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A RESEARCH PROGRAM TO REDUCE INTERIOR NOISE IN GENERAL AVIATION AIRPLANES

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NOISE REDUCTION THROUGH A CAVITY-BACKED FLEXIBLE PLATE

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SUMMARY

A prediction method is found for the noise reduction through a cavity-backed panel. The analysis takes into account only cavity modes in one direction. The results of this analysis are used to find the effect of acoustic stiffness of a backing cavity on the panel behavior. The resulting changes in the noise reduction through the panel are significant. The results of this analysis show good agreement with the results of the method of Guy and Bhattacharya (Ref. 8). The agreement with experimental results obtained with the test facility described in Ref. 1, however, is poor.

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LIST OF SYMBOLS

Symbol	Definition	Dimension
A	Pressure amplitude of incident sound wave	N/m^2
a	Cavity and/or panel height	m
B	Pressure amplitude of reflected sound wave	N/m^2
b	Cavity and/or panel width	m
C	Pressure amplitude of transmitted sound wave	N/m^2
c	Speed of sound in air	m/sec
D	Pressure amplitude of transmitted-reflected wave	N/m^2
D	Bending stiffness	Nm
E	Young's modulus	N/m^2
f	Frequency	Hz
h	Plate thickness	m
j	$\sqrt{-1}$	
k	Wavelength constant	rad/m
ℓ	Cavity length	m
m	Panel modes in x-direction	
m_s	Mass of plate per unit area	kg/m^2
n	Panel modes in y-direction	
NR	Noise reduction	db
P	Acoustic pressure amplitude	N/m^2
p	Acoustic pressure	N/m^2
P_{mn}	Fourier expansion coefficient	N/m^2
p_z	Forcing pressure	N/m^2
r_p	Panel resistance	$kg/m^2.sec$
r_w	Wall resistance	$kg/m^2.sec$

LIST OF SYMBOLS (continued)

Symbol	Definition	Dimension
SPL	Sound pressure level	db
t	Time	sec
TL	Transmission loss	db
u	Particle velocity	m/sec
W	Shape function	m
w	Plate deflection	m
W_{mn}	Fourier expansion coefficient	m
X	Part of shape function in x-direction	$m^{1/2}$
x	Model coordinate	m
x_p	Panel reactance	$kg/m^2.sec$
x_w	Wall reactance	$kg/m^2.sec$
Y	Part of shape function in y-direction	$m^{1/2}$
y	Model coordinate	m
z	Model coordinate	m
$\bar{z}_p$	Normal specific acoustic impedance of panel	$kg/m^2.sec$
$\bar{z}_w$	Normal specific acoustic impedance of wall	$kg/m^2.sec$
α_r	Sound power reflection coefficient	
α_t	Sound power transmission coefficient	
β	Time dependency of displacement	
η	Internal loss factor	
θ	Phase angle	rad
κ	Real part of Eq. 4.34	$kg/m^2.sec$
λ	Imaginary part of Eq. 4.34	$kg/m^2.sec$
λ	Wavelength	m

LIST OF SYMBOLS (continued)

Symbol	Definition	Dimension
ν	Poisson's ratio	
ρ	Density of air	kg/m^3
ω	Angular frequency	rad/sec

Subscripts

e	Effective
H	Homogeneous solution
i	Incident wave
P	Particular solution
r	Reflected wave
ref	Reference
t	Transmitted wave
tr	Transmitted-reflected wave
1	Source side
2	Receiver side

CHAPTER 1

INTRODUCTION

This report describes part of the work performed under a National Aeronautics and Space Administration (NASA) funded research project on the transmission of sound through general aviation airplane structures.

The project started on April 15, 1976, when the Flight Research Laboratory of the University of Kansas began work on the grant for NASA, Langley Research Center, entitled "A Research Program to Reduce Interior Noise in General Aviation Airplanes," NASA Grant No. NSG 1301. The main objectives of this first phase were to develop a noise research team at the University of Kansas and to define a long-range follow-up research program in interior noise.

At the end of April 1977, the ending of the preparation phase, the grant was extended to carry out the continuing work, which was defined and partially prepared during the first phase of the project. During this second phase a test facility was designed and built. In Ref. 1 the test facility is described thoroughly. The latter made it possible to determine the sound transmission loss characteristics of various structural panels and panel treatments.

It was assumed that the receiving chamber, in the tube behind the test panel, was a termination which absorbed all of the sound passing through the test panel. In that way the receiving chamber would affect the transmission of sound through the test specimen in the same manner as an infinite space of air. However, when testing of panels started, several apparent anomalies were observed. One problem which appeared was that the measured resonance frequencies of most test panels did not agree with the calculated resonance frequencies. Another point was negative transmission loss at the fundamental resonance frequency.

It appeared that the receiving chamber behind the panel behaved as a finite cavity and affected the panel behavior. This cavity effect raised the resonance frequencies and caused the negative transmission loss. In spite of the large amounts of absorbing materials in the tube, the receiving chamber did not have the same impedance as an infinite space of air.

In this report an attempt will be made to explain the effect of the cavity on the panel behavior. Expressions will be derived for the prediction of the noise reduction through free and cavity-backed panels. The influence of cavity damping will also be considered.

CHAPTER 2

HISTORIC REVIEW

According to Pretlove (Ref. 2) there are two types of panel cavity systems. The first type is a system in which the effect of the cavity on the panel behavior is negligible. The panel stiffness is considerably greater than the acoustic stiffness, and the fundamental resonance frequencies are not affected by the cavity behind the plate. The second type is a system in which the effect of the cavity on the panel is considerable. This happens when the acoustic stiffness of the cavity is equal to or greater than the stiffness of the panel. The panel mode shapes and natural frequencies change considerably. For this system it is necessary to include the effect of the cavity in the analysis of the noise transmission through the panel in the cavity.

The manner in which the acoustic cavity modes affect the flexible panel depends very much on the length of the cavity. When the length of the cavity is greater than the larger panel dimension, two acoustic cavity depth modes will occur before any other.

The first acoustic cavity mode is called the "open-ended" mode. The depth of the cavity is one quarter of the acoustic wavelength. In this case, the cavity affects the panel as if there were no cavity at all. The acoustic stiffness due to the cavity has dropped to zero at this frequency. The second acoustic cavity mode, which occurs at about twice the frequency of the "open-ended" mode, is the "closed-end" acoustic depth mode. In this case, the acoustic stiffness approximates minus infinity.

When the in vacuo value of the panel natural frequency (the natural frequency of the panel when backed by an infinite space) is less than the "open-ended" acoustic mode frequency, then the cavity acts as a stiffness and the natural frequency of the panel is increased.

The second possibility is that the in vacuo panel resonance frequency falls between the "open-ended" and the "closed-end" acoustic modes. Then the cavity acts as a negative stiffness, and the panel fundamental frequency is decreased.

If the depth of the cavity is less than one-half of the larger panel dimension, then the effect of the transverse acoustic modes of the cavity becomes more important.

If the depth of the cavity becomes less than one-quarter of the larger panel dimension, then the lowest acoustic mode in the frequency scale is the transverse acoustic mode and infinite values of acoustic stiffness occur before zero values.

The problem of a cavity-backed panel has already been studied by several persons. One of the first articles about this subject was written by Dowell and Voss in 1963 (Ref. 3). It treats a case in which the cavity acts as a stiffness to the fundamental panel mode. The effect of the cavity on the second (anti-symmetric) panel mode is that of a negative stiffness. According to this theory the cavity effect becomes more noticeable with decreasing cavity length.

Lyon (Ref. 4) treats the noise transmission problem for the situation in which the panel vibration is not affected by the cavity. The acoustic stiffness because of the cavity is assumed negligible, compared with the stiffness of the panel. The usual practical situation is considered: the in vacuo panel fundamental frequency is lower than the first acoustic cavity mode. The noise reduction study is divided into three frequency ranges: at "low" frequencies, where both the panel and the cavity are stiffness controlled; at "intermediate" frequencies, where the panel is in the resonance controlled region and the cavity is still stiffness controlled; and at "high" frequencies, where both the panel and the cavity are in the resonance controlled region.

Pretlove discusses, in Ref. 2 and 5, the free and the forced vibrations of a panel-backed cavity. The model used by Pretlove considers a cavity with all walls perfect acoustic reflectors, except for the flexible panel. The solution derived in Ref. 2 is exact.

The problem of a room- or cavity-backed panel has also been studied by Kihlman (Ref. 6) and Bhattacharya and Crocker (Ref. 7) using models somewhat similar to the ones used by Pretlove (Ref. 2 and 5) and Dowell and Voss (Ref. 3).

Guy and Bhattacharya (Ref. 8) discuss the influence of a finite cavity on the transmission of sound through a panel backed by that cavity. The model considered consists of an acoustically hardwalled rectangular cavity having at one face a flexible vibrating panel. The same model is also considered by the above-mentioned authors. The effect of panel damping can be introduced. Cavity damping, however, is not considered in this model. The theoretical predictions show good agreement with experimental results and with Pretlove's results (Ref. 5).

One of the latest articles about sound transmission through cavity-backed panels is Ref. 9. Dowell has developed a theoretical model for interior sound fields which are created by flexible wall motion resulting from exterior sound fields. Included in the model are the mass, stiffness and damping of the panel, as well as the mass, stiffness and damping (due to absorbing walls inside the cavity) of the acoustic cavity. Comparisons of the theoretical predictions with experimental results show good agreement. Contrary to Guy and Bhattacharya (Ref. 8), however, Dowell does not compare experimental noise reduction data with theoretical predictions.

Barton (Ref. 10) presents some experimental findings on the effects of panel stiffness and receiving space absorption on low-frequency sound transmission through panels into closed spaces. Also a simple method for predicting the low-frequency noise reduction of a panel backed by a closed, absorbent cavity is presented. The analytical model used by Barton is analogous to that described in Ref. 11 (page 82), except for the absorbing backing wall. The model uses only one panel mode and consists of a rigid, infinite panel supported on springs and backed by a wall having a frequency-dependent impedance. The theoretical predictions show fair agreement with the experimental data.

In Ref. 12 an attempt is made to predict the effect of an absorbing cavity on the panel dynamics. Although the results show good agreement with experimental results, a number of comments must be made. The model used is the same as the one derived in Ref. 8. To account for the absorbing materials in the receiving chamber, a resistive part is added to the cavity impedance, which in Ref. 8

only consists of a reactive part. However, the absorbing materials also cause a change in boundary conditions, and the equations as derived in Ref. 8 (and used in Ref. 12) do not satisfy those boundary conditions. In Ref. 12 an effective cavity length has been used which decreases with increasing frequency. Physically this is not quite right because with increasing frequency the absorbing materials become more and more effective, and less and less sound will be reflected by the walls. This means that the cavity becomes larger with increasing frequency; and at high frequencies no sound at all will be reflected by the walls, and the cavity behaves like an infinite space. Hence, the cavity must become larger with increasing frequency instead of smaller as suggested in Ref. 12.

In the following chapter, the transmission of sound through a free panel will be discussed.

CHAPTER 3

NOISE REDUCTION THROUGH A FREE PANEL

In this chapter the transmission of sound through a panel which is backed by an infinite space will be discussed. This chapter will be an introduction to the one in which the effect of a finite space (cavity) behind the panel will be discussed.

3.1 Transmission of Sound through Panel

When a progressive plane wave in a fluid medium impinges on the boundary of a contiguous second medium, a reflected wave is generated in the first medium and a transmitted wave in the second medium. Here the boundary between the two mediums consists of a thin, flexible panel. The model used is sketched in Figure 3.1.

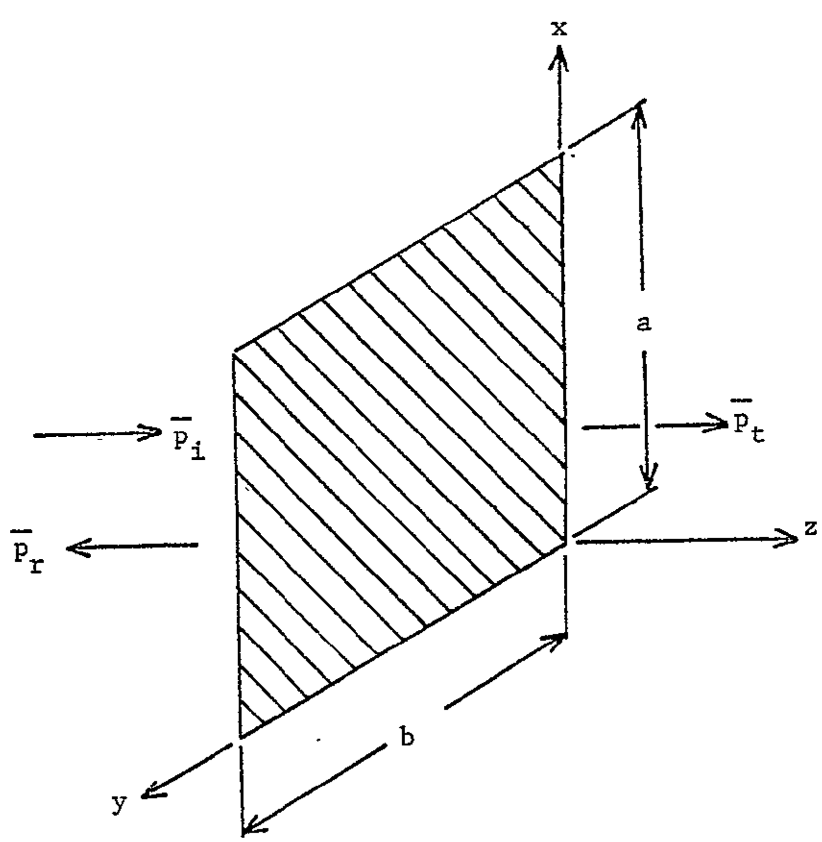

Figure 3.1: Sketch of Panel Geometry

The panel is assumed to be very thin. The length of the sound wave, λ, is much greater than the panel thickness, h. In that case any wave transmission through the panel can be ignored. An acoustic impedance at z = 0 is the only effect of the plate on the acoustic waves.

The panel at z = 0 forms the boundary between medium I, of characteristic impedance $\rho_1 c_1$, and medium II of characteristic impedance $\rho_2 c_2$, where ρ represents the undisturbed equilibrium density of the medium and c the speed of sound of the medium. Consider an incident plane wave traveling in medium I in the positive z-direction. This incident wave may be represented by:

$$\overline{p}_i = A\, e^{j(\omega_i t - k_i z)} \tag{3.1}$$

where A is a real constant representing the pressure amplitude of the wave, ω_i is the angular frequency and:

$$k_i = \frac{\omega_i}{c_1} \tag{3.2}$$

Upon striking the panel which is for convenience at z = 0, a reflected wave:

$$\overline{p}_r = \overline{B}\, e^{j(\omega_r t + k_r z)} \tag{3.3}$$

and a transmitted wave:

$$\overline{p}_t = \overline{C}\, e^{j(\omega_t t - k_t z)} \tag{3.4}$$

are produced. The reflected wave has a frequency ω_r, a complex pressure amplitude $\overline{B}$ and a wavelength constant:

$$k_r = \frac{\omega_r}{c_1} \tag{3.5}$$

The transmitted wave has a frequency of ω_t, a complex amplitude $\overline{C}$ and a wavelength constant:

$$k_t = \frac{\omega_t}{c_2} \tag{3.6}$$

There are two boundary conditions that must be satisfied at all times and at all points on the panel surface. The conditions are continuity of pressure and continuity of particle velocity at $z = 0$. These two conditions can be combined and become instead a condition of continuity of their ratio:

$$\frac{\overline{p}_i + \overline{p}_r}{\overline{u}_i + \overline{u}_r} = \overline{z}_p + \frac{\overline{p}_t}{\overline{u}_t} \tag{3.7}$$

at $z = 0$, where $\overline{z}_p$ represents the normal specific acoustic panel impedance. The incident, reflected and transmitted wave are assumed to be plane progressive waves. The sum of the incident and reflected wave can be a standing wave, however. For plane progressive waves the following expressions are valid for the particle velocity of the incident, reflected and transmitted wave, respectively:

$$\overline{u}_i = \frac{\overline{p}_i}{\rho_1 \, c_1} \tag{3.8}$$

$$\overline{u}_r = - \frac{\overline{p}_r}{\rho_1 \, c_1} \tag{3.9}$$

$$\overline{u}_t = \frac{\overline{p}_t}{\rho_2 \, c_2} \tag{3.10}$$

The normal specific impedance $\overline{z}_p$ of the panel may be written as follows:

$$\bar{z}_p = r_p + j\, x_p \tag{3.11}$$

where r_p represents the panel resistance and x_p the panel reactance. The normal specific acoustic impedance is defined as the complex ratio of the driving acoustic pressure to the resulting velocity of the plate surface. Substitution of the Eqs. 3.8, 3.9, 3.10 and 3.11 in Eq. 3.7 results in the following expression:

$$\frac{\bar{p}_i + \bar{p}_r}{\bar{p}_i - \bar{p}_r}\, \rho_1\, c_1 = r_p + j\, x_p + \rho_2\, c_2 \tag{3.12}$$

or:

$$\frac{\bar{p}_r}{\bar{p}_i} = \frac{(r_p + \rho_2\, c_2 - \rho_1\, c_1) + j\, x_p}{(r_p + \rho_2\, c_2 + \rho_1\, c_1) + j\, x_p} \tag{3.13}$$

By taking the magnitude of both sides of Eq. 3.13 the following expression can be derived:

$$\frac{B}{A} = \left[\frac{(r_p + \rho_2\, c_2 - \rho_1\, c_1)^2 + x_p^{\,2}}{(r_p + \rho_2\, c_2 + \rho_1\, c_1)^2 + x_p^{\,2}}\right]^{1/2} \tag{3.14}$$

See Appendix A for the complete derivation of Eq. 3.14.

