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PHOTON REFLECTION AT THE BOUNDARY OF AN INVERTED MEDIUM

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All measurement values are expressed in the International System of Units (SI) in accordance with NASA Policy Directive 2220.4, paragraph 4.

ABSTRACT

The index change near a quantum transition is shown to cause significant feedback into a gain medium. Criteria are given for the onset of self-lasing in an extended material. The distortion of a reflected pulse is also considered.

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This communication discusses the reflective effects that anomalous dispersion produces in a resonant medium. Near a quantum transition, the susceptibility of a medium as a function of frequency is composed of both real and imaginary parts, both of which are nonlinear in frequency. Depending on the inversion state of the population (gain or absorption), this susceptibility causes a change in the index of refraction (positive or negative). As a function of frequency, the index may be expressed by

$$n(\omega, \bar{r}) = n_o + \delta n(\omega, \bar{r}), \tag{1}$$

where n_0 is the index very far from the transition.

It can be shown that, for a homogeneously broadened transition, the index change, $\delta n(\omega, \bar{r})$, is given by:[1]

$$\delta n(\omega, \bar{r}) = \frac{n_o \bar{\gamma}(\omega_o)(\omega - \omega_o)\ c}{\omega_o \Delta\nu_h \left[1 + \frac{(\omega - \omega_o)^2}{\Delta\nu_h^{\ 2}} + \frac{|\mu_{ij}|^2\ |E(\bar{r})|^2\ T}{\hbar^2\ \Delta\nu_h} \right]} \ . \tag{2}$$

In equation 2, $\bar{\gamma}(\omega_0)$ is the line-center gain or absorption coefficient, $|\mu_{ij}|$ is the dipole matrix element connecting the two states, and $\frac{c\epsilon_0 n_0 |E(\bar{r})|^2}{2}$ is the spatially dependent intensity, $\Delta\nu_h$ is the homogeneous width of the transition, ω_0 is the resonance frequency, and T is a characteristic level lifetime.

When the field in the medium is well below the saturation intensity,

$$|E|^2 \ll \frac{16 n_o h \omega_o^3 t_{sp}}{\epsilon_o c T \Delta\nu_h} \; * ,$$

the index change may be approximated by:

$$\delta n(\omega) = \frac{n_o c \gamma(\omega_o)(\omega - \omega_o)}{\omega_o \Delta\nu_h \left[1 + \frac{(\omega - \omega_o)^2}{\Delta\nu_h^2} \right]} \; . \qquad (3)$$

Examination of equation 3 shows that the difference in the index that a low-level pulse of radiation sees can be as great as $2\delta n(\omega, \bar{r})$ when it goes from an inverted part of the medium to an equilibrium part. Such cases arise in most experiments that involve pumping of a medium. Moreover, even uniformly pumped regions that produce gain at some ω will have index changes relative to free space or air. The point to be made here is that this index change can result in significant reflection effects and can cause feedback into the gain medium. This reflection may be characterized as dielectric reflection from a nonmagnetic medium.

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* t_{sp} is the spontaneous radiative lifetime for the $i \rightarrow j$ transition.

Using Fresnel's equations governing the reflection and refraction at the interface of two media of indexes, n_0 and $n_0 \pm \delta n$; the electric field reflection coefficients are given by:[2]

$$r_{\parallel} = \frac{(n_o \pm \delta n)\cos\theta - n_o \cos\theta'}{(n_o \pm \delta n)\cos\theta + n_o \cos\theta'} \quad , \tag{4a}$$

$$r_{\perp} = \frac{n_o \cos\theta - (n_o \pm \delta n)\cos\theta'}{n_o \cos\theta + (n_o \pm \delta n)\cos\theta'} \quad . \tag{4b}$$

The symbols $\parallel$ and $\perp$ represent parallel and perpendicular polarizations at the interface, respectively, while θ and θ' are the incident and refracted angles.

