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ACCURACY OF RESULTS WITH NASTRAN MODAL SYNTHESIS

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SUMMARY

A new method for component mode synthesis has been developed for installation in NASTRAN Level 17.5. An introduction and summary of the method was presented at the 1977 NASTRAN Colloquium [Ref. 1], but actual results were unavailable at that time. This paper serves as a continuation to Reference 1 by presenting results obtained from the new methoc and comparing these results with existing modal synthesis methods.

INTRODUCTION

The modal synthesis system developed by Universal Analytics, Inc. (UAI) for NASTRAN is a new development which provides for the benefits inherent in existing methods but eliminates the restrictions and computational drawbacks associated with other methods. In Reference 1 it was postulated that the new method was sufficiently general to duplicate the results of other, more restricted, methods simply by choosing different types of normal modes or vector recovery procedures. The test problems described herein have been selected for direct comparison with other published results. The goal of this effort was to determine the relative accuracy of the UAI method with its different options.

The use of structural modes as generalized degrees of freedom in dynamic models originated in the analog computer field where structures were combined with aeroelastic and control system models. The first applications to digital computers were simple extensions of the analog techniques. This so-called classical approach proved both highly restrictive and limited in accuracy. Many different approaches have been developed in recent years having increased accuracy and more generality in solving large-order structure dynamics problems.

Although the previous methods used in component mode synthesis vary considerably in both approach and application, they may be grouped into two distinct categories. The first category contains all of the methods using a Rayleigh-Ritz approach in which the component degrees of freedom represent the deflections of normal modes and static deflection shapes. The second category contains methods in which the component degrees of freedom are actual physical displacements plus a set of modal coordinates. Here, the classical method has been improved by adding flexibility coefficients to the matrices to account for the effects of a truncated set of modes.

The theoretical development of the NASTRAN modal synthesis system is being issued in Reference 2, the Level 17.5 Theoretical Manual. In this development the "residual flexibility" approach is used as a starting point but the end results become very similar to the Rayleigh-Ritz approach. The new method, in effect, is related to both categories of modal synthesis and shows that the differences between them are more related to computational procedures than in theoretical basis.

The two test problems described below were selected for comparison with several advanced mode synthesis methods. The problems also provide a comparison of the various options that will be available in the NASTRAN system which are summarized below:

1. The boundary conditions used to obtain component modes are not restricted. Free, constrained, and partially free modes may be used.

2. Inertia relief displacement shape functions may be included as degrees of freedom as a user option. These provide for exact static response of free bodies and more accuracy for low frequency response.

3. In the vector recovery process, after a system solution has been obtained, a "mode acceleration" procedure which calculates "improved" displacements is available.

4. A full set of error check procedures are available to assess the acturacy of the results. These include printout of the equilibrium forces, energy checks of truncated modes, and direct evaluation of the participation of the modal coordinates.

The test problems and their results are summarized below, followed by a summary of the conclusions which follow from the evaluation of the tests.

NOMENCLATURE

u - Physical Displacement

G - Guyan Reduction Transformation Matrix

K - Stiffness Matrix

M - Mass Matrix

ℓ - Length

x - Spacial Coordinates

T - Kinetic Energy

V - Potential Energy

ε - Error Ratio

ξ - Generalized Displacement of a Mode

ρ - Density

ϕ - Eigenvector

ω - Radian Frequency

EXACT ROD PROBLEM

A convenient test problem for modal synthesis evaluation was used by Rubin [Ref. 3] to compare various methods, including his own new method. The problem, illustrated in Figure 1 consists of a single rod with extensional motion. Rather than solve the problem with finite elements, a set of closed form integral solutions may be used to obtain the modal synthesis matrix coefficients. In effect, the results will simulate a problem with an infinite number of elements. This procedure will elminate the finite element errors and will allow analysis of errors resulting only from the modal synthesis formulation.

The problem solved by Rubin uses the free-free modes of the rod to fomulate a component mode substructure. The solution matrix is then constrained to obtain cantilever modes. If the end degree of freedom were included in the normal formulation it could be attached to another structure directly. The errors in the results will occur because the sine wave solutions for the cantilever rod must be approximated by the dissimilar cosine waves of the free rod.

