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A Temperature Dependent Fatigue Failure Criterion for Graphite/Epoxy Laminates

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Introduction

Advanced composite materials are very attractive to the designer of high performance structures because they offer exceptionally high strength-to-weight and stiffness-to-weight ratios. Yet many questions concerning their performance remain to be answered. One of the most important is the question of the long-term durability under conditions of fatigue loading. The fatigue behavior of composite materials is complicated not only by the large number of variables involved in the fatigue process itself but also by the many variables involved in the construction and fabrication of these materials.

Most of today's design procedures are based upon static strength criteria with little consideration given to the influences of dynamic or fatigue loading. After a composite structure is designed, that is the number and orientation of the plies have been established, the composite is tested to determine the structure's response to dynamic loading. This procedure is very expensive and time consuming. Thus, a more rational design methodology must be developed which will allow the designer to initially consider the long-term response of a composite structure to dynamic loading. The development of a reliable fatigue failure criterion would be a significant step in that direction.

The potential extension of a static failure criterion to conditions of fatigue loading has been successfully demonstrated (1-3). For the general case of a static loading state imposed on a laminate, the stress field in each lamina can be calculated using one of a number of lamination theories for composites (4). By simple transformation techniques the stresses in each lamina can be expressed in terms of the principal directions of the lamina. Once the principal stresses are determined, a static failure criterion is used to establish potential laminae failures. Static failure criteria include a number of parameters, many of which must be determined experimentally. In the case of fatigue loading a similar failure criterion can be used (3). The parameters of the criterion, however, must involve not only static parameters but also fatigue functions. These fatigue functions normally express degradation in the strength of a lamina as a function of the number of loading cycles under a defined loading pattern. If any one of the fatigue conditions is changed, such as temperature, loading pattern, frequency of cycling or environment (moisture, radiation, chemical, etc.), both the static parameters and the fatigue functions may be influenced. The present paper is concerned with the initial steps in the application of a failure criterion to the fatigue behavior of an advanced composite material. Particular emphasis is placed on the influences of temperature on both the static parameters and the fatigue functions involved in the criterion and upon how these influences may be predicted through the development of proper shifting factors.

Background

In a composite material consisting of both fibers and matrix, failure of a lamina under tension-tension fatigue loading can be expressed (3) in terms of either fiber failure:

$$ {}_p\sigma_A^{\,c} \ge \sigma_A^{\,u} = \sigma_A^{\,s}(T)\; f_A(R,N,n,T) \tag{1} $$

or in terms of matrix failure:

$$ \left(\frac{{}_p\sigma_A^{\,c}}{\sigma_m^{\,u}}\,\frac{E_m}{E_A}\right)^2 + \left(\frac{{}_p\sigma_T^{\,c}}{\sigma_T^{\,u}}\right)^2 + \left(\frac{{}_p\tau^{\,c}}{\tau^{\,u}}\right)^2 \ge 1 \tag{2} $$

where subscript A indicates fiber direction; subscript p indicates lamina; superscript c indicates cyclic loading; superscript u indicates critical; superscript s indicates static; σ is stress; T is temperature; R is the ratio of fatigue minimum load to maximum load; N is the number of applied cycles; n is the speed or frequency of cycling; f is a fatigue failure function; subscript m indicates matrix; E is elastic modulus; and τ is shear stress.

From eqn. (1) fiber failure in a lamina will occur when ${}_p\sigma_A{}^c$, the applied cyclic stress in the fiber direction equals or exceeds $\sigma_A{}^u$, the critical fatigue strength of the fiber. The critical fiber fatigue strength will be a function of both $\sigma_A{}^s$, a static strength parameter which may be a function of temperature, and f_A, a fatigue function which may be a function of the load ratio R, the number of applied cycles N, the frequency of cycling n, and temperature T.

For matrix failure, the sum of the squares of the three terms of eqn. (2) must be equal to or greater than one. The first term of this equation represents the stress in the matrix material in the fiber direction, and is the ratio of the applied cyclic stress in the lamina, ${}_p\sigma_A{}^c$, to the critical fatigue strength of the matrix material, $\sigma_m{}^u$, times the ratio of the modulus of the matrix, E_m, to that of the fiber, E_A. For the advanced composite materials considered here, the modulus of the fiber is much greater than that of the matrix. Therefore, the ratio of the two moduli becomes very small and the first term of eqn. (2) can be neglected. Thus, the failure criterion of the matrix becomes:

$$\left(\frac{{}_p\sigma_T{}^c}{\sigma_T{}^u}\right)^2 + \left(\frac{{}_p\tau^c}{\tau^u}\right)^2 \geq 1 \tag{3}$$

where subscript T indicates the transverse direction of the lamina. This means that sum of the squares of the ratio of ${}_p\sigma_T{}^c$, the transverse stress in the lamina perpendicular to the fiber direction, to $\sigma_T{}^u$, the transverse critical fatigue strength of the matrix and of ${}_p\tau^c$, the shear stress in the lamina, to τ^u, the critical fatigue shear strength of the matrix, must be greater than or equal to one. As in eqn. (1), the critical fatigue strengths are the static strength parameters times the fatigue functions:

$$\sigma_T{}^u = \sigma_T{}^s f_T(R,N,n,T)$$

$$\tau^u = \tau^s f_\tau(R,N,n,T)$$

Although interlaminar failure will not be dealt with directly in this paper, for completeness we will give the failure criterion for interlaminar failure. Interlaminar failure, which is simply matrix failure between the laminae, can be expressed as (3):

