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MODELING STRUCTURAL DAMPING FOR SOLIDS HAVING DISTINCT SHEAR AND DILATATIONAL LOSS FACTORS

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SUMMARY

For steady state time harmonic problems (rigid format 8), the NASTRAN program as currently configured treats internal structural damping through the introduction of a single structural element damping coefficient that typically is viewed as the ratio of the complex to real modulus of elasticity (E^i/E^r). For problems dealing with two or three dimensional dynamic linear viscoelasticity (e.g. a Kelvin-Voigt viscoelastic model), the present NASTRAN capability cannot directly handle this situation wherein two independent damping coefficients are required to properly model the dissipation phenomenon. A technique is presented whereby the user can adapt the standard versions of NASTRAN (without resorting to either DMAP and/or FORTRAN coding changes) for the purpose of treating this class of problem.

INTRODUCTION

This paper is concerned with the solution to 1, 2, or 3 dimensional steady state (time harmonic) structural response problems wherein part or all of the structure is comprised of a linear viscoelastic material. In particular, attention is focused on the representation of the viscoelastic dissipation (or equivalently sound absorption) properties of this class of materials. Typically, rubber-like materials fall into the category of interest. In situations where the driving frequencies of the applied loading is large, the effect of the energy dissipation characteristics on the overall dynamic response (particularly in wave propagation problems) can be significant. Consequently, it is important that the material physical properties are modeled as accurately as possible. In this paper, the Kelvin-Voigt viscoelastic model is selected wherein the corresponding continuous field equations are given by (ref. (1))

$$\left(1 + \frac{\tilde{\mu}^i}{\mu^r}\frac{\partial}{\partial t}\right)\nabla^2\bar{u} + \left(\frac{\lambda^r+\mu^r}{\mu^r}\right)\left(1 + \frac{\tilde{\lambda}^i + \tilde{\mu}^i}{\lambda^r + \mu^r}\frac{\partial}{\partial t}\right)\nabla(\nabla\cdot\bar{u}) = \frac{\rho}{\mu^r}\frac{\partial^2\bar{u}}{\partial t^2} \qquad (1)$$

where λ^r,μ^r are the usual Lame´ elastic constants; $\tilde{\lambda}^i$, $\tilde{\mu}^i$ are their viscous counterparts; ρ denotes the mass per unit volume, $\bar{u}$ is the displacement vector; t is time and ∇ is the vector "del" operator ($\partial/\partial x$, $\partial/\partial y$, $\partial/\partial z$). The steady state harmonic version of equation (1) is obtained by substituting $\bar{u} = \bar{u}_o e^{+i\omega t}$ into equation (1) which results in the form

$$\tilde{c}_s^2 \nabla^2 \bar{u}_o + \tilde{c}_d^2 \nabla(\nabla \cdot \bar{u}_o) - \tilde{c}_s^2 \nabla(\nabla \cdot \bar{u}_o) = \omega^2 \bar{u}_o \tag{2}$$

where $\tilde{c}_s$ and $\tilde{c}_d$ are the complex wave speeds given by the expressions

$$\tilde{c}_s^2 = c_s^2(1+i\eta_s)$$
$$\tilde{c}_d^2 = c_d^2(1+i\eta_d) \tag{3}$$

In equations (3), c_s and c_d are the usual elastic shear and dilatational wave speeds, and η_s and η_d are the associated shear and dilatational dissipation constants; these four constants are related to the constants originally employed in equation (1) through the relations:

$$c_s = \sqrt{\mu^r/\rho} \qquad c_d = \sqrt{(\lambda^r + 2\mu^r)/\rho} \tag{4}$$

$$\eta_s = \mu^i/\mu^r \qquad \eta_d = (\lambda^i + 2\mu^i)/(\lambda^r + 2\mu^r) \tag{5}$$

where λ^i and μ^i are two independent frequency dependent viscoelastic constants which are determined experimentally. The λ^i, μ^i constants are related to the $\tilde{\lambda}^i$, $\tilde{\mu}^i$ constants of Eq (1) through the relations $\lambda^i = \omega\, \tilde{\lambda}^i$ and $\mu^i = \omega\, \tilde{\mu}^i$. Depending on one's viewpoint, the pair of parameter λ^i, μ^i (or alternatively η_s, η_d) can be viewed as the two independent parameters which describe the dissipation characteristics of the viscoelastic media. Further, the independent constants λ^r, μ^r (or alternatively c_p, c_s) can be viewed as the two independent constants which define the elastic characteristics of the media.

