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Gravitational Spectra from Direct Measurements

C. A. Wagner and O. L. Colombo

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ABSTRACT

A simple rapid method is described for determining the spectrum of a surface field (in spherical harmonics) from harmonic analysis of direct (in situ) measurements along great circle arcs. The method is shown to give excellent overall trends (smoothed spectra) to very high degree from even a few short arcs of satellite data. Three examples are taken with perfect measurements of satellite tracking over a planet made up of hundreds of point-masses using (1) altimetric heights from a low orbiting spacecraft, (2) velocity (range rate) residuals between a low and a high satellite in circular orbits, and (3) range-rate data between a station at infinity and a satellite in a highly eccentric orbit.

In particular, the smoothed spectrum of the Earth's gravitational field is determined to about degree 400(50 km half wavelength) from 1° × 1° gravimetry and the equivalent of 11 revolutions of Geos 3 and Skylab altimetry. This measurement shows there is about 46 cm of geoid height (rms world-wide) remaining in the field beyond degree 180.

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INTRODUCTION

The spectrum of a two-dimensional field in spherical harmonics is conveniently expressed by the so-called degree variances of its fluctuations. We demonstrate here that the trend of the spectrum is far easier to find than the actual field. For example the anomalous gravitational potential external to a planet can be written as:

$$T(r, \phi, \lambda) = \frac{\mu}{r} \sum_{\ell=2}^{\infty} \sum_{m=0}^{\infty} \left(\frac{R}{r}\right)^{\ell} \bar{P}_{\ell m}(\sin \phi) \left\{ \bar{C}_{\ell m} \cos m\lambda + \bar{S}_{\ell m} \sin m\lambda \right\}, \qquad (1)$$

where r is the distance to the center of mass, R is the radius of the smallest sphere enclosing all the mass, ϕ and λ are planet centered latitude and longitude, μ is the planet's gaussian mass, $\bar{P}_{\ell m}$ are fully normalized associated legendre polynomials (Heiskanen and Moritz, 1967, p. 31) and $\bar{C}_{\ell m}$, $\bar{S}_{\ell m}$ are their associated coefficients. The potential degree variances are:

$$\sigma_{\ell}^2 = \sum_{m=0}^{\ell} \left(\bar{C}_{\ell m}^2 + \bar{S}_{\ell m}^2 \right). \qquad (2)$$

We note that this power spectrum (and others simply related to it) is the same for any choice of reference pole for the field. Potential fluctuations on any surface great circle always has maximum wave number ℓ with respect to the $(2\ell + 1)$ terms of that 'line' of the spectrum.

The internal density source for both the external field or its summary spectrum is of course not unique. But the mass integral representation of the field coefficients together with other physical evidence puts important constraints on the source spectrum. From Hotine, 1969, p. 159:

$$(\overline{C}_{\ell m}, \overline{S}_{\ell m}) = \frac{(3/4\pi R^3)}{\overline{\rho}(2\ell + 1)} \int_{r=0}^{R} (r/R)^{\ell}\, dr \int_{\lambda,\phi} \rho \overline{P}_{\ell m}(\sin\phi) \left\{\cos m\lambda, \sin m\lambda\right\} dA, \quad (3)$$

where dA is an area element, ρ is the density and $\overline{\rho}$ is the Earth's average density. For example, the severe radius factor in (3) means that the deep density anomalies can have little affect on the high degree field if they are of reasonable size (e.g., Jeffreys, 1962) or are not to give rise to unreasonably large shearing stresses (e.g., Higbie and Stacey, 1971a, b).

Kaula (1966, p. 98) first gave the simplest expression of the Earth's gravitational spectrum (from autocovariance analysis of gravimetry):

$$\sigma_\ell^2 = (2\ell + 1)(10^{-5}/\ell^2)^2,$$

a result which we will later show is a fair overall approximation, though too high up to about degree 60.

In recent years many attempts have been made to interpret the source of this spectrum using mineralogic, seismologic, thermal as well as stress evidence (e.g., Kaula, 1963; Higbie and Stacey, 1971b; Bott, 1971; Allan, 1972; Lambeck, 1976; Dziewonski, 1977). Kaula, 1977 concludes that the smoothness of the spectrum is probably as significant as its general form. (This smoothness may be due to 'dynamic' compensation at depth as a result of convective processes.)

Our goal here is more modest; to show that simple harmonic analysis of 'central' arcs of 'field' information will rapidly elucidate the spectrum. Our method complements Kaula's (1959) classical technique of autocovariant analysis (see also Heiskanen and Moritz, 1967, pp. 252-259; Tscherning and Rapp, 1974). But the new method does not require global information and is more rapid (with access to the fast Fourier transform).

METHOD

We observe from (1) that harmonic analysis of potential data on the equator or any latitude circle of a spherical surface will yield Fourier coefficients [for wave numbers n = m]

$$(T_{n,\cos}, T_{n,\sin}) = \left(\frac{\mu}{r}\right)\sum_{\ell=n}^{\infty}\left(\frac{R}{r}\right)^{\ell} \bar{P}_{\ell n}(\sin\phi)\left\{\bar{C}_{\ell n}, \bar{S}_{\ell n}\right\} \tag{4}$$

The expected value of the one dimensional power spectrum for this circle data is

$$E(T^2_{n,\cos} + T^2_{n,\sin}) = \left(\frac{\mu}{r}\right)^2\sum_{\ell=n}^{\infty}\left(\frac{R}{r}\right)^{2\ell} \bar{P}^2_{\ell n}(\sin\phi)\left\{E\bar{C}^2_{\ell n} + E\bar{S}^2_{\ell n}\right\}, \tag{5}$$

assuming that the field coefficients are chosen from an uncorrelated population. But this is a weak assumption and in any case we really seek the average power spectrum for a given field over all possible great circles (or small circles of the same latitude). Consider the case for great circle data. We assert (without proof here) that:

$$<(T^2_{n,\cos} + T^2_{n,\sin})> = 2\left(\frac{\mu}{r}\right)^2 \sum_{\ell=n}^{\infty\,(\ell-n\ \text{even})}\left(\frac{R}{r}\right)^{2\ell} \bar{P}^2_{\ell n}(o)\,\delta^2_{\ell}, \tag{6}$$

where

$$\delta^2_{\ell} = \frac{\sigma^2_{\ell}}{(2\ell+1)},$$

the mean square coefficient by degree.

Note that for coefficients chosen from an uncorrelated population with $E\bar{C}^2_{\ell m} = E\bar{S}^2_{\ell m} = \delta^2_{\ell}$, (6) follows from (5) because the degree variance of an harmonic field is invariant under rotation. We choose the equator to evaluate the associated legendre polynomials because of the simplicity and stability of the formulas for them there. For example, from Abramowitz and Stegun, 1964, we find:

$$\bar{P}^2_{\ell+2,n}(o) = \left[\frac{(\ell-n+1)(\ell+n+1)(2\ell+5)}{(\ell-n+2)(\ell+n+2)}\right]\bar{P}^2_{\ell,n}(o) \qquad (7)$$

Using the spherical approximation of Bruns' formula [Heiskanen and Mortiz, 1967, pp. 84-85] the average great circle geoid height (N) spectrum is:

$$<(N^2_{n,\cos} + N^2_{n,\sin})> = 2R \sum_{\ell=n}^{\infty(\ell-n\ \text{even})} \bar{P}^2_{\ell n}(o)\, \delta^2_\ell \ . \qquad (8)$$

Since the gravity anomaly field at the surface of the Earth is given by [e.g., Heiskanen and Moritz, 1967, p. 89]:

$$\Delta g \doteq \gamma \sum_{\ell=2}^{\infty} \sum_{m=0}^{\ell} (\ell-1)\, \bar{P}_{\ell m}(\sin\phi) \left\{ \bar{C}_{\ell m} \cos m\lambda + \bar{S}_{\ell m} \sin m\lambda \right\},$$

the average of the one dimensional power spectra for all great circle arcs of gravity anomaly data is:

$$<(\Delta g^2_{n,\cos} + \Delta g^2_{n,\sin})> = 2\gamma^2 \sum_{\ell=n}^{\infty(\ell-n\ \text{even})} \bar{P}^2_{\ell n}(o)\,(\ell-1)^2\, \delta^2_\ell , \qquad (9)$$

where

$$\gamma = \frac{\mu}{R^2}.$$

Similarly it is easily shown [from the radial derivative of (1) and comparison with (6)] that the average spectrum for anomalous radial acceleration external to the sphere is:

$$<(\Delta\ddot{r}^2_{n,\cos}, \Delta\ddot{r}^2_{n,\sin})> = 2\gamma^2 \sum_{\ell=n}^{\infty (\ell-n \text{ even})} \left(\frac{R}{r}\right)^{2(\ell+2)} (\ell+1)^2 \bar{P}^2_{\ell n}(o)\, \delta^2_\ell . \qquad (10)$$

Of more general interest (than the radial) is the anomalous line-of-sight acceleration with respect to a fixed external direction. For example the (residual) tracking data returned from space probes or through high-low satellite-satellite exchange contains this kind of information. The line-of-sight of the signal actually changes slowly during the tracking of the probe or low satellite due to the planet's rotation or the fact that the high satellite is not at infinity. But these defects actually enhance the averaging of aspects necessary to achieve good global results.

