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MAGNETIC FIELD IN THE AIR GAP OF A HYSTERESIS CLUTCH WITH CYLINDRICAL ROTOR

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MAGNETIC FIELD IN THE AIR GAP OF A HYSTERESIS CLUTCH WITH CYLINDRICAL ROTOR /63*

Yu. A. Yermolin

Recently, electromagnetic clutches (EMC) have been finding increasing applications in various power-driven devices and in systems of automatic control. The principle on which these clutches operate has been thoroughly described in the literature [1, 2]. Their design is not complicated, and they possess highly stable characteristics. In many instances, EMC compete successfully with other analogous devices, e.g., with electromagnetic-particle clutches (EPC). Without dwelling on the various applications of EMC, we shall show, however, that in the case of pre-assigned overall dimensions, successful design in most instances determines the maximum moment transmitted by the clutch. From this point of view, there is great practical interest in EMC with a rotor in the form of a full cylinder. The schematic diagram of this clutch is presented in Figure 1.

The inductor 1 is made of cast steel. The complete rotor 2, made of magnetically-hard material, is placed in the air gap of the inductor and is seated rigidly on the shaft 3. The control coil 4 receives d.c. current through the conductring ring 5, and generates a magnetic flux, whose path is shown by broken lines in Figure 1. Due to the cylindrical slots 6, the magnetic flux, which passes across the air gap, splits up and forms poles. Thus, at each point of the air gap within the boundaries of the division into poles, induction occurs. The value and direction of this induction are functions of the coordinates of each such point.

When the distribution law for the induction in the air gap is /64
known, all characteristics of the EMC can be determined. However, it is quite complicated to find this law for the gap configuration under consideration (c.f. Fig 1). Successful solution of this self-contained problem can only be accomplished by approximation, using a number of

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[1]Numbers in margin indicate pagination in foreign text.

assumptions.

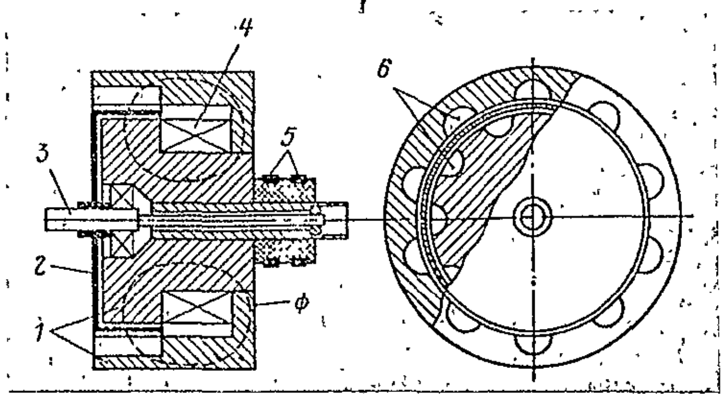

Figure 1

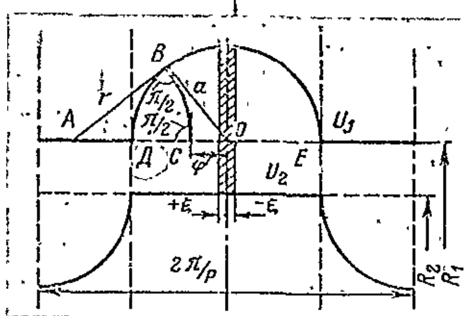

Figure 2

Since the axial dimension of the gap is somewhat larger than the radial, the field can be considered as plane-parallel and a solution can be sought for the LaPlace equation relative to the scalar magnetic potential U on the plane

$$\nabla U = 0. \quad (1)$$

Let us consider the evolute of the air gap having p poles within the boundaries of one polar division (Figure 2).

Let us assume that to the upper boundary there correspond the potential U_1, and to the lower, the potential U_2, and let us determine the value of the magnetic potential on the straight line DE. Analysis of the results of modeling on electrically-conducting paper and considerations of symmetry make it possible to assume that the field within the boundaries of a cylindrical slot can be described with sufficient accuracy by circular arcs of radius r, as shown in Figure 2.

As a first approximation, we shall assume that the magnetic potential at any point on the line DE differs from U_1 by an amount which is proportinal to the length of the magnetic line of force l passing through this point. From the triangles OBA and ABC, we obtain:

$$l(\varphi)=a\left(\frac{\pi}{2p\varphi}-\frac{2p}{\pi}\varphi\right)\operatorname{arctg}\frac{2p}{\pi}\varphi, \tag{2}$$

where a is the radius of the cylindrical slot, and φ is the moving co-ordinate.

As shown above, we shall assume:

$$\Delta U_1=k_1 l(\varphi). \tag{3}$$

Let us determine the coefficient k_1 from the following considerations. It is obvious that between $+\xi$ and $-\xi$ when $|\xi| \to 0$ (Figure 2), we may regard the field as formed by two coaxial circular cylinders of radii (R_1 + a) and R_2 with potentials U_1 and U_2, respectively. It is well-known that the expression for the scalar magnetic potential of such a field has the form [3]:

$$U=M_1+N_1\ln\rho, \tag{4}$$

where M_1 and N_1 are constants, and ρ is the moving radius.

