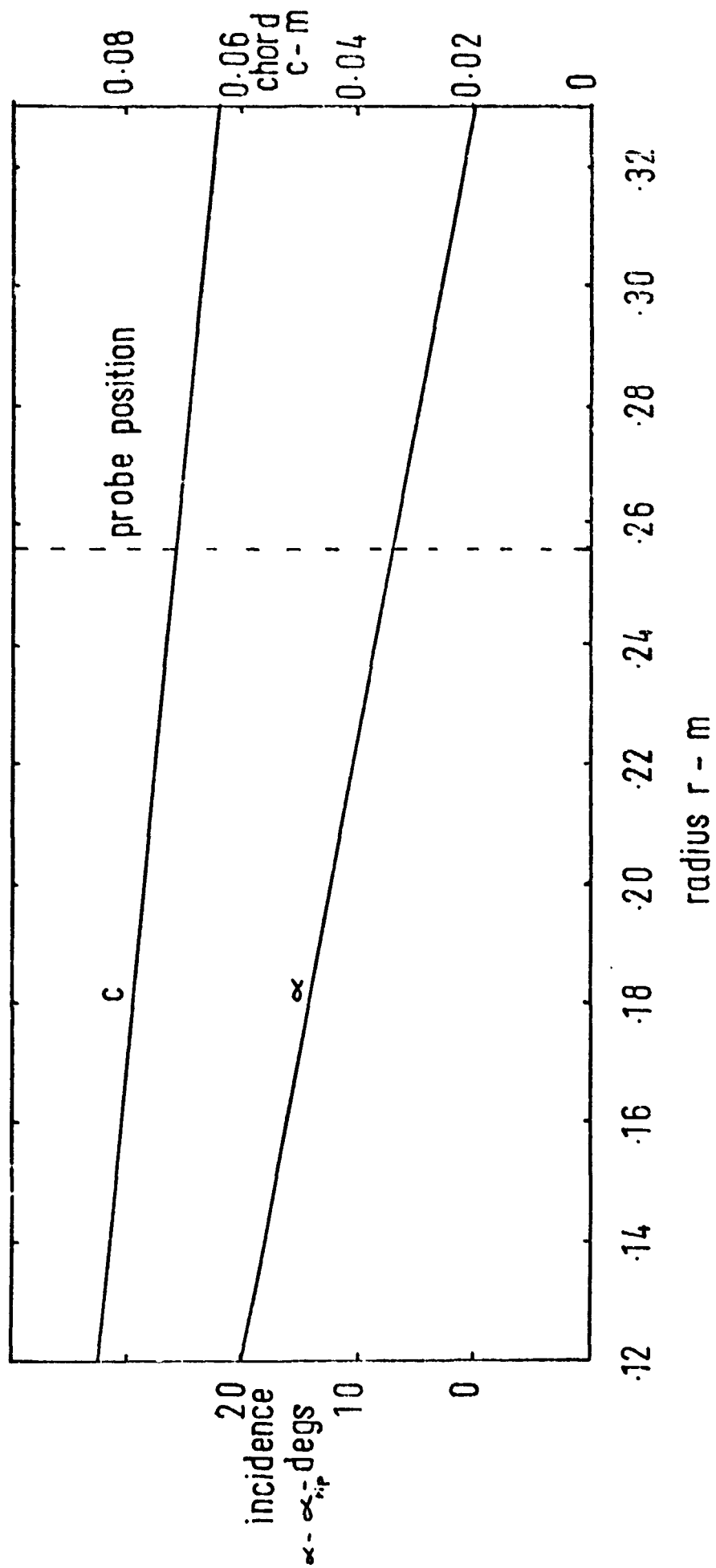


Figure 3

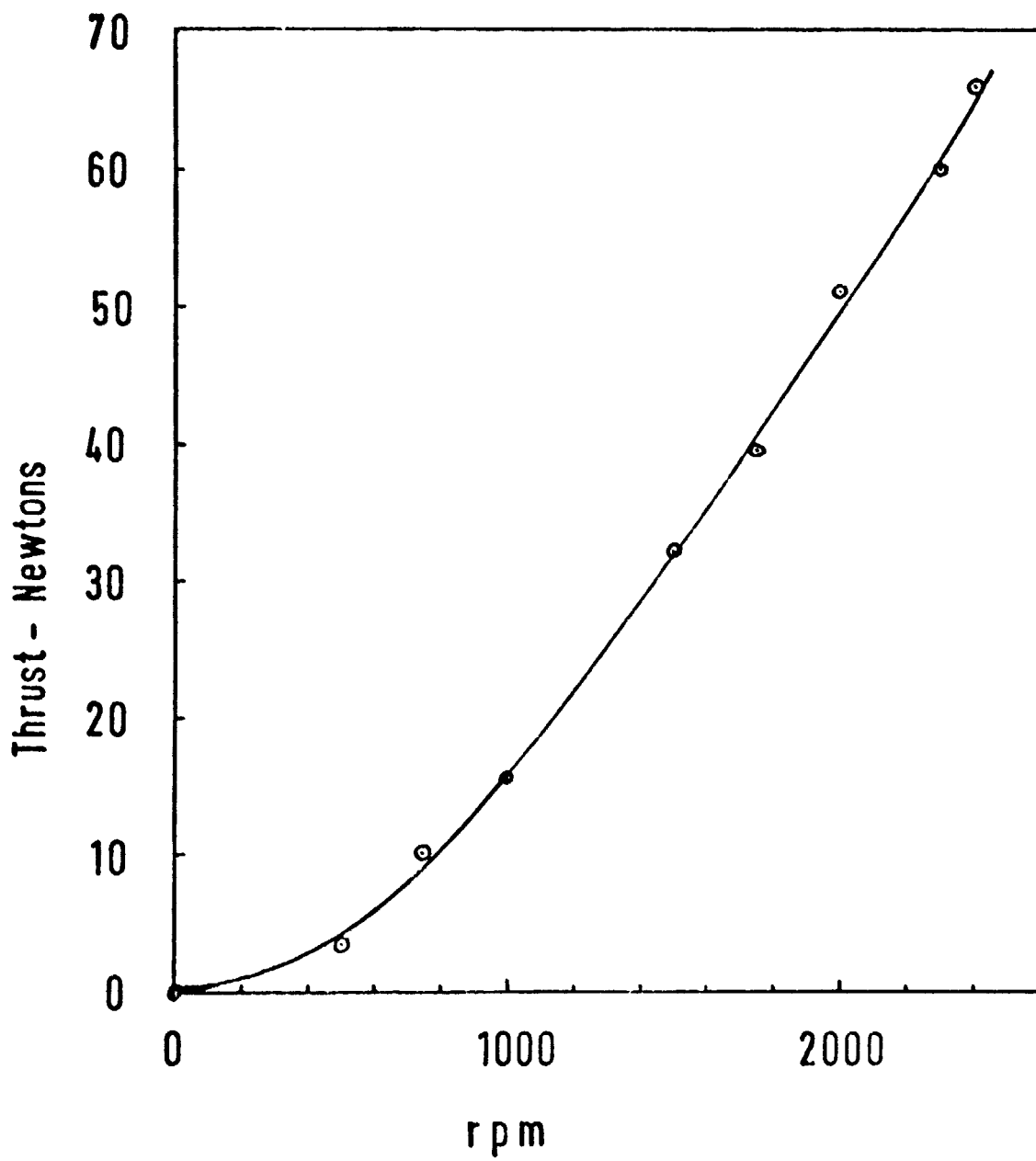
Fan Rig

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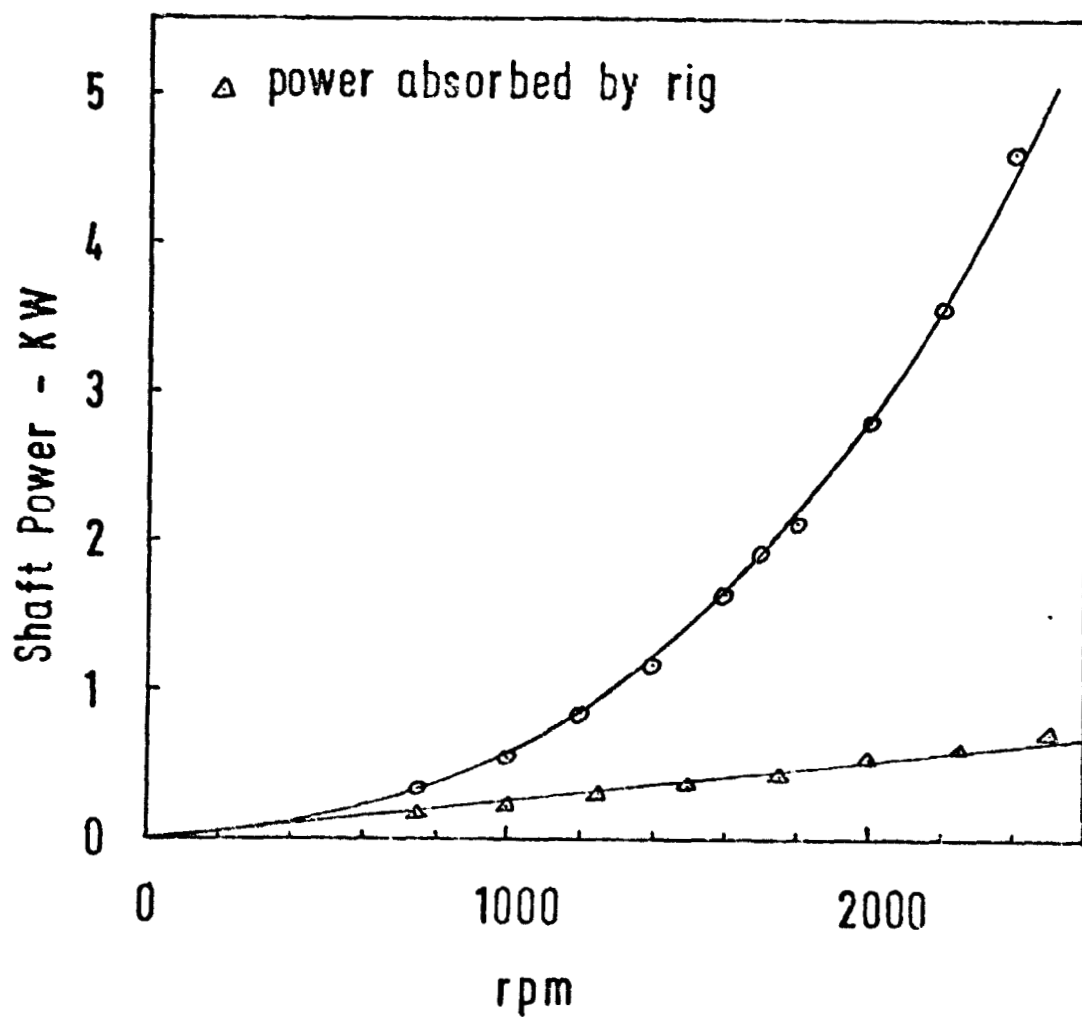
Blade Data

Figure 4



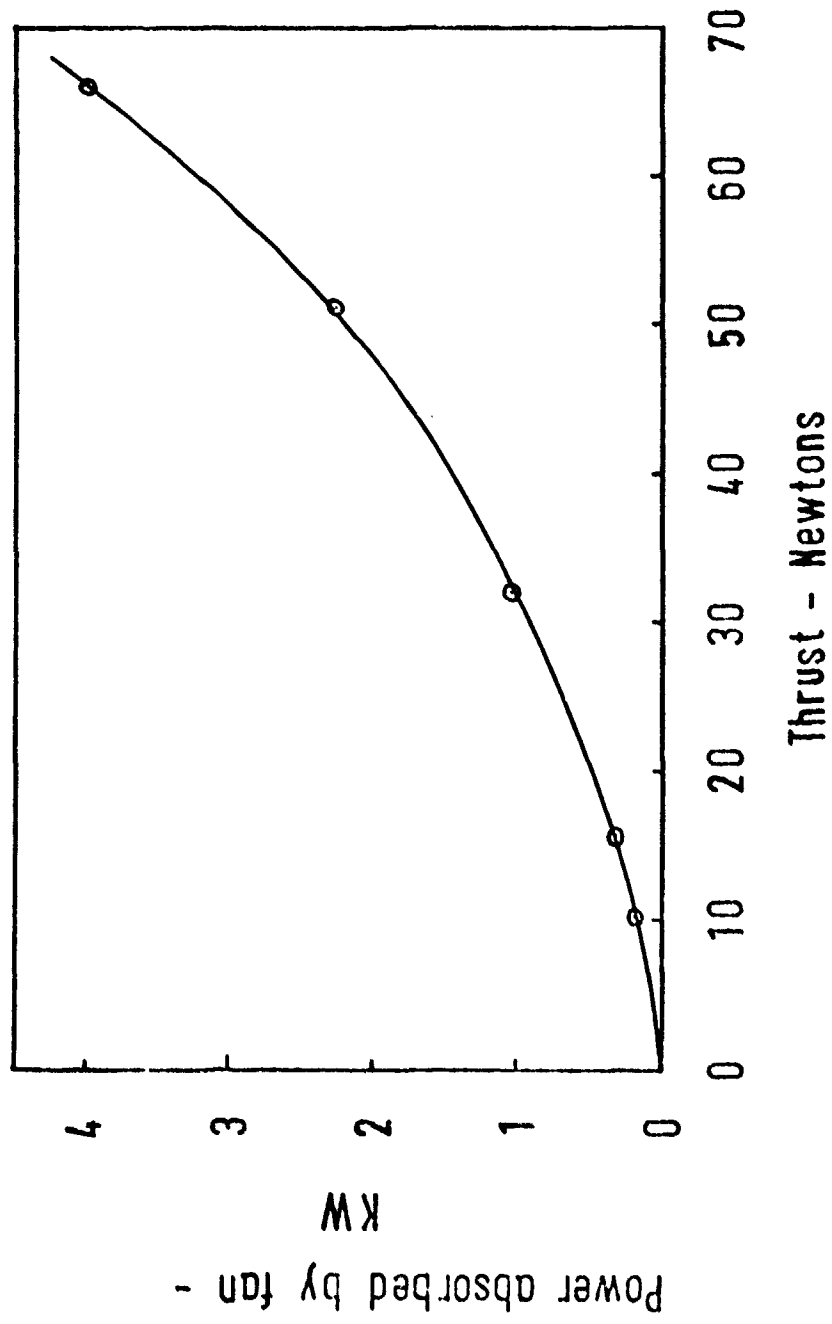
Thrust Produced by Fan (7 Blades)

Figure 5



Power Absorbed by Fan (7 Blades)