The gravitational acceleration in the direction of the planet's spin axis (Z) provides a convenient starting point for the "fixed" direction spectrum. From Hotine (1969, p. 180, 21.140/141) we find these accelerations are:

$$A_z = \frac{\mu}{r^2} \sum_{\ell=0}^{\infty} \sum_{m=0}^{\ell} \left(\frac{R}{r}\right)^{\ell} \bar{P}_{\ell m}(\sin\phi) \left[\bar{a}_{\ell,m} \cos m\lambda + \bar{b}_{\ell,m} \sin m\lambda \right] , \qquad (11)$$

where

$$(\bar{a}_{\ell m}, \bar{b}_{\ell m}) = \left(\frac{R}{r}\right)^{-1} \left[(2\ell-1)(\ell^2 - m^2)(2\ell+1)^{-1} \right]^{1/2} (\bar{C}_{\ell-1,m}, \bar{S}_{\ell-1,m}) \qquad (12)$$

Again, the average of all great-circle spectra of this (single direction) field can be expressed simply in terms of the mean square coefficient by degree for $(\bar{a}, \bar{b})$ as in (6), (9) and (10). Furthermore, we can show from (12) that this variance is:

$$\delta^2_\ell(\bar{a}, \bar{b}) = \left(\frac{R}{r}\right)^{-2} \left[\frac{\ell(2\ell-1)^2}{3(2\ell+1)} \right] \delta^2_{\ell-1} \qquad (13)$$

Since $\delta^2_{\ell-1}$ is invariant for any position of the z axis, the average of every fixed direction (D) spectrum is the same. We find:

$$<(\ddot{D}^2_{n,\cos} + \ddot{D}^2_{n,\sin})> = 2\gamma^2 \sum_{\ell=n}^{\infty (\ell-n \text{ even})} \left(\frac{R}{r}\right)^{2(\ell+1)} \left[\frac{(2\ell-1)^2 \ell}{3(2\ell+1)}\right] \overline{P}^2_{\ell,n}(o)\, \delta^2_{\ell-1} \cdot \quad (14)$$

Comparing (14) to (10) the average fixed direction spectrum of accelerations is no less than 2/3 of the radial for realistic smooth gravitational spectra such as Kaula's rule. However, if the probe is observed entirely cross track (normal to its orbit plane) the acceleration spectrum is weaker still since the leading tesseral harmonic is absent. In this case only, (12) gives the average cross track (CT) spectrum directly (for all "equatorial" great circle tracks observed from a pole; $\phi = 0$):

$$<(\ddot{CT}^2_{n,\cos} + \ddot{CT}^2_{n,\sin})> = 2\gamma^2 \sum_{\ell=n+2}^{\infty (\ell-n \text{ even})} \left(\frac{R}{r}\right)^{2(\ell+1)} [(2\ell-1)(\ell^2-n^2)(2\ell+1)^{-1}]\, \overline{P}^2_{\ell,n}(o)\, \delta^2_{\ell-1} \cdot (15)$$

Finally, the data type most frequently encountered in space tracking is range rate or relative velocity between the observer and the probe. Since acceleration (A) is the derivative of velocity (V) we have used the harmonics of tracks of V data to estimate the corresponding wave numbers of the A spectrum:

$$[A^2_{n,\cos} + A^2_{n,\sin}] = \left(\frac{2\pi n}{P}\right)^2 [V^2_{n,\cos} + V^2_{n,\sin}] , \qquad (16)$$

where P is the period of the probe's orbit.

Inversion

Imagine that the average circular "track" power spectra (P_n^2) are available for a certain data type. All of these are related to an infinite sum of every other degree variance of the spherical harmonic field starting at the degree equal to the wave number of the track spectra (or one less for "fixed" direction spectra).

$$P_n^2 = \sum_{\ell = n}^{\infty (\ell - n \text{ even})} S^2(\ell, n) \, [\delta_\ell^2 \text{ or } \delta_{\ell - 1}^2] \tag{17}$$

where S^2 are sensitivity factors for the various data types calculable from (8), (9), (10), (14) or (16).

In practice this infinite system is solved iteratively for the spherical harmonic variances δ_ℓ^2 from high to low degree. It starts from a sufficiently high degree beyond the significant power spectrum so that the truncation error (from assuming $\delta^2_{> \max \ell} \equiv 0$) is negligible.

In the "real world" probes do not follow great circles over planets nor do observers view them from fixed directions over complete (global arcs). But global power from non-global arcs can be estimated from well known data "windows." Furthermore, trajectories of close satellites of even rapidly rotating planets are often close enough to great circles and observing directions constant enough for this simple methodology to be useful.

SIMULATIONS

We have constructed a model (spherical) Earth, rotating with 266 randomly chosen point masses added in three layers (at 20,150 and 1000 km depth) so as to give fairly realistic degree variances (figure 1). In the first simulation, ten (1/6 revolution) arcs of perfect geoid height altimetric data (every 10 seconds) from a 200 km circular orbiting satellite (115° inclination) has been taken (figure 2). The average track-power spectrum (rms) for these arcs are given in figure 3 together with the values expected from the degree variances for this model [calculated from (3)] using (8). The fidelity of these averages is remarkable considering the field's high local roughness and its limited sampling here. The global power estimates were made from residuals to simple end-matching trend lines, using a transfer function P^2(global) = P^2(measured)/6, for the 1/6 revolution data [Wagner, 1977, Appendix C]. A smooth power law of the form $An^{B + C (\log n)}$ has been fitted to these averages and smooth mean square coefficients estimated from back-solving equation (8). The results (figure 1) confirm the overall accuracy of the method even to very high degree.

Spectral analysis has been made of four (1/2 revolution) arcs of perfect range rate data (taken every 10 seconds) between a stationary satellite at 6.61 Earth radii and the 200 km satellite (figure 2). Average velocity power at altitude falls off steeply (figure 4). Starting at about 60 cycles, the leakage or edge effect of the "rectangular window" begins to dominate the spectrum. Using residuals to quadratic trend curves which match the first derivatives as well as the data at the ends, the average power spectrum is dramatically improved to about 170 cycles. Some aliasing and leakage is evident at still higher frequencies.

Again a smooth power function has been fit to the average spectral data (using the linear trend results up to 60 cycles and the quadratic to 166 cycles). Inversion of this function [from (14) and (16)] is reasonably accurate up to about 100 cycles (figure 1). Above 100 cycles the smooth variances are systematically high, a result due to the lack of coverage of the back side of the planet.

It seems likely that unless accuracies within 10% are required, it will not be necessary to correct for Earth rotation or the 17° variation of direction to the low satellite from synchronous orbit.

Finally, we have a promising result using a single pass of simulated range rate data from a Venus-orbiter type mission. Here we tracked a highly eccentric 24-hour orbiter from a truly fixed direction normal to the orbit plane. We used the same model planet as before but without rotation (figure 2). The significant data is highly compressed (in time) around periapsis (200 km altitude) and our analysis is of a stretched out version of the data at equal intervals of true anomaly (central angle θ). Figure 5 shows the line-of-sight acceleration of this probe between $\pm 30°$ of periapsis (data every .36°). The power spectrum (using the "rectangular" window) shows high frequency leakage beginning at about 150 cycles, but is otherwise quite regular (figure 6). We have estimated the spectrum for a circular orbit at various radii along this trajectory [using (14) and (15)] but no single spectrum or average of spectra "fits" the trend of this data satisfactorily over all frequencies. However, the expected circular orbit cross track spectrum is an improvement over the corresponding general fixed direction spectrum at high frequencies ($n > 50$). Examination of trajectories at different longitudes shows that while the mid-frequency spectrum in figure 6 is indeed higher than average, no simple circular orbit spectrum can accurately represent all frequencies. Nevertheless, the circular orbit spectra are fairly good first approximations.

SMOOTH SPECTRUM OF THE GEOPOTENTIAL

We have determined the smooth spectrum of the geopotential to high degree from average track spectra using: (1) satellite altimetry over the oceans and (2) 1° X 1° global gravimetry. There is remarkable agreement between these two independent methods.