$$\left(\frac{{}_{p'}\sigma_z{}^c}{\sigma_z{}^u}\right)^2 + \left(\frac{{}_{p'}\tau_d{}^c}{\tau_d{}^u}\right)^2 \geq 1 \tag{4}$$

where subscript p' indicates interlaminar; subscript z indicates the direction perpendicular to the plane of the laminae; and subscript d indicates the direction parallel to the plane of the laminae. Here ${}_{p'}\sigma_z{}^c$ is the normal stress perpendicular to the plane of the laminae and ${}_{p'}\tau_d{}^c$ is the shear stress in the interlaminar region. Again, the terms $\sigma_z{}^u$ and $\tau_d{}^u$ are the corresponding critical fatigue strengths given by the static strength parameters times the fatigue functions:

$$\sigma_z^u = \sigma_z^s f_z(R,N,n,T)$$

$$\tau_d^u = \tau_d^s f_d(R,N,n,T)$$

The three failure criteria developed in eqns. (1), (3) and (4) relate to the three possible modes of failure in a laminate--fiber failure, matrix failure and interlaminar failure. Failure can of course occur in each mode separately or in all three modes simultaneously. Optimum design would be the latter situation. The conditions for failure of any laminate can be examined by resolving the applied external loads on the laminate into the principal stresses acting on the laminae. These stresses are then compared to the critical stress for each failure mode through a knowledge of the respective static strength parameters and fatigue functions. As we have seen, the fatigue functions are dependent upon a number of variables. Assuming all variables are kept constant with the exception of the number of applied cycles, N, the five fatigue functions (f_i at i = A, T, τ, z, d) can be determined experimentally from the S-N (applied cyclic stress versus number of cycles to failure) curves. In this case the fatigue functions can be written as;

$$f_i(N) = \frac{\sigma_i^u}{\sigma_i^s} \qquad i = A,\ T,\ \tau,\ z,\ d \tag{5}$$

If the temperature is changed, there will be a slightly different set of fatigue functions again determined experimentally from the S-N curves at that temperature. The temperature dependence of the fatigue functions can then be expressed by the relationship

$$f_i(N,T) = \frac{\sigma_i^u(T)}{\sigma_i^s(T)} \tag{6}$$

If the change in the S-N curves with temperature is known to be continuous, the fatigue functions of eqn. (6) can be written as a set of master functions given by;

$$a_i(T)\,f_i(N,T_0) = \frac{\sigma_i^u(T)}{\sigma_i^s(T)} \tag{7}$$

where T_0 is the reference temperature and $a_i(T)$ are the shifting factors with temperature for the specific principal stresses considered. This general form of the fatigue functions reduces the number of required functions to five for the three modes of failure. The exact form of the shifting functions, $a_i(T)$, must be determined experimentally.

For a class of viscoelastic materials called thermorheological simple materials (5), a relationship is known to exist between temperature and the values of the viscoelastic moduli. This relationship is called the W. L. F. Equation (5) and is valid for polymeric materials near the glass-transition temperature. It is expressed as

$$\log a_T = \frac{-8.86(T-T_0)}{101.6 + T-T_0} \tag{8}$$

where a_T is a shift factor which is the ratio of the time to cause a given change in moduli at some temperature over the time to cause a similar change at a reference temperature, T_0, expressed in absolute scale (°K). For polymeric materials in the glassy state this relationship becomes an Arrhenius-type function having the form

$$\log a_T = \frac{\Delta H}{2.303\,R}\left(\frac{1}{T} - \frac{1}{T_0}\right) \qquad (9)$$

where ΔH is the energy of activation for the process (being of the range of 10 t0 20 Kcal/mole) and R is the gas constant. By use of the applicable equation the time dependent viscoelastic modulus of a material can be expressed at any temperature in terms of a shifting factor and its behavior at a reference temperature. Some investigators have found a similar behavior for static strength of polymeric materials (6). If this is the case, in a composite laminate having polymeric materials as the matrix, the influence of temperature may come about through both its influence on the fatigue functions as well as its influence on the static strengths.

The principal static strengths in a laminate at any temperature, $\sigma_i^s(T)$, can be related to the static strengths at a reference temperature, $\sigma_i^s(T_0)$, by the relationship;

$$\sigma_i^s(T) = a_i^s \sigma_i^s(T_0) \qquad (10)$$

where a_i^s are simply shifting factors. Substituting eqn. (10) into eqn. (7) we obtain

$$a_i(T) f_i(N,T_0) = \frac{\sigma_i^u(T)}{a_i^s \sigma_i^s(T_0)} \qquad (11)$$

which is nothing more than a shifting of the S-N curve;

$$\sigma_i^u(T) = a_i^s \sigma_i^s(T_0) a_i f_i(N,T_0) \qquad (12)$$

The interpretation of eqn. (12) in terms of a shifting of the S-N curve is illustrated in Fig. 1. As can be seen from this figure, the vertical shift due to the change in the static strength with temperature is given by a_i^s and the rotation of the S-N curve, which is due to the change in fatigue function with temperature, is given by a_i. For a given set of laminae once the functions a_i^s and a_i are determined experimentally, it becomes possible to construct a fatigue-failure-surface--a three dimensional surface formed by stress amplitude, number of cycles to failure, and temperature--for any of the five principal stresses in the laminae. Additionally, the shifting functions can be used to predict the long-term fatigue behavior of a composite at low temperatures from short-term behavior at elevated temperature.