Consider the situation of a low-level pulse propagating through a gain medium that abruptly interfaces with a medium of index 1. Assuming that the pulse strikes the interface at normal incidence and that $n_0 \approx 1$, from equations 3 and 4, the reflection coefficient is given by:

$$r \approx \frac{-\gamma(\omega_o)(\omega - \omega_o)\, c}{2\omega_o \Delta\nu_h \left[1 + \frac{(\omega - \omega_o)^2}{\Delta\nu_h^{\,2}} \right]} \quad . \tag{5}$$

When the frequency spread of the pulse is narrow ($\omega - \omega_0 \ll \Delta\nu_h$) and centered about ω_0; equation 5 can be approximated by:*

$$r \approx \frac{-\gamma(\omega_o)(\omega - \omega_o)\ c}{2\Delta\nu_h\ \omega_o} \quad . \tag{6}$$

It is clear from equation 6 that the pulse that is reentering the gain medium will be weighted in the frequency domain and therefore will not exhibit the same temporal characteristics of the pulse that made the last pass and continued through the interface. The reflected pulse can be examined in terms of its temporal characteristics by choosing as a prereflection pulse shape:

$$E_{in}(t) = E_o\, e^{-\alpha t^2}\, e^{i\omega_o t} \quad . \tag{7}$$

The foregoing pulse can be Fourier-transformed into:

$$\tilde{E}_{in}(\omega) = \frac{E_o}{2}\left(\frac{1}{\pi\alpha}\right)^{1/2} \exp\left[\frac{-(\omega - \omega_o)^2}{4\alpha}\right] \quad . \tag{8}$$

The pulse reentering the gain medium is then given by:

$$E_r(t) = \frac{-E_o\,\gamma(\omega_o)\ c}{4\omega_o\,\Delta\nu_h\sqrt{\pi\alpha}} \int_{-\infty}^{+\infty} (\omega - \omega_o)\ e^{\frac{-(\omega - \omega_o)^2}{4\alpha}}\ e^{i\omega t}\ d\omega \quad . \tag{9}$$

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*Such situations often arise when working with laser pulse and spontaneous emission amplification.

Expression 9 results in an intensity:

$$I_r(t) = \frac{E_o^2 c^3 n_o \epsilon_o \alpha^2 \gamma^2(\omega_o)}{2 \omega_o^2 \Delta\nu_h^2} \left[t^2 e^{-2\alpha t^2} \right] \quad (10)$$

The reflected pulse is distorted and has two lobes separated in time by $\frac{1}{\sqrt{\alpha}}$. When this signal reappears after passage through the gain medium, the output will exhibit a ringing type of behavior. Figure 1 shows pulses $I_{in}(t)$ and $I_r(t)$.

Examination of equation 6 shows that the effect described would be strongest in media that exhibit high gains and narrow linewidths. A good example is optically pumped CH_3F gas. Using the 9-μm, P(20) line of a CO_2 laser, CH_3F can be excited by the Q branch transition, Q(1, 12, 2). This results in gain at the $J = 12 \rightarrow J = 11$, $\Delta K = 0$ submillimeter wavelength (496 μ) transition. The system described exhibits gains on the order of 5 m^{-1} and a homogeneous linewidth of 3 MHz at pressures of about 0.1 torr.[3,4] The values stated result in a power reflection coefficient of $R \approx 10^{-3}\%$ for $\omega - \omega_o = \Delta\nu_h$. For other gases, in which the gain transition has extremely large values (~100 m^{-1}), R values as high as a few tenths of a percent may be obtained.

An interesting consequence of this effect is the possible self-lasing of a gas because of the described reflection coefficient. A simplified approach to establishing the criteria for laser oscillation is one of equating the round-trip

