NASA

Technical Memorandum 79647

The Dynamical Halo in the Variation of Cosmic - Ray Path Length with Energy

Frank C. Jones

September 1978

National Aeronautics and
Space Administration

Goddard Space Flight Center
Greenbelt, Maryland 20771

(NASA-TM-79647) THE DYNAMICAL HALO AND THE VARIATION OF COSMIC-RAY PATH LENGTH WITH ENERGY (NASA) 22 p HC A02/MF A01 CSCL 03B

N78-34027

Unclas

G3/93 34199

The Dynamical Halo and the Variation of Cosmic-Ray Path Length With Energy

Frank C. Jones
Laboratory for High Energy Astrophysics
NASA/Goddard Space Flight Center
Greenbelt, Maryland 20771

Abstract

It is shown that the dynamical halo model offers a natural explanation for the form of the variation of the comsic-ray path length with energy. The variation above ~ 1 GeV/nucleon can be understood as due to the variation of the diffusion coefficient, and hence the resident time, with energy. The flattening of the curve below 1 GeV/nucleon is seen to mark a transition to a convection dominated regime where the diffusion coefficient is no longer the determining parameter. A fit to the observations yields a halo outflow velocity of 8 km sec^{-1}. An attempt is made to determine the overall scale of the halo and the diffusion coefficient using recent ^{10}Be flux measurements but the data do not agree well enough to pin down these variables to within less than four or five orders of magnitude.

1. Introduction

It has been long known that cosmic rays travel through 5-6 gm cm^{-2} of interstellar matter before escaping from the galaxy. In the last few years, however, it has become evident that the amount of matter traversed, or grammage, is not constant but is energy dependent. Above 1 GeV per nucleon the mean leakage path appears to decrease with energy as $E^{-\alpha}$ where $\alpha \approx 0.3-0.5$ (Juliusson et al. 1972; Smith et al 1973). Below 1 GeV per nucleon the evidence is somewhat unclear but if one focuses ones attention on the abundance ratios of those nuclei that are believed to be purely secondary to the parent nuclei their is evidence that the mean leakage path is a constant $\sim$ 5-6 gm cm^{-2}. This evidence is not conclusive, however, and various authors have differing opinions regarding this matter (Ormes and Freier 1978; Lezniak and Webber 1978; Fontes et al. 1977). For the present we shall adopt the view that the independance (or weak dependance) of the cosmic-ray grammage below a few GeV is real. The following disucssion must therefore be regarded as tentative and subject to further investigation of the observational situation..

The behavior of the mean leakage path above 1 GeV per nucleon is usually discussed within the framework of the "leaky box" model with an escape probability that increases with energy. Since most theories of charged particle propagation is disordered magnetic fields predict a diffusion coefficient that increases with particle energy (strictly speaking, rigidity) an energy dependent escape probability, or equivalently, residence time is to be expected.

For example, in the case of simple, one dimensional diffusion in a region of linear size D the mean residence time for particles is $\tau \approx D^2/\kappa$ where κ is the diffusion coefficient.

The main difficulty with such a model is its inability to explain the relative independence of the mean path length on energy for particles below $\sim$ 1 GeV/nucleon. There is no reason to expect the diffusion coefficient to exhibit this behavior therefore some additional ingredient is required. In this note I shall demonstrate that the concept of a dynamical, outward flowing galactic halo can supply this missing ingredient.

The notion that a tenous halo about our galaxy could have a profund effect on the observed properties of cosmic rays is certainly not a new one. (See, for example, the references in Stecker and Jones 1977.) Until recently, however, the halo has been regarded as a static diffusion volume that acted only as a passive storage region where cosmic rays could spend a great portion of their lifetimes without encountering very much matter. It is now appreciated that if the material of the halo is moving it can carry the cosmic rays with it and this connective motion can significantly affect observed parameters of the cosmic rays (Jokipii 1976, Ownes and Jokipii 1977a, b). As we shall now see one of the parameters that can be affected is the mean path length traversed by the cosmic rays.

In the ensuing discussion we shall consider a highly simplified model of the galaxy. The disk of the galaxy that contains the cosmic ray sources and all of the matter that contributes to thier grammage will be represented by an infinitesimally thin disk of infinite extent. The halo will extend in the X direction, perpendicular to the disk, to an outer free escape boundary at $X = \pm D$. The outflow in the halo will be represented by a cosmic-ray convection velocity, V in the positive X direction for positive X and in negative X direction for negative X.