The total external applied cyclic stress in a laminate, σ^c, can be resolved into two stress components--a mean stationary stress, σ^m, and an alternating stress, σ^a. The principal stresses in a laminate can be expressed by the relationship

$$[\sigma^c] = [\sigma^m] + [\sigma^a] \qquad (13)$$

where [ ] denote an array of the stress matrix. For the case of a lamina with the stress applied in the x directions, eqn. (13) becomes

$${}_p\sigma_x^{\,c} = {}_p\sigma_x^{\,m} + {}_p\sigma_x^{\,a} \qquad (14)$$

In addition to external stresses, there can also exist thermal stresses induced in the laminate due to differences in thermal expansion between the fibers and the matrix and between the various laminae. For the resin-based matrix materials considered here, these stresses relax with time, particularly at the higher temperatures. In general these stresses will not be considered.

For an elastic material the stresses in any lamina of a laminate will depend upon the elastic moduli of all of the laminae, their orientations, thicknesses and locations within the laminate. Specifically, for symmetrically balanced laminates subjected to uniaxial loading, the stress in a lamina in the direction of loading, x, will be given by the relationship;

$$ {}_p\sigma_x^{\,c} = \frac{{}_\ell\sigma_x^{\,c}}{{}_\ell E_{xx}} \left({}_pQ_{xx} - {}_\ell\nu_{xy}\, {}_pQ_{xy} \right) \tag{15} $$

where p and ℓ denote lamina and laminate, respectively, and ${}_pQ_{xx}$ and ${}_pQ_{xy}$ are the stiffnesses and ν_{xy} is the poisson's ratio in the x-y directions. Resolving this stress, ${}_p\sigma_x^{\,c}$, the principal stresses in a lamina are given by the relations

$$ {}_p\sigma_A^{\,c} = {}_pk_{xx}\; {}_p\sigma_x^{\,c} \tag{16a} $$

$$ {}_p\sigma_T^{\,c} = {}_pk_{yy}\; {}_p\sigma_x^{\,c} \tag{16b} $$

$$ {}_p\tau^{c} = {}_pk_{xy}\; {}_p\sigma_x^{\,c} \tag{16c} $$

where

$$ {}_pk_{xx} = \frac{1}{2}\left\{\left[1 + \sec 2\theta - \frac{(\mu + \sec 2\theta)\tan^2 2\theta}{\eta + \tan^2 2\theta}\right] + \left[1 - \csc 2\theta - \frac{(\mu - \csc 2\theta)\tan^2 2\theta}{\eta + \tan^2 2\theta}\right] \frac{{}_p\nu_{xy} - \nu_{xy_\ell}}{{}_pQ_{xx}/{}_pQ_{yy} - {}_p\nu_{xy}\,\nu_{xy_\ell}}\right\} \tag{17a} $$

$$ {}_pk_{yy} = \frac{1}{2}\left\{\left[1 - \sec 2\theta + \frac{(\mu + \sec 2\theta)\tan^2 2\theta}{\eta + \tan^2 2\theta}\right] + \left[1 + \csc 2\theta + \frac{(\mu - \csc 2\theta)\tan^2 2\theta}{\eta + \tan^2 2\theta}\right] \frac{{}_p\nu_{xy} - \nu_{xy_\ell}}{{}_pQ_{xx}/{}_pQ_{yy} - {}_p\nu_{xy}\,\nu_{xy_\ell}}\right\} \tag{17b} $$

$$ {}_pk_{xy} = -\frac{1}{2}\left\{\frac{(\mu + \sec 2\theta)\tan 2\theta}{\eta + \tan^2 2\theta} - \left[\frac{(\mu - \csc 2\theta)\tan 2\theta}{\eta + \tan^2 2\theta}\right] \frac{{}_p\nu_{xy} - \nu_{xy_\ell}}{{}_pQ_{xx}/{}_pQ_{yy} - {}_p\nu_{xy}\,\nu_{xy_\ell}}\right\} \tag{17c} $$

$$ \mu = \frac{1 - E_A/E_T}{1 + 2\nu_A + E_A/E_T} \tag{18a} $$

$$ \eta = \frac{E_A/G_A}{1 + 2\nu_A + E_A/E_T} \tag{18b} $$

where θ is the angle between the load axis and the fiber direction in the lamina.

For the viscoelastic case considering the mean applied stress and the alternating applied stress components of the cyclic stress, eqns. (17) and (18) must be reformulated. For the relationship expressing mean stress, the elastic moduli must be replaced by moduli which are time and temperature dependent. Equations (15), (17) and (18) are rewritten to include the following substitutions;

$$E_A,\ E_T,\ \nu_A,\ G_A,\ \nu_{xy},\ Q_{xx},\ Q_{xy} \rightarrow E_A(t,T),\ E_T(t,T),\ \nu_A(t,T),$$

$$G_A(t,T),\ \nu_{xy}(t,T),\ Q_{xx}(t,T),\ Q_{yy}(t,T) \qquad (19)$$

For the relationship expressing the applied alternating stress, the elastic moduli must be replaced by the complex moduli. Equations (15), (17) and (18) are rewritten to include the following substitutions;

$$E_A,\ E_T,\ \nu_A,\ G_A,\ \nu_{xy},\ Q_{xx},\ Q_{yy} \rightarrow E_A(i\omega,T),\ E_T(i\omega,T),\ \nu_A(i\omega,T),\ldots \qquad (20)$$

