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EFFECT OF WEIGHTING ON TIME SIDELOBE SUPPRESSION*

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SUMMARY

Weighting is a well known technique for shaping the compressed pulse waveform in radar processing. Usually, weighting is applied to the transfer function and has the effect of sacrificing the time mainlobe width (resolution) in exchange for decreasing the height of the neighboring sidelobes. This paper reports on simulations of weighting in the time domain, as used to shape the time-compressed pulse waveform. The digital input radar data is 32 bit I,Q, and simulates data from a point target as imaged by a Seasat-A type system. Weighting functions tested include stepped-amplitude distributions, (with 1 through 5 steps), and the cosine-squared plus pedestal distribution. Effects treated include mainlobe broadening, peak energy reduction, the integrated sidelobe ratio, signal to noise ratio, and nearest sidelobe suppression.

1.0 INTRODUCTION

Pulse compression in radar is commonly accomplished through matched filtering. A radar point target return $s(t)$ is correlated against a τ-shifted version of its conjugate $s^*(t)$ over a time interval T, to yield an output at time τ:

$$\tilde{s}(\tau) = \int_{-T/2}^{T/2} s(t)s^*(t - \tau)\, dt. \qquad (1)$$

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The estimate $\tilde{s}(\tau)$ yields a prediction of the return from a target at the location specified by τ. The function $|\tilde{s}(\tau)|$ usually has multiple peaks, and thus creates detection ambiguities when there is large signal dynamic range. For example, if $s(t)$ is a linear FM signal, $\tilde{s}(\tau)$ approximates a sinc function with high (-13.2 db) sidelobes.

The relation (1) may also be expressed in the frequency domain. Set

$$s_r(t) = s^*(-t).$$

Then (1) becomes

$$\tilde{s}(\tau) = \int_{-T/2}^{T/2} s(t)\, s_r(\tau - t)\, dt \tag{2}$$

$$= s \otimes s_r(\tau).$$

If capital letters denote fourier transforms, then (2) implies

$$\tilde{S}(w) = S(w) \cdot S_r(w). \tag{3}$$

The inverse transform of $\tilde{S}(w)$ is of course $\tilde{s}(\tau)$. In (3) we can multiply $S_r(w)$ by a weighting function $W(w)$ to obtain a new output S_1:

$$S_1(w) = S(w) \cdot S_r(w) \cdot W(w). \tag{4}$$

The inverse transform, $s_1(\tau)$, is now only an approximation to $\tilde{s}(\tau)$. The goal of frequency domain weighting is to select a function W in (4) such that $s_1(\tau)$ is a good local approximation to $\tilde{s}(\tau)$, but has lower sidelobes.

It is known that weighting of the time domain signal can produce similar effects, particularly if the signal is linear FM. In time weighting the weighting function is multiplied with the return signal

$$s_1(t) = s(t)\ w(t)$$

or with the reference function $s^*(t)$

$$s_1^*(t) = s^*(t)\ w(t),$$

prior to filtering.

This paper reports on simulations of time-weighting, performed on digital SAR data of the type expected from satellites such as SEASAT-A.

It is shown that use of a cosine-squared-plus-pedestal time-domain weighting scheme can result in sidelobe suppression down to -36 db, with mainlobe broadening of less than 30% for the simulated data.

Section 2 gives a brief description of the frequency domain weighting approach, which is then used to motivate the ideas on time weighting presented in section 3. Finally, section 3 contains the numerical results extrapolated from the simulations.

2.0 FREQUENCY DOMAIN WEIGHTING

Let the signal given by eq. (5) be received at the radar receiver at time t.

$$s(t) = \begin{cases} \exp\left\{ j\left(f_o t + \frac{bt^2}{2}\right)\right\} & \text{for } -T \leq t \leq T \\ 0 \text{ elsewhere.} \end{cases} \tag{5}$$

The matched filter response at time τ is

$$\hat{s}(\tau) = \int_{-T}^{T-\tau} e^{j\left(\frac{bt^2}{2} + f_o t\right)}\ e^{-j\left(\frac{b}{2}(t+\tau)^2 + f_o(t+\tau)\right)} dt. \tag{6}$$