The altimetry consists of 81 arcs of sea surface heights (R. Stanley, private communications, 1977; McGoogan, Leitao and Wells, 1975) over all the world's oceans estimated by the radar altimeter aboard the Skylab IV and Geos 3 spacecrafts (figure 7). The individual heights represent averages over 15 to 20 km but a few arcs have been sampled at up to 70 km spacing (Wagner, 1977; Wagner, 1978). The original data is noisy (±15 to 50 cm) and is contaminated with low frequency orbit error (1 to 10 m; n = 0, 1, 2 cycles/rev.). The orbit error and most of the geoid signal to 20 cycles has been removed by fitting the heights to the geoid from Goddard Earth Model 7 (Wagner et al., 1977) and empirical terms with these low frequencies. Group averages of the (Gem 7) residual spectra for these arcs are shown in figure 8. The "noise" tail starting at about 750 cycles corresponds to an average "white noise" level of 21 cm.

We have also evaluated the spectrum of sea surface topography (departures from the level geoid). Topography which changes with time has been evaluated from (1): eleven Geos 3 altimeter arc overlaps (within 10 km, having 1¼ and 2½ month time separation); (2) a number of profiles over sea level air pressure maps (e.g., Strahler, 1967) interpreted as an inverted barometer on the sea surface; (3) a 20-minute Geos 3 profile in the Pacific of luni-solar tide heights (Estes, 1977); and (4) similar profiles over bi-weekly averages of topography for the East Australian current (Boland and Hamon, 1970). Actually (2) and (4) include "permanent" effects in the lower frequencies.

The "permanent" topography has been estimated from (1), four long profiles of Defant's (1961) map of the North Atlantic [in: Apel, 1976] and (2), five similar profiles over annual averages of surface topography for the Pacific (Wyrtki, 1975). Group averages for these spectra are displayed on figure 8. Note that at frequencies higher than 50 cycles the dynamic (time varying) topography dominates the departure spectrum. The altimeter-measured departure spectrum is fairly flat from 50 to 500 cycles. But above 500 cycles we have not evaluated enough data to properly distinguish it from the (noise-reduced) surface height spectrum. Using a composite average of these departure spectra and subtracting from the noise-reduced sea height spectrum, an estimated smooth geoid-track spectrum has been derived.

The inversion of the smooth geoid-track spectrum according to (8) is shown in figure 9 along with the same result from Wagner, 1978 using only about 1/2 of this data. The stability of the solution appears to be excellent. It is also confirmed by separate solutions from independent data-arc sets.

An entirely different data set, 38,000 1° X 1° mean gravity anomalies (R. H. Rapp, private communication, 1977) has also been analyzed by this method. The remaining 26,800 global 1° X 1° anomalies have been estimated by averaging over the 8 adjacent 1° blocks centered on the missing data. All missing blocks were first filled in with 5° X 5° mean values (Rapp, 1977).

Harmonic analysis of the 180 global meridional arcs of this data set was performed with the fast Fourier transform (Oppenheim and Schafer, 1975). The average power spectrum from this analysis is shown in figure 10. The overall comparison is excellent with the spectrum expected from the previous analysis of marine altimetry. In making this comparison it is essential to use [in (9)] geopotential variances reduced for the effects of 1° X 1° smoothing (Pellinen, 1966; Rapp, 1977, pp. 18-20). The anomaly power remains high for high degree, and the track spectra at n are sensitive to all geopotential variances above n of the same parity. Thus the smoothing effect on 1° anomalies is seen to extend back as far as 10 cycles.

We interpret the excess of track power above 100 cycles to be due principally to aliasing because no "noise tail" is obvious. (It should be present at an average white noise level above 12 mgals.)

We estimate the aliasing in a declining power spectrum, at and below the Nyquist limit (here, N = 180 cycles) to be given by:

$$(P_n^2)_{aliased} = (P_n^2)_{expected} \left[1 + \left(\frac{2N}{n} - 1 \right)^{-2B} + \left(\frac{4N}{n} - 1 \right)^{-2B} + \ldots \right] \quad (18)$$

where,

$$(P_n^2)_{expected} = (An^{-B})^2, \quad B > 1/2 \,. \quad (19)$$

The lower limit on the slope of the "true" spectrum (B) ensures that the total signal is finite.

From figure 10, we estimate that B = 1 is the correct slope of the spectrum near the Nyquist limit, somewhat flatter above and steeper below. (The final result is fairly insensitive to this estimate. We also note that for this slope the effect of the second fold of frequencies is only 11% of the first at n = N.) Using (18) we have corrected the average track spectrum for aliasing. Then, using the odd and even frequencies separately (in conjunction with a smooth extrapolation of the power beyond n = 180 to account for the truncation) we have found both odd and even degree variances from inversion of (9). The result for individual degree variances however is too widely varying to be realistic. Recall that the track spectrum should be smoother than the field spectrum since it is formed from an infinite sum of the degree variances. Evidently the meridional arcs do not yield a fair sampling of the field at all frequencies, only showing north-south variations and over-emphasizing the polar regions. Nevertheless, averaging the variances in 10 degree groups ($\ell > 20$) gives a smoother result (figure 9) agreeing remarkably well with the marine geoid spectrum ($\ell \geqslant 25$) and ($\ell < 20$) the recent Goddard Earth Model 10B (Lerch et al., 1978).

Also shown on figure 9 is an early modification of Kaula's rule from covariance analysis of gravimetry (Kaula, 1969). The significant change from $10^{-5}/\ell^2$ occurs for $\ell > 90$ showing greater power than the "rule," which we confirm from $\ell > 60$ to at least $\ell = 400$.

In summary, the gravitational spectrum of the Earth [from extensive harmonic analysis of track spectra of marine altimetry ($\ell > 20$), surface gravimetry ($\ell \leq 180$) as well as from least squares solutions for satellite tracking, and broader altimetry and gravimetry data ($\ell \leq 36$)] is given closely by:

$$\delta_\ell = a\ell^{-b} \tag{20}$$

where:

$$\begin{aligned}
10^6 a &= 12.6 \;, & b &= 2.198 \;; & \ell &\leq 10 \\
10^6 a &= 50.1 \;, & b &= 2.797 \;; & 10 &< \ell \leq 20 \\
10^6 a &= 0.430 \;, & b &= 1.209 \;; & 20 &< \ell \leq 50 \\
10^6 a &= 2.74 \;, & b &= 1.681 \;; & 50 &< \ell \leq 150 \\
10^6 a &= 18.5 \;, & b &= 2.063 \;; & 150 &< \ell \leq 400 \\
10^6 a &= 23.1 \;, & b &= 2.1 \;; & \ell &> 400
\end{aligned} \tag{21}$$

This spectrum has been used to calculate the total geoid power above each complete degree (rms, world-wide):

$$P_G(\ell_{max}) = R\left[\sum_{\ell=\ell_{max}+1}^{\infty} \sigma_\ell^2\right]^{1/2} . \tag{22}$$

The same power from Kaula's rule is closely approximated by $(64/\ell_{max})$ m for $\ell_{max} > 10$ (e.g., Chovitz, 1973). Figure 11 shows that even if the geopotential is known perfectly to (180, 180) about 46 cm of power remains in the geoid undulations. It should be emphasized that the piecewise spectra of (21) are smooth and that variations for individual degrees can be as great as 25% (see figure 9).

SUMMARY AND CONCLUSIONS

An efficient method has been found to determine spherical harmonic spectra directly from harmonic analysis of track data. The method is a "short cut" complement to the complete covariance analysis of data on a sphere. It does not require global data and can make use of the fast Fourier transform. The method has been developed for a wide variety of satellite tracking (as well as surface) data types for application to the determination of gravitational spectra. Included is a simple algorithm to allow for aliasing. In particular, the smooth spectrum of the geopotential has been derived from global surface gravimetry and the equivalent of 11 revolutions of marine satellite altimery. Important points to note are:

(1) The present application is only strictly for probe data on great circle tracks;

(2) if the tracks are global, individual degree variances can be found but their accuracy depends on sufficient number and distribution of tracks;

(3) if the tracks are not global, smooth (degree-averaged) spectra are found;

(4) results from non-global tracks within about 10% of great circles show smooth spectra with no significant distortions;

(5) results from highly elliptical tracks (rectified) do show significant distortions but may still be useful.

ACKNOWLEDGMENTS

We are grateful to Guy Germana for stimulating discusssions of techniques for harmonic analysis. He performed the fast Fourier transform of the gravimetry data. We also thank Michael Graber for help with the representation of aliasing.