For fiber reinforced polymeric matrix composites it has been shown (7) that the moduli in the fiber direction, E_A and ν_A, are not time or temperature dependent. Consequently, for symmetrically balanced laminates E_A, ν_A, ${}_{\ell}E_{xx}$ and ${}_{\ell}\nu_{xy}$ will not be time or temperature dependent. All other moduli are time and temperature dependent and must be determined experimentally. The principal stresses to be used in the failure criteria are established through substitution into eqn. (16a) giving;

$$_p\sigma_Q^{\ m} = {}_pk_{ij}(t,T)\,{}_p\sigma_x^{\ m}\ H(t) \qquad Q = A,T,\tau$$

$$_p\sigma_Q^{\ a} = {}_pk_{ij}(i\omega,T)\left({}_p\sigma_x^{\ a}\right)_{max} \qquad i,j = x,y \qquad (21)$$

Here, $H(t)$ is the Heavyside function and $({}_p\sigma_x^{\ a})_{max}$ is the maximum amplitude of the alternating stress from the mean stress.

Materials and Procedures

The advanced composite material selected for this study was a graphite/epoxy system of Union Carbide T300 fibers in a Narmco 5208 matrix. This material was chosen because of its extensive use in NASA's Aircraft Energy Efficiency (ACEF) Program (8). The material was fabricated from prepreg tapes by Lockheed Missiles and Space Co., Sunnyvale, California. Unidirectional laminae panels, angle-ply laminate panels and symmetrically balanced laminate panels were fabricated from the prepreg tapes. Each panel consisted of eight layers with an average fiber volume fraction of 65 percent. All test specimens had the dimensions shown in Fig. 2. Specimens were cut from the unidirectional panels in the direction of the fibers and at angles to the fibers of 10°, 15°, 30°, 45° and 90°. These specimens served to characterize the properties of the lamina. From the angle-ply panels specimens were cut such that the long axis of the specimens made angles with the fibers of ±15°, ±30°, ±45°, ±60° and ±75°. These specimens served two purposes: (1) to study interlaminar failure and (2) to compare the strength of laminae containing a number of small cracks with that observed in unidirectional off-axis specimens which fail upon the initiation of the first crack. Specimens were cut from the symmetrically balanced panels such that their major axis lie parallel to one of the fiber directions while the other fiber directions were symmetric about this direction. These specimens were defined as $[0°,\pm15°,0°]_s$, $[0°,\pm30°,0°]_s$, $[0°,\pm45°,0°]_s$, $[0°,\pm60°,0°]_s$, $[0°,\pm75°,0°]_s$ and $[0°,\pm90°,0°]_s$. These specimens were used as a check on the predictions of the theoretical analysis on the behavior characteristics observed in the unidirectional, off-axis specimens and in the angle-ply specimens. Additionally, the symmetrically balanced specimens represent the general type of loading behavior of a laminate where the loads in the fiber direction, perpendicular to the fiber direction and inplane shear can be calculated.

All specimens were tested in fatigue under tension-tension cycling, at R (ratio of minimum applied load to maximum applied load) equal to 0.1, and at frequencies between 10 and 30 cycles/sec using an electro-hydraulic, servo-controlled test system. Tests were conducted under conditions of load control and both load and strain were monitored during each test. S-N curves were developed for each specimen set at three temperatures—25°C (77°F), 74°C (165°F) and 114°C (237°F). Load was applied to the specimens through specially designed grips which facilitated specimen alignment and load transfer and supported the specimen sides in an effort to eliminate specimen bending during deformation. All specimens were soaked at temperature for one hour prior to testing.

Results and Discussion

Static Behavior

The stress-strain behavior and static strengths of unidirectional off-axis, angle-ply and symmetrically balanced laminates were measured at three different temperatures (25°C, 74°C and 114°C). Typical curves observed at selected temperatures are shown in Figs. 3 through 5. In Fig. 3 the typical stress-strain curves up to fracture are shown for unidirectional laminates (lamina) having different angles between the applied load and the fibers. It is seen that neither the strength nor the modulus of the on-axis (0°) specimens were influenced by temperature. This is typical of fiber-dominated composite behavior. A slight deviation in the fiber angle of 10° is sufficient to observe matrix dominated behavior and as a result both the strength and modulus are seen to be influenced by temperature. This temperature dependence is also seen for the 30° lamina; however, at 90° the strength of the lamina is so low that the influence of temperature is nearly indistinguishable.

The stress-strain curves of angle-ply laminates loaded along the symmetric axis are shown in Fig. 4. Because these laminates behave in a matrix dominated manner, all specimens showed a temperature influence on both strength and modulus. The magnitude of the temperature influence appears different for the two properties and thus, these properties appear to respond separately to temperature. The failure strength of angle-ply laminates is of course different from that observed for unidirectional off-axis laminates (Fig. 3). While the latter failed by the propagation of the first crack through the matrix, across the specimen, the former failed after the accumulation of a great number of cracks resulting in a much higher failure strength. Finally, it is interesting to note the form of the stress-strain curves for the $[\pm15°]_{2S}$ laminate. This angle-ply laminate has a very low inplane shear stress and transverse stress and thus, the curves are essentially linear. The laminate finally failed by delamination as the result of interlaminar stresses before the nonlinearity of the inplane shear could be observed.

