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THERMAL STRESS ANALYSIS OF CERAMIC STRUCTURES WITH NASTRAN ISOPARAMETRIC SOLID ELEMENTS

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SUMMARY

This paper presents a study of the performance of the NASTRAN level 16.0 twenty node isoparametric bricks (CIHEX2) to thermal loading. A free ceramic plate was modelled using twenty node bricks of varying thicknesses. The thermal loading for this problem was uniform over the surface with an extremely large gradient through the thickness. No mechanical loading was considered. Temperature-dependent mechanical properties were considered in this analysis. The NASTRAN results are compared to one-dimensional stress distributions calculated by direct numerical integration.

INTRODUCTION

In attempting to analyze a ceramic radome no information was available concerning the sensitivity of the twenty node brick to large aspect ratio or large thermal gradients. The free plate was identified as an appropriate problem to examine this sensitivity. The thermal gradient of interest (Fig. 1) is severe and generates significant compressive hoop stress spikes immediately beneath the surface. A model containing sufficient elements near the surface to adequately predict the detailed in-depth response of a full radome model was infeasibly large. Therefore, this study was made to determine the effects of lesser numbers of elements through the thickness on the stress predictions. Also studied was the degradation of the stresses with increase in the ratio of the length of the surface side to the element thickness (aspect ratio).

The basic model used in this study was a 3 in. by 3 in. by 1/4 in. plate modelled with a 4 x 4 x 4 element grid. Various element spacings through the thickness were examined starting with uniform spacing and gradually allowing the surface elements to shrink in-depth while increasing the thickness of the rear elements. Variable Mechanical Properties were used for this analysis (Fig. 2). A total of four cases were examined.

The NASTRAN twenty node bricks showed little sensitivity to high aspect ratios (tests were run to 94:1) for this loading. Stresses, as always with this element, are less uniform than could be desired; but, when averaged in a reasonable fashion the correlation between NASTRAN'S stresses and one-dimensional numerical solutions is good provided any one element does not span more than one inflection point in the stress curve.

SYMBOLS

ρ = mass density

c_p = specific heat at constant pressure

T = temperature field

t = time

z = location through the depth

z+h is the heated surface

k = the thermal conductivity

σ = stress

f = σ_{xx}

E = young's modulus

ν = poisson's ratio

α = thermal expansion coefficient

2h = plate thickness

THERMAL LOADING

The thermal loading is produced by the application of a suddenly applied uniform heat flux over one face of the plate while the other face is maintained adiabatic. The resulting transient temperature distribution is obtained by the implicit finite difference numerical solution of the one dimension variable property heat conduction equation.

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial k}{\partial z} \frac{\partial T}{\partial z} + k \frac{\partial^2 T}{\partial z^2} \tag{1}$$

with initial conditions for t<0 T=530°R

and boundary conditions for t>0

$$\begin{aligned} z = +h \qquad & -k \frac{\partial T}{\partial z} = 88 \text{ BTU/in}^2\text{-sec} \\ z = -h \qquad & -k \frac{\partial T}{\partial z} = 0 \end{aligned} \tag{2}$$

Sublimation of the ceramic is modelled with an Arrhenius type function which was derived from test data.

ONE-DIMENSIONAL STRESS DISTRIBUTION

The one-dimensional stress distribution was derived by considering a free plate of arbitrary planform, constant thickness, and with temperature dependent isotropic properties. The plate is thermally loaded with a temperature gradient through the thickness only. It is assumed that the stress field, away from the edges, is also only a function of the thickness coordinate and that all out of plane stresses are zero.

$$\sigma_{xx} = \sigma_{yy} = f(z);\ T = T(z)$$
$$\sigma_{xz} = \sigma_{xy} = \sigma_{yz} = \sigma_{zz} = 0 \tag{3}$$

The boundary conditions are chosen such that the resultant force and moment produced by f(z) are zero over the edges. Under the above assumptions, the equations of equilibrium are satisfied identically. The stress distribution is obtained by direct integration of the compatibility equations and application of the boundary conditions

$$f = \frac{E}{1-\nu}\left[-\alpha T + (D-Bz)\frac{N_T}{H} + (Az-B)\frac{M_T}{H}\right] \tag{4}$$

where

$$A = \int_{-h}^{h} \frac{E}{1-\nu}\,dz \qquad B = \int_{-h}^{h} \frac{E}{1-\nu}\,z\,dz$$

$$D = \int_{-h}^{h} \frac{EZ^2}{1-\nu}\,dz \qquad N_T = \int_{-h}^{h} \frac{\alpha ET}{1-\nu}\,dz \tag{5}$$

$$M_T = \int_{-h}^{h} \frac{\alpha ET}{1-\nu}\,z\,dz \qquad H = AD-B^2$$

The above integrals were numerically evaluated by the trapezoidal method utilizing the predicted temperature distribution and variable thermal properties (Fig. 2).