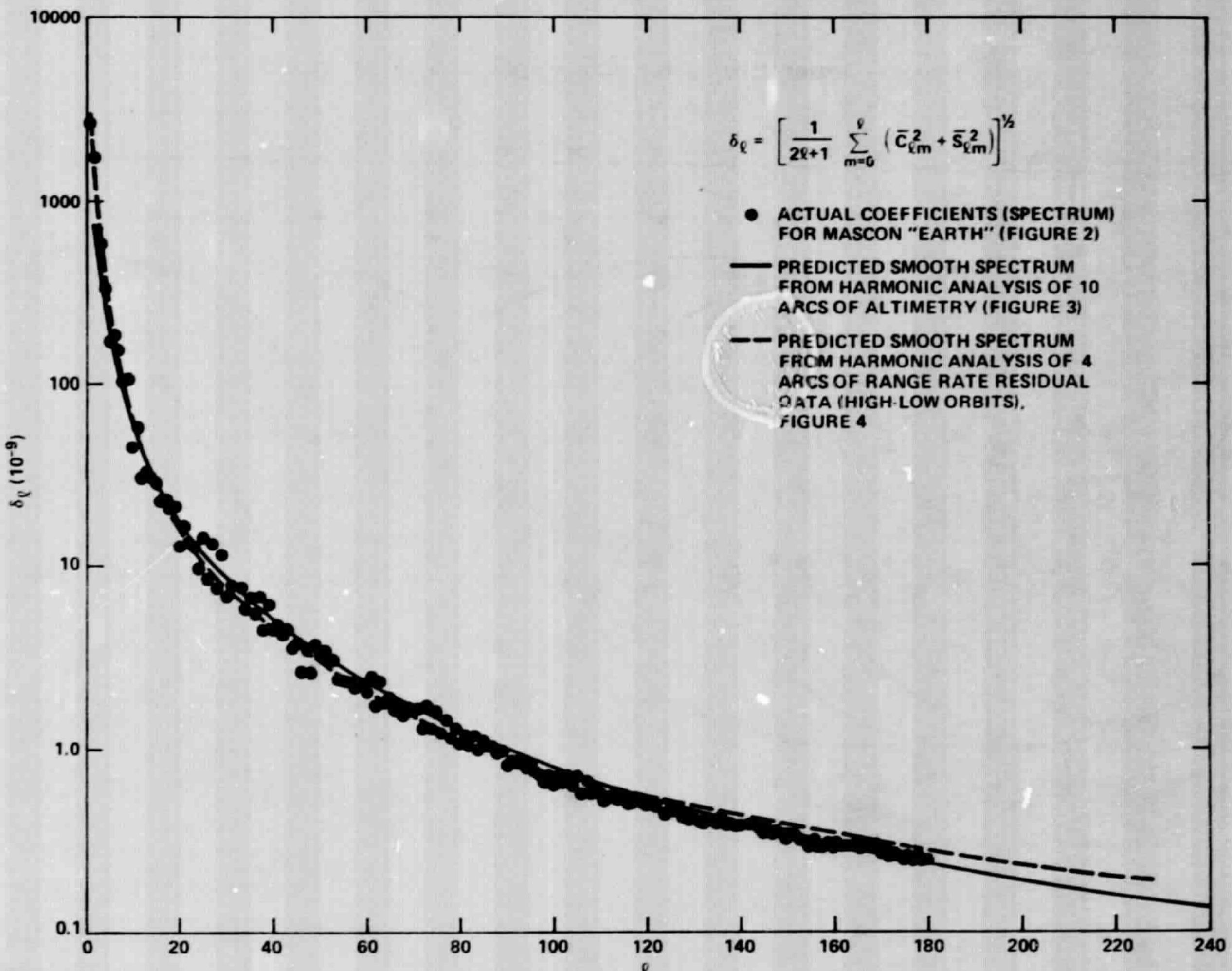

Figure 1. RMS Gravitational Coefficient by Degree, for Model Earth

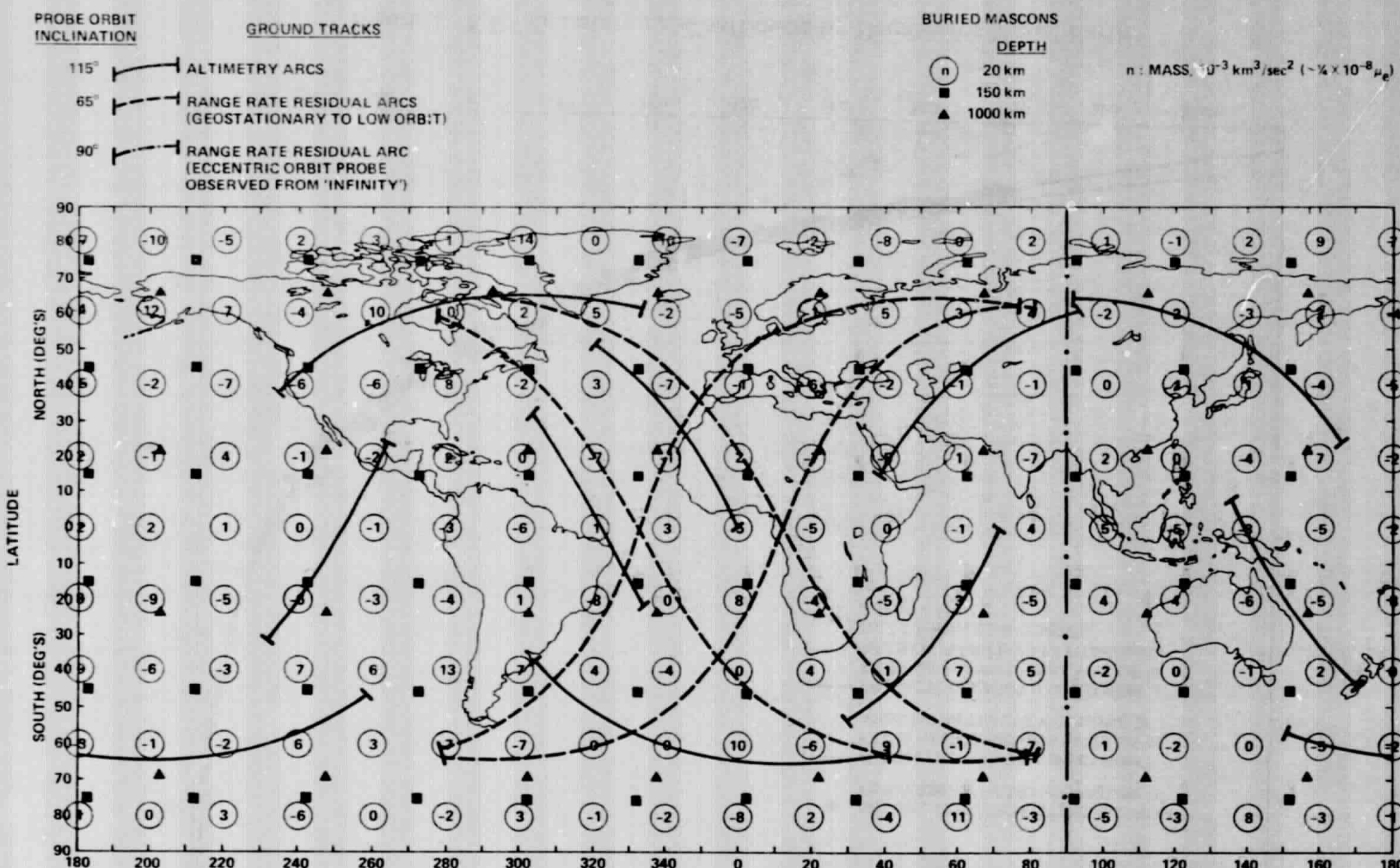

Figure 2. Point Mass and Ground Track Distribution – Model Earth