Stress-strain curves of the symmetrically balanced laminates loaded in the direction of the symmetric axis are shown in Fig. 5. As these laminates have one half of their fibers in the direction of loading, the influence of temperature on the moduli is seen to be negligible. The strengths of these laminates are, however, influenced significantly by temperature. Because one half of the fibers are symmetric about the loading axis, the first crack does not propagate to failure but instead a number of cracks are initiated prior to laminate failure. Additionally, it is interesting to note that laminates having laminae oriented at high angles (60°, 75° and 90°) exhibit essentially identical behavior, suggesting that these laminae contribute little or nothing to the overall strength of the laminate.

The static strength parameters, $\sigma_T{}^s$ and τ^s, have been calculated from the data obtained on unidirectional laminates (Fig. 3) and angle-ply laminates (Fig. 4) and are shown in Table I. For unidirectional laminates these parameters were calculated using the static form of eqn. (3) where the fatigue functions, f_T and f_τ, are equal to one and the applied stress is the static stress rather than the cyclic stress. For the angle-ply laminates eqn. (3) was used in a similar manner with the stress in the laminae resolved using the static forms of eqns. (16) through (18). From the data on unidirectional laminates both the transverse tension and shear strength parameters were reduced by approximately 19 percent with increasing temperature from 25°C to 114°C. From the data on angle-ply laminates the strength parameters are seen in Table I to be greater than those observed for the unidirectional off-axis laminates. Additionally, an 18 percent reduction in the transverse strength and a 24 percent reduction in the shear strength is seen to occur upon increasing the temperature from 25°C to 114°C. As discussed previously, the lower values of the strength parameters for unidirectional, off-axis laminates is the result of the first crack initiating in the weakest portion of the laminate and propagating to failure. Obviously, the parameters determined using unidirectional data are conservative.

Table I. Static Strength Parameters

	Static Strength, σ^s (MPa)		
T(°C)	25	74	114
σ_A	1510	1510	1510
τ	74	65	56
σ_T	44	40	36

Fatigue Behavior

The tension-tension (R=0.1) fatigue behavior of unidirectional, angle-ply and symmetrically balanced laminates were studied in an effort to establish the influences of temperature and to verify the application of the relevant failure criterion. The observed fatigue behavior of unidirectional laminates is shown in Figs. 6 and 7. As with static strength behavior, unidirectional laminae loaded in the fiber direction exhibit a high fatigue strength which is relatively independent of the number of applied cycles (Fig. 6). The influence of temperature on the fatigue properties of this lamina was not investigated because it has been shown (Fig. 3) that its mechanical properties are essentially independent of temperature. The typical fatigue behavior of unidirectional, off-axis specimens tested at 25°C and 114°C is shown in Fig. 7. As can be seen, the data for these laminae exhibit significant scatter. This is most probably the result of the first crack propagating to failure as discussed earlier for the static failure of these laminates. Both temperature and off-axis angle are seen in Fig. 7 to have a significant influence on the fatigue strength of these laminates--increasing both the temperature and off-axis angle lowers the fatigue strength substantially.

The results of fatigue tests conducted on angle-ply laminates at three temperatures are shown in Figs. 8 and 9. As can be seen from these figures, both fiber angle and temperature have significant influences on the fatigue curves. In general the fatigue strength decreases with increasing fiber angle and increasing temperature. An anomaly appears to exist for the ±60° and ±75° angle-ply laminates where there is little or no difference between the data at 74°C and 114°C. Although there is much less scatter in the data for angle-ply laminates compared with the results obtained on unidirectional laminates, it is still difficult to determine whether this apparent anomaly is the result of data scatter or is in fact the result of a nonlinear temperature dependence.

The three fatigue functions--f_A, f_τ, f_T--calculated from the slopes of the S-N curves (Figs. 6 through 9) using eqn. (5) are listed in Table II. It now becomes possible to formulate to a first approximation shifting factors using the parameters tabulated in Tables I and II. From the static strength data the static strength shifting parameter, a_i^s, can be approximated by the simple ratio of temperatures, T_0/T. Thus, to a first approximation we may write;

$$\tau = \tau_0 \frac{T_0}{T}, \quad \sigma_T = \sigma_{T_0} \frac{T_0}{T} \tag{22}$$

where τ_0 and σ_{T_0} are the static strength parameters at some reference temperature, T_0. The fatigue functions can also be shifted through the use of eqn. (7) to yield

$$f_\tau = a_\tau(T)(C_{\tau_0} - b_{\tau_0} \log N)$$

$$f_T = a_T(T)(C_{T_0} - b_{T_0} \log N) \tag{23}$$

where the subscript "0" represents values at some reference temperature. Combining eqns. (22) and (23), we obtain the overall fatigue shifting relationships;

$$\tau^u = \tau_0{}^s \frac{T_0}{T} a_\tau(T)(C_{\tau_0} - b_{\tau_0} \log N)$$

$$\sigma_T{}^u = \sigma_{T_0}{}^s \frac{T_0}{T} a_T(T)(C_{T_0} - b_{T_0} \log N) \tag{24}$$

These equations have the general form of eqn. (12) where to a first approximation the shifting factor a^s is given by the ratio of temperatures, T_0/T. Using the experimentally determined parameters of Tables I and II and eqns. (24) and taking 25°C (298°K) as the reference temperature, the temperature dependence of the shifting factors, a_τ and a_T, are shown in Fig. 10. Although the data is sparce, the temperature dependence of a_T appears to be essentially linear and a_τ appears to be hyperbolic with the shear strength far more strongly influenced by temperature than the transverse tension strength.

