THEORY ON ACOUSTIC SOURCES*

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SUMMARY

A theory is described for the radiation emission from acoustic multipole sources. The sources can be stationary or moving at speeds including supersonic and experience stationary or moving disturbances. The effect of finite source distributions and disturbances is investigated as well as the manner in which they interact. Distinction is made between source distributions that respond as a function of time and those that respond as a function of space.

It is found that motional amplification, for a point source, is given by $|1 - M\cos\sigma|^{-(N+2)}$, where N is the multipole order. Also, that motional attenuation from finite source distributions can be as high as $|1 - M\cos\sigma|^{3z}$, where z depends on the shape of the source distribution (typically between 1 and 2).

INTRODUCTION

In the last few years, there has been much interest in the noise from high speed sources. Particularly in the new generation of jet engines and in helicopter rotors in high speed flight. One knows from experience that moving sources produce a change of pitch as they pass, that whips crack, and that supersonic sources go bang. What we do not know in detail is how these sources radiate.

One of the first published works on isolated sources in motion was that by Oestreicher (ref. 1) in 1951. Using physical arguments involving the time it takes for an emitted sound to travel along a moving source, Oestreicher deduced that the effective source length in the direction of motion should change by (ε^{-1}).† By assuming, without justification, that the effective source density (strength per unit effective length of the source) was unchanged, he concluded that the total source strength and, therefore, the radiation would change by ε^{-1}. Although this model now appears inaccurate, it was useful at the time in that it predicted some kind of an acoustic beaming effect, when $M\cos\sigma = 1$.

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†$\varepsilon = |1 - M\cos\sigma|$, where M is the source Mach number and σ is the observation angle made with the direction of motion. ε is referred to as a zero and ε^{-1} as a pole, infinity, or singularity. ε^{-1} is the inverse of ε.

Soon after Oestreicher, Lighthill (ref. 2), using formal methods, again interpreted that the effective source dimension should change by ε^{-1}. He also found that each space derivative for the higher order sources produced a further ε^{-1}. However, Lighthill, like Oestreicher, assumed that the effective source strength density (not the actual source strength) was unchanged by motion and that the radiation magnitude change was affected by the effective source volume change. Lighthill, therefore, applied the ε^{-1} singularity in blanket fashion to all types of sources. This made the total singularity power for a multipole of order N, where N is the number of space derivatives equal to N + 1, i.e., $(\varepsilon^{-1})^{N+1}$.

Unlike Lighthill's approach, which uses a moving-stationary coordinate transform (Jacobian) to obtain the radiation from moving sources, the following theory simulates source motion by using a stationary source region of appropriately phased elements.

The main difference between this and Lighthill's theory is that the actual source strength in this theory is assumed to be unchanged by motion and that motional amplification is affected entirely by temporal transform changes between moving and stationary sources; this is contrary to the assumption that motional amplification is caused by an apparent source volume change and, therefore, total source strength change as in references 1 and 2. There is, of course, an effective source volume change through motion; but, according to the following theory, this produces motional attenuation only, the converse to what Oestreicher and Lighthill had originally thought.

In this present theory, it is first found necessary to distinguish clearly between multipole sources (sources constructed from an array of discrete mass displacement sources), aerodynamic sources (sources with source strengths in terms of physical quantities such as fluid mass flow and forces applied to the propagating fluid), and finite sources (sources whose acoustic wavelength is small compared to the source distribution size). When this is done, motional amplification in terms of the multipole source is found to be $(\varepsilon^{-1})^{N+2}$ where N is the number of dipoles forming the source, i.e., the result is one degree higher than Lighthill's series $(\varepsilon^{-1})^{N+1}$. For an aerodynamic source, the theory gives $(\varepsilon^{-1})^{p}$, where p = 2, 1, 1, 2, for a mass displacement (fundamental source), mass injection (simple source), force, and stress source, respectively.

It can be seen that both theories are in agreement as far as the simple source is concerned. (Lighthill makes no comment regarding the fundamental source.) However the higher order aerodynamic sources appear to have a singularity less than those predicted by Lighthill. Note that a multipole source and an aerodynamic source in this theory can have the same multipole order but a different singularity power.

Finally, motional attenuation for finite sources is found to be $(\varepsilon)^{z_a + z_b + z_c}$, where the z's are the shape orders of the source distributions in the three coordinate directions. Typically, the z values are between 1 and 2 once the acoustic wavelength is less than the distribution size. Thus motional amplification is only possible if the singularity power $p > z_a + z_b + z_c$.

THEORY DESCRIPTION

The theory is described in detail in reference 3; only a brief description of the main points of the theory is given below.

The radiation equations are first written in terms of multipole analysis, except that the mass displacement source is used as the fundamental acoustic source. For stationary sources, it is not important whether the fundamental source or the time derivative of the fundamental source (simple source) is used. However, for motional effects, it is important to make the distinction. To reduce the complexity of the radiation equations, far field approximations are applied. This enables the directivity effects between acoustic poles and source distributions to be effectively separated.

Source motion is then taken into account by using the concept of a source region. Here, a stationary region is defined with respect to the stationary propagating fluid. Source motion is simulated within the region by allowing the phase of the source region density function to vary with space in such a way as to produce waves of finite phase velocity to pass across the region. Using this model, individual source distributions and disturbances can be accommodated with relative velocities between each other and the stationary coordinate system. At all times the analysis remains in the far field with respect to the stationary coordinate system thus avoiding the use of stationary to moving coordinate transforms such as Jacobian and Lorentz.

To evaluate the radiation from the source region, the source distribution and disturbance functions are Fourier analyzed into spatial harmonics (modes). The resulting double Fourier summation is then simplified into sum and difference mode pairs. The radiation from a single mode is then evaluated, and the radiation from all the modes of the source region summed. In this way, the radiation from any complex source region structure can be evaluated.

It is found that each mode has a characteristic radiation pattern. The directivity has a dominant lobe corresponding to when the mode phase speed, resolved in the direction of the observer, equals the speed of sound, plus finer radiation details at least 14 dB lower than the dominant lobe. Beautiful and complex radiation patterns can result, therefore, depending on the modal content and thus the source region structure.

Fortunately, for most source situations of interest, the finer radiation details cancel and only the dominant mode radiation need be considered. This reduces the complexity of the problem considerably. In fact, in the dominant mode solution, the source region can be considered simply as a "black box" frequency changer. Source frequencies f_s go in, and radiation frequencies f_r come out, with the simple relationship between the two of ε^{-1}.

This simple frequency changing action, however, has a more profound effect. It results in the source radiation emission, for the fundamental source, being increased in the direction of motion by ε^{-2}. The effective multipole leverage (distance between poles) is increased by ε^{-1} and therefore the acoustic interference between poles is reduced giving a further amplification factor of ε^{-N}. Thus the total multipole motional emission is changed by $(\varepsilon^{-N})^{N+2}$.

To convert a multipole source (constant fundamental source strength) into an aerodynamic source (constant aerodynamic source strength), it is found that the transform $(\varepsilon^{-1})^n$ is valid, where n = 0, 1, 2, 2 for the mass displacement, mass flow, applied force, and stress source, respectively. Therefore the motional amplification for an aerodynamic source becomes $(\varepsilon^{-1})^p$ where $p = N + 2 - n$, thus p = 2, 1, 1, 2 for sources of the type m, q, f_i, and t_{ij}, respectively.