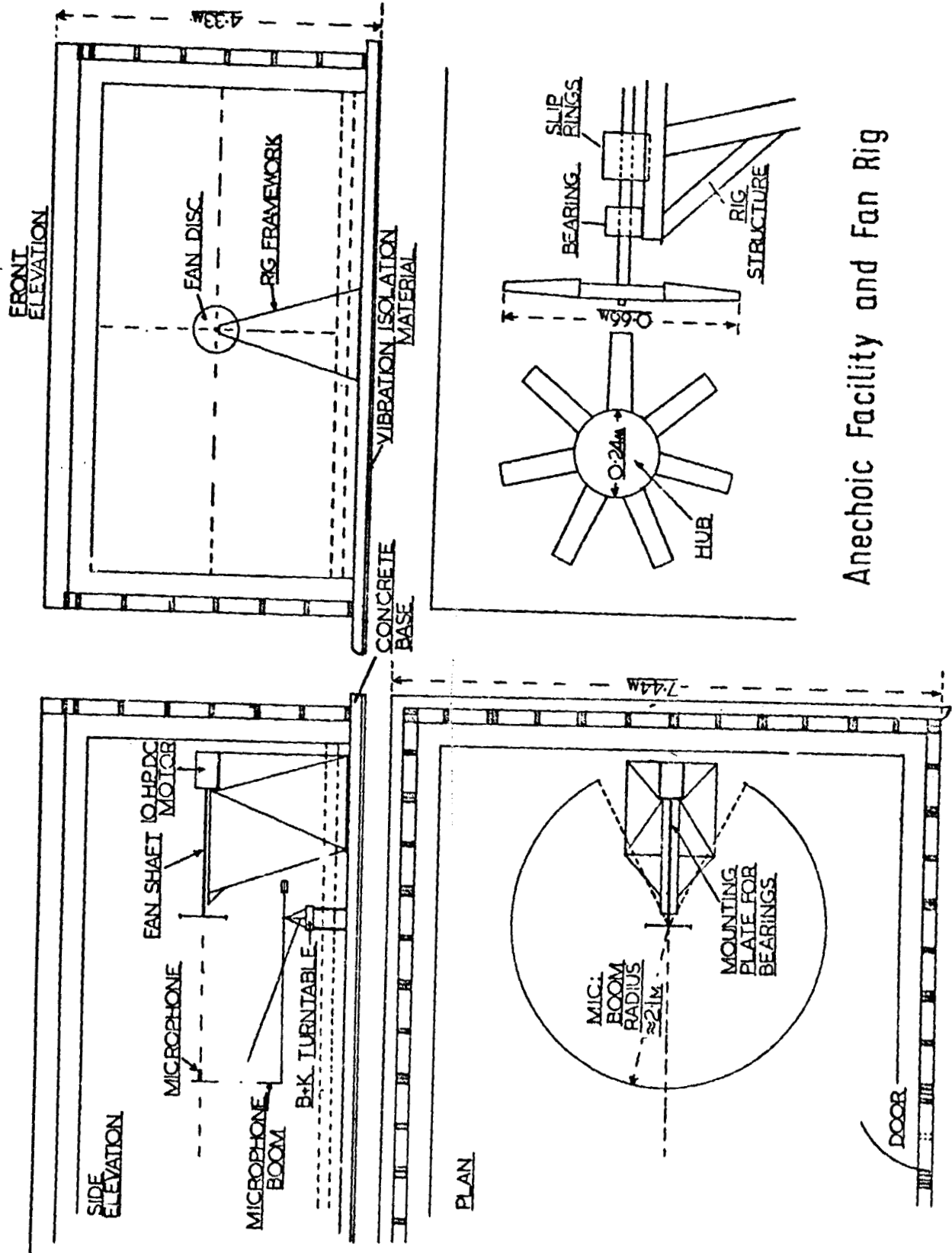
Figure 6



Efficiency of Fan (7 Blades)

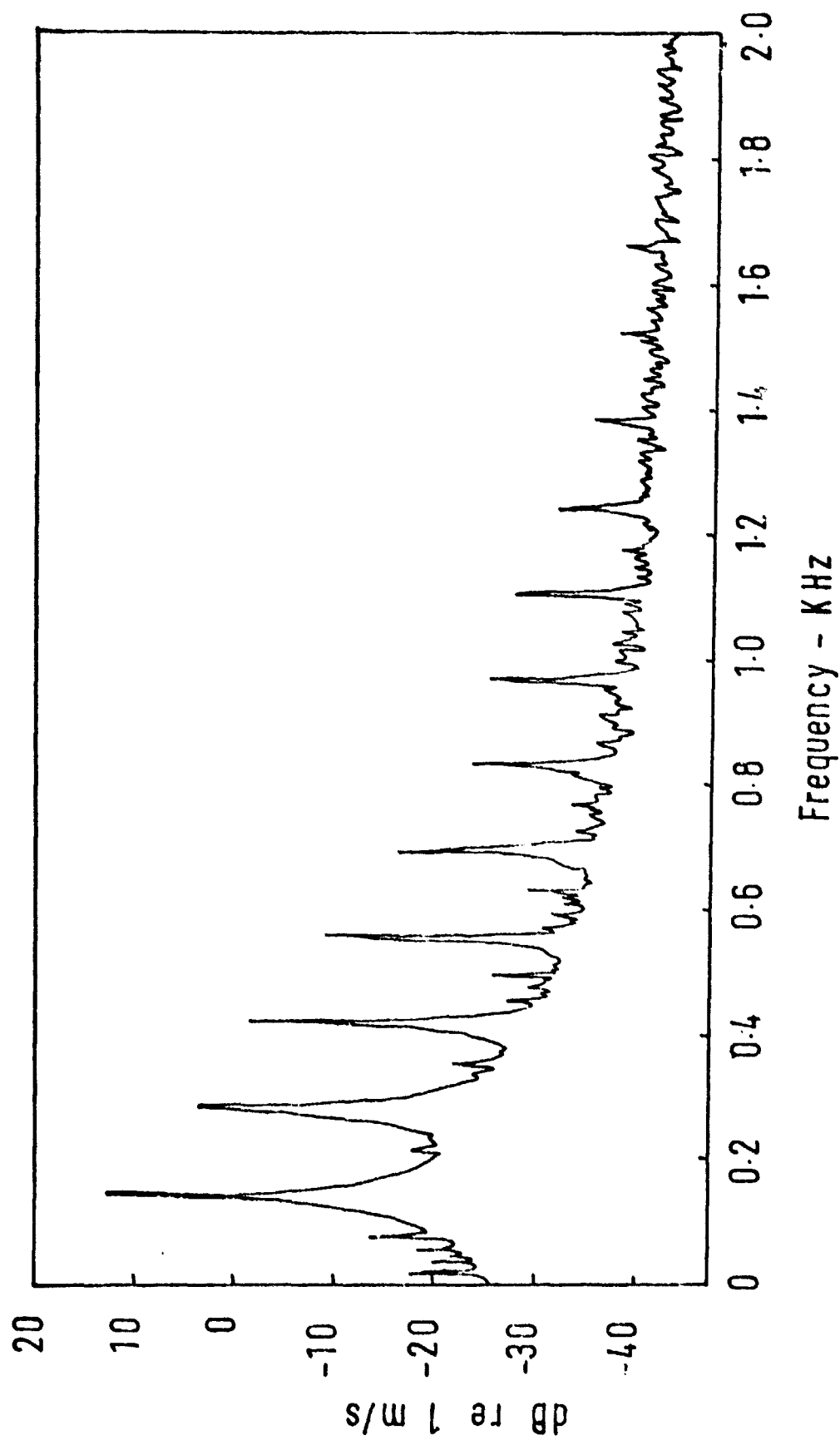
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Anechoic Facility and Fan Rig

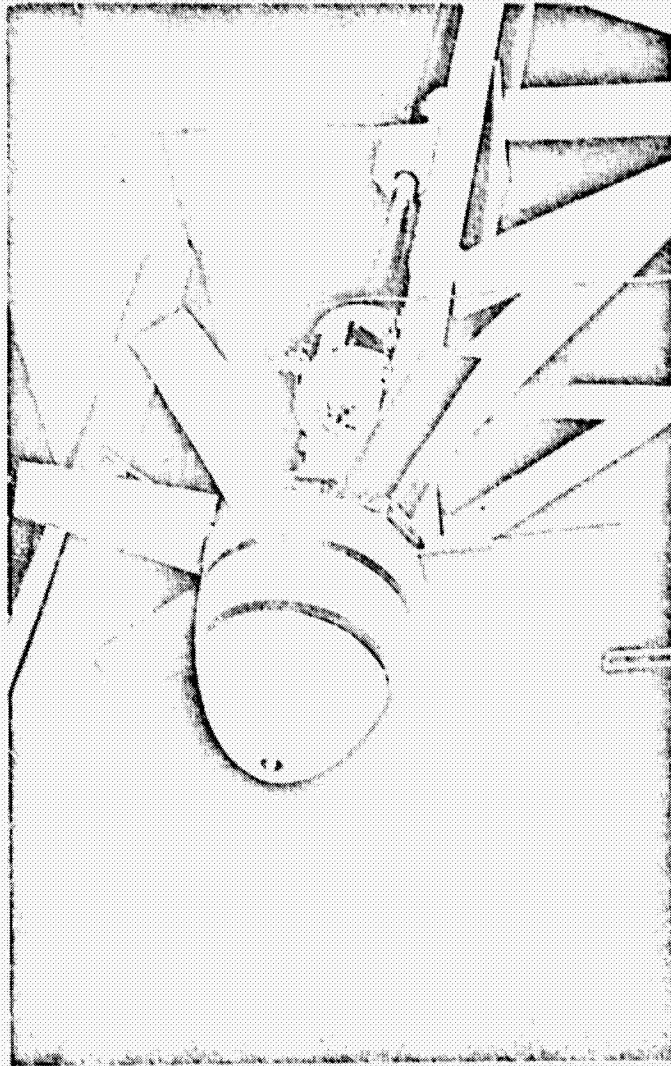
Figure



Stationary Hot Wire Spectrum - 1200 rpm

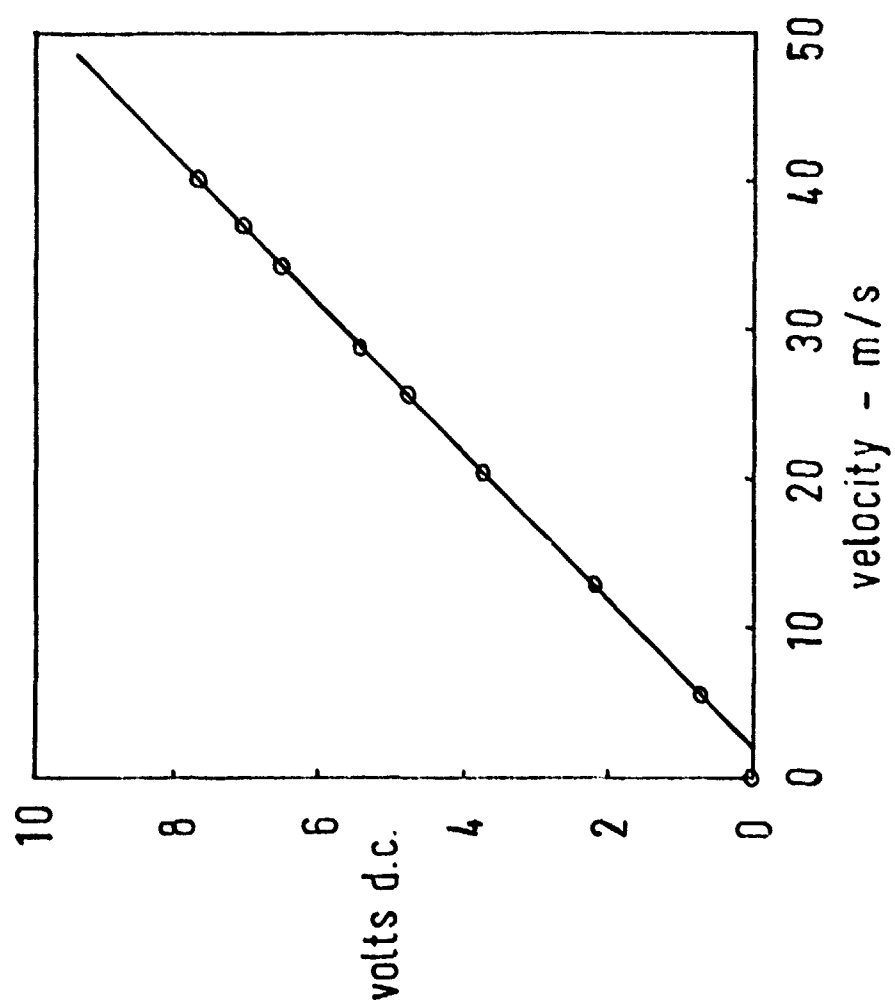
Figure 9

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Rotating Hot Wire and Slip Ring Assembly