Substitution of Eqs. 3.1 and 3.3 in Eq. 3.13 results in (at z = 0):

$$\frac{B\, e^{\,j(\omega_r t + \theta_r)}}{A\, e^{\,j\omega_i t}} = \tag{3.16}$$

$$\frac{r_p^{\,2} + 2r_p\, \rho_2\, c_2 + (\rho_2\, c_2)^2 - (\rho_1\, c_1)^2 + x_p^{\,2} + j\, 2\, x_p\, \rho_1\, c_1}{(r_p + \rho_2\, c_2 + \rho_1\, c_1)^2 + x_p^{\,2}}$$

When the assumption is made that the frequency of the incident wave is identical to the frequency of the reflected wave:

$$\omega_i = \omega_r = \omega_1$$

then the phase angle θ_r can be derived from Eq. 3.16 and may be written as:

$$\theta_r = \arctan \left\{ \frac{2\, x_p\, \rho_1\, c_1}{(r_p + \rho_2\, c_2)^2 - (\rho_1\, c_1)^2 + x_p^{\;2}} \right\} \tag{3.17}$$

At z = 0 also the following boundary condition must be satisfied: the particle velocities normal to the surface are equal. This is true when the panel damping is zero. This boundary condition can be written as follows:

$$\bar{u}_i + \bar{u}_r = \bar{u}_t \tag{3.18}$$

Substitution of Eqs. 3.8, 3.9 and 3.10 in Eq. 3.18 results in:

$$\frac{\bar{p}_i - \bar{p}_r}{\rho_1\, c_1} = \frac{\bar{p}_t}{\rho_2\, c_2} \tag{3.19}$$

When the assumption is made:

$$\omega_i = \omega_r = \omega_t = \omega$$

Eq. 3.19 becomes:

$$A - \bar{B} = \frac{\rho_1\, c_1}{\rho_2\, c_2}\, \bar{C} \tag{3.20}$$

or:

$$\frac{\overline{C}}{A} = \frac{\rho_2\ c_2}{\rho_1\ c_1}\left(1 - \frac{\overline{B}}{A}\right) \tag{3.21}$$

The ratio $\overline{B}/A$ follows from Eq. 3.13. Substitution of this expression in Eq. 3.21 results in:

$$\frac{\overline{C}}{A} = \frac{\rho_2\ c_2}{\rho_1\ c_1}\left\{\frac{2\ \rho_1\ c_1}{(r_p + \rho_2\ c_2 + \rho_1\ c_1) + j\ x_p}\right\} \tag{3.22}$$

By taking the magnitude of both sides of Eq. 3.22, the following expression is obtained:

$$\frac{C}{A} = \frac{2\ \rho_2\ c_2}{\sqrt{(r_p + \rho_2\ c_2 + \rho_1\ c_1)^2 + x_p{}^2}} \tag{3.23}$$

With the above expressions, the transmission loss and the noise reduction of a free panel can be determined. The transmission loss of a panel can be written as:

$$TL = 10 \log \frac{1}{\alpha_t} \tag{3.24}$$

where the sound power transmission coefficient:

$$\alpha_t = \frac{C^2}{A^2} \tag{3.25}$$

Transmission loss is defined as 10 times the logarithm, to base 10, of the ratio of sound power incident on the panel to the sound power transmitted by the panel (Ref. 13). Transmission loss is a useful analytic concept, but its measurement is hampered by the lack of acoustic watt meters. Sound power can be derived from measurements of sound pressure only under certain limited circumstances, e.g. when the acoustic waves are plane. The formation of a reflected wave with the same frequency as the incident wave will generate a pattern of standing waves in the fluid. These standing waves make it difficult if not impossible to measure the pressure amplitude of the initially incident plane progressive wave.

Instead of transmission loss, noise reduction is a more useful term. Noise reduction is defined as the difference in sound pressure level at the source side and the receiver side of the panel (Ref. 13):

$$NR = SPL_1 - SPL_2 \tag{3.26}$$

where SPL_1 and SPL_2 represent the sound pressure level at the source side and the receiver side of the panel, respectively. The sound pressure level SPL is defined as:

$$SPL = 10 \log \frac{P_e^{\,2}}{P_{ref}^{\,2}} \tag{3.27}$$

where P_e is the measured effective pressure of the sound wave (root-mean-square pressure) and P_{ref} is the effective reference pressure. A reference pressure $P_{ref} = 2 \times 10^{-5}$ N/m^2 is commonly used for computing sound pressure levels in air. The effective pressure amplitude may be written as:

$$P_e = \frac{P}{\sqrt{2}} \tag{3.28}$$

where P is the peak pressure amplitude. Now Eq. 3.26 can be written as follows:

$$NR = 10 \log \frac{P_1^{\,2}}{P_2^{\,2}} \tag{3.29}$$

where P_1 and P_2 are root-mean-square (rms) pressure amplitudes at the source side and the receiver side, respectively.

At the source side at a fixed position z_1, the acoustic pressure p_1 is the sum of the real part of the incident wave and the reflected wave:

$$p_1 = A \cos (\omega_i t - k_i z_1) + B \cos (\omega_r t + k_r z_1 + \theta_r) \tag{3.30}$$

where θ_r is a phase angle defined as:

$$\overline{B} = B\, e^{j\theta_r} \tag{3.31}$$

This angle θ_r measures the amount by which the reflected pressure leads or lags the incident pressure. To obtain the rms pressure, first the square of p_1 (Eq. 3.30) has to be taken:

$$p_1^{\,2} = \frac{A^2 + B^2}{2} + \frac{A^2 \cos 2\, (\omega_i t - k_i z_1)}{2} + \frac{B^2 \cos 2\, (\omega_r t + k_r z_1 + \theta_r)}{2} + 2\, A\, B \cos (\omega_i t - k_i z_1) \cos (\omega_r t + k_r z_1 + \theta_r) \tag{3.32}$$

Secondly, the time average of $p_1^{\,2}$ (Eq. 3.32) has to be taken. The time average of a function is defined as:

$$\langle f \rangle = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{+T} f\,(t)\, dt \tag{3.33}$$

The time average of each of the single cosines in Eq. 3.32 is zero. It can be shown that the time average of the cosine product is also zero when the frequencies ω_i and ω_r are not identical (see Appendix B).

The rms pressure for a combination of two tones of different frequencies is:

$$P_1 = \sqrt{\frac{A^2 + B^2}{2}} \quad (\omega_i \neq \omega_r) \tag{3.34}$$

Under the condition $\omega_i = \omega_r = \omega_1$ the rms pressure amplitude becomes:

$$P_1 = \sqrt{\frac{A^2 + B^2 + 2\ A\ B\ \cos\ (\theta_r + 2\ k_1\ z_1)}{2}} \tag{3.35}$$

This is shown in Appendix C.

At the receiver side (position z_2) of the free panel, only the transmitted wave exists. The acoustic pressure p_2 is the real part of the transmitted wave:

$$p_2 = C\ \cos\ (\omega_t t - k_t\ z_2 + \theta_t) \tag{3.36}$$

where θ_t represents the phase difference between the incident wave and the transmitted wave or:

$$\bar{C} = C\ e^{j\theta_t} \tag{3.37}$$

The rms pressure amplitude at the receiver side can be derived and is:

$$P_2 = \frac{C}{\sqrt{2}} \tag{3.38}$$

Substitution of Eqs. 3.35 and 3.38 in Eq. 3.29 results in the following expression:

$$\mathrm{NR} = 10 \log \frac{A^2 + B^2 + 2\, A\, B \cos\,(\theta_r + 2\, k_1\, z_1)}{C^2} \tag{3.39}$$

in the case $\omega_i = \omega_r = \omega_1$. The expression may also be written in the following manner:

$$\mathrm{NR} = 10 \log \frac{A^2}{C^2} \left\{ 1 + \frac{B^2}{A^2} + 2\, \frac{B}{A} \cos\,(\theta_r + 2\, k_1\, z_1) \right\}. \tag{3.40}$$

where the ratio B/A follows from Eq. 3.14, C/A follows from Eq. 3.23 and the phase angle θ_r from Eq. 3.17. In Eq. 3.40 z_1 indicates the source microphone position. The noise reduction of a free panel is a function of the panel resistance, panel reactance, frequency of the sound, the source microphone position and the density and speed of sound of the air at both sides of the panel.

3.2 Panel Impedance of a Free Flexible Panel with Simply Supported Boundary Conditions

The differential equation of forced motion of an isotropic flat plate is obtained from Ref. 14 and may be written as:

$$\bar{D}\ \nabla^2\nabla^2\ w\ (x,\ y,\ t) + m_s\ \frac{\partial^2\ w\ (x,\ y,\ t)}{\partial t^2} = p_z\ (x,\ y,\ t) \qquad (3.41)$$

where x and y are Cartesian coordinates in the plane as indicated by Figure 3.1. The complex flexural rigidity $\bar{D}$ can be computed with the following expression:

$$\bar{D} = \frac{\bar{E}\ h^3}{12(1 - \nu^2)} = \frac{E\ h^3}{12(1 - \nu^2)}\ (1 + j\eta) = D\ (1 + j\eta) \qquad (3.42)$$

where η = the internal loss factor of the plate

h = plate thickness [m]

ν = Poisson's ratio of plate material

E = Young's modulus [N/m^2]

m_s = mass of plate per unit area [kg/m^2]

w = plate deflection [m]

p_z = forcing pressure [N/m^2]

The first term on the left-hand side of Eq. 3.41 represents the structural stiffness of the plate. The second term at the same side represents the structural inertia. On the right-hand side of Eq. 3.41 there is the pressure loading.

Aluminum, steel and plexiglass have, according to Ref. 13, a very low value for the internal loss factor η. The internal loss factor for this type of materials is lower than 0.01. Here the assumption will be made that the effect of the structural damping is negligible.

3.2.1 Free Vibration of Simply Supported Plate

For the case of a freely vibrating plate, the external force, p_z, is zero and the differential equation of motion becomes:

$$D \nabla^2\nabla^2 w (x, y, t) + m_s \frac{\partial^2 w (x, y, t)}{\partial t^2} = 0 \qquad (3.43)$$

where $\nabla^2\nabla^2$ represents the two-dimensional Laplacian operator:

$$\nabla^2\nabla^2 = \frac{\partial^4}{\partial x^4} + 2 \frac{\partial^4}{\partial x^2 \, \partial y^2} + \frac{\partial^4}{\partial y^4} \qquad (3.44)$$

The solution of Eq. 3.43 can be put in the following form:

$$w (x, y, t) = W (x, y) \beta (t) \qquad (3.45)$$

$$W (x, y) = X (x) Y (y) \qquad (3.46)$$

The time-dependency of the displacement β (t) is assumed to be harmonic:

$$\beta (t) = e^{j\omega t} \qquad (3.47)$$

The solution w (x, y, t) must satisfy the boundary conditions of the motion at t = 0.

These initial conditions of the plate are: $(w)_{t = 0}$ and $(\dot{w})_{t = 0}$.

Substitute Eq. 3.45 into the differential equation of motion Eq. 3.43 and use primes to denote differentiation with respect to the variables x and y, while dots indicate differentiation with respect to time t. Now the governing differential equation of free vibration becomes:

$$D X''''(x) Y(y) \beta(t) + 2 D X''(x) Y''(y) \beta(t) +$$
$$D X(x) Y''''(y) \beta(t) - m_s\omega^2 X(x) Y(y) \beta(t) = 0 \qquad (3.48)$$

or:

$$X''''Y + 2\,X''Y'' + X\,Y'''' - \frac{m_s\,\omega^2\,X\,Y}{D} = 0 \qquad (3.49)$$

The boundary conditions are assumed to be simply supported; both the displacement and the edge moment are zero:

$$(w)_{x\,=\,0,\;x\,=\,a} = 0 \qquad \left(\frac{\partial^2 w}{\partial x^2}\right)_{x\,=\,0,\;x\,=\,a} = 0$$

and:

$$(w)_{y\,=\,0,\;y\,=\,b} = 0 \qquad \left(\frac{\partial^2 w}{\partial y^2}\right)_{y\,=\,0,\;y\,=\,b} = 0 \qquad (3.50)$$

The solution of the governing differential equation Eq. 3.43 is obtained by Navier's method as follows:

$$W(x,\;y) = X(x)Y(y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn}\,\sin\frac{m\pi x}{a}\,\sin\frac{n\pi y}{b} \qquad (3.51)$$

which expression satisfies all the above stated boundary conditions. Now Eq. 3.49 may be written as follows:

$$\left(\frac{m^4\pi^4}{a^4} + 2\,\frac{m^2\pi^2}{a^2}\,\frac{n^2\pi^2}{b^2} + \frac{n^4\pi^4}{b^4}\right) - \frac{m_s}{D}\,\omega^2 = 0 \qquad (3.52)$$

or:

$$\pi^4\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2 = \frac{m_s}{D}\,\omega^2 \qquad (3.53)$$

which leads to:

$$\omega = \omega_{mn} = \pi^2\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)\sqrt{\frac{D}{m_s}} \qquad (3.54)$$

where $m = 1, 2, 3, \cdots$ and $n = 1, 2, 3, \cdots$

From Eq. 3.54 the natural frequencies of a simply supported plate can be obtained.

3.2.2 Forced Vibration of Simply Supported Plate

The governing differential equation of motion is:

$$D \nabla^2\nabla^2 w (x, y, t) + m_s \frac{\partial^2 w (x, y, t)}{\partial t^2} = p_z (x, y, t) \quad (3.41)$$

The assumption will be made that the forcing pressure on the plate is not a function of x and y, but only a function of time. The forcing pressure is the sum of the incident, the reflected and the transmitted sound pressure:

$$p_z (t) = \bar{p}_i + \bar{p}_r - \bar{p}_t \quad (3.55)$$

The differential equation of motion is linear. This means that the differential equation of motion can be solved separately for $\bar{p}_i$, $\bar{p}_r$ and $\bar{p}_t$. Except for this division, the solution of Eq. 3.41 can also be divided into two parts:

$$w (x, y, t) = w_H (x, y, t) + w_P (x, y, t) \quad (3.56)$$

where w_H represents the solution of the homogeneous form of the equation (Eq. 3.43) while w_P is the particular solution of Eq. 3.41. The homogeneous solution is associated with the free vibration of the plate at its natural frequency. Here it is assumed that the free vibration is successfully damped; consequently, only the particular solution is considered. The total steady-state deflection of the plate is:

$$w_P = \bar{w}_i + \bar{w}_r + \bar{w}_t \quad (3.57)$$

where $\overline{w}_i$, $\overline{w}_r$ and $\overline{w}_t$ represent the complex deflection caused by the incident wave, reflected wave and the transmitted wave, respectively.

In accordance with Navier's solution method, the particular solution of Eq. 3.41 can be written as follows:

$$w_p(x, y, t) = e^{j\omega t} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{3.58}$$

which satisfies the boundary conditions of Eq. 3.50. The lateral load p_z (x, y, t) can also be expanded into a double sine series:

$$p_z(x, y, t) = e^{j\omega t} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} P_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{3.59}$$

Substitution of Eqs. 3.58 and 3.59 in the governing differential equation of motion, Eq. 3.41, yields:

$$D\, W_{mn} \left[\frac{m^4\pi^4}{a^4} + 2\, \frac{m^2\pi^2}{a^2} \frac{n^2\pi^2}{b^2} + \frac{n^4\pi^4}{b^4} \right] - m_s\, \omega^2\, W_{mn} = P_{mn} \tag{3.60}$$

This expression can also be written as follows:

$$W_{mn} = \frac{P_{mn}}{D\, \pi^4 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 - m_s\, \omega^2} \tag{3.61}$$

Substitute Eq. 3.54 in Eq. 3.61 and the result is:

$$W_{mn} = \frac{P_{mn}}{m_s\, (\omega_{mn}^2 - \omega^2)} \tag{3.62}$$

According to Ref. 14 the coefficient of the double Fourier expansion of the lateral load is:

$$P_{mn} = \frac{4}{a\,b} \int_0^a \int_0^b p_z\ (x,\ y)\ \sin \frac{m\pi x}{a}\ \sin \frac{n\pi y}{b}\ d\,x\ d\,y \tag{3.63}$$

Here the lateral load is assumed to be not a function of x and y but a constant:

$$p_z\ (x,\ y) = p_o \tag{3.64}$$

From Eqs. 3.63 and 3.64 the following result can be derived:

$$P_{mn} = \begin{cases} \dfrac{16\ p_o}{\pi^2\ mn} & \text{both m and n are odd} \\ 0 & \text{otherwise} \end{cases} \tag{3.65}$$

As mentioned above, the lateral load can be divided into three separate loads: the incident, the reflected and the transmitted pressure wave. At $z = 0$:

$$p_z\ (t) = p_o\ e^{j\omega t} \tag{3.66}$$

or:

$$p_z\ (t) = A\ e^{j\omega_i t} + \overline{B}\ e^{j\omega_r t} - \overline{C}\ e^{j\omega_t t} \tag{3.67}$$

which expression may also be written in the following manner:

$$p_z\ (t) = A\ e^{j\omega_i t} + B\ e^{j\ (\omega_r t\ +\ \theta_r)} - C\ e^{j\ (\omega_t t\ +\ \theta_t)} \tag{3.68}$$

where θ_r and θ_t are the phase angles between the incident and the reflected wave and the incident and the transmitted wave, respectively.