Table II. Fatigue Functions

	Fatigue Functions, f_i		
T(°C)	25	74	114
σ_A	1-0.033 log N	1-0.033 log N	1-0.033 log N
τ	1.1-0.081 log N	1.15-0.086 log N	1.3-0.092 log N
σ_T	1.12-0.0897 log N	1.13-0.0901 log N	1.14-0.0904 log N

Although interlaminar failure has not been dealt with extensively in the above analysis, the ±15° angle-ply laminate was observed to exhibit interlaminar failure. Because such failures have received considerable attention in recent years, it is believed worthwhile to consider briefly this form of failure. From the stress-strain curve (Fig. 4) for the ±15° angle-ply laminate, the applied static strength is seen to be:

$$\sigma_{\pm 15^\circ}{}^{s} = 686 \text{ MPa}$$

Assuming the ratios between the applied external stress and the interlaminar stresses, $\tau_d{}^s$ and $\sigma_z{}^s$, are constant;

$$\tau_d{}^s, \sigma_z{}^s \alpha\, \sigma_{\pm 15^\circ}{}^{s}$$

the fatigue function for this laminate can be formulated and given by:

$$f_i = 1.08\text{-}0.1007 \log N$$

Some Failure Predictions

The failure criteria, analysis and experimentally determined static and fatigue parameters determined above can be used to predict the failure of the more complex and more useful symmetrically balanced laminates. Examples considered here will be the $[0°,\pm15°,0°]_s$, $[0°,\pm30°,0°]_s$ and the $[0°,\pm90°,0°]_s$ laminates. First we will consider the static strengths. For the $[0°,\pm15°,0°]_s$ laminate the static strength (Fig. 5) is much greater than that observed for the angle-ply laminate (Fig. 4) where failure occurred by delamination. The stress field in this symmetrically balanced laminate is much different than that in the angle-ply laminate and failure is in fact predicted by the 0° laminae alone (Fig. 3). For the $[0°,\pm30°,0°]_s$ laminate, failure is predicted to occur first in the ±30° laminae at 713 MPa as a result of inplane shear. This is the approximate stress where the first crack was observed (Fig. 5). With failure of these laminae the constraint of the 0° laminae no longer applies, the shear stress field is changed and total failure will occur by inplane shear at 919 MPa--in fair agreement with what was observed (Fig. 5). For the $[0°,\pm90°,0°]_s$ laminate, the ±90° laminae do not contribute to the strength of the laminate (Fig. 5) and failure is predicted by the 0° laminae alone (Fig. 3).

The predictions of the fatigue behavior of the symmetrically balanced laminates together with the experimentally observed behavior are summarized in Fig. 11. As can be seen, failure of the $[0°,\pm15°,0°]_s$ laminate is predicted with quite good agreement to occur by delamination and is based upon the parameters observed for the delamination of the ±15° angle-ply laminate. Fatigue failure of the $[0°,\pm30°,0°]_s$ laminate is seen to be reasonably predicted to occur by inplane shear. And failure of the $[0°,\pm90°,0°]_s$ laminate is predicted to occur by fatigue failure of the fibers in the 0° laminae.

Conclusions

The fatigue behavior of a graphite/epoxy composite material was analyzed theoretically and experimentally, with the temperature as a parameter. A rationale was introduced which included the influence of the temperature on the static strength and the fatigue functions. The fatigue functions describe the degradation of the principal strengths with cycles. Introducing temperature dependent shifting factors enables one to characterize the fatigue behavior with a single curve or formula. It was found that for many laminate configurations, use of the laminated plate theory together with the fatigue behavior theory introduced here, gave satisfactory results. For some laminates, where the inplane shear is very high, nonlinear relations must be used. It was also found that the inplane shear is most influenced by the temperature; the transverse tension is less influenced and tension in the fiber direction is not influenced at all. Another important finding is that even though some angle-ply laminates failed by delamination, when they were reinforced with uniaxial laminae, delamination did not occur even when the strain was higher than the critical value.

The theory and experiments presented here could be used to extend the fatigue behavior prediction to a high number of cycles from results at a low number of cycles and elevated temperature.

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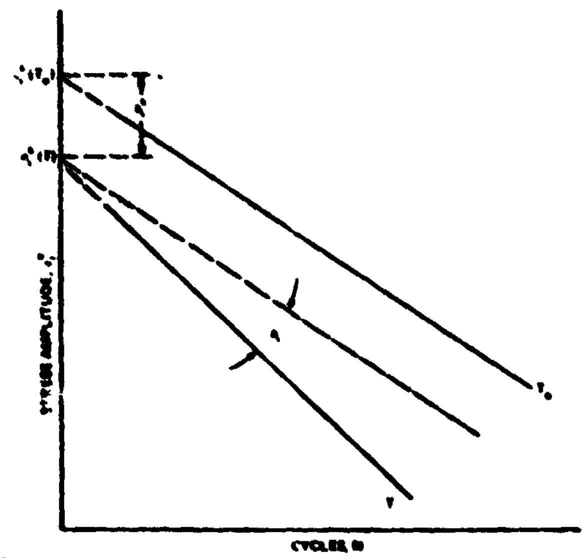

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Fig. 1 - Shifting of an S-N curve to a reference temperature T_0;
a_i^s - static strength shift and a_i - fatigue function shift.