For finite source distributions, the effective source size in the direction of motion is also increased by ε^{-1}; however, this results in an increased acoustic interference across the source distributions and, therefore, an attenuation effect of ε. For radiation wave lengths smaller than the source distribution size, the attenuation can be as great as ε^{3z}, where 3 is for the three source dimensions, if all spatially modulated, and z depends on the source distribution shape (z is typically between 1 and 2).

The degree of motional amplification, therefore, depends on the multipole or aerodynamic order (number of mathematical poles) and the shape and nature of the source distribution (number of zeros z and whether the distributions are time or spatially modulated). Thus it appears that source motion alters the multipole and distribution size, but not the order.

Consider now the disturbance function which generates the source frequencies. Its radiated scale is decreased in the direction of motion by ε, the converse to the distribution size. For a given radiated frequency, the disturbance gives an acoustic beam, the longer the disturbance the sharper the beam. The radiation can be greater for a disturbance of longer duration although the rise and fall time is less. Also, sources experiencing finite steady disturbances in motion radiate finite radiation, including an infinite steady disturbance duration.

Distinction is also made between source distributions that respond as a whole to the disturbance as a function of time, similar to a loudspeaker (time modulated), and distributions whose elements respond individually as a function of space, similar to aerodynamic sources (spatially modulated). At subsonic source speeds, spatially modulated sources are effectively less compact than time modulated sources by a factor of $M \cos\sigma$. That is, the onset of distributed interference across the source distributions can take place at a much lower frequency than time modulated sources for the same source dimensions. The effect is also experienced normal to the direction of motion in the case of a spatially modulated source.

Further, the fascinating situation is considered where the source is stationary and the disturbance is in motion, such as in an open wind tunnel (ignoring flow effects). Although there is no Doppler effect (stationary source), an acoustic beam is again generated at high flow speeds. Compared with a moving source, the logic here appears to be in reverse. For example, the effective source distribution size now decreases in the direction of motion by ε, and the distribution interference now effectively generates the acoustic beam, not attenuates it.

In general, it is found that there is nothing extraordinary regarding accelerating or nonrectilinear source motion. The radiation level at any instant is proportional to the instantaneous velocity and is not dependent on the acceleration of previous or subsequent flight paths.

For example, the notion that steady sources rotating in a circle radiate through centripetal acceleration is, according to this theory, fallacious. The same sources travelling in a straight line will radiate similarly.

It should be pointed out that the theory derived in this paper is based on the observed fact that acoustic propagation is simple, i.e., its propagation is at all times relative to the propagating fluid and not with respect to some coordinate system, moving or otherwise, as in the propagation of electromagnetic waves (relativity theory). Thus, there are no relativistic time or space changes; all space changes in this theory are related to the radiated scale (wavelength).

ILLUSTRATIONS

Figure 1 shows the relationship between an observer and the source region. X_i is the observation point and Y_i is a source point within the source distribution. For far field radiation conditions, i.e., observation distances large compared with the source distribution size, $X_i >> Y_i$, $|X_i| \simeq R$ and R becomes the representative distance of all the source elements in the distribution.

Figure 2 illustrates how the radiation at the observer, for far field radiation conditions, can be represented by the product of two interference effects. K_w represents the interference between monopoles, $\tilde{H}$ represents the acoustic interference across the source distribution and $K_w\tilde{H}$ represents the total interference effect. This is a particularly useful concept as we can treat the multipole and distributive acoustic properties separately.

Figure 3 gives some examples of the multipole directivity function K_w. $\frac{X_i}{R}$ are the direction cosines made with the observer and the multipole directional vectors. This multipole interference effect between poles is well known, and nothing further need be said except to emphasize that these properties are independent of source motion. We now need to concentrate on the $\tilde{H}$ function, which is not so simple, particularly for sources in motion.

Figure 4 illustrates how the source and disturbance motion is accommodated within a stationary source region. Motion is accomplished by allowing the phase of the acoustic density function h_r to vary across the source region. In this manner, h_r can be made to contain information regarding the number, shape, and speed of the source distributions and disturbances. $\tilde{H}$ is then the summation of h_r across the source region (total radiation activity).

Figure 5 shows how the h_r function is represented by two functions, a disturbance function h_w and a source distribution function h_a. The h_w function is the instantaneous value of the summation of h_r across the source distribution at each source position x. The h_a function describes how this summation value is distributed across each source.

In Figure 6, the h_w and h_a functions are Fourier analyzed into spatial harmonics. The resulting double Fourier summation generates a modulated mode system shown in the center figure. This system is then simplified further into simple mode pairs as indicated on the right. Thus any complicated source region structure can be represented by the summation of simple modes, the radiation from which depends on the mode amplitude and radiation efficiency of each mode.

Figure 7 illustrates how simple spatial functions can be converted into spectrum functions, χ, where χ is a nondimensional Fourier coefficient whose maximum value is unity. Each of the disturbance and distribution shape functions 15 converted into a disturbance and distribution spectrum function χ_s and χ_{mB}, respectively. The mode amplitude then depends on the product spectrum function $\chi_s\chi_{mB}$. Thus, a knowledge of the amplitude, duration, and shape of the spatial functions will give an idea of the mode spectrum amplitude.

Figure 8 concerns the other controlling factor which determines the radiation, i.e., the mode radiation efficiency. Here the radiation is summed for each mode travelling across the source region of dimension d. The radiation depends on the radiation frequency η/d, the number of mode wavelengths ξ across d, and the mode speed η/ξ.

Figure 9 shows the typical radiation directivity for each mode. The mode interference function χ_ξ has a maximum value of unity (dominant lobe); this corresponds to when the mode speed, resolved in the direction of the observer, equals the speed of sound plus finer radiation details at least 14 dB lower. Thus the total radiation from a source region is composed of many such mode radiation directivities, one for each mode generated.

Figure 10 illustrates three specifying characteristics of a radiating source. Figure 10(a) shows the directivity for a single source frequency, f_s, which gives rise to whole family of modes, each mode giving a directivity similar to that in figure 9. Here it can be seen that the directivity envelope containing the dominant lobe is controlled by the distribution spectrum function χ_{mB} only. Figure 10(b) shows the directivity for a given radiation frequency f_r from a complete disturbance. Here the directivity envelope is given by both the disturbance and distribution spectrum functions χ_s, χ_{mB}. Figure 10(c) shows the radiated spectrum at a given observation angle σ. In this case each source frequency generates a passband of discrete frequencies with a dominant center frequency given by χ_ξ. The spectrum envelope (dominant frequencies) is given by $\chi_s\chi_{mB}$. Thus it can be seen that the disturbance and distribution spectrum functions, for a given source speed, control the major acoustic properties of a radiating source.

Figure 11 summarizes the radiation properties for three different source situations. The first column depicts the properties of a moving inphase source experiencing a stationary disturbance (time modulated). Column 2 is for a moving source whose elements respond individually to the disturbance as a function of space (spatially modulated). The last column shows the properties of a stationary modulated source experiencing a moving disturbance. The disturbance can be a simple sinusoidal disturbance, in which case the radiation directivity is given in row a . Or the disturbance can be some arbitrary periodic disturbance. In this case, the directivity for a single radiation frequency is given in row b , and the resulting spectrum at a given observation angle is shown in row c . These directivity and spectrum envelopes indicate the dominant lobe and radiation frequencies only; the finer radiation details given in figures 9 and 10 are not shown.