The current version of NASTRAN can treat a solid media governed by equation (2) by using 2 and 3 dimensional elements (e.g. CTRMEM, CQDMEM, CTRAPGR, CTETRA, CTRIARG to name a few) in conjunction with both rigid format 8 and the introduction of a loss factor (e.g. the GE input variable on a MAT1 or MAT2 card; this is referred to as the "structural element damping coefficient" in the NASTRAN manual). Unfortunately, however, the user can introduce only one independent loss factor. A single, rather than two, independent loss factor can properly represent the Kelvin-Voigt model if the relation

$$\frac{\lambda^i}{\lambda^r} = \frac{\mu^i}{\mu^r} = \eta_E \tag{6}$$

is met. Substituting equation (6) into equations (5) in conjunction with the fact that the Young's modulus, E, is related to μ, λ through $E = \mu(3\lambda+2\mu)/(\lambda+\mu)$ yields the relations

$$\eta_d = \eta_s = \eta_E = E^i/E^r \tag{7}$$

Thus, the more limited single parameter loss factor case can directly be implementated in NASTRAN by assuming equation (6) holds and setting GE = η_E on a MAT 1 or MAT 2 card as appropriate.

The remainder of this paper is concerned with the situation wherein the viscoelastic physical constants are such that $\eta_d \neq \eta_s$ (i.e. the special case implied by equations (6) are not satisfied). A technique is presented which permits the user to treat this more general case without having to resort to either DMAP instructions and/or modified FORTRAN coding modifications to the original NASTRAN program. Because of this general type "fix", the approach also has applications to other finite element programs having the same one parameter type dissipation limitation found in NASTRAN.

IMPLEMENTATION OF TWO PARAMETER LOSS FACTORS

Here we consider finite elements representing equations (2) for the case $\eta_s \neq \eta_d$. The finite element formulation representing equations (2) leads to a complex set of simultaneous equations to be solved of the form (ref. (2))

$$[\bar{K}]\{U\}=\{P\} \tag{8}$$

where $[\bar{K}]=[-\omega^2[M]+i\omega[B]+[K]]$ (9)

where {U} is the vector of nodal displacement amplitudes, {P} is the vector of applied forces, [M], is the assembled mass matrix, [B] is the assembled damping matrix and [K] is the elastic stiffness matrix. The formation of the mass, stiffness and non-structural damping portion of the matrices in Eq. (8) follow exactly the same process used in modeling elasticity type problems, hence will not be commented on here (e.g. see refs. (2,3). Attention is consequently focused on the formation of the structural damping part of the $[\bar{K}]$ matrix; let us define $[\bar{K}]_e^\eta$ as the individual element structural damping portion due to the contribution of the e^{th} element. NASTRAN currently forms $[\bar{K}]_e^\eta$ from the relation

$$[\bar{K}]_e^\eta = i\eta_e[K]_e \equiv i\int_{V_e}\left([C]^T\eta_E[G]_e^r[C]\right)dxdydz \tag{10}$$

where $[K]_e$ is the individual element elastic stiffness matrix, [C] is the corresponding displacement-strain matrix and $[G]_e^r$ is the elastic stress-strain law matrix. For isotropic materials, $[G]_e^r$ is of the form:

$$
\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{Bmatrix} = \underbrace{\begin{bmatrix} (\lambda^r+2\mu^r) & \lambda^r & \lambda^r & 0 & 0 & 0 \\ \lambda^r & (\lambda^r+2\mu^r) & \lambda^r & 0 & 0 & 0 \\ \lambda^r & \lambda^r & (\lambda^r+2\mu^r) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu^r & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu^r & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu^r \end{bmatrix}}_{[G]^r_e} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{31} \\ \varepsilon_{12} \end{Bmatrix} \quad (11)
$$

where σ_{11}, σ_{22} ... and ε_{11}, ε_{22} ... are the element stresses and strains.