In the UAI method, the displacement shapes are

Static Displacements

$$u_1^i(x) = \left[\frac{I}{G}\right]\{u_b\} = u_1 \tag{1}$$

Inertial Relief:

$$u_2^i(x) = k_{ii}^{-1}\left[M_{ib} + M_{ii}G\right]\phi_o\xi_o$$

$$= \frac{\rho\ell^2}{E}\left(x - \frac{x^2}{2\ell}\right)\xi_o \tag{2}$$

Normal Modes:

$$u_k(x) = \{\phi_{ik} - G\phi_{bk}\}\xi_k$$

$$= \left(\cos \frac{\pi k x}{\ell} - 1 \right) \xi_k \qquad k = 1,2,\ldots\infty \tag{3}$$

After normalizing the units, the total displacement at any point on the rod is:

$$u(x) = u_1 + \frac{1}{\ell}\left(x - \frac{x^2}{2\ell} \right) \xi_o + \sum_k \left(\cos \frac{\pi k x}{\ell} - 1 \right) \xi_k \tag{4}$$

Instead of performing matrix transformations the stiffness, [K], and mass, [M], matrices are obtained using the La Grange formulation which states:

$$M_{ij} = \frac{d}{dt} \frac{\partial^2 T}{\partial \dot{q}_i \partial \dot{q}_j} \tag{5}$$

and

$$K_{ij} = \frac{\partial^2 V}{\partial q_i \partial q_j} \qquad , q_i = u_1, \xi_o, \xi_1 \ldots , \tag{6}$$

where the potential energy, V, and the kinetic energy, T, are:

$$V = \int_o^\ell \frac{EA}{2} \left(\frac{\partial u}{\partial x} \right)^2 dx \tag{7}$$

$$T = \int_o^\ell \frac{PA}{2} \dot{u}^2 dx \tag{8}$$

After evaluating the integrals, the stiffness matrix produced by the new method is:

$$[K] = \frac{EA}{\ell}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \\ & \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & \cdots \\ & & \frac{\pi^2}{2} & 0 & 0 & \cdots \\ & & & \frac{4\pi^2}{2} & 0 & \cdots \\ & \text{sym} & & & \frac{9\pi^2}{2} & \cdots \\ & & & & & \ddots \; \frac{(k\pi)^2}{2} \end{bmatrix} \tag{9}$$

The mass matrix is:

$$[M] = PA\ell \begin{bmatrix} 1 & -\frac{1}{3} & -1 & -1 & \cdots & -1 \\ -\frac{1}{3} & \frac{2}{15} & -\left(\frac{1}{3}+\frac{1}{\pi^2}\right) & -\left(\frac{1}{3}+\frac{1}{4\pi^2}\right) & \cdots & -\left(\frac{1}{3}+\frac{1}{(k\pi)^2}\right) \\ & & \frac{3}{2} & 1 & \cdots & 1 \\ & & & \frac{3}{2} & 1 & 1 \\ & (\text{sym}) & & & \ddots & \vdots \\ & & & & & \frac{3}{2} \end{bmatrix} \tag{10}$$

Since the first row corresponds to the displacement at x = 0, the boundary constraint, $u_1 = 0$, requires that the first row and column be deleted for calculation of the cantilever modes.

The results of the modal solution are the frequencies ω_i and generalized displacements ξ_{oi}, ξ_{ki}, k = 1, 2 .... The actual mode shapes may be obtained from equation (4). However, a mode acceleration method (UIMPROVE) available in NASTRAN and also used by Rubin will enhance the vectors. Transforming the matrix equations into equivalent integrals results in the equation:

$$\phi_2(x) = u(0) + \int_o^x \left[\frac{\rho}{E}\int_x^\ell - \ddot{u}(x)dx\right] dx \qquad (11)$$

where $\ddot{u}(x)$ is obtained by multiplying the displacement $u(x)$ in Equation (4) by $-\omega^2$. This results in the mode shape:

$$\phi_2 = \frac{\omega_1{}^2\rho\ell^2}{E}\left\{\left(\frac{\bar{x}}{3} - \frac{\bar{x}^3}{6} + \frac{\bar{x}^4}{24}\right)\xi_o - \sum_k \left[\bar{x} - \frac{\bar{x}^2}{2} + \frac{1}{(\pi k)^2}(1 - \cos\pi k\bar{x})\right]\xi_k\right\} \qquad (12)$$

where $\bar{x} = x/\ell$ is used for simplicity.

