# ANALYSIS AND DESIGN OF A HIGH TIP SPEED, LOW SOURCE NOISE AIRCRAFT FAN INCORPORATING SWEPT LEADING EDGE ROTOR AND STATOR BLADES 

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| 16. Abstract In modern high bypass ratio turbofans, the fan thrust is achieved in a single fan stage, which usually requires supersonic tip speeds of the fan rotor to produce the necessary pressure rise. In such fans, the predominant sources of noise are shocks radiated from the supersonicallymoving rotor blades (called multiple-pure-tone [MPT] noise), and tones radiated from the rotor wake interaction with stator vanes. <br> In this program, two advanced noise reduction concepts were applied to the design of a 1.6 pressure ratio single stage fan. The goal of the design was to reduce the following acoustic sources: multiple pure tone noise, rotor-wake/stator-blade interaction noise, and noise due to operating the rotor in distorted or turbulent inflow. Unique nonradial blading of the rotor and stator was used to achieve these goals. The rotor blade leading edges were swept so that the normal component of flow to the edge is subsonic at all points along the blade span, thus preventing the occurrence of leading edge shockwaves. The stator vanes were designed to minimize noise generated by rotor wakes incident on the blades by progressively sweeping the vanes from root to tip in order to produce subsonic trace speeds for the unsteady loads along the span. Special aerodynamic and structural design considerations were required to assure the performance and integrity of this unusual blade and vane design. <br> This report sumarizes the physical rationale for the swept blade concepts, the detailed aerodynamic, acoustic, and structural design, and the mechanical assembly of the rig fan. |  |  |  |
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FOREWORD

The purpose of this report is to describe a recently completed program to design and manufacture an experimental transonic fan model featuring novel methods for noise reduction at the source. The program was conducted between 1974-1976 under contract NAS3-18512 issued by NASA Lewis Research Center, with Bolt Beranek and Newman Inc. (BBN) as the prime contractor and AVCO Lycoming as a subcontractor. The contract resulted from a NASA request for proposals (RFP) concerning CTOL aircraft engine fan source reduction concepts. The intent of the RFP was to identify advanced design concepts for reducing both rotor and stator sources which could be implemented with existing aerodynamic and structural design capabilities. The RFP encouraged proposals to reduce noise from high speed single stage fans.

BBN proposed the use of "subsonic leading edge" rotor blades and variably swept stator vanes as the concepts to be investigated. The study and engineering work culminated in the fabrication of a 20 -inch diameter fan stage to be tested for acoustic and aerodynamic performance at the Lewis Research Center, National Aeronautics and Space Administration, Cleveland, Ohio.

Bolt Beranek and Newman Inc. (BBN), Cambridge, Massachusetts, served as the prime contractor with overall program responsiblity, as well as prime technical responsibility for the fan acoustic design, and other areas. The Lycoming Division of AVCO Corporation, Stratford, Connecticut, was a major subcontractor to BBN, with responsibilities in aerodynamic and mechanical design, and manufacture of the fan hardware. Rotor blades and stator vanes were manufactured under subcontract by New England Aircraft Products, Farmington, Connecticut.

Also included in the program were efforts to develop a 3dimensional compressible flow computer program to analyze the flow through the rotor, especially in the vicinity of the leading edge, and the investigation of the feasibility of using porous trailing edges on the stators to reduce broadband noise. The 3-D flow program was discontinued at the time the rotor design was finalized, and the porous edge concept was not used because of the difficulties perceived in manufacture of small vanes from available porous metal materials.

Numerous individuals at BBN and AVCO made significant contributions to this project. Mr. Richard Hayden served as project
manager, and contributed to the acoustic design of the fan as well as other areas. Dr. Donald Bliss served as an associate project manager and had responsibility for the concept of the rotor blade, the rotor acoustic design, and the coordination of the aerodynamic design with AVCO. Mr. Bruce Murray also served as an associate project manager and supervised the mechanical design and manufacturing aspects of the fan. The stator acoustic design was carried out by Dr. K.L. Chandiramani, and Mr. Joseph Smullin. Drs. John McElman and John O'Callahan performed finite element stress analysis of the rotor blades, and Dr. O'Callahan contributed to numerical fluid mechanical analysis of the rotor flow field.

At AVCO Lycoming, Mr. Pierre Schwaar served as the principal investigator and has primary program responsibility for the fan aerothermodynamical design, and for implementing the subsonic rotor leading edge concept and the acoustic design of the stator blades within operational structural constraints. Mr. Herbert Kaehler led AVCO's work on structural analysis and Mr. John Banks supervised mechanical design and manufacturing activities there.

Mr. James G. Lucas of the NASA Lewis Research Center's V/STOL and Noise Division was the NASA Program Manager, and contributed valuable assistance in the mechanical design and manufacturing areas, and in the integration of the fan into NASA's test facilities.

This report has been designated as Bolt Beranek and Newman (BBN) Report No. 3332.

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## SUMMARY

On current generation high bypass ratio turbofan engines, the fan is a predominant noise source which must be controlled to meet future aircraft noise goals. Of the various approaches to turbofan engine noise reduction, the most attractive is reducing the strength of the noise-producing elements at the source, thus avoiding weight and performance penalties associated with various sound sappression approaches.

In modern high bypass ratio turbofans, the fan thrust is achieved in a single fan stage, which usually requires supersonic tip speeds of the fan rotor to produce the necessary pressure rise. In such fans, the predominant sources of noise are shocks radiated from the supersonically-moving rotor blades (called multiple-pure-tone [MPT] noise), and tones radiated from the rotor wake interaction with stator vanes.

In this program, two advanced noise reduction concepts were applied to the design of a 1.6 pressure ratio single stage fan. The goal of the design was to reduce the following acoustic sources: multiple pure tone noise, rotor-wake/stator-blade interaction noise, and noise due to operating the rotor in distorted or turbulent inflow. Unique nonradial blading of the rotor and stator was used to achieve these goals. The rotor blade leading edges were swept so that the normal component of flow to the edge is subsonic at all points along the blade span, thus preventing the occurrence of leading edge shockwaves. The stator vanes were designed to minimize noise generated by rotor wakes incident on the blades by progressivly sweeping the vanes from root to tip in order to produce subsonic trace speeds for the unsteady loads along the span. Special aerodynamic and structural design considerations were required to assure the performance and integrity of this unusual blade and vane design.

The rotor design using a blade concept with shock-free leading edges (except at points of inflection where weak conical shocks occur) is highly flexible in that a large family of blade shapes and leading edge contours may, in general, be used to achieve the noise reduction goal. The swept rotor design is also attractive since it should perform equally well at off-design conditions if it has been designed to perform properly at the highest envisioned rotor speed. The swept edges also are compatible with reducing noise generation due to inflow distortion.

In the final design of the particular rotor ultimately constructed, a reversal of the sweep direction was required near mid-span to minimize stress levels in the blade. Once this was done, aeroacoustical-structural design iterations led to a blade with acceptable stress levels and no additional compromise in acoustic performance beyond the expected weak conical shock at the sweep reversal point.

The aerodynamics of subsonic leading edge rotor cascades with supersonic absolute inflow velocities are not well known, and will clearly require further study.

The concept of forcing the trace speeds of moving load distributions on stator vanes to be subsonic was introduced for the first time in this program. The design of a stator which uses this concept requires a controlled rate of axial sweepback (or circumferential skew), the details of which depend heavily on the rotor wake field which varies with distance from the rotor. The selection of a stator vane number for a given rotor design is done with the familiar cutoff condition in mind; however, supersonic rotor tip speeds make it impossible to completely cut off the radiation at the tips of the stator vanes. No serious aerodynamic or structural problems were associated with the swept stator. The stator acoustic design procedure is now well-defined in terms of flow parameters needed as inputs, but the ability to predict the necessary flow parameters of the rotor wake field is presently limited.

## SECTION 1

## INTRODUCTION

With the advent of high bypass ratio turbofan engines, and the associated decrease in exhaust velocity, the fan stage has become the dominant aircraft engine noise source. Therefore, fan noise reduction is a problem of primary importance in the ongoing effort to evolve quieter aircraft. Furthermore, it is increasingly important that any penalty in operating efficiency incurred by noise reduction methods be minimized.

In general, noise reduction can be achieved in two ways: (1) reduction through the attenuation of propagating sound fields; and (2) reduction of the strength of the noise sources themselves. The first approach typically involves the use of absorptive duct liners and splitters, and possibly basic modifications to the inlet duct geometry. Because add-on features are required, and the duct length may be increased, the penalties associated with this approach are added weight and some direct reduction in aerodynamic efficiency. Furthermore, there may be a degree of noise generation associated with some treatment modifications, such as in-duct splitters, particularly if the inflow to the fan is disturbed.

The second approach, which is the reduction of noise at the source, can be pursued in many ways. The basic fan design parameters can be chosen to give more favorable acoustic behavior. For instance, the tip speed can be reduced, the spacing between the rotor and stator can be increased, and the number of blades and vanes can be altered to prevent the propagation of certain duct modes. Whether these options can be exercised in a given case depends on the design constraints on the performance and size of the system.

Because, in most circumstances, acoustic considerations cannot dictate the choice of basic fan design parameters, other means of noise source reduction are worthy of consideration. These other means of source reduction necessarily involve changes in the aerodynamic design of the blades and vanes. The design changes may occur either within the framework of conventional design practice, such as the use of optimized blade section properties, or may involve the exploration of novel concepts. Although development of all the design data needed for implementing novel concepts for noise source reduction may be initially difficult, the noise reduction potential of a successful concept may greatly exceed the reduction obtained by more conventional means. Of course, the final test of an acoustically successful concept must always
be whether any associated penalties in performance, complexity, and system integration can be overcome or, at least, justified in relation to the benefits.

The subsonic leading edge rotor is implemented by tailoring (sweeping) the rotor leading edge to the mean inflow such that subsonic Mach number flow is achieved normal to the leading edge along the entire span, thus preventing shock generation. Previous use of partially-swept transonic rotors was done in an effort to reduce transonic drag rise and thus improve stage efficiency. Swept stators have been previously used to reduce noise, but the design concept implemented here involves tailoring the leading edge shape to a detailed estimate of the rotor wake field incident upon the stator.

The remainder of this report is organized to describe in detail the rationale for selection of the particular concepts (Sections 2 and 3), details of the design procedure used on the swept rotor blades (Section 5) and stator vanes (Section 6), residual noise sources (Section 7), and facility integration (Section 8). Appendices contain a listing of aerothermodynamic design parameters (App. A), a discussion of geometric considerations for subsonic leading edge rotor blades (App. B), a detailed discussion of acoustical considerations in the stator design (App. C), discussion of empirical estimates of fan noise levels (App. D), and a useful algorithm for estimating trace speeds of rotor wakes on stator vane leading edges (App. E).

## SECTION 2

## TRANSONIC FAN NOISE SOURCES

This section summarizes the major noise sources and mechanisms encounted with transonic fans. Typical design characteristics of single stage transonic fans are summarized in Table 1.

TABLE 1. TYPICAL CHARACTERISTICS OF SINGLE STAGE TRANSONIC FANS.

| Pressure Ratio Range | $1.4-1.8$ |
| :--- | :--- | :--- | :--- |
| Tip Speed | 300 to $600 \mathrm{~m} / \mathrm{s}(1000-2000 \mathrm{ft} / \mathrm{sec})$ |
| Relative Rotor Tip Mach No. | $1.1-1.8$ |
| Rotor Inlet Hub/Tip Ratio | $.35-.50$ |
| Stator Hub Mach No. | .8 |

The most important noise sources, which involve both the rotor and stator, are:

Shockwave noise from the supersonic portion of the rotor blades, often called multiple pure tone (MPT) noise.

Rotor/stator interaction noise caused by unsteady loading due to aerodynamic interaction (tonal and broadband noise).

Noise caused by unsteady loading on rotor blades interacting with inflow distortions and turbulence (tonal and broadband noise).

A brief elaboration on each of these sources is now provided.

### 2.1 Shockwave Noise

When the relative flow past the rotor becomes supersonic, the propagation of shock waves out of the inlet duct becomes an important noise source. The upstream propagation of waves from blade rows with detached and attached shock wave patterns is shown in Fig. l, (from Ref. l). Because the pressure field must satisfy a periodicity condition, expansion waves occur in the regions between the shock waves.

Several investigations (Refs. 2 through 7) have shown that nonlinear effects are an important factor in the upstream shock propagation process. Because nonlinear attenuation occurs more rapidly for higher initial levels, an increase or reduction of the

(a) Detached Shock Wave Pattern

(b) Attached Shock Wave Pattern

FIG. 1. POSSIBLE SHOCK WAVE CONFIGURATIONS FOR ROTORS IN SUPERSONIC FLOW.
shock strength at the blades does not produce a comparable increase or reduction of levels at the end of the inlet duct, or in the far field. This effect is strongest when the wave train in the duct is well ordered and can be considered nearly one-dimensional in character. The important consequence of this effect is that very substantial levels of source reduction must be achieved to guarantee a worthwhile reduction in level in the far field.

Another important consequence of nonlinear propagation is the redistribution of the shock noise spectrum from blade passage frequency and its harmonics to the rotor shaft rotation frequency and its harmonics. This redistribution occurs because of blade-to-blade differences in the initial strength and position of the shock waves. These blade-to-blade differences are caused by variations in manufacturing tolerances that may affect the circumferential location, setting angle, thickness, and camber of the blades. Because the shock train structure is inherently unstable to perturbations in strength and position, these initial disturbances need not be large. As an example, when periodic variations in shock strength occur, the stronger shocks tend to overtake and dominate the weaker shocks because of nonlinear effects. Because the variations in strength are caused by blade-to-blade differences, they are periodic in the shaft rotation speed. Thus, as the wave train propagates, the harmonics of shaft speed become increasingly important relative to the harmonics of blade passage frequency. Fig. 2 shows the redistribution of energy from blade passage frequency to shaft rotation frequency as the result of an initial amplitude perturbation to one shock in a wave train. Figure 3 shows sketches of typical noise spectra for a subsonic fan, which has no shock noise, and for a supersonic fan, where the tones at the harmonics of shaft speed are clearly present. Clearly the multiple pure tone noise due to shock wave propagation is a major noise problem.

### 2.2 Rotor/Stator Interaction Noise

Unsteady aerodynamic loads on rotor blades or stator vanes produced by the aerodynamic interaction between the rotor and stator are an important source of both tonal and broadband noise. The main causes of the aerodynamic interaction are the interference with the potential flow pressure and velocity fields and the interaction with the viscous and turbulent wakes from upstream blades. The potential field interaction that produces tonal noise at the harmonics of the blade passage frequency can be virtually


FIG. 2. DEVELOPMENT OF A SHOCK TRAIN WITH AN INITIAL DISTURBANCE (from Ref. 5).

a) TYPICAL NOISE SPECTRUM FOR A SUBSONIC TIP SPEED FAN


FREQUENCY
b) TYPICAL NOISE SPECTRUM FOR A SUPERSONIC TIP SPEED FAN

FIG. 3. TYPICAL FAN NOISE SPECTRA FOR SUBSONIC AND SUPERSONIC TIP SPEEDS.
eliminated by providing adequate spacing between the rotor and stator. Increasing the spacing on a high by-pass ratio fan stage is usually practical and does not involve a severe aerodynamic penalty. The interaction of the stator vanes with the "mean component" (steady velocity deficit) of the rotor wakes produces tonal noise at the harmonics of blade passage frequency, while the interaction with the wake turbulence produces broadband noise. Increasing the spacing between the rotor and stator also reduces - but does not necessarily eliminate - this noise source.

### 2.3 Inflow Distortion Noise

The inflow to the fan rotor typically exhibits a degree of spatial nonuniformity and a certain amount of turbulence. Sound is produced by unsteady loads on the rotor blades operating in this disturbed inflow. Steady spatial nonuniformity causes tonal noise to be produced at the harmonics of blade passage frequency, and the presence of turbulence produces broadband noise. However, if the turbulence scales are sufficiently long in the streamwise direction, then many blades will interact with a given disturbance in a similar manner, producing peaks in the noise spectrum at the harmonics of blade passage frequency. Because the basis for this noise source is a random process, the amplitude of these peaks will vary in time in a random manner. Inflow distortions have been shown to be a potentially important noise source in static fan test facilities. Their importance in an actual flight environment is less certain, since the effect of forward motion is usually to reduce certain types of inflow distortion.

## NOISE SOURCE REDUCTION CONCEPTS

In this section, the concepts for the reduction of rotor and stator noise sources are described. A review of the detailed analysis and design procedures associated with the implementation of these concepts in the present program is postponed to the sections later in the report dealing with detailed design.

### 3.1 Rotor Noise Reduction

As discussed in the previous section, two noise sources associated with the rotor are multiple pure tone noise due to shock waves and inflow distortion noise. This section describes a concept which has the potential to substantially reduce multiple pure tone noise. As an additional advantage, this concept will also help reduce the problem of inflow distortion noise.

In principle, upstream-propagating shockwave noise can be reduced by designing for careful alignment of the relative velocity, w, with the suction surface near the rotor blade leading edge, as shown in Fig. 4a. However, completely shockfree entry into the blade row cannot be achieved in conventional blading because of the finite thickness of the blade leading edge. The effect of thickness is 1llustrated in Fig. 4b. Moreover, since the relative inflow direction varies with the operating conditions, the proper alignment cannot be maintained in off-design operation, nor in the presence of inflow distortions. Thus, this concept presents several practical difficulties for application to aircraft fans which do not operate at a single design point.

A different approach to obtain shockfree entry into a blade row is now described. It is believed that this approach does not suffer from the shortcoming of the more conventional approach just described. Consider a blade whose leading edge is swept relative to the local inflow velcocity vector. The leading edge would in general appear swept when viewed from the side and skewed when viewed from the front. If the leading edge is swept such that the Mach number of the relative flow component normal to the leading edge is everywhere subsonic, a shockless leading edge results. In wing theory, this is referred to as a "subsonic leading edge in supersonic flow" (See, for instance, Ref. 8). In rotating applications, the radial variation in relative Mach number makes it possible, in principle, to completely avoid upstream shock wave propagation by using leading edge and surface generating line sweep which varies from hub to tip. In practice, structural constrains force some design compromises. In the present design, the structural

(a) Two-Dimensional Leading Edge Design Without Upstream Waves

(b) The Effect of Leading Edge Thickness

FIG. 4. SHOCKLESS LEADING EDGE DESIGN AND THE EFFECT OF THICKNESS.
compromise entails the presence of a train of conical shocks upstream of the rotor associated with a sweep discontinuity in the leading edge. From the standpoint of preventing shock noise, the design can be made insensitive to operating conditions, relative flow alignment, and inlet distortions by designing the sweep distribution for the highest relative inflow Mach number to be expected; thus ensuring a lower subsonic normal Mach number component for off-design conditions. This insensitivity is considered to be a major asset.

The underlying aerodynamic idea is now reviewed. Figure 5a shows a swept wing of infinite extent subject to an incident supersonic flow. Since there is no spanwise variation in the wing geometry, the axial component has no effect. The aerodynamic forces are determined entirely by the component of the flow normal to the wingspan. If the component normal to the span is subsonic, then there are no shock waves associated with the flow over this wing. Of course, to be completely shockless, the normal component must be sufficiently subsonic that transonic flow effects do not occur in the normal flow plane. The only effect of the axial component is in the structure of the viscous boundary layer on the wing surface, but this is not related to the presence or absence of shock waves. The same ideas are applicable, of course, to an infinite span sweptback cascade. Fig. 5b shows a finite span wing sweptback to have subsonic leading edges. The aerodynamic behavior is now considerably more complicated. In particular, the presence of conical shocks at the front and rear of the wing root and at the rear of the tips is unavoidable. These isolated points on the wing are discontinuities in the otherwise subsonic edges. The conical shocks are, however, weaker than their twodimensional counterparts and, because of their three-dimensional nature, decrease in strength with distance from their point of origin.

The application of a subsonic leading edge to a fan blade is illustrated in Fig. 5c. This illustration is simplified to its essential form, showing only the radial change in Mach number. The actual process is nonplanar because of the change in direction of the inflow with radial location. The particular case illustrated applies to a transonic fan, since part of the incident flow is subsonic. Then the leading edge can be made completely shockless even though the blade is of finite extent assuming that one is able to predict and accommodate the effects of spanwise flows (Ref. 9).

The local leading edge sweep at each radial station is chosen to be greater than the Mach angle of the local flow, i.e., the swept edge must lie within the local Mach cone. This assumes that the normal flow to the leading edge is everywhere subsonic. Because of the gradient in Mach number, the incident flow is subsonic at the base of the blade so a shock cannot emanate from this point (unlike the wing root in Fig. 3b). Hence, the blade leading edge can be entirely shockless, except for the effects of aerodynamic interference between the blade tip and the shroud which produce conical shocks. If the fan were completely supersonic, a conical shock should also occur at the root of the blade. By designing the leading edge and the other generating lines of the forward portion of the blade surface to be subsonic for the situation that produces maximum relative flow Mach number, the edge will remain subsonic under all other operating conditions. The blade leading edge would usually be designed to have a constant normal velocity (Mach number) component at all points along the span at radii (from the hub) greater than that at which the critical normal Mach number, $M_{\text {w }}$.fit is reached. The critical Mach number is that normal Mach numberitel) at which thickness effects would cause the flow to become transonic.

In addition to sweepback, Figs. 6a, 6b, and 6 c show swept forward and compound sweep blades that are also possible configurations. All of the blade configurations must have a conical shock at the tips caused by aerodynamic interference with the shroud. The compound sweep blade will also have a weak conical shock at the discontinuity in sweep, which is positioned somewhere along the leading edge (assuming the discontinuity lies in the region of supersonic relative inflow). Although the compound sweep blade has the acoustic penalty of introducing a weak conical shock, it offers other definite advantages. Structural considerations provide the most severe constraint to the design of high speed fans with swept blades. Fairly large excursions of the leading edge are required to implement this concept. It should be noted that the family of threedimensional curves that satisfies the subsonic leading edge condition is not unique and therefore considerable latitude exists to determine structurally optimum shapes. Figure 7 shows the type of conical shock wave pattern for a compound sweep blade. The blade in the sketch closely resembles the design developed during the course of the project being described.

Figure 8 compares the operation of a moderately loaded blade row with and without subsonic leading edges in supersonic flow. As explained above, the subsonic edge region allows shock-free entry into the blade row. The blade rows are identical except for the addition of a subsonic leading edge region in one case. The front surface of the blade must be designed so that any shocks generated on the suction surface of

B. Swept Forward

C. Compound Sweep

FIG. 6. FRONT VIEW OF SOME POSSIBLE BLADE CONFIGURATIONS WITH SUBSONIC LEADING EDGES.


FIG. 7. CONICAL SHOCK FIELD FROM A ROTOR BLADE WITH A COMPOUND SWEEP LEADING EDGE.