This model is schematically illustrated in figure 1. The model is highly simplified in that it ignores any variation of cosmic-ray production, or galactic parameters over the disk of the galaxy. There is evidence, however, that cosmic rays do not travel about the disk for distances much greater than 1 kpc during their lifetime (Stecker and Jones 1977, Ormes and Freier 1978) therefore one would expect that a local, one-dimensional model would be a reasonable representation of reality. The disregard of the disk components finite thickness as well as possible variations of the convective velocity with X are more serious omissions. It is expected, nevertheless, that results obtained with this model will not differ too seriously from those obtained from more detailed models.

In examining this particular model Jokipii (1976) showed that the mean age of the cosmic rays that are observed in the disk depends significantly on the size of the dimensionless parameter $\chi = VD/\kappa$. This parameter being essentially the ratio of the convective velocity to the diffusion velocity, is a measure of the importance of the outward convection. Jokipii showed that the mean age or residence time of the comsic-ray particles observed in the disk approached the limiting forms:

$$\tau \approx D^2/3\kappa, \ \chi \ll 1 \tag{1}$$

$$\tau \approx \kappa/V^2, \ \chi \gg 1. \tag{2}$$

Equation (1) is what one would expect for a simple diffusion picture when convection is unimportant. Equation (2) shows, however, that when convection dominates the mean age becomes independent of the size of the containment volume.

The present author (Jones 1978) gave a simple explanation of this result in terms of the "effective size" of the containment volume, D^*. The concept of the effective size is readily understood if one considers a particle that finds itself a distance x out in the expanding halo. It will take this particle a time of order $\tau = x^2/\kappa$ to diffuse back to the disk. During this time convection will carry it a distance outward equal to $V\tau = Vx^2/\kappa$. If this is greater than the original distance it had to diffuse to return it is very unlikely to make it back. In other words if

$$\frac{Vx^2}{\kappa} > x \tag{3}$$

or

$$x > \frac{\kappa}{V} \approx D^* \tag{4}$$

the particle is effectively lost from the containment volume insofar as observations in the disk are concerned.

Furthermore, using the concept of retrodictive probability (Jones 1978) it is possible to show that those particles that are observed in the disk have spent their entire past history confined to a volume characterized by D^* rather than D. Of course, in the case $D^* > D$ ($X < 1$) the actual boundary of the volume prevails and D^* has no particular significance. It is straightforward to verify that substitution of D^* for D in equation (1) yields equation (2) (to within a constant factor) showing that D^* is indeed the physically relevant confinement scale whenever it is smaller than D.

Turning our attention to the mean path length traversed by the observed cosmic rays it is evident that

$$\bar{\ell} = v\bar{\rho}\tau \tag{5}$$

where ℓ is the mean path length, $\overline{\rho}$ the mean matter density seen by the cosmic rays, τ the mean age of the cosmic rays and v is the speed of the particles. If the matter in the disk is of density ρ and confined to a disk of half thickness a the cosmic rays, confined to a volume of half thickness D′ will see a mean matter density

$$\overline{\rho} = \rho_o a/D' . \qquad (6)$$

The mean lifetime is given by equation (1) using D′ instead of D where D′ will be either D or D^* whichever is appropriate. Combining equations (5), (6) and (1) we obtain

$$\overline{\ell} = \frac{1}{3} v\rho_o a\ D'/\kappa . \qquad (7)$$

We see now that if κ is large enough such that $X \ll 1$ and $D^* > D$, D′ should be replaced by D and the mean path length varies inversely as κ. If κ is small enough for convection to dominate, however, $X \gg 1$ and D′ should be replaced by $D^* = \kappa/V$ and the mean path length is given by

$$\overline{\ell} = \frac{1}{3} v\rho_o a/V \qquad (8)$$

and is independent of κ. If the particles are still relativistic at these energies $\overline{\ell}$ is independent of energy since $v \approx c$ but for non relativistic particles an energy dependence will enter through the particle velocity v.