Concentrating for a moment on the top left hand square, block 1a represents the directivity for a point source experiencing a single source frequency. This directivity is given by $(\varepsilon^{-1})^p$ where $\varepsilon = 1 - M\cos\sigma$ and p is the aerodynamic order. Thus at the Mach angle $\cos\sigma = 1/M, \varepsilon = 0, \varepsilon^{-1} = \infty$, the directivity has a series of poles or infinities of order p giving infinite radiation. Block 2a represents the acoustic interference effect across finite source distributions given by χ_{mB}. This function contains zeros, ε, of order z_a, the power of which depends on the shape of the source distributions. This function, of course, gives zero radiation at the Mach angle. Thus, motional amplification or attenuation can occur depending on the balance of the poles p (aerodynamic order) and on the number of zeros z_a (source distribution shape).

Continuing along the top row of Figure 11, the main difference between a time-modulated source, 1a, and a spatially modulated source, 2a, is that the distribution spectrum function, in 2a, is also operative at right angles to the source motion. Whereas in 1a, at 90^0 to the source motion, χ_{mB} is unity, i.e., there is no distributive source interference effect. In the case of the stationary spatially modulated source experiencing a moving disturbance, 3a, there are no poles in the point source term as the source is stationary. Here the directivity is given by the distributive interference effect χ_{mB} only. At the Mach angle, χ_{mB} has unities of order z_a giving the acoustic beam. Thus the sharpness of the beam is given solely by the shape of the source distributions.

Moving along the second row of Figure 11, the radiation directivity at a given radiation frequency from a periodic disturbance is considered. For a time-modulated source, 1b, the directivity is given by both χ_s and χ_{mB}, where disturbance spectrum function χ_s is responsible for generating the acoustic beam. This function has unities of order z_w where z_w depends on the shape of the disturbance function. Thus the disturbance shape determines the shape (sharpness) of the acoustic beam. The main effect of the disturbance spectrum function χ_{mB} is to attenuate the acoustic beam in the direction of source motion. In the case of the moving spatially modulated source, 2b, the χ_{mB} function is omnidirectional, thus giving no attenuation of the acoustic beaming effect given by χ_s. In the last situation, 3b, χ_s is now nondirectional and the acoustic beaming is given by χ_{mB}. Note that situations 3a and 3b are the same, as the source is stationary making both the source and radiation frequencies identical.

Finally the radiation spectrum characteristics arising from a periodic disturbance are given along the bottom row of Figure 11. The spectrum is the product of the three functions χ_s, χ_{mB} and $(f_r)^p$. The disturbance and distribution spectrum function depress the acoustic spectrum after the "start" of the decay of these functions, given by $(f_r)_w$ and $(f_r)_a$, have been exceeded. The decay rate then depends on the order z_w and z_a of the disturbance and distribution functions, respectively. Thus the acoustic spectrum rises at low frequencies according to the frequency multiplier $(f_r)^p$, where p depends on the aerodynamic order, and finally decays according to $(f_r)^{p-z_w-z_a}$. It can be seen that the spectra can be quite different for each source situation depending on the individual break frequencies $(f_r)_w$ and $(f_r)_a$. Here v is the disturbance speed with respect to the source, u is the source speed and a_o is the speed of sound. Note that in situations 1c and 2c, the radiated disturbance scale w_r in comparison to the actual disturbance scale w, in the direction of motion, is reduced by ε_u and the radiated source distribution size a_r in comparison to the actual size a, is increased in the direction of motion by ε_u^{-1}. In the last situation, 3c, the radiated and actual disturbance scales are the same, and the radiated distribution size is now decreased in the direction of motion by ε_v.

THEORETICAL BASIS

To help indicate the differences between this and other theories, the essential principles on which the theory is based are summarized below. The detailed analysis of the following statements can be found in reference 3.

(a) Fundamental source.

The mass displacement source m is considered as the fundamental or basic acoustic source. The simple source q, usually taken to be the simplest acoustic source, is, in fact, the time derivative of the fundamental source.

$$m = \rho_o v_o \qquad\qquad q = \frac{\partial m}{\partial t} \tag{1}$$

m and q are the mass displacement and mass flow source strengths, respectively; ρ_o is the density of the displaced propagating fluid and v_o is the displaced volume. For a stationary harmonic source of the form

$$m_s = \hat{m} \cos \omega_s t \qquad\qquad \omega_s = 2\pi f_s \tag{2}$$

where $\hat{m}$ is the amplitude and ω_s is the source frequency. The acoustic strength h_s is then

$$h_s = \left(\frac{\partial}{\partial t}\right)^2 m_s = \omega_s^2 m_s \tag{3}$$

(b) Moving fundamental source.

The only basic effect of source motion is to change the radiation frequency, $f_s \rightarrow f_r$. The source strength $\hat{m}$ remains unchanged by motion

$$m_r = \hat{m} \cos \omega_r t \qquad\qquad \omega_r = \varepsilon^{-1} \omega_s \tag{4}$$

The acoustic source strength then becomes

$$h_r = \left(\frac{\partial}{\partial t}\right)^2 m_r = \omega_r^2 m_r = \left(\varepsilon^{-1}\right)^2 \omega_s^2 m_r \tag{5}$$

i.e., the radiation amplitude between a stationary and moving fundamental source, all other things being equal, is

$$\hat{h}_r = \left(\varepsilon^{-1}\right)^2 \hat{h}_s \tag{6}$$

(c) Multipole source.

A multipole source is defined as an array of 2^N discrete equispaced fundamental sources where N is the multipole order (number of dipoles). The acoustic strength is given by

$$h_s = d_i^N \left(\frac{\partial}{\partial t}\right)^{N+2} m_s \tag{7}$$

where d_i is separation distance between monopole sources. The acoustic strength of a moving multipole source is then

$$h_r = d_i^N \left(\frac{\partial}{\partial t}\right)^{N+2} m_r = d_i^N \omega_r^{N+2} m_r = \left(\varepsilon^{-1}\right)^{N+2} d_i^N \omega_s^{N+2} m_r \tag{8}$$

i.e., $$\hat{h}_r = \left(\varepsilon^{-1}\right)^{N+2} \hat{h}_s \quad (\hat{m} \text{ constant}) \tag{9}$$

(d) Aerodynamic source.

If g represents the source strength of an aerodynamic source, such as mass flow q, applied force f_i, and stress t_{ij}, then the acoustic relation between these sources is

$$g_r = d_i^N \left(\frac{\partial}{\partial t}\right)^n m_r \tag{10}$$

N	0	0	1	2
g	m	q	f_i	t_{ij}
n	0	1	2	2

and the acoustic strength in terms of g becomes

$$h_r = d_i^N \left(\frac{\partial}{\partial t}\right)^{N+2} m_r = \left(\frac{\partial}{\partial t}\right)^p g_r \quad , \qquad p = N + 2 - n \tag{11}$$

$$= \omega_r^p g_r = \left(\varepsilon^{-1}\right)^p \omega_s^p g_r \tag{12}$$

i.e., $$\hat{h}_r = \left(\varepsilon^{-1}\right)^p \hat{h}_s \quad (\hat{g} \text{ constant}) \tag{13}$$

(e) Sound pressure.