(a) OPERATION OF A CONVENTIONAL MODERATELY LOADED BLADE ROW

(b) OPERATION OF A BLADE ROW WITH SWEPT ("SUBSONIC") LEADING EDGES

FIG. 8. COMPARISON OF THE OPERATION OF A MODERATELY LOADED BLADE ROW WITH AND WITHOUT SUBSONIC LEADING EDGES.
the blade are formed sufficiently far back that the disturbance is entirely contained in the blade row, even during off-design operation.

Using a swept leading edge also helps reduce the response of the rotor to inflow distortions, because the magnitude of the response is largely determined by the velocity component normal to the leading edge (Ref. 10). The effect of inflow distortion is most important near the tip of the rotor where the relative velocity is highest. Fortunately, the concept for sweeping the blades requires the most sweep near the tip.

### 3.2 Stator Noise Reduction by Leading Edge Sweeping and Blade/ Vane Number Selection

Although increasing the spacing between the rotor and stator leads to some noise reduction, the aerodynamic interaction between the rotor wakes and stator vanes remains an important noise source. Further reduction by conventional means can be achieved by choosing the proper number of blades and vanes to cut off many of the acoustic spinning modes in the duct (Ref.li). When the rotor tip speed is subsonic, the blade and vane numbers can be chosen so that all the spinning modes at blade passage frequency, and at least some of the modes at higher harmonics, are cut off. However, if the rotor tip speed is supersonic, at least one spinning mode at blade passage frequency cannot be cut off, regardless of the choice of blade and vane numbers. Since supersonic spinning speeds often occur on transonic fan designs, other means of stator noise reduction are of considerable interest.

Figure 9a illustrates the interaction of a row of stator vanes with rotor wakes when viewed on a surface of constant radius from the fan axis. The wakes can be described as flow regions with an average velocity $\bar{W}$ lower than the velocity of the adjacent fluid, upon which a turbulent perturbation velocity field $\Delta w$ is superimposed.

Figure 9 b shows a sketch of a three-dimensional wake/vane interaction in a fan. The structure of the viscous, usually turbulent, wakes that trail each rotor blade is complex. However, on the average, these wakes can be considered as being convected with the mean flow in which they are imbedded. The nature of the downstream mean flow is such that the convection process will distort the wakes from their original shape; namely, the downstream flow is distorted both axially and circumferentially across a given radial path, leaving the downstream pattern of the wake disturbance very much altered from the pattern at the rotor trailing edge. Suppose, for instance, the rotor is designed to give a mean flow that has a uniform axial velocity distribution and a free vortex tangential velocity distribution. Assuming

(a) The Interaction of the Stator Vane Row with the Mean and Unsteady Rotor Wake Components as Seen on a Constant Radius Surface.

(b) A Sketch Showing the Three-Dimensional Nature of the Rotor-Wake/ Stator-Vane Interaction.
the wakes are radial at the rotor trailing edge, it is clear that the tangential velocity component will act to skew the wakes over, with the hub region leading the tip region. This situation is illustrated in Fig. 9b. In this case, the interaction of a given wake with a given stator vane does not occur simultaneously all along the stator vane span. Instead, the instantaneous spanwise interaction region of a single rotor wake will extend over only a portion of any one vane and will sweep along the vane leading edge, beginning at the hub and ending at the tip. The skewing of a wake due to convection by the downstream mean flow can be sufficient to involve simultaneously portions of several stator vanes.

The shape of wake and the magnitude of its velocity components vary from hub to tip. To complete this picture of the downstream flow field, one must consider the unsteady velocity components which account for the turbulent structure of the wakes and for any other sources of inhomogenieties in the flow, e.g., inlet flow distortions, large-scale flow instabilities, and blading errors. In general, the statistical properties of these unsteady components can be expected to vary axially, circumferentially, and radially.

Both the mean and unsteady velocity components of the wake flow induce unsteady loads on the stator vanes. The mean component will produce a load distribution that travels from hub to tip, changing shape and amplitude in accordance with the radial variation of the mean flow properties and wake strength, width, and skew. Imposed on this traveling load distribution will be the unsteady effect of the turbulent structure of the wake. The end result of all sources of unsteady loading on the stator vanes is to produce tonal and broadband noise. The tonal noise is usually considered to be the more important noise source. The speed at which the point of interaction of the flow disturbance with the vane travels along the span is called the trace speed.

A particular source of unsteady loading will produce no significant acoustic radiation if it satisfies a subsonic and non-accelerating trace speed criterion along the vane span. The trace speed concept has been previously recognized for the problem of helicopter-blade/vortex interaction by Widnall (Ref. l2) although it has not been generally recognized in the study of fan noise.

The interaction of the wake with the vane produces a load distribution that travels along the vane. Suppose the vane is much longer than an acoustic wavelength. Following the trace of a phase front of this load distribution, acoustic radiation can occur along the vane span if the magnitude of the load changes,
the phase speed changes with time, or if the phase speed is supersonic. For instance, in fan noise analyses the rotor-wake/ stator-vane interaction is usually assumed to be two-dimensional (corresponding to infinite spanwise trace speed). The conditions mentioned above are necessary for radiation but not sufficient. The interaction with the acoustic field produced by the other vanes must also be considered before the actual occurrence of acoustic radiation can be established. Therefore, regions along the stator vane span can be expected to be poor radiators if the phase speeds are subsonic, nearly constant, and local levels do not vary rapidly. Other regions may or may not be efficient radiators depending on the behavior of the distribution of sources elsewhere on the stator. Furthermore, end effects at the hub and tip (within approximately one half an acoustic wavelength of the ends) makes these regions potential radiators. These considerations are discussed in Appendix C, and justified in detail in Bliss, et al., (Ref. 13).

Understanding the rotor-wake/stator-vane interaction and the criteria for radiation from the span of a single vane suggests ways in which the vane configuration can be altered to achieve noise reduction. The vane should be shaped so that loads traveling along the span move at a constant subsonic speed. Assuming that the amplitude of the load distribution, moving with a phase front, is essentially constant, then radiation from the vane span will not occur (except for endeffects). The condition of a constant subsonic spanwise trace speed can be achieved by sweeping or skewing the stator vanes, as illustrated in Fig. 10. In this illustration, the lines of constant phase can be considered to be the intersection of the rotor wakes with the plane of observation (e.g., the r-o plane in Fig. 10a, and the r-z plane in Fig. lob). Except for the effect of shape changes, these lines travel at constant speed (rotational in the $r-\theta$ plane and rectilinear in the $r-z$ plane) because of the rotation of the rotor. The speed at which a phase front traces the leading edge of the stator vane depends on the shape of the leading edge and the shape of the phase front. Clearly the trace speed can be controlled by either sweeping or skewing the stator vane. With this approach, radiation from the stator span can be prevented, leaving only acoustic radiation from end effects at the hub and tip of the vane. Radiation from the hub region can be cut off by the proper choice of blade and vane numoers, provided that the rotation speed of wakes at the hub is subsonic. Since the rotation speed of wakes at the stator tip will usually be supersonic for a transonic fan, the radiation from tip end effects can never be entirely cut off. Note that the rotor

wake pattern rotates with the same angular velocity $\Omega$ as the rotor. Thus, at any given radius at any downstream location between the rotor and stator, the rotation speed of the wake pattern is simply, $\Omega r$, which is different than the swirl velocity component. This can be best visualized from rotor fixed coordinates from which the wake pattern appears "frozen."

Another, but related, way to view the effect of sweeping or skewing the stator vanes is as follows. Tyler and Sofrin (1962) have shown that for a given circumferential mode number, $m$, and hub-to-tip ratio, $v$, the radial structure of an acoustic spinning mode can be described by an infinite series of characteristic functions. The functions in this series differ according to their radial order, $\mu$, i.e., each function has a different number of nodes in the interval between the hub and tip. The spinning speed at which each of these functions begins to radiate is always supersonic and increases with increasing radial order. Therefore, at a given supersonic spinning speed and fixed $m$ and $\sigma$, only a certain number of the functions corresponding to the lowest radial order will not be cut off. Vanes can be skewed or swept so that the number of wakes on a given vane is increased, raising the radial order of the load distribution on the vanes. The acoustic energy is thereby redistributed to higher radial orders, some of which will be cut off. The relationship between duct mode cut off and the constant subsonic trace criterion is discussed by Bliss, et al., (Ref. l3), and in Appendix C.

## SECTION 4

## FAN STAGE DESIGN SUMMARY

An experimental transonic fan stage was designed and constructed using the noise reduction concepts explained in the two preceding sections. The fan uses compound sweep rotor blades designed to have "subsonic leading edges" in the region of supersonic relative inflow. The stator vanes were swept back to achieve a constant subsonic trace speed of rotor wakes along the vane span. Figures lla, b and c show photographs of the actual fan stage. A cross-sectional view of the fan as it will appear when installed in the test facility of NASA Lewis is shown in Fig. 12. As indicated in the illustration, the fan will be tested in both forward and reverse installation arrangements in order to measure both the fore and aft noise characteristics. The design data for the fan stage is summarized in Table 2. In the remainder of the report, the detailed design procedures used in the development of the fan stage are described.

TABLE 2. FAN STAGE DESIGN SUMMARY

## Stage Characteristics:

Stage Pressure Ratio, $P_{4} / P_{1}=1.6$
Mass Flow Rate, $\quad W=31.2 \mathrm{~kg} / \mathrm{s}(68.8 \mathrm{lb} / \mathrm{sec})$
Specific Mass Flow Rate: (referred to annular area at
rotor inlet)
$\mathrm{W}_{\mathrm{as}}=199.03 \mathrm{~kg} / \mathrm{s} \cdot \mathrm{m}^{2} \quad\left(40.76 \mathrm{lb} / \mathrm{sec} \cdot \mathrm{ft}^{2}\right)$
Polytropic Stage Efficiency, $\eta=0.86$

## Rotor:

28 Compound Sweep Blades
Leading Edge Normal Mach Number $=0.91$
Tip Speed $=480 \mathrm{~m} / \mathrm{s}(1575 \mathrm{ft} / \mathrm{sec})$
Relative Tip Inlet Mach Number $=1.593$
Rotor Inlet Tip Radius $=249 \mathrm{~mm}(9.803 \mathrm{in})$
Rotor Inlet Hub-Tip Ratio $=0.442$
Rotor Pressure Ratio, $P_{2} / P_{1}=1.64$
Stator:
59 Swept Back Vanes
Sweep Angle $=25^{\circ}$ At Root, $40^{\circ}$ At Tip
Stator Inlet Mach Number $=0.80$
Stator Pressure Loss $\Delta \mathrm{P}_{3-4} / \mathrm{P}_{3}=.025$


(a) Side View

FIG. 11 concluded PHOTOGRAPHS OF THE EXPERIMENTAL FAN STAGE.

fig．12．FORWARD AND REVERSE INSTALLATION in the w2 fan NOISE test

## SECTION 5

## DETAILED ROTOR DESIGN

This section deals with all aspects of the detailed analysis and design procedures associated with the fan rotor. The aerodynamic design of a transonic rotor having blades with "subsonic" leading edges differs significantly from conventional design practice because the acoustic, aerodynamic, and structural requirements interact strongly with each other from the very beginning of the preliminary design phase. Therefore it was necessary to conduct numerous aerodynamic-acousticstructural design iterations to optimize and finalize a rotor configuration satisfying all the design requirements.

The overall design point data for the rotor are listed in Table 2 of the previous section.

### 5.1 Aerodynamic Design

Certain differences are to be expected in the aerodynamic behavior of a rotor with subsonic leading edges. Since the entry into the blade row is shock free, any shocks that occur must remain within the blade row under all operating conditions because the edge region cannot support a shock system. Furthermore, the effects of sweep may introduce other three-dimensional flow phenomena which are not present in a conventional blade design. Given these facts, the rotor aerodynamic design was undertaken using the best conventional design practice combined with an anticipation of the most important effects of swept edges. The design was carried out primarily with the use of an axisymmetric flow computer program. Conventional (twodimensional) methods for analyzing the flow behavior within the blade row are not really adequate for the three-dimensional case of blades with swept "subsonic" edges. To handle this problem analytically requires a more general approach. Some work was done to adapt a new fully three-dimensional computer code to the analysis of flow through the blades with swept edges, but was discontinued due to schedule requirements.

An important question in the design of a rotor with "subsonic" edges is related to its surge margin. Because the edge region cannot support a shock system it was felt that the surge margin might be reduced. Such a reduction would occur, if the effective rotor operating range were limited by the condition that the shock system remain within the covered cascade region. The flow configuration in which the shock system must remain within the covered cascade, however, does not yield the maximum static pressure rise achievable in a conventional transonic-supersonic rotor. Consequently, in a
stage where surge is not triggered prematurely by the stator flow conditions, a rotor with subsonic leading edges might result in a decrease of the surge margin as compared to a conventional design.

Since the rotor aerodynamic loading essentially depends upon the rotor static pressure rise, the selection of the meridional flow path was the main design step taken to achieve the desired loading levels. Meridional channel conicity and curvature through the rotor section were traded off in several preliminary design attempts. The flow calculations were performed by means of a code which solves the general equation of radial equilibrium on straight axial or slanted stations for the axisymmetric flow case taking into account the radial variation of the blade efficiency. The polytropic efficiency $\eta$ assumed for the rotor blading is shown in Fig. 13, where $\eta$ is derived from

$$
\frac{P_{2}}{P_{1}}=\left(\frac{T_{2}}{T_{1}}\right)^{(\eta)\left(\frac{\gamma}{\gamma-1}\right)}
$$

It was found that the comparatively large channel conicity across the rotor section required by the high design pressure ratio $P_{4} / P_{1}=1.6$ and the wall curvature needed at rotor exit to prevent excessive channel contraction in the free space between rotor and stator, combine to shift the maximum rotor static pressure rise from the tip towards the midspan location, where the shock system has the greatest tendency to move upstream into the uncovered cascade region because of the lower relative inlet Mach number. The main preliminary design effort consequently was directed toward minimizing the static pressure rise at the critical midspan location.

The optimum channel configuration is shown in Fig. 14. The flow conditions are summarized on the Aero design program (R-121) input and output printout attached in Appendix $A$.

Figure 15 a shows the distribution of the rotor static pressure ratio $P_{2} / P_{1}$ over the channel height, together with the relative inlet Mach number $M_{w}$ and the corresponding normal shock pressure ratio $\hat{P}_{1} / P_{1}$, which is roughly equivalent to the static pressure ratio obtained in the front portion of a conventional cascade with a normal shock attached to the leading edge.


FIG. 13. ROTOR POLYTROPIC EFFICIENCY $\eta$

FIG. 14. MERIDIONAL FLOW PATH.

Three types of operating conditions can be distinguished along the rotor blade span.
(a) From the hub to the sonic radius $r_{1}\left(M_{W_{1}}=1\right)$, the rotor static pressure rise is achieved essentially by subsonic relative flow deceleration and centrifugation.
(b) From $r_{1}\left(M_{W_{1}}=I\right)$, to $r_{1}\left(P_{2} / P_{1}=\hat{P}_{1} / P_{1}\right)$ the rotor static pressure rise must be achieved through a channel-contained normal shock or a pseudoshock system followed by subsonic relative flow deceleration. The radial distribution of the rotor static pressure ratio $P_{2} / P_{1}$ determined how far this operating condition extends beyond the sonic radius. In the present case, it extends roughly from the $12 \%$ to the $40 \%$ mass flow streamline, or from $20 \%$ to $53 \%$ of the span, i.e., slightly beyond the point of sweep reversal. The maximum inlet Mach number in this blade section remains below the 1.3 level at which the interaction of normal shock with a turbulent boundary layer produces extensive flow separation ( $\hat{P} / \mathrm{P}=1.8$ ). If minor flow separation does occur in the upper portion of this region, the flow will reattach to the blade because of the large solidity provided in the vicinity of the point of sweep reversal. Consequently, it is expected that the design flow conditions will be obtained over this critical span section by a shock configuration located in the forward, yet still covered portion, of the cascade.
(c) In the upper blade section, $P_{2} / P_{1}$ is smaller than $\hat{P}_{1} / P_{1}$, and the shock system consequently moves progressively toward the rear portion of the cascade. Since no shock is attached to the leading edge, the flow conditions are essentially similar to those in the diverging section of a converging-diverging nozzle in the supersonic off-design operating range.

Figure 15b schematically shows the meridional projection of the rotor blade and the anticipated shock/pseudoshock interception area on its pressure and suction sides. The main question pertains to the rotor surge margin, i.e. the extent to which the tip region will be allowed to increase its pressure ratio beyond the design value by forward shifting of the shock configuration before (1) flow separation occurs at the hub, or (1i) the shock system at midspan is forced into the uncovered cascade region.


FIG. 15(a). ROTOR FLOW CONDITIONS: SPANWISE DISTRIBUTION OF STATIC PRESSURE RATIO AND INLET RELATIVE MACH NUMBER (Design Point).


FIG. 15(b). ROTOR BLADE SHOCK/PSEUDOSHOCK INTERCEPTION AREA (Schematic).

### 5.2 Description of the Aero-Structural Design Interaction Problem

The problem of achieving acceptable stress levels is much more difficult for a rotor with swept leading edges than for a conventional design. The aerodynamic and structural requirements for the rotor blade are therefore closely coupled. Within the aerodynamic constraints, a number of design iterations were required to achieve acceptable stress levels and to optimize the design. The major aerodynamic constraints are that the rotor meet the design performance requirements and that the normal component of flow to the leading edge be maintained at a certain subsonic value. Because of the gradient of relative inflow Mach number, the angle of sweep must increase toward the tip to meet the condition of a subsonic normal component. In the present case, the maximum normal component Mach number was chosen to be 0.91 along these leading edges (actually lower near the hub). The value of 0.91 was chosen as a goal since it represented a normal Mach number sufficiently below sonic to avoid thicknessrelated shocks. Lower values can be chosen, but the severity of the blade leading edge excursions increase as the normal Mach number is lowered. For the fan design tip speed, the excursions of the swept leading edge are large and it was necessary to use a compound sweep configuration to minimize bending stresses. The major variables available to control blade stresses are the location of the sweep reversal point, the local section properties of chord length, maximum thickness, thickness distribution, and the stacking of the blade sections.

Because of the large leading edge excursions, the centers of gravity of the blade sections can no longer be stacked on a radial line. In addition to the centrifugal tensile stresses, large bending moments about both principal axes of inertia of the blade sections were found to occur (Fig. 16). Achievement of acceptable stress levels required the use of a carefully chosen sweep reversal point and the development of an effective nonradial stacking procedure.

Typically, the most critical problem was the bending moments about the minor axis of inertia, and a special stacking procedure was used to minimize these moments. A near-optimum procedure for nonradial stacking is as follows. The blade sections were stacked starting at the tip and moving inward. The addition of each incremental blade section was made so that the center of gravity of the entire portion of the blade above this section falls on the axis of minimum inertia of the new section. The center of gravity of the new upper portion was then reevaluated before the next incremental section was added in the same manner. This procedure nearly minimizes the critical bending stresses around the axis of minimum inertia. The result is not completely optimum because of the


FIG. 16. BLADE CENTRIFUGAL FORCES AND MOMENTS.
complexity of the actual situation in which the stresses are determined by the complex interaction of many effects. Further improvements were made by iterative changes around the result of the above stacking procedure, particularly with the intention of relieving local stress concentrations. To the extent that high stresses arise due to bending around the axis of maximum inertia, these can be relieved largely by changing the location of the sweep reversal point and varying the local section chord and thickness.

### 5.3 Determination of the Subsonic Rotor Leading Edge Geometry

At each leading edge point, the relative Mach vector $M_{w}$
defines a Mach cone. To a prescribed value of the subsonic velocity component $M_{W_{1}}$ perpendicular to the leading edge, there corresponds a coaxial cone with smaller aperture. The subsonic leading edge elements must only satisfy the condition that they lie on such cones. Their direction otherwise is arbitrary.

Referring to Fig. 17, a particular sweep direction can be defined by specifying that each leading edge element be swept in the plane formed by the relative inlet velocity, $W$, and the radius ( $W-r$ ) plane. This yields the shortest leading edge line from hub to tip, since it maximizes the radial projection of every leading edge segment.

Sweeping in the $W-r$ plane however, does not result in a blade with minimum stresses. The resulting stacking of the CG's of the profiles in fact was shown to generate substantial bending moments around their axis of minimum inertia. The main parameter used to minimize bending stresses is the lateral sweep angle, $v$, between the radial plane passing through the leading edge element dl and the $W-r$ plane. The situation is illustrated in Fig. 17. The geometric analysis used for this design is described in Appendix $B$, and only some pertinent results are cited below. They are expressed by the two following equations for the cylindrical coordinates $\theta_{L}$ and $z_{L}$ of the leading edge points in function of the relative flow angle $B_{1}$, the lateral sweep angle $v$, the slope $\varepsilon_{w_{1}}$ of the relative velocity and the projection $\mu^{\prime \prime}$ of the Mach cone angle $\mu$ in the $W-r$ plane (see Appendix B):


5

FIG. 17. SONIC SWEPT LEADING EDGE ELEMENT.

$$
\left.\begin{array}{l}
\theta_{L}(r)=\theta_{L_{1}} \pm \int_{r_{M_{W}}=1}^{r}\left[\frac{\cos \left(\beta_{1}+v\right)}{\tan \left(\mu^{\prime \prime} \quad \pm \varepsilon_{W_{1}}\right)} \cdot \frac{1}{\rho \cos v}\right] d \rho \\
Z_{L}(r)=Z_{L_{1}} \pm \int_{r_{M_{W}}=1}^{r}\left[\frac{\sin \left(\beta_{1}+v\right)}{\tan \left(\mu^{\prime \prime}\right.} \pm \varepsilon_{W_{1}}\right) \cos v
\end{array}\right] d_{\rho} \quad \begin{aligned}
& \varepsilon_{W_{1}}=\sin ^{-1}\left(\frac{V_{r_{1}}}{W_{1}}\right)
\end{aligned}
$$

The relation between $\mu^{\prime \prime}$ and $\mu$ is given by the formula

$$
\begin{equation*}
\tan \mu^{\prime \prime}=\frac{ \pm \sin \varepsilon_{w_{1}} \cos \varepsilon_{w_{1}} \tan ^{2} \nu+\sqrt{\tan ^{2} \mu\left(1+\sin ^{2} \varepsilon_{w_{1}} \tan ^{2} v\right)-\cos ^{2} \varepsilon_{w_{1}} \tan ^{2} v}}{1+\sin ^{2} \varepsilon_{w_{1}} \tan ^{2} v} \tag{4}
\end{equation*}
$$

In the above relations the (+) sign applies for backward, the (-) sign for forward sweep.

The formulae define a sonic leading edge, i.e., leading edge points lying on the Mach cones of the adjacent points. A subsonic leading edge is simply obtained by using in the formulae $\mu$ values corresponding to relative Mach numbers increased by a factor $f=1 / M_{1_{L}}$, i.e. $M_{w_{1}}^{1}=M_{w_{1}} / M_{w_{1}}$ where
$M_{w}$ is the subsonic Mach number of the relative velocity
${ }^{1} \mathrm{~L}$
component perpendicular to the leading edge. This simple relationship is illustrated in Fig. 18.