2. The Diffusion-Convection Equation

The foregoing arguments are not precise and furthermore they neglect the effect of the expanding halo on the particles energy. Since we are interested in the energy dependence of the results it is important not to neglect energy in the derivation. A more correct procedure is to solve the equation for the cosmic-ray particle density $N(t,x,\ell,T)$

$$\frac{\partial N}{\partial t} - \frac{1}{3}\frac{\partial V}{\partial x}\frac{\partial}{\partial T}(TN) + v\rho\frac{\partial N}{\partial \ell} + V\frac{\partial N}{\partial x} - \kappa\frac{\partial^2 N}{\partial x^2} = S(x,T,\ell) \tag{9}$$

where ℓ is the path length, T is the particle kinetic energy and S is the source term that describes the particle injection. Equation (9) differs from the one employed by Jokipii (1976) only in the addition of the term describing the change in path length.

What we desire is a steady state solution of equation (9). A completely general solution would be quite difficult to obtain because the coefficients of the path length and diffusion terms are energy dependent hence the equation is not separable. However, if we assume that the primary energy dependence of the solution is due to a strongly varying injection term, eg. an inverse power law, we may ignore the energy dependence of these coefficients to obtain a reasonable approximate solution.

If we adopt the following

$$\frac{\partial V}{\partial x} = 2V\delta(x) \tag{10}$$

$$\rho(x) = 2\rho_o a\delta(x) \tag{11}$$

$$S(x,T,\ell) = S_o\delta(x)\delta(\ell)T^{-\gamma} \tag{12}$$

we obtain the solution to equation (9) with $\partial N/\partial t = 0$

$$N = \frac{S_o}{2v\rho_o a}\exp(-\ell/\bar{\ell})\frac{(1-\exp[V(x-D)/\kappa])}{(1-\exp[-VD/\kappa])}T^{-\gamma} \tag{13}$$

where the mean path length is

$$\bar{\ell} = \frac{v\rho_o a[1-\exp(-VD/\kappa)]}{V\{1+\frac{(\gamma-1)}{3}[1-\exp(-VD/\kappa)]\}}. \tag{14}$$

If we adopt for the diffusion coefficient κ the form

$$\kappa = \kappa_o \beta R^{1/2}, \tag{15}$$

where $\beta \equiv v/c$ and R is the particle rigidity, we obtain an expression for $\overline{\ell}$ as a function of rigidity;

$$\overline{\ell} = \frac{L_o \beta[1-\exp(-\frac{R_o^{1/2}}{\beta R^{1/2}})]}{1 + (\frac{\gamma-1}{3})[1-\exp(-\frac{R_o^{1/2}}{\beta R^{1/2}})]} . \tag{16}$$

In figure 2 we show a compilation of data after Ormes and Freier (1978) and a fit of expressions (16) where we have chosen the parameters

$$L_o \equiv \rho_o ac/V = 20\text{gm cm}^{-2}$$
$$R_o^{1/2} \equiv VD/\kappa_o = \sqrt{2}\ (\text{GV})^{1/2}.$$

We have also shown the effect of including a retarding potential $\Phi \sim 220$ MeV/nuc at 1 A.U. to approximate the effects of solar modulation. The value of $\Phi = 220$ MeV/nucleon was found by Garcia-Munoz et al. (1977a) to best order their data on cosmic-ray composition.

Choosing $2\rho_o a$ to be 5.16 $M_{\odot}\text{pc}^{-2}$ in the local galactic region, $\overline{\varpi} = 10$ kpc, (Gordon and Burton, 1976) we obtain V = 8 km sec^{-1} for the halo outflow velocity. This velocity, while not large, has observable effects. For if the present interpretation is correct it is responsible for the turn over at low rigidities of the curve of cosmic-ray path length versus rigidity.

3. The Mean Cosmic-Ray Age

We may also determine that the ratio

$$\frac{D}{\kappa_o} = \frac{\sqrt{2}}{8} \times 10^{-5} (\text{GV})^{1/2} \text{sec-cm}^{-1}$$

but to determine D or κ_o separately we require an additional piece of experimental data. The mean age of the cosmic rays would be such an independent quantity and several experimenters (Garcia-Munoz et al 1977b, Webber et al 1977, Hagen et al 1977, Buffington et al 1978) have reported values for this quantity based on measurements of the flux of radioactive ^{10}Be in the cosmic-ray beam. One must not think, however, that the cosmic-ray ages reported by these authors may be applied directly to our present propagation model. As was pointed out by Prishchep and Ptuskin (1975) and Ginzburg and Ptuskin (1978) these ages were derived from the observations within the homogeneous or leaky box model of the galaxy and the same observations can lead to quite different ages when interpreted by a diffusion or diffusion-convection model.