Figure 10



Typical Probe Calibration

Figure 11

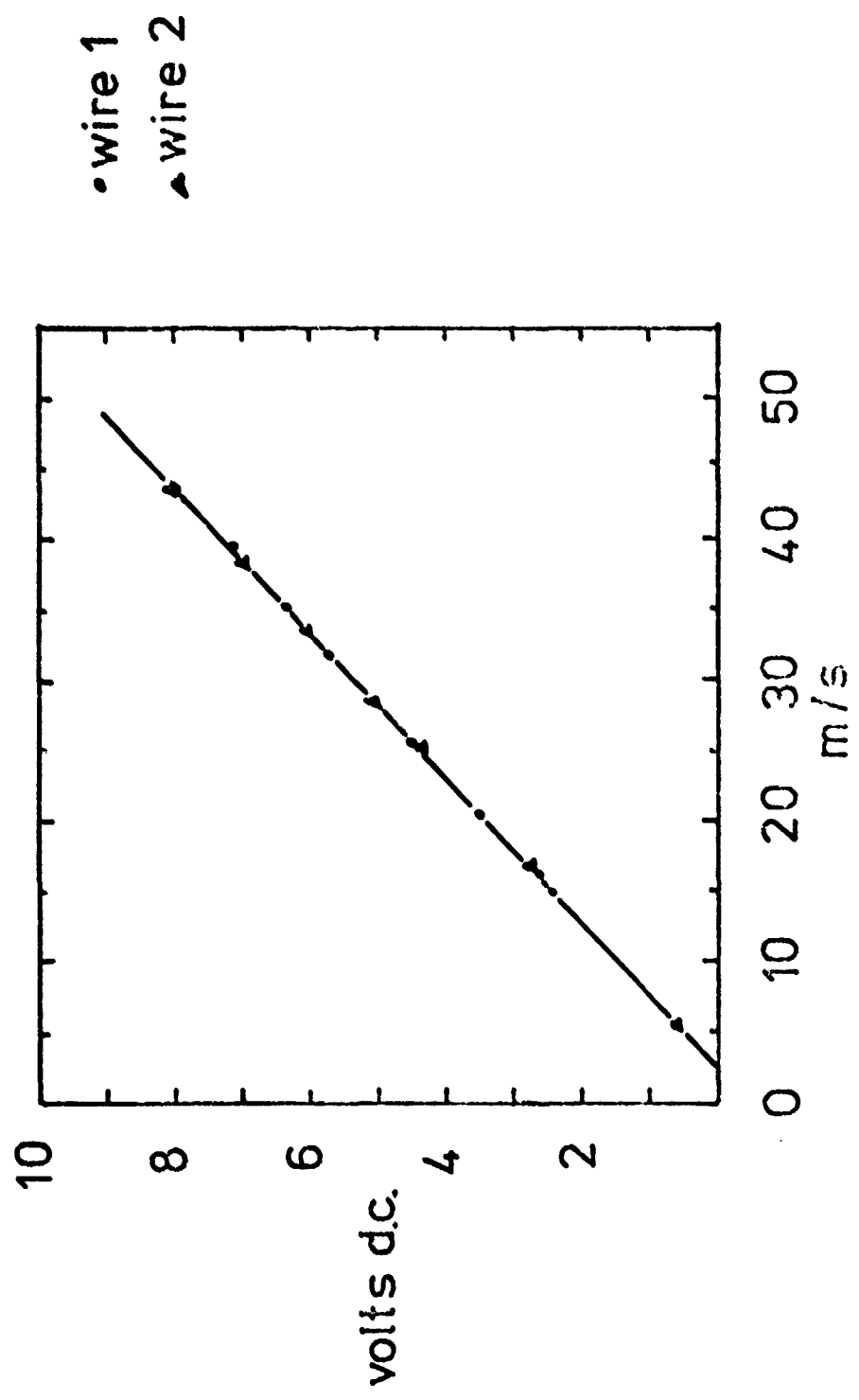
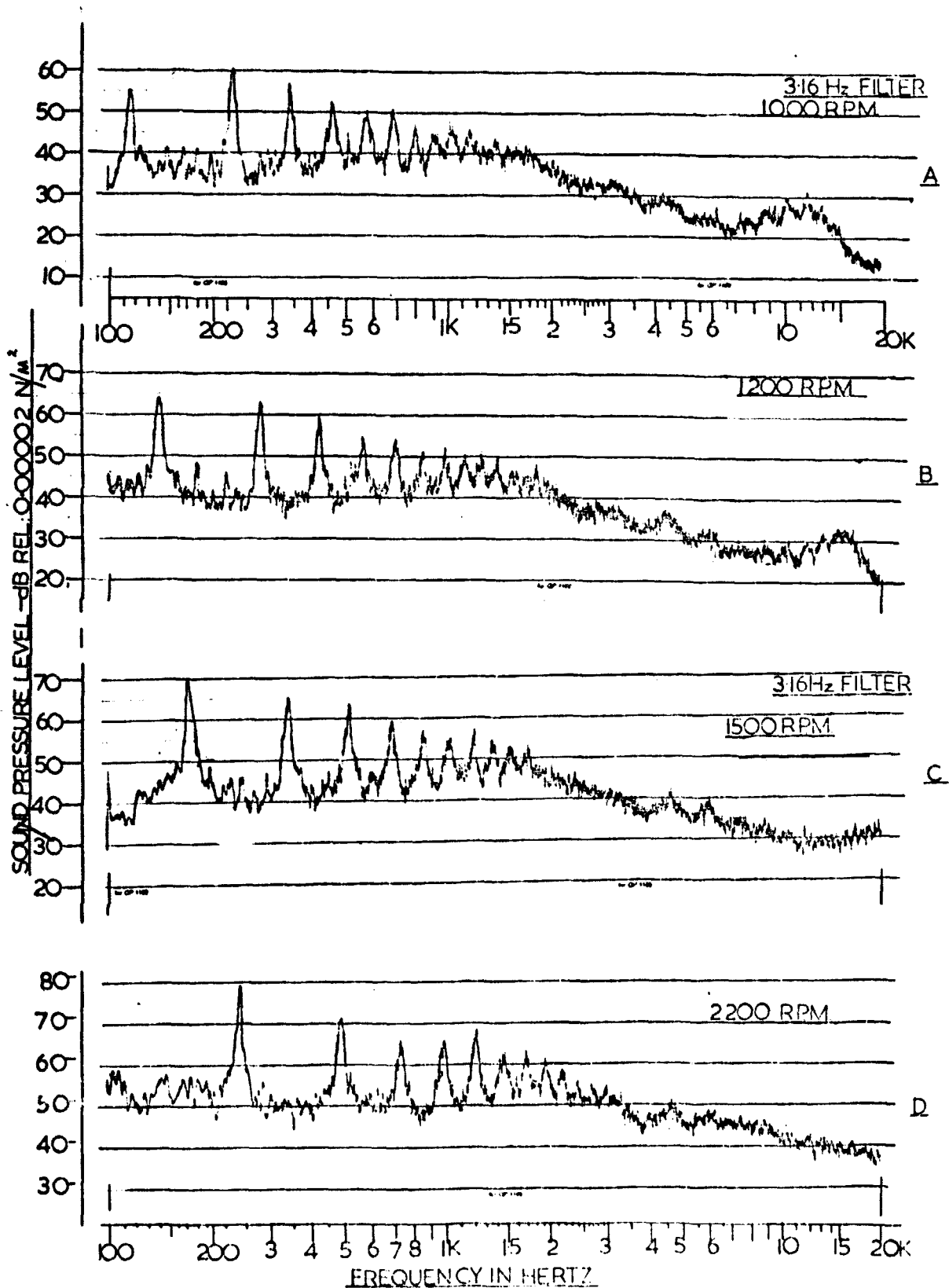


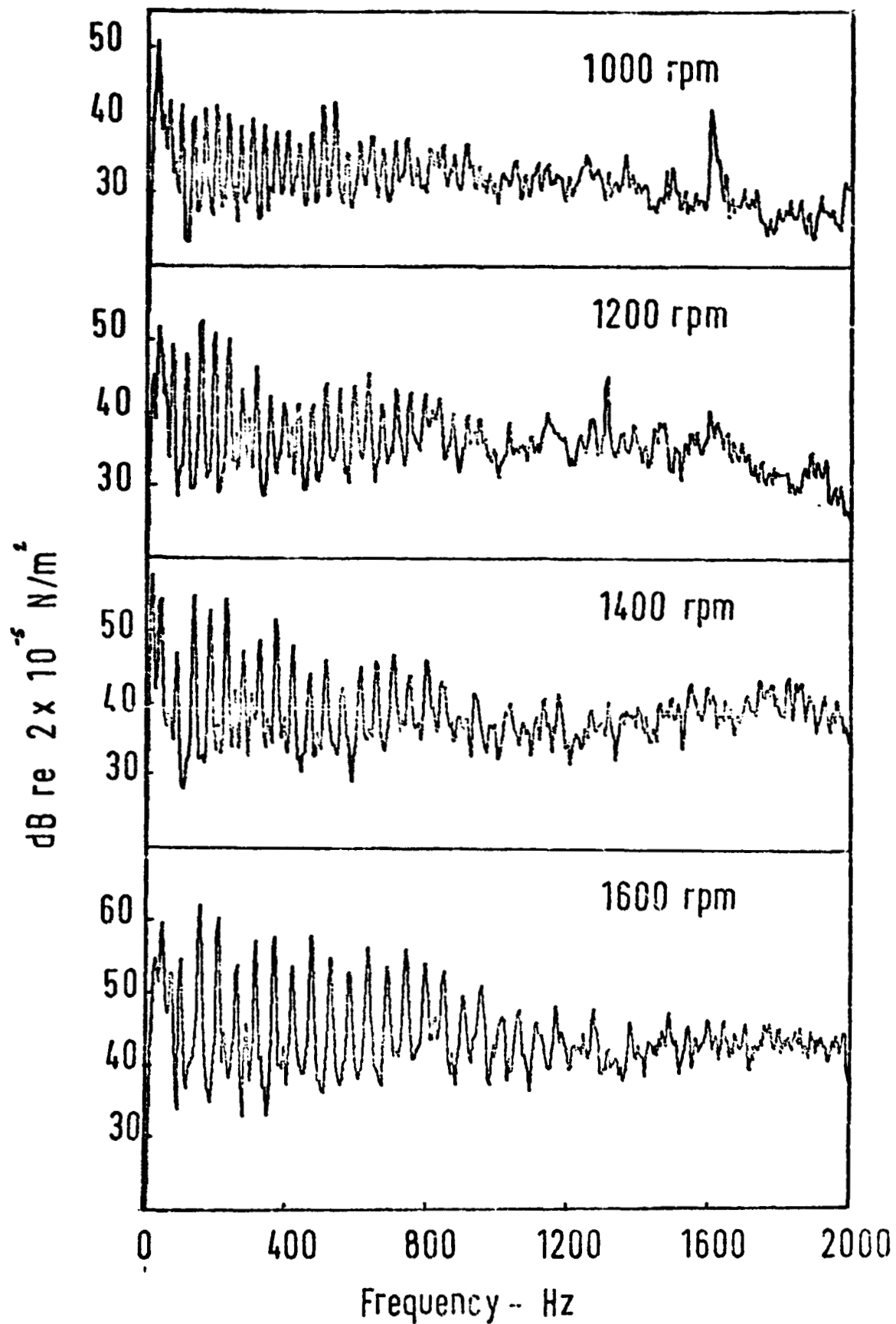
Figure 12

Typical Calibration for Two Probes



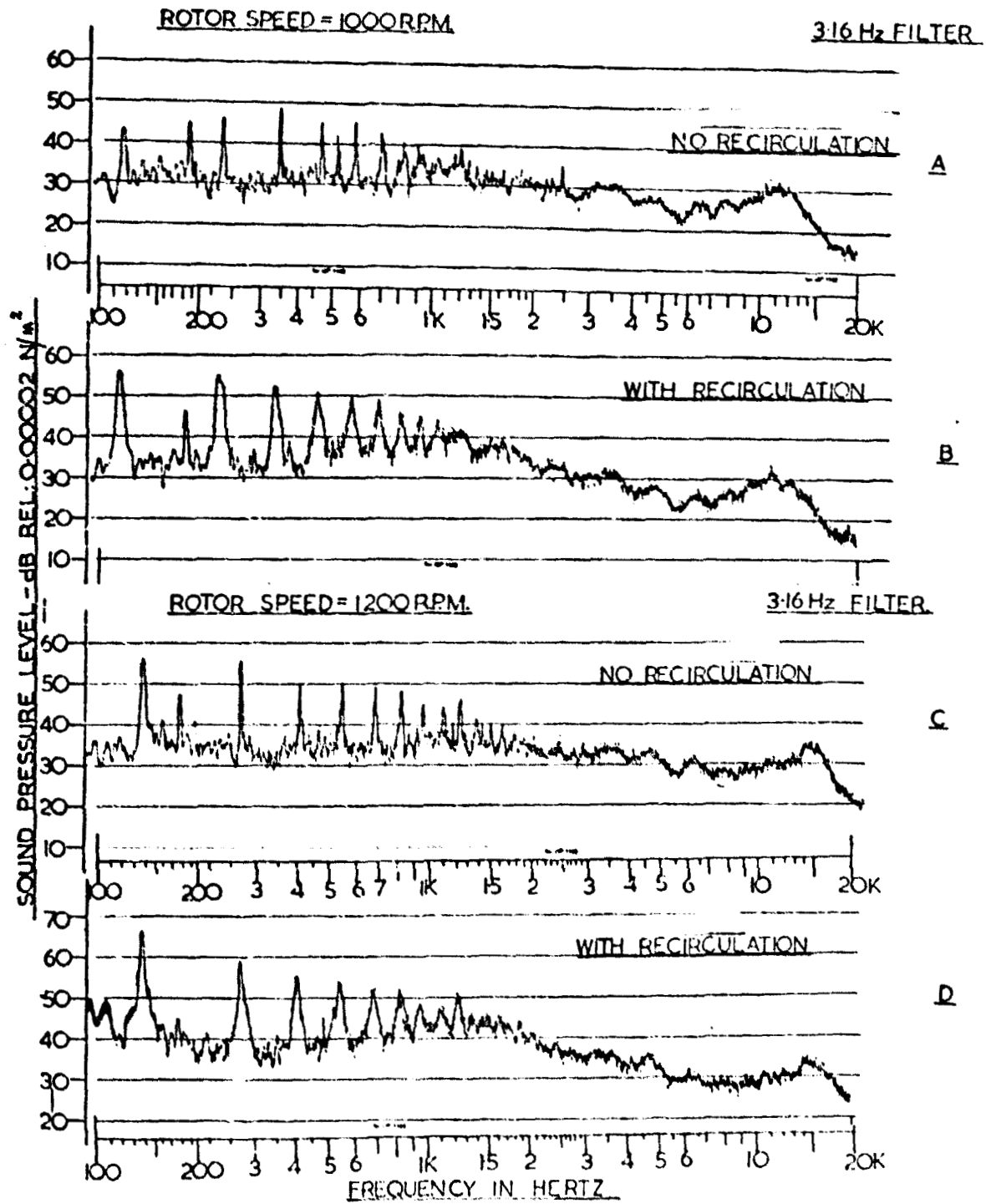
Effect of Speed on Fan Noise (7 Blades)

Figure 13



Effect of Speed on Fan Noise (2 Blades)

Figure 14



Effect of Recirculation on Fan Noise (7 Blades)