The second design parameter used to minimize the bending stresses was the sweep reversal radius. By proper selection of the point of sweep reversal, the center of gravity of the blade can be located in such as way as to project radially on, or near, the axis of maximum inertia of the hub section. From a structural viewpoint, the compound sweep blade of Fig. 6 could be considered as a blade with hub and tip sections designed and stacked according to conventional practice and fitted with an additional front section to materialize the subsonic leading edge configuration. The above CG stacking condition then could be fulfilled by similarly fitting a rear section to restore the symmetry of the mass distribution with respect to the axis of maximum inertia of the profiles. This, however, would maximize the additional blade mass and the elongation of the profile chord lengths required by the compound sweep design, which is structurally and aerodynamically undesirable. Proper selection of the radius of sweep reversal thus is necessary to ensure minimum blade stress and aerodynamic performance penalties. Adjustments of the profile chord lengths can be used only to compensate for a slightly off-optimum location of the sweep reversal point. Accordingly, the optimum stacking should yield hub stresses exceeding those of a conventional blade only by the contribution due to the blade mass added to incorporate the subsonic leading edge configuration.

From the preliminary design iterations, the meridional projection of the subsonic leading edge line and its sweep reversal point were known with sufficient accuracy to define the radial distribution of the relative Mach numbers $M_{w_{1}}(r)$ and the relative flow angles $\beta_{1}(r)$ and $\varepsilon_{w_{1}}(r)$ at the leading edge for final design. Those data were interpolated on the streamlines between stations 9, 10, 11, 12 of the R-l2l flow calculation. (For the axial station nomenclature, refer to Fig. 14 and Appendix A.) Table 3 presents the interpolated inlet Mach numbers $M_{W_{1}}$, together with the selected Mach factors $f$ and the corresponding Mach numbers $M_{W_{1}}$ of the relative velocity component perpendicular to the leading edge and $M_{w_{1}}$ of the relative velocities, introduced in Eqs. 1, 2 and 4.


FIG. 18. SONIC AND SUBSONIC LEADING EDGES.

TABLE 3. Interpolated Aerodynamic Data for Final Subsonic Leading Edge Design

| Leading Edge Radius (mm) | Relative Mach $\mathrm{Nr} . \mathrm{M}_{\mathrm{w}_{1}}$ | Mach Factor f | $\mathrm{M}_{\mathrm{w}_{1}}^{\prime}=\mathrm{f} \mathrm{M}_{\mathrm{w}_{1}}$ | ${ }^{M}{ }^{1}{ }_{1}$ |
| :---: | :---: | :---: | :---: | :---: |
| 110 | . 829 | 1.206 | 1.000 | . 829 |
| 116 | . 859 | 1.170 | 1.005 | . 855 |
| 122 | . 888 | 1.140 | 1.012 | . 877 |
| 129 | . 924 | 1.118 | 1.033 | . 894 |
| 136 | . 959 | 1.107 | 1.062 | . 903 |
| 143 | . 994 | 1.102 | 1.095 | . 908 |
| 150 | 1.028 | 1.100 | 1.131 | . 909 |
| 160 | 1.078 |  | 1.186 |  |
| 170 | 1.127 |  | 1.240 |  |
| 180 | 1.185 |  | 1.304 |  |
| 190 | 1.242 |  | 1.366 |  |
| 200 | 1.302 |  | 1.432 |  |
| 210 | 1.363 |  | 1.499 |  |
| 220 | 1.422 |  | 1. 564 |  |
| 230 | 1.481 |  | 1.629 |  |
| 240 | 1.539 |  | 1.693 |  |
| 249 | 1.588 | 1.100 | 1.747 | . 909 |

It will be seen that forward sweep starts immediately at the hub by setting $M_{W_{1}}^{\prime}=1$, i.e., by requiring that $M_{w_{1}} \equiv M_{w}=$ .829 at the hub section. The selected values of $M_{W_{1}}$ increase gradually to . 91 at approximately $1 / 3$ of the span in accordance with the decreasing thickness and camber of the profiles, and then remain constant up to the tip section.

The cylindrical coordinates in the lateral sweep angle $v$ of the subsonic leading edge line are listed in the outlined columns of Table 4, which reproduces the input/output data of the computerized calculation.

### 5.4 Rotor Blade Profile Definition and Stacking Procedure

The optimum profile stacking configuration can be described as follows: At every blade section along the span, the CG of the upper blade portion projects radially on or near the axis of minimum inertia of that section. This means that the radial projections of the individual CG's of the upper profiles must straddle the axis of minimum inertia of the lower section (subsequently referred to as i-straddling). This is achieved by iterative selection of the lateral sweep angle $v$ along the span. During that iteration, the radial location of the point of sweep reversal initially selected is kept unchanged. When adequate i-straddling is obtained for all blade sections, the CG straddling with respect to the axis of maximum inertia (I-straddling) of the hub section is checked and the radial location of the point of sweep reversal modified accordingly.

The first preliminary design investigations were carried out with double circular arc profiles. In the course of the profile stacking iterations, it appeared that using airfoil sections with CG's shifting progressively backward in the lower span portion with forward leading edge sweep, and forward in the upper portion with backward sweep, i.e., a blade configuration with minimum chordwise excursion of the profile CG's, could substantially contribute to minimize bending stresses.

A simple analytical blade thickness distribution was used to simplify the design iterations involving changes in section properties to help relieve stresses. The thickness distribution is written in the following parametric form

$$
\begin{equation*}
t(x)=k x^{n}(c-x) \tag{5}
\end{equation*}
$$

TABLE 4．CYLINDRICAL COORDINATES AND LATERAL SWEEP ANGLE OF THE SUBSONIC

| chers <br> dyntratno | －pantils | qelative <br> velartive <br> （n／SFCl | qurist vflarity （x／spre） | $\begin{aligned} & \text { R「LATIVE } \\ & \text { MACH } \\ & \text { Vn. } \end{aligned}$ | angle petalil （0Erio．） | $\begin{gathered} \text { ANGLF } \\ \text { NUI } \\ \text { INEGR . }) \end{gathered}$ | $\begin{aligned} & \text { ANGLE } \\ & \text { FPSILON(W) } \\ & \text { (DFGR } 1 \end{aligned}$ | $\begin{aligned} & \text { MNGLE } \\ & \text { MU } \\ & \text { IDEGR. } \end{aligned}$ | ancle MIJ（N－R） （OEGR．） | Function THFTA（R） | $\begin{gathered} \text { FUNCTION } \\ \text { ZIR } \end{gathered}$ | THETA | （METERS） |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1．-1 | n．119nn | 274．07 | 49.30 | 1.0930 | 3x．ace | 15．003 | 14.586 | 93．93？ | 97.790 | 1.51448 | 3.21170 | 3．0 | 0.0 |
| ？．－1 | ヘ．11497 | ？ 84.10 | 34.70 | $1.075 n$ | 36．ron | 15.200 | 13.024 | 84.283 | 84.261 | 1.90150 | 0.27434 | －0．01037 | －0．00148 |
| 3．－1 | 9．1 | 394．0．9 | 55.50 | 1．ci2r | 35.150 | 1．5．0\％9 | 11.676 | 81.168 | 81.129. | 2.03814 | 3.29791 | －n．02220 | －0．00320 |
| 4．-1 |  | 295．5， | 53.59 | $1.133 n$ | 24.193 | 14．3\％ | 10.386 | 75.479 | 75.395 | 2.44191 | 0.35480 | －0．03776 | －0．00547 |
| c．－1 | 0．13477 | 2！ 7.5 ？ | 49.0 \％ | 1．c62n | 33.050 | 13.000 | 8.695 | 70.326 | 70．2n？ | 2.84295 | 0.40108 | －0．C5629 | －0．00813 |
| $\therefore-1$ | $\therefore 1430$ | $3 ? 7$ | $47.9 n$ | 1．－959 | $32 . r 5 n$ | 5． 700 | 7.379 | 65.957 | 65.855 | 3.24653 | 0.41436 | －0．07758 | $-0.01100$ |
| 7．－1 | 2．15nnn | 347.98 | 36.55 | 1.1310 | 31．roo | $5.9 n 0$ | 6.163 | 62.150 | 62.114 | 3.65857 | C． 39872 | －0．10177 | －0．01386 |
| 8．－1 | n．1400n | 256.0 | ？9．50 | 1．196r | 29．550 | 0.693 | 4.753 | 57.476 | 57.476 | 4.11398 | 0.38233 | －0．14279 | －0．01775 |
|  | ¢ ¢ $\underbrace{\text { c }}$ | Q F | Q ¢ 1 L |  |  | －2．n7？ |  |  | 53.740 | 4.40314 | 0.36870 |  |  |
| Q． 1 | 7．17กח | 372． 7 n | 73.71 | 1.2401 | 28．1 10 | 9.300 | 3.535 | 53.751 | 53.596 | 3.06939 | 0.39463 | －0．18346 | －0．02150 |
| 1r． 1 | $\checkmark .180 .1$ | 301.54 | 1R．5n | 1.3740 | 27．20rs | 9． 3 ．n | 2.708 | 50.174 | 49.899 | 3.5943 | 0.44492 | －0． 15055 | －3．01730 |
| 11. | －．190nn | $410.0 n$ | 13.50 | 1.3560 | 26.590 | 7.000 | 1.987 | 47.060 | 46.885 | 3.87483 | 0.48729 | $-0.11363$ | －0．01263 |
| 12.1 | フ．？กกาา | 4.29 .50 | 7.50 | 1.4327 | 25． 900 | 6.000 | 1.701 | 44.293 | 44.131 | 4.24864 | 0.52891 | －0．07302 | －0．00755 |
| 12．！ | －．71ral | 449.5 | 9.59 | 1.4997 | 25.350 | 5.000 | 2.764 | 41.845 | 41.708 | 4.61988 | 0.56784 | －0．02868 | －0．00206 |
| 14．${ }^{1}$ | $\bigcirc . ? 3 n ¢$ | 467．5n | －7．5n | 1． 5647 | 24.850 | 4.200 | －0．919 | 39.746 | 39.633 | 4.98041 | 0.60361 | 0.01932 | 0.00379 |
| 15． 1 | －．23nn | 498．～？ | －16．59 | 1.6299 | 23．988 | 4.307 | －1．946 | 37.870 | 37.752 | 5.33953 | 3.65024 | 2.07390 | 0.0 .1005 |
| 16． 1 | －． 24 กnา | 505．5n | －27．5n | 1.0 .93 .3 | 23.100 | 4.000 | －3．119 | 36.204 | 36.070 | 5． 73638 | 0.70451 | 0.12626 | 0.01682 |
| 17． 1 | n． 249 mm | 591．5n | －37．rr | 1.7477 | 22.200 | 4.900 | －4．069 | 34．918 | $34.77 n$ | 6.08336 | 0.74535 | 0.17946 | 0.02335 |

where $c$ is the chord length and $n$, a shape parameter. By adding a leading and trailing edge thickness,

$$
t_{L E} \equiv t_{T E}=\tau V C
$$

where $\tau$ is the LE and TE thickness factor and $\nu=t_{\max } / C$ the relative blade thickness, a practical blade thickness distribution is obtained. The abcissa for maximum thickness is given by

$$
\begin{equation*}
x_{t_{\max }}=\frac{n c}{n+1} \tag{6}
\end{equation*}
$$

the factor $k$ by

$$
k=\frac{v(1-\tau)}{\left(\frac{n c}{n+1}\right)^{n}\left(\frac{1}{n+1}\right)}
$$

The complete non-dimensionalized formula is

$$
\begin{equation*}
\frac{t}{c}=\tau \nu+\frac{v(1-\tau)}{\left(\frac{n}{n+1}\right)^{n}\left(\frac{1}{n+1}\right)}\left(\frac{x}{c}\right)^{n}\left(1-\frac{x}{c}\right) \tag{7}
\end{equation*}
$$

For $n=1,(x / c)_{t_{\max }}=.5$. Furthermore, the second derivative is constant, so that the resulting profile is essentially a double circular arc profile for small thickness.

For $n>1,(x / c)_{t_{\max }}>1 / 2$ and the profile CG shifts toward the trailing edge. Since the first and second derivatives of the thickness distribution are continuous, the profile curvature is continuous.

Using profiles with circular mean camber lines and $n$ varying from 1 to 1.8 from the hub to the point of sweep reversal, and back to 1 at the tip section, a favorable blade configuration was obtained. However, manufacturing difficulties and the extreme sensitivity to tolerance and foreign object damage of thin profiles with $n>1.5$ lead to the selection of $n=1, i . e$. , essentially double circular arc profiles for final rotor blade design.

The blade cascade geometry was defined by means of conventional procedures and criteria. Figure 19 shows representative streamline velocity triangles, together with the corresponding relative flow deceleration rates $W_{2} / W_{1}$ and static pressure ratios $P_{2} / P_{1}$, the selected cascade solidities $\sigma=c / s$ and the resulting $D$-factor values. The hub and tip cascade solidities are equivalent to those which would have been selected for a conventional design with identical rotor inlet and exit flow conditions. The $30 \%$ streamline velocity triangles are representative of the conditions at the sweep reversal section ( $\mathrm{r}=170 \mathrm{~mm}$ ).

The flow deviation angles $\delta$ at rotor exit were calculated with Carter's empirical formula (Ref. 13)

$$
\begin{equation*}
\delta=m \phi / \sqrt{\sigma} \tag{8}
\end{equation*}
$$

with $m=0.23+0.05 \beta_{2} \quad$ (circular mean camber line). For small camber angles $\phi$, ax Eq. (8) gives unacceptably low deviation angles, especially in transonic cascades with shock-boundary layer flow interaction. A minimum deviation angle of $2^{\circ}$ was arbitrarily assumed and the calculated $\delta$ - values were faired gradually to the minimum value toward the tip section. The actual profiles were defined on coaxial cylinders for the most part of the blade. Three profiles were defined on cones in the hub region to ensure a smooth evolution of the profile geometry toward the conical hub section. Fig. 20 shows the relative inlet and exit angles $\beta_{1}, \beta_{2}$ with the tangential direction and the deviation angles ${ }^{1}{ }^{2}$ used to define the cascade geometry. All profiles were set at a nominal incidence $i=+2^{\circ}$ with respect to the suction surface. The selected profile sections are indicated on Fig. 21. Table 5 lists the profile design data defining the cylindrical and conical sections unwrapped on planes tangent to the cylinders and cones. (While all angles are conserved in the development of cylindrical sections, the profile camber angle is reduced in the developed conical sections by the sector angle formed by the radii passing through the leading and trailing edge points.)

The coordinates of the center of gravity of a cylindrical section are determined by the following simple relations:


FIG. 19. ROTOR VELOCITY TRIANGLES (28 blades).


FIG. 20. RELATIVE ROTOR FLOW AND DEVIATION ANGLES.


TABLE 5. Rotor Blade Profile Data 28 Blades (Developed Cylindrical and Conical Sections).

| Section Radius (mm) | Mean Camber Angle $\Phi\left(^{\circ}\right)$ | Setting Angle $\gamma \quad{ }^{(0)}$ | Chord Length c (mm) | Relative Thickness $\mathscr{(} \%)$ |
| :---: | :---: | :---: | :---: | :---: |
| 110/134 | 28.85 | 62.44* | 64.55 | 10.77 |
| 122/140 | 24.70 | 56.45* | 67.91 | 9.73 |
| 136/145 | 24.95 | 50.93* | 74.73 | 8.03 |
| 150 | 26.10 | 46.05 | 85.20 | 6.18 |
| 160 | 20.90 | 41.95 | 95.20 | 4.95 |
| 170 | 17.00 | 38.60 | 105.90 | 4.00 |
| 180 | 13.70 | 36.05 | 102.30 |  |
| 190 | 10.90 | 33.95 | 97.70 |  |
| 200 | 8.70 | 32.25 | 92.90 |  |
| 210 | 6.90 | 30.75 | 88.20 |  |
| 220 | 5.60 | 29.40 | 83.80 |  |
| 230 | 4.70 | 28.25 | 80.20 |  |
| 240 | 3.90 | 27.05 | 77.20 |  |
| 249 | 3.30 | 25.85 | 75.00 | 4.00 |

*Angle between chord and tangent to the developed section circle at the trailing edge

$$
\begin{equation*}
\theta_{c g}=\theta_{L}+\frac{.5 c \cos \gamma+d \sin \gamma}{r} \tag{9}
\end{equation*}
$$

and

$$
\begin{equation*}
Z_{c g}=Z_{L}+.5 c \sin \gamma-d \cos \gamma \tag{10}
\end{equation*}
$$

where $c$ is the chord length, and $d$ is the distance of the $C G$ to the profile chord in the developed section.

Figure 22 shows the situation for a developed conical section. From the aerodynamic design, the geometric characteristics of the profile, especially the inlet and exit angles $\beta_{1}$ and $\beta_{2}$ between the tangent to the mean camber line at LE
$g \quad g$
and $T E$ and the circumferential direction, are known. Also known are the inlet and exit radii $r_{1}$ and $r_{2}$ and the meridional projection $c_{m}$ of the chord. Hence, from similar triangles in the meridional plane:

$$
R_{I}=\frac{c_{m} r_{1}}{r_{2}^{-r_{1}}} \quad \text { Further, } \quad R_{2}=R_{1}+c_{m}
$$

and with $m=R_{1}$ sin $\psi$ and $\delta R=R_{1}(1-\cos \psi)$

$$
\begin{equation*}
c^{2}=\left(c_{m}+\delta R\right)^{2}+m^{2}=c_{m}^{2}+2 R_{1} R_{2}(1-\cos \psi) \tag{11}
\end{equation*}
$$

In the developed section, the camber angle is

$$
\begin{equation*}
\Phi=\beta_{2_{g}}-\beta_{1_{g}}-\psi \tag{12}
\end{equation*}
$$

and the setting angle is defined by

$$
\begin{equation*}
\sin \gamma=\left(c_{m}+\delta R\right) / c \tag{13}
\end{equation*}
$$

Assuming a circular mean camber line in the developed section,

$$
\begin{equation*}
\beta_{2_{g}}=\gamma+\Phi / 2 \tag{14}
\end{equation*}
$$



FIG. 22. DEFINITION OF CONICAL BLADE SECTIONS.

Equations (11)-(14) determine the four quantities $c, \psi, \Phi$ and $\gamma$. They must be solved by successive iterations. Assuming tentatively $\psi$, equation (ll) gives $c$, equation (12) gives $\Phi$, equation (13) gives $\gamma$, while $\psi$ is iterated until equation (14) is satisfied.

After a profile is superimposed upon the circular mean camber line, CG distance $d$ is known and the coordinates of the center of gravity are determined as follows:

$$
\ell^{2}=\frac{c^{2}}{4}+d^{2}(\text { symmetrical profile }), \varepsilon=\sin ^{-1}(d / \ell) \alpha=90+\gamma-\psi-\varepsilon
$$

Hence, $R_{c g}^{2}=R_{2}^{2}+\ell^{2}-2 R_{2} \ell \cos \alpha$ and from triangle 0-LE-CG:

$$
\sin \psi_{c g}=\ell \sin \alpha / R_{c g}
$$

Finally, $\quad r_{c g}=\frac{r_{1}}{R_{1}} R_{c g}$ and the cylindrical coordinates of the center of gravity are

$$
\begin{align*}
& \theta c g=\theta_{L}+\frac{R_{c g} \psi_{c g}}{r_{c g}}  \tag{15}\\
& z_{c g}=Z_{L}+\left(R_{c g}-R_{1}\right) \cos \lambda \tag{16}
\end{align*}
$$

All CG stacking investigations, including preliminary bending stress evaluations, were carried out manually. However, as will be discussed later, verification of stress levels was carried out using computer programs at BBN and AVCO Lycoming. Figure 23 shows the final stacking of the profile CG's radially projected on the conical hub section, which was investigated by NASTRAN analysis. The corresponding distribution of the lateral sweep angle $v$ is shown on Table 4. The NASTRAN results indicated that the stress distribution at the hub section could be improved by a slight tangential shift of the first two conical sections in the rotation sense. $\Delta \theta_{\text {I }}$ - shifts of -. 008 for the hub and -. 004 for the next section were effected without readjusting the $z$ - coordinates of the leading edge points. Those shifts are indicated on Fig. 23. Provision has been made in the 1 stradding to generate a moment that continuously compensates the moment of the aerodynamic forces, (which are reflected in results hereafter).


FIG. 23. CONICAL HUB SECTION DEVELOPED ONTO PLANE TANGENT TO CONE, WITH SUPERIMPOSED RADIAL PROJECTION OF PROFILE CG's.

The optimum radial distribution of the lateral sweep angle $\nu$ is different for forward and backward leading edge sweep directions. Consequently, a discontinuity of lateral sweep may occur at the sections above and below the point of sweep reversal. This, in turn, results in a high rate of curvature of the blade surface. Since the blade is defined by discrete sections, this appears only as a more or less pronounced concentration of the spanwise curvature of the blade surface in the sweep reversal region. Nevertheless, this local curvature increase generated prohibitive stresses near the trailing edge in several preliminarily generated configurations.

This problem was compounded by the additional bending moment around the I-axis of the section of sweep reversal, due to the rearward location of the CG of the upper blade portion with backward leading edge sweep. The difficulty increases since the sweep reversal was selected initially so as to minimize that moment and it was gradually moved inward from $r_{s r}=188$ to 170 mm , still leaving the blade CG in a forward position with respect to the I-axis of the hub section. The stress concentration problem at the sweep reversal section was solved by means of an elaborate compromise of the profile stacking through that section, involving especially the selection of the critical lateral sweep angle discontinuity. For the final configuration, with $\mathrm{r}_{\mathrm{sr}}=170 \mathrm{~mm}$, this was achieved at a late design state only, the iast optimization step, which would have required the sweep reversal point to be set at 160 mm radius, or the profile chord lengths to be increased in the upper blade section. With the present stacking, the highest stress is $645 \mathrm{~N} / \mathrm{mm}^{2}(93.5 \mathrm{ksi})$, which is adequate for concept demonstration purposes. Figure 21 shows the developed sweep reversal section, together with the radial projections of the profile CG's of the upper blade portion and the leading and trailing edge lines. The upper profile CG's have been stacked to compensate for the aerodynamic moment and to minimize the additional TE tensile stress resulting from the rearward CG position of the upper blade portion.

Whereas the radial projection of the leading edge points indicates a smooth subsonic leading edge line, the trailing edge line does not appear to be as smooth as desirable. For manufacturing the blade was defined by flat sections generated from the blade configuration defined in the cylindrical
coordinates used for the stacking investigations. Any minor irregularities of the trailing edge were smoothed out by a slight increasing of the chord lengths of a few local sections. All profile data are listed in Table 5.

### 5.5 A Review of the Rotor Blade Design Iterations for Stress Optimization

The main objective of the preliminary design effort (see Fig. 24) was to define a stacking configuration that maintains the subsonic leading edge concept, i.e., satisfies the acoustic rotor design requirements with as low a blade stress level as possible. A target design goal of $725 \mathrm{~N} / \mathrm{mm}^{2}$ (105 ksi) maximum steady state stress was sought for the design speed of $18,450 \mathrm{rpm}$. For the selected titanium blade, such a stress level is considered adequate for the demonstration purposes of this program.

