PROGRESSIVE FAILURE OF STRUCTURES*

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SUMMARY

A procedure is presented for determining the nonlinear behavior of structures subjected to extreme loading and the possibility of development of potential for progressive failure. The methodology takes into account the effect of both material and geometric nonlinearities. At a given stage of analysis, the individual components of the structure are checked against predetermined failure criteria. Subsequently, the failing components are removed and the modified structure is analyzed for overall failure. Examples, obtained from a computer program based on the proposed procedure, showing the applicability of the method are presented.

SYMBOLS

Values are given in both SI and U.S. Customary Units. The calculations were made in U.S. Customary Units.

$\{F\},\{F^\circ\}$	nodal applied and equivalent force vectors, respectively
$[K],[K_G],[K_T]$	elastic, geometric, and total system stiffness matrices, respectively
k_2,k_1	slope of inelastic and elastic branch of stress-strain curve, respectively
$[m]$	consistent mass matrix
P_i	ith component of element nodal forces
$\bar{P}_1,\bar{P}_2,\ldots$	normalized stress resultants used in the yield criterion
$\{q\},\{\dot{q}\},\{\ddot{q}\}$	generalized displacement, velocity, and acceleration vectors, respectively

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Δt time increment

Φ plastic potential function

Subscripts:

est estimate

pre previous solution step

INTRODUCTION

The determination of the response of structural systems under externally applied loading, whether of static or dynamic nature, has been always of great concern to the structural engineers, especially when such response has extended into the nonlinear range. However, up to recent years the solution to only few simple problems had been obtained. This is due to the complicated nature of the problem which renders the classical methods of solution inapplicable.

With the advent of high speed computers in the past few decades, a more realistic solution of complex engineering problems has become an attainable goal. Consequently, numerous investigators have turned their attention to the solution methods for nonlinear structural problems. Some of the work done in this respect with regards to beam and frame type structures as well as plate structures can be found in references 1-8. In addition, numerous studies have been reported on the application of the finite element to nonlinear problems. Some of these studies deal primarily with the material nonlinearity while others outline methods for treating general problems. Notable among the first group are the works of Akyuz and Merwin (ref. 9), Argyris (ref. 10), Marcal (ref. 11) and Armen et al.(ref. 12). Among the works concerned with the latter category are the findings of Oden (ref. 13), Stricklin et al.(ref. 14), Marcal and collaborators (ref. 15), Bathe et al.(ref. 16), and Zienkiewicz et al. (ref. 17).

The nonlinear behavior of particular types of elements has also received the attention of the investigators in the field. Some of the works dealing with beam and frame type elements have been cited above. Other studies dealing with beams as well as plate type elements are the works of Toridis and Khozeimeh (ref. 18,19) Akkoush et al.(ref. 20), Marcal et al.(ref. 21), and McNeice (ref. 22).

In recent years some investigators have turned their attention to the question of structural damage and failure as the result of excessive loads and/or ensuing deformations which are well beyond the linear range or the acceptable design levels. In particular the effect of loss of certain supporting elements on the overall behavior of the structure has received due attention. These investigations have been motivated by the observations on the performance of actual structures in which such loss of elements has caused "progressive failure," a chain reaction type behavior, resulting in the collapse of the entire structural system. Recent examples of this type of behavior are the Ronan Point Modular Building collapse in England and the Skyline Towers High-Rise Building collapse in northern Virginia, U.S.A.

The study of existense of potentials for this type of failure is becoming more and more important as the concept of modular and panelized buildings gains in popularity. In this type of structures extensive use is made of premanufactured shear wall and floor panels that are interconnected to act as the basic load carrying systems, providing the three-dimensional rigidity of the building. The successful performance of such buildings depends on the behavior of the basic panels (elements) and the connecting system between the panels. It is, therefore, highly desirable to determine the performance of such structures under extreme loading and environmental conditions, in order to eliminate unsafe design practices. Since also the failure of one or more of the structural components or subassemblies gives rise to potential for progressive collapse and the ensuing disproportionate deformations, in studies dealing with such systems it is desirable to consider the ultimate strength properties of the structure prior, as well as after, the failure of some of its components.