So far we have described, via equations (10) and (11), the form currently in NASTRAN. Next we consider the form of $[\bar{K}]^{\mu}_{e}$ we would like to have in order to implement the two independent parameter formulation. By comparing what we have to what we would like to have, the "fix" to NASTRAN will become evident. We start by making the important observation that the desired general continuous viscoelastic form of equation (2) can be derived from the usual elastic derivation for the dynamic equations of elasticity (ref. (4)). This is accomplished by following the elastic derivation of ref. (4) with the modification that, in place of the usual isotropic stress-strain law (i.e. Eq. (11)), we use

$$
\{\sigma\}=[[G]^r_e + i[G]^i_e]\{\varepsilon\} \quad (12)
$$

where $[G]^i_e$ is the same expression as $[G]^r_e$ except all r superscripts are replaced with i superscripts for the λ, μ entries. Similarly, in the finite element formulation, all we need do is replace the usual $[G]^r_e$ matrix in the stiffness derivation with $[G]^r_e+i[G]^i_e$. Thus, the element stiffness becomes

$$
[K]_e = \int_{V_e} [C]^T[[G]^r_e+i[G]^i_e][C]dxdydz \quad (12)
$$

$$
= \underbrace{\int_{V_e} [C]^T[G]^r_e[C]dxdydz}_{\text{Term 1}} + i \underbrace{\int_{V_e} [C]^T[G]^i_e[C]dxdydz}_{\text{Term 2}} \quad (12b)
$$

Term 1...Usual elastic stiffness contribution to $[\bar{K}]$ of Eq. (8).

Term 2...viscoelastic dissipation contribution to $[\bar{K}]$ of equation (8).

Now we are in a position to build the desired two independent parameter viscoelastic finite element. The total complex element contribution, $[K]_e$, to the assembled $[\tilde{K}]$ matrix in equation (9) is formed in two parts through the creation of an overlapping double viscoelastic element. This double element is shown schematically in Figure (3) (a two dimensional element is shown only for convenience). It consists of two identically shaped element occupying the same physical space and having the same nodal numbers but different element numbers and different material identification cards (MAT1 for 3-d elements and MAT2 for 2-d elements). Our goal is to let one of the overlapping elements form the Term-1 elastic contribution of Eq. (12b) and the other form the Term-2 viscoelastic dissipation contribution. In agreement with the notation of Figure (3), we refer to the first overlapping element as the "elastic element" and the second as the "massless dissipation element". The only part that remains is to define the input constants on the MAT1 (or MAT2) cards so that Term-1 and Term-2 in equations (12b) are properly formed. More specifically, the rationale for the selection of the MAT1 (or MAT2) constants follows from seeking out a set of parameters for the $\eta_E[G]_e^r$ matrix in equation (10), (namely the η_E, and components of the elastic $[G]_e^r$ matrix which are controlled by the user through selection of the input variables on the appropriate MATi cards) that will result in the desired $[G]_e^i$ matrix in Term-2 of equation (12b). The treatment is slightly different for three or two dimensional elements, consequently we treat them one at a time.