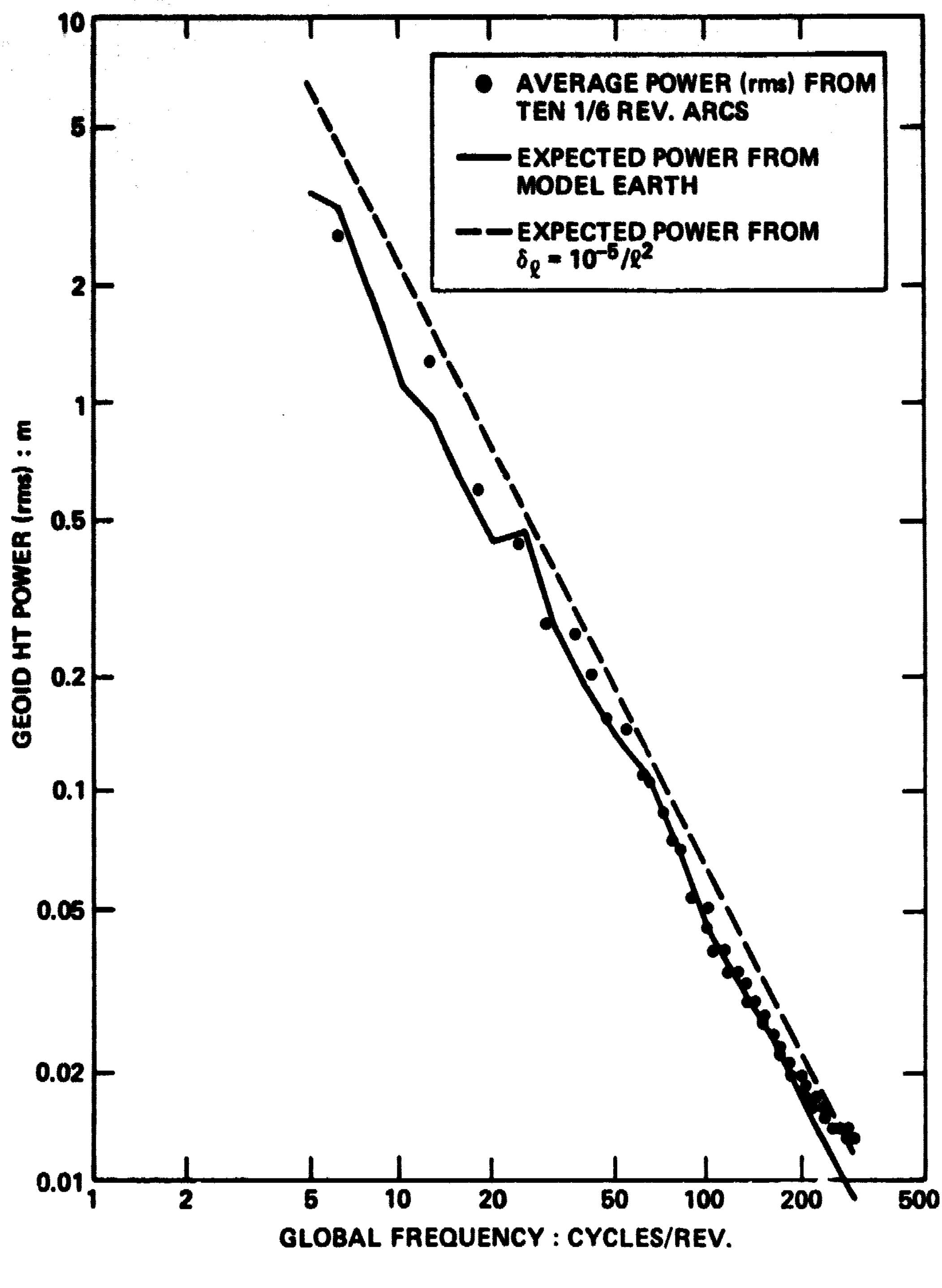

Figure 3. Average Power Spectrum for 10 Arcs of Altimetry over Model Earth

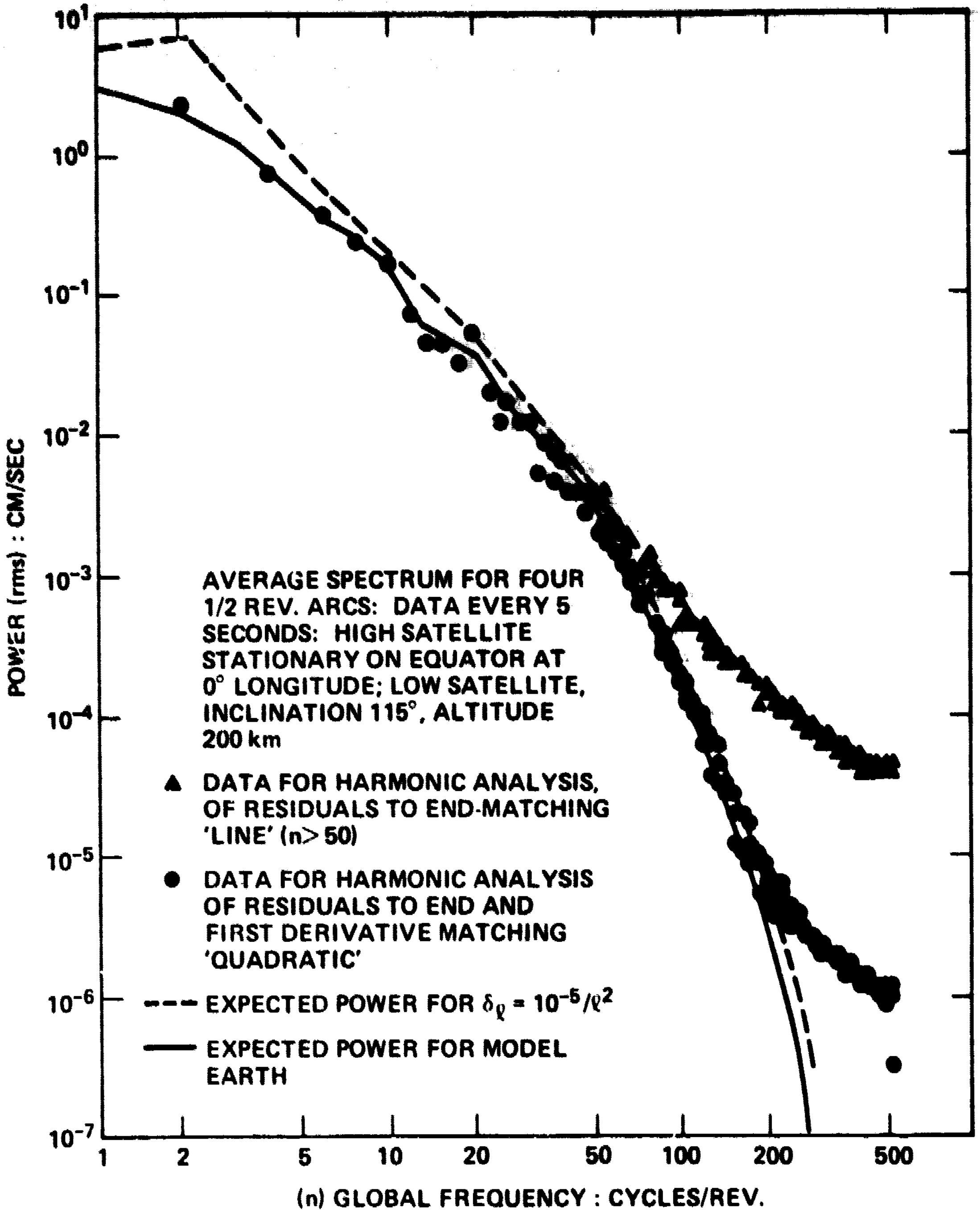

Figure 4. Average Power (rms) in Range Rate between High and Low Orbits over Model Earth