In the present study, a procedure for determining the behavior and the potential for progressive collapse of the structural system subjected to extreme loading is formulated. The structure is modelled as an assemblage of beam and plate type elements and the response is found based on an incremental approach which allows for consideration of both material and geometric nonlinearity. At each stage of the structural deformation, failure criteria pertaining to excessive deformations, strength and stability of the structure are checked and parts of the structure that meet the appropriate failure criteria are removed and the remainder of the structure is checked against overall failure. The entire procedure is incorporated in a computer program.

GENERAL APPROACH

The response of the structural systems when subjected to high intensity loading generally extends into the nonlinear range. Consequently, in the analysis of such systems the effect of both material and geometric nonlinearities must be considered. Of special interest in the analysis is the incidence of abnormal loadings, i.e., loadings against which adequate measures have not been incorporated in the design. Such loadings, although infrequent, may lead to localized structural damage, which in turn may cause a "progressive" chain reaction type failure culminating in structural damages entirely disproportionate to the significance of the initiating cause. Thus to determine the complete nonlinear response of a structural system, its ability to form an alternative path to bridge any local damage must be studied.

The general approach adopted in this study to achieve the above objective is an extension of the work reported in references 18 and 19 and is based on the finite element method coupled with an incremental approach. To this end the structural system is modelled as a collection of beam and rectangular plate type elements. The three-dimensional beam element is used to model the skeletal frames while the plate element which is capable of simulating bending and/or in-plane action can be used effectively in representation of shear walls and floor panels, elements of construction which seem to become more important as trend towards modular, "panelized" construction continues. The detailed properties of the above beam and plate have been reported in references 4 and 18 and will not be repeated here. As shown in reference 23 the basic dynamical equations governing the behavior of a structural system can be obtained through the application of the Hamilton's Principle as applicable to discrete systems. In matrix form, these equations are expressed as

$$[m]\{\ddot{q}\} + ([K] + [K_G])\{q\} = \{F\} + \{F^O\} \tag{1}$$

where

$[m]$ = generalized consistent mass matrix of the structural system

$\{q\}$ and $\{\ddot{q}\}$ = generalized displacement and acceleration vectors, respectively

$[K]$ = generalized elastic stiffness matrix of the structural system

$[K_G]$ = generalized geometric stiffness matrix of the structure

$\{F\}$ = generalized nodal force vector corresponding to the externally applied loads

$\{F^O\}$ = equivalent generalized nodal force vector due to plastic strains, computed in accordance with the "initial stress or strain" method

In case of static loading, the above equation can be reduced to

$$([K] + [K_G])\{q\} = \{F\} + \{F^O\} \tag{2}$$

Based on equation (1) or equation (2), whichever governs the problem, the following incremental procedure to determine the behavior of the structure in the linear and nonlinear range is formulated.

Referring to equation (1) and based on the current configuration and the state of stress of the structural system, the components of the vector of the generalized accelerations, $\{\ddot{q}\}$, are found using the currently applied loads. Subsequently, the generalized displacement vector, $\{q\}$, is determined through a numerical integration procedure. The integration procedure utilized in this study is the Newmark's constant acceleration scheme (i.e., β=0) and can be expressed as

$$\{\dot{q}\}_2 = \{\dot{q}\}_1 + 0.5(\Delta t)[\{\ddot{q}\}_2 + \{\ddot{q}\}_1] \tag{3}$$

$$\{q\}_2 = \{q\}_1 + \Delta t\{\dot{q}\}_1 + 0.5(\Delta t)^2\{\ddot{q}\}_1 \tag{4}$$

In the above equations, Δt is the time step, the subscripts refer to the time stations, and $\{\dot{q}\}$ is the generalized velocity vector.

Having determined the vector $\{q\}$, the totals and the increments of the element nodal displacements and forces are obtained through the appropriate transformations and the current element stiffness matrices. The element nodal forces thus calculated are then used as an estimate to check for inelastic behavior in the element. To this end, the Mises yield criterion expressed in terms of the stress resultants is utilized. The normalized form of this criterion for a beam element is expressible as (ref. 19)

$$\Phi = [\bar{P}_1^2 + \bar{P}_2^2 + \bar{P}_3^2 + \bar{P}_4^2] = Y \tag{5}$$

where Y denotes the "yield value" which may change through straining, and $\bar{P}_1$, $\bar{P}_2$, $\bar{P}_3$, and $\bar{P}_4$ are the normalized form of the axial force, torsional moment, and bending moments about member y and z axes, respectively. A similar expression can be written in the case of a plate element.