In the case that the incident wave excites the panel as only one:

$$p_o\ e^{j\omega t} = A\ e^{j\omega_i t} \tag{3.69}$$

From this expression follows:

$$\begin{aligned} p_o &= A \\ \omega &= \omega_i \end{aligned} \tag{3.70}$$

Secondly, the reflected wave is the only wave which strikes the panel; then:

$$p_o\ e^{j\omega t} = B\ e^{j\ (\omega_r t + \theta_r)} \tag{3.71}$$

or:

$$p_o \cos \omega t = B \cos (\omega_r t + \theta_r) \tag{3.72}$$

$$p_o \sin \omega t = B \sin (\omega_r t + \theta_r) \tag{3.73}$$

From Eqs. 3.72 and 3.73 follows:

$$\begin{aligned} \omega &= \omega_r + \theta_r/t \\ p_o &= B \end{aligned} \tag{3.74}$$

Third and last case is that the transmitted wave is the only wave which hits the panel:

$$p_o\ e^{j\omega t} = -\ C\ e^{j\ (\omega_t t + \theta_t)} \tag{3.75}$$

or:

$$\begin{aligned} \omega &= \omega_t + \theta_t/t \\ p_o &= -\ C \end{aligned} \tag{3.76}$$

Substitution of Eqs. 3.58, 3.70, 3.74 and 3.76 in Eq. 3.57 results in:

$$w_p = e^{j\omega_i t} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn_i} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} +$$

$$e^{j(\omega_r t + \theta_r)} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn_r} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} +$$

$$e^{j(\omega_t t + \theta_t)} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn_t} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \qquad (3.77)$$

where:

$$W_{mn_i} = \frac{16\ A}{m_s\ \pi^2\ m\ n\ (\omega_{mn}^2 - \omega_i^2)} \qquad (3.78)$$

$$W_{mn_r} = \frac{16\ B}{m_s\ \pi^2\ m\ n\ (\omega_{mn}^2 - \omega_r^2)} \qquad (3.79)$$

and:

$$W_{mn_t} = \frac{-\ 16\ C}{m_s\ \pi^2\ m\ n\ (\omega_{mn}^2 - \omega_t^2)} \qquad (3.80)$$

which expressions follow from Eqs. 3.62, 3.65, 3.70, 3.74 and 3.76.

3.2.3 Normal Specific Acoustic Impedance of Panel

The normal specific acoustic impedance $\overline{z}_p$ of a plate is defined as the ratio of the driving acoustic pressure to the resulting velocity of the plate:

$$\overline{z}_p = \frac{\overline{p}_i + \overline{p}_r - \overline{p}_t}{\dot{w}_p} \qquad (3.81)$$

where $\dot{w}_p$ represents the panel velocity. The panel velocity can be derived from Eq. 3.77 and may be written as follows:

$$\dot{w}_p = j\,\omega_i\,\overline{w}_i + j\,\omega_r\,\overline{w}_r + j\,\omega_t\,\overline{w}_t \tag{3.82}$$

Now the panel impedance can be written:

$$\overline{z}_p = \frac{A\,e^{j\omega_i t} + B\,e^{j(\omega_r t + \theta_r)} - C\,e^{j(\omega_t t + \theta_t)}}{j\,(\omega_i\,\overline{w}_i + \omega_r\,\overline{w}_r + \omega_t\,\overline{w}_t)} \tag{3.83}$$

Substitution of Eqs. 3.77, 3.78, 3.79 and 3.80 in Eq. 3.83 results in the following expression:

$$\overline{z}_p = \frac{A\,e^{j\omega_i t} + B\,e^{j(\omega_r t + \theta_r)} - C\,e^{j(\omega_t t + \theta_t)}}{\frac{j\,16}{m_s\,\pi^2}\left\{\omega_i e^{j\omega_i t}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{A}{(\omega_{mn}^2-\omega_i^2)}\psi_{mn} + \omega_r e^{j(\omega_r t+\theta_r)}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{B}{(\omega_{mn}^2-\omega_r^2)}\psi_{mn} - \omega_t e^{j(\omega_t+\theta_t)}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{C}{(\omega_{mn}^2-\omega_t^2)}\psi_{mn}\right\}}$$

(3.84)

where:

$$\psi_{mn} = \frac{1}{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{3.85}$$

To obtain a useful expression, the following assumption will be made:

$$\omega_i = \omega_r = \omega_t = \omega$$

In that case Eq. 3.84 simplifies to the following equation:

$$\overline{z}_p = \frac{1}{\frac{j\,16}{m_s\,\pi^2}\,\omega\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin(m\pi x/a)\,\sin(n\pi y/b)}{m\,n\,(\omega_{mn}^2-\omega^2)}} \tag{3.86}$$

In Section 3.1 the panel impedance is defined as:

$$\bar{z}_p = r_p + jx_p \tag{3.11}$$

Combination of Eq. 3.11 and 3.86 leads to the following result:

$$r_p = 0$$
$$x_p = \frac{-\,m_s\,\pi^2/16\,\omega}{\sum\limits_{m=1}^{\infty}\sum\limits_{n=1}^{\infty}\dfrac{\sin\,(m\pi x/a)\,\sin\,(n\pi y/b)}{m\,n\,(\omega_{mn}^2-\omega^2)}} \tag{3.88}$$

The panel resistance r_p is zero. This results from the assumption made earlier, that the internal damping of the plate is zero.

CHAPTER 4

NOISE REDUCTION THROUGH A CAVITY-BACKED PANEL

In the preceding chapter the transmission of sound through a panel backed by an infinite space has been discussed. In this chapter the effect of a finite space behind the panel will be discussed. As will be shown, this effect is quite noticeable, especially in the case of a panel with a low stiffness.

4.1 Transmission of Sound through Panel

The model which will be considered in this chapter is sketched in Figure 4.1.

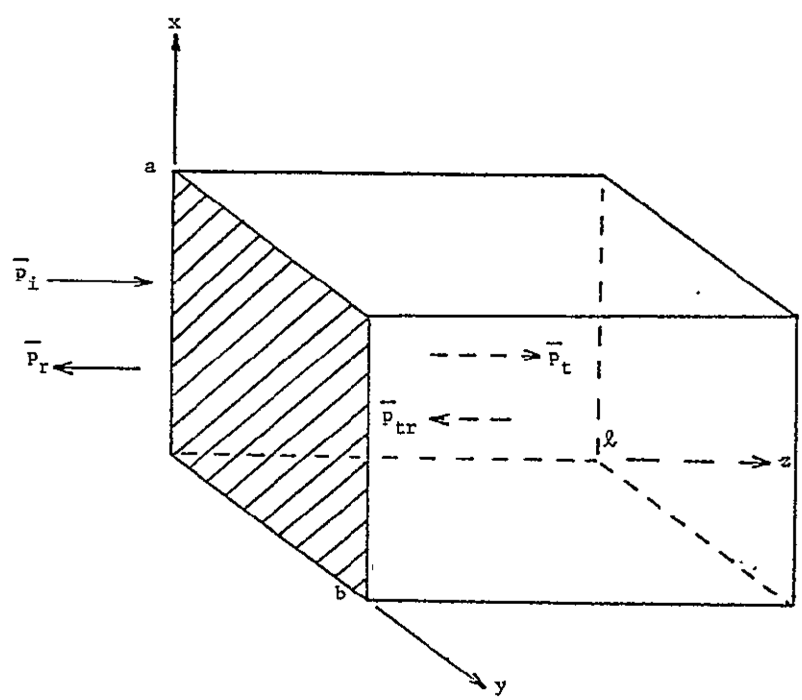

Figure 4.1: Sketch of Panel and Cavity Geometry

At z = 0 the side of the box consists of a flexible panel which has a normal specific acoustic impedance $\bar{z}_p$. At $z = \ell$ the rear wall of the cavity is situated, and this wall has a normal specific impedance $\bar{z}_w$. The model used only considers sound waves in one direction: the z-directions. Thus cavity modes in the x-direction and the y-direction will be omitted. This assumption is reasonable as long as the length (depth) of the cavity is much larger than the width and height of the cavity.

Again there is an incident sound wave in medium I which may be represented by the following expression:

$$\bar{p}_i = A\, e^{j(\omega_i t - k_i z)} \tag{4.1}$$

where A is the amplitude of the sound wave and the wave length constant is:

$$k_i = \frac{\omega_i}{c_1} \tag{4.2}$$

When the sound wave strikes the panel, a reflected wave and a transmitted wave will be produced. The reflected wave may be written as follows:

$$\bar{p}_r = \bar{B}\, e^{j(\omega_r t + k_r z)} \tag{4.3}$$

where $\bar{B}$ represents the complex amplitude of the reflected sound wave and the wavelength constant is:

$$k_r = \frac{\omega_r}{c_1} \tag{4.4}$$

The transmitted wave is:

$$\bar{p}_t = \bar{C}\, e^{j(\omega_t t - k_t z)} \tag{4.5}$$

where $\overline{C}$ is the complex amplitude of the wave and:

$$k_t = \frac{\omega_t}{c_2} \tag{4.6}$$

Now the panel is not backed by an infinite space, as in Chapter 3. Because of the backing wall, a reflected wave will be produced when the transmitted wave strikes the backing wall. This transmitted-reflected wave may be represented by the following expression:

$$\overline{p}_{tr} = \overline{D}\, e^{j(\omega_{tr} t + k_{tr} z)} \tag{4.7}$$

where:

$$k_{tr} = \frac{\omega_{tr}}{c_2} \tag{4.8}$$

and $\overline{D}$ is the complex pressure amplitude of the transmitted-reflected wave.

The normal specific acoustic impedance of the wall at $z = \ell$ is:

$$\overline{z}_w = r_w + j\, x_w \tag{4.9}$$

where r_w represents the resistive component and x_w the reactive component of the wall impedance. Here it will be assumed that the wall is very stiff and does not vibrate, in which case $x_w = 0$. When the wall is a perfect reflector, the impedance of the wall $z_w = \infty$. However, when the wall also consists of absorptive materials, then $z_w = r_w \neq \infty$. In this section the wall impedance will be written as:

$$z_w = r_w \tag{4.10}$$

The separate boundary conditions of continuity of pressure and continuity of particle velocity at $z = \ell$ become a condition of continuity of their ratio:

$$\frac{\overline{p}_t + \overline{p}_{tr}}{\overline{u}_t + \overline{u}_{tr}} = \overline{z}_w \tag{4.11}$$

at $z = \ell$. In Eq. 4.11 $\overline{u}_t$ and $\overline{u}_{tr}$ represent the particle velocity of the transmitted wave and the transmitted-reflected wave, respectively. The transmitted wave and the transmitted-reflected wave both are assumed to be plane progressive waves. In that case the particle velocity of the transmitted wave may be written as:

$$\overline{u}_t = \frac{\overline{p}_t}{\rho_2 c_2} \tag{4.12}$$

while the particle velocity of the transmitted-reflected wave is:

$$\overline{u}_{tr} = -\frac{\overline{p}_{tr}}{\rho_2 c_2} \tag{4.13}$$

Substitution of the Eqs. 4.10, 4.12 and 4.13 in Eq. 4.11 results in the following expression:

$$\frac{\overline{p}_t + \overline{p}_{tr}}{\overline{p}_t - \overline{p}_{tr}} = \frac{r_w}{\rho_2 c_2} \tag{4.14}$$

From Eq. 4.14 the following expression may be derived:

$$\frac{\overline{p}_{tr}}{\overline{p}_t} = \frac{r_w - \rho_2 c_2}{r_w + \rho_2 c_2} \tag{4.15}$$

Substitution of Eqs. 4.5 and 4.7 in Eq. 4.15 leads to the following result:

$$\frac{D\, e^{j(\omega_{tr} t + k_{tr} \ell + \theta_{tr})}}{C\, e^{j(\omega_t t - k_t \ell + \theta_t)}} = \frac{r_w - \rho_2 c_2}{r_w + \rho_2 c_2} \tag{4.16}$$

where θ_{tr} is the phase angle between the transmitted-reflected wave and the incident wave and θ_t represents the phase angle between the transmitted wave and the incident wave. By taking the magnitude of both sides of Eq. 4.16, the following result is obtained:

$$\frac{D}{C} = \frac{r_w - \rho_2 c_2}{r_w + \rho_2 c_2} \tag{4.17}$$

According to Eq. 4.15 the ratio $\overline{p}_{tr} / \overline{p}_t$ is real and does not have an imaginary part. Eq. 4.16 may be written as follows:

$$\frac{D\ (\cos\alpha + j\sin\alpha)}{C\ (\cos\beta + j\sin\beta)} = \frac{r_w - \rho_2 c_2}{r_w + \rho_2 c_2} \tag{4.18}$$

where:

$$\begin{aligned} \alpha &= \omega_{tr} t + k_{tr} \ell + \theta_{tr} \\ \beta &= \omega_t t - k_t \ell + \theta_t \end{aligned} \tag{4.19}$$

Now from Eq. 4.18 the following expression can be derived:

$$\begin{aligned} &\frac{D}{C} (\cos\alpha + j\sin\alpha)(\cos\beta - j\sin\beta) = \\ &\frac{D}{C} \{ \cos\alpha\cos\beta + \sin\alpha\sin\beta + j(\sin\alpha\cos\beta - \sin\beta\cos\alpha) \} = \\ &\frac{r_w - \rho_2 c_2}{r_w + \rho_2 c_2} = \text{real} \end{aligned} \tag{4.20}$$

This means that the imaginary part in the left-hand side of Eq. 4.20 has to be zero:

$$\tan \alpha = \tan \beta \tag{4.21}$$

or:

$$\omega_{tr} t + k_{tr} \ell + \theta_{tr} = \omega_t t - k_t \ell + \theta_t \tag{4.22}$$

When the assumption is made:

$$\omega_t = \omega_{tr} = \omega_2$$

then Eq. 4.22 becomes:

$$\theta_t - \theta_{tr} = \frac{2\ \omega_2\ \ell}{c_2} \tag{4.23}$$

From this expression follows that the phase difference between the transmitted wave and the transmitted-reflected wave is determined by the frequency of the sound wave and the cavity depth. This occurs when the speed of sound of air is assumed to be a constant.

At $z = 0$ the impedance of the incident and reflected wave is identical to the sum of the normal specific acoustic impedance of the plate and the impedance of the transmitted and the transmitted-reflected wave:

$$\frac{\overline{p}_i + \overline{p}_r}{\overline{u}_i + \overline{u}_r} = \overline{z}_p + \frac{\overline{p}_t + \overline{p}_{tr}}{\overline{u}_t + \overline{u}_{tr}} \tag{4.24}$$

where $\overline{z}_p$ represents the panel impedance. This impedance will be written as follows:

$$\overline{z}_p = r_p + j\, x_p \tag{4.25}$$

The Eqs. 4.12 and 4.13 already give the particle velocity of the transmitted and the transmitted-reflected wave. The incident wave and the reflected wave are also assumed to be plane progressive waves. Then the particle velocity of the incident wave is:

$$\overline{u}_i = \frac{\overline{p}_i}{\rho_1 c_1} \tag{4.26}$$

while the particle velocity of the reflected wave may be written as follows:

$$\overline{u}_r = - \frac{\overline{p}_r}{\rho_1 c_1} \tag{4.27}$$

Substitution of Eqs. 4.12, 4.13, 4.25, 4.26 and 4.27 into Eq. 4.24 results in the following expression:

$$\frac{\overline{p}_i + \overline{p}_r}{\overline{p}_i - \overline{p}_r} \rho_1 c_1 = r_p + jx_p + \frac{\overline{p}_t + \overline{p}_{tr}}{\overline{p}_t - \overline{p}_{tr}} \rho_2 c_2 \tag{4.28}$$

or:

$$\frac{\overline{p}_i + \overline{p}_r}{\overline{p}_i - \overline{p}_r} \rho_1 c_1 = r_p + jx_p + \frac{1 + \overline{p}_{tr}/\, \overline{p}_t}{1 - \overline{p}_{tr}/\, \overline{p}_t} \rho_2 c_2 \tag{4.29}$$

The ratio of the transmitted-reflected sound pressure to the transmitted sound pressure at z = 0 is:

$$\frac{\overline{p}_{tr}}{\overline{p}_t} = \frac{D\ e^{j(\omega_{tr} t + \theta_{tr})}}{C\ e^{j(\omega_t t + \theta_t)}} \tag{4.30}$$

With the assumption $\omega_t = \omega_{tr} = \omega_2$ and the use of Eq. 4.16, Eq. 4.30 becomes:

$$\frac{\overline{p}_{tr}}{\overline{p}_t} = \frac{D\, e^{j\theta_{tr}}}{C\, e^{j\theta_t}} = \frac{r_w - \rho_2 c_2}{r_w + \rho_2 c_2}\, e^{-2jk_2\ell} \tag{4.31}$$

Substitution of Eq. 4.31 in Eq. 4.29 results in the following expression:

$$\frac{\overline{p}_i + \overline{p}_r}{\overline{p}_i - \overline{p}_r}\,\rho_1 c_1 = r_p + jx_p + \frac{r_w(1 + e^{-2jk_2\ell}) + \rho_2 c_2 (1 - e^{-2jk_2\ell})}{r_w(1 - e^{-2jk_2\ell}) + \rho_2 c_2 (1 + e^{-2jk_2\ell})}\,\rho_2 c_2 \tag{4.32}$$

or:

$$\frac{\overline{p}_i + \overline{p}_r}{\overline{p}_i - \overline{p}_r}\,\rho_1 c_1 = r_p + jx_p + \frac{r_w + j\,\rho_2 c_2 \tan k_2\ell}{\rho_2 c_2 + j\, r_w \tan k_2\ell}\,\rho_2 c_2 \tag{4.33}$$

The right-hand side, RHS, of Eq. 4.33 may be written as follows:

$$\mathrm{RHS} = r_p + \frac{(1 + \tan^2 k_2\ell)(\rho_2 c_2)^2\, r_w}{(\rho_2 c_2)^2 + r_w^{\,2} \tan^2 k_2\ell} + j\left[x_p + \frac{\{(\rho_2 c_2)^2 - r_w^{\,2}\}\,\rho_2 c_2 \tan k_2\ell}{(\rho_2 c_2)^2 + r_w^{\,2}\tan^2 k_2\ell}\right] \tag{4.34}$$

or:

$$\mathrm{RHS} = \kappa + j\,\lambda \tag{4.35}$$

Eq. 4.33 may now be written as:

$$\frac{\overline{p}_i + \overline{p}_r}{\overline{p}_i - \overline{p}_r}\,\rho_1 c_1 = \kappa + j\,\lambda \tag{4.36}$$

From Eq. 4.36 follows:

$$\frac{\overline{p}_r}{\overline{p}_i} = \frac{\kappa - \rho_1 c_1 + j\,\lambda}{\kappa + \rho_1 c_1 + j\,\lambda} \tag{4.37}$$

By taking the magnitude of both sides of Eq. 4.37 (in the same way as done in Appendix A) the following result is obtained:

$$\frac{B}{A} = \left[\frac{(\kappa - \rho_1 c_1)^2 + \lambda^2}{(\kappa + \rho_1 c_1)^2 + \lambda^2}\right]^{1/2} \tag{4.38}$$

The sound power reflection coefficient is:

$$\alpha_r = \frac{B^2}{A^2} = \frac{(\kappa - \rho_1 c_1)^2 + \lambda^2}{(\kappa + \rho_1 c_1)^2 + \lambda^2} \tag{4.39}$$

where κ and λ follow from Eq. 4.34.