To determine M_1 and N_1, let us solve the system:

$$\left.\begin{aligned} U_1&=M_1+N_1\ln(R_1+a);\\ U_2&=M_1+N_1\ln R_2. \end{aligned}\right\} \tag{5}$$

We obtain

$$\left.\begin{aligned} M&=\frac{U_2\ln(R_1+a)-U_1\ln R_2}{\ln\dfrac{R_1+a}{R_2}};\\ N_1&=\frac{U_1-U_2}{\ln\dfrac{R_1+a}{R_2}}; \end{aligned}\right\} \tag{6}$$

Then the potential U_0 at the point 0 is determined by substituting the values obtained for the constants M_1 and M_2 into (4) with $\rho = R_1$. Further, we find that

$$\Delta U=U_1-U_0=(U_1-U_2)\frac{\ln\dfrac{R_1+a}{R_1}}{\ln\dfrac{R_1+a}{R_2}} \tag{7}$$

and on equating (7) to the expression (3), when φ = 0, we obtain:

$$k_1 = (U_1 - U_2)\frac{\ln\dfrac{R_1+a}{R_1}}{a\ln\dfrac{R_1+a}{R_2}}. \tag{8}$$

On repeating the analogous arguments and calculations for the lower boundary of the air gap, we have:

$$k_2 = -(U_1 - U_2)\frac{\ln\dfrac{R_2}{R_2-a}}{a\ln\dfrac{R_1}{R_2-a}}. \tag{9}$$

To simplify the further calculations, we shall approximate (2) by the function

$$l(\varphi) = a\cos p\varphi. \tag{10}$$

The validity of this approximation is obvious from a comparison of the graphs constructed from (2) and (10) (Figure 3).

Taking into consideration the assumptions made and the accompanying calculations, it turns out to be possible to replace the actual air gap by an annular gap with boundary conditions written in the form:

$$\left.\begin{array}{l} U(\rho = R_1) = \\ = \begin{cases} U_1 & \text{when } \dfrac{\pi}{p} \geqslant \varphi \geqslant \dfrac{\pi}{2p}; \\ & -\dfrac{\pi}{p} \leqslant \varphi \leqslant -\dfrac{\pi}{2p}. \\ U_1 - ak_1\cos p\varphi & \text{when } -\dfrac{\pi}{2p} \leqslant \varphi \leqslant \dfrac{\pi}{2p}; \end{cases} \\ U(\rho = R_2) = \\ = \begin{cases} U_2 & \text{when } -\dfrac{\pi}{2p} \leqslant \varphi \leqslant \dfrac{\pi}{2p}; \\ U_2 + ak_2\cos p\varphi & \text{when } \dfrac{\pi}{p} \geqslant \varphi \geqslant \dfrac{\pi}{2p}; \\ & -\dfrac{\pi}{p} \leqslant \varphi \leqslant -\dfrac{\pi}{2p}. \end{cases} \end{array}\right\} \tag{11}$$

Thus the problem is reduced to that of finding a harmonic function inside the ring when the values of the function are known on the boundaries. This problem is known in literature as the Dirichlet Problem for a Ring [4].

Expanding the boundary conditions (11) in a Fourier series, and /6
without sacrificing generality, placing $U_1 = U$ and $U_2 = 0$, we obtain:

$$\left.\begin{aligned}U(\rho=R_1)&=\Delta U-\frac{ak_1}{\pi}-\frac{ak_1}{2}\cos p\varphi+\\&+\frac{2ak_1}{\pi}\sum_{n=2,4,6\ldots}^{\infty}\frac{(-1)^{\frac{n}{2}}}{n^2-1}\cos np\varphi;\\U(\rho=R_2)&=-\frac{ak_2}{\pi}+\frac{ak_2}{2}\cos p\varphi+\\&+\frac{2ak_2}{\pi}\sum_{n=2,4,6,\ldots}^{\infty}\frac{(-1)^{\frac{n}{2}}}{n^2-1}\cos np\varphi.\end{aligned}\right\}\quad(12)$$

Taking into account the form of the spectrum of the boundary conditions in the expansion (12), we shall seek the solution of the Laplace equation (1) in polar coordinates in the form [4]:

$$\begin{aligned}U(\rho,\varphi)&=A+B\ln\rho+(C_1\rho^{p}+C_{-1}\rho^{-p})\cos p\varphi+\\&+\sum_{n=2,4,6,\ldots}^{\infty}(C_n\rho^{np}+C_{-n}\rho^{-np})\cos np\varphi.\end{aligned}\quad(13)$$