The sound pressure for an aerodynamic source, is then given by

$$SP = \frac{1}{4\pi R} K_N [h_r] \quad , \qquad K_N = \left(\frac{X_i}{R}\ \frac{1}{a_o}\right)^N \tag{14}$$

$$= \frac{1}{4\pi R} K_N \left(\frac{\partial}{\partial t}\right)^p [g_r] \quad , \qquad [g_r] = \hat{g} \cos \omega_r [t] \tag{15}$$

$$[t] = t - \frac{R}{a_o} \ , \quad \omega_r = \varepsilon^{-1}\ \omega_s \tag{16}$$

$$= \frac{1}{4\pi R} K_N \left(\varepsilon^{-1}\right)^p \ \omega_s^p \ [g_r] \tag{17}$$

The $\frac{1}{4\pi R}$ term gives the spherical spreading effect; K_N gives the interference effect between poles; and, $\frac{X_i}{R}$ are the direction cosines made with the source directional vectors and the observer.

Or in a more familiar form:

(1) Wave equation.

$$WE = \frac{\partial^2 m}{\partial t^2} + \frac{\partial q}{\partial t} - \frac{\partial f_i}{\partial x_i} + \frac{\partial^2 t_{ij}}{\partial x_i\ \partial x_j} \tag{18}$$

$$SP = \frac{1}{4\pi R} \left\{ \frac{\partial^2 [m]}{\partial t^2} + \frac{\partial [q]}{\partial t} - \frac{\partial [f_i]}{\partial x_i} + \frac{\partial^2 [t_{ij}]}{\partial x_i\ \partial x_j} \right\} \tag{19}$$

Far field solution (20)

$$\frac{\partial^N [g]}{\left(\partial x_i\right)^N} = \left(-\ \frac{X_i}{R}\ \frac{1}{a_o}\right)^N \left(\frac{\partial}{\partial t}\right)^N [g]$$

$$SP = \frac{1}{4\pi R} \left\{ \frac{\partial^2 [m]}{\partial t^2} + \frac{\partial [q]}{\partial t} + \left(\frac{X_i}{R}\ \frac{1}{a_o}\right) \frac{\partial [f_i]}{\partial t} + \left(\frac{X_i}{R}\ \frac{1}{a_o}\right)^2 \frac{\partial^2 [t_{ij}]}{\partial t^2} \right\} \tag{21}$$

$$= \frac{1}{4\pi R} K_N \left(\frac{\partial}{\partial t}\right)^p [g] \tag{22}$$

Equations (19) and (22) are valid only for a stationary source.

(2) Moving harmonic source.

By using a stationary source region of elements with varying phase, the effect of source motion can be simulated. In this case, the solution (22) above is valid provided

$$g \begin{cases} \text{sta.} & g_s = \hat{g} \cos \omega_s t \\ \text{mov.} & g_r = \hat{g} \cos \omega_r t \qquad \omega_r = \left(\varepsilon^{-1}\right)\omega_s \end{cases} \qquad \begin{matrix} (23) \\ (24) \end{matrix}$$

where sta. and mov. are for stationary and moving sources. This gives

$$SP = \frac{1}{4\pi R} K_N \left(\frac{\partial}{\partial t}\right)^p [g_r] \tag{25}$$

$$= \frac{1}{4\pi R} K_N \left(\varepsilon^{-1}\right)^p \omega_s^p [g_r] \tag{26}$$

(3) Stationary source equivalence for a point source.

$$|h_r| = \left|\left(\frac{\partial}{\partial t}\right)^p g_r\right| = \left|\left(\varepsilon^{-1}\right)^p \left(\frac{\partial}{\partial t}\right)^p g_s\right| \tag{27}$$

$$\hat{SP}_{mov.} = \left(\varepsilon^{-1}\right)^p \hat{SP}_{sta.} \tag{28}$$

$$\hat{SP}_{mov.} = \frac{1}{4\pi R} K_N \left\{\left(\varepsilon^{-1}\right)^2 \frac{\partial^2 [m]}{\partial t^2} + \left(\varepsilon^{-1}\right) \frac{\partial [q]}{\partial t} + \left(\varepsilon^{-1}\right) \frac{\partial [f_i]}{\partial t} + \left(\varepsilon^{-1}\right)^2 \frac{\partial^2 [t_{ij}]}{\partial t^2} \right\}_{sta.} \tag{29}$$

The main difference between equation (29) and other general results is that it makes a clear distinction between the mass displacement source (m) and the mass flow source (q) indicated by the first two terms and that the last two source terms, f_i and t_{ij} in equation (29), have a singularity (ε^{-1}) less than other general results.

PRINCIPLE CONCEPTS

In summary, the main concepts used in the finite source theory are listed below.

(1) Radiation equation.

$$SP = \frac{1}{4\pi R} \cdot K_N \cdot \tilde{H}_r \tag{30}$$

$$K_N = \left(\frac{X_i}{R} \frac{1}{a_o}\right)^N \qquad \tilde{H}_r = \int [h_r] d_x \tag{31}$$

i.e., the radiation equation can be represented by two separate interference effects: (i) the interference between poles, represented by K_N, which is independent of motion; (ii) the interference effect, represented by H_r, across finite source distributions.

(2) Relation between h_r, m_r, and g_r.

$$h_r = d_i^N \left(\frac{\partial}{\partial t}\right)^{N+2} m_r \qquad (32)$$

$$g_r = d_i^N \cdot \left(\frac{\partial}{\partial t}\right)^{n} m_r \qquad (33)$$

$$h_r = \left(\frac{\partial}{\partial t}\right)^{p} g_r \qquad (34)$$

g	N	n	p
m	0	0	2
q	0	1	1
f_i	1	2	1
t_{ij}	2	2	2

$$p + n = N + 2 \qquad (35)$$

h_r = acoustic strength, m_r = fundamental source strength, g_r = aerodynamic source strength.

(3) Disturbance and distribution function.

$$h_r = h_w \cdot h_a \qquad (36)$$

For a source region of finite source distributions and periodic disturbances, the acoustic activity (h_r) can be represented by the product of a disturbance function (h_w) and a distribution function (h_a).

(4) Modal analysis.

$$h_r = h_w h_a = \sum_s h_s \sum_{mB} h_{mB} = \sum_s \sum_{mB} h_\xi \quad , \quad h_\xi = h_s h_{mB} \qquad (37)$$

The disturbance and distribution functions are Fourier-analyzed into a double summation of simple modes (spatial harmonics).

(5) Radiation integral.

$$\tilde{H}_r = \int_d [h_r]\, dx = \sum_s \sum_{mB} \chi_s \chi_{mB} \chi_\xi [H_r] \qquad (38)$$

$$[H_r] = H \cos\left(2\pi f_r [t] + \theta_r + \phi\right), \; H = \left(\frac{\partial}{\partial t}\right)^p G \qquad (39)$$

$$G = \frac{2BE}{d} \int_w |g_w|\, dx \qquad g_w = \int_a |g_r|\, dx \qquad (40)$$

The radiation integral (total acoustic activity) $\tilde{H}_r$ is the summation of the acoustic radiation from all the modes of the source region. χ_s and χ_{mB} are the disturbance and distribution spectrum functions which give the mode amplitude. χ_ξ is the mode acoustic interference function which gives the mode radiation efficiency.

(6) Dominant mode solution.

$$\chi_\xi = 1 \qquad \theta_r = 0 \qquad mB = \mp s\left(\frac{\Delta - M_r \cos\sigma}{\delta - M_r \cos\sigma}\right) \tag{41}$$

$$\tilde{H}_r = \chi_s\, \chi_{mB}\, [H_r] \qquad f_r = \varepsilon_u^{-1} f_s \tag{42}$$

In the dominant mode solution, the mode interference function χ_ξ becomes unity, and the radiation phase angle θ_r becomes zero. Thus, the complexity of the radiation integral $\tilde{H}_r$ is reduced considerably.