At $z = 0$ Eq. 4.37 may also be written as follows ($\omega_i = \omega_r = \omega_1$):

$$\frac{B}{A}\,e^{j\theta_r} = \frac{\kappa - \rho_1 c_1 + j\,\lambda}{\kappa + \rho_1 c_1 + j\,\lambda} \tag{4.40}$$

where θ_r represents the phase angle between the reflected wave and the incident wave. From Eq. 4.40 follows:

$$\frac{B}{A}\,e^{j\theta_r} = \frac{\kappa^2 - (\rho_1 c_1)^2 + \lambda^2 + j2\lambda\rho_1 c_1}{(\kappa + \rho_1 c_1)^2 + \lambda^2} \tag{4.41}$$

With this expression the phase angle θ_r can be derived:

$$\theta_r = \arctan\left\{\frac{2\lambda\rho_1 c_1}{\kappa^2 - (\rho_1 c_1)^2 + \lambda^2}\right\} \tag{4.42}$$

At z = 0 also the following boundary condition must be satisfied: the particle velocities normal to the surface of the plate are equal. This is true when the panel damping is zero. This boundary condition can be written as follows:

$$\overline{u}_i + \overline{u}_r = \overline{u}_t + \overline{u}_{tr} \tag{4.43}$$

Substitution of Eqs. 4.12, 4.13, 4.26 and 4.27 in Eq. 4.43 results in the following expression:

$$\frac{\overline{p}_i - \overline{p}_r}{\rho_1 c_1} = \frac{\overline{p}_t - \overline{p}_{tr}}{\rho_2 c_2} \tag{4.44}$$

Now the assumption will be made:

$$\omega_i = \omega_r = \omega_t = \omega_{tr} = \omega$$

In that case Eq. 4.44 becomes:

$$A - \overline{B} = \frac{\rho_1 c_1}{\rho_2 c_2}(\overline{C} - \overline{D}) \tag{4.45}$$

or:

$$\frac{\overline{C}}{A} = \frac{\rho_2 c_2}{\rho_1 c_1}\frac{(1 - \overline{B}/A)}{(1 - \overline{D}/\overline{C})} \tag{4.46}$$

The ratio $\overline{B}/A$ follows from Eq. 4.37, while $\overline{D}/\overline{C}$ comes from Eq. 4.31. Substitution of these expressions in Eq. 4.46 results in:

$$\frac{\overline{C}}{A} = \frac{\rho_2 c_2}{\rho_1 c_1} \frac{\dfrac{2\rho_1 c_1}{\kappa + \rho_1 c_1 + j\lambda}}{\dfrac{r_w (1 - e^{-2jk_2\ell}) + \rho_2 c_2 (1 + e^{-2jk_2\ell})}{r_w + \rho_2 c_2}} \tag{4.47}$$

or:

$$\frac{\overline{C}}{A} = \rho_2 c_2 (r_w + \rho_2 c_2) \frac{1 + j \tan k_2\ell}{(\kappa + \rho_1 c_1 + j\lambda)(\rho_2 c_2 + j r_w \tan k_2\ell)} \tag{4.48}$$

The denominator of the right-hand side of Eq. 4.48 may be written as follows:

$$(\kappa\rho_2 c_2 + \rho_1 c_1 \rho_2 c_2 - \lambda r_w \tan k_2\ell) + j(\kappa r_w \tan k_2\ell + \rho_1 c_1 r_w \tan k_2\ell + \rho_2 c_2 \lambda) \tag{4.49}$$

Now the magnitude of Eq. 4.48 may be taken. This leads to the following result:

$$\frac{C}{A} = \rho_2 c_2 (r_w + \rho_2 c_2) \sqrt{\frac{1 + \tan^2 k_2\ell}{(\kappa\rho_2 c_2 + \rho_1 c_1 \rho_2 c_2 - \lambda r_w \tan k_2\ell)^2 + (\kappa r_w \tan k_2\ell + \rho_1 c_1 r_w \tan k_2\ell + \rho_2 c_2 \lambda)^2}} \tag{4.50}$$

$$\frac{C}{A} = \rho_2 c_2 (r_w + \rho_2 c_2) \sqrt{\frac{1 + \tan^2 k_2\ell}{\{(\kappa + \rho_1 c_1) + \lambda^2\}\{r_w^2 \tan^2 k_2\ell + (\rho_2 c_2)^2\}}} \tag{4.51}$$

With above expressions the noise reduction through a cavity-backed flexible plate can be determined. Noise reduction is the difference in sound pressure level at the source side of the panel and the receiver side, or:

$$NR = SPL_1 - SPL_2 \tag{4.52}$$

where SPL_1 is the sound pressure level at the source side and SPL_2 the sound pressure level at the receiver side of the panel, inside the cavity. Sound pressure level is defined as:

$$SPL = 10 \log \frac{P_e^{\,2}}{P_{ref}^{\,2}} \tag{4.53}$$

where P_e represents the measured effective pressure (rms) of the sound wave and P_{ref} is the reference effective pressure. Here the rms sound pressure at the source side is P_1, while the rms sound pressure at the receiver side is P_2. Now Eq. 4.52 may be written in the following manner:

$$NR = 10 \log \frac{P_1^{\,2}}{P_2^{\,2}} \tag{4.54}$$

The acoustic pressure p_1 at a point z_1 at the source side of the panel is given by the sum of the real part of the incident sound wave $\overline{p}_i$ and the real part of the reflected sound wave $\overline{p}_r$. Therefore:

$$p_1 = A \cos (\omega_i t - k_i \, z_1) + B \cos (\omega_r t + k_r \, z_1 + \theta_r) \tag{4.55}$$

where θ_r indicates the phase difference between the incident wave and the reflected wave. In the same manner as done in Section 3.1, the rms pressure can be derived.

When the frequency of the incident wave is not the same as the frequency of the reflected wave ($\omega_i \neq \omega_r$), then the rms pressure at the source side may be written as follows:

$$P_1 = \sqrt{\frac{A^2 + B^2}{2}} \tag{4.56}$$

In the case that the frequencies of the incident wave and the reflected wave are identical ($\omega_i = \omega_r = \omega_1$), the rms pressure is:

$$P_1 = \sqrt{\frac{A^2 + B^2 + 2\, AB \cos\,(\theta_r + 2\, k_1\, z_1)}{2}} \tag{4.57}$$

This expression indicates clearly the importance of the microphone position.

At the receiver side of the panel (inside the cavity) the acoustic pressure p_2 at a fixed point z_2 is the sum of the real part of the transmitted sound pressure and the transmitted-reflected sound pressure. Therefore:

$$p_2 = C \cos\,(\omega_t t - k_t z_2 + \theta_t) + D \cos\,(\omega_{tr} t + k_{tr} z_2 + \theta_{tr}) \tag{4.58}$$

where θ_t indicates the difference in phase between the transmitted wave and the incident wave, while θ_{tr} indicates the phase difference between the transmitted-reflected wave and the incident wave. In the case that the frequency of the transmitted sound wave is not identical to the frequency of the transmitted-reflected sound wave, no standing waves will occur inside the cavity and the rms sound pressure is not a function of the microphone position as shown by the following expression:

$$P_2 = \sqrt{\frac{C^2 + D^2}{2}} \quad (\omega_t \neq \omega_{tr}) \tag{4.59}$$

When the frequencies of the transmitted sound wave and the transmitted-reflected sound wave are identical, then the rms sound pressure is:

$$P_2 = \sqrt{\frac{C^2 + D^2 + 2\ CD\ \cos\ (\theta_t - \theta_{tr} - 2\ k_2\ z_2)}{2}} \tag{4.60}$$

Substitution of Eqs. 4.57 and 4.60 in Eq. 4.54 gives the following result:

$$NR = 10\ \log \frac{A^2 + B^2 + 2\ A\ B\ \cos\ (\theta_r + 2\ k_1\ z_1)}{C^2 + D^2 + 2\ C\ D\ \cos\ (\theta_t - \theta_{tr} - 2\ k_2\ z_2)} \tag{4.61}$$

$$NR = 10\ \log \frac{A^2}{C^2}\ \frac{1 + \frac{B^2}{A^2} + 2\ \frac{B}{A}\ \cos\ (\theta_r + 2\ k_1\ z_1)}{1 + \frac{D^2}{C^2} + 2\ \frac{D}{C}\ \cos\ (\theta_t - \theta_{tr} - 2\ k_2\ z_2)} \tag{4.62}$$

where B/A follows from Eq. 4.38, θ_r from Eq. 4.42, D/C from Eq. 4.17, $\theta_t - \theta_{tr}$ from Eq. 4.23 and C/A from Eq. 4.51.

Eq. 4.62 is valid when the frequency of the incident sound wave and the reflected sound wave are identical and the frequency of the transmitted wave is identical to the frequency of the transmitted-reflected wave. However, to derive the expression for the ratio C/A, the assumption is made that the frequency of the sound waves at the source side is the same as the frequency at the receiver side or:

$$\omega_i = \omega_r = \omega_t = \omega_{tr} = \omega$$

In the following sections expressions will be derived for the impedance of the panel and the backing wall.

4.2 Panel Impedance of a Cavity-Backed Flexible Panel with Simply Supported Boundary Conditions

The derivation to obtain the normal specific acoustic impedance of a cavity-backed flexible panel is basically identical to the one in Section 3.2, the impedance of a free flexible panel. In this section, too, the structural damping of the plate will be ignored. According to Ref. 14 structural damping usually has little or no effect on the natural frequencies and on the steady-state amplitude of the plate.

The governing differential equation of motion of an isotropic flat plate is:

$$D \nabla^2\nabla^2 w (x, y, t) + m_s \frac{\partial^2 w (x, y, t)}{\partial t^2} = p_z (x, y, t) \tag{4.63}$$

This equation is based on the small deflection plate theory. In the following sub-sections the free vibration and the forced vibration of a simply supported cavity-backed flexible plate will be discussed.

4.2.1 Free Vibration of Simply Supported Plate

When the plate is vibrating freely, the external force, p_z, is omitted and the differential equation of motion is:

$$D \nabla^2\nabla^2 w (x, y, t) + m_s \frac{\partial^2 w (x, y, t)}{\partial t^2} = 0 \tag{4.64}$$

This expression is identical to Eq. 3.43. Therefore, the solution will also be identical. Eq. 3.54 gives the natural frequencies of a freely vibrating plate, namely:

$$\omega_{mn} = \pi^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right) \sqrt{\frac{D}{m_s}} \qquad (4.65)$$

where m = 1, 2, 3, . . .; n = 1, 2, 3 . . .; D represents the bending stiffness and m_s the surface mass of the plate.

4.2.2 Forced Vibration of a Cavity-Backed Simply Supported Plate

The governing differential equation of motion of the plate is given by Eq. 4.63. The assumption will be made that the plate will be hit by plane waves only. In that case the forcing function is not a function of the coordinates x and y any more, but only a function of time. When discussing the forcing function of the free panel, it was concluded that the forcing pressure was the sum of the incident, reflected and the transmitted sound pressure. However, a cavity-backed panel is also excited by the transmitted-reflected wave or:

$$p_z(t) = (\overline{p}_i + \overline{p}_r) - (\overline{p}_t + \overline{p}_{tr}) \qquad (4.66)$$

The differential equation of motion is linear. This means that Eq. 4.63 can be solved separately for $\overline{p}_i$, $\overline{p}_r$, $\overline{p}_t$ and $\overline{p}_{tr}$. In that case the total solution is the sum of the partial solutions. The total plate deflection can be divided in the transient deflection w_H and the steady-state deflection w_p. Here only the steady-state deflection will be considered. The total steady-state deflection of the plate will be:

$$w_p = \overline{w}_i + \overline{w}_r + \overline{w}_t + \overline{w}_{tr} \qquad (4.67)$$

In the same manner as done in Section 3.2.2 for the free panel, the steady-state deflection of a cavity-backed panel can be written as:

$$w_p = e^{j\omega_i t} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn_i} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} +$$

$$e^{j(\omega_r t + \theta_r)} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn_r} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} +$$

$$e^{j(\omega_t t + \theta_t)} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn_t} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} +$$

$$e^{j(\omega_{tr} t + \theta_{tr})} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn_{tr}} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{4.68}$$

where:

$$W_{mn_i} = \frac{16\ A}{m_s\ \pi^2\ m\ n\ (\omega_{mn}^2 - \omega_i^2)} \tag{4.69}$$

$$W_{mn_r} = \frac{16\ B}{m\ \ \pi^2\ m\ n\ (\omega_{mn}^2 - \omega_r^2)} \tag{4.70}$$

$$W_{mn_t} = \frac{-\ 16\ C}{m_s\ \pi^2\ m\ n\ (\omega_{mn}^2 - \omega_t^2)} \tag{4.71}$$

$$W_{mn_{tr}} = \frac{-\ 16\ D}{m_s\ \pi^2\ m\ n\ (\omega_{mn}^2 - \omega_{tr}^2)} \tag{4.72}$$

With above expressions the normal specific impedance of the panel can be obtained.

4.2.3 Normal Specific Acoustic Impedance of a Cavity-Backed Panel

The normal specific acoustic impedance of a plate is defined as the ratio of the driving acoustic pressure to the resulting velocity of the plate:

$$\bar{z}_p = \frac{\bar{p}_i + \bar{p}_r - \bar{p}_t - \bar{p}_{tr}}{\dot{w}_p} \tag{4.73}$$

The panel velocity $\dot{w}_p$ can be obtained from Eq. 4.68 and may be written as follows:

$$\dot{w}_p = j\omega_i\bar{w}_i + j\omega_r\bar{w}_r + j\omega_t\bar{w}_t + j\omega_{tr}\bar{w}_{tr} \tag{4.74}$$

where $\bar{w}_i$, $\bar{w}_r$, $\bar{w}_t$ and $\bar{w}_{tr}$ are given in Eq. 4.68. The panel impedance may be written as:

$$\bar{z}_p = \frac{A\, e^{j\omega_i t} + B\, e^{j(\omega_r t + \theta_r)} - C\, e^{j(\omega_t t + \theta_t)} - D\, e^{j(\omega_{tr} t + \theta_{tr})}}{j\,(\omega_i\,\bar{w}_i + \omega_r\,\bar{w}_r + \omega_t\,\bar{w}_t + \omega_{tr}\,\bar{w}_{tr})} \tag{4.75}$$

Substitution of Eqs. 4.68, 4.69, 4.70, 4.71 and 4.72 in Eq. 4.75 results in a very complicated expression. However, the assumption:

$$\omega_i = \omega_r = \omega_t = \omega_{tr} = \omega$$

simplifies this expression and makes it more useful. Because of this assumption the panel impedance is:

$$\bar{z}_p = \frac{1}{\dfrac{j\,16}{m_s\pi^2}\,\omega \displaystyle\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{\sin(m\pi x/a)\,\sin(n\pi y/b)}{mn\,(\omega_{mn}^2 - \omega^2)}} \tag{4.76}$$

This expression is identical to the expression of the panel impedance of the free panel (see Eq. 3.86).

The panel resistance r_p and reactance x_p are:

$$r_p = 0 \tag{4.77}$$

$$x_p = -\frac{m_s \pi^2}{16\ \omega} \frac{1}{\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin\ (m\pi x/a)\ \sin\ (n\pi y/b)}{m\ n\ (\omega_{mn}^2 - \omega^2)}}$$

These two expressions are identical to Eq. 3.88.