To determine the constant coefficients in (13), it is sufficient to solve the system:

$$\left.\begin{aligned}&\Delta U-\frac{ak_1}{\pi}-\frac{ak_1}{2}\cos p\varphi+\frac{2ak_1}{\pi}\times\\&\times\sum_{n=2,4,6,\ldots}^{\infty}\frac{(-1)^{\frac{n}{2}}}{n^2-1}\cos np\varphi=A+B\ln R_1+\\&+(C_1R_1^{p}+C_{-1}R_1^{-p})\cos p\varphi+\sum_{n=2,4,6,\ldots}^{\infty}(C_nR_1^{np}+\\&+C_{-n}R_1^{-np})\cos np\varphi,\\&-\frac{ak_2}{\pi}+\frac{ak_2}{2}\cos p\varphi+\frac{2ak_2}{\pi}\times\\&\times\sum_{n=2,4,6,\ldots}^{\infty}\frac{(-1)^{\frac{n}{2}}}{n^2-1}\cos np\varphi=A+B\ln R_2+\\&+(C_1R_2^{p}+C_{-1}R_2^{-p})\cos p\varphi+\sum_{n=2,4,6,\ldots}^{\infty}(C_nR_2^{np}+\\&+C_{-n}R_2^{-np})\cos np\varphi.\end{aligned}\right\}\quad(14)$$

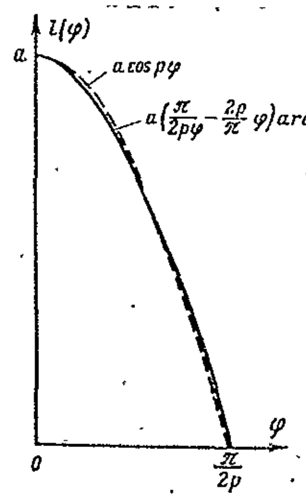

Figure 3

Equating in (14) the amplitudes of the corresponding harmonics on both sides of the equations and solving the system of two equations thus obtained, we have

$$\left.\begin{aligned} &A=\frac{-\frac{ak_2}{\pi}\ln R_1-\left(\Delta U-\frac{ak_1}{\pi}\right)\ln R_2}{\ln\beta};\ B=\\ &=\frac{\Delta U-\frac{a}{\pi}(k_1-k_2)}{\ln\beta};\\ &C_1=\frac{a}{2}\frac{-k_2-k_1\beta^p}{R_2^p(\beta^{2p}-1)};\ C_{-1}=\frac{a}{2}\frac{k_1+k_2\beta^p}{R_1^{-p}(\beta^{2p}-1)};\\ &C_n=\frac{2a}{\pi}\frac{(-1)^{\frac{n}{2}}}{n^2-1}\frac{-k_2+k_1\beta^{np}}{R_2^{np}(\beta^{2np}-1)};\\ &C_{-n}=-\frac{2a}{\pi}\frac{(-1)^{\frac{n}{2}}}{n^2-1}\frac{k_1-k_2\beta^{np}}{R_1^{-np}(\beta^{2np}-1)},\end{aligned}\right\}\qquad(15)$$

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where $\beta = R_1/R_2$.

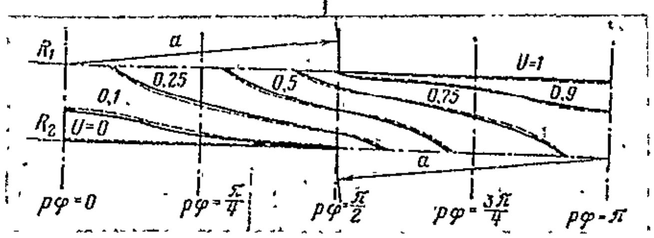

Figure 4.

It should be noted that, in calculating the coefficients k_1 and k_2 from (8) and (9), it is also necessary to set $U_1 = \Delta U$ and $U_2 = 0$.

Substitution of the coefficients (15) and (16) into the expression for the magnetic potential (13) yields the solution of the problem.

The components of the induction with respect to the coordinate axes are connected to the potential $U(\rho, \varphi)$ by the familiar relationships [3]:

$$B_\rho=-\mu_0\frac{\partial U}{\partial\rho};\quad B_\varphi=-\mu_0\frac{1}{\rho}\frac{\partial U}{\partial\varphi}.$$

To test the results obtained on the digital computer "Nairi", the field in the air gap of a hysteresis clutch with the following design parameters was computed: $\underline{R}_1 = 1.75$ cm, $\underline{R}_2 = 1.7$ cm, $\underline{a} = 0.18$ cm, $\rho = 15$. Here in the series (13) under the summation sign five terms were retained. The curves of the equipotential lines obtained (Figure 4, broken lines) were compared with the experimental curves obtained by modelling the field on conducting paper (Figure 4, the solid lines). The results of the comparison make it possible to draw a conclusion about the extent to which computation and experiment are in good agreement.

In conclusion, it should be noted that the expression for the magnetic potential in the form (13) is valid only in the domain bounded by the radii R_1 and R_2, i.e., in the region of greatest interest from the point of view of possible further analysis of EMC, and cannot be used to find the field in the cylindrical slots of the inductor. This circumstance is explained by the fact that in the course of the solution the problem was reduced to the Dirichlet Problem for a Ring.

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