Figure 15

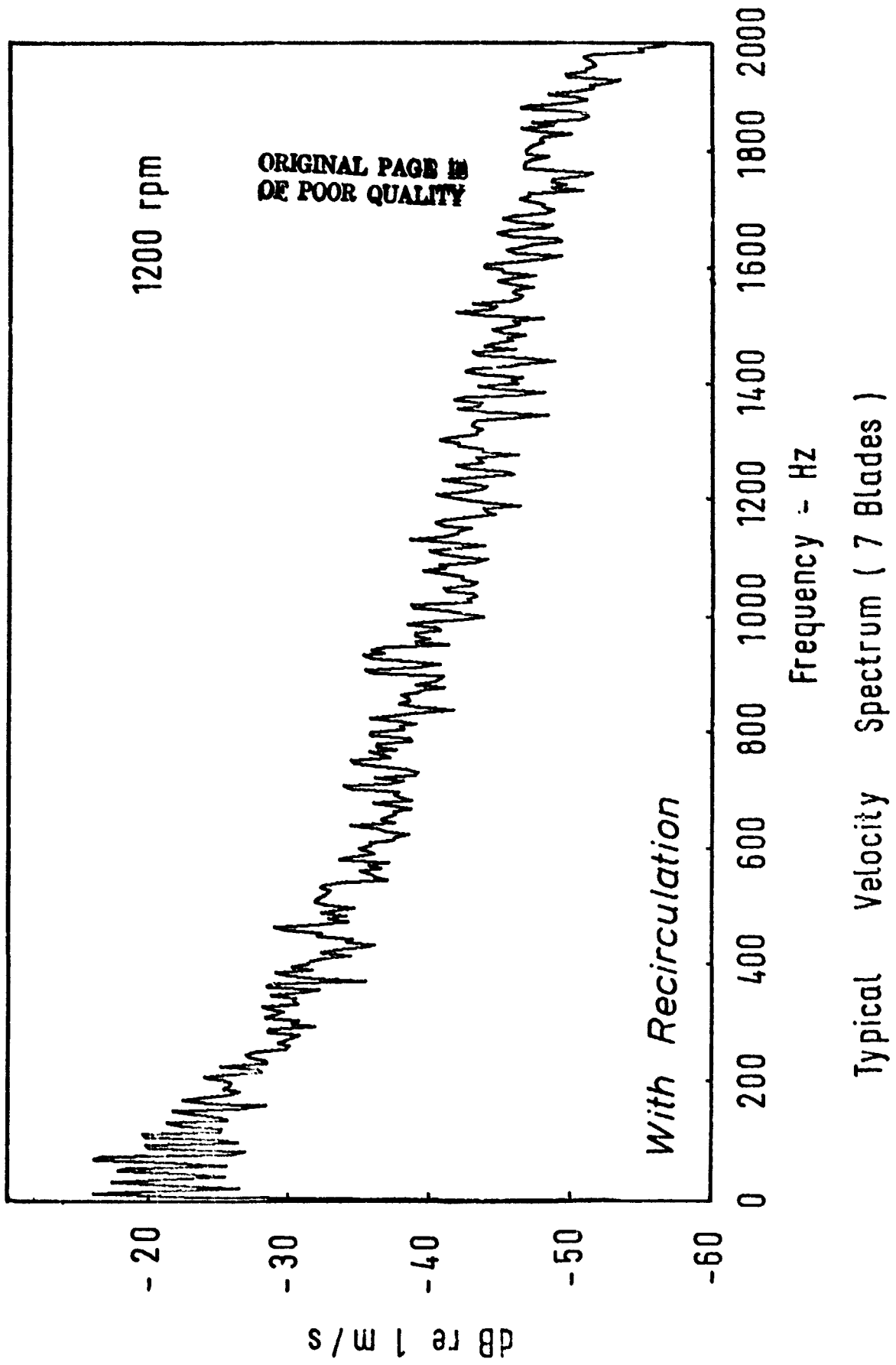
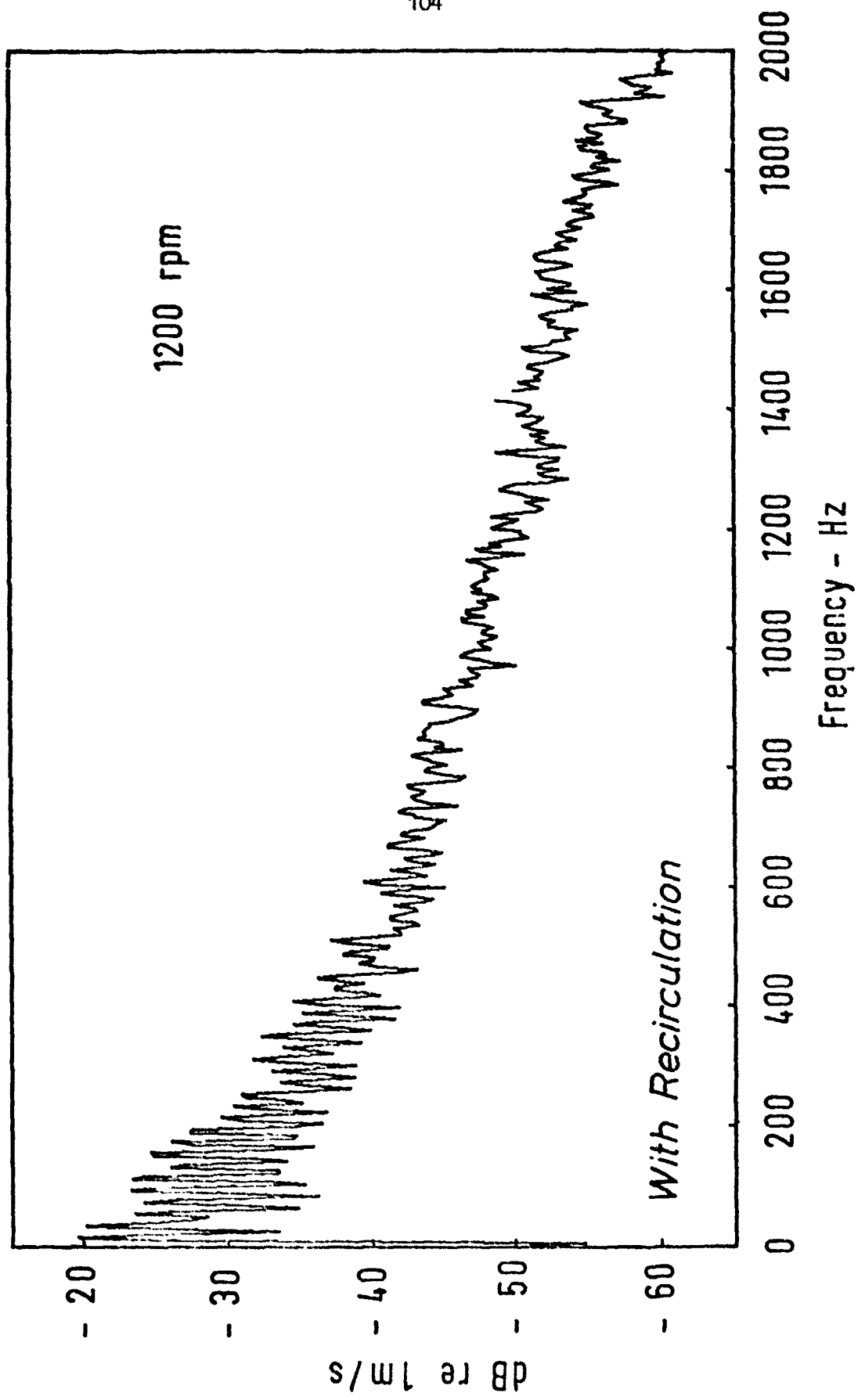


Figure 16



Typical Velocity Spectrum (2 Blades)

Figure 17

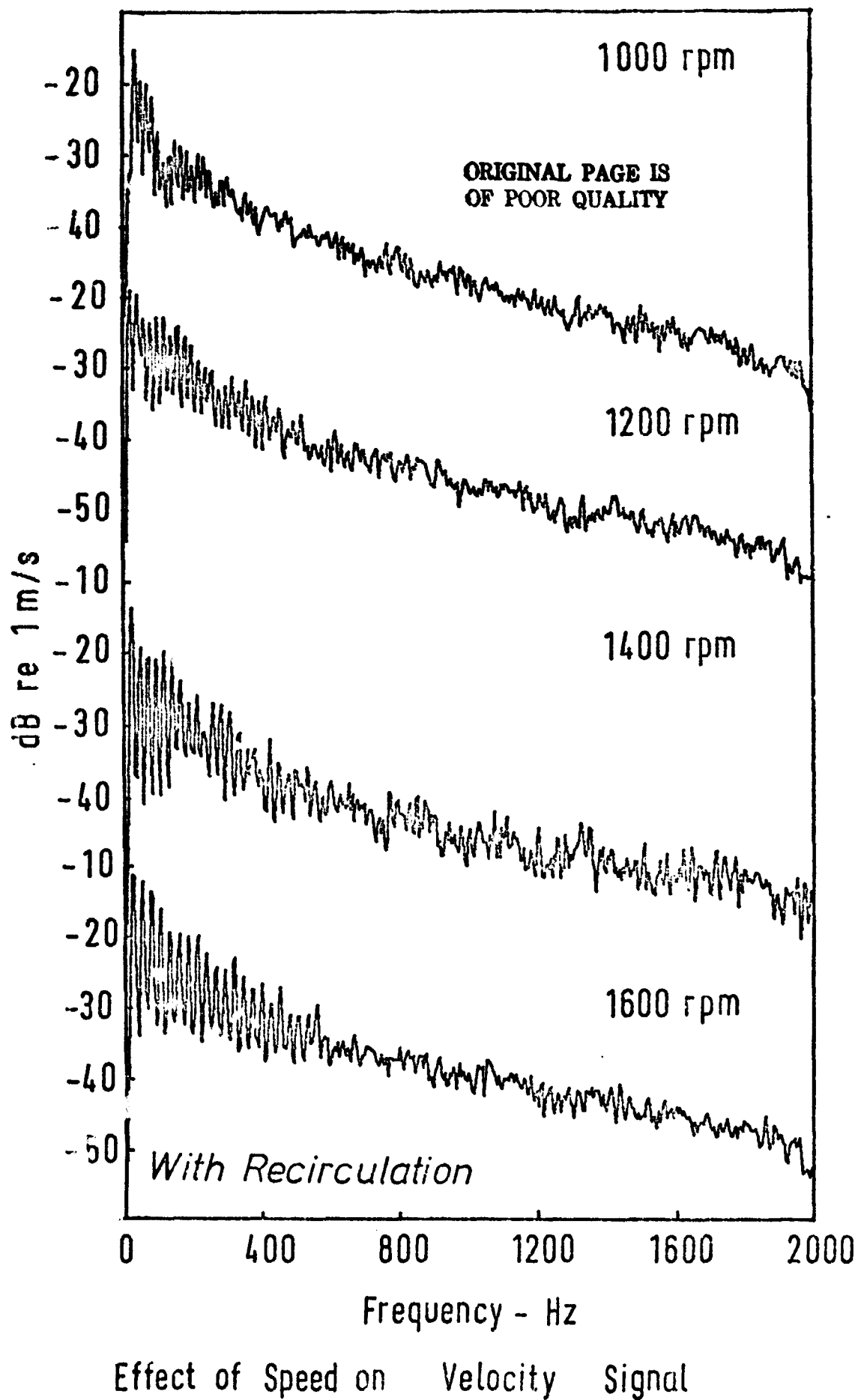
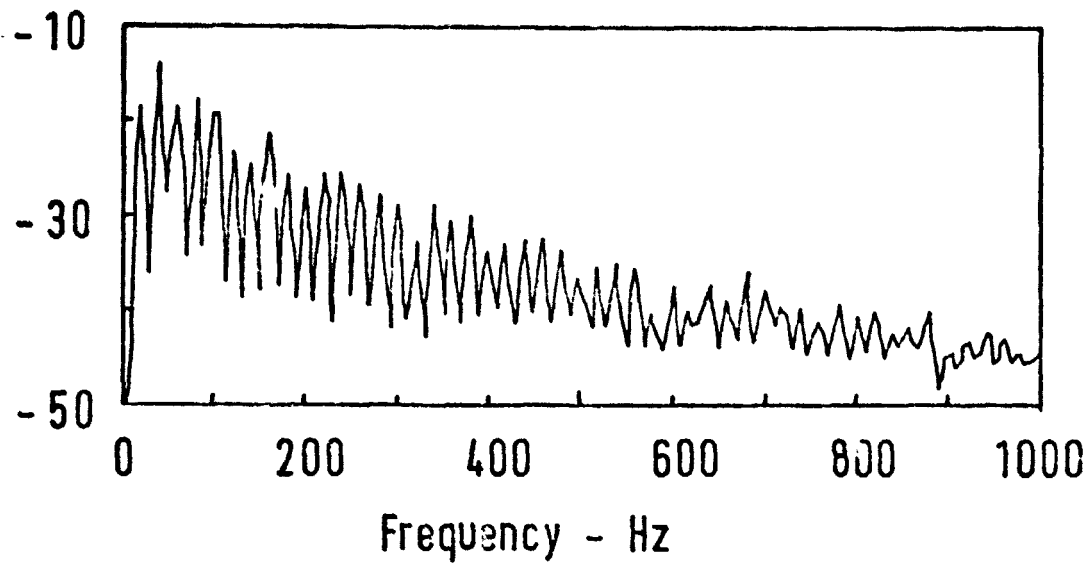
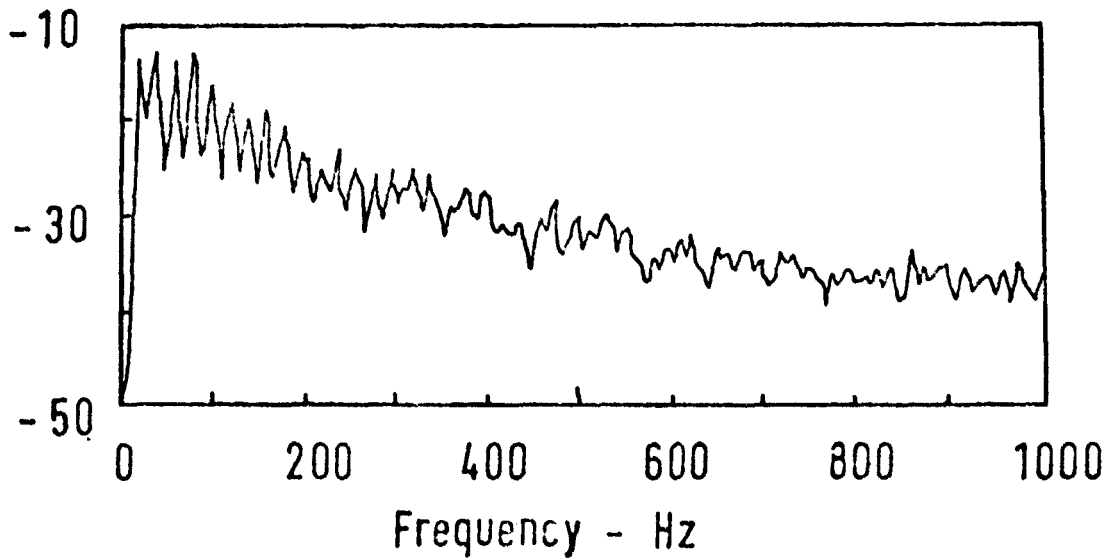


Figure 18



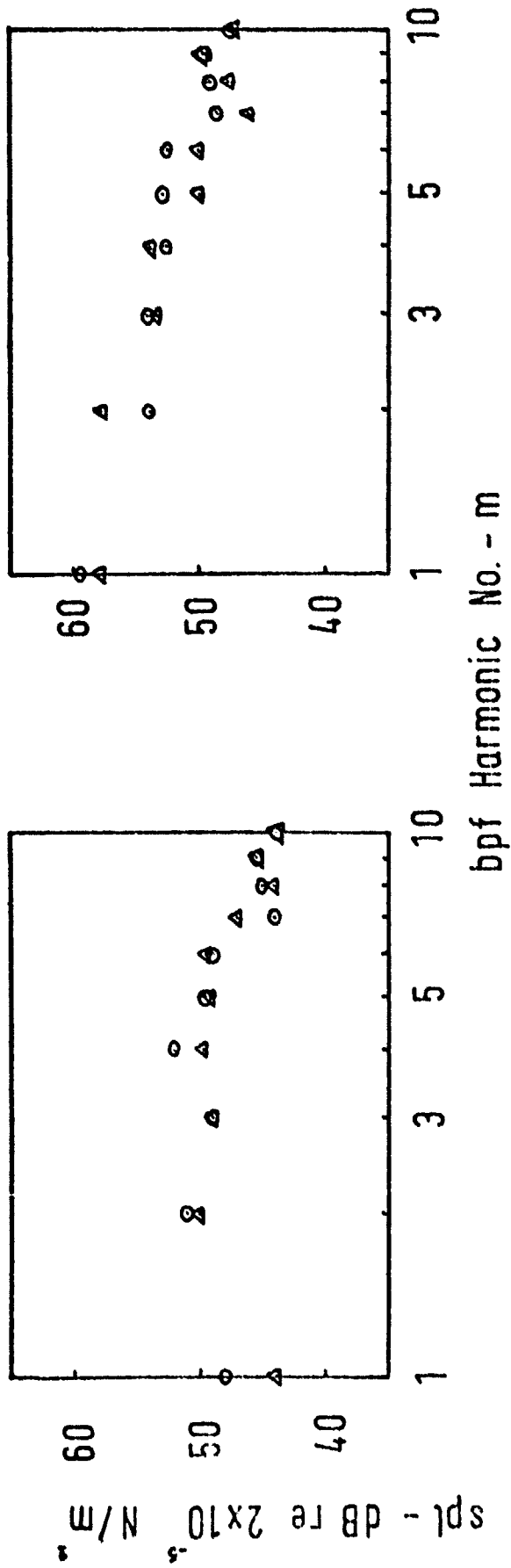
Before Recirculation



With Recirculation

Effect of Recirculation on Velocity Signal (1200 rpm)