Three Dimensional Viscoelastic Elements

- The elastic element contribution is obtained by setting the following parameters on a MAT1 card:

  1) set mass density (RHO) = actual mass density of viscoelastic material

  2) set loss factor (GE) = 0.0 (13)

  3) set $E = \mu^r(3\lambda^r+2\mu^r)/(\lambda^r+\mu^r)$

     $G = \mu^r$

- The massless dissipation element contribution is obtained by setting the following parameters on another MAT1 card specially earmarked for this second overlapping element:

  1) set mass density (RHO) = 0.0 (zero to avoid double counting)

  2) set loss factor $\eta_E \equiv$ (GE) = λ^i/ε (14)

3) set $E = \varepsilon R(3+2R)/(1+R)$

$G = \varepsilon R$ (14) cont'd)

with $R = \mu^i/\lambda^i$

where ε is an arbitrary parameter that is selected small as desired (but not zero). The small ε parameter removes the elastic contribution of the massless dissipation element. A suggested value for ε is that it is on the order of 10,000 times smaller than the larger of the real components λ^r or μ^r.

The interested reader is invited to back substitute the above values for E, G, and GE (e.g. η_E) into the $\eta_E[G]_e^r$ matrix formed by NASTRAN in conjunction with equation (1) where it can be easily verified that the results reduce (independent of the ε choice) exactly to the desired two independent parameter matrix $[G]_e^i$ employed in Eq. (12b). The smallness of ε only effects the unwanted elastic stiffness contribution of the massless dissipation element already accounted for in the "elastic element".

Two Dimensional Elements

- The elastic element contribution is obtained by setting the following parameters on a MAT2 card:

1) set mass density (RHO) = actual mass density of viscoelastic material

2) set loss factor (GE) = 0.0

3) set $G11 = \lambda^r+2\mu^r$

$G12 = \lambda^r$ (15)

$G13 = 0.0$

$G22 = \lambda^r$

$G23 = 0.0$

$G33 = \mu^r$

- The massless dissipation element contribution is obtained by setting the following parameters on another MAT2 card specially earmarked for this second overlapping element:

1) set mass density (RHO) = 0.0

2) set loss factor $\eta_E \equiv (GE) = \lambda^i/\varepsilon$ (16)

3) set $G11 = \varepsilon(1+2\mu^i/\lambda^i)$ (16) cont'd

$G12 = \varepsilon$

$G13 = 0.0$

$G22 = \varepsilon$

$G23 = 0.0$

$G33 = \varepsilon\mu^i/\lambda^i$

where as with the three dimensional element, the ε is an arbitrary parameter that is selected small as desired (but not zero). See 3-d element write-up for a suggested ε value.

DEMONSTRATION PROBLEM

Next we consider a demonstration problem to illustrate the implementation and accuracy of the two independent parameter loss factor viscoelastic elements described in the previous section. The problem is to determine the scattered and transmitted pressures for a water-submerged steel plate, (covered with a constant thickness layer of viscoelastic material), subject to an incident plane wave normal to the plate as illustrated in figure (1). The problem is tractable from a closed form solution point of view, consequently, an independent check on the NASTRAN solution is available. Furthermore, experimental results are also available to further back up the accuracy of the physical representation of the viscoelastic material.

Exact Solution

The exact solution to this problem can initially be treated as an ordinary one dimensional, small deformation wave propagation problem. The effect of viscoelasticity can be introduced by replacing the wave speed c_d with $\tilde{c}_d = c_d\ (1+i\eta_d)^{\frac{1}{2}}$. The solution to the problem illustrated in figure (1) is carried out exactly like the problem given in ref. (5), page 136, except that two finite thick plates (rather than one) is submerged in the fluid. The origin (at $x=0$) is located at the right face of the viscoelastic layer (+x to the left). The back side fluid is denoted as media (4), steel plate as (3), the viscoelastic layer as (2) and the front side fluid as (1). The thickness of the steel plate is ℓ_3 and the viscoelastic layer is ℓ_2. The following relations define the various waves present in the problem:

$$(p_i)_1 = A_1\ e^{i(\omega t-k_1x)} \qquad \text{Incident wave in (1)}$$

$$(p_s)_1 = B_1\ e^{i(\omega t+k_1x)} \qquad \text{Scattered wave in (1)} \qquad (17)$$

$$(p_t)_2 = A_2 \, e^{i(\omega t - k_2 x)} \qquad \text{Transmitted wave in (2)}$$