THREE-DIMENSIONAL NASTRAN ANALYSIS

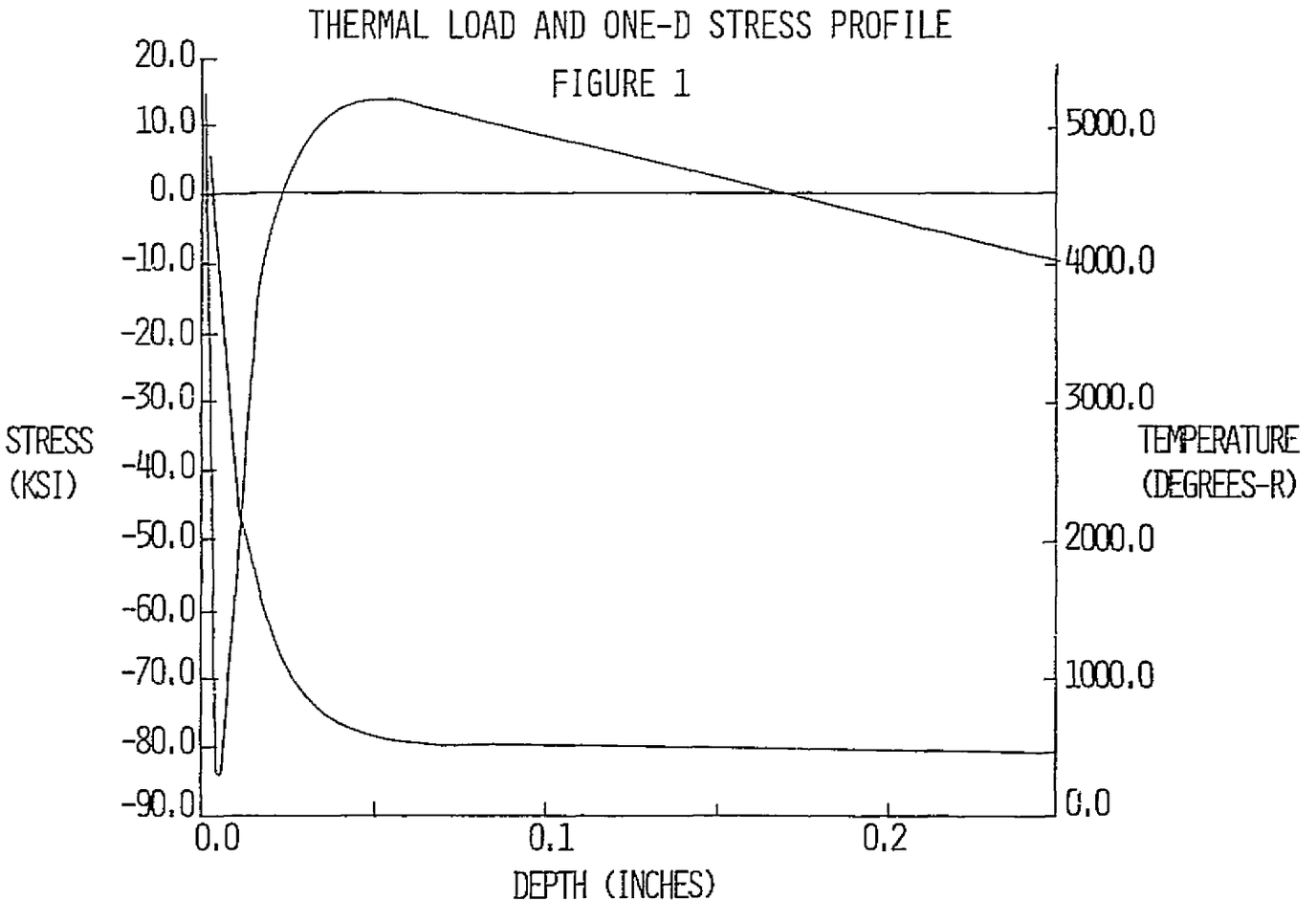
A 3 in. by 3 in. by 1/4 in. ceramic plate was modelled with a 4 x 4 x 4 element grid (Fig. 4). Surface elements of .0625, .05, .02, and .008 inch thickness were used. The temperature distribution through the thickness (Fig. 1) is applied uniformly over the plate surface. The plate is free-free and mechanical properties that vary with temperature are used (Fig. 2). Four integration points per side were used in all elements.

