NASA

Technical Memorandum 79609

Diffuse Fluxes of Cosmic High Energy Neutrinos

F. W. Stecker

JULY 1978

National Aeronautics and
Space Administration

Goddard Space Flight Center
Greenbelt, Maryland 20771

Diffuse Fluxes of Cosmic High Energy Neutrinos

F. W. Stecker
Laboratory for High Energy Astrophysics
NASA/Goddard Space Flight Center

Received _____

1. Introduction

There have been a number of recent papers estimating high-energy neutrino fluxes and spectra from various astrophysical processes (Berezinsky and Zatsepin 1970, Stecker 1973a, Berezinsky and Zatsepin 1977 and references therein, Silberberg and Shapiro 1977, Margolis, Schramm and Silberberg 1978, Eichler 1978.) In this work, I reexamine the problem by 1) presenting the results of a detailed calculation of galactic ν-production in cosmic-ray interactions with interstellar gas and extragalactic ν-production by interactions of ultrahigh energy cosmic rays with the 3K universal background radiation, 2) using these results together with present γ-ray observations to predict neutrino fluxes and event rates, and 3) reexamining some of the assumptions and results of previous work, pointing out significant differences.

2. Production Rates

The first basic production process for high energy cosmic neutrinos is the decay of charged pions produced in cosmic ray interactions with interstellar gas, primarily pp interactions. This process will henceforth be referred to as "pp", although the effects of αp pHe and αHe interactions are included in the calculation. The second process involves the photoproduction of π-mesons by interaction of ultrahigh energy cosmic rays with the 3K universal microwave background radiation (henceforth referred to as γp) and subsequent meson decay. Both of these types of interactions involve the accompanying production of π° mesons and their decay into cosmic γ-rays. Thus, the production rates of cosmic γ-rays and neutrinos are generically linked.

A detailed discussion of the kinematics of the production and decay of secondary particles produced in "pp" interactions may be found in Stecker

1. Introduction

There have been a number of recent papers estimating high-energy neutrino fluxes and spectra from various astrophysical processes (Berezinsky and Zatsepin 1970, Stecker 1973a, Berezinsky and Zatsepin 1977 and references therein, Silberberg and Shapiro 1977, Margolis, Schramm and Silberberg 1978, Eichler 1978.) In this work, I reexamine the problem by 1) presenting the results of a detailed calculation of galactic ν-production in cosmic-ray interactions with interstellar gas and extragalactic ν-production by interactions of ultrahigh energy cosmic rays with the 3K universal background radiation, 2) using these results together with present γ-ray observations to predict neutrino fluxes and event rates, and 3) reexamining some of the assumptions and results of previous work, pointing out significant differences.

2. Production Rates

The first basic production process for high energy cosmic neutrinos is the decay of charged pions produced in cosmic ray interactions with interstellar gas, primarily pp interactions. This process will henceforth be referred to as "pp", although the effects of αp pHe and αHe interactions are included in the calculation. The second process involves the photoproduction of π-mesons by interaction of ultrahigh energy cosmic rays with the 3K universal microwave background radiation (henceforth referred to as γp) and subsequent meson decay. Both of these types of interactions involve the accompanying production of π° mesons and their decay into cosmic γ-rays. Thus, the production rates of cosmic γ-rays and neutrinos are generically linked.

A detailed discussion of the kinematics of the production and decay of secondary particles produced in "pp" interactions may be found in Stecker

(1971). Details of the γp process have also been previously given (Stecker 1968). The γ-ray production spectrum in γp interactions has been calculated by Wdowczyk et al. (1972), Stecker and Bredekamp (1972), and Stecker (1973a).

In the present paper, the "pp" neutrino production spectrum was calculated for pp interactions up to 30 GeV by methods previously employed (Stecker 1971, 1973b). At higher energies, scaling was assumed to hold. At these energies, the parameters of Ganguli and Sreekantan (1976) were adopted for the rapidity distribution of charged pions. These authors have calculated γ-ray spectra which are in good agreement with those of Stecker (1970) for γ-ray energies $\leq$ 10 GeV, however, at higher energies, the assumption of scaling gives larger fluxes and a flatter spectrum than those calculated by Stecker (1971) using an "isobar + fireball" model in "pre-scaling days". It should be noted that the isobar + fireball (I-F) model is equivalent to a "leading-pion" model at high energies, since the isobar carries off $\sim$ 50% of the energy and decays into a "leading pion". Both scaling and the I-F models produce a secondary spectrum which has the same spectral index as the primary spectrum in the high-energy limit. (See Appendix I).

The results of the "pp" production spectrum calculation are shown in Fig. 1. The two neutrino production spectra are given for an interstellar hydrogen density of $1 cm^{-3}$ so that it is really a production rate per hydrogen atom. The upper neutrino curve and the γ-ray production spectrum shown in the figure are calculated using a primary cosmic ray spectrum $I_p(E) = 2.35\ E^{-2.67} cm^{-2} s^{-1} sr^{-1} GeV^{-1}$ for $E_p > 10$ GeV and the lower curve is obtained for a primary spectrum $I_p(E_p) = 2.0\ E^{-2.75} cm^{-2} s^{-1} sr^{-1} GeV^{-1}$ (Ryan, [illegible] and Balasubrahmanyan 1972). The γ-ray spectrum is for all γ-rays

from π^0-decay; the ν-spectrum is for the ν_μ component. Since each pion decay results in one ν_μ, one $\tilde{\nu}_\mu$ and one $\nu_e(\tilde{\nu}_e)$, all of roughly the same energy, the total number of ν's produced (although of four different types) is a factor of 3 higher than that shown for ν_μ's. In this regard, it should be kept in mind that the cross section ratios given in Table 1 apply.

The production rate for γp interactions has been calculated using the method of Stecker and Bredekamp (1972). As an example of the range of uncertainty, two different primary spectra were used, i.e. $I_p(E_p) = 6.4\times10^{-31}(E_p/10^{11})^{-2.6}\,cm^{-2}s^{-1}sr^{-1}GeV^{-1}$ (Linsley 1963) and $I_p(E_p) = 2.4\times10^{-31}(E_p/10^{11})^{-3.24}\,cm^{-2}s^{-1}sr^{-1}GeV^{-1}$ (Andrews et al. 1971). The latest analysis of the ultrahigh energy air-shower data (Hillas, private communication) gives $I_p(E_p) \simeq 4\times10^{-31}(E_p/10^{11})^{-3}\,cm^{-2}s^{-1}sr^{-1}GeV$ in the energy range $10^8 \leq E_p \leq 10^{10}$GeV with indications of a flattening to $I_p(E_p) \simeq 2\times10^{-30}(E_p/10^{11})^{-2.3}$ in the energy range $10^{10} \leq E_p \leq 10^{11}$GeV. The results of the γp calculation are shown in Figure 2 for the two spectra chosen. The right-hand scale of Figure 2 also shows the diffuse background flux from this process obtained by multiplying by the factor $c/(4\pi H_o)$ where H_o, the Hubble constant, is taken to be 50km $s^{-1}Mpc^{-1}$. This flux, which is also shown in the subsequent figures, only holds if the ultrahigh energy primary spectrum is universal, an assumption which is contradicted by the lack of an observed high energy cutoff in the spectrum (Greisen 1966, Zatsepin and Kuzmin 1966, Stecker 1968, 1978a). Thus the fluxes given may be over-estimated and may actually be upper limits.

Figure 3 shows the integral galactic "pp" neutrino production spectrum.