$$(p_s)_2 = B_2 \, e^{i(\omega t + k_2 x)} \qquad \text{Scattered wave in (2)} \qquad \text{(17 cont'd)}$$

$$(p_t)_3 = A_3 \, e^{i(\omega t - k_3 (x - \ell_2))} \qquad \text{Transmitted wave in (3)}$$

$$(p_s)_3 = B_3 \, e^{i(\omega t + k_3 (x - \ell_2))} \qquad \text{Scattered wave in (3)}$$

$$(p_t)_4 = A_4 \, e^{i(\omega t - k_4 (x - \ell_2 - \ell_3))} \qquad \text{Transmitted wave in (4)}$$

where in the fluid p is pressure and in the solid, p is the negative of the stress σ_{11} in the x direction. The k quantities are defined as:

$$k_1 = \omega / c_{d1}$$

$$k_2 = \omega / [c_{d2}(1+\eta_{d2})^{\frac{1}{2}}]$$

$$k_3 = \omega / c_{d3}$$

$$k_4 = \omega / c_{d4}$$

where c_{d1}, c_{d2}, c_{d3}, c_{d4} are the real dilatational wave speeds of the four materials and η_{d2} is the dilatational loss factor (equation (5)) of the visco-elastic layer. The six unknown B_1, A_2, B_2, A_3, B_3, A_4 can be determined from equating pressure and particle velocities at the three interfaces, thus providing six equations to balance the six unknowns. All response variables are referred to the incident wave amplitude A_1, thus A_1 is not considered an unknown in the problem.

The quantities of interest are the scattered pressure in the incident side fluid (media 1) and the transmitted pressure in the back side fluid (media 4). After algebraically solving for the constants we obtain

$$\text{Scatter pressure amplitude} \equiv B_1/A_1 = (C_2 A_{12} - C_1 A_{22})/(A_{21} A_{12} - A_{11} A_{22}) \qquad (19)$$

$$\text{Transmitted pressure amplitude} \equiv A_4/A_1 = (A_{21} A_{32} C_1 - A_{32} A_{11} C_2)/(A_{21} A_{12} - A_{11} A_{22})$$

where $C_1 = -\cos(k_2 \ell_2) + i\, r_{12} \mathrm{Sin}(k_2 \ell_2)$; $r_{12} = \rho_2 \tilde{c}_{d2} / \rho_1 c_{d1}$; $r_{34} = \rho_4 c_{d4} / \rho_3 c_{d3}$

$C_2 = r_{23} i \, \mathrm{Sin}(k_2 \ell_2) - r_{23} r_{12} \cos(k_2 \ell_2)$; $r_{23} = \rho_3 c_{d3} / \rho_2 \tilde{c}_{d2}$; $\tilde{c}_{d2} = c_{d2}(1 + i\eta_{d2})^{\frac{1}{2}}$

$$A_{12} = ([1+r_{34}][\cos(2k_3\ell_3)+i\ \sin(2k_3\ell_3)]/[1-r_{34}])-1$$

$$A_{21} = -r_{23}i\ \sin(k_2\ell_2)-r_{23}r_{12}\ \cos(k_2\ell_2)$$

$$A_{11} = \cos(k_2\ell_2)+i\ r_{12}\ \sin(k_2\ell_2)$$

$$A_{22} = 1 + [1+r_{34}][\cos(2k_3\ell_3)+i\ \sin(2k_3\ell_3)]/[1-r_{34}]$$

$$A_{32} = -2r_{34}[\cos(k_3\ell_3) + i\ \sin(k_3\ell_3)]/[1-r_{34}]$$

Finite Element Solution

The finite element representation of the figure (1) problem is shown in figure (2). The procedure for representing the infinite domain of fluid on the front and back side of the submerged plate is given in detail in ref. (6). Briefly, it consists of using a plane wave boundary condition at the mesh termination of the form $p=\rho c_d\dot{u}$ where $\dot{u}$ is the normal particle velocity and p is the total pressure at the mesh termination. The nodes along the outer boundaries are constrained to move only in the direction of wave propagation. The figure (2) sketch is drawn to scale and represents the actual number of elements used in the problem. All elements employed in the model are comprised of CQDMEM quadralateral elements. The viscoelastic zone is made up from the overlapping double elements described earlier in the paper (e.g. a typical viscoelastic element is illustrated in figure (3)).