The exact solutions for the cantilever rod problem modal frequencies are:

$$\omega_{ex} = \frac{(2n - 1)\pi}{2\ell}\sqrt{\frac{E}{\rho}} \qquad (13)$$

The exact mode shapes are:

$$\phi_{ex} = \text{sine}\,\frac{(2n - 1)\pi x}{2\ell} \qquad (14)$$

The calculated natural frequencies for the synthesized system produce an error ratio ε_ω, defined by the equation:

$$\varepsilon_\omega = \frac{\omega - \omega_{ex}}{\omega_{ex}} \qquad (15)$$

The resulting errors in natural frequency are tabulated in Table 1. These errors match Rubins results for his method exactly. Note that, except for the single degree of freedom problem, the error in the last mode for any matrix size is nearly constant and that the convergence rate for a given order matrix is nearly uniform.

An order of magnitude fit of the frequency errors is produced by the equation:

$$\varepsilon_{\omega n} \sim 0.01\,\frac{\omega_n{}^4}{\omega_k{}^2(\omega_k{}^2 - \omega_n{}^2)} \qquad (16)$$

where ω_k is the frequency of the lowest truncated mode shape. The equation is not accurate for the lower modes of the large order matrices due to computer round-off. Single precision arithmetic produced numerical errors of the order 10^{-7}.

The RMS errors for the calculated eigenvectors are shown in Table 2 for the same order synthesized matrices. Both first and second methods for calculating the vectors were used. The equations used for the vector errors are:

$$\varepsilon_\phi^1 = \frac{1}{\phi_{max}} \sqrt{\frac{1}{\ell} \int_o^\ell (\phi_1 - \phi_{ex})^2 dx} \qquad (17)$$

and

$$\varepsilon_\phi^2 = \frac{1}{\phi_{max}} \sqrt{\frac{1}{\ell} \int_o^\ell (\phi_2 - \phi_{ex})^2 dx} \qquad (18)$$

Both vectors were normalized to unit modal mass.

Note that the improved displacement calculations (ϕ_2) produce much better results when the first order errors are between 10^{-2} and 10^{-6}.

In other words, a good first approximation will produce a better improved solution. A poor first solution, such as the last mode in a set, will result in little improvement. A nearly exact first solution will not improve due to numerical round-off.

The results of this test are nearly identical to Rubin's [Ref. 3] results for his method. The frequency errors fall exactly on the published curves. The displacement errors for the UAI method appear to be better than Rubin's results. However it is suspected that differences in numerical procedures produced these changes. Also the first order displacement results compare with the referenced results for the modified Bamford method used in Reference 3.

TWO COMPONENT TRUSS PROBLEM

This problem has nearly become a standard for the evaluation of modal synthesis methods. It has been used in References 4, 5, and others for comparison between different formulations and procedures. A large quantity of data is available for validation of any new method. The basic problem, shown in Figure 2, consists of two truss substructures. Each substructure is reduced to its normal modes plus any additional shape functions used by a particular method. The trusses are combined at the three common grid points and the unconstrained modes of the combination are obtained.

This problem was solved directly on the UAI modal synthesis system implemented on L16 NASTRAN. Several different options and matrix sizes were tested. The parameters of the test cases are shown in Table 3. The matrix sizes were chosen to provide direct comparison with the results in Reference 4.