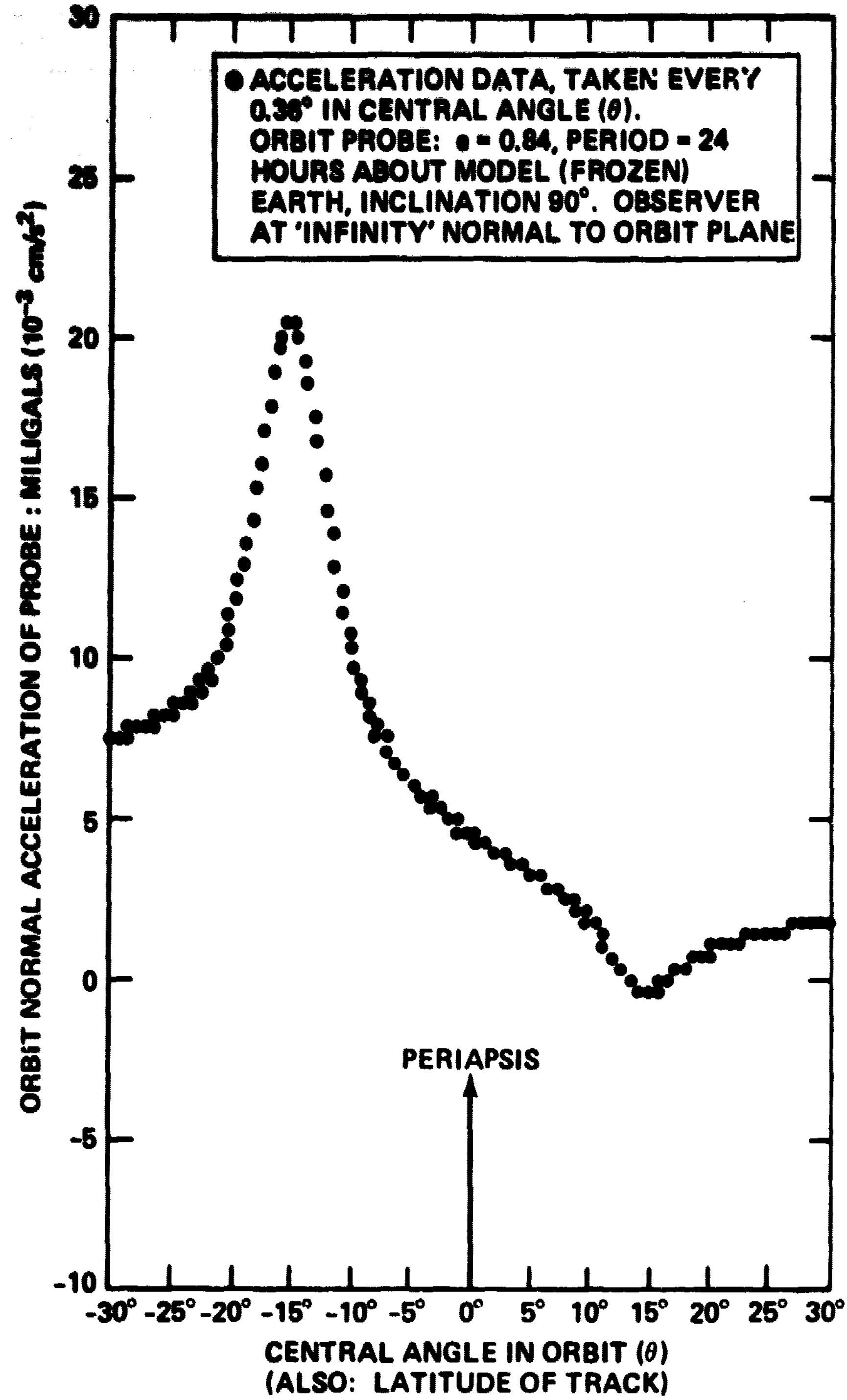

Figure 5. Acceleration of the Line of Sight between an Eccentric Orbiter and a 'Point at Infinity'

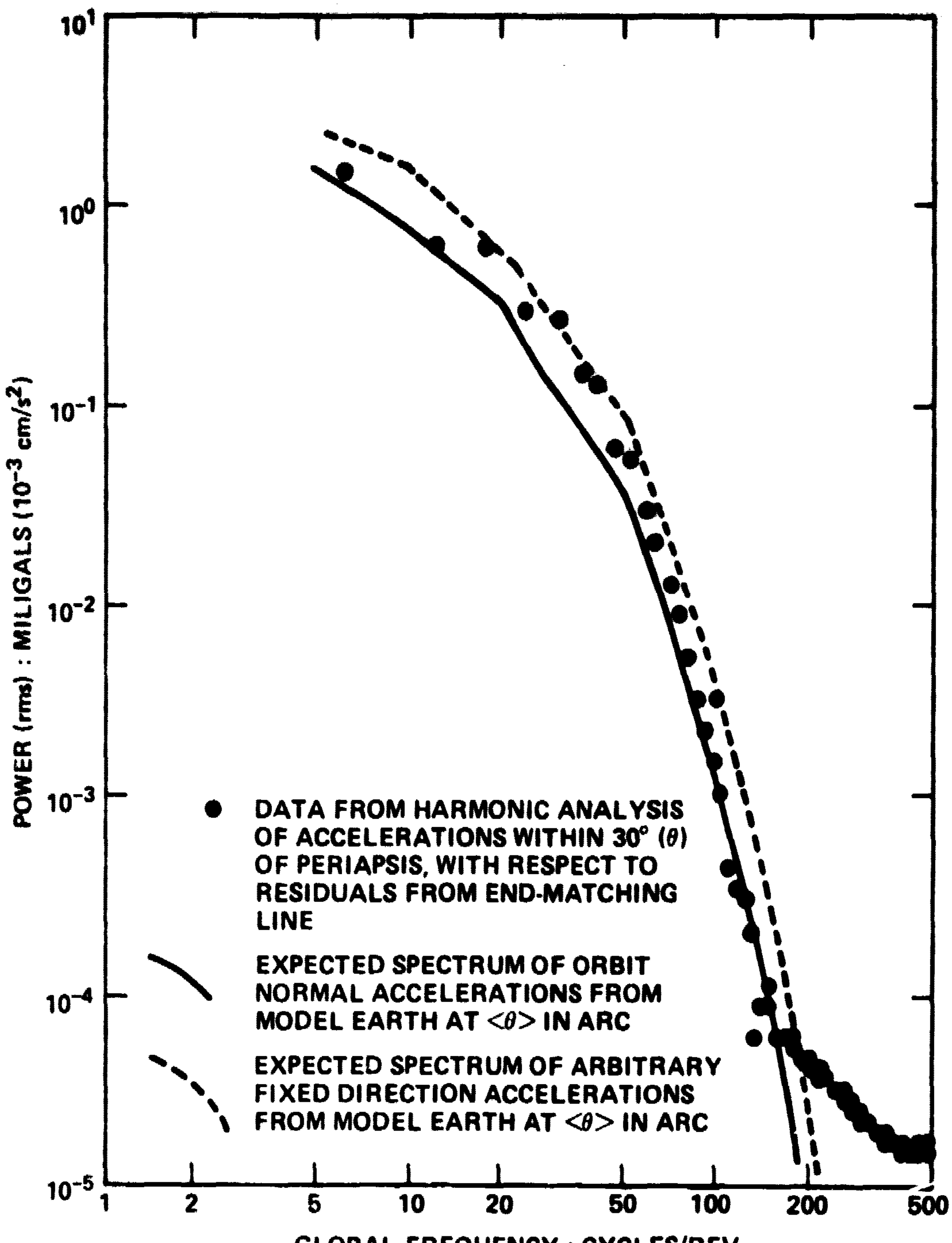

Figure 6. Orbit Normal Acceleration Power (rms) for a Single Pass of an Eccentric Orbiter-Probe

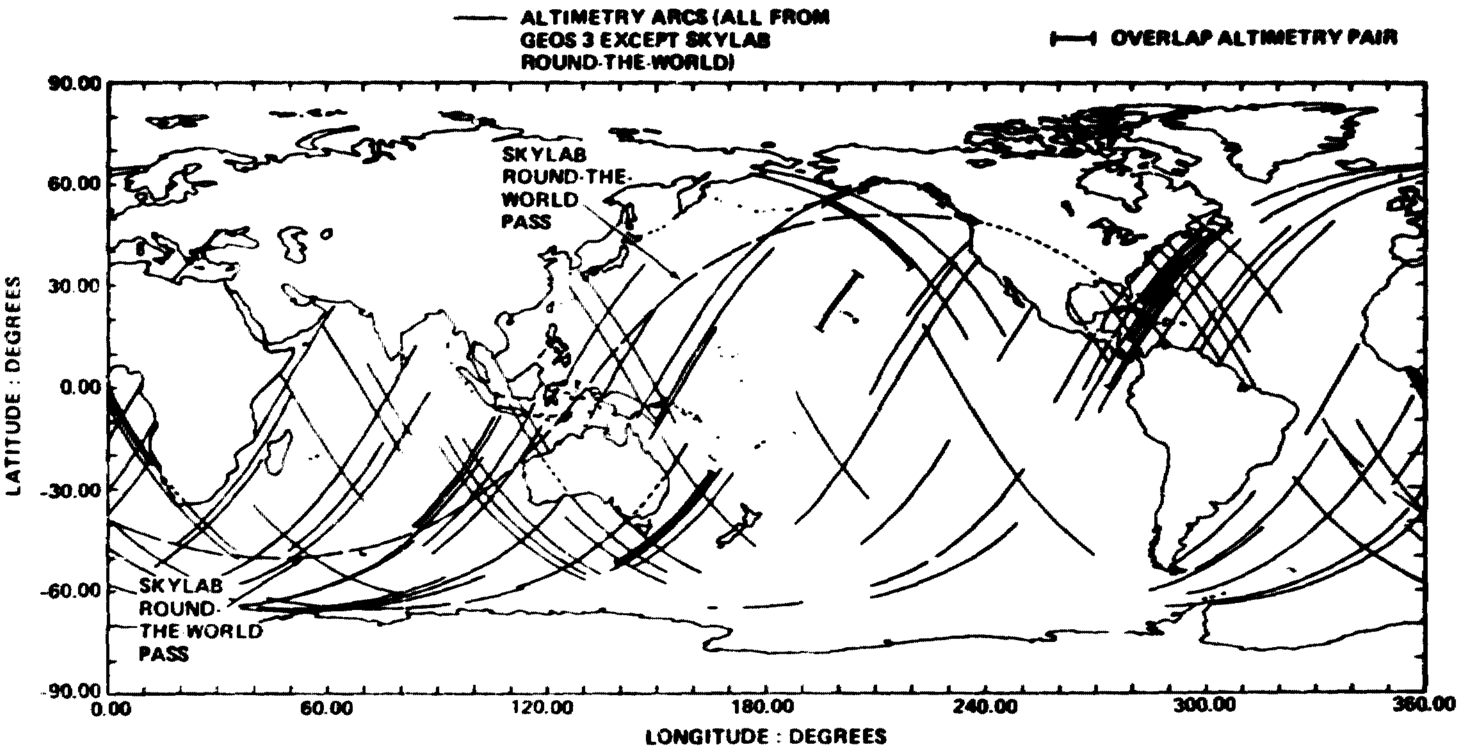

Figure 7. Distribution of Altimetry (81 Arcs and 11 Overlap Pairs) in Evaluation of Geoid Spectrum