(7) Sound pressure.

$$SP = \frac{1}{4\pi R} K_N\, \chi_s\, \chi_{mB}\, [H_r] \tag{43}$$

$$[H_r] = \left(\frac{\partial}{\partial t}\right)^p G \cos\left(2\pi f_r[t] + \phi\right) \tag{44}$$

$$= \frac{1}{4\pi R} K_N \left(2\pi f_r\right)^p \chi_s\, \chi_{mB}\, [G_r] \qquad \text{single } f_r \tag{45}$$

$$= \frac{1}{4\pi R} K_N \left(\varepsilon_u^{-1}\right)^p \left(2\pi f_s\right)^p \chi_s\, \chi_{mB}\, [G_r] \qquad \text{single } f_s \tag{46}$$

The sound pressure (dominant mode solution) for a given radiation frequency is given by equation (45) and for a given source frequency by equation (46). The total radiation fo: a complete disturbance is given by $\sum_s (f_r)$ or $\sum_s (f_s)$.

(8) Motional amplification.

$$\hat{SP}_{mov.} = \left(\varepsilon_u^{-1}\right)^p \hat{SP}_{sta.} \quad \text{(point source)} \tag{47}$$

$$\hat{SP}_{mov.} = \left(\varepsilon_u^{-1}\right)^p \left(\varepsilon_u\right)^z \hat{SP}_{sta.} \qquad \text{(finite source)} \tag{48}$$

$$\left(f_s\right)_{a,b,c} < f_s < \left(f_s\right)_{d_i} \tag{49}$$

$$p > z \qquad z = z_a + z_b + z_c \tag{50}$$

A finite source is considered to be one whose source frequencies are greater than the source distribution cut off frequencies $(f_s)_{a,b,c}$ and less than the multipole leverage cut off frequencies $(f_s)_{d_i}$. Motional amplification, for a finite time-modulated source, occurs only if the aerodynamic order p is greater than the sum of the source distribution order z in the three coordinate directions.

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2. Lighthill, M. J. On sound generated aerodynamically. I. General Theory. A 211, Proc. Roy. Soc., 1952, pp. 564-587.

3. Wright, S. E. Theorie sur les source acoustiques. H/DE-ER 351-91. Bureau D'etudes Aerospatiale Marignane, 1978.

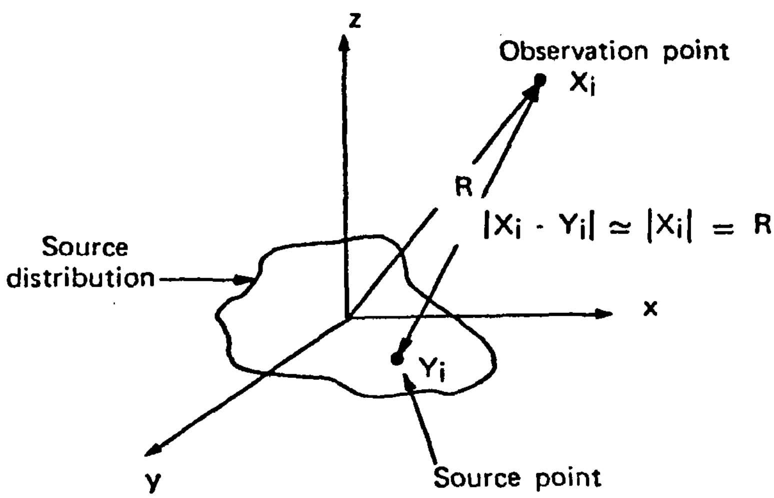


Figure 1.- Source-observer definition.

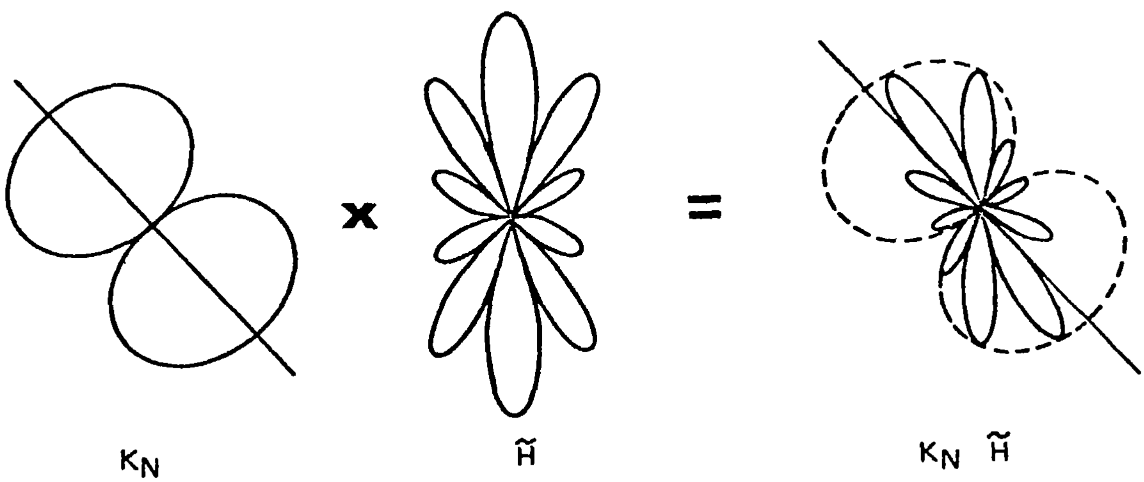
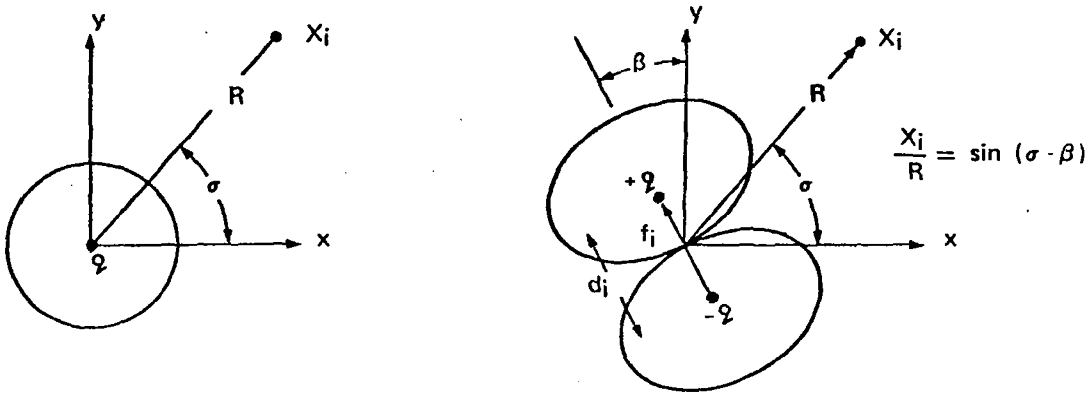
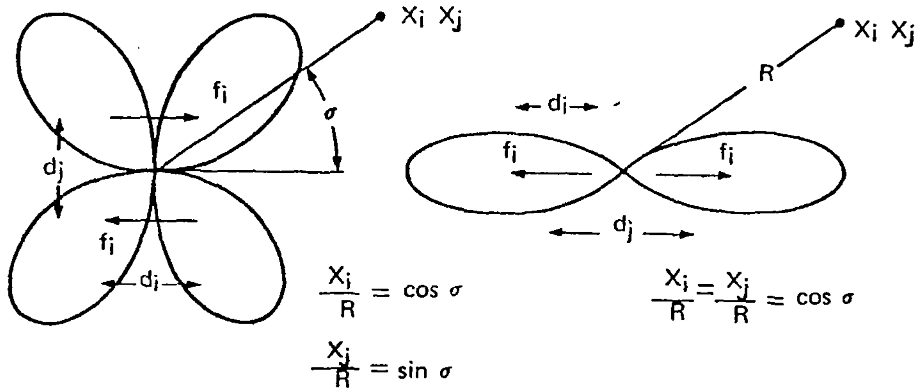


Figure 2.- Separation of source terms into two interference effects.