As a first step in each iteration, both manual and computerized beam-type stress computations were carried out to develop a feel for the iterative stacking procedure and to ensure numerical agreement. The standard AVCO Lycoming blade stress computer program which was used treats the blade as a twisted, rotating cantilevered beam with variable section properties, and takes into account the shroud and aerodynamic forces and the centrifugal restoring moments. All trial blade stacking iterations were analyzed with this program.

Simultaneously, a quick, inexpensive and efficient finite element analysis was used at BBN to verify the results of blade iterations. The program, based on SAP, was operated in conjunction with a blade geometry generator which was based on the family of blade profile shapes, described previously by Eqs. 5-7, which reduce to a minimum the number of parameters required to specify a blade shape; namely, the leading and trailing edge coordinates, the section setting angle and camber, and the profile shape parameters. The program was therefore very well suited for iterative design studies. The purpose of the simultaneous effort was to provide further verification of the beam and manual analysis and to help identify stress concentration, which are neglected in the beam-type stress analysis program and in the manual calculations. These efforts were deemed necessary because the blade configuration differs radically from more conventional designs, and it was uncertain whether conventional design methods would be sufficiently accurate.


FIG. 24. OPTIMIZATION PROGRESS.

A NASTRAN stress analysis program was used by AVCO Lycoming on design iterations which were considered particularly important, and for the final stress computations verification.

The evolution of the maximum blade stress levels as the blade design evolved through the series of trial designs is shown in Fig. 24. The results of the first design substantiated the impractical stress level of a blade with simple forward leading edge sweep. The initial sweep reversal radius (SR) was selected at 195.2 mm . The stacking for trial design 2 was such that the center of gravity of each of the 13 cylindrical blade sections used to define the blade projected radially down onto the axis of minimum inertia (i-axis) of the airfoil section immediately below. For Iteration 3, all section CG's were projected onto the i-axis of the hub section. For Iteration 5, all section CG's above the sweep reversal section were projected onto the SR section i-axis, while the stacking of Iteration 3 was kept for the lower blade sections. As can be seen, the resulting misalignment of the upper blade portion with respect to the hub section produced higher hub stresses. However, this design also showed the lowest stress level for the upper blade portion.

For Iteration 6, the sweep reversal radius was lowered and the misalignment was corrected by introducing a discontinuity of the lateral sweep angle, (i.e., the angle between the sweep direction and the radial plane containing the relative inlet velocity), at the point of sweep reversal. By varying this parameter, a number of stacking combinations involving individual compromises within the upper and lower blade sections, were investigated. Iteration 7 shows the best result obtained with this stacking concept.

With the stress level still substantially beyond the preliminary design goal of 105 ksi , a detailed investigation of the stress pattern in design 7 was performed using the NASTRAN stress program. The excellent correlation which was obtained substantiated the beam-theory analysis method as a useful approach to analyze blade stacking changes.

Subsequent iterations were conducted with the optimum stacking concept described in Sec. 5.4. This stacking satisfies the condition that, at every section along the span, the CG of the entire blade portion above the section projects radially onto the i-axis of the section. As shown by Iteration 8, this reduced the maximum stress level very nearly to the preliminary design target value.

The new stacking concept confirmed the necessity of a lateral sweep angle discontinuity at the point of sweep reversal to achieve proper stacking of the profile CG's across that section. This discontinuity, however, resulted in a rapid change of the spanwise curvature of the blade surface in the trailing edge region, which in turn results in a local stress concentration that was not shown by the simplified analysis. Iterations 1-8 were conducted with double circular arc profiles (DCA). Iterations 9 and 10 used new profiles featuring rearward CG shifts from the hub to the section of sweep reversal, and forward CG shifts from that section to the tip (see previous section). In this way, the CG excursions from a radial line were minimized within the leading and trailing edge envelope and the stresses were reduced to the target level.

Figures 25 and 26 show the moments about the axes of minimum inertia and maximum stress distributions for Design 10 as calculated by the standard blade stress program. The influence of aerodynamic loads and centrifugal restoring moments are also shown. (Design 10 was chosen for further study since this is the design which first indicated stresses below the design goal.)

A detailed investigation of Design 10 was also performed with the NASTRAN program. The results showed local high stresses of 96 ksi at the trailing edge of the sweep reversal section and 110 ksi at the leading edge of the hub section. By slightly increasing the chord length of the sweep reversal section, and slight re-alignment of the conical hub, these stresses were brought down to 84 and 96 ksi , respectively. The NASTRAN finite element representation of this configuration, called Design 10A, is shown in Fig. 27. The stress distributions of the suction and pressure surfaces are shown in Fig. 28.

During the entire iteration process, it was apparent that the radial location of the point of sweep reversal would have to be moved substantially inward from its initially assumed location in order to avoid a large moment about the I-axis of the hub section. Moving the point of sweep reversal inboard, however, increases the bending moment about the I-axis of the sweep reversal section, thereby increasing the tensile stress at the trailing edge of that section. To minimize the local trailing edge stress concentration the radial location of the sweep reversal section was moved inboard cautiously. Even so, the blade CG remained ahead of the hub section I-axis, and resulted in an additional bending stress (on the order of 120 $\mathrm{N} / \mathrm{mm}^{2}$ ) at the hub section leading edge.


FIG. 25. SECTION MOMENT DISTRIBUTION [Preliminary Design 10].


FIG. 26. MAX STRESS DISTRIBUTION [Preliminary Design 10].


FIG. 27. NASTRAN ANALYSIS [Preliminary Design 10A].


FIG. 28. EXAMPLE OF INTERIM RESULTS OF NASTRAN STRESS ANALYSIS MAX SHEAR CRITERION [Preliminary Design 10A].

Prior to the selection and analysis of the final blade design, several intermediate designs were investigated based on local shifts of the CG location within the indificual airfoil profiles. (Noted as Designs 11 through 16 in Fig. 24.) The polynomial blade sections had been evolving toward $n=1$ or a DCA profile. For manufacturing reasons, however, double circular arc profiles were specified for the final design. This raised the stresses to virtually the level of iteration 9, and additional stacking iterations were required to achieve the design objective. In particular, the relative blade thickness was increased from 10.00 to $10.77 \%$ at the hub section. This resulted in a $10 \%$ decrease in the stress level. Additional reductions were achieved through a judicious balancing of the profile stacking in the lower blade portion and lateral sweep angle discontinuity at the sweep reversal section.

A check was performed to see if the DCA profiles allowed adequate flow area margins. On an average basis, the rotor throat passage area has a large margin to sonic throat area because of the comparatively high mean relative inlet Mach number level $\overline{\mathrm{M}}_{\mathrm{W}_{1}}=1.33$ and the positive inlet incidence of $2^{\circ}$ selected for optimum blading efficiency. The throat hub region is most susceptible to local throat choking because of the transonic inlet flow conditions and the higher relative blade thickness. Because of unknown 3-dimensional flow effects, it is difficult to determine local blade stream tube areas and no definite section throat area margins thus were specified for the design. A check, however, was tentatively made for the rotor hub section. On the two-dimensional basis of the developed section of Fig. 23 the ratio of throat to inlet passage width is 1.045. At the throat location, however, the channel height has decreased from 139 to 136.3 mm . Assuming that all individual stream tube heights are reduced in the same proportion, the effective geometric throat/inlet area ratio thus is $A_{m i n} / A_{\text {in }}=$ $1.045 \times 136.3 / 139=1.027$. With a relative inlet Mach number of .825 , the sonic area ratio $A_{i n} / A_{S}$ is 1.0285 , thus $A_{m i n} / A_{S}=$ $1.027 \times 1.0285=1.055,1 . e ., a 5.5 \%$ choke area margin.

In the hub region, the flow has the tendency to be deflected inwards because of the forward leading edge sweep. On the other hand, the increasing density toward the tip at rotor exit combines with the essentially constant axial velocity of free-vortex flow to shift the streamline pattern outwards at rotor exit. Those compensating effects cannot be quantified at the throat location and the comparatively large $5.5 \%$ margin thus was judged adequate to account for the possibility of unfavorable threedimensional effects and for the suction side boundary layer
growth upstream of the throat in the absence of a detached leading edge shock. In summary, in spite of the selection of DCA profiles, the individual rotor section throat margins are adequate.

### 5.6 Final Rotor Blade Stress Analysis

The stress analysis for the final design iteration was performed using NASTRAN. The loads considered in this run were based on the maximum operating speed of $18,450 \mathrm{rpm}$. In addition to the major contribution of the centrifugal load, aerodynamic gas pressure loads, the centrifugal load and the torsional restraints of the part span shroud were applied to the blade. The resulting von Mises effective stress patterns over the pressure and suction surfaces of the blade are shown in Figs. 29a and 29b. An independent verification of these results was performed using the SAP program at BBN.

The maximum stress level of $645 \mathrm{~N} / \mathrm{mm}^{2}(93.5 \mathrm{ksi})$ is at the root near the leading edge on the suction surface. The high stress region of 90 ksi , however, extends only over a small portion of the suction surface (Fig. 29b) and so should not pose a problem for the planned test program. The permissible number of start/maximum speed/stop cycles is approximately 500 , considering a notch condition ( $S C F=3.5$ ) at the juncture of the blade airfoil and the base shroud.

The tendency of the blade to untwist at the shroud location is small since there is only $1 / 2$ degree difference in untwist between the shrouded and unshrouded NASTRAN results. The most significant load on the shroud, therefore, is the bending load due to the centilevered mass. The maximum shroud stress of 78.7 ksi is at the blade-shroud juncture, and is conservative in that the large fairing radius at the juncture was not included in the calculation. Because of the constraining effect of the mid-span shroud, the untwist of the blade at the shroud location is negligibly small. The untwist of the tip section calculated from the NASTRAN results is $.36^{\circ}$, thus increasing the tip incidence from 2 to $2.4^{\circ}$ at the design speed, a value well within the blade incidence design tolerance. However, radial growth of the shroud has not been accounted for and, if such growth occurs, undesirable increased tip incidence angle could result, due to the consequences of shroud sections "unlocking".


FIG. 29a.
Pressure surface


FIG. 29b.
Suction surface

FIG. 29 NASTRAN STRESS ANALYSIS: FINAL ROTOR BLADE DESIGN

The magnitude of the stresses in the fan blade airfoil are acceptable for an experimental program. The computed stresses in this design exceed AVCO Lycoming practice for titanium blades for longtime service operation, but fall within acceptable limits for the planned experimental program.

### 5.7 Rotor Blade Vibration and Flutter

The avoidance of large amplitudes of resonant vibration of the rotor blades over the full operating range is necessary to ensure the structural integrity of the fan. The design procedure included an assessment of the natural frequencies of the rotor blade so that the forced vibration response is minimized, and the self-excited response is eliminated. The design goal for the minimization of forced vibration is ensuring that the rotor blade cannot resonate with the first three rotational orders of excitation due to possible inlet distortions. Although higher excitation orders will exist in the intake, it is considered that these levels will be minimal in the clean inflow expected in the acoustic test facilities and, thus, they will not generate significant resonant stress levels in the blade. The avoidance of self-excited blade vibration flutter is mandatory, since the associated stress levels usually lead to blade failure in a very short time. The two flutter phenomena that were considered in the design are subsonic positive stalled flutter at part-speed operation and supersonic unstalled flutter at design speed. The criteria for avoidance of these flutter conditions are based on extensive experience by the engine manufacturers and are expressed in terms of a reduced velocity parameter: $u / b \omega$, where $u=$ air velocity over the blade ( $\mathrm{m} / \mathrm{sec}$ ), $b=$ blade semichord ( m ), and $\omega=$ frequency of vibration in the flutter mode (rads/sec). The empirical design limit values for this parameter under positive stalled flow are 6.7 and 2.4 for the first bending and first torsion modes, respectively. The supersonic unstalled flutter design limit at first torsion frequency was:

$$
\frac{u}{b w}\left(\frac{M^{2}-1}{M}\right)<1.05 \text {, where } M=\text { Mach number. }
$$

The coefficients are calculated at $3 / 4$ span. (Since supersonic unstalled flutter usually occurs in vibration modes which are predominantly torsional, only this mode is considered.)

A free-standing blade, assumed fixed at the base, was used in the calculation of the resonant frequencies. The natural frequencies for the unshrouded blade are shown in the excitation diagram of Fig. 30. This design is clearly unsatisfactory since the natural frequency of the first bending mode has a second order resonance in the operating speed range. The stall flutter coefficients are 3.44 and 1.53 for bending and torsion, respectively. These values are within the safe limits which were established as design criteria. The supersonic unstalled flutter parameter is 1.16 and exceeds the safe upper limit.

A partspan shroud is required to raise both the first bending and torsion natural frequencies and avoid forced and self-excited vibrations (flutter). As a physical model, the shroud was assumed to restrict the blade motion to a unfform translation at three representative points.

The design analysis was checked by mounting two spare blades in a fixture which clamped at the root and partspan shroud locations. An acoustically coupled exciter was used to vibrate the blade so that the frequencies and mode shapes could be obtained. The comparison between the measured and theoretical static frequencies shown in Fig. 31, is considered good, especially in view of the unusual blade shape. The "measured" frequency line in Fig. 31 is actually the theoretical centrifugal stiffening line originating at the measured static frequencies of the first three modes.

Figure 31 shows the excitation diagram and calculated and measured frequencies for the final airfoil with the partspan shroud located at $64 \%$ of the span ( 201 mm radius). The first bending natural frequency has been raised so that it clears the first three excitation orders in the operating speed range. The fourth excitation order of the first mode, (e.g., four equally spaced front struts) however should be avoided. Based on the measured frequencies, the stalled flutter coefficients are 1.5 and 1.0 for bending and torsion, respectively. The supersonic unstalled flutter coefficient is .75. These values meet the design criteria. The excitation diagram shows that the torsion and bending modes are not coincident in the operating speed range. This ensures that the modes are decoupled.

Strain gauges will be used during the test program to ensure that safe steady and vibrating stress levels are not exceeded. In order to locate the strain gauges appropriately, a vibratory stress survey was conducted using strain gauges during the static vibration tests: Fig. 32 shows the results of this test, normalized for each mode. The vibratory stress distributions, shown as


FIG. 30 RESONANCE DIAGRAM OF FINAL BLADE BEFORE SHROUD WAS ADDED.

PARTSPAN SHROUD LOCATED AT 64\% SPAN ( $\mathrm{r}=201 \mathrm{~mm}$ )


FIG. 31 RESONANCE DIAGRAM OF FINAL (SHROUDED) BLADE.


FIG. 32. MEASURED AND NORMALIZED STRESS DISTRIBUTIONS DURING STATIC VIBRATION TESTS ON BLADE S/N 17.
the combined steady and alternating stresses in the blade, are plotted for each mode in conjunction with calculated steady stresses at each gauge location used. Figure 33 shows the Goodman diagram for the blade material and the vibrating stresses measured in each mode proportioned for the most critical location. From this diagram it is seen that location ' $F$ ' is the most critical location in terms of combined stress in the first mode of vibration. Location ' C' is seen to be the most critical for the second and third modes of vibration. It is therefore recommended that strain gauges at positions ' $C$ ' and ' $F$ ' are used to monitor the steady and vibrating stresses during the rig running.

### 5.8 Attachment and Disk Analysis

The fan disk stresses were computed by a Lycoming finite element program which evaluates the loading variation throughout the disk accounting for the effects of rotation, temperature gradients and elastic-plastic conditions.

Low cycle fatigue (LCF) Iife was evaluated for the significant regions, i.e., the disk serrations, the bolt holes and the disk bore, utilizing statistical minimum fatigue property data for Timkin 17-22AS material. The stress/strain ranges utilized in the life evaluation are the stabilized values corresponding to start/stop excursions to $18,450 \mathrm{rpm}$ design speed.

Stress concentration factors (SCF) were evaluated for those areas of the disk containing a high stress gradient, i.e., the serrations and bolt holes. This was accomplished by ratioing the peak stresses determined by finite element analyses with the nominal stresses in each of the two regions.

Nominal radial and tangential stress distributions for the fan disk are shown in Fig. 34, while the nominal stresses in the serration are shown in Fig. 35. The finite element models for the bolt holes and serrations are handled separately.

Stress distributions about the disk bolt-holes are given in Fig. 36 from which an SCF of 2.06 was calculated, so that the resulting LCF life is in excess of 100,000 "start/stop" cycles based on the material $\mathrm{S}-\mathrm{N}$ data of Fig . 37. It has been concluded that the disk bore also has a calculated life of at least 100,000 cycles.


FIG. 34. DISK STRESSES.


| Part | Location | Type of Stress＊ | Sさrローふ <br> Level <br> （ksi） | Yieid strength |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\begin{aligned} & \text { Disk } \\ & \text { (ksi) } \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Blade } \\ & \text { (ksi) } \end{aligned}$ |
| Blade | $A-A$ | Tensile | 49.20 | － | 100，000 |
|  | C－C | Shear | 21.00 | － | 50，000 |
| Disk | B－B | Tensile | 53.07 | 115，000 | － |
|  | D－D | Shear | 17.36 | 57，500 | － |
|  | E－E | Bearing | 61.03 | － | － |

＊Includes C．F．and Bending Effect
Blade Material：ritanium 6AL－4V Disk Material：17－22AS

Temperature： $200^{\circ} \mathrm{F}$ Speed： $18,450 \mathrm{rpm}$

FIG．35．DISK／BLADE ATTACHMENT STRESSES．


FIG. 36. DISK FINITE ELEMENT STRESS ANALYSIS.

fig. 37. Minimum disk low cycle fatigue life.

Blade root attachment stresses and corresponding material properties are also summarized in Fig. 35. The bending effects in both the root and tenon have been included. The axial width of the base-shrouded dovetail root was determined by a permissible bearing stress of $420 \mathrm{~N} / \mathrm{mm}^{2}(61.0 \mathrm{ksi})$. This is less than the compression yield limit, yet somewhat beyond the level at which fretting can occur under prolonged operation, but which should be satisfactory for a limited experimental program.

From a disk serration finite element analysis, the SCF was calculated to be 3.33 which is consistent with values measured from photo-elastic analyses of similar blade root configurations. The corresponding LCF life is 24,000 "start/stop" cycles based on the appropriate curve of Fig. 37. These fatigue lives are ample for the anticipated program of testing.

## DETAILED STATOR DESIGN

This section describes the detailed design of the fan stator which embodies the stator noise reduction concept described earlier. The stator uses vanes with varying sweepback angle to meet the criterion of a constant subsonic rotor wake trace speed along the stator vane span. The use of circumferential vane skew (lean) was avoided primarily to simplify the manufacturing problem. The stator vane number was chosen to cut off the radiation from residual sources due to end effects in the hub region of the vanes at blade passage frequency. The corresponding residual sources at the tip cannot be cut off because the spinning speed of wake disturbance pattern is supersonic at the tip.

To determine the proper vane sweep angle distribution, the rotor wakes were assumed to be convected with the mean flow. The spatial location of the wake centerline surfaces could then be computed from the mean flow properties by integration downstream from initial points on the rotor trailing edge. Since the rotor wake pattern spins fixed with respect to the rotor, it is possible to find leading edge lines whose shape is such that their point of intersection with the rotor wake centerlines travels at constant speed. Moving medium effects were taken into account in the actual calculation of a vane leading edge shape (see Appendix C for details). The trace speed was made constant and subsonic relative to the local flow velocity vector at all points on the vane span. The stator vane sweep distribution was designed to have an effective spanwise trace speed corresponding to a Mach number of 0.8 for the traveling load distribution.

The fundamental acoustical analysis which underlies the stator design concept is presented in Appendix C. In the remainder of this section, the methods for determination of the vane leading edge shape, and vane number are described, and the aerodynamic design considerations for the stator are reviewed.

### 6.1 Acoustic Aspects of Stator Design

The major noise producing mechanism of the stator is the interaction between the stator blades and the wakes shed by the rotor. This interaction causes fluctuating lift at the stator blades; the fluctuating lift in turn can be a potential source of noise. The fluctuating lift is restricted essentially near the stator blade leading edges (SBLE); this fact is made
abundantly clear from the analytical work of Filotas (Ref. lo). It is also well known (e.g., Lighthill, Ref. 15) that any fluctuating lift, whether at the leading or trailing edge, whether acoustically compact or not, whether in a stationary or moving acoustic medium, acts as a dipole source of sound.

However, irrespective of the nature of the sources (i.e., whether monopole, dipole or quadrupole, etc.), there are certain aspects of acoustics of stationary and uniformly moving media which need to be considered before approaching the specific task of stator design and related acoustic problems. Discussion of these fundamental aspects is provided in Appendices C.1 and C.2, and their application to the stator design of this fan is described below.

### 6.1.1 Criteria for non-radiation

Acoustic wavelengths at rotor blade passage frequency are small compared to the stator blade span. In this case, the criterion for non-radiation due to unsteady forces is that the trace phase velocity of the force disturbance be subsonic relative to the local gas flow. Skewing, or sweeping of the stator blade, increasing the separation between rotor and stator, and shaping the rotor blade are techniques which can be used to reduce the phase trace speed.

Proper modification of leading edge profiles can reduce the phase trace speed along the leading edge and also the relative angle between that velocity and the local flow. Both effects are important as it is the trace velocity relative to the local gas-properties flow which must be kept subsonic.

Each individual wake shed by a rotor blade suffers a lag in the circumferential direction. The net effect of this lag on the nature of impingement of the wake on an unswept SBLE is that the wake hits the SBLE at the hub first and the impingement process propagates radially outwards towards the SBLE tip with a spanwise varying phase or trace velocity $c_{o}(r)$. Sweeping back the SBLE enhances this phase lag effect, in the sense that the spanwise trace velocity of wake impingement is reduced. A criterion along the lines of Eqs. (C.62) and (C.63) is used to
guarantee that the wake trace Mach number $m_{o}$ is less than $m_{u}$ everywhere along the swept back SBLE. The nature of the wake phase lag and the calculation of the SBLE sweep angle is described more fully below. The successful analysis of a rotor wake tracing along the leading edge of a stator requires understanding of a set of transformations between stator-fixed coordinates and moving medium coordinates. The derivation of the trace velocity in stator-fixed coordinates, and subsequent Gallilean transformations to gas-fixed coordinates is given in Appendix E.

### 6.1.2 Estimate of rotor viscous wake

Estimates of the magnitude of the rotor viscous wake at the leading edge of the stator have been made. The method of estimation involved modeling the rotor blade wakes as the wakes behind isolated airfoils. The method is somewhat crude, as it ignores the interference between wakes and the axial pressure gradient. The variation of angle of attack at the stator which results from the estimated velocity fluctuations is as much as lo degrees from the mean. Experience with axial flow turbomachinery wakes indicates that the estimated rotor wake amplitudes at the stator leading edge are likely to predict higher resultant angle of attack fluctuations than will exist in the actual rotor wake. This is due to the higher rate of decay of rotor blade viscous wakes in turbomachines when compared to isolated viscous wakes in free flow (see, for example, Lakshminarayana and Raj; Ref. 16).