3. Diffuse Cosmic Neutrino Flux

Figures 4 and 5 show the differential and integral neutrino fluxes from various sources. In figure 4, the crosshatched region marked γp is obtained from the curves shown in Figure 2. The hatched region marked pp(G.C. $|b| \leq 10°$) is for galactic ν-production coming from the galactic central region defined by galactic longitude $330° \leq \ell \leq 40°$ and latitudes $|b| \leq 10°$. The dashed line is the flux computed using the isobar-fireball (I-F) model (Stecker 1971), which is probably too low. The lines bounding the hatched region are obtained using the two "pp" production spectra shown in Figure 1. All galactic "pp" curves are noramlized by assuming that 1/2 of the galactic γ-radiation above 100 MeV observed by SAS-2 (Fichtel et al. 1978) is from π^0-decay and by relating the γ-ray and ν production using the results given in Figure 1. Spectral measurements indicate that $\sim$ 50% of the galactic γ-rays observed above 100 MeV are most probably from cosmic-ray electron bremsstrahlung with no associated ν-production. The curve marked PCR (UL) is an upper limit on the ν-flux from primordial (or "cosmological") cosmic-ray interactions obtained under the assumption that the extragalactic γ-ray background (8×10^{-6} cm^{-2} s^{-1} sr^{-1} above 100 MeV with an E^{-3} power law spectrum) is from interactions of cosmic-rays at high redshifts (Stecker 1969). A detailed comparison of the calculated and observed spectra (Stecker 1974, 1978b, Montmerle 1977) indicates that the γ-ray background is most probably not from PCRs so that the curve shown is an upper limit. Note that this upper limit is many orders of magnitude below the cosmological neutrino upper limit given by Berezinsky and Zatsepin (1977 and references therein).

The curves in Figure 4 marked H(ATM) and V(ATM) are the horizontal and vertical atmospheric background fluxes given by Osborne, Said and

Wolfendale (1965), which indicate the background problems involved in detecting cosmic neutrinos.

The integral fluxes in Figure 5 show galactic "pp" fluxes for the central region $330° \leq \ell \leq 40°$ (GC) for $|b| \leq 2°$ and $|b| \leq 10°$, the anticenter region (AC) for $|b| \leq 2°$ and the region of the galactic poles (GP). All of these spectra were normalized using the SAS-2 γ-ray data as presented by Fichtel et al. (1978) except for the GP curve which was obtained from Fig. 3 using a mean galactic path length of hydrogen gas $\langle nL \rangle = 3 \times 10^{20} cm^2$. The atmospheric flux shown in Fig. 5 is from Zatsepin and Kuzmin (1962). The insert shows the galactic latitude distribution of neutrinos calculated for energies above 1 TeV. The curve marked "other galaxies" is obtained by assuming a density of normal galaxies of $3 \times 10^{-76} cm^{-3}$ (Felten 1977 and private communication) and assuming that they have the same γ-ray liminosity as our galaxy which is assumed to radiate 1/2 of its γ-ray flux from π°-decay. Note that since the predicted γ-ray spectrum from normal galaxies is considerably flatter than the observed extragalactic background, so that the limit on the γ-ray flux from other galaxies due to π°-decay is probably $\sim 2 \times 10^{-6} cm^{-2} s^{-1} sr^{-1}$ above 100 MeV, the ν-background from π°-decay cannot be significantly higher than the curve marked "other galaxies".

4. Event Rates

The νN cross section rises linearly with E_ν and is $\sim 0.7 \times 10^{-38} E_\nu cm^2$ at accelerator energies (Barish 1978). The $\tilde{\nu} N$ cross sections are $\sim 1/3$-0.4 of this value (see Table 1), although there is now evidence of an increase in this ratio to ~ 0.6 at $E_\nu \sim 100$ GeV (Barish 1978), possibly due to a new flavor b-quark with $m_b \sim 5$ GeV (Barnett 1976). Above a critical energy

$$E_\nu^c = M_W^2 / 2M_p, \tag{1}$$

where M_W is the mass of the intermediatee vector boson and M_p is the proton mass, the energy dependence of the cross section levels off to a logarithmic one (Bjorken and Paschos 1970). In the unified gauge theory of Weinberg and Salam, $M_W = 37.3 \csc \theta_w$ where θ_w is a parameter of the theory (Weinberg 1974). The best experimentally determined value at present is $\sin^2\theta_w \simeq 0.3$ (Barish 1978) giving $M_W \simeq 70$ GeV. The resulting νN cross section as a function of E_ν is shown in Fig. 6.

Using the νN cross section from Fig. 6 for $M_W = 70$ GeV (solid line) and a strictly linear cross section ($M_W = \infty$ dashed line) together with the calculated cosmic ν-fluxes, expected event rates for ν_μ's falling on a 1 km^3 seawater detector were calculated and are given in Fig. 7. The dotted line marks the level corresponding to 1 event/yr. The upper "pp" curve is for the flux from the galactic central region using the production spectrum of Fig. 3 which is derived from the upper ν-curve of Fig. 1. The curve marked $\langle pp \rangle_{GAL}$ is for the galactic "pp" neutrinos averaged over 4π sr. Thus, on the average, one would expect an event rate of ~ 1 yr^{-1} from galactic ν_μ's of energy >10 TeV. If ultra high energy cosmic rays are universal (not likely) and the νN cross section rises linearly up to $\sim 10^9$GeV ($M_W \geq 5 \times 10^4$GeV, again not likely) this rate could again be reached for $E_\nu > 10^9$GeV from γp interactions. It is more likely, however, that at energies high enough to see steady diffuse fluxes of cosmic neutrinos with a 1 km^3 detector above the atmospheric background, the event rates will be very low.

5. Comparison With Previous Calculations

Figure 8 shows the predicted fluxes of <u>both</u> ν_μ and $\tilde{\nu}_\mu$ coming from "pp" interactions in the galactic anticenter direction as well as the flux of

$\nu_\mu+\tilde{\nu}_\mu$ from the γp process. The curves are shown for the present work and the results of Silberberg and Shapiro (1977) and Margolis et al. (1978). The calculations of Berezinsky and Zatsepin (1977), when adjusted for the anticenter region, would be similar to the results of Silberberg and Shapiro (1977). The results obtained here, as shown in Fig. 8, give pp fluxes which are roughly an order of magnitude below those obtained by Shapiro and Silberberg for the energy range $10^2 GeV \leq E_\nu \leq 10^7 GeV$ and which are, on the average, roughly two orders of magnitude below those given by Margolis et al. over the energy range $10^3 GeV \leq E_\nu \leq 10^8 GeV$ for galactic "pp" neutrinos. The γp flux calculated here is two orders of magnitude below that given by Margolis et al. for the 10^6-10^8GeV energy range. (See Appendix II).

There are three probable reasons why the fluxes given here are lower than those given previously by other authors.

1) Somewhat steeper, but more accurate, primary spectra were used in the present calculation.

2) An erroneous flattening appears in the pp results of Margolis et al. (1978) in the 10^2-10^4GeV energy range. (See Appendix I).

3) Perhaps most importantly, our pp neutrino fluxes were normalized to the 100 MeV γ-ray observations of the Galaxy, making use of the generic relation between charged and neutral pion production (see Fig. 1).

Finally, it should be noted that existing observations of the extragalactic γ-ray background flux and spectrum place severe limits on cosmological cosmic-ray pion production (Stecker and Silk 1969, Stecker 1971) and therefore call into question high redshift production burst models such as those discussed by Berezinsky and Smirnov (1975) and Berezinsky and Zatsepin (1977). In particular, the upper limits given by these authors for the cosmological

pp and pγ ν-fluxes are too high. The "pp" upper limit is that given in Figs. 4 and 5 marked PCR(UL) (see Section 3). The pγ upper limit is not related to the X-ray background as proposed by Berezinsky and Smirnov (1975) because the X-ray background cannot be related to photopion production as originally proposed by Strong et al. (1973) since the shape of the γ-ray background spectrum above 10 MeV has the form of $\sim E^{-3}$ whereas the model of Strong et al would give a spectrum of the form $\sim E^{-2}$. Furthermore, the Hillas (1967) model on which these cosmological calculations are based, implies a γ-ray background flux above that observed unless the mean intergalactic gas density at present is $\leq 10^{-10} cm^{-3}$ (Stecker 1971, 1975)[1], well below that implied by recent x-ray observations of galactic clusters, and unless galaxy formation occurred at an extremely efficient rate at redshifts $z \geq 15$, an assumption which creates problems of compatibility with radio and optical observations (Rees 1972).

The flux of ν's from other normal galaxies is estimated here in Fig. 5 and discussed in Section 3. Further discussion of possible extragalactic ν-fluxes is given by Silberberg and Shapiro (1977).

[1]The earlier limit of $10^{-9} cm^{-3}$ was obtained using older data. The presently given limit of $10^{-10} cm^{-3}$ is obtained using the results recently given by Fichtel et al (1978).