Figure 19



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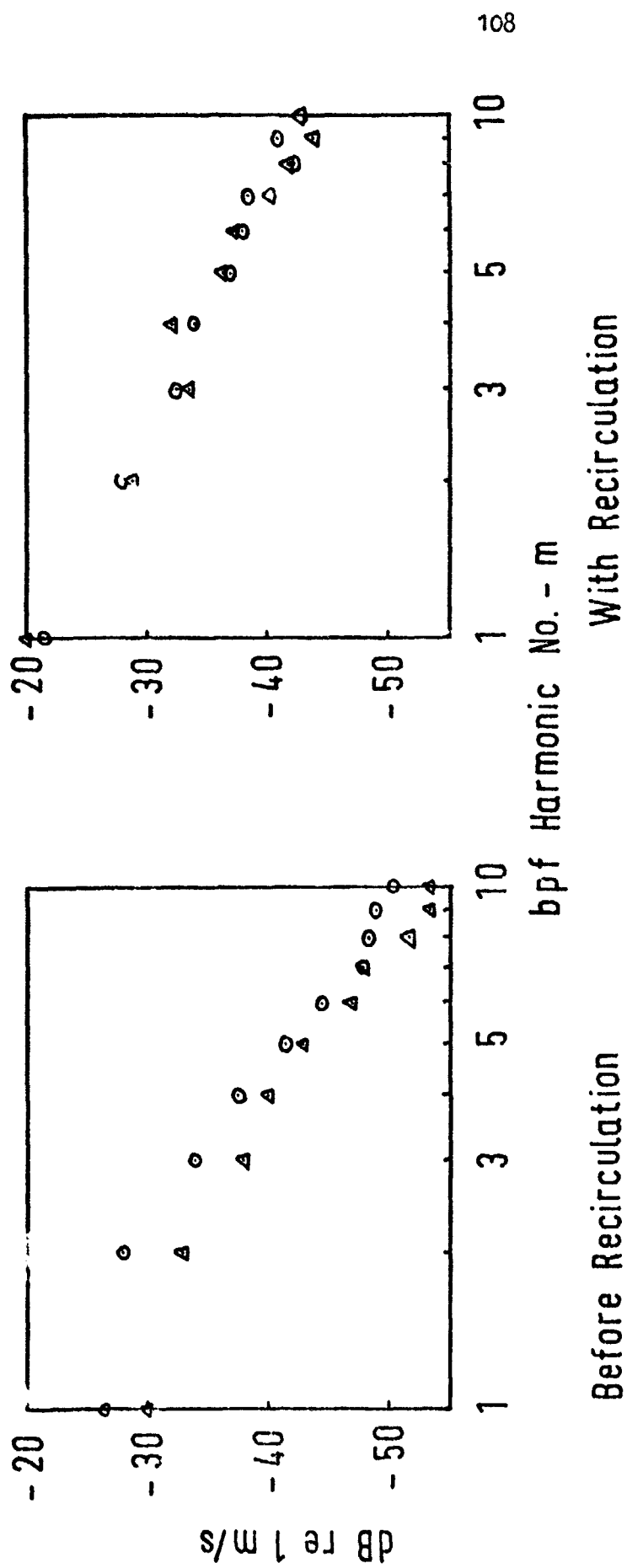
Before Recirculation

With Recirculation

○ Run 1, ▲ Run 2 .

Acoustic Data - 7 Blades, 1200 rpm

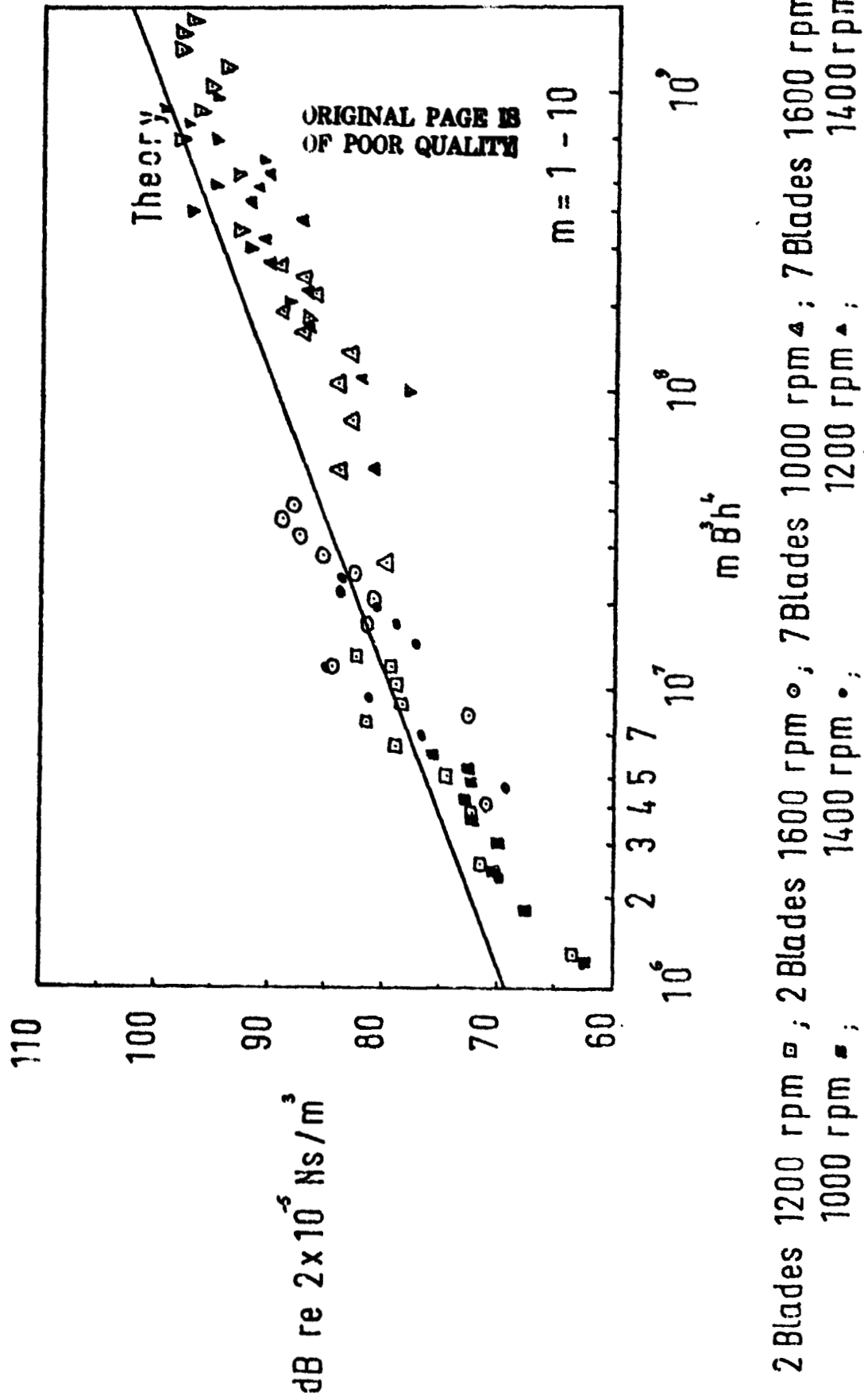
Figure 20



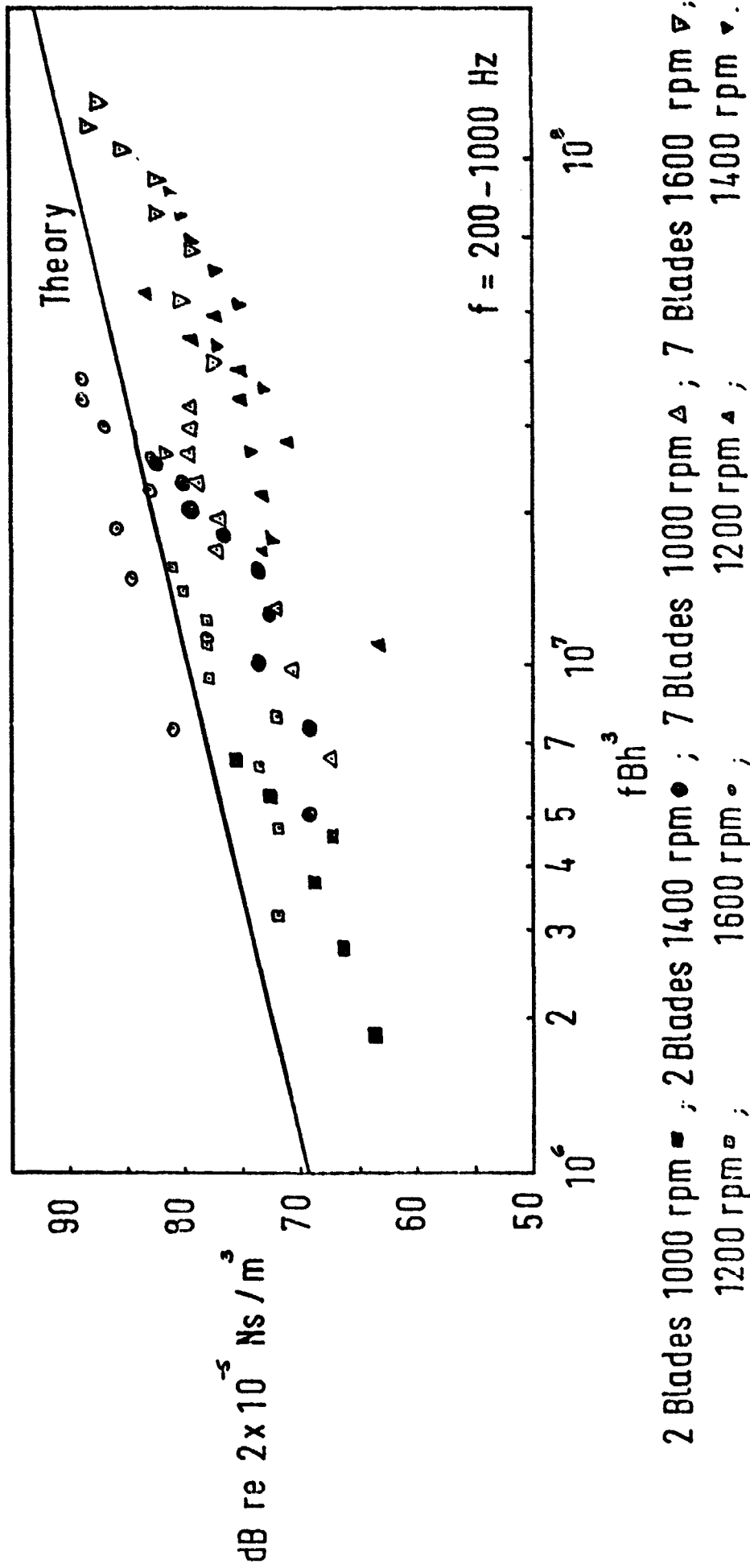
○ Run 1, △ Run 2.

Aerodynamic Data - 7 Blades, 1200 rpm

Figure 21

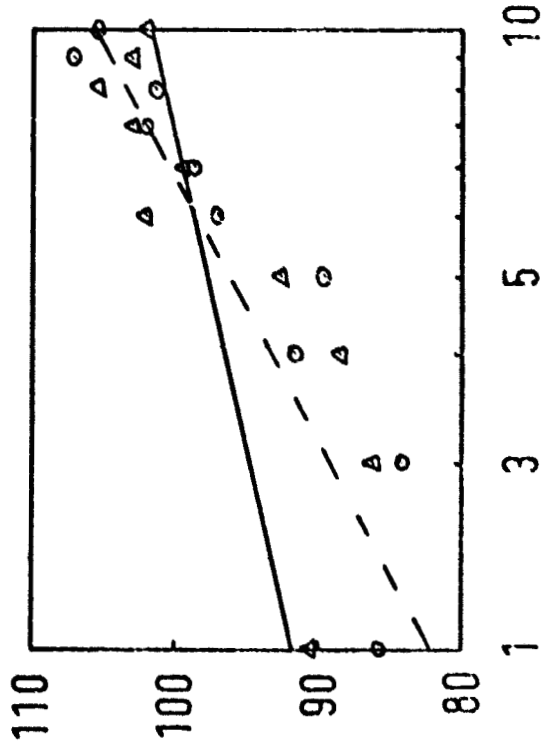


Discrete Frequency Aero-Acoustic Transfer Function (With Recirculation)

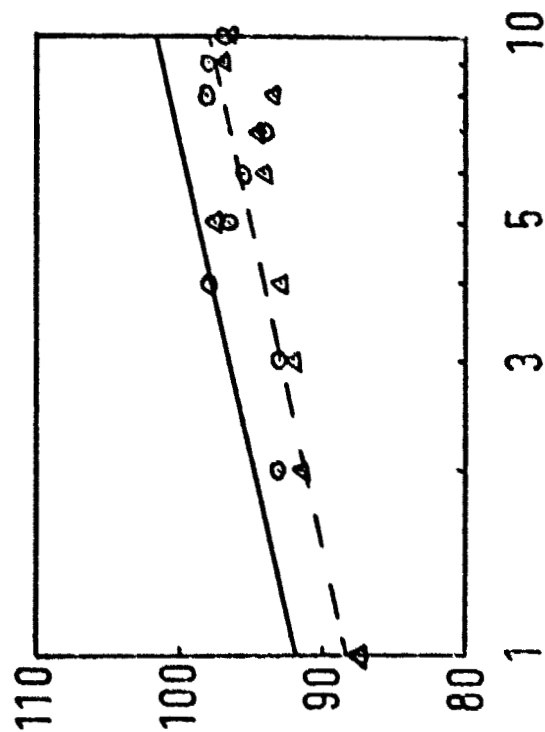


Broad Band Aero - Acoustic Transfer Function (With Recirculation)

Figure 23



Before Recirculation



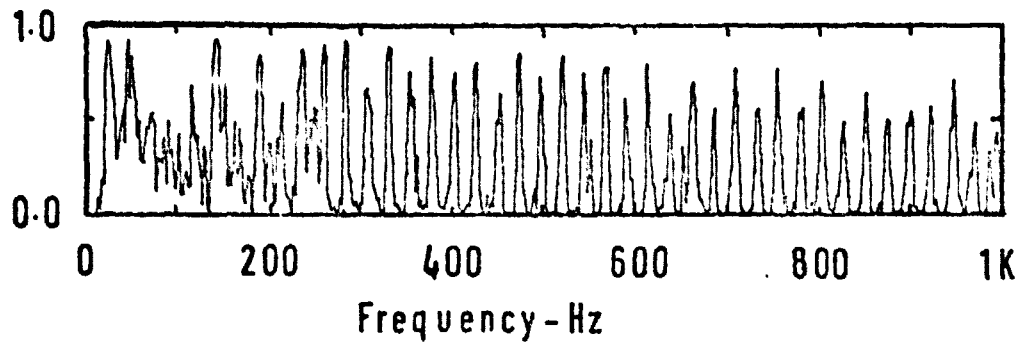
With Recirculation

Run 1 $\circ$; Run 2 Δ ; Least Squares Fit - - ; Theory — .