If any element is undergoing inelastic deformation, its corrected force components and the corresponding contributions to the vector $\{F^o\}$ must be determined. This is done through a simplified approximate procedure known as "Proportioning Method" (ref. 19). In this approach, if Φ_{est} denotes the plastic potential function assuming elastic behavior, then an estimate of the increment of the plastic potential function due to current load/time increment, $d\Phi_{est}$, is found as

$$d\Phi_{est} = \Phi_{est} - \Phi_{pre} \tag{6}$$

where Φ_{pre} refers to the plastic potential function at the end of previous step. Then assuming a bilinear stress-strain relation and utilizing the "universal" stress-strain curve, the corrected plastic potential function, Φ, is determined as

$$\Phi = \Phi_{pre} + \frac{k_2}{k_1} d\Phi_{est} \tag{7}$$

in which k_1 and k_2 represent the slopes of the elastic and inelastic branches of the stress-strain curve, respectively. The basis of this simple procedure is explained in detail in reference 19.

Having found the final value of the plastic potential function, the ith component of the nodal forces, P_i, is determined by a proportioning process, i.e.,

$$P_i = \frac{P_{i_{est}}}{\Phi_{est}} \Phi \tag{8}$$

where $P_{i_{est}}$ is the estimate of the force component assuming elastic behavior. Furthermore, in the analysis the "Average Force Model" (ref. 19) is utilized. In the development of this model it is assumed that the entire element undergoes plastic deformations if the plastic potential determined from average value of stress resultants acting at the nodes exceeds the current yield value.

As is well known, the geometric nonlinearities can be attributed to two causes, namely the effect of large rotations and the contributions due to nonlinear strain-displacements (P-Δ) effects. The latter effects are accounted for in the present analysis through the inclusion of the geometric stiffness matrix, $[K_G]$. The entries to this matrix are directly affected by changes in the axial or in-plane force components acting on the elements of the structure. Consequently, this matrix is continuously updated in the solution process to reflect changes in the internal forces of the structure. To account for the effect of large rotations with its inherent change in geometry the total stiffness matrix, $[K_T]$, defined as the sum of elastic and geometric stiffness matrices, i.e.,

$$[K_T] = [K] + [K_G] \tag{9}$$

as well as the mass matrix and the equivalent force vector are regenerated through the use of the current transformation relations based on the deformed configuration of the structure.

In addition, before any new solution step is attempted, the modified configuration of the structure is determined. This is accomplished by checking the individual elements for excessive inelastic deformation and attainment of its ultimate strength which necessitates the removal of such elements from further consideration in the analysis. Also, the entire structure is checked for stability and functionality and all portions of the structural system that fail to meet the above requirements are also removed from consideration. Furthermore, if any modification is made to the structure, the appropriate system matrices are reformulated based on the latest configuration of the structure.

The entire procedure outlined above is incorporated in a general purpose computer program. The macro flow chart depicting the sequence of the operations is presented in Figure 1. As seen in the figure and based on the foregoing discussion, the procedure allows for removal of elements and nodes from further consideration. In case of an element, this is done by placing its number on the list of inactive elements and thus neglecting its contribution to system matrices in the subsequent solution steps. The removal of a given node entails two operations. First, all the elements incident to the node must be removed through the aforementioned procedure. Second, all the degrees of freedoms associated with the node must be eliminated to prevent the structural stiffness matrix from becoming singular. This is done in this study by introduction of artificial constraints at the node so that no degrees of freedom are assigned to the node in the subsequent analysis cycles. It should also be noted that in the case of static loading the analysis cycle refers to an increment of load rather than time.

