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EXPERIMENTAL CLEAN COMBUSTOR PROGRAM - PHASE III

Turbulence Measurement Addendum

-- FINAL REPORT --

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by
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1.0 SUMMARY

Airflow turbulence parameters were measured in the high-pressure, high-temperature flow stream leaving the compressor of an operating gas turbine engine. A water-cooled hot film turbulence probe was used to determine the turbulence intensity and length scale in the compressor exit flow stream of the CF6-50 engine. Data were obtained only at idle operating conditions. At engine power levels above idle conditions, durability problems and erratic data readings were encountered with the turbulence measurement probes. Turbulence measurements were made at three radial immersions at a point 15.5-cm downstream of the compressor outlet guide vanes in the compressor exit diffuser. The passage height at this point is 5.54 cm. Data reduction was accomplished with a Time-Data Fast Fourier Transform (FFT) system. This system finds the power spectral density distribution (PSD) of a large number of data samples using a direct Fourier transform algorithm and finds the autocorrelation parameter for these data by doing an FFT analysis of the PSD curves for a series of time delay intervals. At idle operating conditions, the measured turbulence intensity values varied from about 4.8 percent to about 5.6 percent of the average stream velocity at the measurement point, and the length scale values varied from 5.64 cm to 6.95 cm. A large proportion of the turbulent energy at the measurement plane is concentrated at frequencies below one kilohertz.

These turbulence data can be used to simulate compressor exit flow conditions in combustion system test rigs when testing low emissions combustion systems at engine idle points. To simulate flow conditions at high-power engine cycle points, additional turbulence data are needed at the high-power engine operating conditions. A considerable amount of probe development work will be required to develop turbulence measurement probes that can withstand the very severe environment in the compressor exit flow stream at high-power engine operating conditions.

2.0 INTRODUCTION

The purpose of this program was to measure the turbulence intensity and scale in the compressor exit flow stream of an operating gas turbine engine. This program was conducted as an addendum to, and concurrently with, Phase III (Contract NAS3-19736) of the NASA/GE Experimental Clean Combustor Program (Reference 1).

Compressor exit turbulence data are required for the development of lean burning, premixed, prevaporized combustion systems that have low levels of nitrogen oxide (NO_x) emissions. Experimental studies have demonstrated the feasibility of achieving very low NO_x emissions levels with lean burning, premixed, prevaporized combustors (References 2 and 3). Rapid dispersion of liquid fuel droplets in premixer ducts may be accomplished by turbulent diffusion of the fuel droplets across the premixer duct airflow. Also, high-intensity turbulence improves fuel droplet evaporation rates. The systematic development of very lean burning, premixed, prevaporized combustion systems to the point of practical application to advanced gas turbine engines requires quantitative knowledge of compressor exit turbulence parameters and the effects of turbulence intensity and scale on fuel preparation, the premixing process, and fuel droplet evaporation.

Prior to this program, compressor exit turbulence test data have not been available, probably because compressor exit flow conditions represent a very severe environment for turbulence measurement instrumentation. Flow velocities, temperatures, and pressure levels are very high. Also, high vibration levels and solid particles in the airstream can destroy fragile instrumentation very quickly.

Ruggedized cooled film probes were used in this program to measure CF6-50 compressor exit turbulence properties at three different engine idle condition test points. Data were also obtained with this probe at the 30 percent and at the 85 percent engine power settings but, unfortunately, the quality of the data for these conditions was not acceptable. These measurements were made by using an electric motor-driven probe actuator to set three "equal area" radial immersions in the compressor exit diffuser flow stream at an axial location 15.5-cm downstream of the compressor exit plane.

These turbulence data can be used to help simulate compressor exit flow conditions in development test programs for advanced combustion systems. These results also demonstrate the feasibility of using ruggedized cooled film probes to make turbulence measurements in the high-pressure, high-temperature, and high-velocity environment of an operating gas turbine engine.

3.0 EXPERIMENTAL EQUIPMENT AND PROCEDURES

This turbulence measurements program was conducted as a part of the NASA-sponsored engine test program to test the Double Annular Low Emissions Combustor (Reference 1). The engine used for this Experimental Clean Combustor Program (ECCP) test program was a General Electric CF6-50 high bypass turbofan. Cycle data at the compressor exit station for four CF6-50 ground level operating conditions are presented in Table 1. Pressures and temperatures at the compressor exit plane range from 2.9 atmospheres and 429 K at idle conditions up to 29.8 atmospheres and 821 K at full-power conditions.

The turbulence measurement probe was installed in an existing instrumentation port in the CF6-50 engine compressor rear frame to obtain the turbulence data. A cross-sectional view of the CF6-50 compressor exit region, showing the location of this hole, is illustrated in Figure 1. This port is located 15.5-cm downstream of the compressor exit plane, near the plane of the frame strut leading edges, but at a circumferential spacing that is well removed from the aerodynamic influence of the strut contours. At each engine test condition, the sensing element of the probe was moved to three radial immersion points in the compressor exit flowpath. Each immersion point was located at the center of one-third of the flowpath cross-sectional area.

A water-cooled hot film probe design was selected as the primary sensor for the turbulence measurements. This cooled film probe, which was supplied by Thermo-Systems, Inc., is illustrated in Figure 2. The internal cooling fluid provides a built-in heat sink which allows measurements in extremely high-temperature gases. The maximum frequency response of this probe is about 14 kilohertz. The cooled film sensor is formed from a ceramic coating on a metal tube with a thin platinum film on the surface. Gold plating isolates the short platinum sensing element. The water coolant is discharged into the hot gas stream.