By elementary calculation, we have

$$\tilde{s}(\tau) = 2Te^{-jf_o\tau} \cdot \frac{\sin\left(bT\tau - \frac{b}{2}\tau^2\right)}{bT\tau} \tag{7}$$

Usually, the range of interest is such that τ is small compared to T, so that the approximation

$$\tilde{s}(\tau) \doteq 2Te^{-jf_o\tau} \frac{\sin(bT\tau)}{bT\tau} \tag{8}$$

$$= 2T \text{ sinc } (bT\tau) \times (\text{phase factor})$$

is valid. Ignoring the phase factor, we see that $\tilde{s}(\tau)$ peaks at $\tau = 0$. Thereafter, sidelobes 13.2 db below the peak intensity occur near $\tau = \pm 3\pi/2bT$.

These sidelobes can mask weaker peaks corresponding to neighboring targets, as shown in Figure 1.

By slightly mismatching the filter response, (and hence sacrificing signal to noise ratio and mainlobe width), the sidelobe levels of the filter output can be decreased. The process is most easily understood when mismatching is

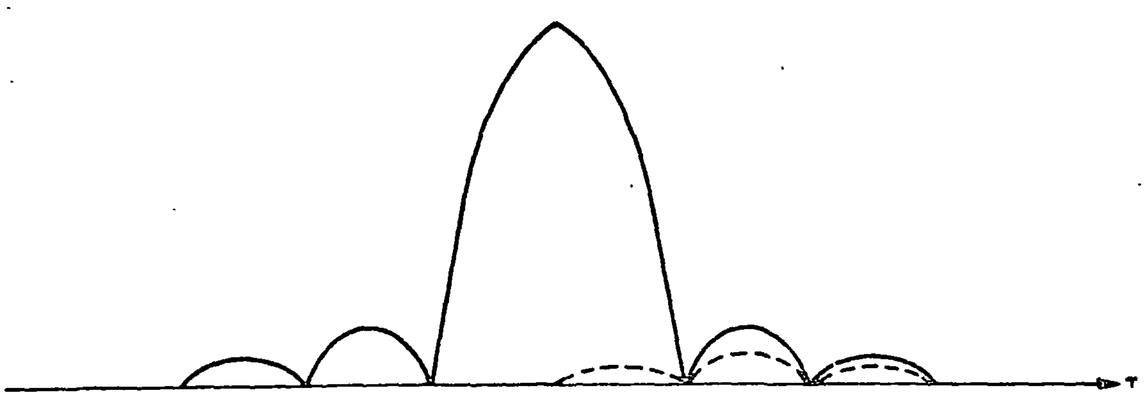

Fig. 1. Sidelobes Masking Weak Peaks

accomplished by weighting in the frequency domain [5]. Let $s(t)$ represent the output of a matched filter. Its fourier transform is given by

$$S(w) = \int_{-\infty}^{\infty} s(t) \exp\{-j2\pi wt\}\, dt. \tag{9}$$

(We assume $S(w) = 0$ for $|w| > B/2$). By the fourier shift theorem the function

$$S_1(w) = S(w)\left[1 + a_n \cos \frac{2\pi n w}{B}\right] \tag{10}$$

has an inverse fourier transform given by

$$s_1(t) = \frac{a_n}{2}\, s\left(t + \frac{n}{B}\right) + s(t) + \frac{a_n}{2}\, s\left(t - \frac{n}{B}\right). \tag{11}$$

(Here n is an integer and a_n is arbitrary).

Thus $s_1(t)$ is the original matched filter output with paired echos superimposed at a time offset of $\pm n/B$. The echos are scaled by a factor of $a_n/2$. In the case where $s = \hat{s}$, the sinc function in (8), then $\hat{S}(w)$ is a rectangle function (except for a shift factor due to the phase offset). Taking $n = 1$, we obtain from (8):

$$\hat{S}_1(w) = \hat{S}(w)\left[1 + a_1 \cos \frac{2\pi w}{B}\right] \tag{12}$$

and

$$\hat{s}_1(t) = \frac{a_1}{2}\, \hat{s}\left(t + \frac{1}{B}\right) + \hat{s}(t) + \frac{a_1}{2}\, \hat{s}\left(t - \frac{1}{B}\right)$$

$$= \frac{a_1}{2} \operatorname{sinc}\left(BT\left(t + \frac{1}{B}\right)\right)$$

$$+ \operatorname{sinc}(BTt) + \frac{a_1}{2} \operatorname{sinc}\left(BT\left(t - \frac{1}{B}\right)\right).$$

(Here B is proportional to T, by virtue of (8)). When the three sinc functions are added, a new "quasi-sinc" function results with lower sidelobes. (See figure 2).