### 6.1.3 Computation of rotor wake distortion

Contours of constant phase for rotor wakes at different axial locations were computed by use of a stepwise integration of the phase lag of the wake relative to a point in the rotor, as a function of radius. Cylindrical helical flow was assumed (radial flow velocity was assumed to be zero). Axial and tangential velocities used in this calculation were provided by AVCO's aerodynamic design program (Appendix A). Contours of constant phase calculated at several stations downstream of the rotor are shown in Fig. 38 (see Fig. 14 and Appendix A for Station Locations). Contours of constant phase versus axial location on cylindrical surfaces, shown in Fig. 39 and contours of constant phase in the axial/radial plane, shown in Fig. 40 , were derived by cross-plotting from Fig. 38.



FIG. 39. PHASE vs AXIAL LOCATION ON CYLINDRICAL SURFACES.


FIG 40. CONTOURS OF CONSTANT PHASE IN Y-Z PLANE.

Figure 41 illustrates, in stationary coordinates, the flow and blade motion geometry and the equations used in the calculation of the constant phase contours. The two terms $\theta(\alpha)$ and $\theta(\omega)$ in the calculations represent the angular translation of a fluid particle and the angular rotation of the rotor, respectively, in the time required for the fluid particle to flow from axial Station 2 to Station 2. The trace velocity, relative to local flow, for a number of constant-sweep-angle stators is shown in Fig. 42 The very high trace velocities near the tips result from reduced wake "windup" in that region, thus illustrating the limited effectiveness of constant angle swept stators.

### 6.1.4 Mach . 78 leading edge stator

A blade leading edge sweep profile for trace speeds less than Mach 0.8 was developed for the final rotor and flow path design using an iterative method to achieve a nearly uniform trace velocity. The blade has a minimum sweep angle of 25 degrees at approximately $1 / 3$ of the span from the root. Sweep at the root and tip are 30 and 40 degrees, respectively. Figure 43 shows the sweep profile as well as the trace and acoustic speeds as a function of radius.

The calculations assume a rotor blade reference axis at the axial location $12-1 / 2 \mathrm{~mm}$. forward of the root at Station 13 . It also assumes a stator leading edge which is radial, when prajected in the $r, \theta$ plane, and has its root 12.5 mm . downstream of Station 15.

The inflow-induced radiation from an array of such variablyswept stator vanes is now restricted to the tip regions when discontinuities occur. The limitation of such radiation depends upon proper choice of the number of swept leading edge stators, which in turn depends upon the rotor blade number, rotation speed, and moving medium acoustical considerations. The computation of vane numbers is discussed below.




### 6.2 Analysis For Determination Of Number Of Stator Blades

This section determines the appropriate minimum number $V$ of stator blades so that the acoustic noise radiated from the stator at the rotor blade passage frequency $f_{r}$ is minimized.

Since the swept back SBLE derived in the above section is of finite extent, the end effects at the SBLE hub and tip from the wake/SBLE interactions remain as potential sources of noise. The aim here is to seek a partial circumferential cancellation of these end sources. Thus, two discrete circumferential arrays exist, one at the SBLE hub and the other at the SBLE tips. Since the circumferential phase velocity $c$ is higher at the tip than at the hub, one concentrates on the discrete circumferential array made up of uncancelled sources at the SBLE tips. Also, as discussed in Appendix $C$, the discussion of a discrete array must be limited to only one frequency $\omega_{0}$. Choosing $\omega$ to correspond to the fundamental rotor harmonic, (i.e., to o the rotor blade passage frequency $f_{r}$ ), one obtains

$$
\begin{equation*}
\omega_{0}=\Omega \mathrm{B}, \tag{17}
\end{equation*}
$$

where $\Omega$ is the shaft rotation in radian $/ \mathrm{sec}(\Omega \approx 1940 \mathrm{rad} / \mathrm{sec})$ and $B$ is the number of rotor blades $(B=28)$. The blade passage frequency $f_{r}$ is given by

$$
\begin{equation*}
f_{r}=\frac{\omega_{0}}{2 \pi} \approx 8600 \mathrm{~Hz} \tag{18}
\end{equation*}
$$

For the circumferential phase velocity $c_{o}$

$$
\begin{equation*}
c_{0}=\Omega r_{t} \tag{19}
\end{equation*}
$$

where $r_{t}$ is the radius at the stator $\operatorname{tip}\left(r_{t} \approx 0.24 \mathrm{~m} \approx 0.79 \mathrm{ft}\right)$. The corresponding Mach number $m_{0}$ is therefore given by

$$
\begin{equation*}
m_{0}=\frac{c_{0}}{c} \approx 1.27 \tag{20}
\end{equation*}
$$

With reference to results of Appendix C.2, (Eq. C.60); $m_{1}$ is the Mach number of the gas flow parallel to the array, and $m_{r}$ is the gas Mach number normal to the array. Since the array under consideration is oriented circumferentially, the gas Mach number $m$ in the circumferential direction plays the role of $m_{1}$ and the gas Mach number $m_{\text {a }}$ in the axial direction plays the role of $m_{r}$; the radial gas ${ }^{\text {Mach }}$ number, normal to the duct walls, is to a good approximation zero. Thus, we have

$$
\begin{align*}
& m_{1} \equiv m_{c} \approx 0.353  \tag{2I}\\
& m_{r} \equiv m_{a} \approx 0.582 \tag{22}
\end{align*}
$$

Note that $m_{c}$ is directed the same way as the shaft rotation $\Omega$ or the phase Mach number $m_{0}$. Hence, first one would like to find from Eqs. C. 62 and C. 63 whether $m_{0}<m_{u}$ - one of the two necessary conditions for no radiation to occur. Substituting the quoted values in Eq. C.62, one finds that

$$
\begin{equation*}
m_{0}<m_{u}, \text { for } \frac{\pi}{2}<\alpha \leq \frac{\pi}{4} \text {, } \tag{23}
\end{equation*}
$$

in other words, the condition for no radiation is satisfled for angles $\alpha$ that are sufficiently remote from the axial direction.

In order to satisfy the second condition for (partial) cancellation of radiation from the fundamental rotor harmonic, it is required that

$$
\begin{equation*}
\frac{2 \pi}{d_{t}}>2 \frac{2 \pi}{\lambda_{r}} \frac{\left(1-m_{a}^{2} \cos ^{2} \alpha\right)^{1 / 2}}{\left(1-m_{c}^{2}-m_{a}^{2} \cos ^{2} \alpha\right)} \tag{24}
\end{equation*}
$$

where $d_{t}$ is the circumferential spacing between two adjacent SBLE tips The spacing, $d_{t}$, is related to the number of stator blades .by the relation

$$
\begin{equation*}
d_{t}=\frac{2 \pi r_{t}}{v} \tag{25}
\end{equation*}
$$

The right hand side of $\mathrm{Eq} .(24)$ is the radiation span ( $k_{a_{0+}}-k_{a_{0-}}$ )
obtained from Eq. C. 60 , where $2 \pi / \lambda$ has been substituted for $k_{a}$, $\lambda_{r}$ being the acoustic wavelength at frequency $f_{r}$. $a_{0}$ Taking sound speed $c$ in the gas to be about $365 \mathrm{~m} / \mathrm{s}$ ( $1200 \mathrm{ft} / \mathrm{sec}$ ), the wavelength at blade passage frequency is

$$
\begin{equation*}
\lambda_{\mathrm{r}}=\frac{\mathrm{c}}{\mathrm{f}_{\mathrm{r}}} \approx 0.14 \mathrm{ft} \approx 0.043 \mathrm{~m} \tag{26}
\end{equation*}
$$

Thus, substituting Eq. 25 in Eq. 24 , the velocity is

$$
\begin{equation*}
V>\frac{4 \pi r_{t}}{\lambda_{r}} \frac{\left(1-m_{a}^{2} \cos ^{2} \alpha\right)^{1 / 2}}{\left(1-m_{c}^{2}-m_{a}^{2} \cos ^{2} \alpha\right)} \tag{27}
\end{equation*}
$$

Since the first necessary condition (Eq. 23 ) is satisfied only for a restricted range of angles $\alpha$, it would not pay to find the maximum possible value of $V$ for arbitrary $\alpha$. Instead, Eq. 27 is evaluated for $\alpha=\pi / 4$ (as $\alpha$ goes from $\pi / 2$ to $\pi / 4$ to $0, V$ evaluated from Eq. 27 increases), and the result is

$$
\begin{equation*}
v \geq 92 \tag{28}
\end{equation*}
$$

One can now examine the application of traditional analyses (e.g., Tyler and Sofrin (Ref. ll)) of noise generated by rotorstator interaction, the anlysis that is used primarily for lowspeed compressors (i.e., analysis is based on stationary medium acoustics) that involve subsonic circumferential phase speeds (i.e., $\Omega r_{t}<c$ ) and are acoustically compact (i.e., $d_{t}<\lambda_{r} / 2$ ).

An arbitrary component (say, the predominant component of wake velocity deficit pattern that generates fluctuating lift at SBLE) $a(x, r, \theta, t)$ of rotor-generated flow field near the stator may be decomposed into circumferential harmonics as follows

$$
\begin{equation*}
a(x, r, \theta, t)=\sum_{n=-\infty}^{+\infty} A_{n}(x, r) e^{1[n B(\theta-\Omega t)]}, \tag{29}
\end{equation*}
$$

where $x$ and $r$ are the axial and radial locations (and for our case of interest denote the locations of SBLE tips) and $\theta$ is the circumferential angle.

The noise sources (in particular, the fluctuating lift $\ell$ generated at SBLE tips) at the stator due to the $n t h$ rotor harmonic may then be viewed as composed of a sum of stator/rotor harmonics mn. A typical interaction harmonic $I_{m n}(s, r, \theta, t)$ may be written as

$$
\begin{equation*}
L_{m n}(x, r, \theta, t)=L_{m n}(x, r) e^{i(m \theta-n B \Omega t)} \tag{30}
\end{equation*}
$$

where

$$
\begin{equation*}
m=n B+k V, \tag{31}
\end{equation*}
$$

and where $k$ can assume arbitrary integral values (positive, negative or zero).

The circumferential phase velocity $c_{0}(r)_{m n}$ associated with Eqs. 30 and 31 can be written as

$$
\begin{equation*}
c_{0}(r)_{m n}=\frac{n B \Omega r}{n B+k V}, \tag{32}
\end{equation*}
$$

Similarly, from Eq. 19, the circumferential phase velocity for all the rotor harmonics $n$ is $\Omega r$ and assumes the value $\Omega r_{t}$ at the SBLE tips. This same value is recovered for the interaction modes from Eq. 32, for the stator fundamental mode, i.e., for the case $k=0$.

Figure 44. depicts the situation in terms of these rotorstator interaction harmonics. The harmonics lie at the intersections of vertical lines passing through the $n B$ axis for $n=0, \pm 1, \pm 2 \ldots$ and horizontal lines passing through the $m$ axis for $k=0, \pm 1, \pm 2 \ldots$ The rotor fundamental tone occurring at the blade passage frequency $f_{r}$ (see Eqs. 17 and 18.) corresponds to vertical straight lines passing through $n= \pm 1$ (i.e., $n B= \pm 28$ ). Similarly, $n t h$ rotor harmonic corresponds to frequency $\mathrm{nf}_{\mathrm{r}}$. The fact that attention was turned to the stator fundamental harmonic at frequency $f_{r}$ (see Eqs. 19 and 20 ) means that the $k=0$ stator mode was examined at $f_{r}$. From Eq. 23 , one finds that this stator fundamental harmonic barely escapes radiation. From Eq. 32 , it can be seen that the same situation applies to stator fundamental harmonics (i.e., $k=0$ modes) for all rotor harmonics (i.e., arbitrary $n$ ). Thus, the straight line in Fig. 44 passing through these $k=0$ modes separates the radiating and non-radiating harmonics.

The criterion of Eq. (24) was applied to prevent the next candidate stator harmonics ( $k= \pm 1$ modes for $n= \pm 1$ ) from radiating at the blade passage frequency $f r(o n z y)$. The straight line joining these $k=-1$, +1 moders thus also separates the radiating and non-radiating harmonics. The flow-induced assymmetry in radiation span along wavenumber is reflected in Fig. 44 by assymmetry of radiating and non-radiating harmonics around $m$ and $n B$ axes. Incidentally, stator harmonics lying in the upper right and lower left quadrants of the $m, n B$ plane possess circumferential velocities that are oriented in the same direction as shaft rotation $\Omega$, and the harmonics lying in the upper lef't and lower right quadrants possess velocities that are oriented in the direction opposed to $\Omega$.

Finally, note that the relatively high number $V$ of stator blades indicated by Eq. 28 may cause design problems of aerodynamic nature. For example, relatively high solidity at the hub, particularly for the scale model fan, is unacepceptable. Therefore, a compromise number of 59 was selected for $V$. Such a choice ensures circumferential cancellation at the SBLE hub, but not at the tip. In other words, with reference to $\operatorname{Fig} .44, k= \pm 1$ modes would radiate from the SBLE tips.


FIG. 44. SKETCH OF RADIATING AND NON-RADIATING ROTOR/STATOR INTERACTION OF HARMONICS ( $n B, m$ ).

### 6.3 Stator Aerodynamic Design Considerations

The stator is characterized by a backward leading edge sweep varying from 25 to $40^{\circ}$ in the meridional plane. The axial spacing between rotor trailing edge (TE) and stator leading edge (LE) varies approximately from two to three rotor hub chords along the span.

A minimum number of 59 blades was specified by acoustic considerations. This, in conjunction with a tentatively selected hub cascade solidity $\sigma_{\text {hub }}=2$, resulted in a chord length of approximately 30 mm .

The meridional contour of the stator was shown in Fig. 14. Radial station 17 crosses the leading edge, 19 the trailing edge, and 18 crosses both the leading and trailing edges. Radial equilibrium along those stations is markedly influenced by the varying degree of stator turning, resulting in peculiar tangential velocity distributions that have been input in R-121, together with the corresponding total pressure loss distributions. The flow conditions from rotor exit station 13 to stator inlet stations 16, 17, 18, and 19, have been calculated according to constant rotor exit momentum $V_{u} \cdot r$ specified along the streamlines.

The meridional flow pattern (Fig. 14) shows the radial streamline shifts induced by the swept back stator configuration, especially in the hub region, where $V_{u} \cdot r$ is large and has a strong effect on radial equilibrium. In the axisymmetric flow case, the streamlines approaching the leading edge are deflected inboard. Looking at the lower portion of station 17, the flow at the hub section has already undergone the major part of its turning, and $V_{u} \cdot r$ thus increases markedly from the hub to the $30 \%$ streamline on that station. This substantial departure from free-vortex flows generates an increase of the axial velocity component toward the hub and a corresponding increase of the mass flow density $\rho V_{x}$, in turn resulting in inboard streamline shifts between leading edge and station 17 . This characteristic pattern is found along the entire span, but disappears gradually toward the tip section because of the decreasing value of the $V_{u}^{2} / r$-term.

The meridional streamline curvature term $V_{m}^{2} / R_{c}$ has a strong effect in this flow field region. The determination of $R_{c}$ however is very approximative even with the spline-on spline procedure
used in R-121, so that the interpolated values of the flow conditions at the stator leading edge cannot be expected to be smooth. Fig. 45 shows the radial distribution of the inlet angles $\alpha_{3}$ determined by $V_{x}$-interpolation and constant $V_{u} \cdot r$ along the streamlines, and the smooth distribution assumed for blading design. The maximum smoothing error does not exceed $1-1.5^{\circ}$, which is well within the accuracy that can be expected from the axisymmetric analysis.

Figure 45 also shows the stator exit flow deviation angles $\delta$ calculated with Carter's formula [Equation (8), circular mean camber lines]. The blade sections are stacked with the leading edge in a meridional plane. All profiles were set at $0^{\circ}$ nominal incidence. Table 6 lists the profile data defining plane sections perpendicular to the radius in the leading edge plane.


FIG. 45. STATOR INLET FLOW AND DEVIATION ANGLES.
$\forall 1 \forall 0$ ヨO甘าタ yOL甘1S •9 ヨาタ甘1

| 59 Blades |  |  |  |  |  |  |  | （Plane Sections） |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Section Radius（mm） | 147 | 154 | 160 | 171 | 191 | 210 | 228 | 236 | 242 | 248 |
| Mean Camber Angle $\Phi\left({ }^{\circ}\right)$ | 47.80 | 44.90 | 43.00 | 40.80 | 38.90 | 38.40 | 40.00 | 43.1 | 46.50 | 50.20 |
| Setting Angle $\gamma\left({ }^{\circ}\right.$ ） | 73.90 | 75.05 | 75.80 | 76.70 | 77.55 | 77.90 | 77.50 | 76.55 | 75.45 | 74.30 |
| Chord Length c（mm） | 31.3 | 31.3 | 31.5 | 31.85 | 33.05 | 34.80 | 36.90 | 37.9 | 38.75 | 39.60 |
| Diffusion Factor D | ． 396 |  |  |  |  |  |  |  |  | ． 288 |
| Rel．Thickness v（\％） | 6.0 | － | － | － | － | － | － | － | － | 6.0 |
| Axial Coordinate of L．E． $\mathrm{z}_{\mathrm{L}}(\mathrm{mm}) *$ | 0 | 4.1 | 7.8 | 14.0 | 25.1 | 35.0 | 47.4 | 53.7 | 58.7 | 64.0 |

＊Leading Edge swept back in a meridional plane．

## SECTION 7

## COMMENTS ON RESIDUAL NOISE SOURCES AND NOISE LEVELS OF THE SWEPT ROTOR AND STATOR FAN STAGE

The object of the Low Source Noise Fan Program is to design fan components for minimal noise generation. This has been done by first using physical models for each of the component noise mechanisms, and calculating the appropriate parameters from the particular baseline fan design, then modifying the component geometry to minimize noise generation. All sources of noise cannot be eliminated, and indeed all sources have not been attacked in this study.

### 7.1 Residual Sources for a Fan outside the Laboratory Environment.

As has been previously discussed in detail, the compound sweep required on the rotor blades for structural reasons will lead to a conical shock at the sweep reversal point. However, in some future fan designs, the location of the sweep reversal point at a radius less than that at which the critical relative Mach number occurs will eliminate the source of noise. Rotor discrete frequency mechanisms which cannot be eliminated include the so-called Gutin noise sources associated with steady loads and thickness noise. However, the non-radial blading may cause these mechanisms to excite high order duct modes and thus reduce the radiation to the far field. Rotor broadband mechanisms are relatively poorly understood quantitatively (in the absence of inflow turbulence), and thus are difficult to attack at the source. Shock/turbulence interaction in the channel may cause some forward radiated noise, and quite likely causes aft-radiated broadband noise.

Stator noise mechanisms are much better understood and can be attacked with much more confidence than some rotor mechanisms. The uncancelled tip radiation (calculated in Appendix $C$ ) is the only discrete-frequency mechanism inherently associated with the subsonic trace speed swept stator concept, assuming that the rotor wake field can be accurately specified. Stator broadband mechanisms not attacked by the swept leading edge include vortex shedding and flow separation at the trailing edge.

Other broadband noise from an installed fan comes from the exhaust jet and duct boundary layer turbulence interaction with the lip of the fan duct.

### 7.2 Prediction of Noise Levels and Noise Reduction of the Swept Rotor and Stator Fan.

Despite intensive research efforts in the past twenty years which have.led to a good understanding of noise mechanisms and scaling laws, the ability to predict fan noise for an arbitrary design on a component-by-component basis is quite limited. For conventional fans, useful semi-empirical correlations of data have been made using scaling laws which are based on assumed mechanisms. Thus, for conventional fans, one can predict within a few dB the expected sound power and directivity. However, the applicability of those correlations to a fan of unconventional component design is doubtful.

For the subject fan design, the prediction of residual noise from the rotor requires the knowledge of the strength of the conical shock upstream of the rotor, which is not presently known due to the cessation of activity on the 3-D compressible flow program. The stator discrete noise has been calculated directly for basic principles and is presented in Appendix C.

However, the main noise source of interest, rotor multiple pure tones cannot be reliably estimated without detailed information on shock structure and duct propagation characteristics. In the interest of providing an estimate of the benefits of eliminating MPT noise, a computer program published by Burdsall et al., (Ref. 20) was exercised (see Appendix D for details). The results summarized below for a full scale (a $40,000 \mathrm{lb}$ thrust) counterpart of the 20 inch fan built in this program, show that elimination of the shock-generated MPT's reduces the overall and perceived noise levels by $4-6 \mathrm{~dB}$, and reduces the tonal content in the $1 / 3$ octave band containing the blade passage frequency by about 10 dB .

TABLE 7. ORDER-OF-MAGNITUDE EMPIRICAL ESTIMATE OF NOISE LEVELS FROM FULL SCALE SINGLE STAGE FAN.

| Spectrum Component | Overall <br> PWL (dB re $10^{-12} w$ ) | $\begin{gathered} 0 A S P L *\left(0150^{\prime}\right) \\ \mathrm{dB}\left(\mathrm{re} 2 \times 10^{-5} \mathrm{~N} / \mathrm{m}^{2}\right) \end{gathered}$ | PNL* (@150.') |
| :---: | :---: | :---: | :---: |
| M.P.T. (conventional blades) | 152-154 | 104-106 | 117-119 |
| B.P. Tone | 143 | 95 | 108 |
| Broadband Mechanisms | 150 | 103 | 114 |
| TOTAL |  |  |  |
| with MPT's | 153.5-155.2 | 105.3-107.2 | 118-120 |
| without MPT's | 150 | 103 | 114 |

[^0]
## SECTION 8

## MECHANICAL DESIGN ASPECTS AND FACILITY INTEGRATION

The fan rig is built to conventional standards and is designed to interface with the NASA $W-2$ and $W-8$ test facilities. The $W-2$, acoustic facility is arranged for the measurement of forward and rearward radiated noise; thus, the rig casings have flanges at both ends which mate with the facility mounting flanges. The manner in which this is accomplished is shown schematically in Fig. 12. In the reverse flow mode, for backward radiated noise, an additional flow path adaptor supplied by NASA and not shown in Fig. 12 is fitted to the fan outlet flanges. In the $W-8$ facility the fan is mounted on its rear flange with the flow entering from the bellmouth. All detailed performance measurements of the fan will be made in the $W-8$ facility.

A flow path adaptor fits over the facility bearing housing to control the fan outlet flow and into this adaptor is fastened the inner shroud of the stator vane assembly. The outer shroud of the stator vane assembly is located in the fan casing and provision is made for axial adjustment of the stator by relocating the spacers at the inner and outer shrouds. The fan outer casing is split in the vertical plane for assembly purposes. The section of the outer casing in the area of the blade tips is relieved and an abradable shroud lining is installed to prevent blade tip damage in case of tip rubs. Figure 46 shows the engineering cross-section of the fan which details all the major components. Strain gauges will be applied to the rotor blades, the wires being led down the front and rear faces of the disc. In the W-2 facility the slipring is installed at the driven end of the rig shaft and, thus, the strain gauge wiring will pass down the length of the hollow shaft. When the fan is running in the $W-8$ aerodynamic facility, the strain gauge wiring will be led forward through the driveshaft adaptor which is fitted in place of the spinner support cone. A static fairing is installed over the slipring to provide a smooth flow profile into the fan, in place of the spinner.