4.3 Normal Specific Acoustic Impedance of the Backing Wall

In Ref. 15 (page 448) is mentioned that the simplest and most direct procedure to introduce cavity damping is to consider the absorbing characteristics of the wall as being determined by its normal specific acoustic impedance, $\bar{z}_w$, the ratio of pressure to normal velocity at the surface of the wall. Ref. 15 expresses the normal specific acoustic impedance of a material relative to air by the expression:

$$\bar{z}_w = (r_n + j\, x_n)\, \rho_2 c_2 \tag{4.78}$$

Many of the materials, however, used for wall surfaces are characterized by $r_n >> x_n$ and $r_n >> 1$. The imaginary part of the wall impedance may be omitted in most cases.

In the model used in Ref. 9, the absorption characteristics are modeled by an impedance z_w. For representative acoustic materials:

$$\frac{z_w}{\rho_2 c_2} = 1 - 10 \tag{4.79}$$

In Ref. 9 also experimental results are presented for the damping ratio of the cavity as function of the frequency. The cavity wall absorption causes the damping. The curve obtained for the damping ratio of the cavity suggests that the cavity damping ratio at a constant cavity depth varies as the inverse of the frequency squared. This result, as mentioned by Dowell, is inconsistent with the theory. No explanation can be given for this damping result, other than this behavior indicates a more complex damping mechanism than that of the panel. Also in this reference the wall reactance is assumed to be much smaller than the wall resistance.

In Ref. 10 the cavity has an absorptive wall, too. The impedance of the absorptive wall is assumed to be $\rho_2 c_2$ at high frequencies. At low frequencies, however, the absorption material does not have any effect and the normal specific impedance of the wall will approach infinity as the frequency tends to zero. Therefore, a real relationship for z_w is assumed that varies from infinity at zero frequency to $\rho_2 c_2$ at high frequencies:

$$z_w = \rho_2 c_2 \{ 1 + (\frac{\omega_{100}}{2\omega})^2 \} \tag{4.80}$$

where ω_{100} represents the first cavity depth mode:

$$\omega_{100} = \frac{\pi c_2}{\ell} \tag{4.81}$$

where ℓ represents the cavity length (depth).

CHAPTER 5

RESULTS AND COMPARISONS

In Chapter 3 the transmission of sound through a free panel has been discussed, while in Chapter 4 the transmission of sound through a cavity-backed panel has been examined. As can be expected, these two cases do not stand on their own. The noise reduction through a free panel can be seen as a special case of the noise reduction through a cavity-backed panel. The cavity behind a free panel can be considered to have an infinite size, or the backing wall has the same impedance as air:

$$r_w = \rho_2 c_2 \tag{5.1}$$

where $\rho_2 c_2$ is called the characteristic impedance (resistance) of the air at the receiver side. Substitution of Eq. 5.1 in Eq. 4.17 results in:

$$\frac{D}{C} = 0 \tag{5.2}$$

or the pressure amplitude of the transmitted-reflected wave is zero, which means no reflection from the backing wall. Substitution of Eq. 5.1 into Eq. 4.39 gives the following result for the sound power reflection coefficient:

$$\alpha_r = \frac{B^2}{A^2} = \frac{(r_p + \rho_2 c_2 - \rho_1 c_1)^2 + x_p{}^2}{(r_p + \rho_2 c_2 + \rho_1 c_1)^2 + x_p{}^2} \tag{5.3}$$

which expression is identical to Eq. 3.14. The same happens when Eq. 5.1 is used in Eq. 4.42, the expression for the phase angle:

$$\theta_r = \arctan \left\{ \frac{2\, x_p\, \rho_1 c_1}{(r_p + \rho_2 c_2)^2 - (\rho_1 c_1)^2 + x_p^2} \right\} \tag{5.4}$$

This equation is the same as the one derived for a free panel, Eq. 3.17. As a last check Eq. 5.1 can be substituted into Eq. 4.51. This results in the following expression for the sound power transmission coefficient:

$$\alpha_t = \frac{C^2}{A^2} = \frac{4(\rho_1 c_1)^2}{(r_p + \rho_2 c_2 + \rho_1 c_1)^2 + x_p^2} \tag{5.5}$$

which expression is identical to Eq. 3.23, the sound power transmission coefficient of a free panel.

With Eq. 3.24 the transmission loss of a free panel can be computed. In Figure 5.1 the transmission loss for an 0.635 mm. (0.025 in.) thick aluminum panel is shown. The size of the panel is 0.4572 x 0.4572 m.^{2} (18 x 18 in.^{2}). The number of panel modes considered is seven. The material values given in Kinsler and Frey (Ref. 15) are used. The fundamental resonance frequency of the simply supported panel will be at 14.98 Hz. At that frequency the transmission loss of the panel will be zero. This will also happen at the higher odd, odd panel modes. At high frequencies, in the mass-controlled region, the transmission loss increases by 6 db for a doubling of frequency, as predicted by the "mass-law" (Ref. 13):

$$TL \approx 10 \log \left[1 + \left(\frac{\omega m_s}{2\rho_2 c_2}\right)^2\right] \tag{5.6}$$

According to this expression an 0.635 mm. thick aluminum panel will have a transmission loss, TL, of 28.3 db at a frequency of 2000 Hz.

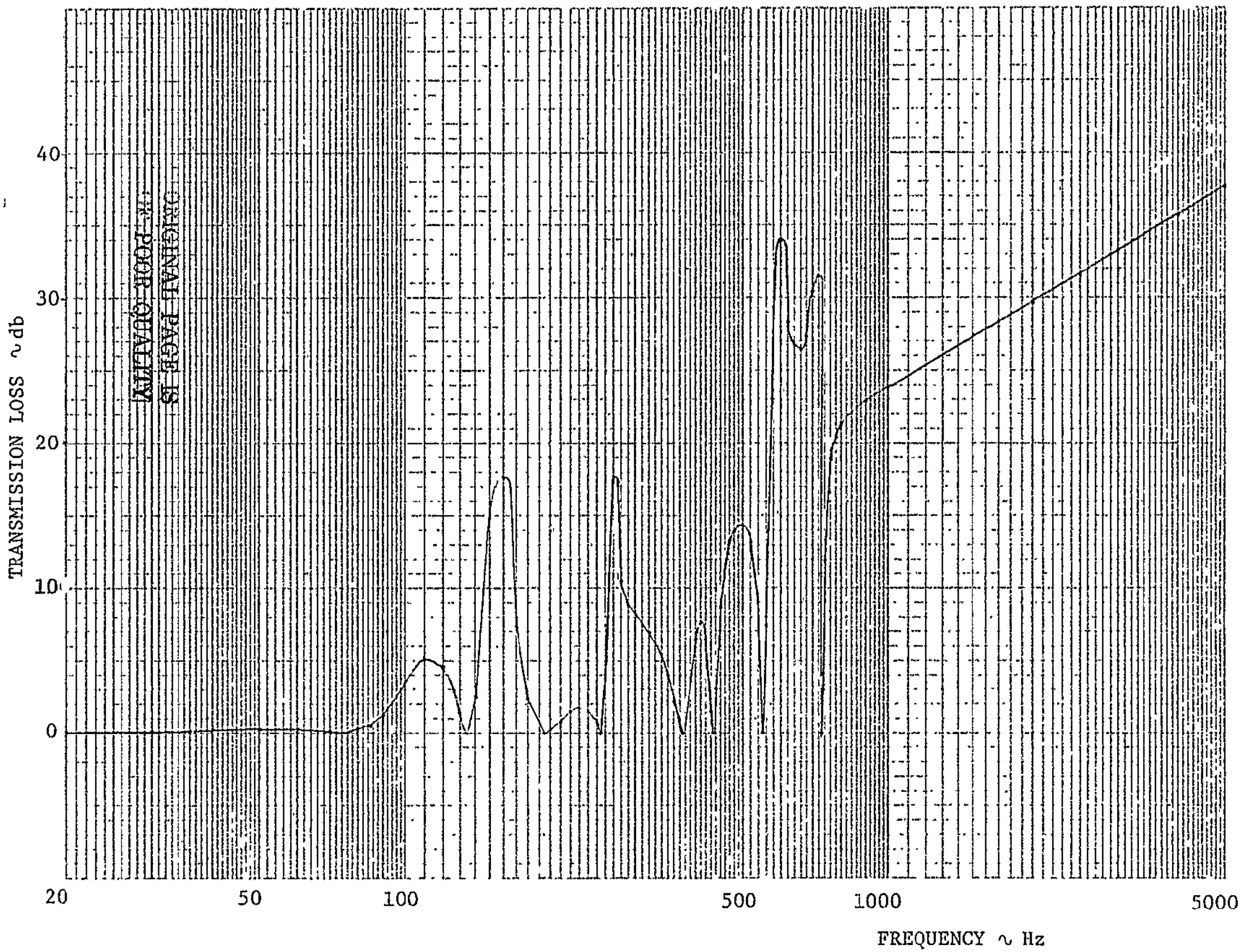

Figure 5.1: Transmission loss through an 0.635mm thick simply supported free aluminum panel of 0.4572 x 0.4572m^2

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The value computed with Eq. 3.24 compares well with the above value, as shown in Figure 5.1. Not shown in Figure 5.1 but demonstrated in Table 5.1 is the behavior of a free 1.016 mm. (0.040 in.) thick aluminum panel at low frequency.

Table 5.1: Sound Transmission through an 1.016 mm. Thick Aluminum Panel

FREQ. (HZ)	NR. (DB)	TL. (DB)	RW. (RAYLS)
1.00	23.56	17.59	415.03
2.00	17.54	11.75	415.03
3.00	14.04	[illegible].51	415.03
4.00	11.57	6.27	415.03
5.00	9.67	4.[illegible]7	415.03

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Low frequency means that the frequency is far below the fundamental resonance frequency of the panel. At low frequencies, in the stiffness-controlled region, the transmission loss will increase by 6 db for each halving of frequency. This behavior, too, is well predicted by the method of Chapter 3.

The noise reduction prediction which can be obtained with the expressions of Chapter 4 are compared with the results of the prediction method of Guy and Bhattacharya (Ref. 8). This method is briefly discussed in Chapter 2, while the expressions which are derived in Ref. 8 are recalled in Appendix D. In Appendix E a Fortran IV time-sharing computer program of the prediction model of Guy and Bhattacharya is given. This model, however, is only valid for perfect reflecting cavities; thus, cavity damping is not considered. To make a comparison between the two models possible:

$$r_w = 1,000,000 \quad kg/m^2.sec \tag{5.7}$$

has been entered in the model of Chapter 4. In that case the backing wall can be considered to be a perfect reflector. In Appendix F a Fortran IV time-sharing program of the prediction model discussed in Chapters 3 and 4 is given. By using both computer programs, Figure 5.2 has been generated. Figure 5.2 shows noise reduction curves of the following model:

Panel:	thickness	0.635 mm. (0.025 in.)
	material	aluminum
	length along x-axis	0.4572 m. (18 in.)
	length along y-axis	0.4572 m.
Cavity:	length along x-axis	0.4572 m.
	length along y-axis	0.4572 m.
	length along z-axis	2.7 m. (106.3 in.)

The source and the receiver microphones both are situated at the axial-axis of the cavity. The source microphone position is 0.01 m. in front of the panel, while the receiver microphone is placed 0.216 m. behind the panel, inside the cavity. Also in this case the material values given in Ref. 15 are used. For both models 7 unsymmetrical (odd, odd) panel modes are considered, and for the model of Guy and Bhattacharya 2 cavity modes are taken into account.

Especially above the frequency of 100 Hz the agreement between the results of both models is very good. Below the frequency of 100 Hz, the results of the model discussed in this report show fair agreement with the results of the model of Guy and Bhattacharya (Ref. 8).

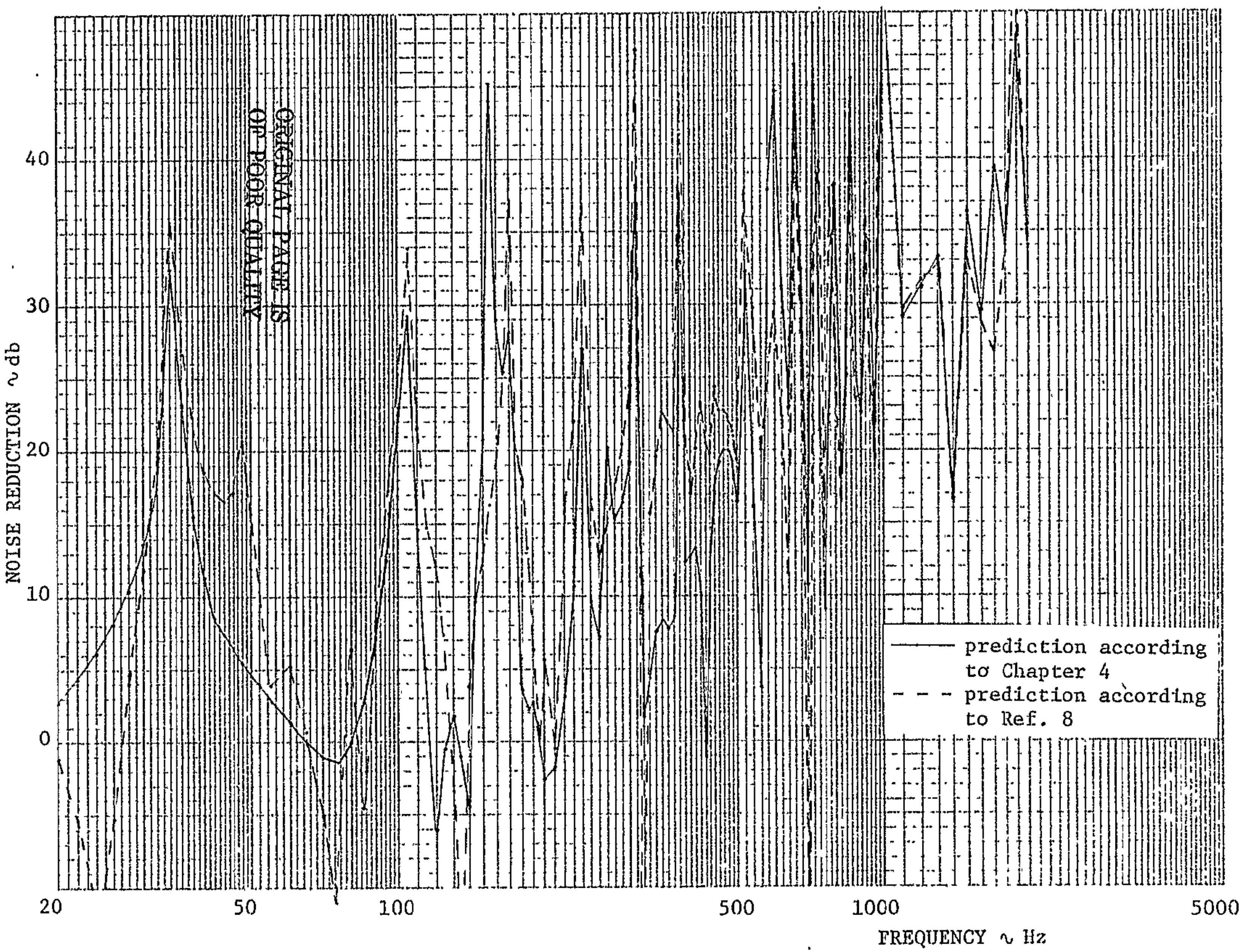

Figure 5.2: Noise reduction through an 0.635mm thick simply supported cavity-backed aluminum panel of 0.4572 x 0.4572m^2

In Ref. 8 a comparison is shown between experimental results, results of the model of Ref. 8 and results of the prediction method of Ref. 5. These results show good agreement with each other. This means that the behavior of a panel backed by a perfect reflecting cavity is fairly well predicted by the model of Chapter 4.

In Figure 5.3 an experimental noise reduction curve is shown of a lead vinyl panel. The flexural rigidity of lead vinyl is very low, which means that the fundamental resonance frequency is very low, or:

$$\omega_{11} \simeq 0 \tag{5.8}$$

In Figure 5.3, however, the noise reduction curve shows a major dip at 45 Hz. This frequency indicates the first axial cavity mode. Substitution of this frequency in the following expression:

$$f_{100} = \frac{c_2}{2\ell} \tag{5.9}$$

results in a value for the "effective" cavity length. The "effective" cavity length appears to be $\ell = 3.8$ m., which value is quite different from the actual cavity length $\ell = 2.7$ m. Around the frequency of 700 Hz, severe dips and peaks start showing up in the noise reduction curve. This indicates that the cavity modes in x- and y-directions (see Figure 4.1) start showing up. The frequency of 750 Hz indicates an 0, 2, 0 cavity mode. This is the first cavity mode which will show up because of the position of the receiver microphone.