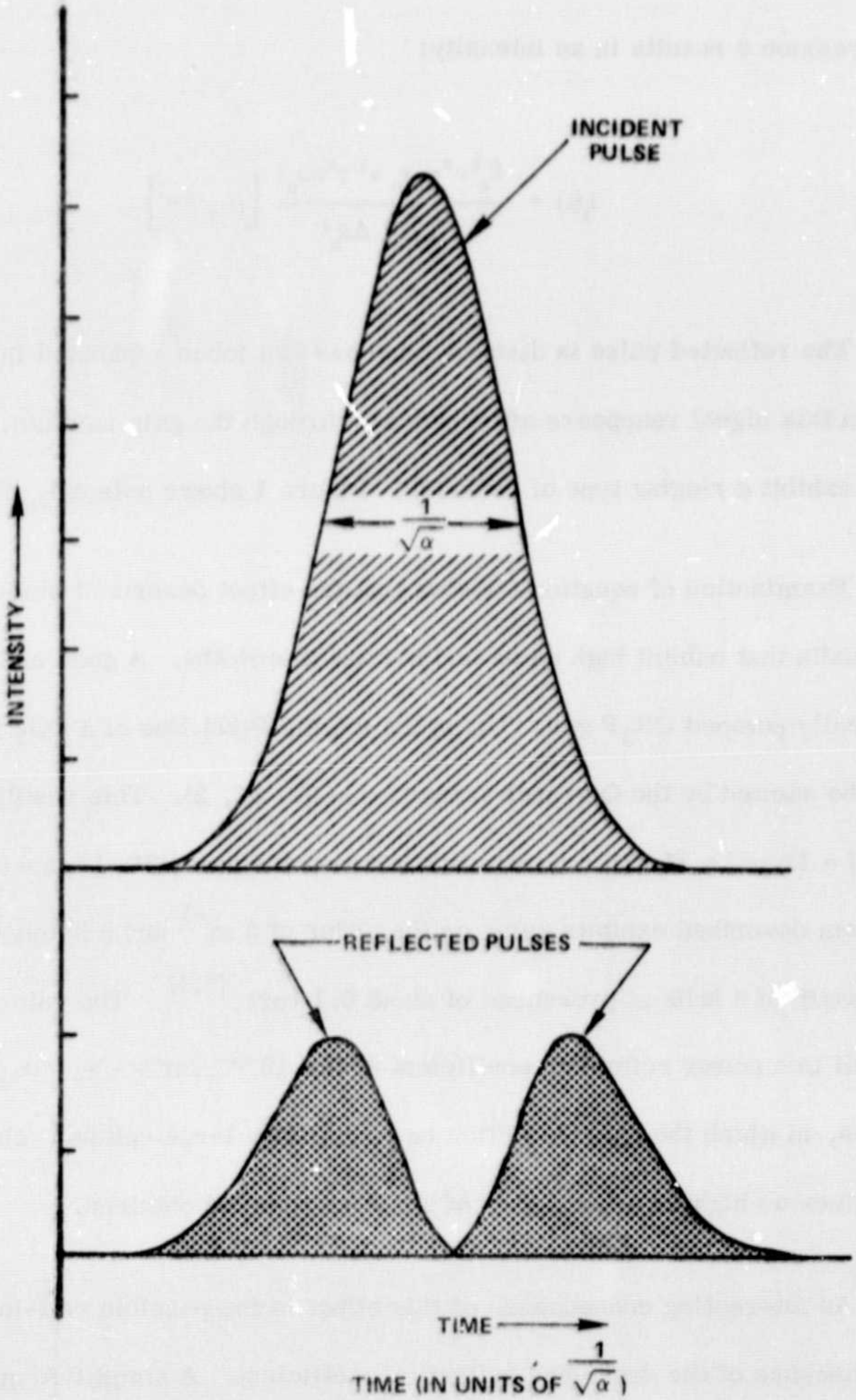

Figure 1. Incident and Reflected Pulse Shapes

losses encountered by a photon packet to the threshold gain needed. Calling the threshold gain, γ_t, and the active length of the medium, ℓ,

$$\gamma_t = -(1/\ell)\ \ln(r^2) \quad . \tag{11}$$

Use of the CH_3F parameters given requires a threshold gain-length product of ≈ 11.5 for oscillation. This fact suggests that any collection of molecules that exhibits some gain, γ°, can begin self-lasing if its predominant dimension, ℓ°, is such that:

$$\ell^{o} > \left(\frac{2}{\gamma^{o}}\right) \ln \left(\frac{\gamma^{o}(\omega - \omega_o)\ c}{2\omega_o\, \Delta\nu_h \left[1 + \frac{(\omega - \omega_o)^2}{\Delta\nu_h^{\,2}} \right]} \right) . \tag{12}$$

In conclusion, the anomalous dispersion near a transition has been shown to produce reflective effects at the interface of a gain medium and free space. The effect is strongest for high-gain, narrow-linewidth transitions. This could result in the self-oscillation of a gain medium by providing feedback for spontaneous emission. Moreover, the criterion is effectively one of dimension for a given transition. Such effects could play an important role in phenomena such as maser clouds and superfluorescence.

In addition, it has been shown that, because of its frequency dependence, this reflection filters out certain frequency components and therefore changes the temporal pulse characteristics. This causes the return envelope to be two pulses of nearly the same width as the incident pulse. In turn, these pulses will be amplified and narrowed after a second pass. This type of behavior is very much like the ringing observed in superradiance experiments on HF gas.[5]