To obtain meaningful quantities to apply to our model we must return to a more primitive concept with respect to the ^{10}Be measurements. The concept of surviving fraction f is not completely model independent since it requires a calculation of the flux of ^{10}Be that would be observed if it were stable against radioactive decay. As we show in the Appendix the path length distribution function is what is needed to compare one model to another. Furthermore, if two different models predict the same path length distribution for stable isotopes they will predict the same ratios of these stable isotopes. Since the leaky box model and the present model both predict an exponential path length distribution we may take the surviving fraction f to be a sufficiently primitive notion that it may be used without modification in either model.

Although the various experimenters use the same galactic propagation model they do not use exactly the same calculation procedures, spallation cross sections, or solar modulation models to compute the non-decay flux of ^{10}Be and hence the surviving fraction. Therefore, solely for the sake of bringing the various experimental results into a common framework, we shall compare the reported values of the $^{10}Be/^{9}Be$ flux ratios to these calculated by Raisbeck and Yiou (1977) for the case of no decay of the ^{10}Be. These authors calculate a value of 0.6 for the no decay $^{10}Be/^{9}Be$ flux ratio at 1 A.U. from the sun, virtually independent of energy.

The surviving fractions are then simply given by

$$f = (^{10}Be/^{9}Be)\ \text{obs.}/0.6.$$

For propagation models that yield an exponential path length distribution, as does our model, we have

$$f = \frac{1/\bar{\ell}_{ND} + \sigma}{L/\ell_D + \sigma} \tag{17}$$

where $\bar{\ell}_{ND}$ is the mean path length neglecting decay of the particles, $\bar{\ell}_D$ is the mean path length including the effects of decay, and σ is the collision probability per unit path length.

Equation (9) may be easily modified to include the effects of radioactive decay to obtain

$$\bar{\ell}_D = \frac{2\beta c \rho_0 a}{V}\left[\frac{1}{F} + \frac{2(\gamma-1)}{3}\right]^{-1} \tag{18}$$

where

$$F = \frac{\tanh\left(\frac{\chi\delta}{2}\right)}{\delta + \tanh\left(\frac{\chi\delta}{2}\right)} \tag{19}$$

$$\chi = \frac{VD}{\kappa},\quad \delta = \left(1 + \frac{4\kappa}{V^2\gamma\tau_o}\right)^{1/2}$$

and $\gamma\tau_o$ is the relativisticly dilated decay time of the ^{10}Be, τ_o = 2.16 Myr.

Since we have previously chosen values for χ, d.ca/V, and V one can insert equation (18) in equation (17) and solve the resulting expression for δ and hence κ. In table 1 we show the various model parameters that can be deduced from the reported $^{10}Be/^{9}Be$ ratios. It is immediately obvious that the comsic-ray lifetimes are considerably longer than those deduced for the leaky box model, in some instances as much as two orders of magnitude longer. It is equally obvious that the present experimental situation does not define these parameters very sharply, the allowed values, including errors, of some quantities range over four or more decades. It appears that we must await a considerable sharpening of the experimental situation before we can have believable numbers for these important parameters. At present the result of Stecker and Jones (1977) limiting the value of $D^{*} \leq 3$ kpc imposes as firm a constraint as any derived from observations of the flux of ^{10}Be.

4. Conclusion

We have seen that the variation of cosmic-ray path length with energy may be understood within the context of a simple diffusion-convection model with a dynamical galactic halo. A fit to the observations of the various model parameters gives rather directly an outflow velocity of ~ 8 km sec^{-1}. This velocity, while quite small, is, nevertheless, responsible for the turn over at low energy of the cosmic-ray path length versus energy curve. This fit also gives a value of $D/\kappa = 1.9\times10^{-6}$ sec-cm^{-1} for a rigidity R = 2GV where we assume $\kappa = \kappa_o \beta R^{1/2}$.