Thus weighting by $W(w) = [1 + a_1 \cos 2\pi w/B]$ can lower the time sidelobes. In the radar case, we would actually be confronted with a superposition of sinc functions at the unweighted filter output. Each sinc function would be centered on its own resolution element. However, by the linearity of the fourier transform operation, one weighting function applied to the superposition spectrum serves to suppress the sidelobes of all sinc functions at once.

3.0 TIME DOMAIN WEIGHTING FOR TIME SIDELOBE SUPPRESSION

Time domain weighting is accomplished by multiplying the return signal or time reference function by a properly designed weighting function. Since the weights can easily be incorporated as part of the time domain reference function, the technique is a promising one for application to real-time spacecraft on board processors. The effect of time weighting on time output can often be easily determined [1].

For example, let the radar signal s(t) be the linear FM signal given in (5). Then, in a rough manner of speaking, there is a one-to-one correspondence between time and frequency. That is, each time interval Δt corresponds to a frequency interval Δw in the spectrum. Hence a time weighting function W(t) would have about the same effect as the frequency weighting function W(t = w). Thus a first order estimate of the effects of time weighting by W(t) may be obtained through

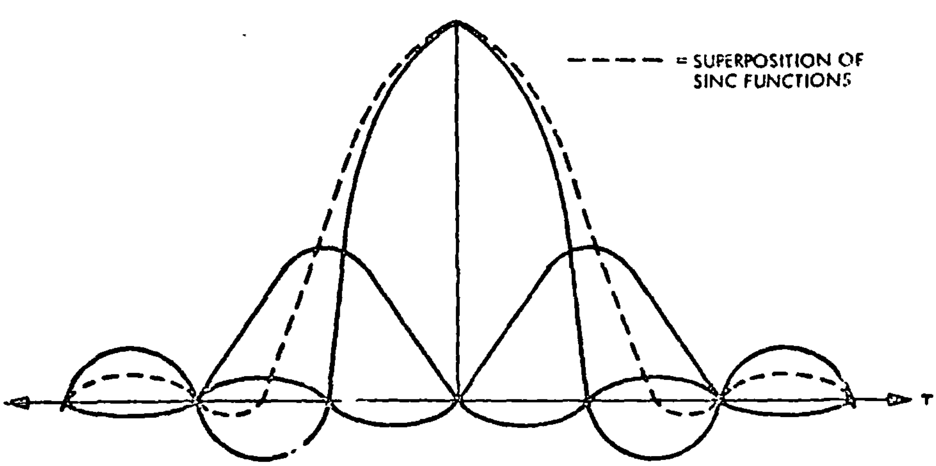

Fig. 2. Superposition of Sinc Function and Scaled Echo

frequency weighting with $W(v)$. More precisely, it is known that for the linear FM signal (5), the filter time output $g(t)$ resulting from a time weighting $w(t)$, satisfies

$$|g(t)| = \sqrt{\frac{b}{2\pi}} \left| \int_{-T}^{T} w(\tau) \exp \{jbt\tau\} \, d\tau \right|. \tag{13}$$

For further details, see [1] p. 187.

In SAR processing, the azimuth aperture is generated by a sensor moving linearly with constant velocity over the aperture length. Thus, to a reasonable approximation, the azimuth matched filter function is a linear FM chirp, and the above time weighting approximations (e.g. (13)) apply. However, the azimuth chirp is a sampled (discrete) function, and the results are somewhat different. Therefore, to discover the effects of weighting with discrete time functions, actual simulations were performed. The weightings were applied along the azimuth direction of simulated SEASAT-A SAR data. Each data input point was quantized to 4 bits, as was each reference and weighting coefficient. The ground spacing between adjacent data input points was around 4 meters. A total of 4096 input points was used to process a 4-look output data point. Thus each look required filtering of 1024 points, or 1024 separate weighting coefficients. The spacing between output data points was around 16 meters. The simulated input data was designed to represent the return from a point target so that the effects of weighting could be easily observed. The output image was 16 (range) by 512 (azimuth) points.