References

Andrews, D., Edge, D. M., Evans, A. C., Reid, R. J. O. and Tennent, A. M. 1971. Proc. 12th Int'l. Conf. on Cosmic Rays, 993.

Barish, B. C. 1978. Phys. Rpts. 39C, 279.

Barnett, R. M. 1976. Phys. Rev. Lett. 36, 1163.

Berezinsky, V. S. and Smirnov, A. Yu. 1975. Ap. and Space Sci. 32, 461.

Berezinsky, V. S. and Zatsepin, G. T. 1970. Sov. J. Nuc. Phys. 11, 111.

Berezinsky, V. S. and Zatsepin, G. T. 1977. Sov. Phys. Usp. 20, 361.

Bjorken, J. D. and Paschos, E. A. 1970. Phys. Rev. D1, 3151.

Eichler, D. 1978. Ap. J. 222, 1109.

Eichten, T., et al. 1973a. Phys. Lett. 46B, 274.

Eichten, T., et al. 1973b. Phys. Lett. 46B, 281.

Felten, J. E. 1977. Astron. J. 82, 861.

Fichtel, C. E., Simpson, G. A. and Thompson, D. J. 1978. Ap. J. 222, 833.

Ganguli, S. N. and Sreekantan, B. V. 1976. J. Phys. A9, 311.

Greisen, K. 1966. Phys. Rev. Lett. 16, 748.

Hillas, A. M. 1967. Phys. Lett. 24A, 677.

Linsley, J. 1963. Phys. Rev. Lett. 9, 126.

Margolis, S. H., Schramm, D. N. and Silberberg, R. 1978. Ap. J. 221, 990.

Montmerle, T. 1977. Ap. J. 216, 620.

Osborne, J. L., Said, S. S. and Wolfendale, A. W. 1965. Proc. Phys. Soc. London 86, 93.

Rees, M. J. 1972. in External Galaxies and Quasi Stellar Objects (ed. D. E. Evans) Reidel, Dordrecht, 407.

Ryan, M. J., Ormes, J. F. and Balasubrahmanyan, V. K. 1972. Phys. Rev. Lett. 28, 1497.

Silberberg, R. and Shapiro, M. M. 1977. Proc. 15th Int'l. Cos. Ray Conf., Plovdiv 6, 237.

Stecker, F. W. 1968. Phys. Rev. Lett. 21, 1016.

Stecker, F. W. 1969. Ap. J. 157, 507.

Stecker, F. W. 1970. Ap. and Space Sci. 6, 377.

Stecker, F. W. 1971. Cosmic Gamma Rays, Mono Book Corp., Baltimore.

Stecker, F. W. 1973a. Ap. and Space Sci. 20, 47.

Stecker, F. W. 1973b. Ap. J. 185, 499.

Stecker, F. W. 1975. in Origin of Cosmic Rays (ed. J. L. Osborne and A. W. Wolfendale) Reidel Pub. Co., Dordrecht, 267.

Stecker, F. W. 1978a. Comments on Ap. 7, 129.

Stecker, F. W. 1978b. Nature 273, 4

Stecker, F. W. and Bredekamp, J. 1972. NASA Rpt. X-641-72-480.

Stecker, F. W. and Silk, J. 1969. Nature 221, 1229.

Wdowczyk, J., Tkaczyk, W., and Wolfendale, A. W. 1972. J. Phys. A5, 1419.

Weinberg, S. 1976. Rev. Mod. Phys. 46, 255.

Zatsepin, G. T. and Kuzmin, V. A. 1962. Sov. Phys. JETP 14, 1294.

Zatsepin, G. T. and Kuzmin, V. A. 1966. Zh. Eksp. Theor Fiz. (Pisma) 4, 114.

Table 1. νN Cross Section Ratios

Ratio	Experimental Value	Theoretical Value
$\sigma_{\nu_e N}/\sigma_{\nu_\mu N}$	1.26 ± 0.23	1
$\sigma_{\tilde{\nu}_e N}/\sigma_{\tilde{\nu}_\mu N}$	1.32 ± 0.32	1
$\sigma_{\tilde{\nu}_\mu N}/\sigma_{\nu_\mu N}$	0.40 ± 0.12	$\sim 1/3$*
$\sigma_{\tilde{\nu}_e N}/\sigma_{\nu_e N}$	0.38 ± 0.02	$\sim 1/3$*

*valence quarks dominant

(Eichten et al. 1973a, b).

Figure Captions

Fig. 1. Differential production spectra of neutrinos and γ-rays from the decay of pions produced by interactions of cosmic-rays in our galactic neighborhood with interstellar gas having a mean hydrogen density of 1 atom per cm^3. The γ-ray curve and the upper neutrino curve are calculated for cosmic rays having a spectral index of 2.67 between 10 and 3×10^6 GeV; The lower neutrino curve is for a cosmic ray spectrum with index 2.75. The spread in the curve is indicative of the uncertainty in such calculations.

Fig. 2. Calculated differential neutrino production spectra and background fluxes from photopion production by ultrahigh energy cosmic rays interacting with the 3K universal microwave background radiation.

Fig. 3. The integral ν-production spectrum obtained from the upper neutrino curve in Fig. 1. The high energy cosmic ray spectrum used is from a recent analysis of Hillas (private communication).

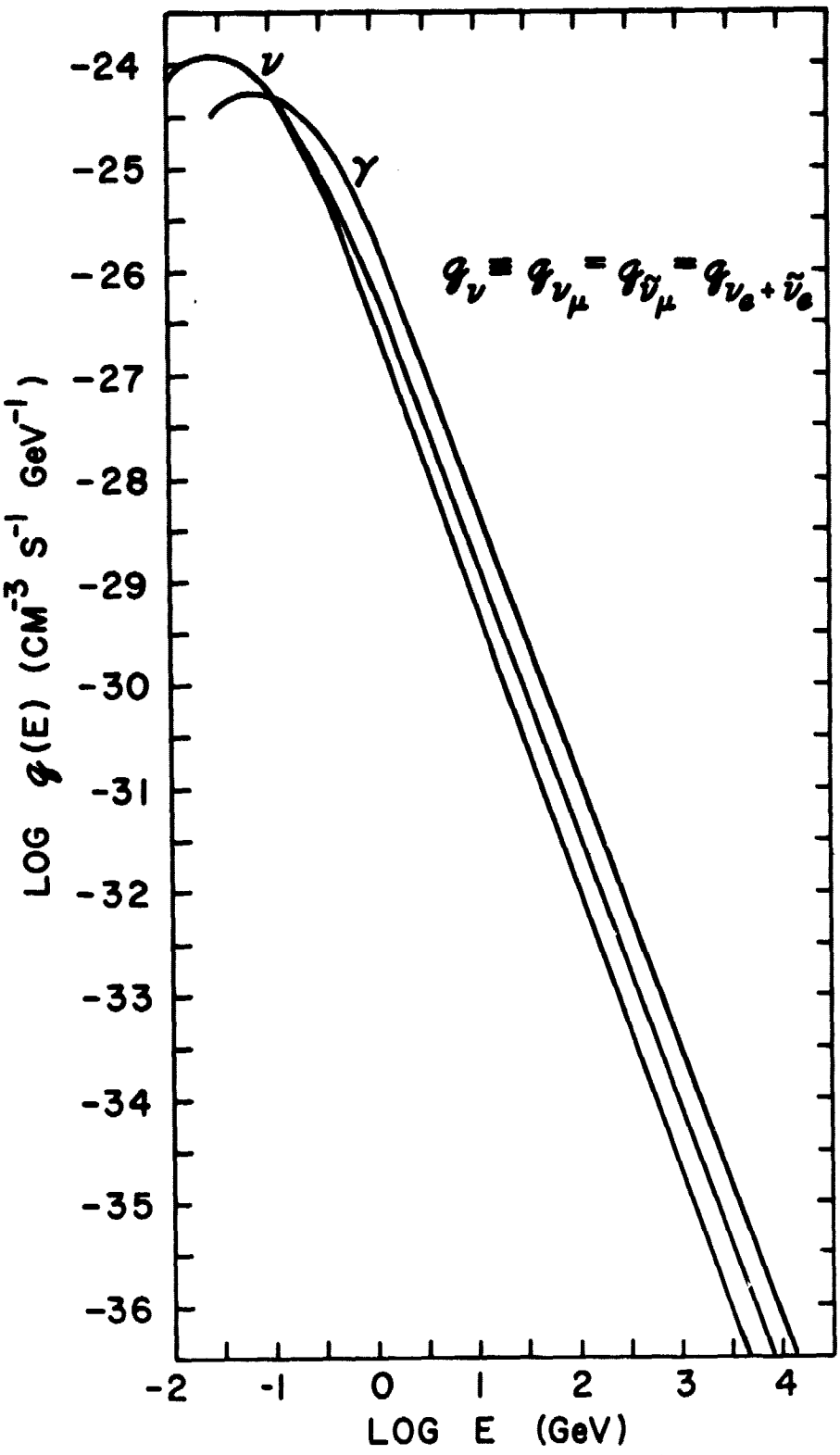
Fig. 4. Differential background ν-fluxes.