TABLE 1

Galactic parameters derived from Cosmic-ray ^{10}Be Measurements

Authors†	K.E. (MeV/Nucleon)	f* Surviving fraction	D* (kpc)	T (Myr)	κ (10^{27} cm^2-sec^{-1})
A	80	$.14 \pm .07$	$125^{+44}_{-7.5}$	$560^{+1.9\times10^3}_{-334}$	$31.2^{+110}_{-18.6}$
B_1	145-200	$.33^{+.18}_{-.20}$	$2.2^{+16.8}_{-1.6}$	$79.9^{+610}_{-56.3}$	$5.5^{+41.9}_{-3.9}$
B_2	200-245	$.13^{+.10}_{-.11}$	$19.6^{+1.1\times10^3}_{-14.2}$	$640^{+3.5\times10^4}_{-464}$	$48.9^{+2.7\times10^3}_{-35.4}$
C	250	$.62 \pm .27$	$0.40^{+1.7}_{-.31}$	$12.4^{+53.6}_{-9.6}$	$.984^{+4.3}_{-.763}$
D	100-1500	$.37 \pm .18$	$2.8^{+11.3}_{-1.9}$	61^{+246}_{-41}	$6.98^{+28.2}_{-4.64}$

† A-Garcia-Munoz et al. (1977b), B. Webber et al. (1977), C. Hagen et al. (1977), D. Buffington et al. (1978)

* The ^{10}Be-^{9}Be ratio reported by these authors may be obtained by multiplying the surviving fraction f by 0.6.

FIGURE CAPTIONS

Figure 1. Schematics of model of galactic cosmic ray propagation.

Figure 2. Measurements of path length vs. rigidity from various authors compiled by Ormes and Freier (1978) with curve of expression (16). See text for explanation of parameters. Data are from: open cirlces, Ormes et al. (1977); solid circles, Caldwell (1977); Crosses, Garcia-Munoz et al. (1977a); open triangle, Smith et al. (1973); Square, Júliusson (1974); closed triangle, Webber, Lezniak, and Kish (1973); hexagon, Shapiro et al. (1973).

REFERENCES

Buffington, A., Orth, C. D., and Mast, T. S. 1978, LBL-7551 preprint.

Caldwell, J. H. 1977, Ap. J., 218, 269.

Fontes, P., Meyer, J. P., and Perron C. 1977, Proc. 15th Int. Conf. Cosmic Rays, Plovdiv, 2, 234.

Garcia-Munoz, M., Mason, G. M., and Simpson, J. A. 1977a, Proc. 15th Int. Conf. Cosmic Rays, Plovdiv, 1, 301.

——1977b, Ap. J., 217, 859.

Ginzburg, V. L., and Ptuskin, V. S. 1976, Rev. Mod. Phys., 48, 161.

Gordon, M. A., and Burton, W. B. 1976, Ap. J., 208, 346.

Gahen, F. A., Fisher, A. J., and Ormes, J. F. 1977, Ap. J. 212, 262.

Jokipii, J. R. 1976, Ap. J., 208, 900.

Jones, F. C. 1978, Ap. J., 222, 1097.

Júliusson, E. 1974, Ap. J., 191, 331.

Júliusson, E., Meyer, P. and Müller, D. 1972, Phys. Rev. Letters, 29, 445.

Lezniak, J. A., and Webber, W. R. 1978, Ap. J., 223, 676.

Ormes, J. F., and Freier, P. 1978, Ap. J., 222, 471.

Ormes, J. F., Hagen, F. A., and Maehl, R. C. 1977, private communication.

Owens, A. J., and Jokipii, J. R. 1977a, Ap. J., 215, 677.

——1977b, Ap. J., 215, 685.

Prishchep, V. L., and Ptuskin, V. S. 1975, Ap. and Sp. Science, 32, 265.

Raisbeck, G. M., and Yiou, F. 1977, Proc. 15th Int. Conf. Cosmic Rays, Plovdiv, 2, 203.

Shapiro, M. M., Silverberg, R., and Tsao, C. H. 1973, Proc. 13th Int. Conf. Cosmic Rays, Denver, 1, 578.

Smith, L. H., Buffington, A., Smoot, G. F., Alverez, L. W., and Wahlig, M. A. 1973, Ap. J., 180, 987.

Stecker, F. W., and Jones, F. C. 1977, Ap. J., 217, 843.

Webber, W. R., Lezniak, J. A., and Kish, J. C. 1973. 13th Int. Conf. Cosmic Rays, Denver, 1, 248.

Webber, W. R., Lezniak, J. A., Kish, J. C., and Simpson, G. A. 1977, Ap. Letters, 18, 125.