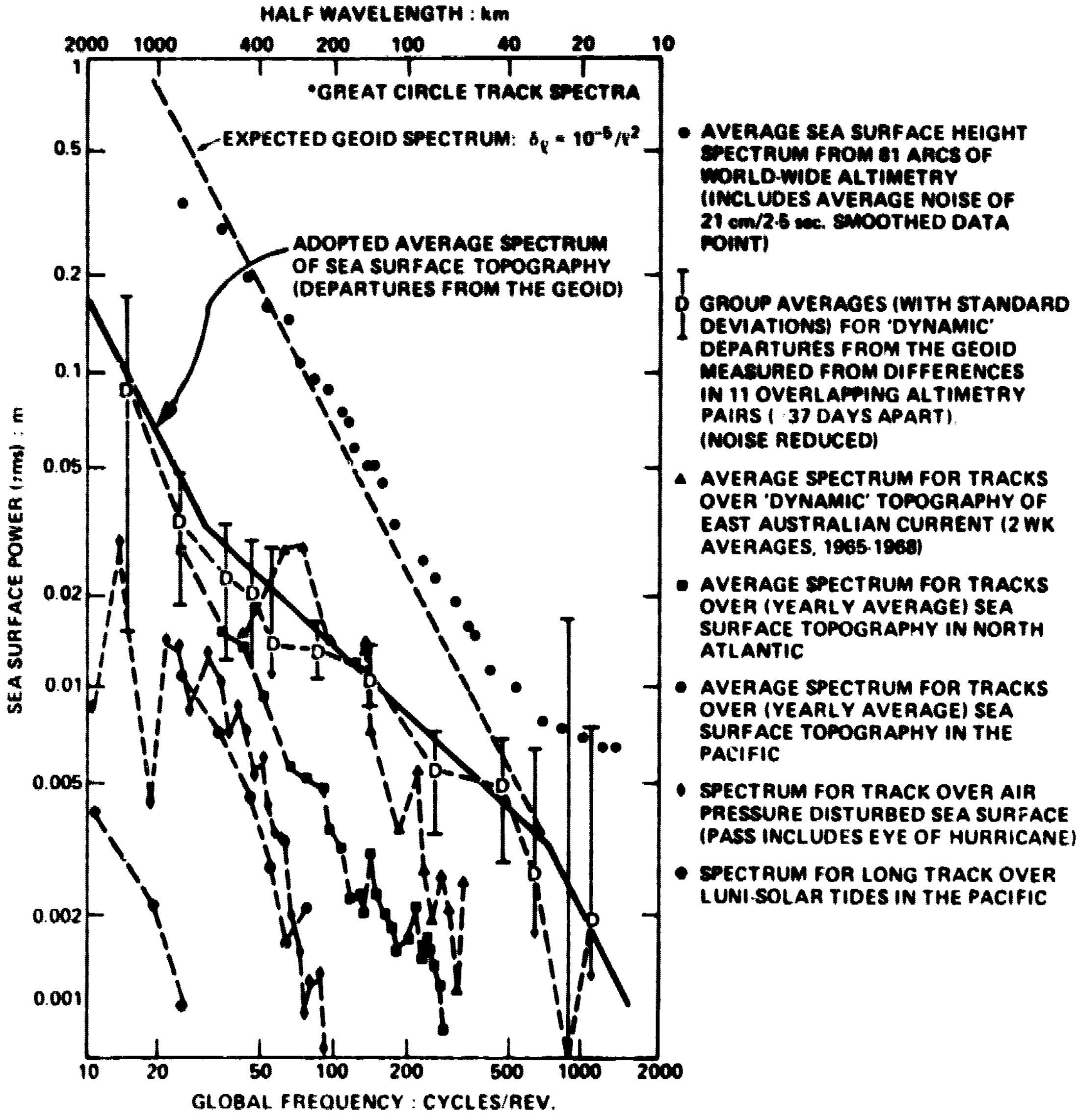

Figure 8. Sea Surface Power Spectrum (rms) from Satellite Altimetry and Oceanographic Data*

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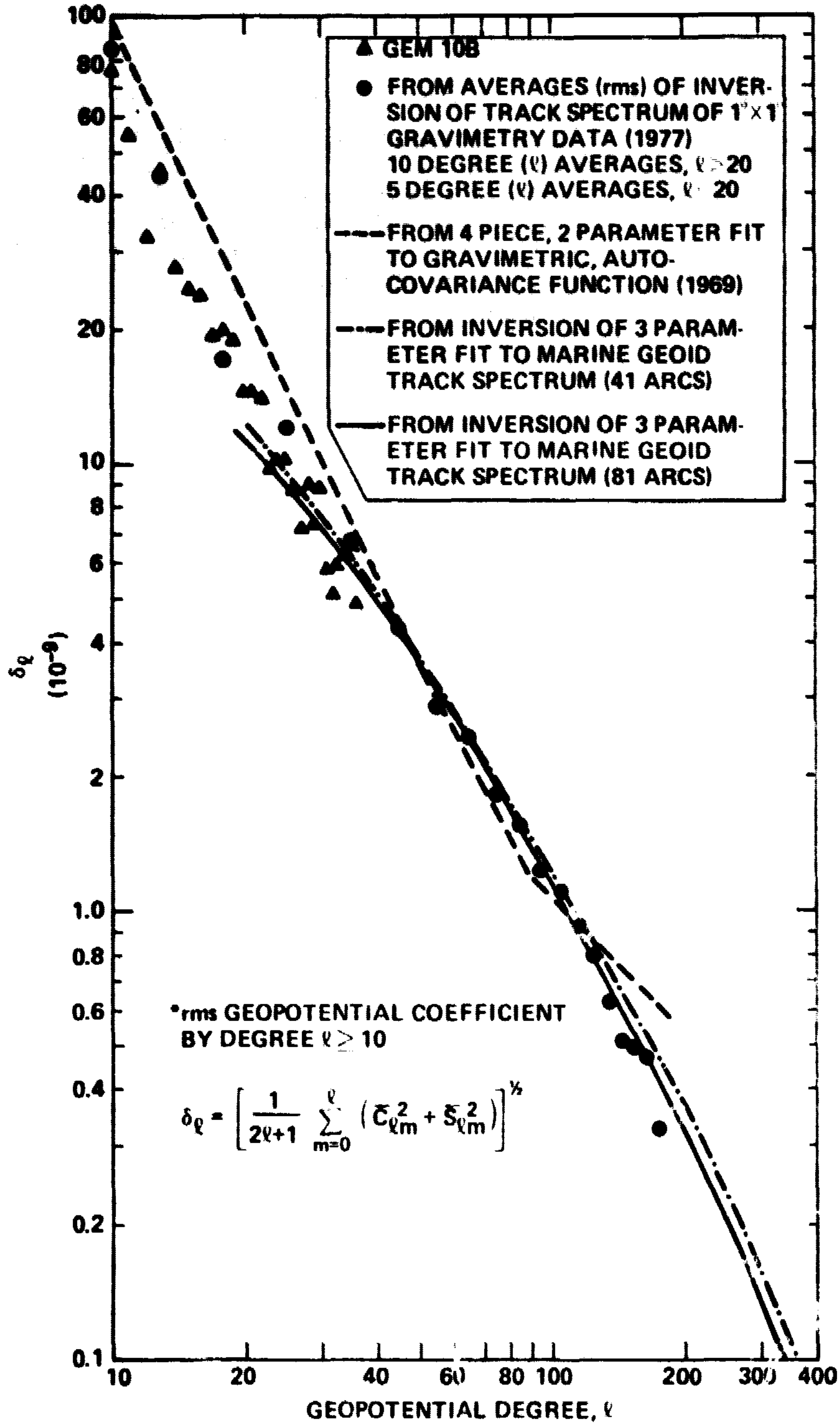