(a) K_0 simple source.

(b) K_1 dipole.



(c) K_2 lateral and longitudinal quadrupole.

Figure 3.- Equivalence of aerodynamic sources in terms of simple sources.

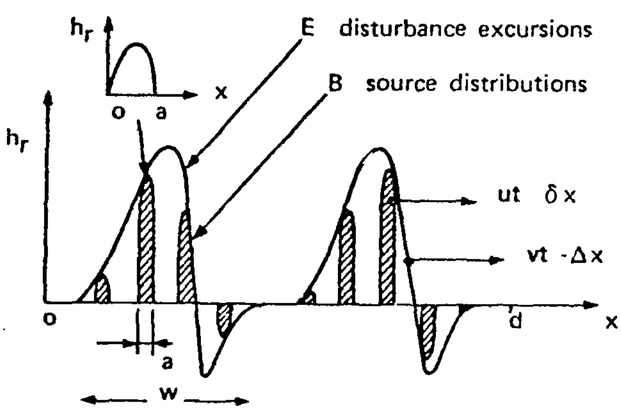


Figure 4.- Source region description.

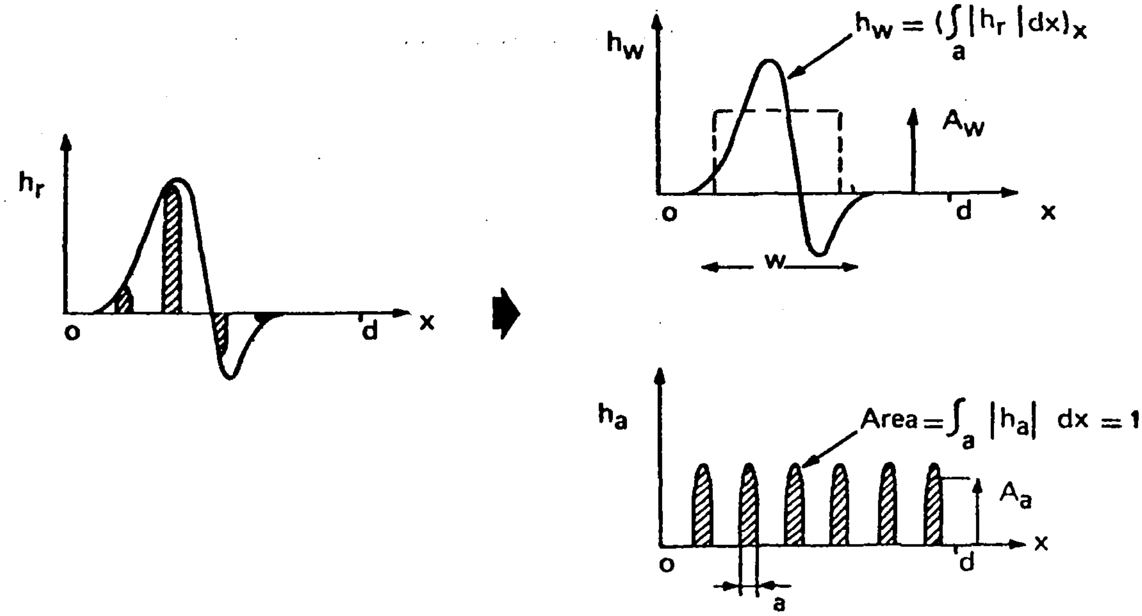


Figure 5.- Representation of density function by a disturbance and distribution function.

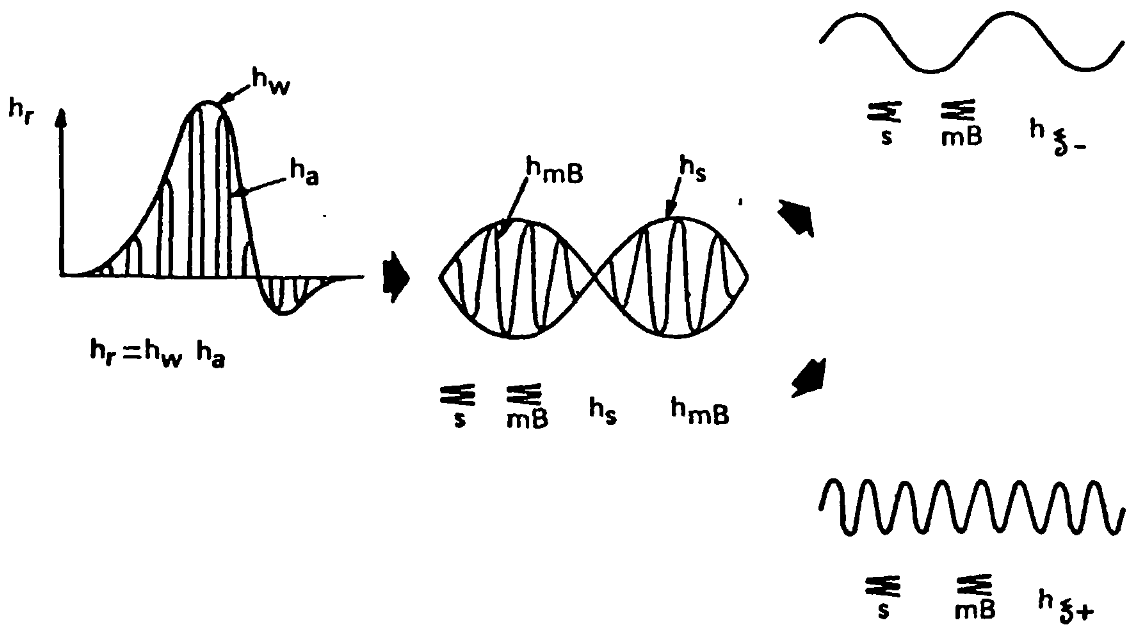
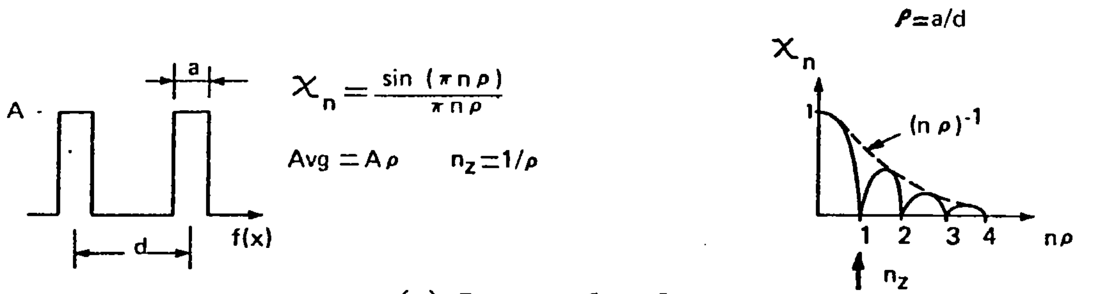
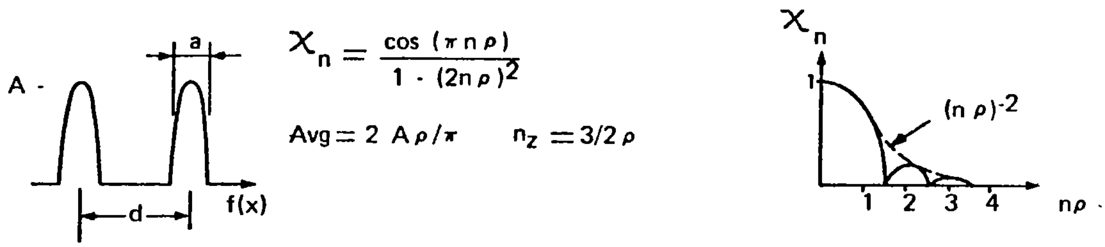