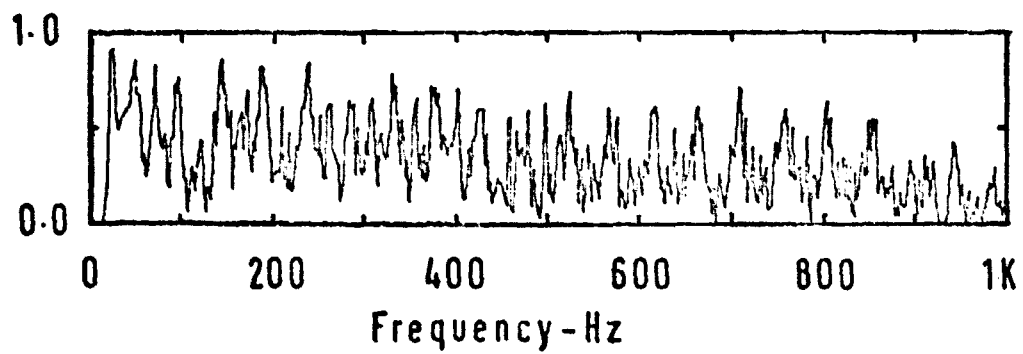
Effect of Recirculation on Discrete Frequency Aero - Acoustic Transfer Function -
7 Blades, 1600 rpm

Figure 24

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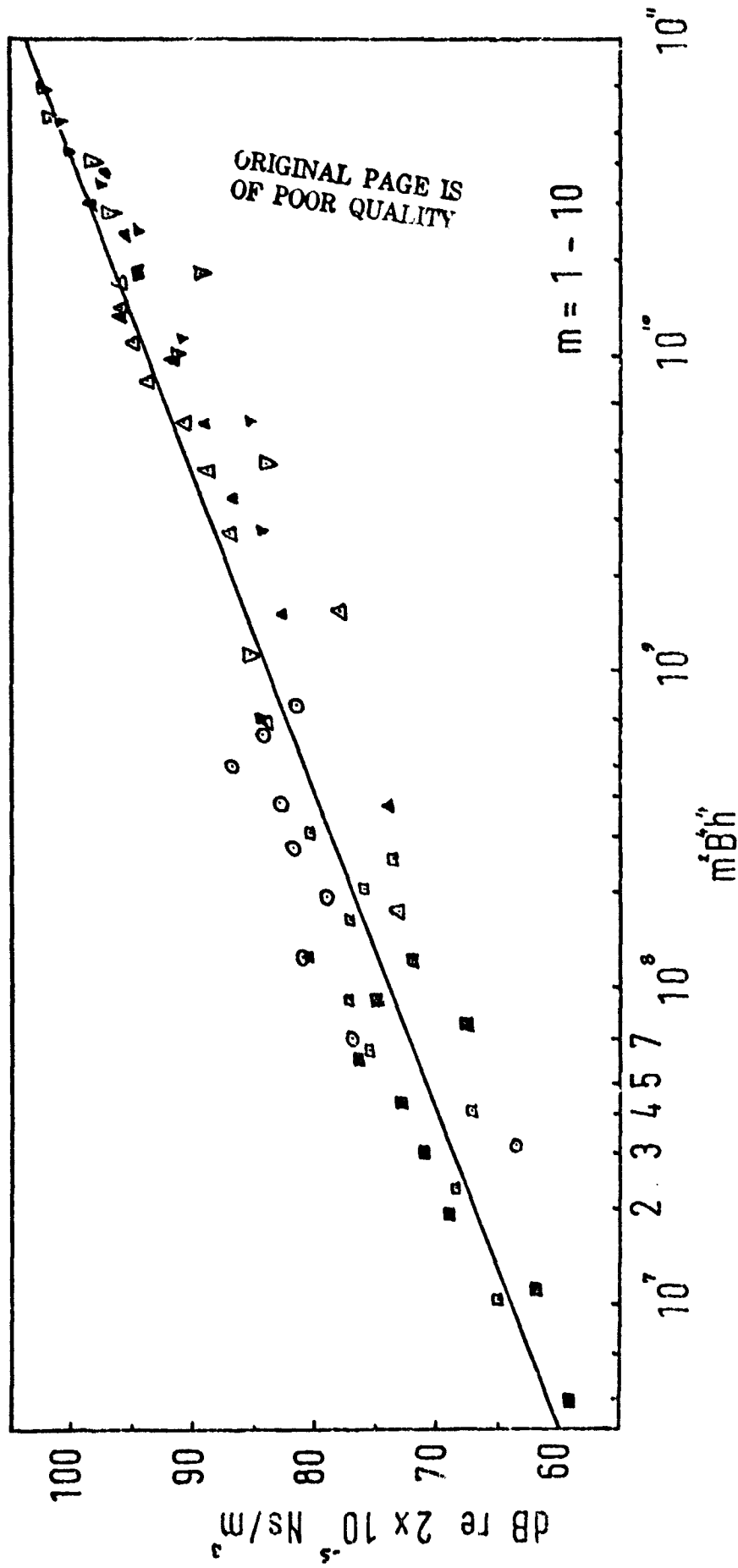
a) Before Recirculation



b) With Recirculation

Typical Coherence Functions - 2 Blades, 1400 rpm

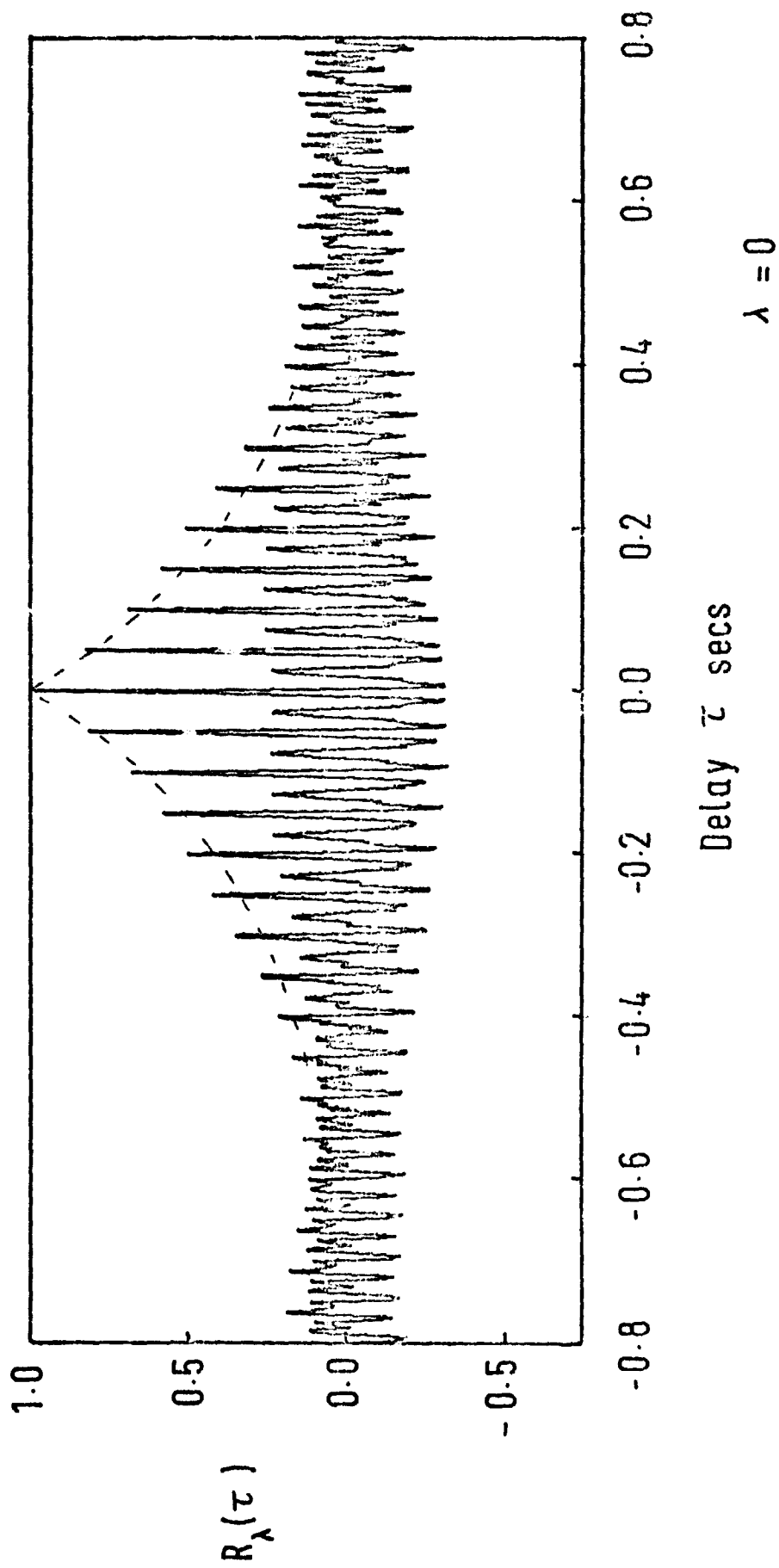
Figure 25



2 Blades 1000 rpm ■ ; 2 Blades 1600 rpm ○ ; 7 Blades 1000rpm ▲ ; 7 Blades 1600 rpm ▽,
1200rpm □ ; 1200rpm ▲ 1400rpm ▼.

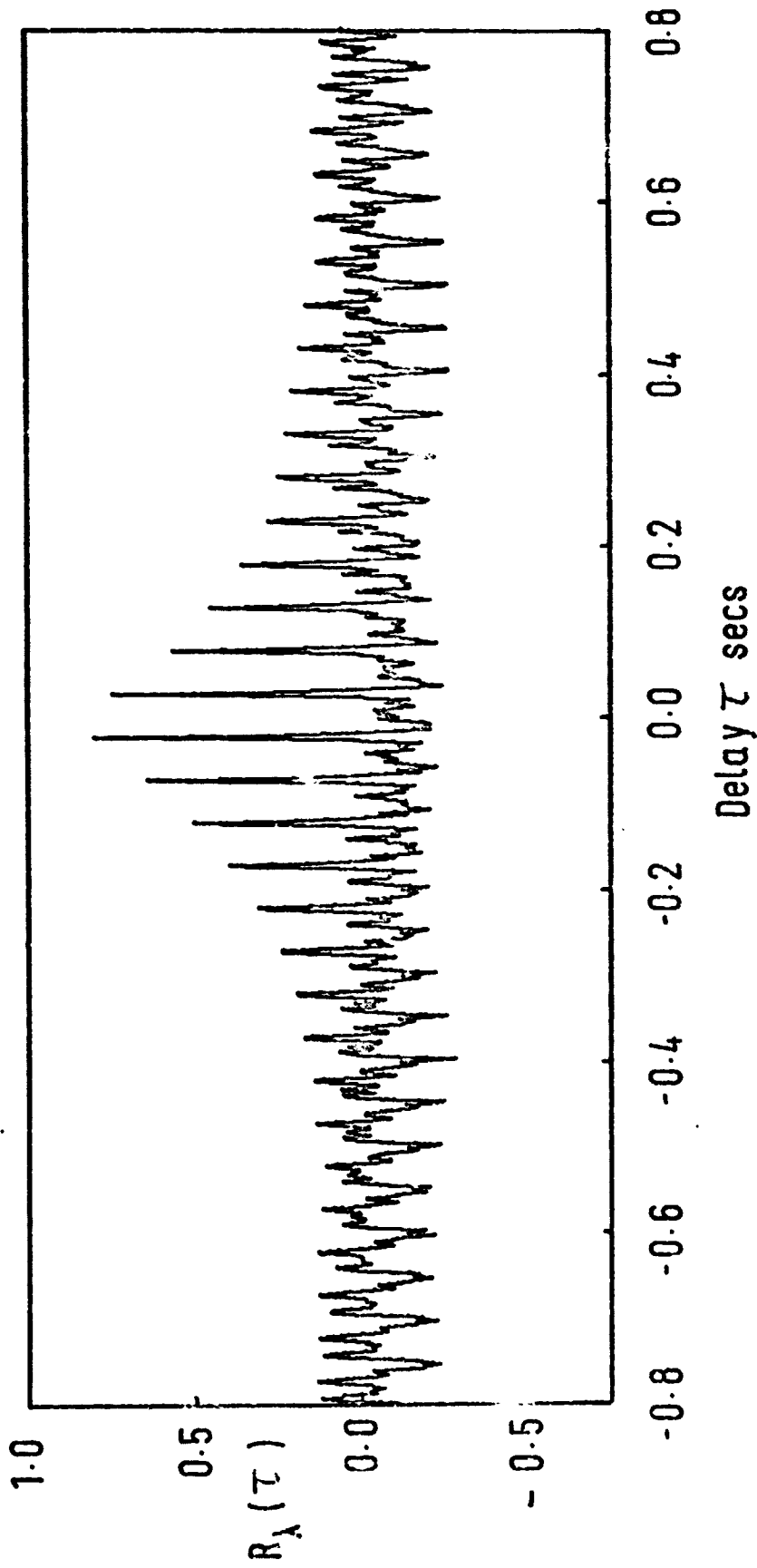
Discrete Frequency Aero - Acoustic Transfer Function (B) Recirculation)