FAILURE CRITERIA

A structural system or portions of it are said to have failed if certain prescribed conditions are violated. These may be based on strength requirements of individual parts of the structure or due to excessive displacements. Obviously, one of these possible modes of failure is instability. In general, instability is induced in the structural system composed of various members if the state of stress and deformation is such as to cause the system to lose its stiffness. This can come about if the axial or in-plane forces reach a critical value (buckling mode). Alternatively, the failure of a segment of the structure, perhaps through excessive deformation and formation of plastic hinges, may cause other portions or subassemblies of the structural system to undergo rigid body motion. However, irrespective of which mode of instability is encountered, the problem of stability can be formulated as the eigenproblem given by

$$([K] - \lambda[K_G]) = 0 \tag{10}$$

However, as has been pointed out by Gallagher (ref. 24), in the case of rigid body motion only, the total stiffness matrix will have eigenvalues of zero magnitude, and the corresponding eigenvectors will represent the rigid body modes. This fact is used to advantage in the present study to check for the potential of occurrence of rigid body motion whenever the determinant of the regenerated total stiffness matrix, $[K_T]$, approaches zero. If such rigid body motion occurs, the parts of the structure undergoing such motion are removed from consideration for the remaining time/load increments of the study.

Deformations are also of great importance as a criterion for determination of acceptable structural behavior. Traditionally, the formation of sufficient number of plastic hinges has been used as a measure of structural failure. However, since displacements and plastic deformations generally become extremely large before a structure becomes a true mechanism, failure criteria based on a count of plastic hinges are unsatisfactory. On the other hand, it is known that the distortions (displacements and rotations) in the structure increase a great deal just before the collapse load is reached. Therefore, a failure criterion based on the magnitude of structural distortions is more appropriate, especially if account is taken of the ability of the structure to strain-harden. This is indeed the approach adopted in this study. Unfortunately, very limited quantitive information is available in the literature on this subject; rather the investigators in the field have stressed the need for the experimental determination of such limits. In the present study, this problem is circumvented by requiring the input of the above information for a given structure based on the best available data and professional judgement.

The effect of inelastic deformations in a member or part of the structure can also be taken into account through the concept of the ductility factor or ductility ratio. Different definitions for the ductility ratio with respect to curvilinear and bilinear hysteresis curves have been reported in the literature (ref. 25-28). In the present study, the ductility factor is defined as the ratio of the maximum permissible or useful strain (or generalized strain/displacement) to the corresponding value at first yield. This factor is then used as a measure of failure in a structural component.

NUMERICAL RESULTS

To demonstrate the applicability of the proposed method, solutions to several structures have been obtained. Some typical results are reported herein. It should be mentioned that although the precedure is applicable to both beam and plate type structures, currently, only the beam elements have been fully incorporated in the computer program.

Example 1

The first example considered is a two story, two bay skeletal frame with dimensions as shown in Figure 2. All the girders are W 10x11.5 steel sections while the columns, with exception of the lower level interior column, are made up of W 8x20 sections. The lower level interior column is M 7x5.5 and all the steel is assumed to have a bilinear stress-strain relation with a yield point of 249 MN/m^2(36 ksi) with the slope of the inelastic branch being 0.01 times the corresponding value for the elastic branch. The columns are modelled by 3 equal elements per story and the girders are subdivided into 4 equal elements per bay. The loading consists of a uniform dead load, W1, distributed over the girders and a live load, W2, as shown in Figure 2. In the analysis, the distributed loads are replaced with equivalent nodal forces. The dead load is applied in 3 increments of 7.78 kN/m(0.53 kip/ft) each. This is then followed by application of live load increments of the same magnitude until the structure fails completely. The failure limits are set at 15.25 cm(6 in) and 0.2 radians for nodal displacement and rotation, respectively.

Figures 3A-F depict the sequence of structural modification due to propagation of failure. In Figure 3A the lower story inner column fails due to its ultimate strength being exceeded. Upon further loading, the girders start to fail due to strength requirements (Fig. 3B,C). Continuation of loading the structure leads to excessive displacements which necessitate the removal of a node (Fig. 3D) which in turn leads to rigid body motion and removal of further portions of the structure (Fig. 3E) and ultimate failure (Fig. 3F).

Example 2

The second example considered demonstrates the effect of a weak exterior column coupled with lateral loads. In this example the same frame as in the previous case is used except that the lower story right-hand side is considered to be a weak column (i.e., M 7x5.5 section) instead of the middle column. In addition, concentrated loads as shown in Figure 4A are applied to the structure. The loading sequence consists of 3 load increments of W1 = 7.78 kN/m(0.53 kip/ft) followed by 11 live load increments, W2, of the same magnitude. This is then followed by 6 load increments of H = 1.34 kN(0.3 kip). The failure pattern of this structure is depicted in Figures 4B to 4E. As can be observed, in this case the failure is not as extensive as in the previous example.