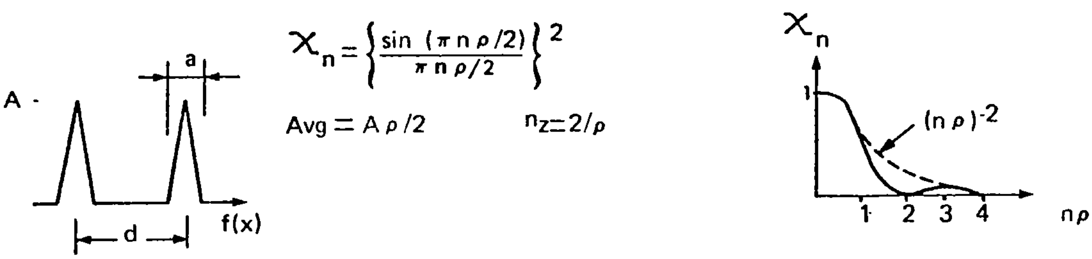
Figure 6.- Decomposition of density function into simple mode pairs.



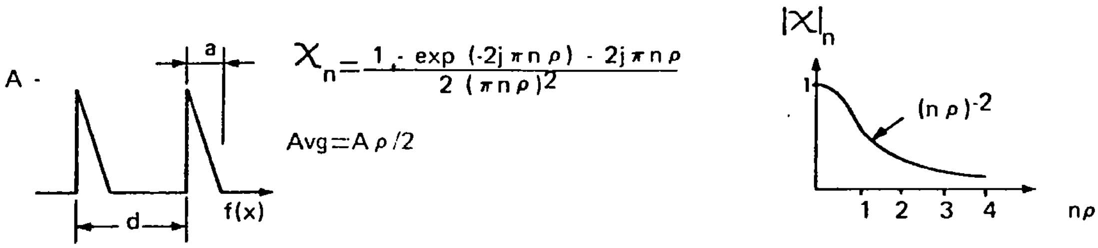
(a) Rectangular f_n.



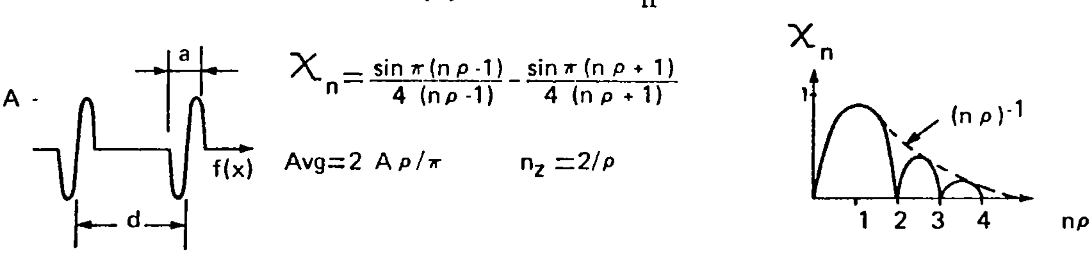
(b) Half cosine f_n.



(c) Triangular f_n.



(d) Sawtooth f_n.



(e) Full sine f_n.

Figure 7.- Spectrum function χ_n for some simple shape functions.

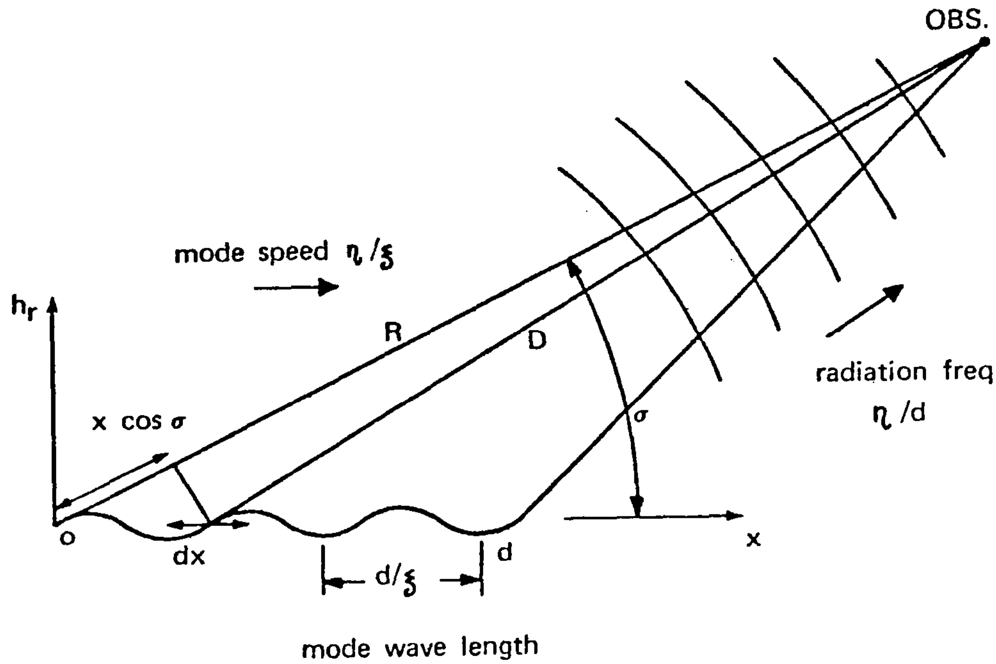


Figure 8.- Mode radiation details.