The stress output from NASTRAN for this element contains a stress value for each node point in each element. These stresses are extrapolated from the values calculated at the integration points and are rarely continuous from one element to the next. The stresses are averaged at each node and these stresses are further averaged for several central locations on the plate.

The stresses for the first two element spacings (Fig. 4) are unacceptable in the first quarter of the structure. The last two spacings adequately describe the stress field (Fig. 5).

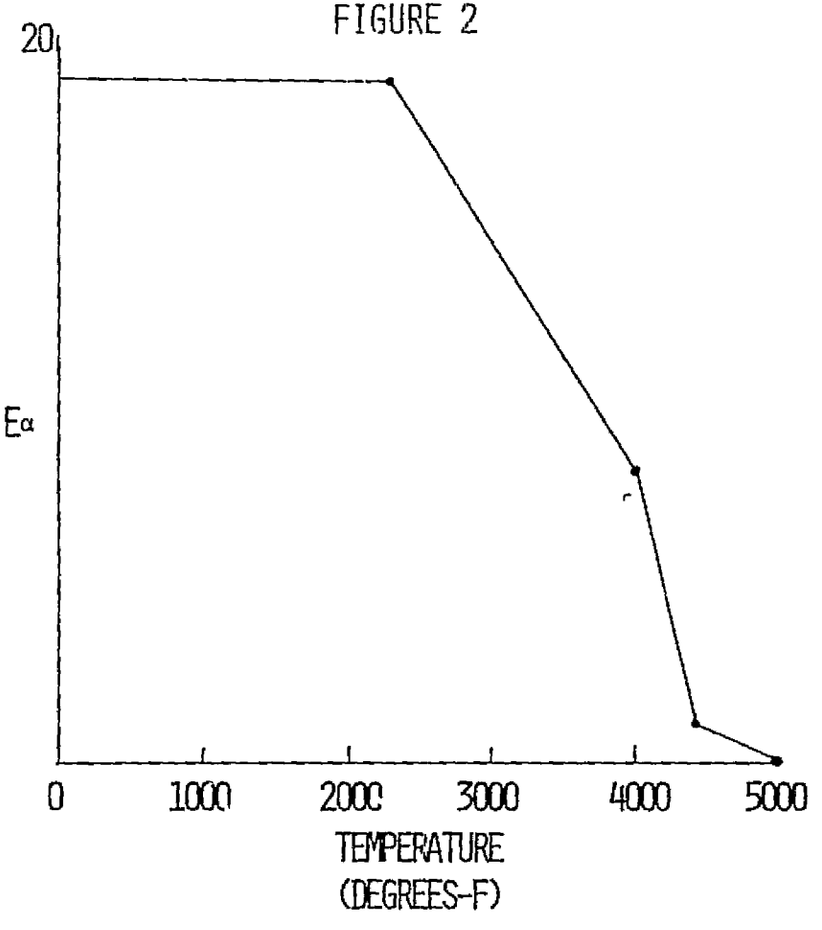
CONCLUDING REMARKS

The twenty node brick adequately describes thermal stress fields provided the stress field is sufficiently known to ensure proper element spacing in regions of high stresses. Aspect ratio does not appear to be critical in this element provided the only significant thermal gradient is in the direction of the smallest dimension.


THERMAL LOAD AND ONE-D STRESS PROFILE
FIGURE 1
20.0
10.0
0.0
-10.0
-20.0
-30.0
-40.0
-50.0
-60.0
-70.0
-80.0
-90.0
STRESS
(KSI)
5000.0
4000.0
3000.0
2000.0
1000.0
0.0
TEMPERATURE
(DEGREES-R)
0.0
0.1
0.2
DEPTH (INCHES)