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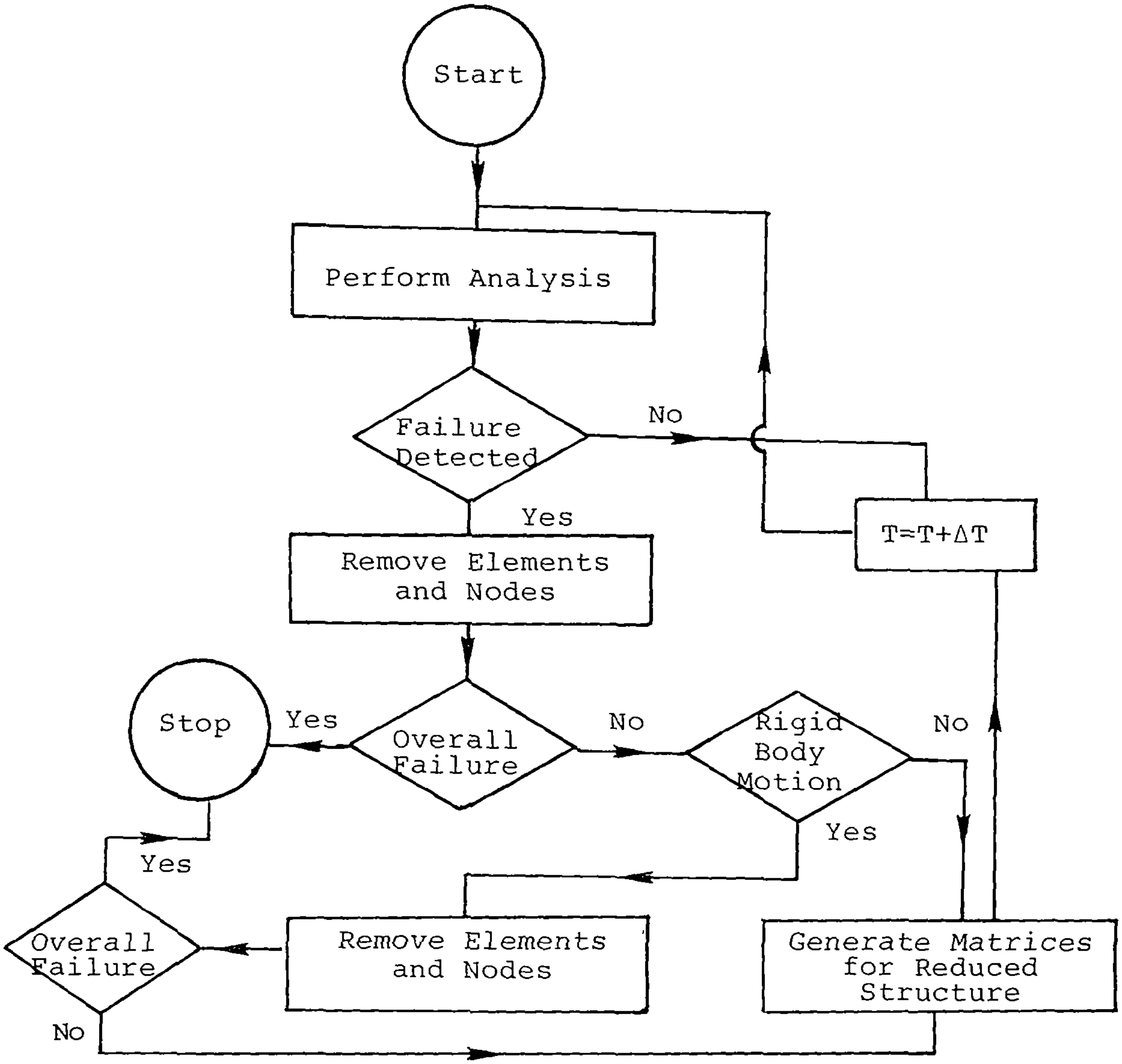


Figure 1.- Macro flow chart for progressive failure.