Figure 9. Geopotential Spectrum*

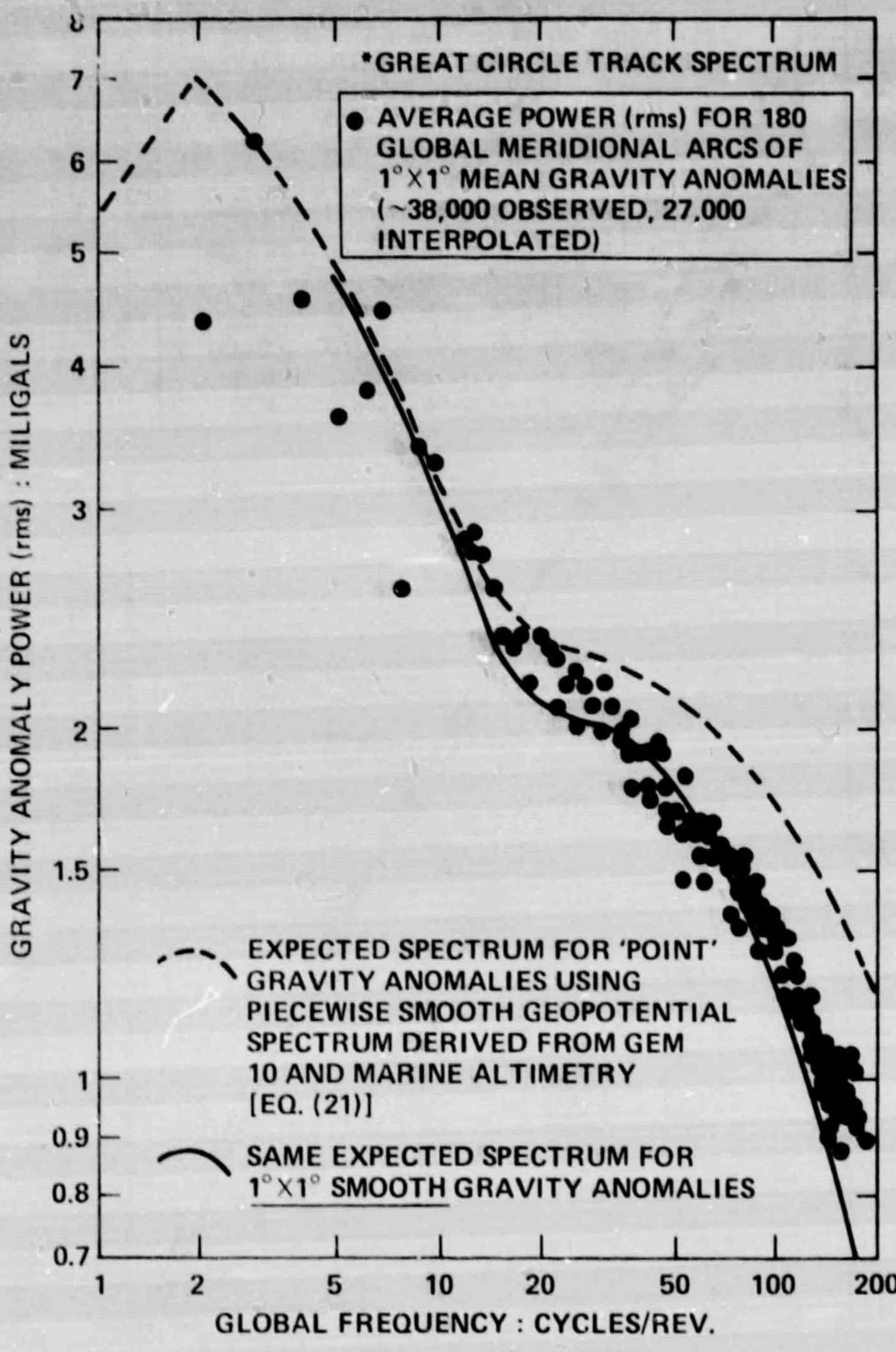

Figure 10. Power Spectrum for 1° X 1° Mean Gravity Anomalies*

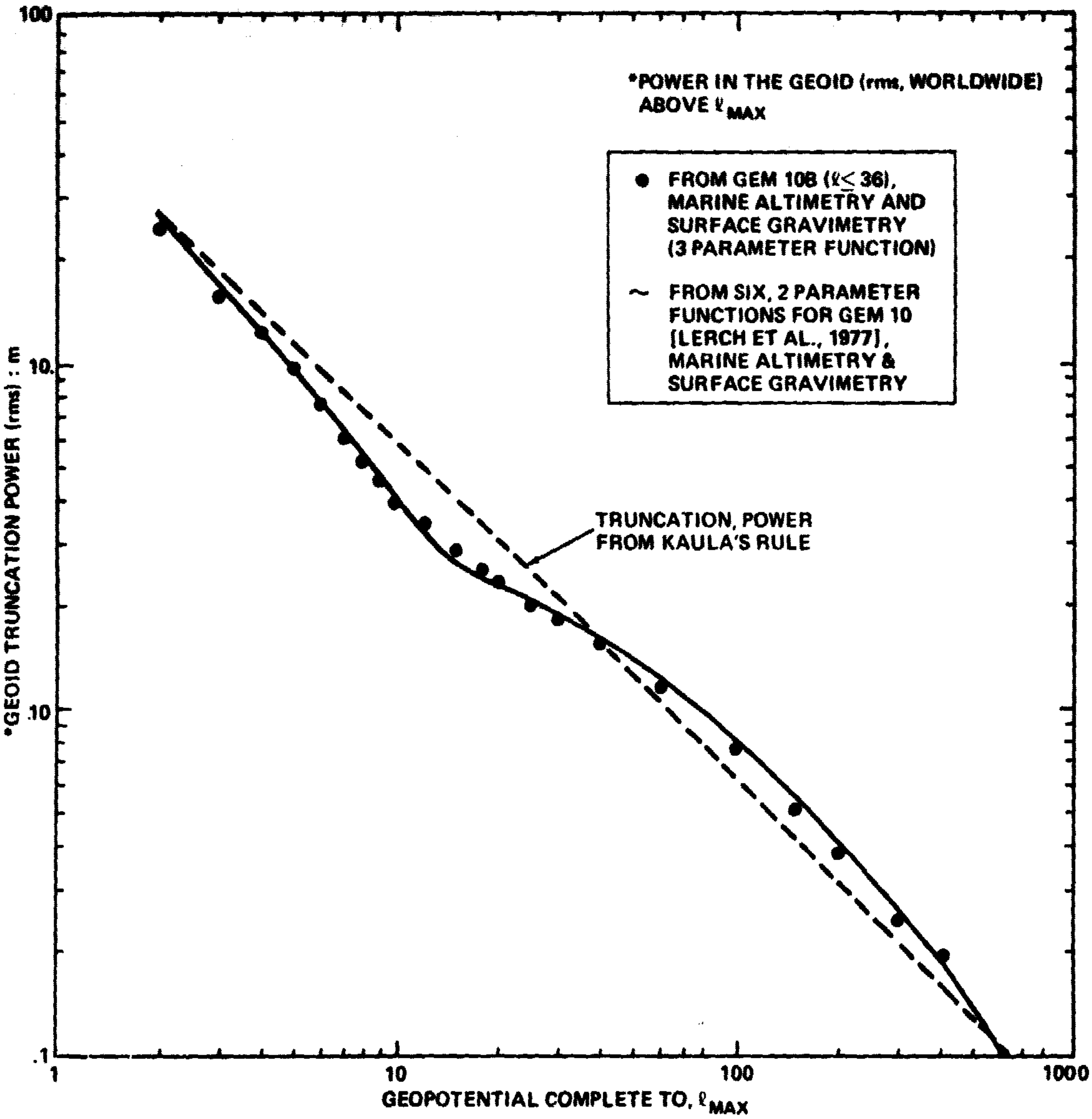

Figure 11. Geoid Truncation Power*

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16. Abstract

A simple rapid method is described for determining the spectrum of a surface field (in spherical harmonics) from harmonic analysis of direct (in situ) measurements along great circle arcs. The method is shown to give excellent overall trends (smoothed spectra) to very high degree from even a few short arcs of satellite data. Three examples are taken with perfect measurements of satellite tracking over a planet made up of hundreds of point-masses using (1) altimetric heights from a low orbiting spacecraft, (2) velocity (range rate) residuals between a low and a high satellite in circular orbits, and (3) range-rate data between a station at infinity and a satellite in a highly eccentric orbit.

In particular, the smoothed spectrum of the Earth's gravitational field is determined to about degree 400 (50 km half wavelength) from 1° × 1° gravimetry and the equivalent of 11 revolutions of Geos 3 and Skylab altimetry. This measurement shows there is about 46 cm of geoid height (rms world-wide) remaining in the field beyond degree 180.

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