Comparative Results

The demonstration problem (both analytical and finite element) was evaluated with the following set of physical constants

Table 1 - DEMONSTRATION PROBLEM PHYSICAL CONSTANTS

MATERIAL	λ^r psi	μ^r psi	λ^i psi	μ^i psi	ρ lb-sec^2/in^4
Water	345,600.	0.0	0.0	0.0	.000096
Steel	17,307,000.	11,538,000.	0.0	0.0	.000735
Viscoelastic Material	86,703.	115.9	41,736.8	11.6	.0003599

The viscoelastic constants were evaluated by W. Madaigosky at NSWC. For the NASTRAN computer run, the water and steel plate properties were entered on a MAT2 card. The elastic entries G11, G12, etc. correspond to the column and row data in the first, second and sixth (columns and rows) of the $[G]_e^r$ matrix of equation (11). The viscoelastic data were entered on two separate MAT2 cards corresponding to the procedure described by equations (15) and (16). An ε parameter of $\varepsilon = 4.16$ was used in the actual run. The non-dimensional results are shown in Table 2.

The significance of the added viscoelastic layer on the scattered and transmitted pressure is seen by comparing the results of Tables 2 and 3, wherein the viscoelastic layer has the effect of absorbing a significant amount of the incident energy. The accuracy of the NASTRAN results for both the case with the viscoelastic layer and without the layer are quite good. As expected, the finite element solution accuracy falls off as the incident frequency increases. As pointed out by ref. (7), for problems of this type, at least 8 elements per wave length are needed to accurately model elastic waves in the media. Note in Table 2 that as the incident frequency approaches the 8 elements per wave length limit, the accuracy is starting to deteriorate. The 8 element per wave length limit suggested by ref. (7) was in the absence of structural damping; perhaps the results presented here suggests that more than 8 elements per wave length are required for structural damping (e.g. 12 elements per wave length).

CONCLUDING REMARKS

The procedure outlined here provides the NASTRAN user with an expanded structural damping capability, thus permitting the user to specify two independent loss factors η_s, η_d via the construction of the "overlapping double viscoelastic element" process described in this paper. The demonstration problem shows good accuracy of the procedure relative to the exact solution for the same problem. It is acknowledged that there is some added solution time due to the added calculation time for the formation of the stiffness of the second overlapping element; however, this is a relatively insignificant amount in comparison to the overall solution time of the problem. The double element uses the same node numbers for both the "elastic element" and the superimposed "massless dissipation element", consequently, the matrix size or bandwidth properties are not affected at all.

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Table 2 - COMPARATIVE SOLUTION RESULTS
(with viscoelastic layer present)

Nondimensional frequency; $\omega \ell_3/c_{d1}$	NASTRAN Scattered Pressure ratio	Exact Scattered Pressure ratio (equation (19))	NASTRAN transmitted pressure ratio	Exact transmitted pressure ratio	Experimental Scattered pressure	Elements per wave length in elastomer
.6545	.401	.398	.273	.273	-	158.
3.272	.191	.190	.028	.028	-	31.6
6.545	.108	.104	.006	.007	-	15.8
9.817	.122	.113	.003	.003	.12	10.5
13.089	.133	.113	.002	.003	.12	7.9

Note that ℓ_3/ℓ_2 = 2.2727 (ratio of steel plate-to-viscoelastic layer thickness)

Table 3 - COMPARATIVE SOLUTION RESULTS
(without viscoelastic layer present)

Nondimensional frequency; $\omega \ell_3/c_{d1}$	NASTRAN Scattered Pressure ratio	Exact Scattered Pressure ratio (equation (19))	NASTRAN transmitted pressure ratio	Exact transmitted pressure ratio	Experimental Scattered pressure
.654	.930	.928	.374	.373	-
3.27	.995	.996	.090	.090	-
6.54	.998	.998	.067	.067	-
9.82	.996	.994	.113	.113	-
13.09	.957	.952	.306	.307	-