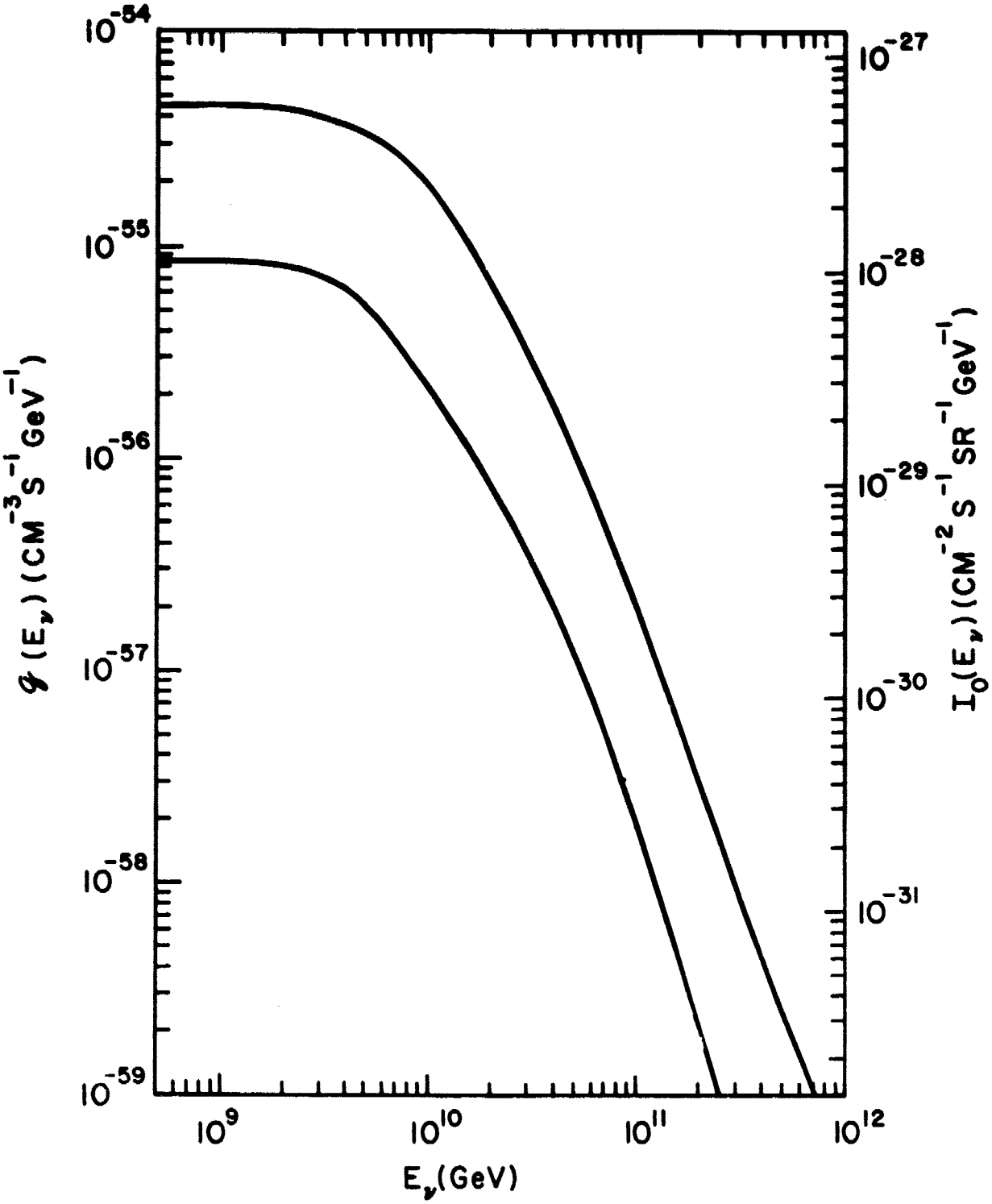
Fig. 5. Integral ν-fluxes and the galactic latitude distribution of neutrinos with energy above 1 TeV in the direction of the inner galaxy.

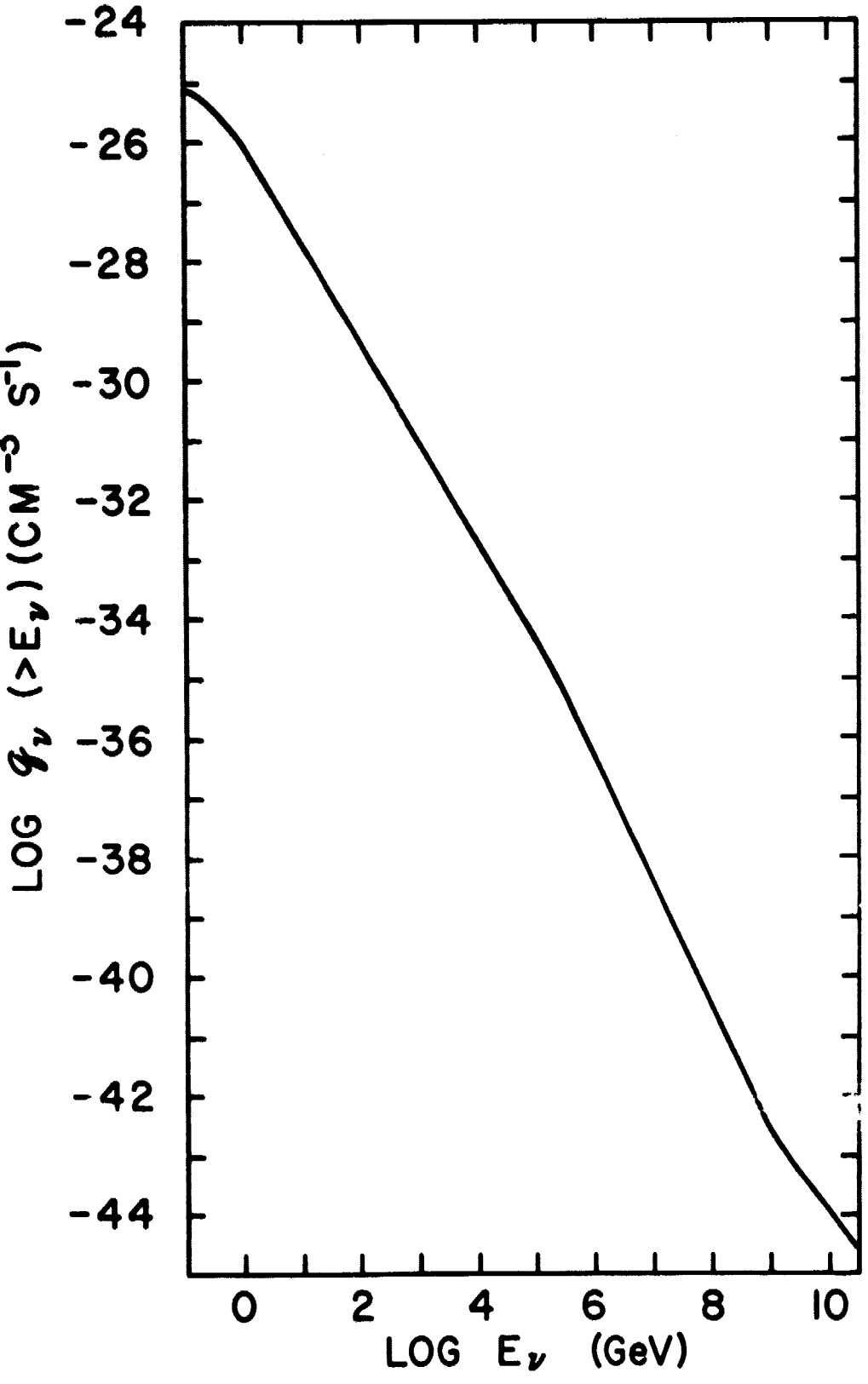
Fig. 6. Cross section for νN interactions assuming an intermediate vector boson mass $M_W = 70$ GeV.