In Figure 5.4 an experimental noise reduction curve is compared with results of the prediction model of Chapter 4. The panel/cavity model used is almost the same as the one mentioned before in this

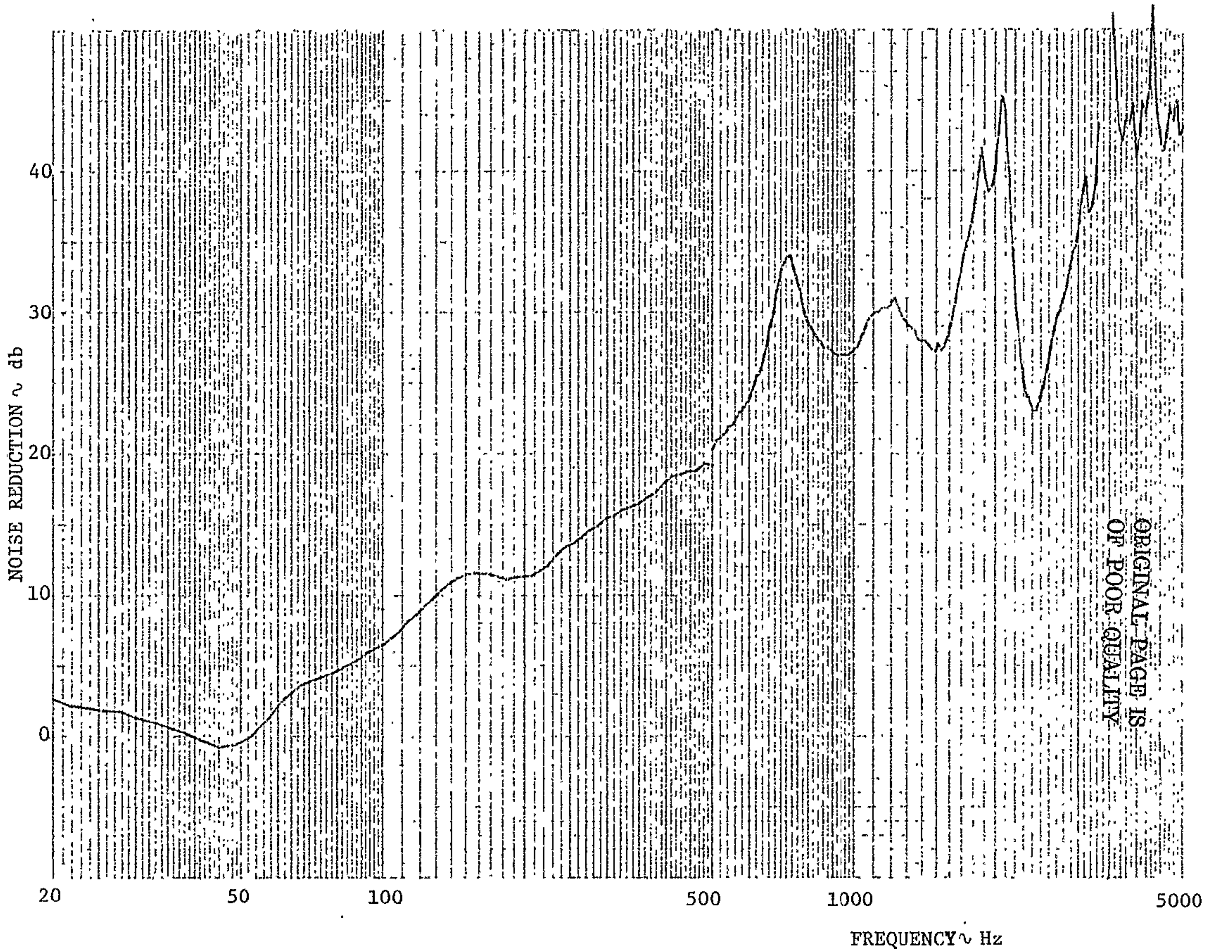

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Figure 5.3: Noise reduction through a lead vinyl sheet (experimental result)

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section. The only differences are that effective cavity length is used instead of the real length and that the impedance of the backing wall is a function of the frequency:

$$r_w = \rho_2 c_2 \{ 1 + (\frac{\omega_{100}}{2\omega})^2 \} \tag{5.10}$$

The agreement between the experimental and the predicted noise reduction curve is poor. One cause for this poor agreement in the unknown behavior of the absorbing materials in the receiving chamber. An attempt has been made to model this behavior with Eq. 5.10. However, the results generated with this expression, but also with other expressions, compare poorly with the experimental results. A second cause is that the panel resonance frequencies show up too strong. Adding panel damping to the model will weaken the influence of the panel resonances. A third cause is that the boundary conditions for the model are simply supported, while the boundary conditons for the experimental results are somewhere between simply supported and clamped.

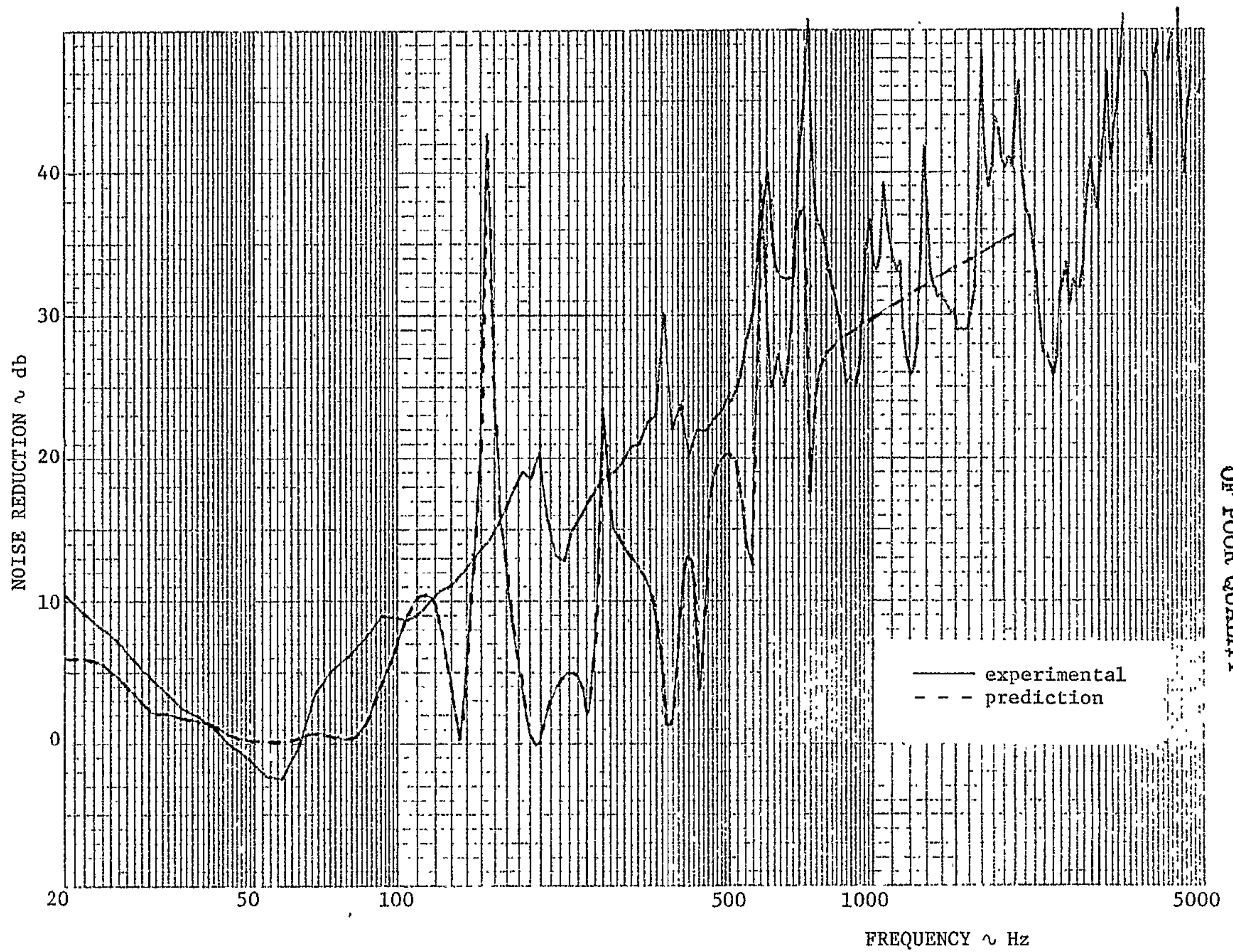

Figure 5.4: Noise reduction through an 0.635mm thick aluminum panel backed by a damped cavity (panel/cavity dimensions $0.4572 \times 0.4572 \times 2.7 m^3$)

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CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

In this report a method has been discussed to predict the noise reduction through a cavity-backed flexible plate. The transmission of sound through a panel backed by a finite space is completely different from the transmission of sound through a panel backed by an infinite space. Figures 5.1 and 5.2 provide a good indication of this effect.

A comparison of the results obtained with the prediction method of Guy and Bhattacharya (Ref. 8) and the results obtained with the method of this report show good agreement. Experimental results obtained with the test facility of Ref. 1, and results of the prediction method of Chapter 4 show poor resemblance. This, however, must be attributed partly to the experimental results.

In the first place the boundary conditions of the test panels in the test facility are not known exactly. The edge conditions are somewhere between clamped and simply supported. Secondly, the behavior of the absorbing materials in the receiving chamber is not known. Thirdly, the cavity is not sealed airtight. A small air leak can change the panel behavior considerably.

In the prediction method no panel damping has been included. Although the internal damping in thin aluminum and steel plates is not much, it has its effect on the size of the peaks and dips in the noise reduction curve. Damping decreases the size of the peaks and dips. According to Figure 5.4 the influence of the panel resonances is too great. The use of the panel impedance equation derived by

Barton (Ref. 10) will omit the effect of the panel resonances in the noise reduction curve. Better results may be expected then.

The prediction method derived in Chapters 3 and 4, however, is very useful in the investigation of 1) the effect of a change in the angle of incidence of the sound waves on the transmission of sound through a panel, and 2) the effect of a difference in air pressure across a cavity-backed panel. In the first case, Kinsler and Frey (Ref. 15) indicate how to incorporate the angle of incidence of the sound waves into the expressions of Chapters 3 and 4. In the latter case, the normal specific acoustic impedance of the flexible panel also becomes a function of the pressure differential across the panel. The forcing function in the differential equation of motion of the plate will consist of a sound pressure term and an air pressure term. In Ref. 16 the effect of internal pressurization is discussed in the case of a curved plate.

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APPENDIX A

DERIVATION OF EQ. 3.14

APPENDIX A

DERIVATION OF EQ. 3.14

The following expression is known at $z = 0$:

$$\frac{\overline{p}_r}{\overline{p}_i} = \frac{(r_p + \rho_2 c_2 - \rho_1 c_1) + j\,x_p}{(r_p + \rho_2 c_2 + \rho_1 c_1) + j\,x_p} \tag{A.1}$$

This expression may also be written as follows:

$$\frac{B\,e^{j(\omega_r t + \theta_r)}}{A\,e^{j\omega_i t}} = \frac{(r_p + \rho_2 c_2 - \rho_1 c_1) + j\,x_p}{(r_p + \rho_2 c_2 + \rho_1 c_1) + j\,x_p} \tag{A.2}$$

or:

$$\frac{B\,(\cos\alpha + j\,\sin\alpha)}{A\,(\cos\beta + j\,\sin\beta)} = \frac{a + jb}{c + jb} \tag{A.3}$$

where:

$$\begin{aligned} \alpha &= \omega_r t + \theta_r \\ \beta &= \omega_i t \\ a &= r_p + \rho_2 c_2 - \rho_1 c_1 \\ b &= x_p \\ c &= r_p + \rho_2 c_2 + \rho_1 c_1 \end{aligned} \tag{A.4}$$

From Eq. A.3 the following equation can be derived:

$$\frac{B}{A} = \frac{(a + jb)(c - jb)(\cos\beta + j\,\sin\beta)(\cos\alpha - j\,\sin\alpha)}{c^2 + b^2} \tag{A.5}$$

or:

$$\frac{B}{A} = \frac{\{ac + b^2 + jb(c - a)\}\{\cos(\alpha - \beta) - j\,\sin(\alpha - \beta)\}}{c^2 + b^2} \tag{A.6}$$

which results in:

$$\frac{B}{A}=\frac{(ac+b^2)\cos(\alpha-\beta)+b(c-a)\sin(\alpha-\beta)+j\{b(c-a)\cos(\alpha-\beta)-(ac+b^2)\sin(\alpha-\beta)\}}{c^2+b^2} \tag{A.7}$$

Taking the magnitude of the right-hand side of Eq. A.7 leads to the following result:

$$\frac{B}{A}=\frac{1}{c^2+b^2}\sqrt{(ac+b^2)^2+(bc-ab)^2} \tag{A.8}$$

or:

$$\frac{B}{A}=\sqrt{\frac{a^2+b^2}{c^2+b^2}} \tag{A.9}$$

Substitution of Eqs. A.4 in Eq. A.9 results in:

$$\frac{B}{A}=\sqrt{\frac{(r_p+\rho_2c_2-\rho_1c_1)^2+x_p^2}{(r_p+\rho_2c_2+\rho_1c_1)^2+x_p^2}} \tag{A.10}$$

APPENDIX B

TIME AVERAGE OF COSINE PRODUCT:

Different Frequencies

APPENDIX B

TIME AVERAGE OF COSINE PRODUCT:

Different Frequencies

In this appendix it will be shown that the time average of the function:

$$f(t) = 2AB \cos(\omega_i t - k_i z_1) \cos(\omega_r t + k_r z_1 + \theta_r) \qquad (B.1)$$

is zero, if $\omega_i \neq \omega_r$. The time average of a function is given as:

$$< f > = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{+T} f(t)\, dt \qquad (B.2)$$

The integral:

$$\int_{-T}^{+T} \cos(\omega_i t - k_i z_1)\cos(\omega_r t + k_r z_1 + \theta_r) dt =$$

$$\int_{-T}^{+T} \{\cos \omega_i t \cos k_i z_1 + \sin \omega_i t \sin k_i z_1\}$$

$$\{\cos \omega_r t \cos(k_r z_1 + \theta_r) - \sin \omega_r t \sin(k_r z_1 + \theta_r)\}\, dt =$$

(1) $\cos k_i z_1 \cos(k_r z_1 + \theta_r) \int_{-T}^{+T} \cos \omega_i t \cos \omega_r t\, dt +$

(2) $-\cos k_i z_1 \sin(k_r z_1 + \theta_r) \int_{-T}^{+T} \cos \omega_i t \sin \omega_r t\, dt +$

(3) $\sin k_i z_1 \cos(k_r z_1 + \theta_r) \int_{-T}^{+T} \sin \omega_i t \cos \omega_r t\, dt +$

(4) $-\sin k_i z_1 \sin(k_r z_1 + \theta_r) \int_{-T}^{+T} \sin \omega_i t \sin \omega_r t\, dt$ (B.3)

The solution of integral (1) is:

$$\int_{-T}^{+T} \cos \omega_i t \cos \omega_r t \, dt = \left[\frac{\sin(\omega_i + \omega_r)t}{2(\omega_i + \omega_t)} + \frac{\sin(\omega_i - \omega_r)\, t}{2(\omega_i - \omega_r)} \right]_{-T}^{+T}$$

$$= \frac{\sin(\omega_i + \omega_r)T}{(\omega_i + \omega_r)} - \frac{\sin(\omega_i - \omega_r)T}{(\omega_i - \omega_r)} \qquad \text{(B.4)}$$

The solution of integral (2) is:

$$\int_{-T}^{+T} \cos \omega_i t \sin \omega_r t \, dt = \left[\frac{\cos(\omega_i - \omega_r)t}{2(\omega_i - \omega_r)} - \frac{\cos(\omega_i + \omega_r)t}{2(\omega_i + \omega_r)} \right]_{-T}^{+T} = 0 \qquad \text{(B.5)}$$

In that case the solution of integral (3) is:

$$\int_{-T}^{+T} \sin \omega_i t \cos \omega_r t \, dt = 0 \qquad \text{(B.6)}$$

The solution of integral (4) is:

$$\int_{-T}^{+T} \sin \omega_i t \sin \omega_r t \, dt = \left[\frac{\sin(\omega_i - \omega_r)t}{2(\omega_i - \omega_r)} - \frac{\sin(\omega_i + \omega_r)t}{2(\omega_i + \omega_r)} \right]_{-T}^{+T}$$

$$= \frac{\sin(\omega_i - \omega_r)T}{(\omega_i - \omega_r)} - \frac{\sin(\omega_i + \omega_r)T}{(\omega_i + \omega_r)} \qquad \text{(B.7)}$$

The solution of Eq. B.3 becomes:

$$\cos k_i z_1 \cos(k_r z_1 + \theta_r) \left\{ \frac{\sin(\omega_i + \omega_r)T}{(\omega_i + \omega_r)} - \frac{\sin(\omega_i - \omega_r)T}{(\omega_i - \omega_r)} \right\}$$

$$- \sin k_i z_1 \sin(k_r z_1 + \theta_r) \left\{ \frac{\sin(\omega_i - \omega_r)T}{(\omega_i - \omega_r)} - \frac{\sin(\omega_i + \omega_r)T}{(\omega_i + \omega_r)} \right\} \qquad \text{(B.8)}$$

According to Eq. B.2, Eq. B.8 has to be divided by the factor 2T.