Figure 26



Typical Velocity Auto - Correlation Function - With Recirculation, 1200 rpm

Figure 27

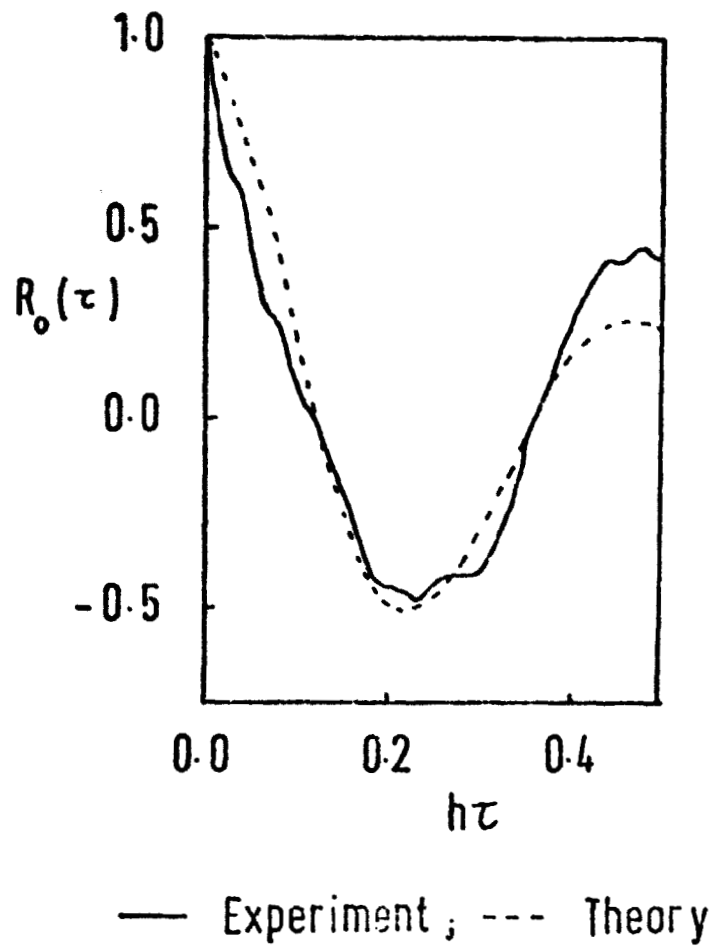


7 Blades $\lambda = 4$

Typical Velocity Cross - Correlation Function - With Recirculation, 1200 rpm

Figure 28

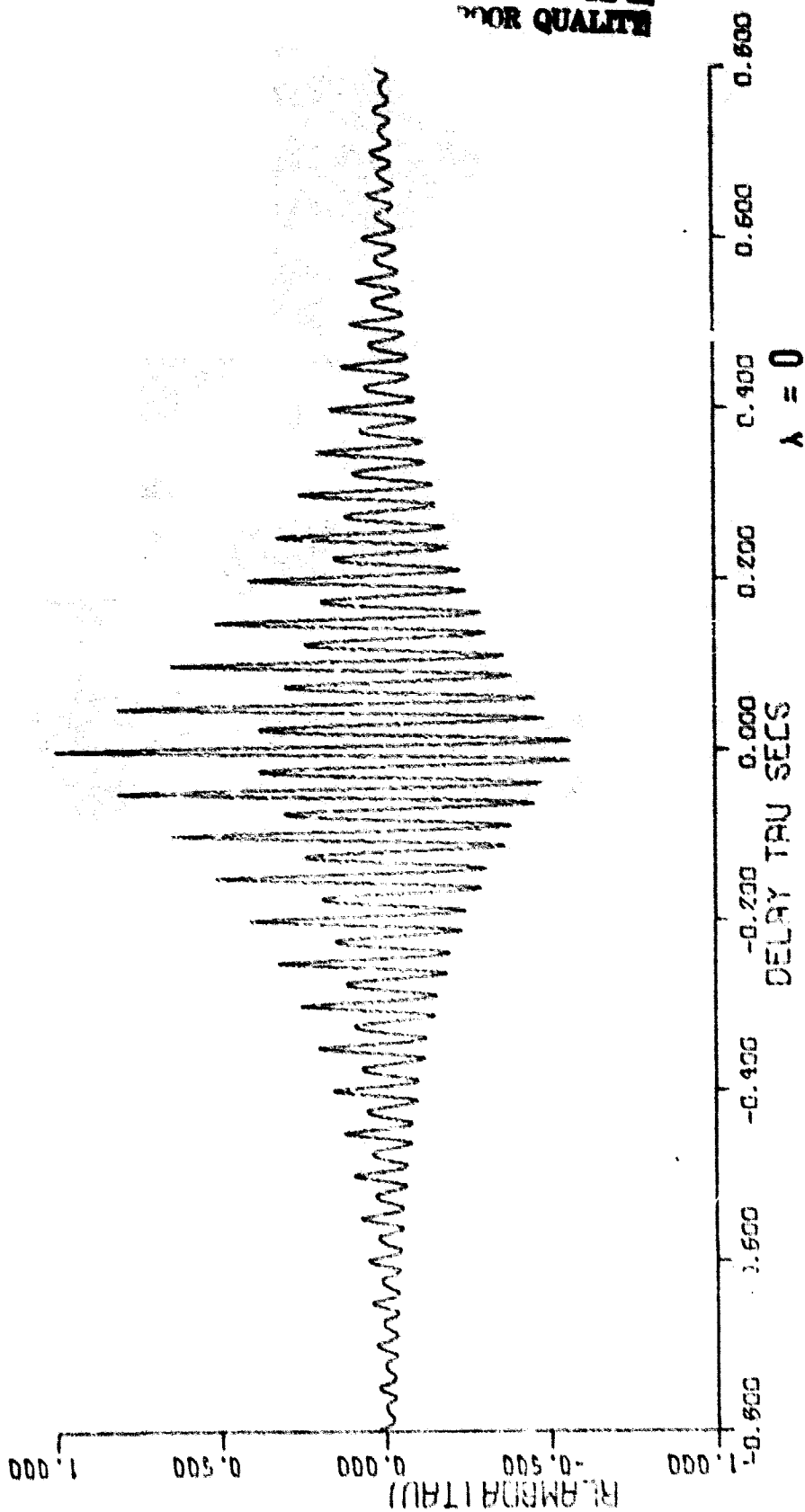
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Velocity Autocorrelation and Pulse Shape
Function

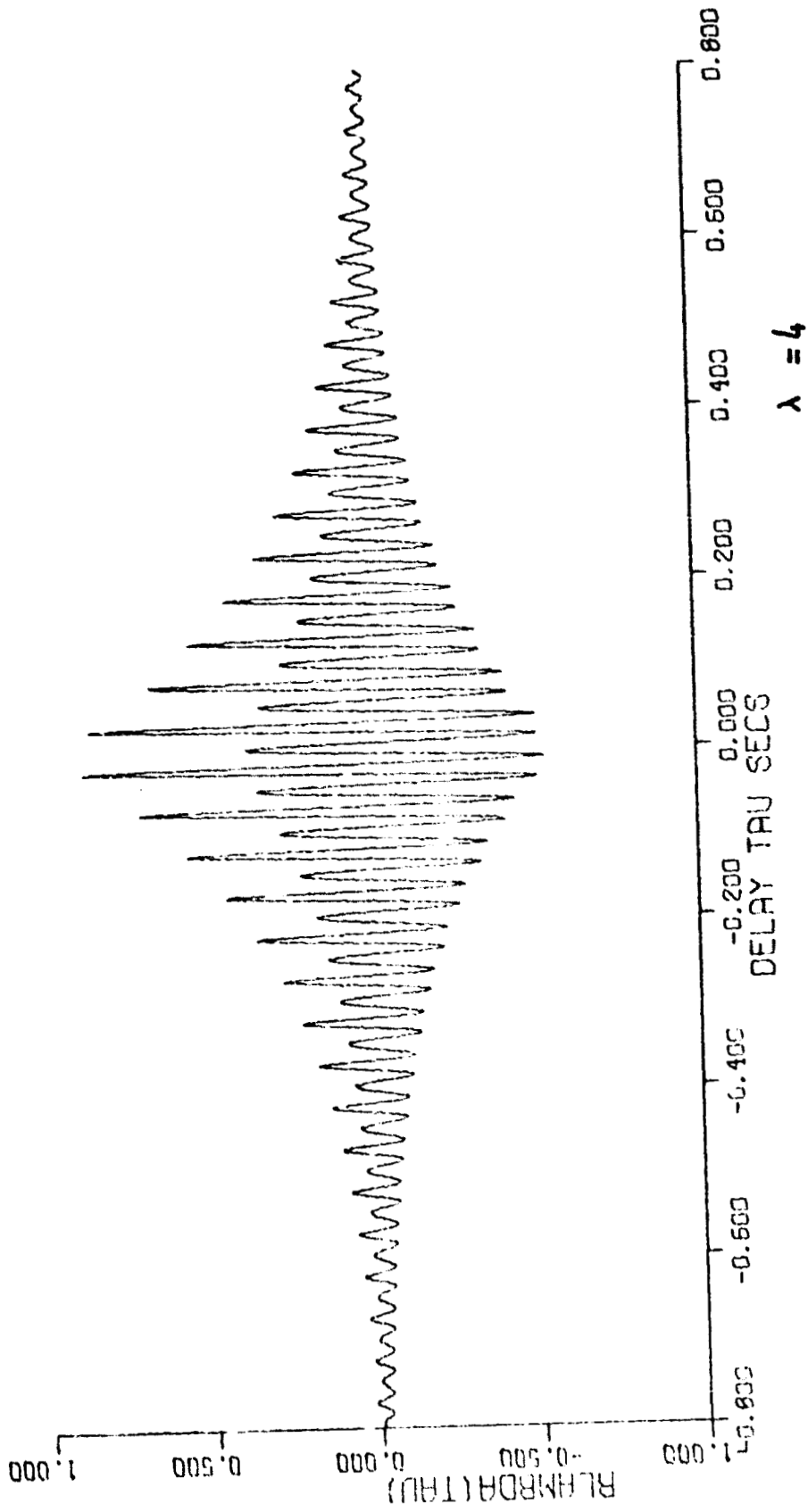
Figure 29

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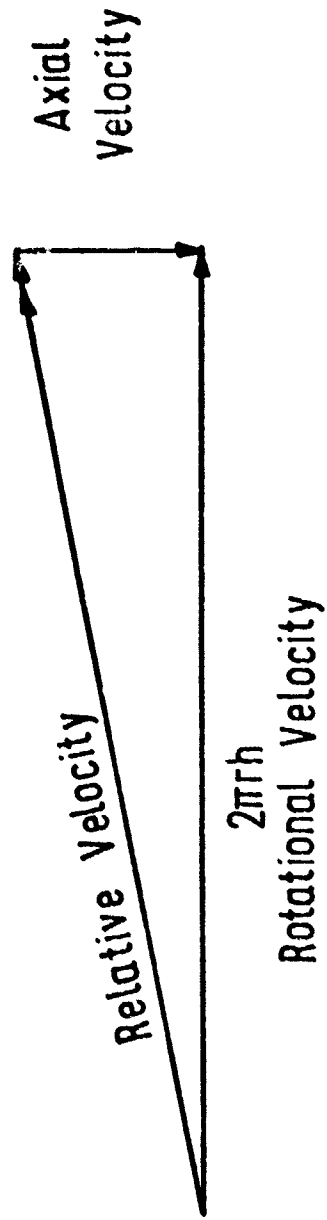
Theoretical Velocity Auto - Correlation Function