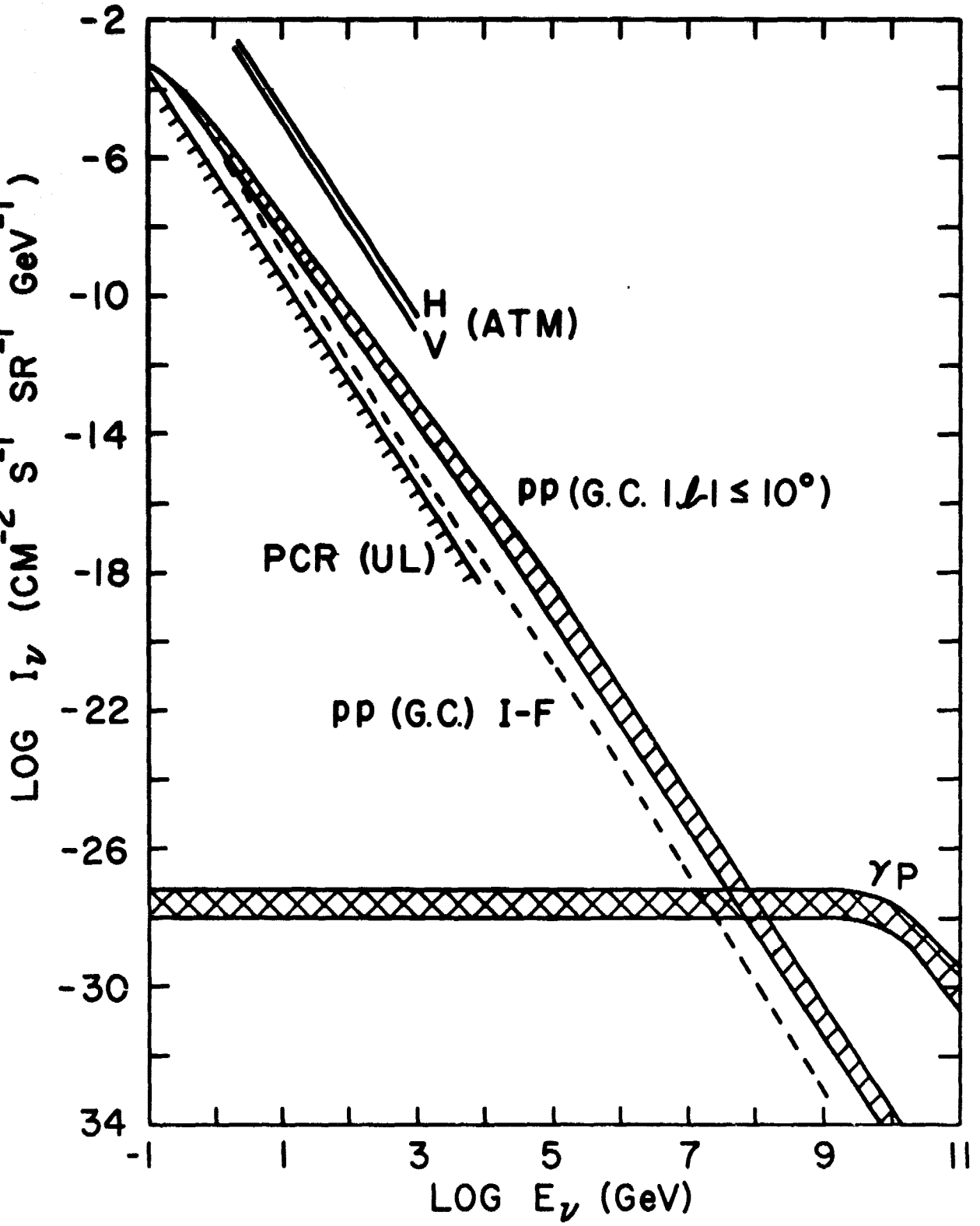
Fig. 7. Neutrino event rates for a detector utilizing 1 km^3 of water.

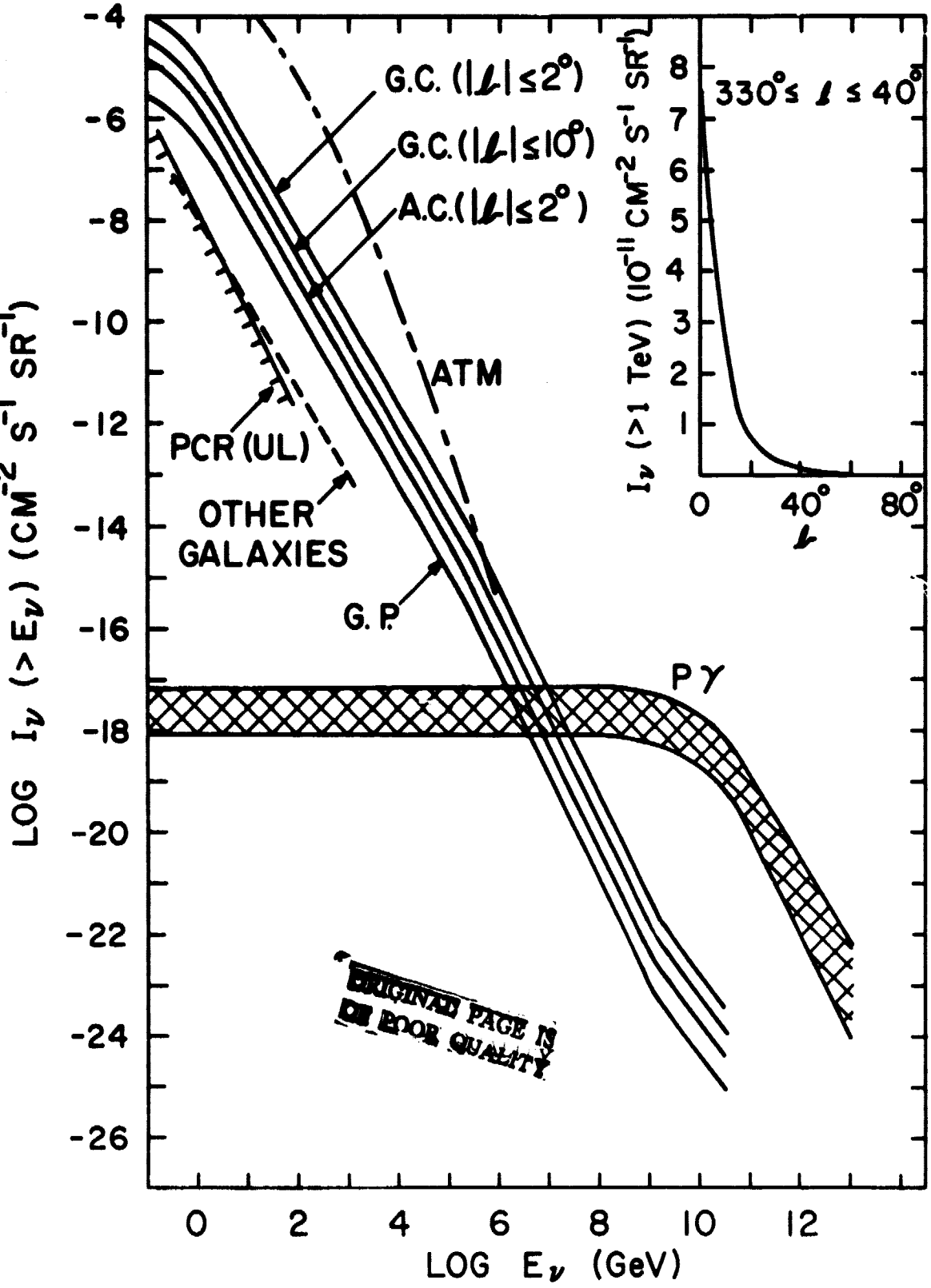
Fig. 8. Comparison of present predicted neutrino fluxes with previous calculations by other workers.