Then the limit for $T \to \infty$ has to be taken:

$$< f > = \lim_{T \to \infty} \frac{2AB}{2T} \cdot (\text{Eq. B.8}) = 0 \tag{B.9}$$

APPENDIX C

TIME AVERAGE OF COSINE PRODUCT:

Identical Frequencies

APPENDIX C

TIME AVERAGE OF COSINE PRODUCT:

Identical Frequencies

The time average of the function:

$$f(t) = 2AB \cos(\omega_i t - k_i z_1) \cos(\omega_r t + k_r z_1 + \theta_r) \qquad (C.1)$$

is not equal to zero when:

$$\omega_i = \omega_r = \omega_1$$

Then Eq. C.1 becomes:

$$f(t) = 2AB \cos(\omega_1 t - k_1 z_1) \cos(\omega_1 t + k_1 z_1 + \theta_r) \qquad (C.2)$$

The time average of f(t) is defined as:

$$\langle f \rangle = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{+T} f(t)\, dt \qquad (C.3)$$

The integral:

$$\int_{-T}^{+T} \cos(\omega_1 t - k_1 z_1)\cos(\omega_1 t + k_1 z_1 + \theta_r)dt =$$

$$\int_{-T}^{+T} \{\cos \omega_1 t \cos k_1 z_1 + \sin \omega_1 t \sin k_1 z_1\}$$

$$\{\cos \omega_1 t \cos (k_1 z_1 + \theta_r) - \sin \omega_1 t \sin(k_1 z_1 + \theta_r)\}dt =$$

$$(1) \quad \cos k_1 z_1 \cos(k_1 z_1 + \theta_r) \int_{-T}^{+T} \cos^2 \omega_1 t\, dt +$$

$$(2) \quad -\cos k_1 z_1 \sin(k_1 z_1 + \theta_r) \int_{-T}^{+T} \frac{\sin 2\omega_1 t}{2}\, dt +$$

$$(3) \quad \sin k_1 z_1 \cos(k_1 z_1 + \theta_r) \int_{-T}^{+T} \frac{\sin 2\omega_1 t}{2} dt +$$

$$(4) \quad -\sin k_1 z_1 \sin(k_1 z_1 + \theta_r) \int_{-T}^{+T} \sin^2 \omega_1 t \, dt \qquad \text{(C.4)}$$

The solution of integral (1) is:

$$\int_{-T}^{+T} \cos^2 \omega_1 t \, dt = \left[\frac{t}{2} + \frac{\sin 2\omega_1 t}{4\omega_1} \right]_{-T}^{+T} = T + \frac{\sin 2\omega_1 T}{2\omega_1} \qquad \text{(C.5)}$$

The solution of integral (2) and (3) is:

$$\int_{-T}^{+T} \frac{\sin 2\omega_1 t}{2} = \left[- \frac{\cos 2\omega_1 t}{4\omega_1} \right]_{-T}^{+T} = 0 \qquad \text{(C.6)}$$

The solution of integral (4) is:

$$\int_{-T}^{+T} \sin^2 \omega_1 t \, dt = \left[\frac{t}{2} - \frac{\sin 2\omega_1 t}{4\omega_1} \right]_{-T}^{+T} = T - \frac{\sin 2\omega_1 T}{2\omega_1} \qquad \text{(C.7)}$$

The total solution of Eq. C.4 is:

$$\cos k_1 z_1 \cos(k_1 z_1 + \theta_r)\{T + \frac{\sin 2\omega_1 T}{2\omega_1} \} +$$

$$- \sin k_1 z_1 \sin(k_1 z_1 + \theta_r)\{T - \frac{\sin 2\omega_1 T}{2\omega_1} \} \qquad \text{(C.8)}$$

The time average:

$$\langle f \rangle = \lim_{T \to \infty} \frac{2AB}{2T} \cdot \text{(Eq. C.8)} \qquad \text{(C.9)}$$

$$< f > = AB\{\cos k_1z_1 \cos(k_1z_1 + \theta_r) - \sin k_1z_1 \sin(k_1z_1 + \theta_r)\} \tag{C.10}$$

$$< f > = AB \cos(k_1z_1 + k_1z_1 + \theta_r) \tag{C.11}$$

$$< f > = AB \cos(2\, k_1z_1 + \theta_r) \tag{C.12}$$

APPENDIX D

NOISE REDUCTION PREDICTION METHOD

BY GUY AND BHATTACHARYA

APPENDIX D

NOISE REDUCTION PREDICTION METHOD

BY GUY AND BHATTACHARYA

The model considered by Guy and Bhattacharya is shown in Figure D.1 and consists of an acoustically hard walled rectangular cavity having at one face (x = 0) a flexible vibrating plate.

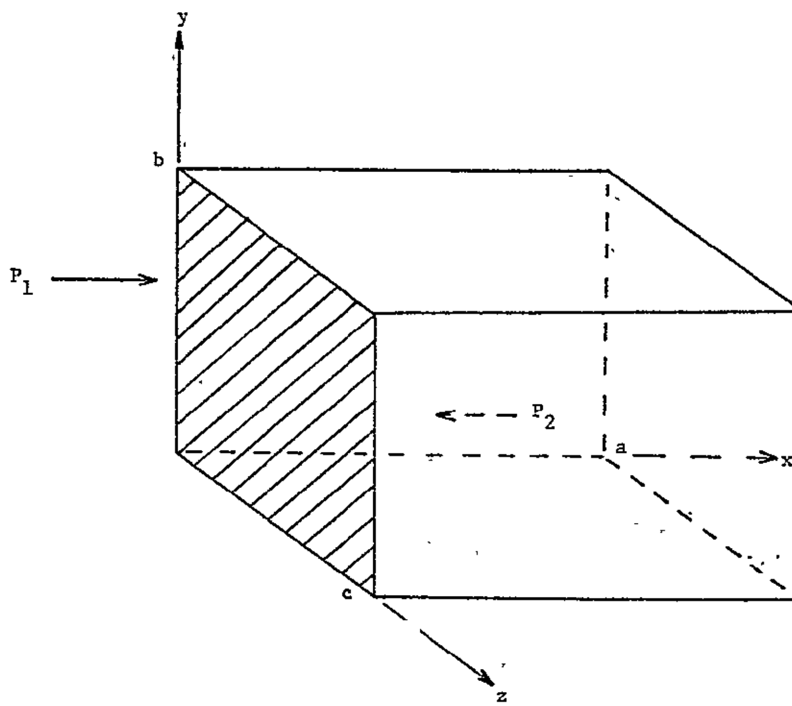

Figure D.1: Sketch of Panel and Cavity Geometry

The following expression is derived for the acoustic pressure within the cavity:

$$P_2(x, y, z, t) = \sum_{\substack{q=1\\r=1\\m=0\\n=0}}^{\infty} K.K'.\ P_{1qr}\ \frac{B_{mn_{qr}}[Z_{mn}(0)cav]\cosh[(a-x)\alpha(t)]}{[Z_{mn}(0)cav + Z_{qr}pan]\cosh[a\alpha(t)]}\cos(\frac{m\pi}{b}\,y)\cos(\frac{n\pi}{c}\,z)e^{j\omega t} \tag{D.1}$$

where:

$K = 0.5$ for $m = 0$ $\qquad$ $K = 1$ for $m > 0$

$K' = 0.5$ for $n = 0$ $\qquad$ $K' = 1$ for $n > 0$

The assumption that the pressure wave on the surface of incidence, P_1, is a plane wave makes the following simplification possible:

$$P_{1qr} = \begin{cases} \dfrac{P_1}{qr} \dfrac{16}{\pi^2} & \text{for } q \text{ and } r \text{ odd} \\ 0 & \text{otherwise} \end{cases} \tag{D.2}$$

The coupling coefficient $B_{\substack{mn\\qr}}$ may be written as follows:

$$B_{\substack{mn\\qr}} = \frac{4}{\pi^2} \frac{q}{(m^2 - q^2)} \frac{r}{(n^2 - r^2)} [\cos(q\pi)\cos(m\pi) - 1][\cos(r\pi)\cos(n\pi) - 1] \tag{D.3}$$

The cavity impedance $Z_{mn}(0)\text{cav}$ is given as follows:

$$\omega_{mn} \geq \omega: \quad Z_{mn}(0)\text{cav} = j\ \rho_o\ \omega \frac{\coth[a\alpha(t)]}{\alpha(t)} \tag{D.4}$$

where:

$$\alpha(t) = \frac{1}{c_o} \sqrt{\omega_{mn}^2 - \omega^2} \tag{D.5}$$

$$\omega_{mn} < \omega: \quad Z_{mn}(0)\text{cav} = -\ j\ \rho_o\ \omega \frac{\cot[a\alpha(t)]}{\alpha(t)} \tag{D.6}$$

where:

$$\alpha(t) = \frac{1}{c_o} \sqrt{\omega^2 - \omega_{mn}^2} \tag{D.7}$$

According to Guy and Bhattacharya the following expression can be used for the natural frequencies of the cavity:

$$\omega_{mn} = \pi c_o \sqrt{\left(\frac{m}{b} \right)^2 + \left(\frac{n}{c} \right)^2} \qquad \text{(D.8)}$$

The expression for the panel impedance is:

$$Z_{1qr} \text{ pan} = j \frac{\rho h}{\omega} (\omega^2 - \omega_{qr}^2) \qquad \text{(D.9)}$$

where ω_{qr} is the panel natural frequency, which may be obtained from the following expression in the case of simply supported boundary conditions:

$$\omega_{qr} = \pi^2 \left[\left(\frac{q}{b} \right)^2 + \left(\frac{r}{c} \right)^2 \right] \sqrt{\frac{D}{\rho h}} \qquad \text{(D.10)}$$

where D represents the bending stiffness of the plate and ρ and h the density and the thickness of the plate.

When the acoustic pressure within the cavity is measured at an arbitrary point on the axial centerline of the cavity, $P_2(x, b/2, c/2, t)$, then the term:

$$\cos \left(\frac{m\pi}{b} y \right) \cos \left(\frac{n\pi}{c} z \right) \text{ for } m, n = 0, 1, 2, 3, \ldots \qquad \text{(D.11)}$$

can be changed into:

$$\cos \left(\frac{m\pi}{2} \right) \cos \left(\frac{n\pi}{2} \right) \text{ for } m, n = 0, 1, 2, 3, \ldots \qquad \text{(D.12)}$$

or:

$$(-1)^{\frac{m+n}{2}} \text{ for } m, n = 0, 2, 4, \ldots \qquad \text{(D.13)}$$

Substitution of the expressions mentioned above in Eq. D.1 and by taking the magnitude of this expression, the noise reduction can be computed:

$$NR = 20 \log \frac{P_1}{P_2} \tag{D.14}$$

Appendix D gives a computer program of this method.

Table D.1: List of Symbols

Symbol	Definition	Dimension
a	Room dimension in x-direction	m
b	Room dimension in y-direction	m
$B_{\substack{mn \\ qr}}$	Coupling coefficient between room and panel modes	
c	Room dimension in z-direction	m
c_o	Velocity of sound in air	m/sec
D	Modulus of rigidity	Nm
h	Thickness of panel	m
j	$\sqrt{-1}$	
K	Constant	
K'	Constant	
m	Room mode in y-direction	
n	Room mode in z-direction	
P_1	Acoustic pressure on the surface of incidence	N/m^2
P_2	Acoustic pressure within the cavity	N/m^2
q	Panel mode in y-direction	
r	Panel mode in z-direction	
t	Time	sec
x	Model coordinate	

Table D.1: List of Symbols (continued)

Symbol	Definition	Dimension
y	Model coordinate	
z	Model coordinate	
$Z_{mn}(0)cav$	Cavity impedance of an (m, n) cavity mode at the surface of the panel at the receiver side	$kg/m^2.sec$
$Z_{qr}pan$	Panel impedance of the (q, r) panel mode	$kg/m^2.sec$
ρ	Density of panel material	kg/m^3
ρ_o	Density of air	kg/m^3
ω	Forced angular frequency	rad/sec
ω_{mn}	Natural frequency for room mode (m, n)	rad/sec
ω_{qr}	Natural frequency for panel mode (q, r)	rad/sec

APPENDIX E

TIME-SHARING PROGRAM OF METHOD OF

APPENDIX D

APPENDIX E

TIME-SHARING PROGRAM OF METHOD OF APPENDIX D

In this appendix a Fortran IV time-sharing computer program will be given, which computes the noise reduction through a cavity-backed flexible panel as function of the frequency. The method used in this program is discussed in Appendix D, and the definition of the variables can be found in Table D.1. Table E.1 provides a listing of the variables used in this program, while Figure E.1 shows the flowchart. Figures E.2, E.3 and E.4 give the listing of the program.

Table E.1: Variable Names in Program

Symbol	Definition	Dimension
A	Room dimension a in x-direction	m
ALPHA	Eqs. D.5 and D.7	m^{-1}
B	Room dimension b in y-direction	m
BMNQR	Coupling coefficient $B_{\substack{mn\\qr}}$	
BNR	Noise reduction NR	db
C	Room dimension c in z-direction	m
CA	Velocity of sound c_o in air	m/sec
CONST1	Constant K	
CONST2	Constant K'	
CONST3	$16/qr\pi^2$	
COSHD	$\mathrm{Cosh}[a\alpha(t)]$	

Table E.1: Variable Names in Program (continued)

Symbol	Definition	Dimension
COSHN	$\text{Cosh}[(a - x)\alpha(t)]$	
D	Bending stiffness D	Nm
DUM1	$(a - x)\alpha(t)$	
DUM2	$a \cdot \alpha(t)$	
E	Young's modulus E	N/m^2
F	Frequency f	Hz
FACTOR	Eq. D.1 without summation sign	
H	Plate thickness h	m
ICMAX	Number of cavity modes	
IPMAX	Number of panel modes	
IHIGH	Maximum frequency	Hz
ILOW	Minimum frequency	Hz
ISTEP	Increase in frequency	Hz
OMG	Angular frequency ω	rad/sec
OMGMN	Natural frequency ω_{mn} for room mode	rad/sec
OMGQR	Natural frequency ω_{qr} for panel mode	rad/sec
PI	π	
Q	Panel mode q in y-direction	
R	Panel mode r in z-direction	
RM	Room mode m in y-direction	
RN	Room mode n in z-direction	
ROA	Density of air ρ_o	kg/m^3
V	Poisson's ratio ν	
X	Receiver microphone position in x-direction	m

Table E.1: Variable Names in Program (continued)

Symbol	Definition	Dimension
ZMNCAV	Cavity impedance $Z_{mn}(0)cav$	$kg/m^2.sec$
ZQRPAN	Panel impedance $Z_{qr}pan$	$kg/m^2.sec$

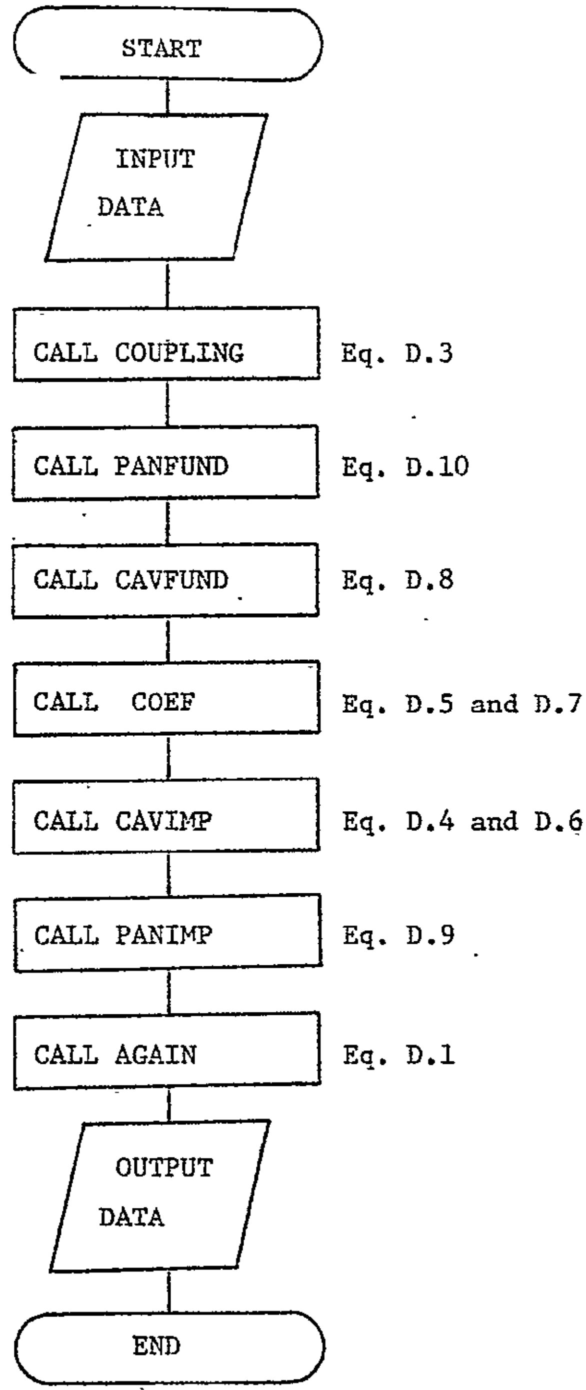

Figure E.1: Flowchart of Program

```
10   PRINT:"PANEL THICKNESS H IN METERS"
20   READ:H
30   PRINT:"CAVITY LENGTH A IN METERS"
40   READ:A
50   PRINT:"CAVITY HEIGTH B IN METERS"
60   READ :B
70   PRINT:"CAVITY WIDTH C IN METERS"
80   READ:C
90   PRINT:"RECEIVER MIKE POSITION Y IN METERS"
100  READ:Y
110  PRINT:"MAX. NUMBER OF PANEL MODES (INT.)"
120  READ:IPMAX
130  PRINT:"MAX. NUMBER OF CAVITY MODES (INT.)"
140  READ:ICMAX
150  PRINT:"MIN., MAX., AND STEP OF FREQ. IN HZ (INT.)"
160  READ:ILOW,IHIGH,ISTEP
170  PRINT:"DENSITY OF PLATE MAT. IN KG/M**3"
180  READ:ROP
190  PRINT:"YOUNG'S MODULUS IN N/M**2"
200  READ:E
210  PRINT:"POISSON'S RATIO OF MAT."
220  READ:V
230  PI=3.14159265
240  ROA=1.21
250  CA=343.
260  D=E*H*H*H/(12*(1-V*V))
270  PRINT:"       FREQ.(HZ)    NR.(DB)"
280  DO 70 LF=ILOW,IHIGH,ISTEP
290  F=LF
300  OMG=2*PI*F
310  TOT=0.
315  IZERO=0
320  DO 10 IC=1,IPMAX,2
330  DO 20 IR=1,IPMAX,2
340  DO 30 M=IZERO,ICMAX,2
350  DO 40 N=IZERO,ICMAX,2
355  CONST1=1.
356  CONST2=1.
360  IF (M.LT.1) CONST1=0.5
370  IF (N.LT.1) CONST2=0.5
380  Q=IC
390  R=IR
400  RM=M
410  RN=N
420  COMMON/ONE/Q,R,RN,RM,PI,B,C,D,ROP,H,CA
430  COMMON/TWO/A,Y
440  CONST3=16/(Q*R*PI*PI)
```