The weighting functions selected for simulation were the stepped amplitude distributions and the cosine - squared plus pedestal function [3], [4], [5].

The stepped amplitude distributions are given in table 1 for 1 through 5 steps. (See also figure 3). These distributions were chosen for their optimality in antenna pattern adjustment, and their ease of implementation [3]. Note that T is proportional to N = 1024, the number of weights.

TABLE 1

STEPPED AMPLITUDE TIME WEIGHTING FUNCTIONS

	a_1	a_2	a_3	a_4	a_5	b_1	b_2	b_3	b_4	b_5
	1	...	...	...	...	1	...	...	...	...
2	0.5	0.5	...	...	...	1	0.55	...	...	...
3	0.35	0.35	0.30	...	...	1	0.625	0.350	...	...
4	0.25	0.25	0.25	0.25	...	1	0.78	0.56	0.34	...
5	0.300	0.225	0.235	0.170	0.070	1	0.72	0.54	0.36	0.18

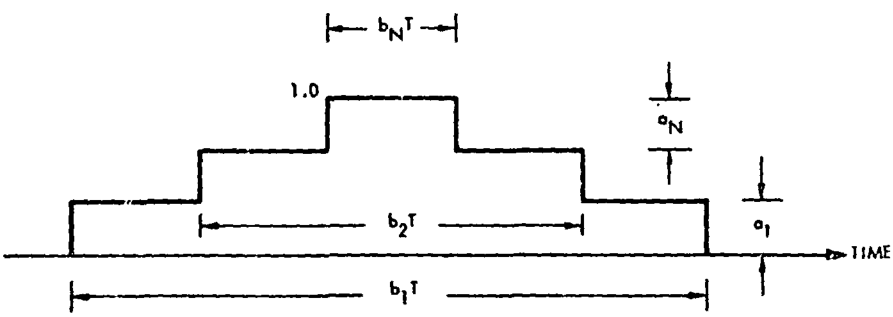

Fig. 3. Stepped Amplitude Time Weighting for Time Waveform Shaping

The cosine squared-plus-pedestal distribution [5] is, through a trigonometric identity, equivalent to the weighting given by the right-hand factor in eq. (10). It is defined by

$$W(t) = H + (1 - H) \cos^2 (\pi t/T) \tag{14}$$

where

H = pedestal height ($0 \leq H < 1$)

T = pulse timewidth.

For the simulation, we select H = 0.08 to approximate Hamming weighting, which produces the lowest sidelobes attainable with this type weighting in the frequency domain [5]. The resulting weighting is similar to the Taylor approximation to the physically unrealizable optimum Dolph - Chebyshev weighting [2], [6], and represents a practical approach for digital processing.

Table 2 gives the total energy in decibels in the 3 center lines about the peak. (By "line" is meant a range line of 16 data points centered around the azimuth peak. See figure 4.)

TABLE 2

PEAK ENERGY AND INTEGRATED SIDELOBE RATIO

Weighting Technique	Total Energy in 3 Center Lines	Peak Energy
Rectangular	103.109	100.067
2 step amplitude	103.458	99.667
3 step amplitude	103.514	99.415
4 step amplitude	103.549	99.359
5 step amplitude	103.552	99.362
$\cos^2$ + pedestal	103.478	98.748

Note: Total energy in 512 lines is 103.671 db for each type of weighting.