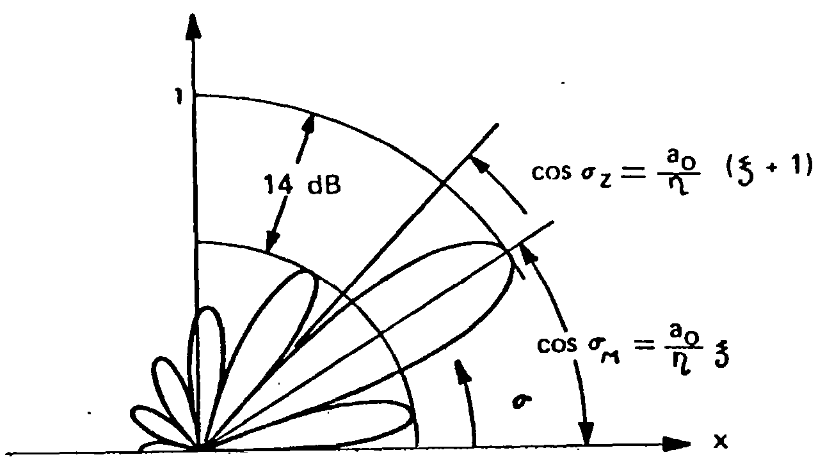
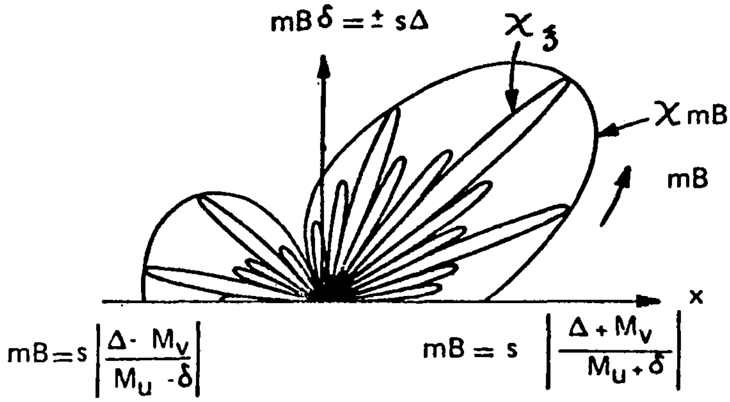
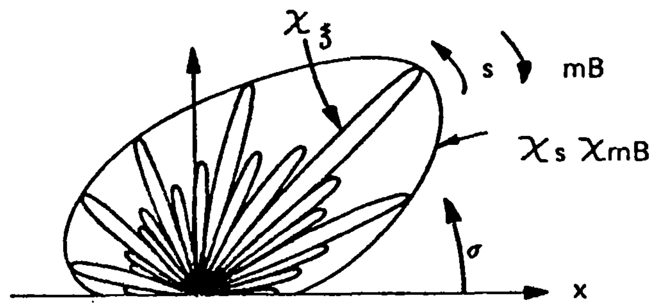


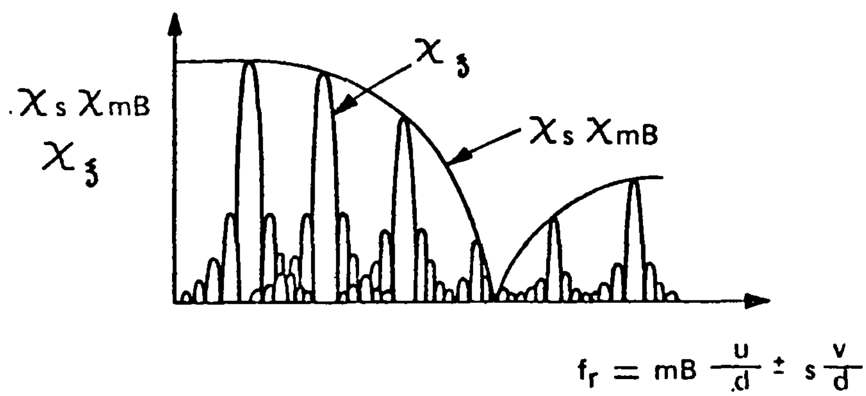
Figure 9.- Directivity properties of the mode interference function χ_ξ.



(a) Directivity for given f_s.



(b) Directivity for given f_r.



(c) Spectrum for given σ.

Figure 10.- Three specifying characteristics of a radiating source.

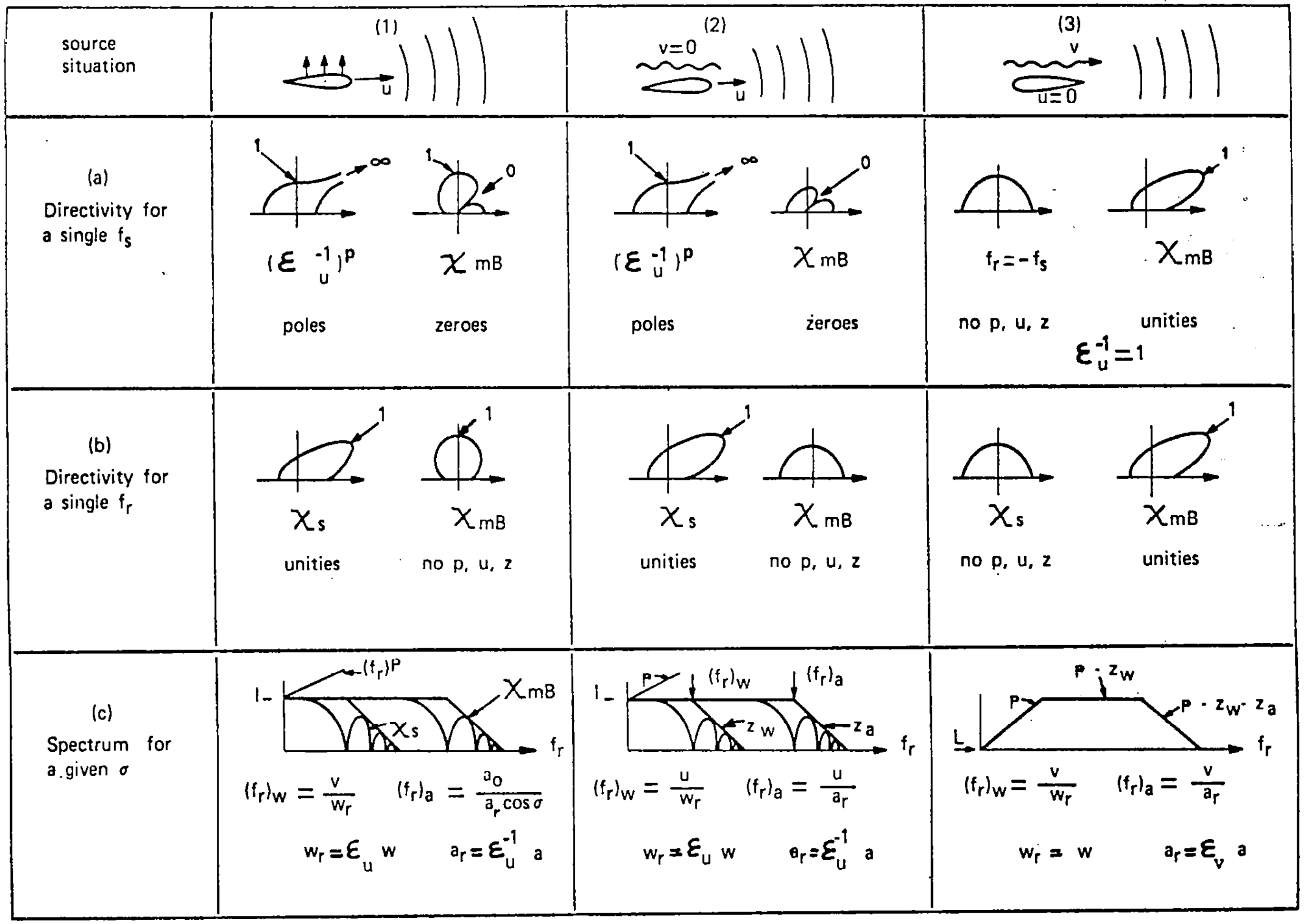


Figure 11.- Summary of time and spatially modulated source radiation characteristics.