FIG. 46. DETAILED CROSS-SECTION OF FAN RIG.

## SECTION 9

## CONCLUDING REMARKS

A research program was undertaken to try to demonstrate that source noise reduction concepts which are based upon full and rigorous application of fundamental aeroacoustic principles can be implemented on turbofans in the currently-operating range of tip speeds and pressure ratios, utilizing the existing design and manufacturing capabilities of the aircraft engine industry, without serious compromise of the noise reduction concept.

The subsonic leading edge rotor blade concept has significant potential as a practical solution to rotor-generated noise due to its inherent lack of sensitivity to off-design-point operating conditions, and the large family of detailed edge and generating surface contours available for fans in various speed ranges. The aerodynamic behavior of subsonic leading edge rotors in supersonic absolute inflow velocities is largely unknown at this point in time. However, it is believed that the characterizations of such flow fields, to the extent necessary in developing actual engines using the subsonic leading edge rotor principle, would at this time require considerably less effort than has been expended historically in understanding aerodynamic behavior of conventional rotors.

The subsonic trace speed stator vane concept can be implemented through application of moving medium acoustic principles and a knowledge of the details of the rotor wake field, the lack of the latter being a current limitation. However, the subsonic trace speed concept can, in principle, be successfully implemented by use of conservative assumptions about the rotor wake field.

## APPENDIX A

## COMPUTER LISTING OF AEROTHERMODYNAMIC PARAMETERS FOR FINAL ROTOR, STATOR \& FLOW PATH DESIGN

## APPENDIX A: DETAILED AEROTHERMODYNAMIC DATA

This appendix contains a computer listing of the aerothermodynamic data for the final design of the fan.

The first four pages, A-6 to A-9 are input data to AVCO Lycoming Program Rl2l at various axial stations. All units in the $S I$ system and headings on the columns are self-explanatory. The three parameters in the left hand column are:

```
TOT PRESS = Total Pressure Ratio
TOT TEMP = Total Temperature Ratio
VU = Absolute Tangential Velocity Component of the Air (m/sec)
```

The remaining pages are detailed output at the various axial stations, the non-obvious terms of which are defined below.

| Coded Term | Meaning | Units |
| :---: | :---: | :---: |
| A STATIC | ambient sound speed | $\mathrm{m} / \mathrm{sec}$ |
| A TOTAL | sound speed based on total temperature |  |
| ALPHA BAR | $\sin ^{-1}(\mathrm{Vm} / \mathrm{V})=$ angle of flow made by $V$ in tangential direction measured on a cone | degrees |
| ALPHA | $\sin ^{-1}(\mathrm{Vx} / \mathrm{V})=$ angle made by projection of absolute velocity vector (on a cylinder) | degrees |
| BETA | angle that the relative velocity vector makes with a cylinder | degrees |
| V | absolute velocity | $\mathrm{m} / \mathrm{s}$ |
| VM | meridional component of V | $\mathrm{m} / \mathrm{s}$ |
| VR | radial component of V | $\mathrm{m} / \mathrm{s}$ |
| S-VALUE | radial length measured along a station cut (origin at hub) | m |
| \% SPAN | percent radial distance compared to full span measured from hub | --- |
| VX | axial component of absolute velocity | $\mathrm{m} / \mathrm{s}$ |
| VU | tangential component of absolute velocity | $\mathrm{m} / \mathrm{s}$ |
| W | relative velocity | $\mathrm{m} / \mathrm{s}$ |
| WU | tangential component of relative velocity | $\mathrm{m} / \mathrm{s}$ |
| MV | Mach number of absolute velocity $V$ | --- |
| MVX | Mach number of VX | --- |
| MVM | Mach number of VM | --- |


| Coded Term | Meaning | Units |
| :---: | :---: | :---: |
| R-ADC | streamline radius of curvature in meridional plane | m |
| RHO | fluid density | $\mathrm{Kg} / \mathrm{m}^{3}$ |
| ROTOR EFF | polytropic rotor efficiency |  |
| S-VALUE | radial length measured along a station cut (origin at hub) | m |
| STAT PRESS | static pressure | bars |
| STAT TEMP | static temperature | - Kelvin |
| $\left.\begin{array}{l} \mathrm{TO} / \mathrm{TO} \\ (\mathrm{TO} / \mathrm{TO}) \mathrm{T} \end{array}\right\}$ | Similar definitions for the stagnation temperatures. |  |
| TOT PRESS | total pressure ratio |  |
| TOT TEMP | total temperature | - Kelvin |
| U | rotational speed | $\mathrm{m} / \mathrm{s}$ |
| V | absolute velocity | $\mathrm{m} / \mathrm{s}$ |
| VM | meridional component of $V$ | $\mathrm{m} / \mathrm{s}$ |
| VR | radial component of $V$ | $\mathrm{m} / \mathrm{s}$ |
| VU | tangential component of absolute velocity | $\mathrm{m} / \mathrm{s}$ |
| VX | axial component of absolute velocity | $\mathrm{m} / \mathrm{s}$ |
| W | relative velocity | $\mathrm{m} / \mathrm{s}$ |
| WU | tangential component of relative velocity | $\mathrm{m} / \mathrm{s}$ |
| X-VALUE | axial location of station re:origin (station 4) | m |

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## APPENDIX B

GEOMETRIC CONSIDERATIONS FOR SUBSONIC LEADING EDGES ON TRANSONIC ROTOR BLADES

## APPENDIX B: GEOMETRIC CONSIDERATIONS FOR SUBSONIC LEADING EdGES ON TRANSONIC ROTOR bLADES

We first note that a simple leading edge configuration is obtained by sweeping each leading edge element dl in the plane formed by the local relative velocity $W_{1}$ and the radius ( $W_{1}-r$ plane). This plane intersects the Mach cone along two generatrices that form the Mach cone angle $\mu$ with $W$. Any other plane through the apex cuts the cone along generatrices forming a smaller angle $\mu "$ with W in the $\mathrm{W}-\mathrm{r}$ projection. Since the radial projection of dl is essentially proportional to sin $\mu "$, it follows that the simple case defined above yields the shortest possible swept blade length for a given annulus height and a given relative velocity distribution $W(r)$. For structural reasons, however, the leading edge must be swept aside from the $W_{1}-r$ plane.

The general situation is shown in Fig. B.l (Refer also to Fig. 17).
la shows the projection on a plane perpendicular to the radius passing through leading edge point $P$. The velocity triangle is projected in that plane for visualization convenience.
lb shows the projection on the $W-r$ plane, with the Mach lines forming the Mach angle $\mu$ with $W$. In general, $W$ forms an angle $\varepsilon_{w_{1}}$ with plane la.
lc shows the projection on a plane perpendicular to W , intersecting the Mach cone along circle c.
ld shows the projection on a meridional plane.


FIG B-1 SONIC SWEPT LEADING EDGE ELEMENT

On la, the leading edge element $d l$ has the projection $d l^{\prime}$ and the radial planes passing through di and $W$ form the angle $v$, which is a design parameter to be selected so as to minimize the blade bending stresses. The resulting lateral sweep component din appears also on projection $1 c$ and causes the Mach angle $\mu$ between $d l$ and $W$ to project into the $W-r$ plane $l b$ with a smaller aperture $\mu^{\prime \prime}$.

From la

$$
\begin{aligned}
& d x_{t}= \pm d l^{\prime} \cos (\beta+v) \\
& d z= \pm d l^{\prime} \sin (\beta+v)
\end{aligned}
$$

In the above and the following relations, the top signs denote backwards, the bottom signs forward sweep.

Since

$$
\begin{align*}
& d I^{\prime}=\frac{d m}{\cos v}=\frac{d \rho}{\tan \left(\mu^{\prime \prime} \pm \varepsilon_{w_{1}}\right) \cdot \cos \nu} \\
& {\left[\frac{d x_{t}}{d z}\right]=\frac{\left[\begin{array}{l}
\cos \\
\sin
\end{array}\right](\beta+\nu)}{\tan \left(\mu^{\prime \prime \pm} \varepsilon_{w_{1}}\right)} \quad \frac{1}{\cos v} d \rho } \tag{B.1}
\end{align*}
$$

From 1 lb and Ic

$$
\begin{equation*}
\tan \mu^{\prime \prime}=\frac{\mathrm{ds} \cos \omega}{\mu}=\tan \mu \cdot \cos \omega \tag{Br}
\end{equation*}
$$

and since

$$
\begin{aligned}
& \sin \omega=\frac{d n}{d s}=\frac{d n}{d m} \frac{d m}{d l^{\prime \prime}} \frac{d l^{\prime \prime}}{d s} \frac{\cos \omega}{\cos \omega}= \\
& =\tan \nu \cdot \cos \left(\mu^{\prime \prime} \pm \varepsilon_{w}\right) \frac{\cos \omega}{\sin \mu^{\prime \prime}}
\end{aligned}
$$

Therefore,

$$
\begin{equation*}
\cos \omega=\frac{1}{\sqrt{1+\frac{\tan ^{2} \nu}{\sin ^{2} \mu^{\prime \prime}} \cos ^{2}\left(\mu^{\prime \prime} \pm \varepsilon_{w}\right)}} \tag{B.3}
\end{equation*}
$$

which is introduced in Eqn. B.2, yielding

$$
\begin{equation*}
\left.\tan ^{2} \mu^{\prime \prime}=\frac{\tan ^{2} \mu}{1+\frac{\tan ^{2} v}{\sin ^{2} \mu^{\prime \prime}}} \cos ^{2}\left(\mu^{\prime \prime} \pm \varepsilon_{i v}\right)\right) \tag{B.4}
\end{equation*}
$$

Developing $\cos \left(\mu^{\prime \prime} \pm \varepsilon_{w}\right)$, Eq. B. 4 is reduced to a quadratic equation for tan $\mu^{\prime \prime}$. The solution is
$\tan \mu^{\prime \prime}=\frac{ \pm \sin \varepsilon_{W} \cos \varepsilon_{W} \tan ^{2} v+\sqrt{\tan ^{2} \mu\left(1+\sin ^{2} \varepsilon_{W} \tan ^{2} v\right)-\cos ^{2} \varepsilon_{W} \tan ^{2} \nu}}{1+\sin ^{2} \varepsilon_{W} \tan ^{2} \nu}$
(Only the (+) sign is valid in front of the radical, since tan $\mu^{\prime \prime}$ must tend toward $\tan \mu$ when $\nu$ tends toward 0 ).

We define the blade profiles and their stacking in cylindrical coordinates. The angular abcissa of leading edge point $P(r)$ then is

$$
\begin{align*}
& \theta_{L}(r)=\theta_{L_{1}}+\int_{r_{M=1}}^{p} \frac{d x_{t}}{\rho}=\theta_{L_{1}} \pm \int_{r_{M=1}}^{p} \frac{\cos (\beta+\nu)}{\tan \left(\mu^{\prime \prime} \pm \varepsilon_{W_{1}}\right)} \frac{1}{\cos v} \frac{d \rho}{\rho}(B .6) \\
& Z_{L}(r)=Z_{L_{1}}^{\prime} \int^{r} \frac{\sin (\beta+v)}{\tan \left(\mu^{\prime \prime} \pm \varepsilon_{W_{1}}\right) \cos v} d \rho  \tag{B.7}\\
& r_{M=1}
\end{align*}
$$

where

$$
\begin{equation*}
\varepsilon_{W_{1}}=\arcsin \frac{V_{r}}{W_{1}} \tag{B.8}
\end{equation*}
$$

Eqs. (B.6) and (B.7), together with (B.5) and (B.6), determine the cylindrical coordinates of the profile leading edges, for a blade with sonic leading edge.

With the section profile data, the stacking of the centers of gravity is determined, and the blade bending stresses can be calculated. However, it is advisable to iterate the leading edge coordinates until a favorable alignment of the CG's is achieved, prior to the calculation of stresses.

By optimum selection of the lateral sweep and of the radial location of the point of sweep reversal, it is expected that the additional stresses affecting the subsonic leading edge blade will be reduced to:
(a) Additional centrifugal stresses from the added blade mass necessary to materialize the subsonic leading edge configuration.
(b) Bending stresses from moments without substantial component in the direction of the axes of minimum inertia.

## APPENDIX C <br> FUNDAMENTAL ACOUSTICAL ASPECTS OF STATOR DESIGN

## APPENDIX C

## FUNDAMENTAL ACOUSTICAL ASPECTS OF STATOR DESIGN

## C. 1 Continuous and Discrete Line Sources in a Stationary Acoustic Medium

Continuous Line Source
Consider a line monopole source of the type

$$
q(x, t)=Q_{0} e^{i\left(k_{0} x-\omega_{0} t\right)},
$$

where $Q_{O}$ is the source strength per unit length. The line source of Eq. C.l represents one wave traveling along the $x$-axis (see Fig. C.l) with a velocity $c_{o}$ given by

$$
\begin{equation*}
c_{0}=\omega_{0} / k_{0} \tag{C.2}
\end{equation*}
$$

One is interested (only) in the far field pressure $p(x, y, z, t)$ radiated by the line source. Define

$$
\begin{equation*}
r=\left(y^{2}+z^{2}\right)^{\frac{1}{2}} \tag{c.3}
\end{equation*}
$$

Consider the case (referred to as Case No. 1) where the source velocity $c_{0}$ is supersonic, i.e.,

$$
\begin{equation*}
\left|c_{0}\right|>c, \tag{c.4}
\end{equation*}
$$

or equivalently,

$$
\begin{equation*}
\left|k_{0}\right|<k_{a_{0}}=\left|\omega_{0}\right| / c \tag{C.5}
\end{equation*}
$$

Here $c$ is the sound speed for the medium and $k_{a}$ is the acoustic wavenumber at frequency $\omega_{0}$. For this case, the ${ }^{\circ}$ far field pressure $p(x, r, t)$ is non-zero; in other words the line source can radiate acoustic power. More specifically,


Fig. C.l. SKETCH OF A Line monopole source.

$$
\begin{equation*}
p(x, r, t)=\text { constant } \times \frac{1}{\left(k_{r} r\right)^{\frac{1}{2}}} e^{i\left(k_{o} x+k_{r} r-\omega_{o} t\right)} \tag{C.6}
\end{equation*}
$$

where the radial wavenumber $k_{r}$ is given by

$$
\begin{equation*}
k_{r}=\left(k_{a_{0}}^{2}-k_{o}^{2}\right)^{\frac{1}{2}} \tag{C.7}
\end{equation*}
$$

Since $k_{0}{ }^{2}<k_{a_{0}}^{2}$ (see Eq. C.5), $k_{r}$ is real and the sound is propagated radially outwards from the $x$-axis.

Now consider the alternate case (Case No. 2) where the source velocity $c_{o}$ is subsonic, i.e.,

$$
\begin{equation*}
\left|c_{o}\right|<c, \tag{C.8}
\end{equation*}
$$

or equivalently,

$$
\begin{equation*}
\left|k_{0}\right|>k_{a_{0}} \tag{C.9}
\end{equation*}
$$

For this case, the far field pressure $p(x, r, t)$ is zero; in other words the line source cannot deliver any acoustic power. More specifically,

$$
\begin{equation*}
p(x, r, t,)=0 \tag{C.10}
\end{equation*}
$$

This happens because the radial wavenumber $k_{r}$ is imaginary,

$$
\begin{equation*}
k_{r}=+i\left(k_{0}^{2}-k_{a_{0}}^{2}\right)^{\frac{1}{2}} \tag{C.11}
\end{equation*}
$$

and the near field pressure decays exponentially in the radial direction.

Let us reconsider the above results in terms of the spatial Fourier transform $\tilde{q}\left(k_{1}, t\right)$ of Eq. (C.l). First, define the general Fourier transforms.

$$
\begin{align*}
& \tilde{q}\left(k_{1}, t\right)=\frac{1}{2 \pi} \int q(x, t) e^{-1 k_{1} x} d x  \tag{C.12}\\
& q(x, t)=\int \tilde{q}\left(k_{1}, t\right) e^{1 k_{1} x} d k_{1} \tag{C.13}
\end{align*}
$$

Unless stated otherwise, the limits of integration are always to be taken from $-\infty$ to $+\infty$. For later use, the temporal Fourier transforms shall also be needed, defined as follows.

$$
\begin{align*}
\tilde{q}\left(k_{1}, \omega\right) & =\frac{1}{2 \pi} \int \tilde{q}\left(k_{1}, t\right) e^{1 \omega t} d t  \tag{c.14}\\
& =\frac{1}{(2 \pi)^{2}} \iint q(x, t) e^{-1\left(k_{1} x-\omega t\right)} d x d t  \tag{C.15}\\
q(x, t) & =\iint \tilde{q}\left(k_{1}, \omega\right) e^{1\left(k_{1} x-\omega t\right)} d k_{1} d \omega  \tag{C.16}\\
& =\int \tilde{q}(x, \omega) e^{1 \omega t} d \omega \tag{C.17}
\end{align*}
$$

Substituting $q(x, t)$ of Eq. (C.I) into Eq. (C.12),

$$
\begin{equation*}
\tilde{q}\left(k_{1}, t\right)=Q_{0} e^{-i \omega_{0} t} \delta\left(k_{1}-k_{0}\right) \tag{C.18}
\end{equation*}
$$

where $\delta$ is the Dirac delta function. Figure C. 2 illustrates $\tilde{q}\left(k_{1}, t\right)$ for Case No. 1 (radiation) and for Case No. 2 (no radiation). The "radiation span" along the wavenumber $k_{1}$ is centered around the wavenumber $k_{1}=0$ and ranges from $-k_{a_{0}}$ to $+\mathrm{k}_{\mathrm{a}}{ }^{\circ}$. This radiation span is shown in Fig. C. 2 as a
$\mathrm{a}_{0}$ shaded strip.
Let us reformulate the condition of no radiation in terms of velocities and Mach numbers. The two extremes $-k_{a_{0}}$ and $+k_{a_{0}}$ lowest and the highest velocities that the source wave can have


FIG. C.2. CASES OF RADIATION (No. 1) AND NO RADIATION (No. 2) ILLUSTRATED IN TERMS OF THE WAVENUMBER $k_{2}$.
for no radiation to occur (Case 2). These extreme velocities and Mach numbers are,

$$
\begin{align*}
& c_{\ell}=\frac{\omega_{0}}{-k_{a_{0}}}=-c \\
& m_{\ell}=\frac{c_{\ell}}{c}=-1  \tag{C.19}\\
& c_{u}=\frac{\omega_{0}}{+k_{a_{0}}}=+c \\
& m_{u}=\frac{c_{u}}{c}=+1 \tag{C.20}
\end{align*}
$$

The source wave Mach number $m_{0}$ is defined as:

$$
\begin{equation*}
m_{0}=\frac{c_{0}}{c} \tag{C.21}
\end{equation*}
$$

Thus, the condition of no radiation (Case 2 ) becomes

$$
\begin{equation*}
m_{\ell}<m_{0}<m_{u} \tag{c.22}
\end{equation*}
$$

Next, consider a spatially frozen but otherwise arbitrary pattern $q(x, t)$ traveling, as before, with fixed velocity $c_{0}$ in the $x$-direction. Thus,

$$
\begin{equation*}
q(x, t)=Q\left(x-c_{0} t\right) \tag{C.23}
\end{equation*}
$$

In contrast to Eq. (C.I), for which there was one wavenumber $\mathrm{k}_{\mathrm{O}}$, one frequency $\omega_{0}$, and one velocity $c_{0}$, for Eq. (C.23) there is a range of wavenumbers $k_{1}$, a corresponding range of frequencies $\omega$, and one velocity $c_{o}$ Using Eq. (C.15), the Fourier transform of $q(x, t)$ of Eq. (C.23) is

$$
\begin{align*}
\tilde{q}\left(k_{1}, \omega\right) & =\frac{1}{(2 \pi)^{2}} \iint Q_{1}\left(x_{0} t\right) e^{-1\left(k_{1} x-\omega t\right)} d x d t  \tag{c.24}\\
& =\frac{1}{2 \pi} \int \tilde{Q}(k) e^{-1 k_{1} c_{0} t+i \omega t} d t  \tag{C.25}\\
& =\tilde{Q}(k) \delta\left(\omega-k_{1} c_{0}\right) \tag{C.26}
\end{align*}
$$

where

$$
\begin{equation*}
\tilde{Q}(k)=\frac{1}{2 \pi} \int Q(x) e^{-i k_{1} x} d x \tag{C.27}
\end{equation*}
$$

Figure C. 3 shows the straight lines along which $\tilde{q}\left(k_{1}, \omega\right)$ of Eq. (C.26) is non-zero. Analogous to Fig. C.2, straight ines corresponding to Case 1 (radiation) and Case 2 (no radiation) are illustrated. Notice that as frequency $\omega$ increases, the radiation span $2 \mathrm{k}_{\mathrm{a}}$ over wavenumber $\mathrm{k}_{1}$ also increases linearly, But, as long as the source velocity magnitude $\left|c_{o}\right|$ is subsonic, there is no radiation to the far field. Incidentially, the upper right and the lower left quadrants of $\omega-k_{1}$ plane correspond to positive phase velocities, (i.e., velocities along increasing $x$ ), whereas the upper left and the lower right quadrants of $\omega-k_{1}$ plane correspond to negative phase velocities.

This completes the discussion of a continuous frozen pattern of line sources in a stationary acoustic medium. Consideration of the fact that the acoustic medium is moving uniformly, will simply alter the radiation span along $k_{1}$, as will be discussed in Sec. C.2. However, a frozen convecting pattern along $x_{1}$, will or will not radiate, by exactly analogous rules as developed here, i.e., in terms of the convection or phase velocity $c_{o}$ of the pattern.

Finally, note that for a continuous convecting line source of finite length, even if the convection velocity $c$ is subsonic, there will be inevitable radiation from the two ends of the line source. For low enough frequencies, the two ends may be less than half an acoustic wavelength apart, in which case there may


FIG: C.3. CASES OF RADIATION AND NO RADIATION FOR A SPATIALLY FROZEN ARBITRARY PATTERA, ILLUSTRATED IN THE $k_{2}, \omega$ PLANE .
be partial cancellation from the two end sources. For higher frequencies, the two end sources will radiate independently. This last remark is discussed more fully below when discrete line sources are considered in a stationary acoustic medium. However, the SBLE is regarded as a continuous line array, and a typical rotor wake impinging on it has a local convection velocity $c_{o}$ along the span of the SBLE. Thus the preceding discussion of continuous line sources, or rather its related extension in Sec. C.2, where account is taken also of the moving-medium acoustics, is applied to determine the SBLE sweep; the criterion that is applied is in terms of the spanwise local velocity of the rotor wake along the SBLE.