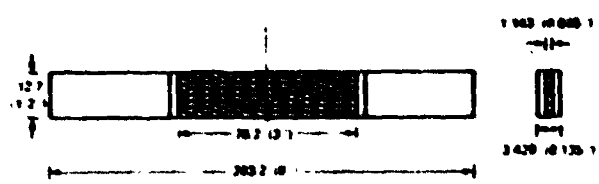

Fig. 2 - Test specimen with end tab.

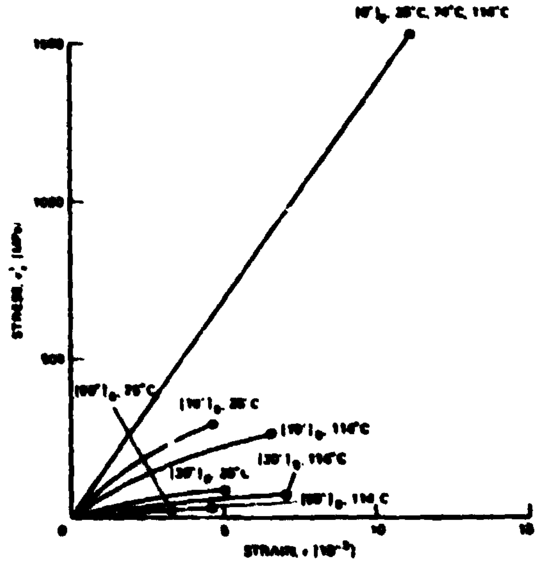

Fig. 3 - Stress-strain curves of unidirectional graphite/epoxy (T300/5208)
lamina at various off-axis angles and temperatures.

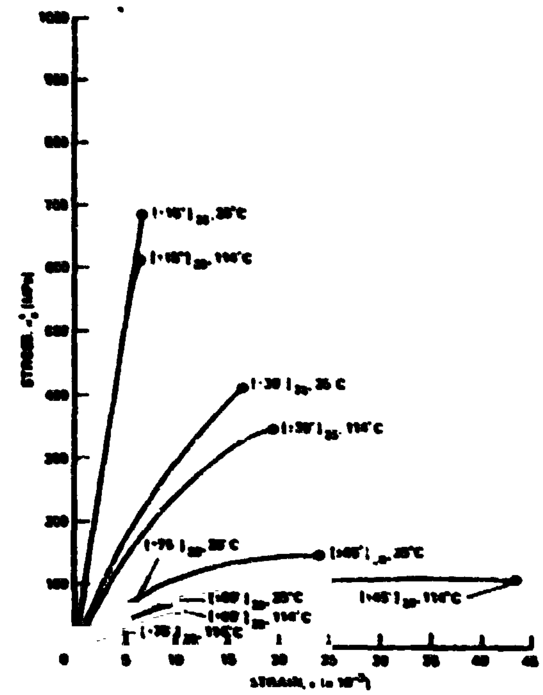

Fig. 4 - Stress-strain curves of various angle-ply graphite/epoxy (T300/5208) laminates at different temperatures.

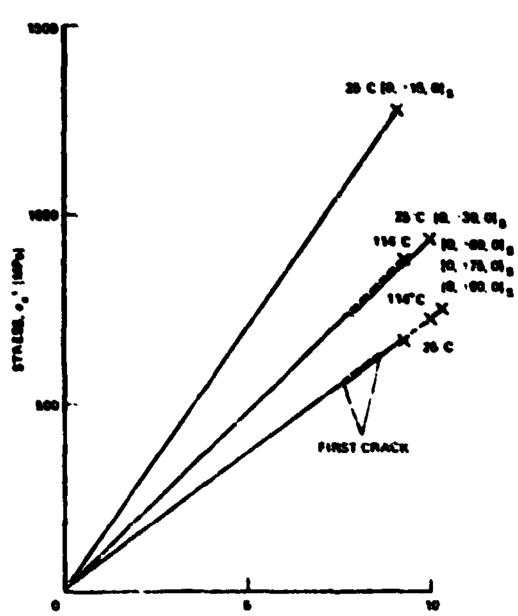

Fig. 5 - Stress-strain curves of various symmetrically balanced graphite/epoxy (T300/5208) laminates at different temperatures.

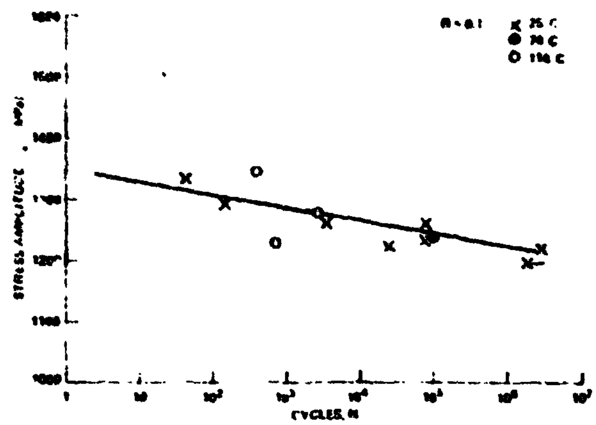

Fig. 6 - S-N curve of unidirectional lamina loaded in the fiber direction.