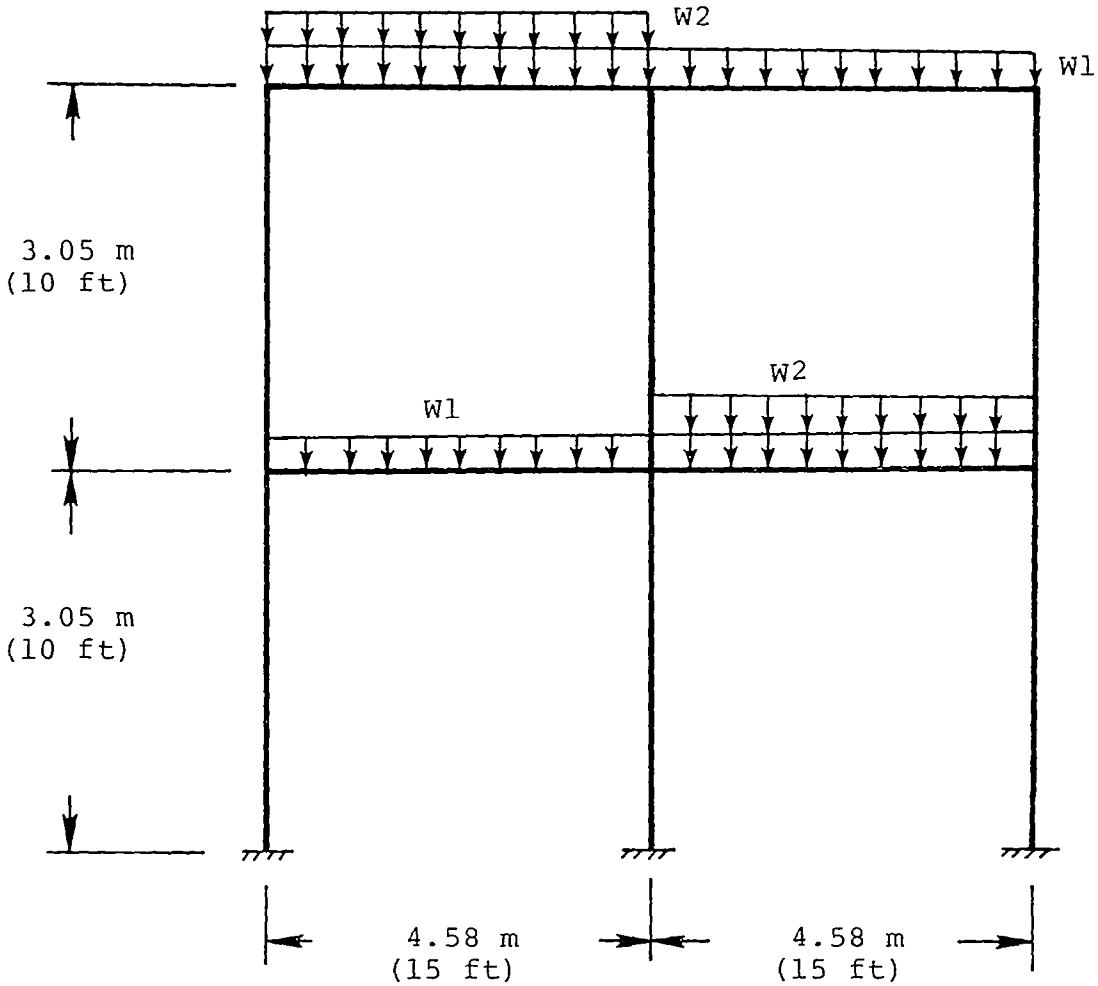
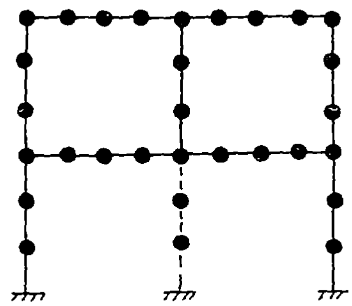
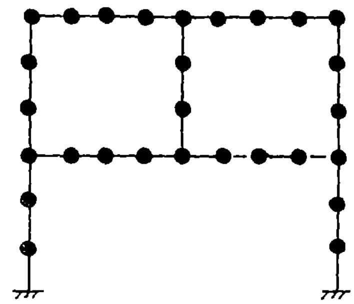


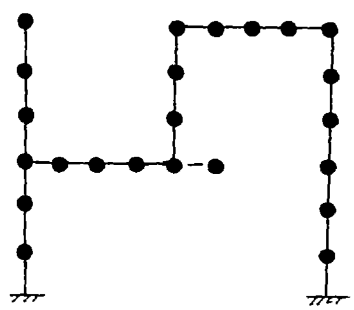
Figure 2.- Example structure.