Discrete Line Source
Now consider an array of equispaced coherent monopoles, spaced d apart (see Fig. C.4), where:

$$
\begin{equation*}
x_{j}=d_{j} \quad, \quad j=0, \pm 1, \pm 2 \ldots \tag{C.28}
\end{equation*}
$$

In analogy with Eq. (C.I), consider one wavenumber $k_{0}$, one frequency $\omega_{0}$ and the corresponding phase velocity $c_{0}$. Thus, the source number $j$ has the strength $q\left(x_{j}, t\right)$ given by

$$
\begin{equation*}
q\left(x_{j}, t\right)=Q_{0} e^{i\left(k_{o} x_{j}-\omega_{0} t\right)} \delta\left(x-d_{j}\right) \tag{C.29}
\end{equation*}
$$

The entire source strength can be written as

$$
\begin{equation*}
q(x, t)=Q_{0} \sum_{j=-\infty}^{+\infty} e^{i\left(k_{0} x-\omega_{0} t\right)} \delta(x-d j), \tag{c.30}
\end{equation*}
$$

and the phase velocity $c_{0}$, as before, is given by

$$
\begin{equation*}
c_{0}=\omega_{0} / k_{0} . \tag{C.31}
\end{equation*}
$$

Eq. (C.12) is used to find the spatial Fourier transform of $q(x, t)$ of $E q$. (C.30),


FIG. C.4. SKETCH OF AN ARRAY OF POINT SOURCES.

$$
\begin{equation*}
\tilde{q}\left(k_{1}, t\right)=\frac{Q_{0}}{2 \pi} \int e^{-i k_{1} x} e^{i\left(k_{0} x-\omega_{0} t\right)} \sum_{j=-\infty}^{+\infty} \delta(x-d j) d x \tag{C.32}
\end{equation*}
$$

$$
\begin{equation*}
=\frac{Q_{0}}{2 \pi} e^{-i \omega_{o} t} \sum_{j=-\infty}^{+\infty} e^{-i\left(k_{1}-k_{o}\right) d, j} \tag{C.33}
\end{equation*}
$$

$$
\begin{equation*}
=\frac{Q_{0}}{d} e^{-i \omega_{0} t} \sum_{m=-\infty}^{+\infty} \delta\left(k_{o}-k_{1}-\frac{2 \pi m}{d}\right) \tag{c.34}
\end{equation*}
$$

$\tilde{q}\left(k_{1}, t\right)$ thus consists of an infinite string of Dirac delta functions equispaced along the wavenumber $k_{1}$, the spacing between two adjacent delta functions being $2 \pi / d$. It is only the "fundamental mode" or harmonic at $k_{1}=k_{0}$ (for $m=0$ In Eq. C.34) that corresponds to the trace velocity $c_{0}$ of Eq. (C.31). The remaining infinite harmonics correspond to infinite other velocities. The rule of radiation (Case l) or no radiation (Case 2) is exactly the same as the one developed for the continuous array and illustrated in Fig. (C.2). If the fundamental mode or any harmonic(s) lie within the radiation span $\left(-k_{a_{0}}, k_{a_{0}}\right.$ ), radiation will occur from the fundamental mode or from the harmonic(s) lying within the radiation span.

However, a more interesting and new feature of the discrete array is the classification based on a different criterion. That classification is as follows:

$$
\begin{align*}
& \text { Case } A ; \frac{2 \pi}{d}>2 k_{a_{0}}, \text { or } d<\frac{\lambda_{a_{o}}}{2}  \tag{C.35}\\
& \text { Case } B, \frac{2 \pi}{d}<2 k_{a_{0}}, \quad \text { or } d>\frac{\lambda a_{0}}{2} \tag{C.36}
\end{align*}
$$

For Case $A$, the spacing $2 \pi / d$ along wavenumber $k$ between harmonics is greater than the radiation span $2 k_{a_{o}}$, since $k_{a_{o}}=2 \pi / \lambda_{a_{o}}$, where $\lambda_{a_{0}}$ is the acoustic wavelength at frequency $\omega_{o}$, spacing d is smaller than half the acoustic wavelength. For

Case B, the opposite situations occur in the wavenumber and spatial descriptions.

The importance of these two cases is depicted in the next few figures. Figure C. 5 describes Case Al (the numbers 1 and 2 denote the older classification, $l$ corresponds to radiation occurring, 2 corresponds to radiation not occurring). The radiation occurs from the fundamental mode at $k_{1}=k_{0}$, but since $2 \pi / d>2 k_{a_{0}}$, no other harmonic can radiate. Figure C. 6 also describes ${ }^{0}$ Case Al, however, this time a harmonic, and only one harmonic radiates. Figure C. 7 shows the Case A2, a situation one would hope to achieve. The fundamental mode at $k_{1}=k_{o}$ lies just outside the radiation span, and no harmonic lies within the radiation span, hence no radiation occurs. Note that for this desired situation, the constraint of Eq. (C.8) (or equivalently of Eq. C.22) as well as the constraint of Eq. (C.35) applies.

Finally, Fig. C. 8 shows Case B1. There is no Case B2. Radiation must occur through same mode(s), whether the phase velocity $c_{o}$ is subsonic or supersonic. In other words, arranging for Case B, i.e., having array spacing d greater than half the wavelength, is basically a poor design.

Note that in contrast to the continuous line array for which the discussion related to Fig. C. 2 for frequency $\omega_{0}$ could be generalized to discussion related to Fig. C. 3 for all frequencies, the discussion of the discrete array presented above cannot be similarly generalized to all frequencies. This is because the array spacing $d$ is in general fixed, whereas the radiation span $2 k$ increases linearly with increasing $\omega_{o}$. Thus, Case A for ${ }_{0}$ frequency $\omega_{0}$ is bound to merge into Case $B$ at some higher frequency.

Aside from the re-definition in Sec. C. 2 of the radiation span in wavenumber $k_{i}$, induced by consideration of moving-medium acoustics, the above discussion of a discrete array is applied to determine the number of stator blades, the spacing d corresponding to the circumferential spacing between two adjacent stator tips, and frequency $\omega_{0}$ corresponding to the blade passage frequency.


FIG. C.5. CASE AI FOR A DISCRETE ARRAY; RADIATION FROM THE FUNDAMENTAL HARMONIC AT $k_{o}$.


## FIG. C.6. CASE AI; RADIATION FROM A HARMONIC OTHER THAN THE FUNDAMENTAL.



FIG. C.7. CASE A2; NO RADIATION.


FIG. C.8. CASE BI; INEVITABLE RADIATION.

## C. 2 Acoustics of a Moving Medium

The only task that needs to be performed in this section is
 wavenumber $k$, gets modified due to the $a_{o} a_{o}$ fact that the acoustic medium is moving uniformly with subsonic velocity $u_{i}=\left(u_{1}, u_{2}, u_{3}\right)$, where $u_{1}, u_{2}, u_{3}$ are the velocity components in the $x, y$ and $z$ directions.

Once again, consider the line monopole source of Fig. C.l, with Eqs. (C.l) through (C.5) still applicable. In addition to the radial coordinate $r$ of Eq. (c.3), the corresponding radial vector $\underline{r}$ is defined as

$$
\begin{equation*}
\underline{r}=(y, z) \tag{C.37}
\end{equation*}
$$

The far field pressure $P(x, \underline{r}, t)$ now must satisfy the following field equation, (Morse and Ingard, 1964),

$$
\begin{equation*}
\nabla_{p}^{2}+k_{a_{0}}^{2}\left[1+\frac{i}{\omega_{0}}\left(u_{1} \frac{\partial}{\partial x}+u_{2} \frac{\partial}{\partial y}+u_{3} \frac{\partial}{\partial z}\right)\right]^{2} p=0 \tag{C.38}
\end{equation*}
$$

For $u_{1}=u_{2}=u_{3}=0$, Eq. (C.38), reduces to the usual Helmholtz equation applicable for a stationary acoustic medium. In analogy with Eq. (C.ll), a criterion is needed that the radial wavenumber $k_{r}$ must satisfy for no radiation to occur to the far field (i.e., Case 2). However, in analogy with generalization of Eq. (C.3) to Eq. (C.37), a radial wavenumber vector $k_{r}$ is defined as

$$
\begin{equation*}
\underline{k_{r}}=\left(k_{2}, k_{3}\right) \tag{C.39}
\end{equation*}
$$

where $k_{2}$ and $k_{3}$ are the wavenumber components of the radially outwards propagating wave.

Now the important phase aspect of the far field pressure $P(x, \underline{r}, t)$ (for a given value of $\underline{r}$ in the farfield) is given by the correspondingly generalized version of Eq. (C.6)

$$
\begin{equation*}
P(x, \underline{r}, t)=\text { constant } \times e^{i\left(k_{0} x-k_{r} \cdot \underline{r}-\omega_{0} t\right)}, \tag{c.40}
\end{equation*}
$$

where

$$
\begin{equation*}
\underline{k_{r}} \cdot \underline{r}=k_{2} y+k_{3} z \tag{C.41}
\end{equation*}
$$

The following definitions are required:

$$
\begin{align*}
& k_{r}=\left|k_{r}\right|=\left(k_{2}^{2}+k_{3}^{2}\right)^{1 / 2}  \tag{c.42}\\
& u_{r}=\left|u_{r}\right|=\left(u_{2}^{2}+u_{3}^{2}\right)^{1 / 2}  \tag{c.43}\\
& k_{2}=k_{r} \sin \alpha_{k},  \tag{c.44}\\
& k_{3}=k_{r} \cos \alpha_{k},  \tag{C.45}\\
& u_{2}=u_{r} \sin \alpha_{u},  \tag{C.46}\\
& u_{3}=u_{r} \cos \alpha_{u}, \tag{c.47}
\end{align*}
$$

so that

$$
\begin{equation*}
\underline{k_{r}} \cdot \underline{u_{r}}=k_{2} u_{2}+k_{3} u_{3}=k_{r} u_{r} \cos \alpha, \tag{C.48}
\end{equation*}
$$

where

$$
\begin{equation*}
\alpha=\left(\alpha_{k}-\alpha_{u}\right) \tag{C.49}
\end{equation*}
$$

Thus, $\alpha$ is the angle between the radial wavenumber vector $k_{r}$ (or the radial location vector ( $y, z$ ) of observation in the far= field) and the radial flow vector $u_{r}$.

Now, substituting Eq. (C.40) into Eq. (C.38) gives the following relation,

$$
\begin{equation*}
-k_{r}^{2}-k_{o}^{2}+k_{a_{0}}^{2}\left[1-\frac{l}{\omega_{0}}\left(u_{1} k_{0}+u_{r} k_{r} \cos \alpha\right)\right]^{2}=0 \tag{c.50}
\end{equation*}
$$

which can be rewritten as a quadratic equation in $k_{r}$ as follows

$$
\begin{equation*}
A k_{r}^{2}+B k_{r}+C=0 \tag{c.51}
\end{equation*}
$$

where

$$
\begin{align*}
A & =\frac{k_{a_{0}}^{2}}{\omega_{0}^{2}} u_{r}^{2} \cos ^{2} \alpha-1=m_{r}^{2} \cos ^{2} \alpha-1  \tag{C.52}\\
B & =2 \frac{k_{a_{0}}^{2}}{\omega_{0}^{2}} u_{1} k_{0} u_{r} \cos \alpha-2 \frac{k_{a_{0}}^{2}}{\omega_{0}} u_{r} \cos \alpha \\
& =2\left(m_{1} m_{r} k_{o} \cos \alpha-k_{a_{0}} m_{r} \cos \alpha\right) \tag{C.53}
\end{align*}
$$

and

$$
\begin{align*}
c & =\left(-k_{0}^{2}+k_{a_{0}}^{2}-2 \frac{k_{a_{0}^{2}}^{2}}{\omega_{0}} u_{1} k_{0}+\frac{k_{a_{0}^{2}}^{u_{1}^{2}}}{\omega_{0}^{2}} k_{0}^{2}\right) \\
& =\left(-k_{0}^{2}+k a_{0}^{2}-2 k_{a_{0}}^{m} k_{0}+m_{1}^{2} k_{0}^{2}\right) \tag{C.54}
\end{align*}
$$

In the above equations appropriate Mach numbers are introduced by division of velocities by the sound speed $c=\omega_{0} / k_{a_{0}}$.

For the stationary acoustic medium, the condition on radial wavenumber magnitude $k_{r}$, for no radiation to occur (Case 2), was that $k_{r}$ be imaginary (see Eq. C.ll). In analogy with that requirement for no radiation to occur, it is required that $k_{r}$ of Eq. (C.5l) be complex (with positive imaginary part). That will happen if and only if

$$
\begin{equation*}
A C-B^{2} / 4>0 . \tag{C.55}
\end{equation*}
$$

Now, the left hand side of Eq. (C.55) does not contain $k_{r}$, but is a quadratic form in $k_{0}$, the wavenumber of the source wave. Thus, Eq. (C.55) can be rewritten as

$$
\begin{equation*}
D K_{0}^{2}+E k_{0}+F>0, \tag{C.56}
\end{equation*}
$$

where

$$
\begin{align*}
& D=1-m_{1}^{2}-m_{r}^{2} \cos ^{2} \alpha  \tag{C.57}\\
& E=2 k_{a_{0}}^{m}  \tag{C.58}\\
& F=-k_{a_{0}}^{2} \tag{C.59}
\end{align*}
$$

The minimum value of $D$ occurs for $\alpha=0$ or $\pi$ (i.e., when $k_{r}$ and $u_{r}$ are coincident or oppositely directed. This minimum value $D_{\text {minm }}$ is given by

$$
D_{\text {minm }}=1-m_{1}^{2}-m_{r}^{2}=1-m^{2}>0,
$$

where $m$ is total flow Mach number. Since $D_{\text {minm }}$, and therefore $D$ is always positive, the left hand side of minm Eq. (C.56) is positive for large $\left|k_{0}\right|$ (i.e., for $k_{0} \rightarrow \pm \infty$ ), being dominated by the first term $D k_{0}^{2}$. This behavior, incidently, is consistent with the inequality expressed by Eq. (C.56). Recalling that the inequality of Eq. (C.56) is the condition on $k_{0}$ for no radiation to occur (i.e., Case 2), the radiation will, in fact, take place for a range of wavenumbers $k_{0}$ of relatively small magnitude. This range, the radiation span along axial wavenumber $k_{1}$, is determined by the two real roots $k_{a_{0+}}$ and $k_{a_{0-}}$ of the quadratic form of Eq. (C.56)

$$
\begin{equation*}
k_{a_{0 \pm}}=k_{a_{0}} \frac{-m_{1} \pm\left(1-m_{r}^{2} \cos ^{2} \alpha\right)^{1 / 2}}{\left(1-m_{1}^{2}-m_{r}^{2} \cos ^{2} \alpha\right)} \tag{C.60}
\end{equation*}
$$

In analogy with Fig. C.2, this radiation span is shown as a shaded strip in Fig. C.9. The center $0^{\prime}$ of the span is shifted to the left by the amount $\mathrm{k}_{\mathrm{a}} \mathrm{m}_{1} / \mathrm{D}$ (with D given by Eq. (C.57) from the origin $0\left(k_{1}=0\right)$. $a_{0}$ This shift resulting from first (common) term on the right hand side of Eq. (C.60), is interpreted as a Galilean shift. The equal intervals ( $k_{a_{0-}}, 0^{\prime}$ ) and $0^{\prime}, k_{a_{0+}}$ ), resulting from the second terms on the right ${ }^{\circ} o_{-}$hand side of $a_{0}{ }^{\text {, }}$ Eq. (C.60) are interpreted as Lorentz half-spans.

Also shown in Fig. C. 9 is $\tilde{q}\left(k_{1}, t\right)$ for a phase wave whose phase velocity co is supersonic in the fixed coordinate system (or equivalently whose $k_{o}$ is less than $k_{a_{o}}$ ), yet since $k_{0}$ lies outside the radiation span, the $\quad a_{o}$ particular phase wave illustrated will not radiate to the farfield.

Reformulation of the condition of no radiation in terms of Mach numbers can be done along exactly similar lines as done in Eqs. (C.19), (C.20) and (C.21). Thus, the upper and lower permissible Mach numbers $m_{u}$ and $m_{\ell}$ are given by

$$
\begin{align*}
& m_{l}=\frac{\left(1-m^{2}-m_{r}^{2} \cos ^{2} \alpha\right)}{-m_{1}-\left(1-m_{r}^{2} \cos ^{2} \alpha\right)^{1 / 2}}=m_{1}-\left(1-m_{r}^{2} \cos ^{2} \alpha\right)^{1 / 2},  \tag{c.6I}\\
& m_{u}=\frac{\left(1-m_{1}^{2}-m_{r}^{2} \cos ^{2} \alpha\right)}{-m_{1}+\left(1-m_{r}^{2} \cos ^{2} \alpha\right)^{1 / 2}}=m_{1}+\left(1-m_{r}^{2} \cos ^{2} \alpha\right)^{1 / 2}, \tag{c.62}
\end{align*}
$$

and for no radiation to occur, the following must be satisfied:

$$
\begin{equation*}
m_{\ell}<m_{0}<m_{u} \tag{c.63}
\end{equation*}
$$



FIG. C.9. SKETCH OF RADIATION SPAN ALONG WAVENUMBER $k_{1}$ FOR A MOVING ACOUSTIC MEDIUM.

Extension similar to that from Figs. C. 2 to C. 3 can also be easily performed for the present case; as a result of the medium motion, the acoustic "cones" of radiation in the $\omega-k_{1}$ plane will be asymmetrical about the $\omega$ axis.

Finally, the entire discussion of discrete arrays in Sec. C. 2 can be applied here with the newly defined radiation span.

## C. 3 An Estimate Of Overall Power Radiated From The Stator

Figure C. 10 shows the wake velocity deficits as viewed in time at one SBLE near the tip. There are fich deficits per second, where $f_{r}=8600 \mathrm{~Hz}$ is the rotor blade passage frequency. Each individual ${ }^{r}$ wake deficit, $v(t)$ has an approximately Gaussian shape around its peak deficit value $\mathrm{v}_{\mathrm{O}}$, thus

$$
\begin{equation*}
v(t) \approx v_{0} e^{-t^{2} / 2 T^{2}} \tag{C.64}
\end{equation*}
$$

where the "standard deviation" $T$, and the maximum deficit $\mathrm{v}_{\mathrm{O}}$ are
estimated to be:

$$
\begin{align*}
& \mathrm{T} \approx 9.6 \times 10^{-6} \mathrm{sec} .  \tag{c.65}\\
& \mathrm{v}_{0} \approx 44 \mathrm{~m} / \mathrm{sec}(144 \mathrm{ft} / \mathrm{sec}) \tag{c.66}
\end{align*}
$$

The maximum deficit, $v$ corresponds to $10^{\circ}$ change in angle of attack. The time interval $\tau$ between consecutive deficits is given by

$$
\begin{equation*}
\tau=\frac{1}{f_{r}} \approx 1.16 \times 10^{-4} \mathrm{sec} . \tag{C.67}
\end{equation*}
$$

Note that $\tau$ is about an order of magnitude greater than $\mathbb{T}$, in other words the wake deficits are narrow in time when compared to their rate of arrival.

The above data regarding the wake velocity deficits was developed from Kemp and Sears (Ref. 18). The description of the wake velocity deficit in a spatial coordinate, $x$, can be obtained by assuming that the wakes arrive at the SBLE tips as (locally) frozen spatial patterns, being convected along with the local gas speed V, where

$$
\begin{equation*}
v=\left(m_{c}^{2}+m_{a}^{2}\right)^{1 / 2} c \approx 195 \mathrm{~m} / \mathrm{s}(64 \mathrm{I} \tag{C.68}
\end{equation*}
$$ ft/sec)



FIG. C.10. SKETCH OF TIME HISTORY OF WAKE VELOCITY DEFICITS AS THEY IMPINGE ON A SINGLE SBLE TIP.
$m_{c}$ and $m_{a}$ being given by Eqs. 21 and 22 . Thus, the spatial picture of a wake velocity deficit is obtained by the transformation,

$$
\begin{equation*}
x=U t . \tag{c.69}
\end{equation*}
$$

For estimating the overall acoustic power radiated from the stator it is convenient, as shown below, to integrate the results in the time domain. However, in order to get a qualitative understanding of the situation in the frequency domain we discuss briefly the Fourier transform $\tilde{v}(\omega)$ of $v(t)$ of Eq. (C.64).

$$
\begin{align*}
\tilde{v}(\omega) & =\frac{1}{2 \pi} \int v(t) e^{i \omega t} d t  \tag{C.70}\\
& =\frac{T v_{0}}{(2 \pi)^{1 / 2}} e^{-\omega^{2} T^{2} / 2} \tag{C.71}
\end{align*}
$$

The Fourier transform $\tilde{v}^{\prime}(\omega)$ of the sequence of pulses of Fig. C. 10 may then be written as

$$
\begin{align*}
\tilde{v}^{\prime}(\omega) & =\frac{1}{2 \pi} \int v^{\prime}(t) e^{i \omega t} d t \\
& =\frac{1}{2 \pi} \int \sum_{j=-\infty}^{+\infty} v(t-\tau j) e^{i \omega t} d t  \tag{C.72}\\
& =\tilde{v}(\omega) \omega_{r} \sum_{n=-\infty}^{+\infty} \delta\left(\omega-n \omega_{r}\right) \tag{C.73}
\end{align*}
$$

where $\omega_{0}$ is the blade passage frequency in radians/sec (see
Eqs.

$$
\begin{equation*}
\omega_{0}=2 \pi f_{r}=\frac{2 \pi}{\tau}=\Omega B \tag{C.74}
\end{equation*}
$$

Thus, as expected, the frequency content of the rotor wake velocity deficits $v^{\prime}(t)$ consists of the various rotor harmonics $n$. Since $\tilde{v}(\omega)$ does not decay appreciably with increasing frequency (the "standard deviation" of $\tilde{v}(\omega)$ is $1 / T$ ) the higher rotor harmonics at $n= \pm 2$, $\pm 3$, etc. have nearly the same strength or amplitude as the fundamental harmonic at $n= \pm 1$.