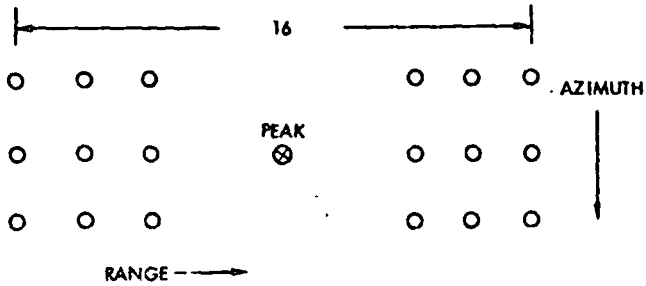

Fig. 4

Since the total energy in each 16 x 512 image was normalized to 103.671 db, the energy in the center lines reveals in a rough way how much energy is left over for sidelobe generation. This data can of course be used to compute the "integrated sidelobe ratio" in decibels, defined by

$$\text{Integrated Sidelobe Ratio} = 10 \log_{10} \left(\frac{\text{total energy outside mainlobe}}{\text{total energy in mainlobe (3 center lines)}} \right)$$

The results are summarized in table 3, for both 4 and 32 bit quantization, and give a fair estimate of the 2-dimensional integrated sidelobe ratio.

TABLE 3

INTEGRATED SIDELOBE RATIO, DB FOR VARIOUS WEIGHTINGS

Weighting	Integrated Sidelobe Ratio	
	4 bits	32 bits
Rectangular (no weighting)	- 12.1 db	- 12.8 db
2 - step amplitude	- 13.3 db	- 14.3 db
3 - step amplitude	- 14.5 db	- 16.1 db
4 - step amplitude	- 15.6 db	- 17.8 db
5 - step amplitude	- 15.7 db	- 18.0 db
$\cos^2$ + pedestal	- 17.3 db	- 21.7 db

Table 2 also gives an indication of loss in signal to noise ratio in terms of peak energy. Since each weighted output has the same total energy, the peak energy in the second column indicates the loss in SNR rather accurately. The greatest loss (1.319 db) relative to the unweighted case occurs with $\cos^2$ + pedestal weighting.

Table 4 is used to derive the mainlobe broadening resulting from weighting. As expected, the worst broadening is seen to occur with $\cos^2$ + ped weighting, with only a 3 db suppression 16 meters from the peak. In this case the 3 db resolution is degraded to 30 meters.

The values in Table 4 can be used to derive fairly precise values for the resolution achieved with each weighting scheme, in the following manner. The output filter function is modelled as a sinc function which is closely tracked in the range of interest by a quadratic. Using Lagrange's interpolation polynomial, with the peak and 2 nearest neighbors as input points, the distance to the 3 db mainlobe threshold is calculated. The results are shown in table 5.

TABLE 4

SUPPRESSION AT NEAREST NEIGHBOR, DB

Weighting Technique	Suppression, db	
	min	max
Rectangular	9	9
2 step amplitude	5	6
3 step amplitude	5	5
4 step amplitude	5	5
5 step amplitude	5	5
$\cos^2$ + pedestal	3	4

TABLE 5

RESOLUTION ACHIEVED WITH WEIGHTING

Weighting	Resolution
Rectangular (No weighting)	24 m
2 - step amplitude	27.2 m
3 - step amplitude	28.8 m
4 - step amplitude	28.8 m
5 - step amplitude	28.8 m
Cos^2 + pedestal	30.4 m

Finally, table 6 gives the sidelobe suppression actually achieved with each weighting scheme. Each technique generated 2 sidelobes of generally different heights; hence the min and max suppressions are both tabulated. The best suppression is achieved by cos^2 + pedestal weighting, with a -36 db measurement at the highest sidelobe. (The sidelobe asymmetry results from the true peak occurring between two data output points).

TABLE 6

NEAREST SIDELOBE SUPPRESSION, DB

Weighting Technique	Sidelode Suppression, db	
	min	max
Rectangular	20	-24
2 step amplitude	22	25
3 step amplitude	26	29
4 step amplitude	30	33
5 step amplitude	29	33
Cos^2 + pedestal	35	40

4.0 CONCLUSION

Weighting in the time domain for time sidelobe suppression presents an attractive alternative to frequency domain weighting, especially if time-domain azimuth processing is to be accomplished. The simulations show that $\cos^2$ + pedestal weighting achieves 36 db sidelobe suppression with a SNR loss of less than 1.5 db, and resolution loss of around 25%.

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