The results were compared with a single-structure NASTRAN execution to obtain the percentage errors of the frequencies. These errors are shown in Tables 4 and 5 along with results from Hintz [Ref. 4]. In all cases an excellent correlation was obtained between the NASTRAN results and the results of the equivalent formulations used by Hintz The only deviation occurred when the errors became too small to calculate when the NASTRAN printout truncated the difference in results. In each case the results are not shown where the eigenvector became unrecognizable and/or the natural frequencies changed in sequence.

It is important to note that the cases using free component modes, with no inertia relief effects, produced very poor results. This is due to the fact that the free modes approximate half waves while the cantilever modes approximate quarter waves. The shapes of the first modes of the combination apparently are difficult to approximate by a set of higher order shapes. The inertia relief shapes supply these smooth functions. Their contribution is most significant in the lower frequency modes.

Also indicated in the tables, by dashed lines, are the lowest truncated frequencies for the component modes used in the analysis. The results indicate that in the inertia relief cases, this frequency is a good indication of the limit for valid results. When only normal and constraining modes are used, this frequency has some significance, but does not indicate possible errors due to poor approximation of the actual mode shapes.

As a further check the problem was executed using 36 elastic degrees of freedom (case 9). This case also correlated with the Reference 4 results, having 29 modes with a frequency error of less than 5%. Nearly all of the first 15 modes were calculated to values exact to the last digit of the printout. However this case should not be considered as a typical example since only 60 degrees of freedom existed in the original structure. The typical application of modal synthesis would result in a matrix size with a much smaller fraction of the original matrix size.

CONCLUSIONS

As was postulated in Reference 1, the new modal synthesis method to be available in NASTRAN is capable of simulating the results and accuracy of any of the current state-of-the-art modal synthesis methods. The differences in the results occur from selecting different types of component modes and types of solution vector recovery processing. Furthermore, it was observed from these tests that the frequencies of the truncated component normal modes are a significant indicator of the upper limit of valid combination modes.

Although the new system provides accuracies equal to or better than any other advanced method, it also eliminates the restrictions that are imposed by the other formulations. The UAI method does not require unconstrained modes required by the Rubin and McNeal [Ref. 6] formulations. The method conveniently uses the actual boundary grid points as degrees of freedom (as in the Rubin and MacNeal methods) as opposed to the conventional Rayleigh-Ritz methods, in which an actual boundary displacement coordinate must be expressed as a combination of mode displacements. Furthermore, it allows any choice of mode shapes, including modes fixed at non-boundary points, partially free modes, and user supplied vectors.

The results for both test problems indicate that the inertia relief option is recommended for most cases. The number of calculations to obtain these shape functions is small relative to the modal calculations. A maximum of six extra degrees of freedom per component are added to the system. Results from the second test problem indicate that one should not replace modal coordinates with the inertia relief components since this will lower the effective frequency range.

The use of the "improved displacement" options in the solution vector recovery process appears to be less dramatic in its effectiveness. This option will be most effective when the first order vectors are reasonably valid and accurate stress data are required.

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TABLE 1. MODE FREQUENCY ERRORS, $\varepsilon\omega$, VERSUS MATRIX ORDER - FREE ROD PROBLEM

MODE NO.	MATRIX ORDER						
	1	2	3	4	6	8	12
1	6.6-3	9.5-5	8.5-6	1.21-6	6.1 -7	5.5 -6	3.0-6
2		3.1-2	1.1-3	1.88-4	1.78-5	-1.21-6	-2.1-5
3			4.4-2	2.4 -3	1.73-4	3.4 -5	-2.1-5
4				5.2 -2	8.9 -4	1.5 -4	-3.3-5
5					4.1 -3	5.1 -4	-2.9-5
6					5.9 -2	1.5 -3	-4.4-6
7						4.9 -3	7.4-5
8						6.2 -2	2.8-4
9							7.5-4
10							1.8-3
11							4.8-3
12							5.9-2