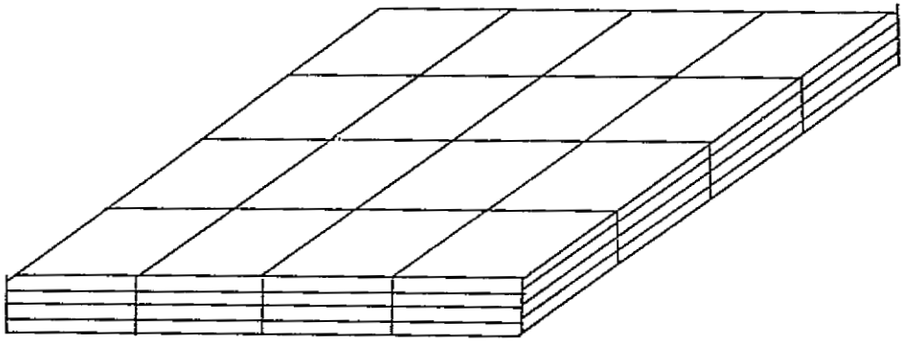
VARIABLE MECHANICAL PROPERTIES

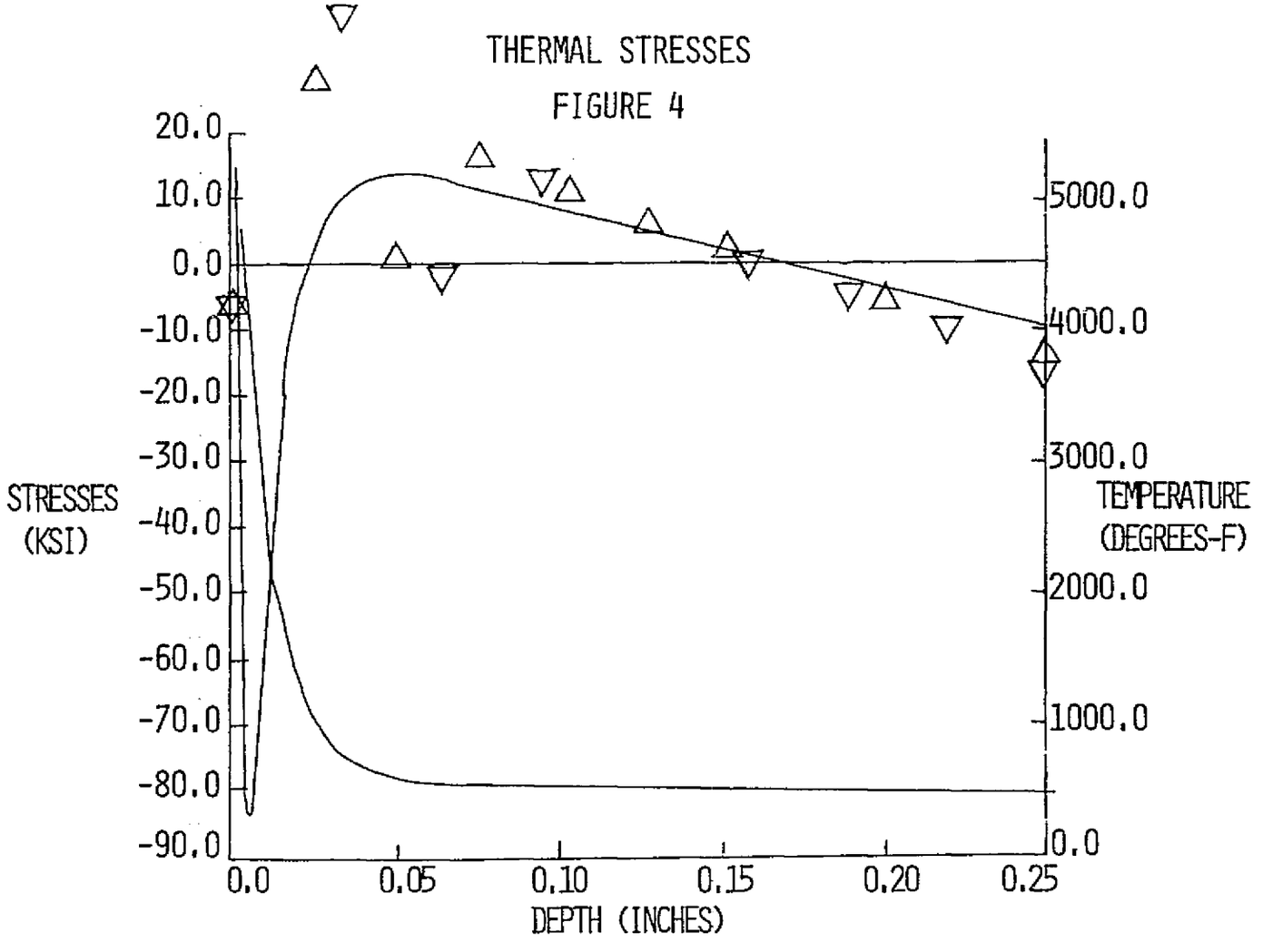
FIGURE 2

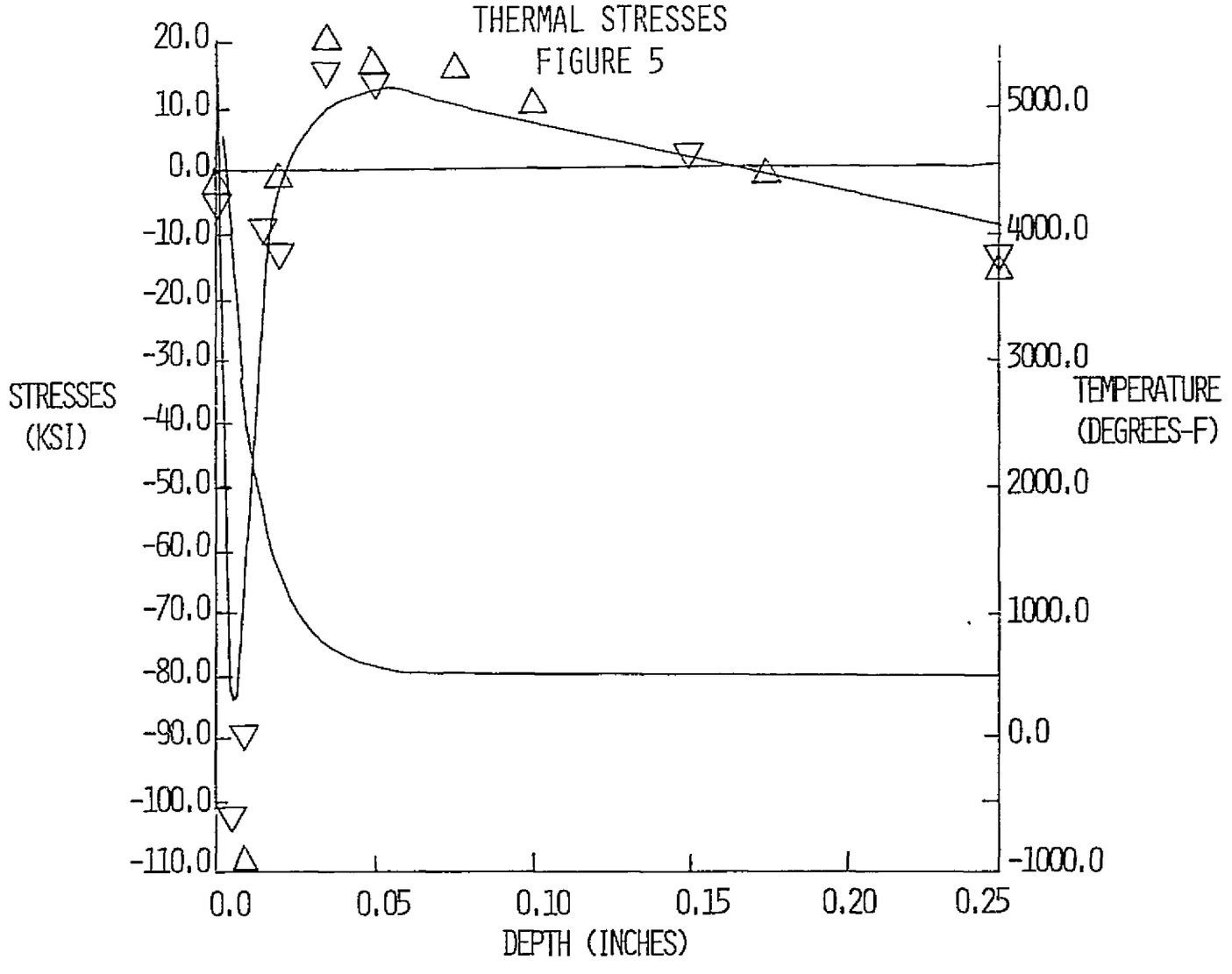


FINITE ELEMENT MODEL

FIGURE 3






THERMAL STRESSES
FIGURE 5
20.0
10.0
0.0
-10.0
-20.0
-30.0
-40.0
-50.0
-60.0
-70.0
-80.0
-90.0
-100.0
-110.0
STRESSES
(KSI)
5000.0
4000.0
3000.0
2000.0
1000.0
0.0
-1000.0
TEMPERATURE
(DEGREES-F)
0.0
0.05
0.10
0.15
0.20
0.25
DEPTH (INCHES)