Appendix

Consider a point in the matter disk of the galaxy. The density of particles of the i^{th} species that have traversed between ℓ and $\ell + d\ell$ gm/cm^2 of matter, neglecting collisions, is given by the path length distribution function $P_i(\ell)d\ell$. If the particles are injected at $\ell = 0$ with a rate-density S_i we may write down the value of $P_i(0)$ provided the sources are all in the matter disk.

Clearly

$$P_i(0) = S_i \, d\tau \tag{A1}$$

where $d\tau$ is the time required for a particle to traverse $d\ell$ of matter,

$$d\tau = (d\ell/dt)^{-1} \, d\ell = (v\rho_o)^{-1} d\ell$$

thus $P_i(0) = S_i(v\rho_o)^{-1}$. If, in addition, the path length distribution function is an exponential function as in the present model or in the leaky box model then we must have

$$P_i(\ell) = \frac{S_i}{v\rho_o} \exp(-\ell/\bar{\ell}_i) \tag{A2}$$

where all of the dependence on the diffusion coefficient, distance to the boundary, decay time for radioactive species etc. must be carried by the mean path length $\bar{\ell}_i$.

If we now consider the effects of collisions we observe that if σ_i is the collision probability per unit path length the probability that a particle which has traveled a length ℓ has not undergone a collision is given by $\exp(-\sigma_i \ell)$. Since a collision will transmute a particle of species i into one or more particles of a different species the actual path length distribution for species i particles is given by

$$P_i(\ell) \text{ (actual)} = \exp(-\sigma_i \ell) \, P_i(\ell) \tag{A3}$$

and the total density of such particles in the disk is just

$$N_i = \int_0^\infty \exp(-\sigma_i \ell)\, P_i(\ell) d\ell$$

$$= \frac{S_i}{v\rho_o} (1/\bar{\ell}_i + \sigma_i)^{-1} \qquad \text{(A4)}$$

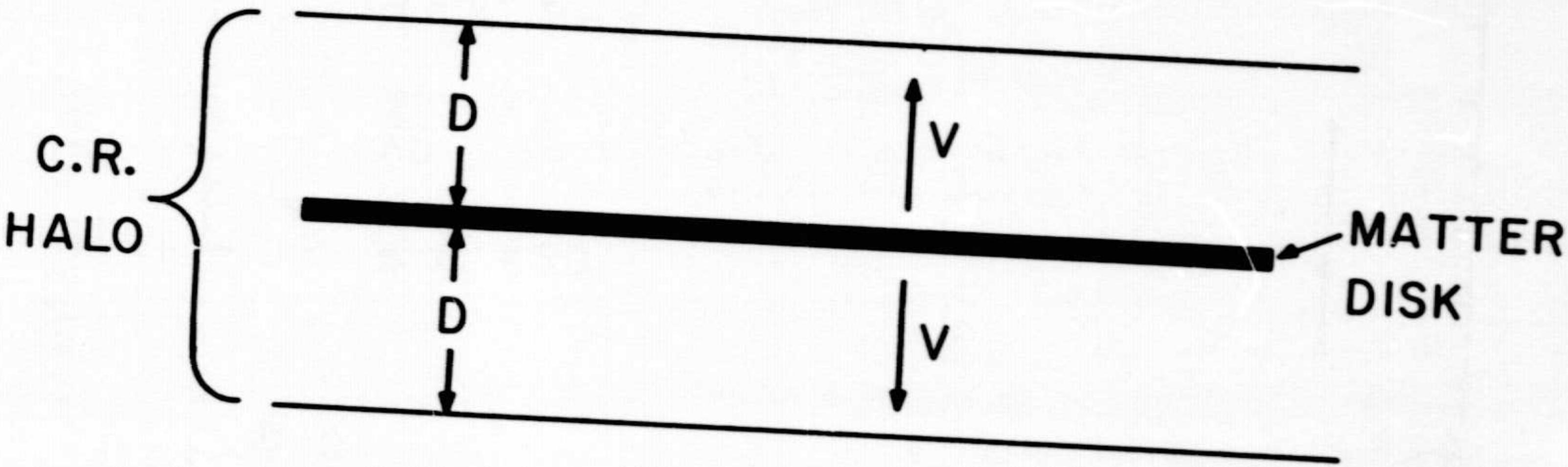
if P_i is an exponential function.