TABLE 2. EIGENVECTOR RMS ERRORS VERSUS
MATRIX ORDER - FREE ROD PROBLEM

MODE NO.	MATRIX ORDER						
	1	2	3	4	6	8	12
1	1.31-2 (2.69-2)*	3.18-5 (4.70-4)	3.06-6 (9.76-5)	6.16-7 (3.25-5)	1.81-7 (6.97-6)	1.35-7 (2.32-6)	2.23-7 (6.52-7)
2		9.81-3 (2.78-2)	1.03-3 (3.79-3)	1.66-5 (1.05-3)	1.87-5 (2.01-4)	9.71-6 (6.72-5)	5.00-6 (1.25-5)
3			2.15-2 (4.30-2)	3.36-3 (7.81-3)	2.16-4 (1.11-3)	5.44-5 (3.27-4)	5.50-5 (6.00-5)
4				3.19-2 (5.37-2)	1.65-3 (4.10-3)	2.77-4 (1.039-3)	1.11-4 (1.73-4)
5					9.19-3 (1.51-2)	1.12-3 (2.76-3)	2.04-4 (4.05-4)
6					4.79-2 (6.79-2)	4.06-3 (8.91-3)	5.46-4 (8.34-4)
7						1.56-2 (2.12-2	9.55-4 (1.62-3)
8	ε_{ω}^{2} (ε_{ω}^{1})					6.01-2 (1.34-2)	2.01-3 (3.06-2)
9							3.96-3 (5.96-3)
10							9.12-3 (1.21-2)
11							2.46-2 (2.93-2)
12							7.57-2 (8.20-2)

*( ) without UIMPROVE

TABLE 3. TEST CASE PARAMETERS
FOR THE TWO COMPONENT TRUSS PROBLEMS

CASE	NUMBER OF MODAL COORDINATES	NUMBER OF INERTIA RELIEF COORDINATES	TYPE OF COMPONENT MODE	TOTAL ELASTIC DOF
1	9	0	Free	12
2	3	6	Free	12
3	17	0	Free	20
4	11	6	Free	20
5	9	0	Cantilever	12
6	3	6	Cantilever	12
7	17	0	Cantilever	20
8	11	6	Cantilever	20
9	27	6	Cantilever	36

TABLE 4. PERCENT FREQUENCY ERRORS WITH 12 ELASTIC DEGREES OF FREEDOM TWO COMPONENT TRUSS PROBLEM

ELASTIC MODE NO.	NASTRAN CASE				REF. 4 RESULTS		
	1 Free Modes	2 Free w/I.R.	5 Cant. Modes	6 Cant. w/I.R.	Free w/I.R.	Hurty	Cant. w/I.R.
1	24.27	.006	.011	.00039	.006	.011	.00067
2	3.28	.021	.013	.194	.019	.013	.187
3	10.41	.737	.031	.737	.074*	.031	.743
4	4.51	.147	.150	2.93	.150	.155	2.94
5	2.47	1.82	.197	10.83	1.68	.190	10.3
6	4.50	6.45	.184	17.06	6.55	.184	16.9
7	1.00	16.02	6.49		16.8	7.39	
8	4.87		6.44				
9	0.75						
10							
11							