Now, reverting back to the time domain analysis, the fluctuating lift $\ell(t)$ generated at the leading edge of a SBLE tip from impingement of one wake deficit $v(t)$ is given by

$$
\begin{equation*}
\ell(t)=\int_{0}^{\infty} v\left(t^{\prime}\right) h\left(t-t^{\prime}\right) d t^{\prime}, \tag{C.75}
\end{equation*}
$$

where $h(t)$ is the unit impulse response function derived from Küssner's function [Bisplinghoff et al., Ref. 19]. Since the essential uncancelled fluctuating lift is restricted to a relatively small segment of the SBLE span near the tip, use of Küssner's function, valid for low aspect ratio, is readily justified for the present calculation. Since Küssner's function gives the lift due to a sharp-edged gust (i.e., due to a gust that is a unit step function), the unit impulse response function $h(t)$ is obtained by differentiating the Küssner's function. $h(t)$, so obtained, is given by

$$
\begin{equation*}
h(t)=C_{L}\left[\frac{0.13}{2 \tau_{b}} e^{-0.13 t / \tau_{b}}+\frac{1}{2 \tau_{b}} e^{-t / \tau_{b}}\right] \tag{C.76}
\end{equation*}
$$

where $\tau_{b}$ is the time taken by the gust to travel the (swept) semichord b.

$$
\begin{equation*}
\tau_{b}=b / U \approx 1.33 \times 10^{-4} \mathrm{sec} \tag{C.77}
\end{equation*}
$$

The above estimate of $\tau_{i}$ is based on $b \approx 0.034 \mathrm{~m}$ ( 0.11 ft ), and $U$ of Eq. C.68. The lift coefficient $C_{L}$ is given by

$$
\begin{equation*}
C_{L} \approx 2 \pi \rho U b \frac{\lambda_{r}}{4} \tag{C.78}
\end{equation*}
$$

where $\rho$ is the medium density ( $2.4 \times 10^{-3} \mathrm{lb}-\mathrm{sec}^{2} / \mathrm{ft}^{4}$ $\approx 1.24 \mathrm{Kg} / \mathrm{m}^{3}$ ) and $\lambda_{r}$ is the acoustic wavelength at blade passage frequency $f_{r}$ (Eq. 26). Note that in Eq. C.78, $\lambda_{r} / 4$ denotes a rough estimate of the SBLE span near the tip from which the uncancelled fluctuating lift is estimated to radiate. Now, this choice of $\lambda_{r} / 4$ is suitable (only) for the rotor fundamental harmonic at ${ }^{r}$ frequency $\omega_{r}$. For the higher rotor harmonics of Eq. C. 73, correspondingly smaller spanwise length scales would be more appropriate. However, in the time domain analysis that is being pursued, the choice of $\lambda_{r} / 4$ in Eq. C. 78 is taken to apply to all the rotor harmonics, therefore the resulting estimate of the overall (i.e., frequency-integrated) radiated power is liable to be conservative.

Substituting Eqs. (C.64) and (C.76) into Eq. (C.75), enables calculation of fluctuating lift $\ell(t)$ at a single SBLE tip due to the impingement of a single wake velocity deficit $v(t)$. Since the minimum time constant $\tau_{b}$ of $h(t)$ is much larger than the time constant or "standard" deviation" $T$ of $v(t)$ [compare Eqs. (C.65) and (C.77)], for evaluating $\ell(t)$ from Eq. (C.75), one can justifiably approximate $v(t)$ of Eq. (C.64) as follows,

$$
\begin{equation*}
v(t) \approx v_{0}(2 \pi)^{1 / 2} T \delta(t) . \tag{C.79}
\end{equation*}
$$

The constant ( $2 \pi)^{1 / 2} \mathrm{~T}$ in Eq. (C.79) is introduced so as to make the total "area" (in other words, the integral $f v(t) d t$ the same for Eqs. (C.64) and (C.79). From Eq. (C.70) and (C.71), note that this area is equal to $\left.2 \pi \tilde{v}(\omega)\right|_{\omega=0^{\circ}}$. Substituting Eq. (C.79) into Eq. (C.75), gives

$$
\begin{equation*}
\ell(t)=v_{0}(2 \pi)^{1 / 2} T h(t) . \tag{C.80}
\end{equation*}
$$

From the point of view of generation of steady lift, the stator blade chord is expected to be oriented parallel to the flow velocity $U$ at its leading edge, so that a zero mean angle of attack is ensured. Hence, the wake-deficit-induced fluctuating lift of Eq. C. 80 is oriented normal to the flow velocity $U$. The acoustic intensity $I(t)$ radiated by this "transverse" dipole (i.e., the direction of fluctuating lift is normal to flow) is given by [Lighthill, Ref. 15; Morse and Ingard, Ref. 17],

$$
\begin{equation*}
I(t)=[2 \dot{\ell}(t)]^{2} \frac{1}{12 \rho \pi c^{3}} \quad G_{2}(m), \tag{C.81}
\end{equation*}
$$

where $\dot{\ell}(t)=d / d t \ell(t)$ and $G_{2}(m)$ is a function of the flow Mach number $\mathrm{m}=\mathrm{U} / \mathrm{c}$,

$$
\begin{equation*}
G_{2}(m)=\frac{3}{4}\left[\frac{2}{m^{2}\left(1-m^{2}\right)}-\frac{1}{m^{3}} \text { \&n } \frac{1+m}{1-m}\right] \tag{C}
\end{equation*}
$$

The factor 2 appearing with $\dot{\ell}(t)$ in Eq. C. 81 accounts for the baffling effect due to the presence of the duct wall (assumed to be acoustically rigid) enclosing the SBLE tip.

The radiated acoustic energy $E$, associated with the intensity $I(t)$ of Eq. C. 81 is given by

$$
\begin{equation*}
E=\int_{0}^{\infty} I(t) d t \tag{C.83}
\end{equation*}
$$

From Eqs.C.76, C. 80 and C.81, we note that the only time dependent factor of $I(t)$ involves $h(t)$ of Eq. C.76, hence, the integral to be evaluated is

$$
\begin{equation*}
\int_{0}^{\infty} \dot{h}(t)^{2} d t=C_{L}{ }^{2} \frac{0.133}{\tau_{b}^{3}}, \tag{C.84}
\end{equation*}
$$

thus, substituting Eqs. C.80, C. 81 and C. 84 into Eq. C. 83 we get

$$
\begin{equation*}
E=\frac{2}{3} \frac{G_{2}(m)}{\rho c^{3}}\left(v_{0} T C_{L}\right)^{2} \frac{0.133}{\tau_{b}^{3}} \tag{C.85}
\end{equation*}
$$

Now, E is the acoustic energy radiated from a single SBLE tip due to impingement of a single wake velocity deficit $v(t)$. Hence, the acoustic power radiated from a single SBLE tip is Ef $r$, where $f r$ is the rate of impingement of wake deficits on the SBLE tip.

Next, assume that the $V$ individual SBLE tips radiate more or less incoherently (an assumption particularly valid for higher rotor harmonics $n$ of Eq. C.73). Hence, the power radiated from the $V$ stator tip sources is $E f_{r} V$.

Finally, even though the calculation for the power radiated from the stator leading edge sources at the hub is not carried out separately, because of closer circumferential spacing between these hub sources, the total power radiated from the stator hub is likely to be considerably less than that from the stator tip. The total power $I I$ radiated from the stator is conservatively estimated to be given by

$$
\begin{equation*}
\Pi=2 E f_{r} V \tag{c:86}
\end{equation*}
$$

Substituting in Eqs. (C.85) and (C.86) the numerical values quoted for various parameters (with $V=59$ ) gives

$$
\begin{equation*}
\Pi=130.5 \mathrm{~dB} \text { re } 10^{-12} \text { watt. } \tag{C.87}
\end{equation*}
$$

## APPENDIX D

NOTES ON EMPIRICAL CALCULATION OF FAN NOISE LEVELS

# APPENDIX D: NOTES ON EMPIRICAL CALCULATION OF FAN NOISE LEVELS 

As mentioned in Section 7, all components of the rotor and stator noise spectrum could not be calculated from basic considerations. The previous Appendix gives a noise level calculation for the residual stator noise sources. In the interest of determining what reduction in levels the swept rotor might be expected to cause, Burdsall's empirical correlation (Ref. 19), was exercisedfor both the actual fan model and a "full scale" counterpart. The parameters required in Burdsall's routine are given in Table D-l. Figure D-l summarizes the narrow band power levels and spectra for the various components, (SPL arbitrarily computed at $150 \mathrm{ft} ., 60^{\circ}$ from rotor axis) showing the predominance of MPT'S. Of course, the details of the MPT spectrum vary from fan to fan due to their origin in manufacturing tolerances. Fig. D-2 shows a typical comparison of Burdsall's prediction with measured data, indicating a fairly large fluctuation in harmonic levels around the mean line of the prediction. In Fig. D-l, it is clear that according to this scheme, elimination of MPT's would reduce the tone levels considerably. However, note that in Fig. D-l, the line is an envelope of discrete frequency levels at various multiples of rotation rate while the broadband spectrum is continous. Thus, integration into constant percentage bandwidths, and into overall levels will lead to the MPT and broadband levels being very nearly identical.

As a final point, it is interesting to note that the power level of the BPF tone (non-MPT noise) is $\sim 10 \mathrm{~dB}$ above the predicted level for the swept stator as described in Appendix C.
TABLE D-I INPUT PARAMETERS FOR EMPIRICAL NOISE PREDICTION




## APPENDIX E

ALGORITHM FOR DERIVATION OF STATOR LEADING EDGE TRACE VELOCITY IN STATOR FIXED COORDINATES

APPENDIX E. ALGORITHM FOR DERIVATION OF STATOR LEADING EDGE TRACE VELOCITY IN STATOR FIXED COORDINATES

The geometry of a rotor wake as it reaches the stator is given in Fig. E-1. Refer to Fig. 9b in the text for 3-dimensional representation. The following four steps give the rotor wake shape and local trace velocity for both an unswept stator (or at the inlet plane of a swept stator), and for swept stators. Aerodynamic reaction on the rotor path by the stator is not taken into account.


FIG. E-1. GEOMETRY FOR CALCULATION OF ROTOR WAKE SHAPE AND TRACE SPEED ON STATOR VANES.

1) Unswept, Unskewed Stator

The skew of the rotor wake at stator plane is $\alpha(r)$. The local angle between the wake and the radial direction may be derived from:

$$
\tan \theta_{w}=r^{\delta \alpha} / \delta r
$$

The trace velocity in the radial and axial directions is respectively

$$
\mathrm{V}_{\mathrm{T}_{\mathrm{B}_{\mathrm{R}}}}(\mathrm{r})=(\mathrm{wr}) / \tan _{\mathrm{w}}(\mathrm{r})
$$

and

$$
\mathrm{V}_{\mathrm{T}_{\mathrm{B}_{\mathrm{x}}}}=0
$$

where:

$$
\begin{aligned}
\mathrm{V}_{\mathrm{T}_{\mathrm{B}_{\mathrm{R}, \mathrm{x}}}}= & \text { the trace velocity in blade fixed coor- } \\
& \text { dinates for radial and axial directions } \\
& \text { respectively. }
\end{aligned}
$$

## 2) Add Sweep $\mu_{B}$

The rotor wake in the conical surface of the swept leading edge is changed, as follows.

$$
\mathrm{V}_{\mathrm{T}_{\mathrm{B}_{\mathrm{R}}}}(\mathrm{r})=\frac{\mathrm{wr}}{\tan \theta_{\mathrm{w}}(r)} \quad \text { where } \quad \tan \theta^{\prime}{ }_{\mathrm{w}}=r \frac{\delta \alpha^{\prime}}{\delta r}
$$

and

$$
\mathrm{V}_{\mathrm{T}_{\mathrm{B}}}^{\prime}(r)=\mathrm{V}_{\mathrm{T}_{\mathrm{B}_{\mathrm{R}}}} \quad \tan \mu_{\mathrm{B}}(\mathrm{r}) \quad \text { where } \mu_{\mathrm{B}}(\mathrm{r})=\text { local blade sweep angle. }
$$

$$
\alpha^{\prime}(r)=\alpha(r)+\Delta x \frac{\delta \alpha(m)}{\delta x},
$$

where $\Delta x$ is the downstream displacement of the leading edge caused by sweep.

$$
\tan \theta^{\prime}(r)=r \frac{\delta\left[\alpha(r)+\Delta x \frac{\delta \alpha(r)}{\delta x}\right]}{\delta r}
$$

$$
=r \frac{\delta \alpha(r)}{\delta r}+r \Delta x \frac{\delta^{2} \alpha(r)}{\delta x \delta r}+r \frac{\delta \Delta x}{\delta r} \frac{\delta \alpha(r)}{\delta x}
$$

$$
=\tan \theta_{w}+\Delta x \frac{\delta \tan \theta w}{\delta x}+r \frac{\delta \alpha(r)}{\delta x} \tan \mu_{B}
$$

$$
\tan \theta_{W}^{\prime}(r)=\tan \theta_{W}+r \frac{\delta \alpha(r)}{\delta x} \tan \mu_{B}
$$

3) Trace Velocity for Swept Stator in Stator-Fixed Coordinates The radial component of trace velocity is

$$
\mathrm{V}_{\mathrm{T}_{\mathrm{B}_{\mathrm{R}}}}(\mathrm{r})=\frac{\mathrm{wr}}{\tan \theta_{\mathrm{W}}^{*}+r \frac{\delta \alpha(r)}{\delta \mathrm{x}} \tan \mu_{\mathrm{B}}}
$$

where $\tan \theta_{W}^{*}(r, x)=\tan \theta_{W}+\Delta x \frac{\delta \tan w}{\delta x} \quad$.

The axial component is:

$$
\mathrm{V}_{\mathrm{T}_{\mathrm{B}}}(\mathrm{r})=\frac{\mathrm{wr} \tan \theta_{\mathrm{B}}}{\tan \theta_{\mathrm{W}}^{*}+\mathrm{r} \frac{\delta \alpha(\mathrm{r})}{\delta \mathrm{x}}{\tan \mu_{\mathrm{B}}}^{\tan }}
$$

where $\theta_{W}^{*}$ is determined from cross-plots of the wake path in the $r, \alpha$ plane at various axial locations, and $\delta \alpha / \delta r=$ the local wake helix pitch angle determined from wake crossplots in the meridional plane.
4) Transformation of Trace Velocity Amplitude From StatorFixed Coordinates to Gas-Fixed Coordinates

$$
\begin{array}{r}
\mathrm{V}_{\mathrm{T}_{\mathrm{B}_{\mathrm{R}}, \mathrm{x}}} \mid{ }^{\mathrm{V}_{\mathrm{T}_{\mathrm{G}}} \mid} \\
\left|\mathrm{V}_{\mathrm{T}_{\mathrm{G}}}\right|=\left[\left(\mathrm{V}_{\left.\left.\mathrm{T}_{\mathrm{B}_{\mathrm{x}}}-\mathrm{V}_{\mathrm{B}_{\mathrm{x}}}\right)^{2}+\mathrm{V}_{\mathrm{T}_{\mathrm{B}_{\mathrm{R}}}^{2}}+\mathrm{V}_{\mathrm{G}_{\mathrm{C}}}^{2}\right]^{1 / 2}}\right.\right.
\end{array}
$$

where $V_{G_{x, c}}$ is the gas velocity in the axial and circumferential directions, respectively.

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## LIST OF SYMbOLS

| A | = | axial |
| :---: | :---: | :---: |
| $\mathrm{A}_{\text {in }}$ | $=$ | inlet area (to rotor passage) |
| $A_{\text {min }}$ | = | geometric throat area |
| $A_{s}$ | $=$ | area to choke the flow |
| $a_{0} t / \lambda$ | $=$ | normalized distance |
| b | $=$ | blade semichord |
| B | = | number of blades |
| c | $=$ | chord length; sound speed |
| $c_{0}$ | = | phase or trace velocity; sound speed |
| CG | $=$ | center of gravity |
| ${ }^{\text {C }}$ | = | lift coefficient |
| D | = | diffusion factor |
| DCA | $=$ | double circular area (blade profile) |
| d | $=$ | distance from blade section c.g. to pressure surface; circumferential spacing between adjacent stator tips |
| de | = | swept leading edge element |
| E | $=$ | acoustic energy (radiated from a single SBLE tip) |
| f | $=$ | Mach factor ( $=1 / M_{W_{1}}$ ); frequency |
| $\mathrm{f}_{\mathrm{r}}$ | $=$ | blade passage frequency of rotor blade |

## LIST OF SYMBOLS (cont.)

| $\Delta F_{j}$ | $=$ | centrifugal force at center of a blade volume element located at $j$ |
| :---: | :---: | :---: |
| $\mathrm{G}_{2}(\mathrm{~m})$ | = | function of flow Mach number |
| h | = | unit impulse response function |
| I | = | acoustic intensity |
| j | = | source number |
| k | = | wavenumber; constant |
| $k_{r}$ | = | radial wavenumber |
| L | = | harmonic |
| $\ell$ | $=$ | distance from section $c . g$. to leading edge; fluctuating lift |
| LCF | = | low cycle fatigue |
| LE | $=$ | leading edge |
| $L_{m n}$ | $=$ | rotor/stator interaction harmonic |
| m | $=$ | circumferential mode number; component of Mach number; function in deviation angle formula |
| M | $=$ | Mach number; moment |
| $M_{\text {w }}$ | $=$ | relative flow Mach number component |
| $\mathrm{M}_{\mathrm{w}_{1}}$ | = | relative inlet Mach number |
| $M_{w_{1}}{ }^{\prime}$ | = | Mach number required for a subsonic edge to achieve sonic $\left(=M_{W_{1}}\right)$ |
| $M_{w_{1}}$ | $=$ | component of Mach number which is always normal to the leading edge ( $=1$ for sonic LE; < 1 for subsonic LE |

## LIST OF SYMBOLS (Cont.)

| ${ }^{M_{w_{1}}}{ }^{\prime}$ |  | Mach number required to make a subsonic LE a sonic LE |
| :---: | :---: | :---: |
| $\Delta M_{i j}$ | $=$ | moment of $j$ force about c.g. of section $i$ |
| N | = | rotation rate |
| n | = | shape parameter for polynomial blade forms harmonic number |
| P | = | total pressure; static pressur |
| P or p | = | far field acoustic pressure |
| P | = | location of leading edge point |
| P/P | = | pressure ratio |
| $\hat{P}$ | $=$ | static pressure after normal shock |
| PNL | = | Perceived Noise Level |
| q | $=$ | source strength |
| A | $=$ | monopole source strength per unit length |
| R | $=$ | distance from origin |
| $\mathrm{R}_{\mathrm{c}}$ | $=$ | radius of curvature of streamline |
| $r$ | = | radial distance |
| s | $=$ | circumferential blade spacing |
| SAP | $=$ | Structural Analysis Program |
| SCF | $=$ | Stress Concentration Factor |
| SBLE | = | Stator Blade Leading Edge |
| SR | $=$ | sweep reversal |

LIST OF SYMBOLS (Cont.)

| t | = | thickness; time |
| :---: | :---: | :---: |
| $t(x)$ | = | thickness distribution |
| T | $=$ | standard deviation (time) |
| TE | $=$ | trailing edge |
| U or $u$ | = | air velocity; wheel speed |
| v | = | velocity |
| $\mathrm{v}_{0}$ | $=$ | peak velocity defect |
| V | = | number of stator vanes; velocity |
| $\Delta \mathrm{V}_{j}$ | = | volume element of a blade |
| w | = | ```mass flow rate; velocity; inlet relative velocity``` |
| $\bar{W}$ | = | average velocity |
| $\mathrm{W}_{\text {as }}$ | = | specific mass flow |
| w/w | = | flow deceleration rate |
| w | = | relative velocity |
| $\Delta \mathrm{x}$ | = | downstream displacement of SBLE caused by sweep |
| x | = | linear distance |
| $\mathrm{z}, \mathrm{z}_{\mathrm{L}}$ | = | axial coordinate of leading edge |

## GREEK

| $\alpha$ | $=$ | angle of attack; angle between radial wave vector $k_{r}$ and radial flow vector $u_{r}$; local Mach angle; wake displacement angle from radial |
| :---: | :---: | :---: |
| $\alpha_{3}$ | $=$ | stator inlet angle |
| B | = | relative flow angle; exit angles of flow |
| $\gamma$ | = | setting angle; ratio of specific heats |
| $\delta$ | $=$ | flow deviation angles; Dirac delta function |
| $\delta_{3}$ | = | stator deviation angle |
| $\varepsilon$ | = | slope of the relative flow velocity; slope between lines connecting section LE and CG and a line connecting LE with lower surfaces coordinate at mid-chord $\left(\varepsilon_{W_{1}}=\sin ^{-1} V_{r^{\prime}} / w_{1}\right)$ |
| $\varepsilon_{j}$ | = | centerline-projected displacement of the $c . g$.'s of an airfoil section at $r_{j}$ relative to one at $r_{i}$ |
| $\lambda$ | = | acoustic wavelength |
| $\lambda$ | = | acoustic wavelength at blade passage frequency |
| $\eta$ | $=$ | polytropic state efficiency |
| $\theta$ | $=$ | circumferential angle |
| $\theta_{\text {w }}$ | $=$ | angle between radius and rotor wake centerline, unswept stator |
| $\theta_{\text {w }}{ }^{\prime}$ | = | angle between radius and rotor wake centerline, swept stator |

## LIST OF SYMBOLS (Cont.)

| $\theta_{L}$ |  | angular abscissa of leading edge point |
| :---: | :---: | :---: |
| $\mu$ | = | Mach cone angle; radial order of acoustic modes |
| ${ }^{\text {B }}$ | = | stator sweep angle |
| $\mu^{\prime \prime}$ | = | projection of the Mach cone angle on the w-r plane |
| $v$ | = | hub-to-tip ratio; lateral sweep angle; section thickness ratio ( $t_{\max } / c$ ); relative thickness |
| II | = | acoustic power |
| $\rho$ | $=$ | density |
| $\sigma$ | = | sweep angle; stress; cascade solidity |
| $\tau$ | $=$ | shear stress; LE and TE thickness factors; time interval between successive events |
| $\tau_{b}$ | $=$ | time for a gust to travel the swept semichord (b) |
| $\phi$ | $=$ | local camber angle; angle between leading and trailing edge along the unwrapped conical surface |
| $\Phi$ | = | mean camber angle |
| $\omega$ | = | radian frequency; wheel rotation speed |
| $\omega_{r}$ | = | radian frequency of blade passages |
| $\Omega$ | $=$ | shaft rotation frequency |

## LIST OF SYMBOLS (Cont.)

## SUBSCRIPTS

| A | $=$ axial; along blade leading edge |
| :--- | :--- |
| a | $=$ acoustic |
| ax | $=$ axial |
| B | $=$ blade-fixed coordinates |
| c | $=$ circumferential |
| crit | $=$ critical |
| cg | $=$ center of gravity |
| G | $=$ defect |
| G | $=$ geometric |
| i | $=$ component parallel to blade array |
| I, | $=$ indices of spatial coordinates |
| L | $=$ normal to leading edge; leading edge |
| l | $=$ mawer |
| M | $=$ minimum |
| m | $=$ radial |
| minm | $=$ |

## LIST OF SYMBOLS (Cont.)

| r | $=$ radial; component normal to blade array |
| ---: | :--- |
| T | $=$ tangential; wake trace |
| t | $=$ tip |
| u | $=$ upper; tangential |
| w | $=$ tangential |
| x | $=$ chordwise distance from LE; direction normal |
| $y$ | $=$ Cartesian coordinate normal to z-axis |
| $z$ | $=$ axial |
| $\infty$ | $=$ freestream relative |
| $1,2,3$ | $=$ y, and z directions |
| 1 | $=$ rotor inlet station |
| 2 | $=$ rotor exit station |
| 3 | $=$ stator inlet station |
| 4 | $=$ stator exit station |


[^0]:    *Valid in the forward-radiated direction at azimuths from $40-80^{\circ}$ from fan axis ( $\pm 3 \mathrm{~dB}$ ); to scale to greater distances, subtract $20 \log r / 150$, where $r$ is distance in feet.

[^1]:    
    