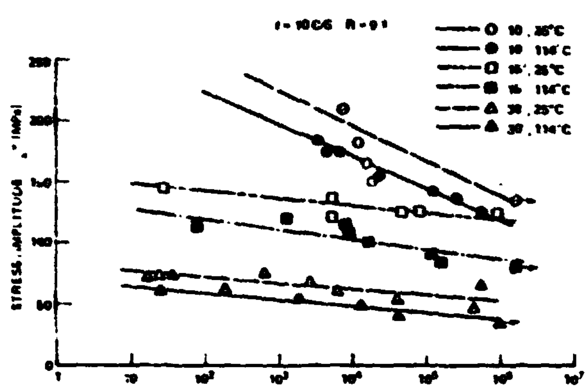

Fig. 7 - S-N curves of various off-axis unidirectional laminae at different temperatures.

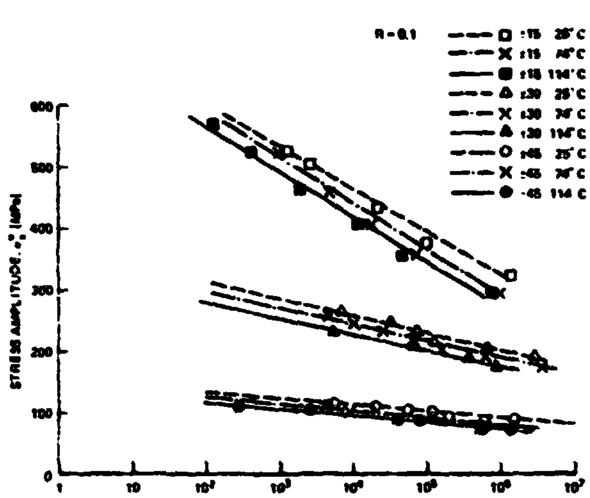

Fig. 8 - S-N curves of some angle-ply laminates at different temperatures.

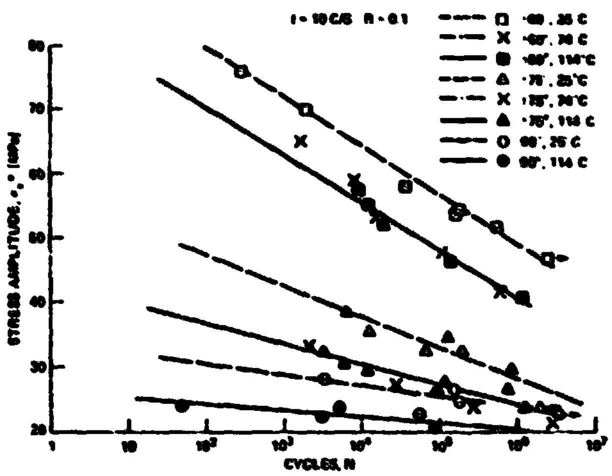

Fig. 9 - S-N curves of some angle-ply laminates at different temperatures.

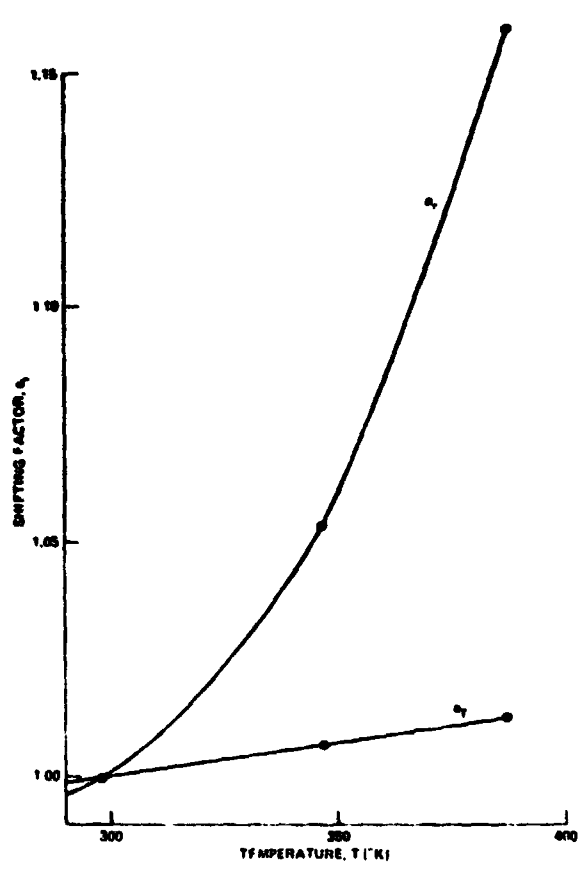

Fig. 10 - Shifting factors for inplane shear, a_τ, and transverse tension, a_T.

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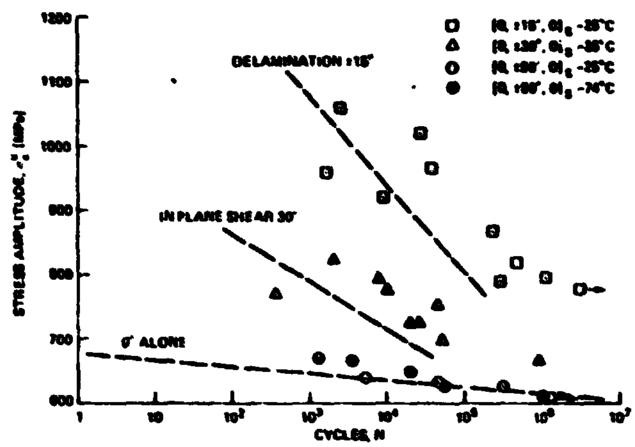

Fig. 11 - S-N curves of some multidirectional symmetrically balanced laminates.