Figure E.2 : Listing of Program

```
      CALL COUPLNG(CENTCR)
      CALL PAIFUND(CONGCR)
      CALL CAVFUND(SCHGNH)
      CALL COEF(OMG,CMGNH,ALPHA,CA)
      CALL CAVTMP(OMG,ALPHA,ZMTCAV,TCA,A,SMCNH)
      CALL PANTMP(OMG,CONGCR,RCP,L,ZCRESN)
      DUM1=(A-X)*ALPHA
      DUM2=(X*ALPHA)
      IF (OMGM.LT.OMG) GOTO 100
      COSHM=COSH(DUM1)
      COSHD=COSH(DUM2)
      GOTO 110
  100 COSHM=COS(DUM1)
      COSHD=COS(DUM2)
  110 CALL AGAIN(CONST1,CONST2,CONST3,BENTCR,ZMTCAV,COSHM,
     1             COSHD,ZCRPAN,FACTOR)
      TOT=TOT+FACTOR
   50 CONTINUE
   30 CONTINUE
   20 CONTINUE
   10 CONTINUE
      TOTP=ABS(TOT)
      BNR=20*ALOG10(1/TOTP)
      WRITE (6,1000) F,BNR
 1000 FORMAT(5X,2F10.2,/)
   70 CONTINUE
      STOP
      END
      SUBROUTINE COUPLNG(CENTCR)
      COMMON/ONE/Q,R,PM,RM,PI,B,C,D,RCP,H,CA
      CENTCR=(Q*Q*R/(PI*PI*(RM*RM-Q*Q))*
     1        (RM*RM-R*R)))*(COS(Q*PI)*COS(RM*PI)-1)*
     2        (COS(R*PI)*COS(RM*PI)-1)
      RETURN
      END
      SUBROUTINE PAIFUND(CONGCR)
      COMMON/ONE/Q,R,PM,RM,PI,B,C,D,RCP,H,CA
      CONGCR=PI*PI*(Q*Q/(B*B)+R*R/(C*C))*
     1          SQRT(D/(RCP*H))
      RETURN
      END
      SUBROUTINE CAVFUND(SCHGNH)
      COMMON/ONE/Q,R,PM,RM,PI,B,C,D,RCP,H,CA
      SCHGNH=PI*CA*SQRT(PM*PM/(B*B)+RM*RM/(C*C))
      RETURN
      END
      SUBROUTINE COEF(SOMG,SCMGNH,SALPHA,SCA)
      IF (SCMGNH.LT.SOMG) GOTO 20
```

Figure E.3 : Listing of Program (Continued)

```
      [illegible] LPHA=SQRT([illegible])
      GOTO [illegible]
   50 SALPHA=SQRT(SOMG*SOMG-[illegible])/SC[illegible]
   [illegible] CONTINUE
      RETURN
      END
      SUBROUTINE CAVIMP([illegible],SALPHA,SIMPCA,[illegible],[illegible],[illegible])
      IF ([illegible].LT.SOMG) GOTO [illegible]
      SIMPCA=[illegible]*SOMG/(SALPHA*TANH(S[illegible]*SALPHA))
      GOTO [illegible]
   [illegible] SIMPCA=-[illegible]*SOMG*COS(S[illegible]*SALPHA)/(SIN([illegible]*SALPHA)*SALPHA)
   [illegible] CONTINUE
      RETURN
      END
      SUBROUTINE PA[illegible]P([illegible],[illegible],SRCP,SH,SZCRPA)
      SZCRPA=(SRCP*SH/SOMG)*([illegible]-[illegible])
      RETURN
      END
      FUNCTION COSH(U)
      COSH=(EXP(U)+EXP(-U))/2
      RETURN
      END
      SUBROUTINE AGAIN(SCCNS1,SCCNS2,SCCNS3,SFACTOR,SIMPCA,
     *                 SCOSHN,SCOSHD,SZCRPA,SFACTOR)
      COMMON/TWC/P,N
      SIGN=(-1)**((N+1)/2)
      SFACTOR=SCCNS1*SCCNS2*SCCNS3*[illegible]/
     *        ((SIMPCA-SZCRPA)*SCOSHD)
      RETURN
      END
```

Figure E.4 : Listing of Program (Continued)

APPENDIX F

TIME-SHARING PROGRAM OF METHOD OF CHAPTER 4

APPENDIX F

TIME-SHARING PROGRAM OF METHOD OF CHAPTER 4

In this appendix a Fortran IV time-sharing computer program will be given to predict the transmission loss of a panel and the noise reduction through a panel. The panel can be either free or cavity backed (I = 1 stands for cavity-backed; I $\neq$ 1 stands for a free panel). In the case of a free panel, some of the input data are not relevant and will not be used by the computer program. These data are: CL and Z2. Table F.1 explains the different variables of the program. Figure F.1 gives a flowchart of the program, and Figures F.2, F.3 and F.4 give a listing of the program.

Table F.1: Variable Names in Program

Symbol	Definition	Dimension
A	Room dimension a in x-direction	m
ALABD	Imaginary part λ of Eq. 4.34	kg/m^2.sec
ALPHAC	D^2/C^2, see Eq. 4.17	
ALPHAR	B^2/A^2, see Eq. 4.39	
ALPHAT	C^2/A^2, see Eq. 4.51	
B	Room dimension b in y-direction	m
BNR	Noise reduction NR	db
CA1	Velocity of sound in air at source side c_1	m/sec
CA2	Velocity of sound in air at receiver side c_2	m/sec

Table F.1: Variable Names in Program (continued)

Symbol	Definition	Dimension
CAPPA	Real part κ of Eq. 4.34	$kg/m^2.sec$
CL	Room dimension ℓ in z-direction	m
D	Bending stiffness D	Nm
E	Young's modulus E	N/m^2
F	Frequency f	Hz
H	Plate thickness h	m
IHIGH	Maximum frequency	Hz
ILOW	Minimum frequency	Hz
ISTEP	Step in frequency	Hz
MAX	Maximum number of panel modes	
OMG	Angular frequency	rad/sec
OMG1	Fundamental cavity mode in z-direction	rad/sec
PI	π	
ROA1	Density of air at source side ρ_1	kg/m^3
ROA2	Density of air at receiver side ρ_2	kg/m^3
ROC1	Characteristic impedance of air at source side $\rho_1 c_1$	$kg/m^2.sec$
ROC2	Characteristic impedance of air at receiver side $\rho_2 c_2$	$kg/m^2.sec$
ROP	Density of plate material ρ	kg/m^3
RP	Panel resistance	$kg/m^2.sec$
RW	Wall resistance	$kg/m^2.sec$
TAN	Tan	
THETAR	Phase angle of reflected wave θ_r	rad

C J

Table F.1: Variable Names in Program (continued)

Symbol	Definition	Dimension
TL	Transmission loss	db
X	Receiver microphone position in x-direction	m
XP	Panel reactance	$kg/m^2.sec$
Y	Receiver microphone position in y-direction	m
Z1	Source microphone position in z-direction	m
Z2	Receiver microphone position in z-direction	m

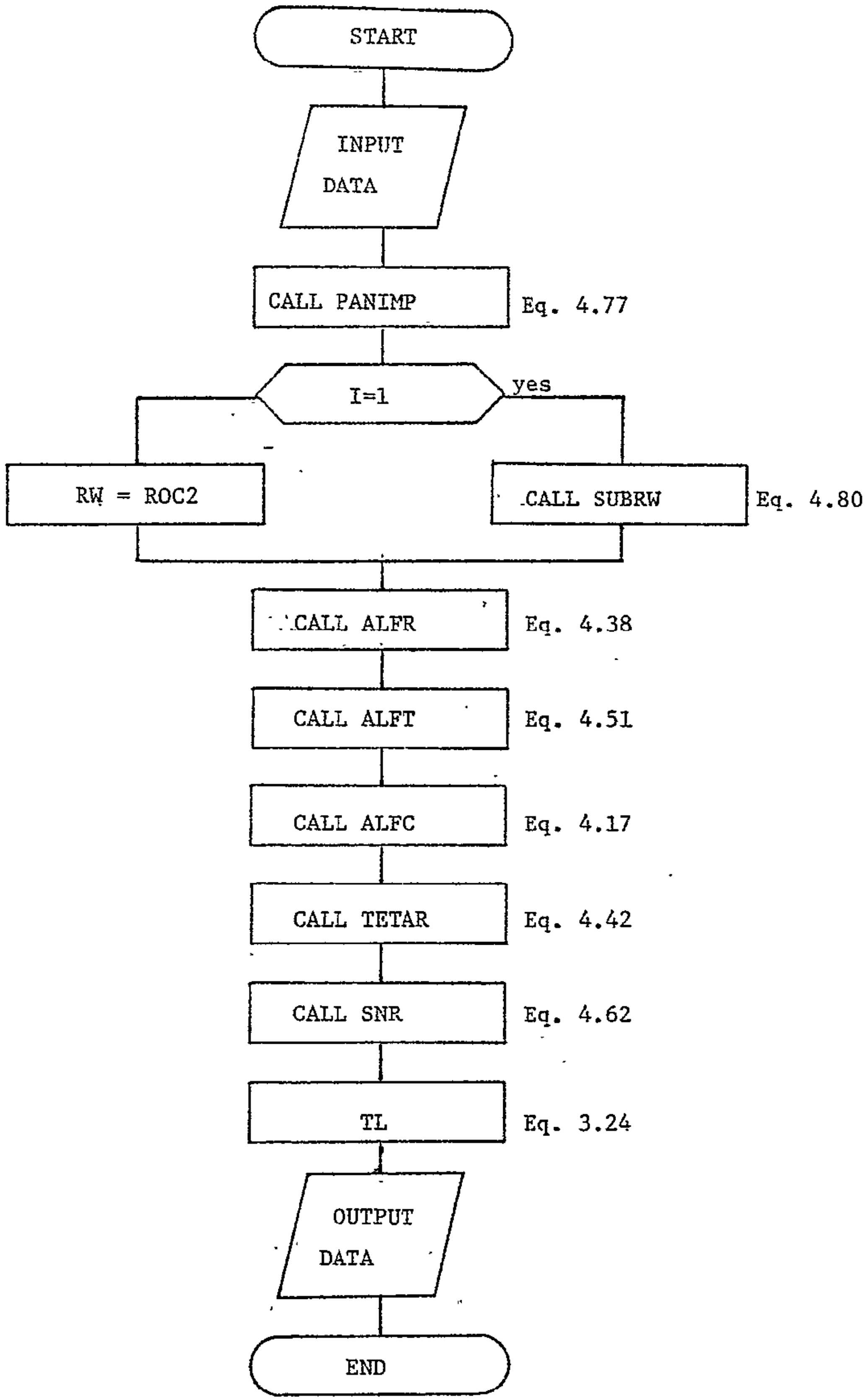

Figure F.1: Flowchart of Program

```
10  PRINT:"PANEL THICKNESS H IN METERS"
20  READ:H
30  PRINT:"YOUNGS MODULUS IN N/M**2"
40  READ:E
50  PRINT:"POISSONS RATIO ..."
60  READ:V
70  PRINT:"DENSITY OF PLATE MAT. IN KG/M**3"
80  READ:ROP
90  PRINT:"CAVITY HEIGTH A IN METERS"
100 READ:A
110 PRINT:"CAVITY WIDTH B IN METERS"
120 READ:B
130 PRINT:"CAVITY LENGTH CL IN METERS"
140 READ:CL
150 PRINT:"RECEIVER MIKE POS. IN X-DIR. IN METERS"
160 READ:X
170 PRINT:"RECEIVER MIKE POS. IN Y-DIR. IN METERS"
180 READ:Y
190 PRINT:"SOURCE MIKE POS. Z1 IN METERS"
200 READ:Z1
210 PRINT:"RECEIVER MIKE POS. Z2 IN METERS"
220 READ:Z2
230 PRINT:"CAVITY BACKED PANEL ? YES=1"
240 READ:I
250 PRINT:"MIN., MAX., AND STEP OF FREQ. IN HZ (INT.)"
260 READ:ILOW,IHIGH,ISTEP
270 PRINT:"MAX. NUMBER OF PANEL MODES (INT.)"
280 READ:MA
290 RCA1=1.21
300 RCA2=1.21
310 CA1=343.
320 CA2=343.
330 PI=3.141592654
340 RCC1=RCA1*CA1
350 RCC2=RCA2*CA2
360 CYG1=PI*CA2/CL
370 D=E*H**3/(12*(1-V**2))
380 COMMON/ONE/A,B,D,ROP,H,X,Y,PI,MA
390 COMMON/TWO/Z1,CA1,Z2,CA2,CL
395 PRINT:"        FREQ.(HZ)     NR.(DB)    TL.(DB)   RW.(RAYLS)"
400 DO 10 IF1=ILOW,IHIGH,ISTEP
410 F=IF1
420 OMG=2*PI*F
430 CALL PANIMP(OMG,RP,XP)
440 RW=RCC2
450 IF (I.EQ.1) CALL CLBP(OMG,CYG1,RCC2,RW)
460 CALL NLEP(RP,XP,OMG,RW,...,RCC1,RCC2,...)
```

Figure F.2 : Listing of Program

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```
      CALL ALFT(SRW,SROC1,SROC2,STAN,SCAPPA,SALAPD,SALPHT)
      CALL ALFC(SRW,SROC2,SALPHC)
      CALL TETAF(SLAMD,ROC1,CAPPA,THETAF)
      CALL SUR(ALPHAR,ALPHAT,ALPHAC,OMG,THETAF,PWR)
      TL=10*ALOG10(1/ALPHAT)
      WRITE (6,1000) F,PWR,TL,R
 1000 FORMAT (5X,4F10.2,/)
   10 CONTINUE
      STOP
      END
      SUBROUTINE RALFAP(SOMG,SRP,SXP)
      COMMON/ONE/A,B,D,ROP,H,X,Y,PI,MAX
      TOT=0.
      DO 40 M=1,MAX,2
      DO 50 N=1,MAX,2
      AM=M
      AN=N
      SOMGMN=PI*PI*(AM*AM/(A*A)+AN*AN/(B*B))*
     *        SQRT(D/(ROP*H))
      ARC1=AM*PI*X/A
      ARC2=AN*PI*Y/B
      FUNC=(SIN(ARC1)*SIN(ARC2))/(AM*AN*(SOMGMN*SOMGMN-
     *     SOMG*SOMG))
      TOT=TOT+FUNC
   50 CONTINUE
   40 CONTINUE
      SXP=-ROP*H*PI*PI/(16*SOMG*TOT)
      SRP=0.
      RETURN
      END
      SUBROUTINE ALFR(SRP,SXP,SOMG,SRW,STAN,SROC1,SROC2,
     *                SCAPPA,SALAPD,SALPHR)
      COMMON/TWO/Z1,CA1,Z2,CA2,CL
      STAN=SIN(SOMG*CL/CA2)/COS(SOMG*CL/CA2)
      DEN=SROC2*SROC2+SRW*SRW*STAN*STAN
      SCAPPA=SRP+SROC2*SROC2*SRW*(1+STAN*STAN)/DEN
      SALAPD=SXP+(SROC2*SROC2-SRW*SRW)*SROC2*STAN/DEN
      SALPHR=((SCAPPA-SROC1)**2+SALAPD*SALAPD)/
     *       ((SCAPPA+SROC1)**2+SALAPD*SALAPD)
      RETURN
      END
      SUBROUTINE ALFT(SRW,SROC1,SROC2,STAN,SCAPPA,SALAPD,SALPHT)
      SALPHT=SROC2*SROC2*(SRW+SROC2)*(SRW+SROC2)*(1+STAN*STAN)/
     *       (((SCAPPA+SROC1)**2+SALAPD*SALAPD)*
     *       (SRW*SRW*STAN*STAN+SROC2*SROC2))
      RETURN
      END
      SUBROUTINE ALFC(SRW,SROC2,SALPHC)
```

Figure F.3 : Listing of Program (Continued)

```
      SALPHC=((SRM-SRCC2)/(SRM+SRCC2))**2
      RETURN
      END
      SUBROUTINE SUPRM(SOMG,SOMG1,SRCC2,SRM)
      SRM=SRCC2*(1+(SOMG1/(2*SOMG))**2)
      RETURN
      END
      SUBROUTINE SNR(SALPHB,SALPHT,SALPHC,SOMG,[illegible],[illegible])
      COMMON/ENG/Z1,CA1,Z2,CA2,CL
      COEF=(1+SALPHB+2*SQRT(SALPHB)*COS(STHETB+2*SOMG*Z1/CA1))/
     1    (SALPHT*(1+SALPHC+2*SQRT(SALPHC)*COS(2*SOMG*CL/CA2-
     2    2*SOMG*Z2/CA2)))
      SNCIDB=10*ALOG10(COEF)
      RETURN
      END
      SUBROUTINE TETAB(SALAPP,SRCC1,SCAPPA,STHETB)
      STHETB=ATAN(2*SALAPP*SRCC1/(SCAPPA*SCAPPA-
     1    SRCC1*SRCC1+SALAPP*SALAPP))
      RETURN
      END
```

Figure F.4 : Listing of Program (Continued)

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