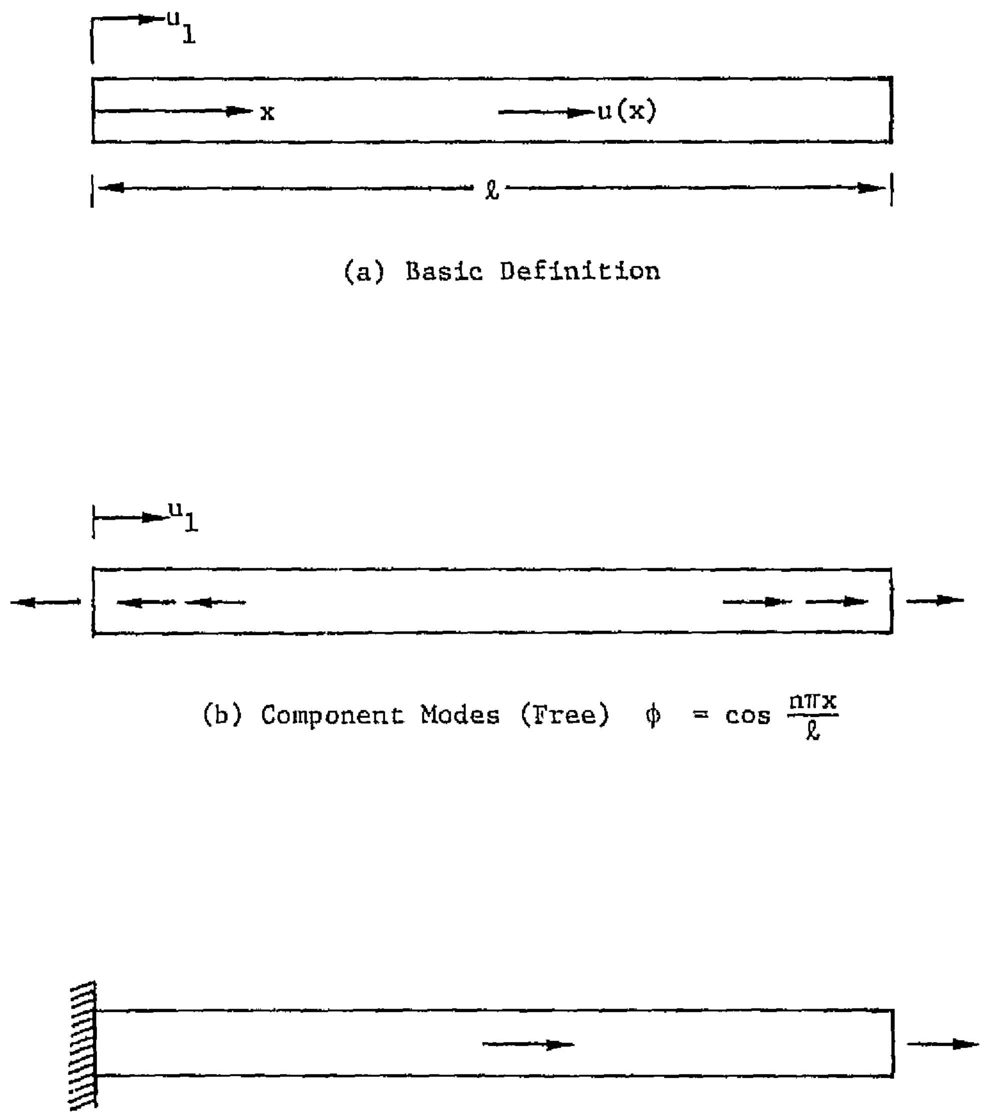
- - - indicates freq. of first truncated mode

*Suspected typo error

TABLE 5. PERCENT FREQUENCY ERRORS WITH 20 ELASTIC DEGREES OF FREEDOM
TWO COMPONENT TRUSS PROBLEM

ELASTIC MODE NO.	NASTRAN CASE				REF. 4 RESULTS		
	3 Free Modes	4 Free w/I.R.	7 Cant. Modes	8 Cant. w/I.R.	Free w/I.R.	Hurty (Cant. Modes)	Cant. w/I.R.
1	8.92	.00034	.00043	.00034	.000017	.00074	$9x10^{-9}$
2	1.21	.00902	.0017	0	.000061	.0018	$3x10^{-6}$
3	7.67	.0135	.0098	.0061	.0138	.0096	.00584
4	1.08	.00023	.0096	.00002	.00024	.0092	.00002
5	6.00	.00083	.033	0	.00081	.034	.0014
6	0.85	.0020	.0098	.00060	.0020	.0103	.00054
7	0.61	.080	.947	.268	.083	.941	.264
8	1.58	.0071	.122	.021	.0068	.117	.018
9	.084	.00098	.59	.54	.00093	.80	.69
10	.030	.0041	.36	.40	.0045	.20	.25
11	.90	.021	.33	.98	.022	.30	1.03
12	3.30	.428	.49	12.3	.134	.28	11.1
13	4.01	5.35	.16		5.33	.14	
14	.244	7.87	.77		7.15	.72	
15	1.10		2.37			2.63	
16	-.59		12.15			11.4	
17	6.69						