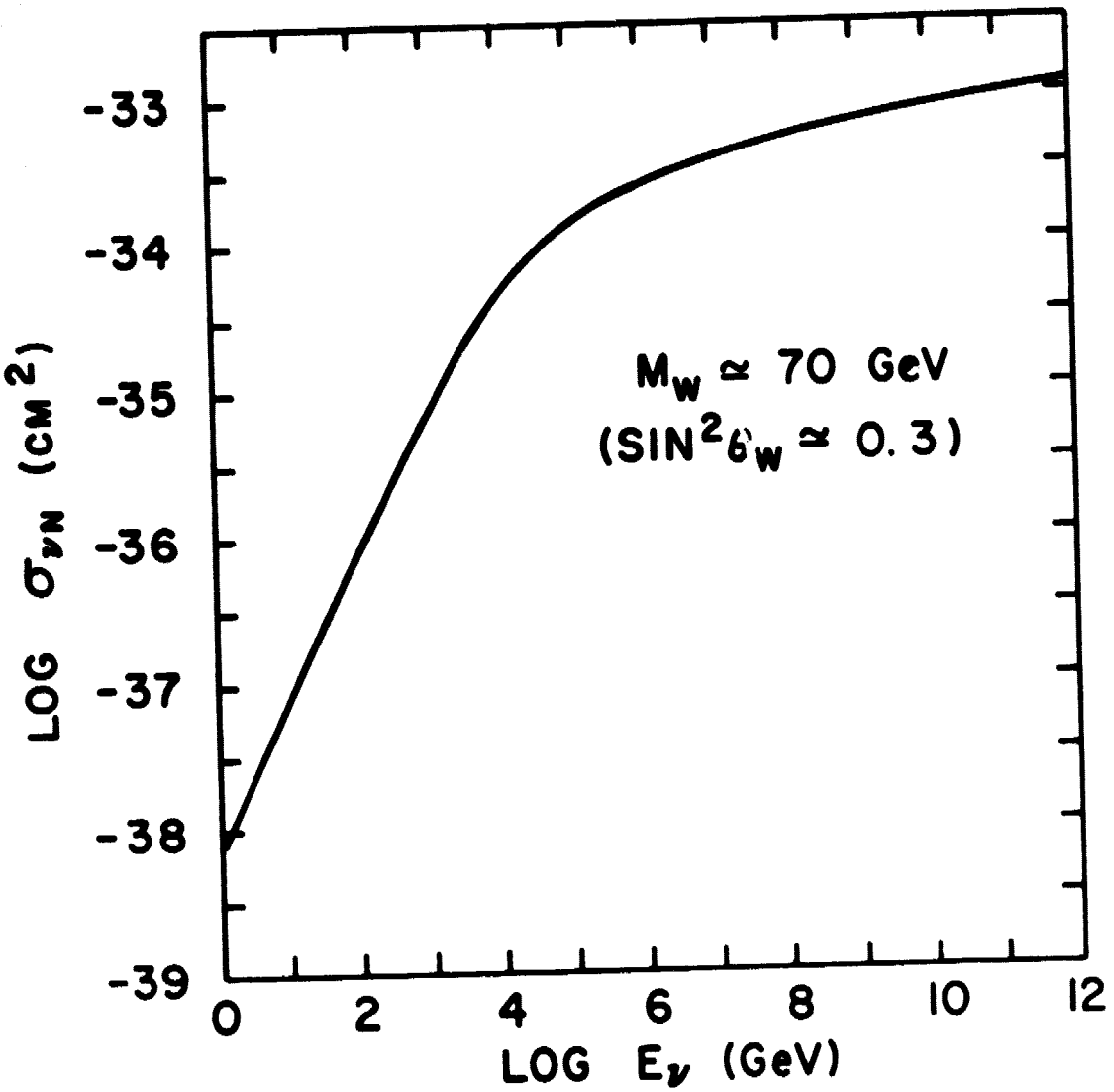


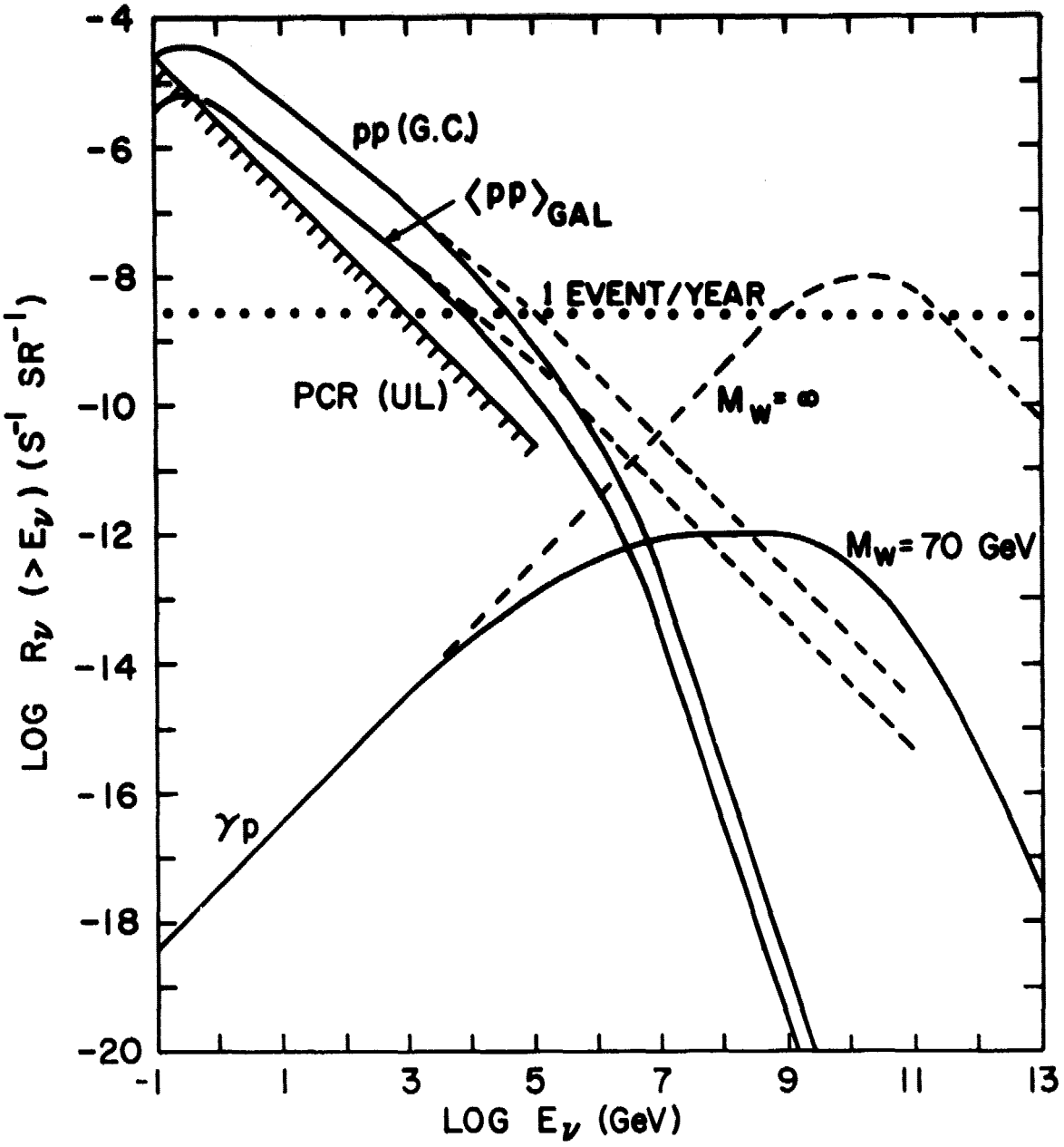

10^{-54}
10^{-55}
10^{-56}
10^{-57}
10^{-58}
10^{-59}
$\phi(E_\nu)$ $(CM^{-3}S^{-1}GeV^{-1})$
10^{-27}
10^{-28}
10^{-29}
10^{-30}
10^{-31}
$I_0(E_\nu)(CM^{-2}S^{-1}SR^{-1}GeV^{-1})$
10^9
10^{10}
10^{11}
10^{12}
E_ν(GeV)