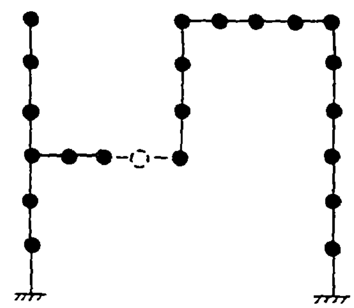
(A) W1 = 23.34 kN/m; W2 = 46.68 kN/m.



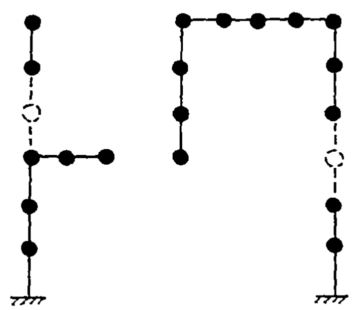
(B) W1 = 23.34 kN/m; W2 = 54.46 kN/m.



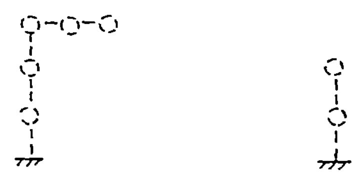
(C) W1 = 23.34 kN/m; W2 = 85.58 kN/m.



(D) W1 = 23.34 kN/m; W2 = 101.1 kN/m.

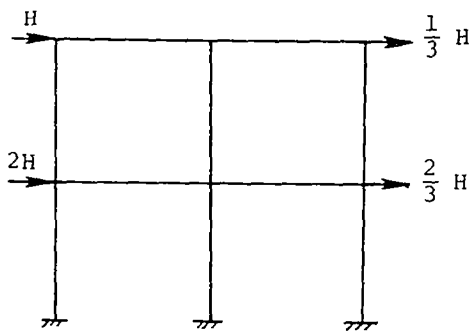


(E) W1 = 23.34 kN/m; W2 = 108.9 kN/m.

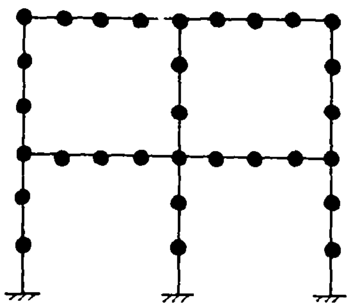


(F) W1 = 23.34 kN/m; W2 = 116.7 kN/m.

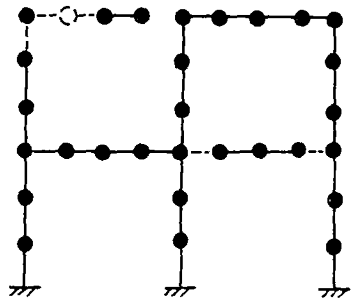
Figure 3.- Progression of failure in example 1.



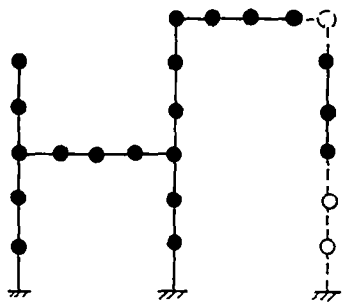
(A) Lateral loads.



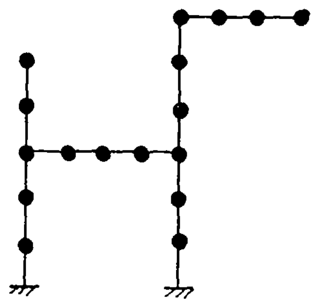
(B) W1 = 23.34 kN; W2 = 70.02 kN.



(C) W1 = 23.34 kN; W2 = 77.8 kN; H = 0 kN.



(D) W1 = 23.34 kN; W2 = 85.58 kN; H = 0 kN.



(E) W1 = 23.34 kN; W2 = 85.58 kN; H = 8.01 kN.

Figure 4.- Pattern of failure in example 2.