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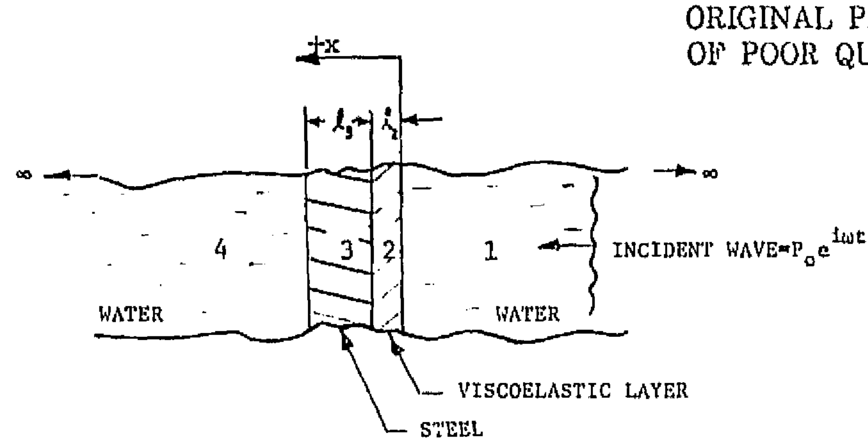

Figure 1. Submerged Plate With Viscoelastic Layer

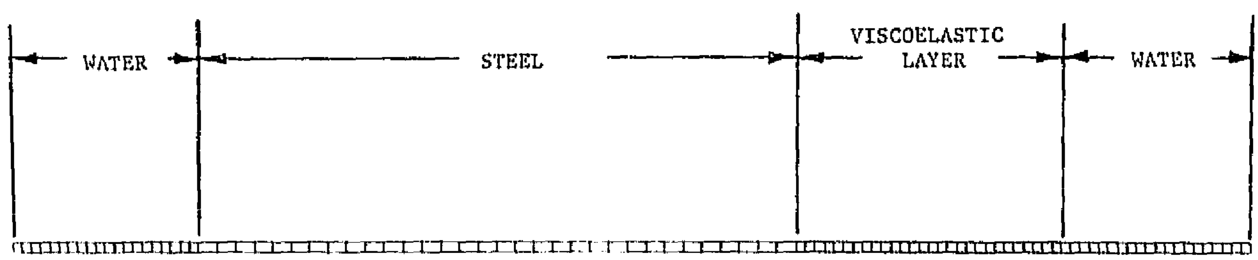

Figure 2. One Dimensional Finite Element Model

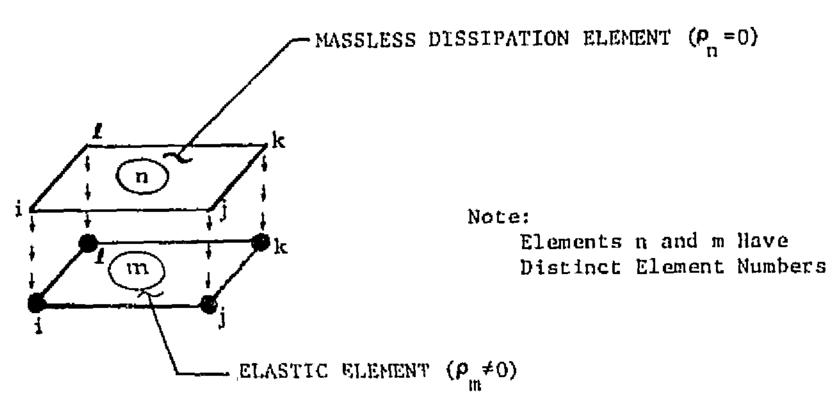

Figure 3. Overlapping Double Viscoelastic Element
(Quadralateral Element Example)