Figure 30



Theoretical Velocity Cross-Correlation Function

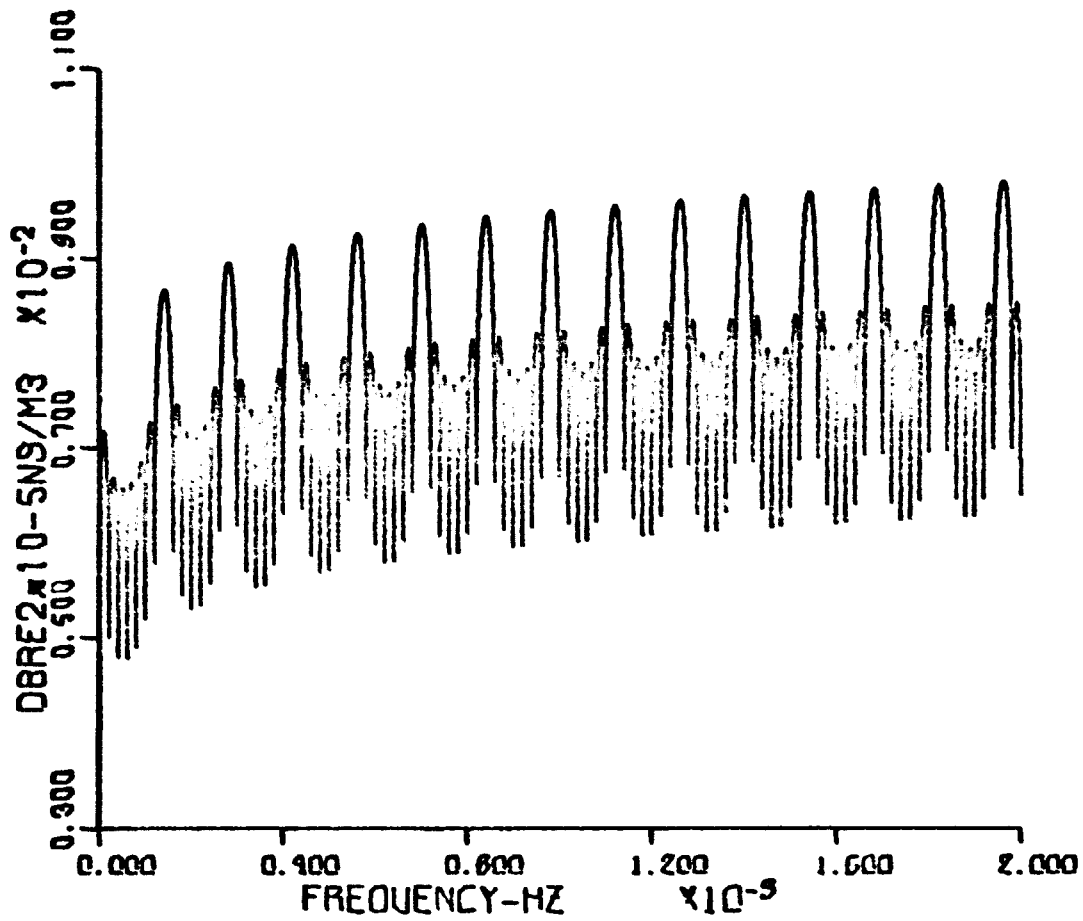
Figure 31



$$\text{Radius } r = 0.256 \text{ m}$$

Velocity Triangle for Fan Blade

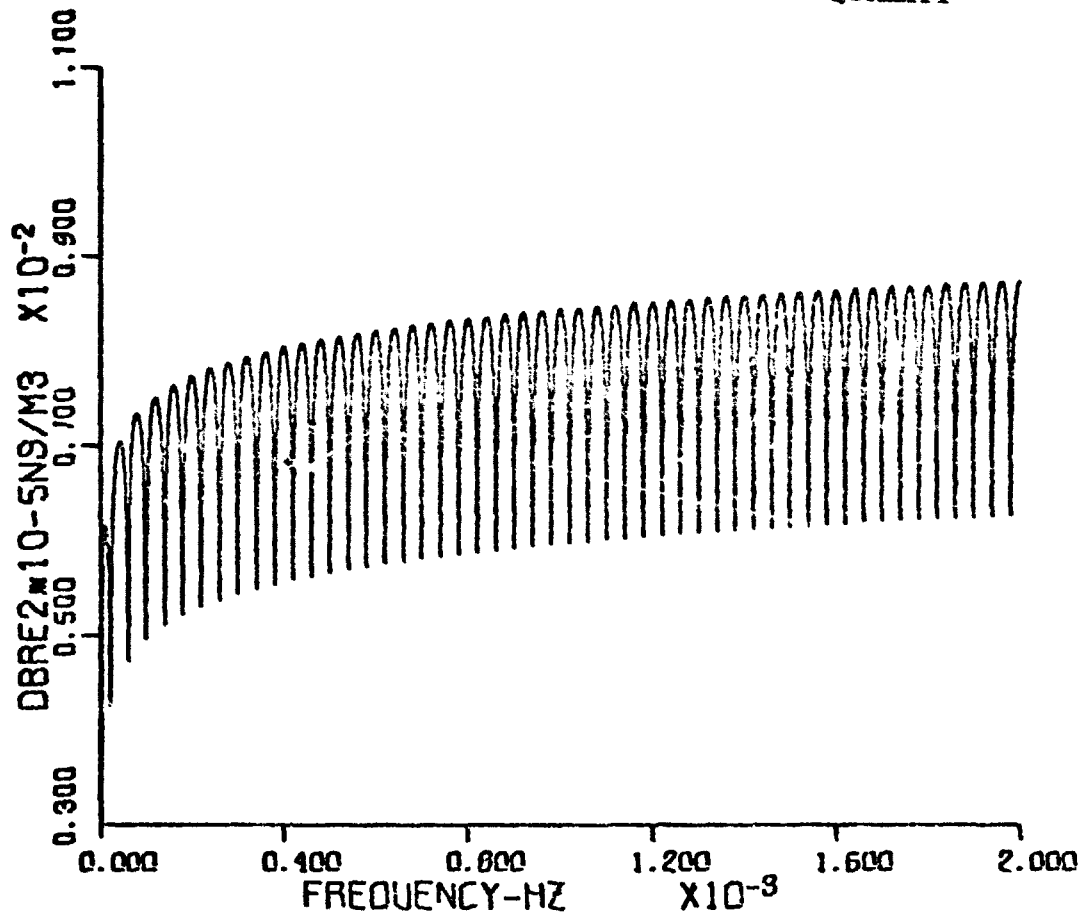
Figure 32



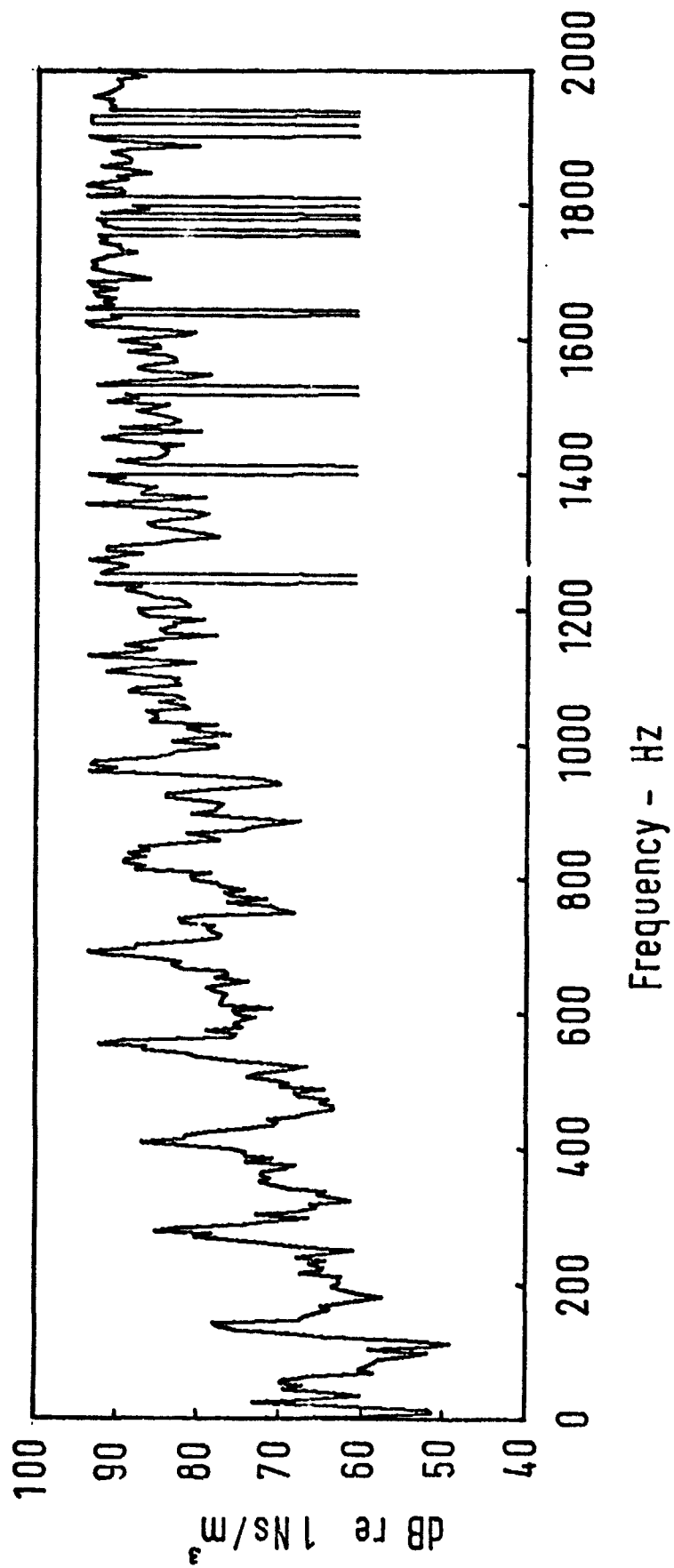
Theoretical Aero - Acoustic Transfer Function -
7 Blades, 1200 rpm

Figure 33

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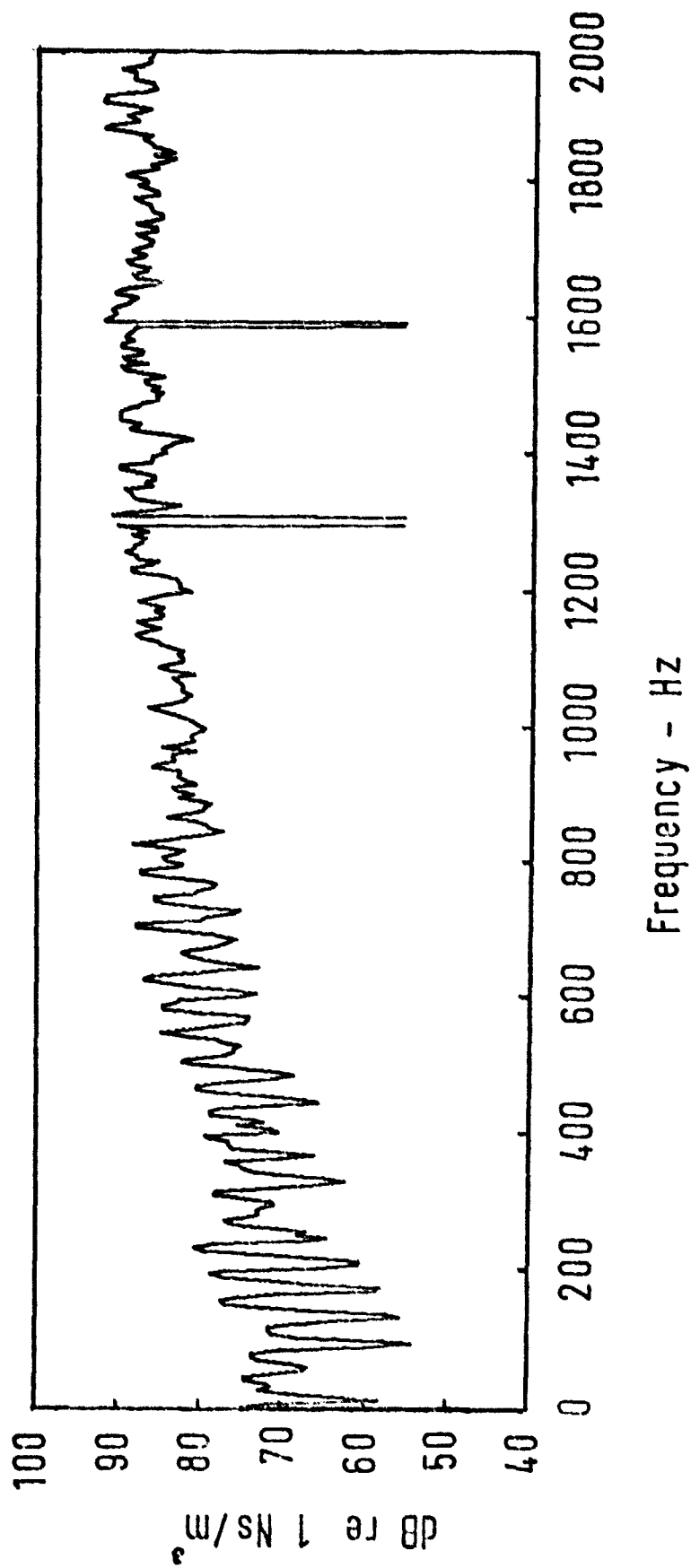


Theoretical Aero - Acoustic Transfer Function -
2 Blades, 1200 rpm



Experimental Aero - Acoustic Transfer Function - 7 Blades, 1200 rpm

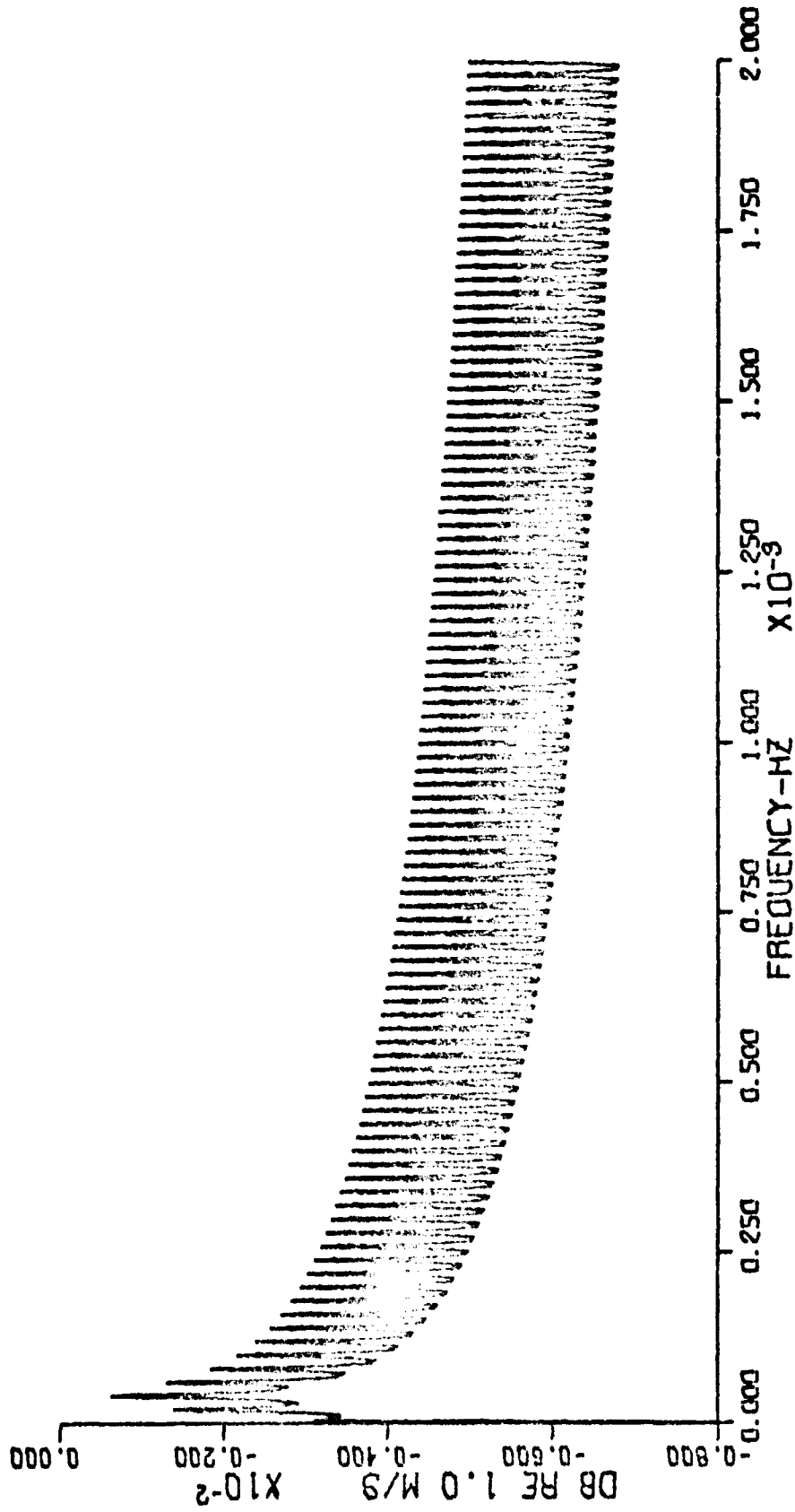
Figure 35



Experimental Aero-Acoustic Transfer Function - 2 Blades, 1200 rpm

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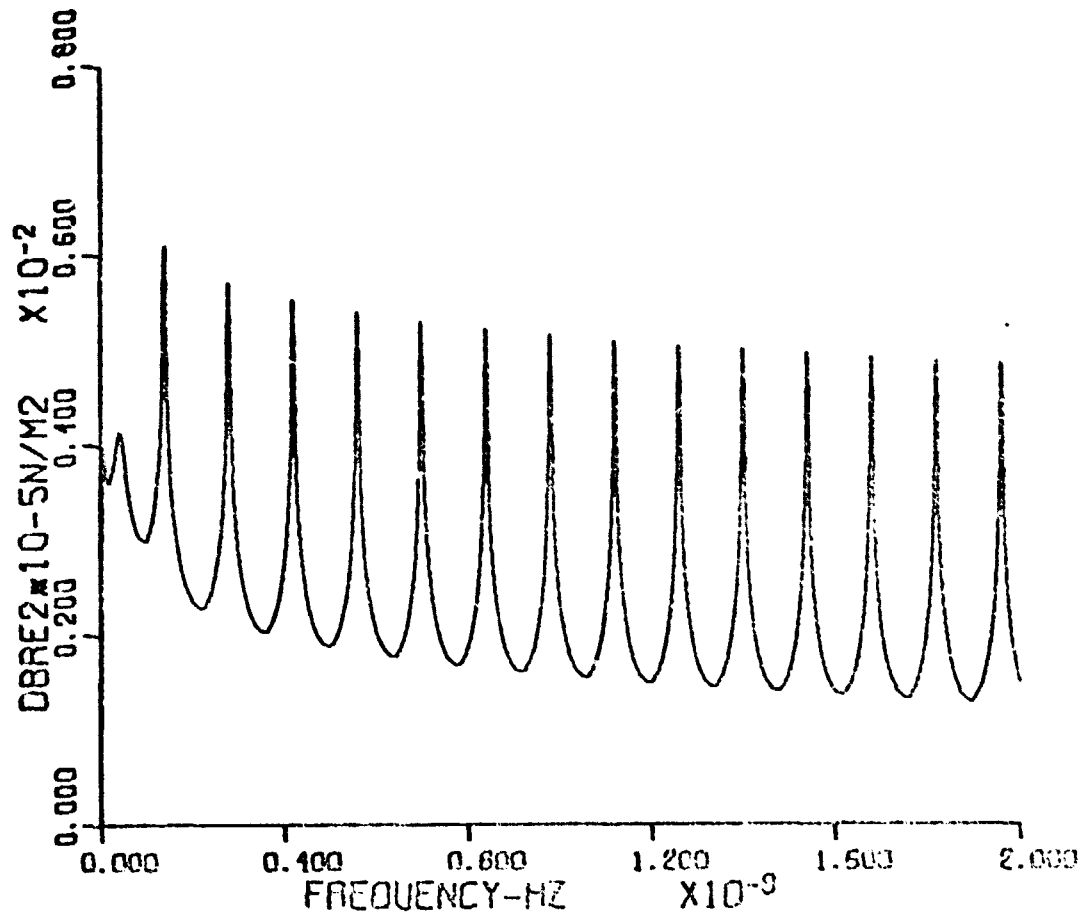
Figure 36



Theoretical Velocity Spectrum - With Recirculation, 1200 rpm

Figure 37

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Theoretical Acoustic Spectrum - With Recirculation,
7 Blades, 1200 rpm

Figure 38