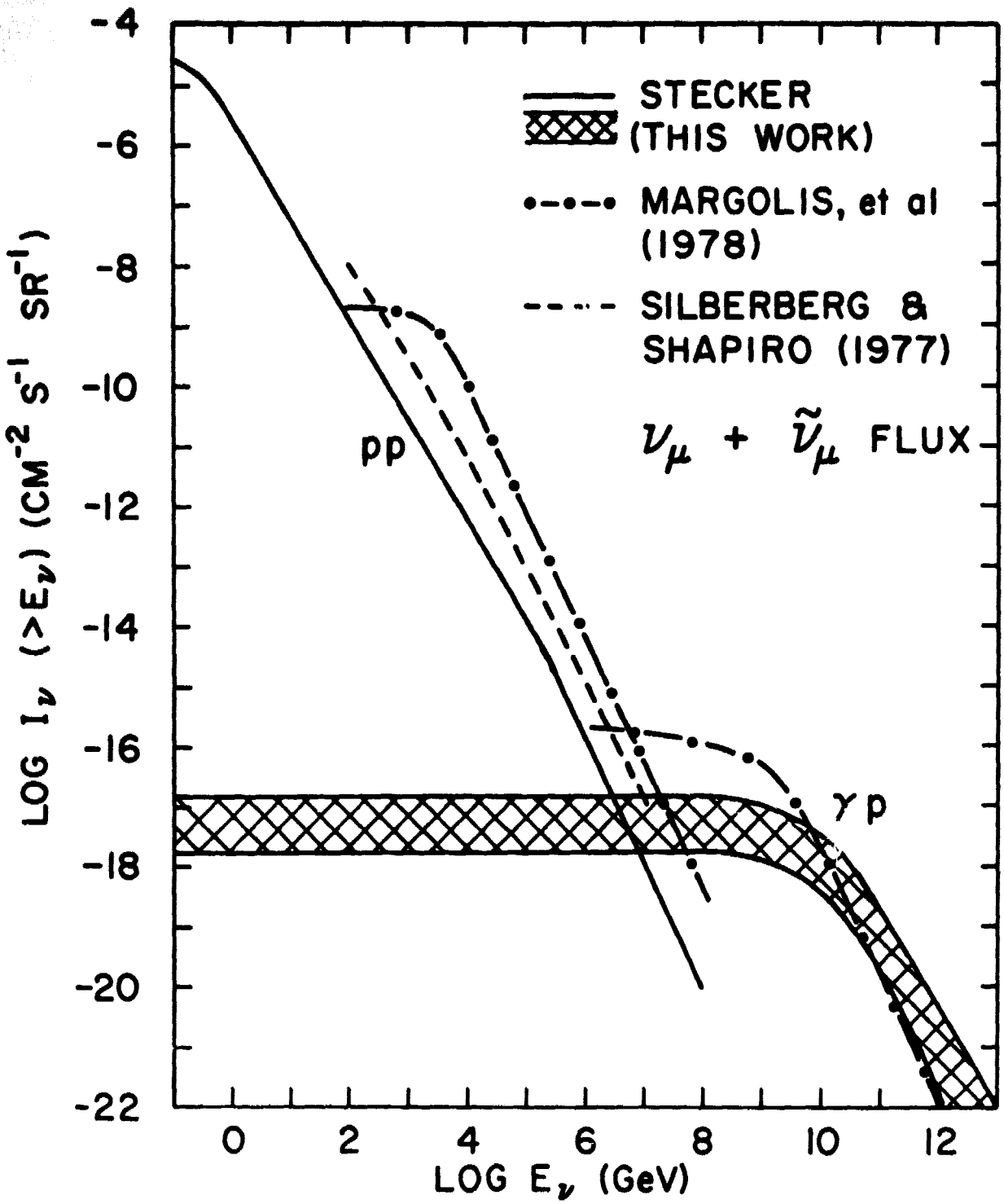

-24
-26
-28
-30
-32
-34
-36
-38
-40
-42
-44
LOG ϕ_ν (>E_ν) (CM^{-3} S^{-1})
0
2
4
6
8
10
LOG E_ν (GeV)


-2
-6
-10
-14
-18
-22
-26
-30
34
-1
1
3
5
7
9
11
LOG I_ν (CM^{-2} S^{-1} SR^{-1} GeV^{-1})
LOG E_ν (GeV)
ν_H (ATM)
PP (G.C. $|l| \leq 10°$)
PCR (UL)
PP (G.C.) I-F
γP


G.C. (|l|≤2°)
G.C.(|l|≤10°)
A.C.(|l|≤2°)
ATM
PCR (UL)
OTHER GALAXIES
G.P.
Pγ
330°≤ l ≤40°
Iν (>1 TeV) (10⁻¹¹ CM⁻² S⁻¹ SR⁻¹)
l
LOG Iν (>Eν) (CM⁻² S⁻¹ SR⁻¹)
LOG Eν (GeV)
ORIGINAL PAGE IS OF POOR QUALITY


-33
-34
-35
-36
-37
-38
-39
LOG $\sigma_{\nu N}$ (CM^2)
0
2
4
6
8
10
12
LOG E_ν (GeV)
$M_W \simeq 70$ GeV
$(SIN^2\theta_W \simeq 0.3)$


pp (G.C.)
⟨pp⟩GAL
1 EVENT/YEAR
PCR (UL)
Mw = ∞
Mw = 70 GeV
γp
LOG Rν (>Eν) (S⁻¹ SR⁻¹)
LOG Eν (GeV)
-4
-6
-8
-10
-12
-14
-16
-18
-20
-1
1
3
5
7
9
11
13


STECKER
(THIS WORK)
MARGOLIS, et al
(1978)
SILBERBERG &
SHAPIRO (1977)
$\nu_\mu + \tilde{\nu}_\mu$ FLUX
pp
γp
LOG I_ν (>E_ν) (CM^{-2} S^{-1} SR^{-1})
LOG E_ν (GeV)
-4
-6
-8
-10
-12
-14
-16
-18
-20
-22
0
2
4
6
8
10
12

Appendix I

High Energy Behavior of the Pion-Decay γ-ray and Neutrino Spectrum

Let us assume a high energy primary cosmic-ray spectrum of the power-law form

$$I_p(E_p) = K E_p^{-\Gamma} \tag{A1}$$

interacting with cosmic gas having a density n and producing secondaries observed along a line-of-sight of effective length L.

We further assume an effective production cross section for pions of average energy E_π produced by primaries of energy E_p

$$\sigma(E_\pi|E_p) = C\,\zeta_o E_p^{\,a}\,\delta(E_\pi - X_o E_p^{\,b}) \tag{A2}$$

where ζ_o and X_o are constants. Such a production function would occur for a pion multiplicity ζ of the form $\zeta(E_p) = \zeta_o E_p^{\,a}$. The secondary pion spectrum is proportional to the integral over dE_p of the product $\sigma(E_\pi|E_p)I(E_p)$.

In the high energy limit, γ-rays and neutrinos from pion decay would then have a power law form with a spectral index

$$\Gamma' = \frac{(\Gamma+b)-(a+1)}{b} \tag{A3}$$

In the case of γ-rays, the spectrum would be

$$I(E_\gamma) = \frac{2\langle nL\rangle K\zeta_o C}{(\Gamma+b)-(a+1)}\, X_o^{\left(\frac{\Gamma-a-1}{b}\right)} E_\gamma^{-[(\Gamma+b)-(a+1)]/b} \tag{A4}$$

(Stecker 1971). The corresponding expression for neutrinos is somewhat more complicated.

Again, in the high energy limit, some constant fraction of the primary energy E_p goes into pions. This fraction is typically of the

order of 50%-60%, but in any case, it cannot exceed unity. It then follows that

$$b = 1-a, \quad 0 \le a \le 1, \quad 0 \le b \le 1. \tag{A5}$$

Thus, equation (A3) reduces to

$$\Gamma' = \frac{\Gamma-2a}{1-a} = \Gamma + \frac{a}{1-a}(\Gamma-2) \tag{A6}$$

In order for the total energy in cosmic rays to be finite, in the high energy limit Γ must be >2. In fact, observationally this is the case. Also, since $0 \le a \le 1$, it follows that

$$\frac{a}{1-a}(\Gamma-2) \ge 0, \text{ thus } \Gamma' \ge \Gamma \tag{A7}$$

i.e., the secondary spectrum has a spectral index which exceeds or equals that of the primaries.