- - - indicates freq. of first truncated mode



(a) Basic Definition

(b) Component Modes (Free) $\phi = \cos \frac{n\pi x}{\ell}$

(c) Solution Modes $\phi = \sin \frac{(2n - 1)\pi x}{2\ell}$

FIGURE 1. EXACT ROD PROBLEM

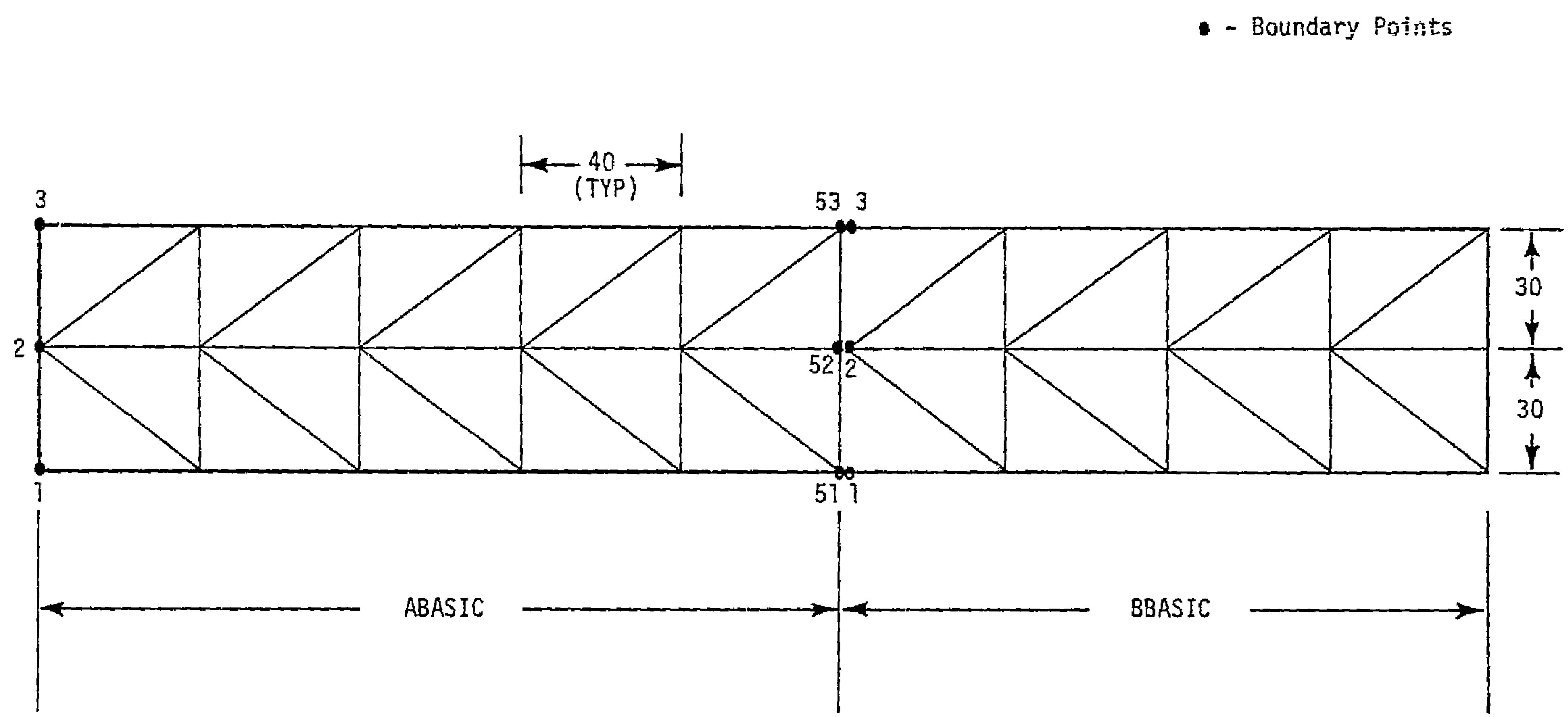


Figure 2. Nine-cell truss basic substructures.