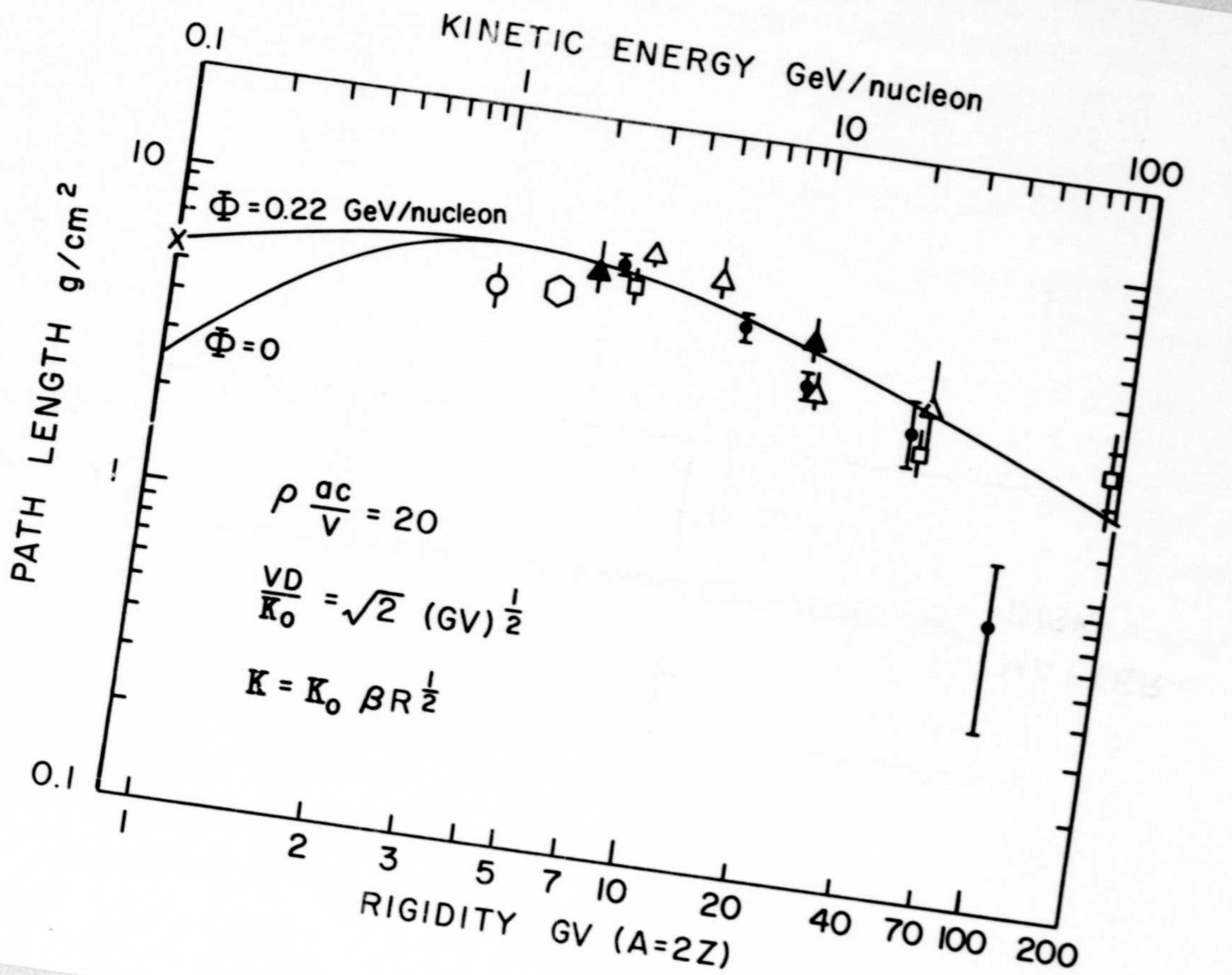
If one considers the expected density of radioactive particles including the effects of decay N_D and ignoring decay N_{ND} the difference will be entirely due to the difference in $\bar{\ell}$ and hence the surviving fraction

$$f \equiv N_D/N_{ND} = \frac{(1/\bar{\ell}_{ND} + \sigma)}{(1/\bar{\ell}_D + \sigma)} . \qquad \text{(A5)}$$

ADDRESS

Frank C. Jones
Laboratory for High Energy Astrophysics
NASA/Goddard Space Flight Center
Code 660
Greenbelt, MD 20771


C.R.
HALO
D
D
V
V
MATTER
DISK



ORIGINAL PAGE IS
OF POOR QUALITY