In the case $a = 0$, $b = 1$ corresponding to the dominance of a leading pion or scaling, we find $\Gamma' = \Gamma$ and the spectral index of the secondaries equals that of the primaries. It follows from eq. (A7) that the spectral index of the secondaries should not be smaller than that of the primaries as Margolis, et al. obtain in their scaling model calculation for the energy range 10^2-10^4 GeV.

It is well known that if π° mesons are generated with a spectrum $g(E_\pi)$, the γ-ray spectrum from their decay has the form

$$g(E_\gamma) = \int_{E_\gamma + \frac{m_\pi^2}{4E_\gamma}}^{\infty} dE_\pi \frac{g(E_\pi)}{(E_\pi^2 - m_\pi^2)^{1/2}} \to \int_{E_\gamma}^{\infty} dE_\pi \frac{g(E_\pi)}{E_\pi} \tag{A8}$$

(e.g. Stecker 1971) in the high energy limit $E_\gamma \gg m_\pi$. Thus if $g(E_\pi) =$

$K_\pi E_\pi^{-\Gamma_\pi}$, $g(E_\gamma) = K_\gamma E_\gamma^{-\Gamma_\gamma}$

where $K_\gamma = K_\pi/\Gamma_\pi$, $\Gamma_\gamma = \Gamma_\pi$. (A9)

It can also be easily demonstrated that the neutrinos from $\pi \to \mu\nu$ decay also have the same spectral index as the primaries. In this case, the cms energy of the neutrino is

$$E_\nu^* = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} = 29.8 \text{ MeV}. \quad \text{(A10)}$$

Defining $\eta \equiv E_\nu^*/m_\pi$, it follows that in the limit $E_\nu \gg m_\pi$

$$g(E_\nu) = \frac{1}{2\eta} \int_{E_\nu/2\eta}^{\infty} dE_\pi \frac{g(E_\pi)}{E_\pi} = \frac{1}{2\eta} \int_{2.34E_\nu}^{\infty} dE_\pi \frac{g(E_\pi)}{E_\pi} . \quad \text{(A11)}$$

Again, for a power law pion spectrum of the form $g(E_\pi) = K_\pi E_\pi^{-\Gamma_\pi}$, the neutrino spectrum is of the form $g(E_\nu) = K_\nu E_\nu^{-\Gamma_\nu}$ where

$$K_\nu = \frac{(2.34)^{-(\Gamma_\pi - 1)}}{\Gamma_\pi} K_\pi, \quad \Gamma_\nu = \Gamma_\pi. \quad \text{(A12)}$$

The $\pi \to \mu\nu$ decay kinematics is similar to the $\Sigma^0 \to \Lambda\gamma$ decay kinematics discussed in Stecker (1971). The 3-body $\mu \to e\nu\nu$ decay is more complicated but the result is also $\Gamma_\nu = \Gamma_\pi$ (see again, Stecker 1971).

Appendix II

Approximate γp Neutrino Source Function

The main source of ultrahigh energy neutrinos is photomeson production interactions between ultrahigh energy cosmic rays and blackbody photons of the 2.7K microwave background radiation - i.e., interactions of the type $\gamma + p \rightarrow \pi + n$. To check on the numerical calculations presented in this paper, it is useful to derive an approximate source function for the production of neutrinos in photomeson interactions with microwave blackbody photons. We assume all the photons to be at the average energy $\varepsilon_0 \simeq 2.7kT \simeq 6.4\times10^{-4}$ eV so that

$$n_{bb}(\varepsilon) = n_{bb}\delta(\varepsilon - \varepsilon_0), \qquad n_{bb} \simeq 400\,\mathrm{cm}^{-3}. \tag{B1}$$

The energy of the photon in the cosmic-ray proton rest system is

$$\varepsilon' = (E_p/M_p)\varepsilon_0(1-\cos\theta). \tag{B2}$$

There is a large peak in the photomeson production cross-section at $\varepsilon' \simeq 0.35\ M_p$ due to the $\Delta(1238)$ resonance and since the cosmic-ray spectrum drops off rapidly with increasing energy, most of the pion production occurs at this resonance energy. Thus, from Equation (B2) we make the approximation

$$\sigma(\varepsilon') \simeq \sigma_0\delta\left[\chi - \frac{0.35\ M_p{}^2}{\varepsilon_0 E_p}\right] \equiv \sigma_0\delta(\chi - E_0/E_p), \tag{B3}$$

where

$$\chi \equiv 1 - \cos\theta, \quad E_0 \equiv \frac{0.35\ M_p{}^2}{\varepsilon_0} \simeq 4.8\times10^{11}\,\mathrm{GeV}$$

and

$$\sigma_0 \simeq 2\times10^{-28}\,\mathrm{cm}^2.$$

Then the source function for neutrino production from $\pi \to \mu\nu$ decay may be written in the form

$$q(E_\nu) = 4\pi/2\eta \int_{E_\nu/2\eta}^{\infty} \frac{dE_\pi}{E_\pi} \; n_{bb} \int_0^2 x \, dx \int_0^x dE_p I(E_p)\sigma(E_p,x) f(E_\pi|E_p).$$

We assume all the pions to be produced at the average energy

$$<E_\pi> = \frac{E_p m_\pi^2 + 2\varepsilon' M_p}{2 M_p^2 + 2\varepsilon' M_p}. \tag{B5}$$

If we use Equation (B4), Equation (B5) further reduces to

$$<E_\pi> \simeq \frac{E_p^2}{2} \; \frac{m_\pi^2 + 0.7 M_p^2}{1.7 M_p^2} \simeq E_p/5, \tag{B6}$$

so that the distribution function is approximated by

$$f(E_\pi|E_p) \simeq \delta(E_\pi - E_p/5). \tag{B7}$$

If we further specify the differential cosmic-ray proton spectrum by a power-law of the form $I(E_p) = K_p E_p^{-\Gamma}$, Equation (B4) reduces to

$$q(E_\nu) \simeq 2\pi n_{bb}\sigma_o K_p/\eta \int_{E_\nu/2\eta}^{\infty} dE_\pi/E_\pi \int_0^2 x dx \int_{(E_o/2)}^{\infty} dE_p E_p^{-\Gamma} \delta(x - E_o/E_p)\delta(E_\pi - E_p/5)$$

$$= 10\pi\sigma_o n_{bb} K_p E_o/\eta \int_{\max[E_o/2, 5E_\nu/2\eta]}^{\infty} dE \; E^{-(\Gamma+2)} \tag{B8}$$

The solution to Equation (B8) may then be written in the simple form

$$q(E_\nu) = \begin{cases} Q_\nu, & E_\nu \le E_1 \\ Q_\nu (E_\nu/E_1)^{-(\Gamma+1)}, & E_\nu \ge E_1, \end{cases} \tag{B9}$$

where $E_1 \equiv 5E_o/\eta \simeq 2\times10^{10}$ GeV and

$$Q_\nu \simeq 2.35\times10^{-23} K_p (E_o/2)^{-\Gamma}/(\Gamma+1) \tag{B10}$$

For example, if $I(E_p) = 2.5 \times 10^{-2} E_p^{-2.6}$ (Linsley 1963)

$$Q_\nu = 4.2 \times 10^{-55} \text{cm}^{-3} \text{s}^{-1} \text{GeV}^{-1}. \quad \text{(B11)}$$

Equations (B9) and (B11) may be directly compared with the upper curve in figure 2 as a check on the order-of-magnitude of the calculations since our numerical results differ significantly from those of Margolis et al. (1978). The numerical results shown in Fig. 2 include the effect of meson production above the $\Delta(1236)$ resonance. This has the effect of producing a flatter spectrum than $E_\nu^{-(\Gamma+1)}$ at the highest energies, one more like $E^{-\Gamma}$. (Please note that eqs. (8)-(10) of Stecker (1973) contain an error in the spectral index but the effect on the results of that paper are not significant as indicated by the more accurate numerical results of Stecker and Bredekamp (1972)).