Although this probe design is very sturdy and can withstand high velocity particle-laden flows at high temperatures and high pressure levels, several probe failures occurred at the engine operating conditions. These failures involved separation of the cooled support tube from the sensor tube at the soldered joint between the tubes and a consequent loss of electrical continuity. To remedy this problem, the joint was reinforced by wrapping several loops of very fine wire around the tubes at the joint. The loops of wire were held in place with high-temperature epoxy cement. This "ruggedized" version of the cooled film probe was then used to measure the engine turbulence parameters.

Platinum-iridium hot wire probes were also tested during this program. Specifications and quantities of the hot wire and cooled film probes procured for this program are presented in Table 2. The small diameter hot wires did not survive at engine test conditions. One hot wire probe with a one-half

Table 1. CF6-50C Reference Engine Cycle.

Cycle Point	Idle	Approach	Climb	Takeoff
Engine Thrust, kN	7.42	67.27	190.5	224.2
% Thrust	3.31	30.0	85.0	100.0
W_3, kg/s	16.56	57.28	108.0	119.6
P_{T_3}, MPa	0.300	1.197	2.616	2.983
T_{T_3}, K	437	632	792	826
T_{T_4}, K	838	1136	1525	1617
W_{comb}, kg/s	13.93	48.17	90.81	100.6
W_{fuel}, kg/s	0.1526	0.6645	1.953	2.376

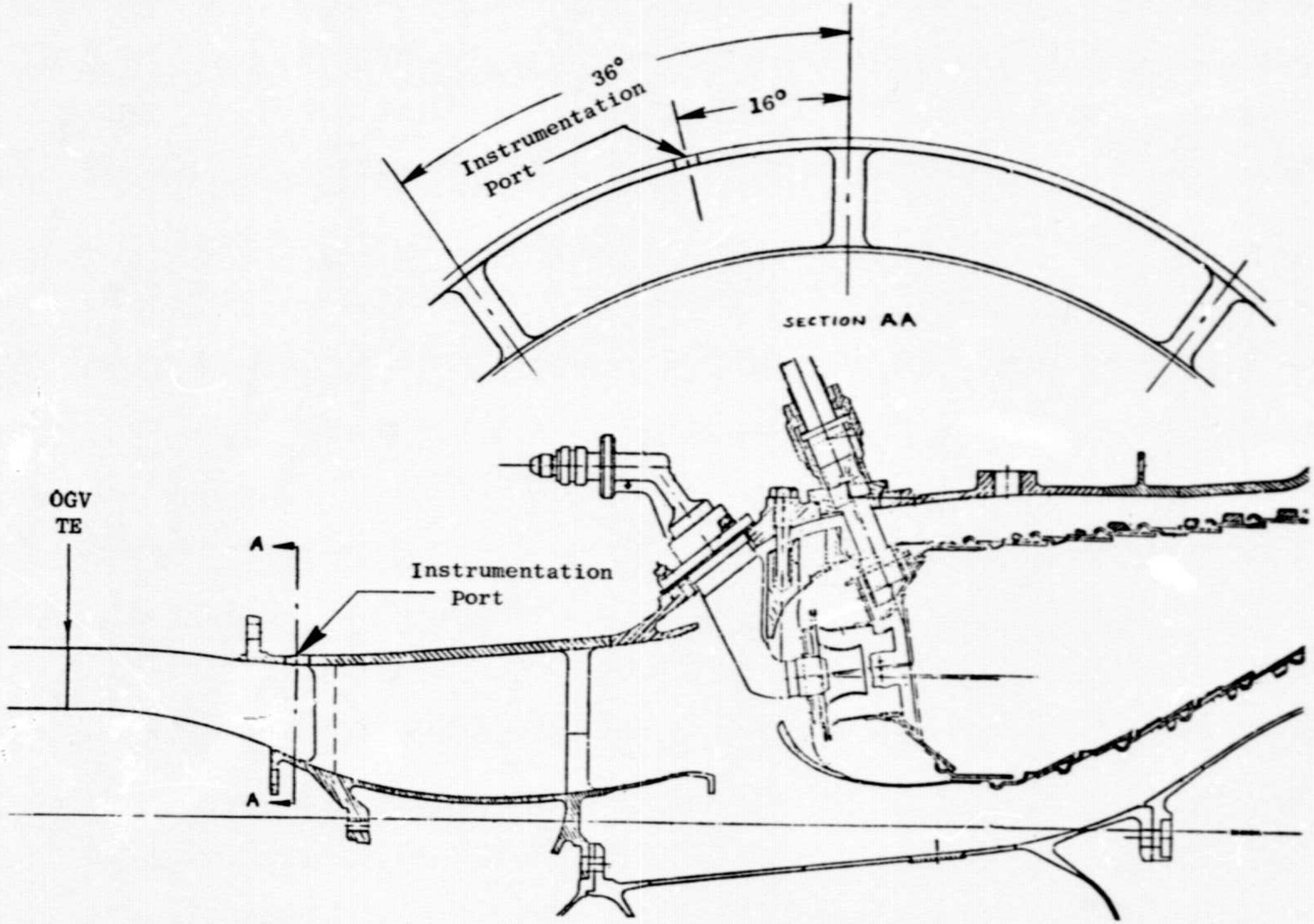
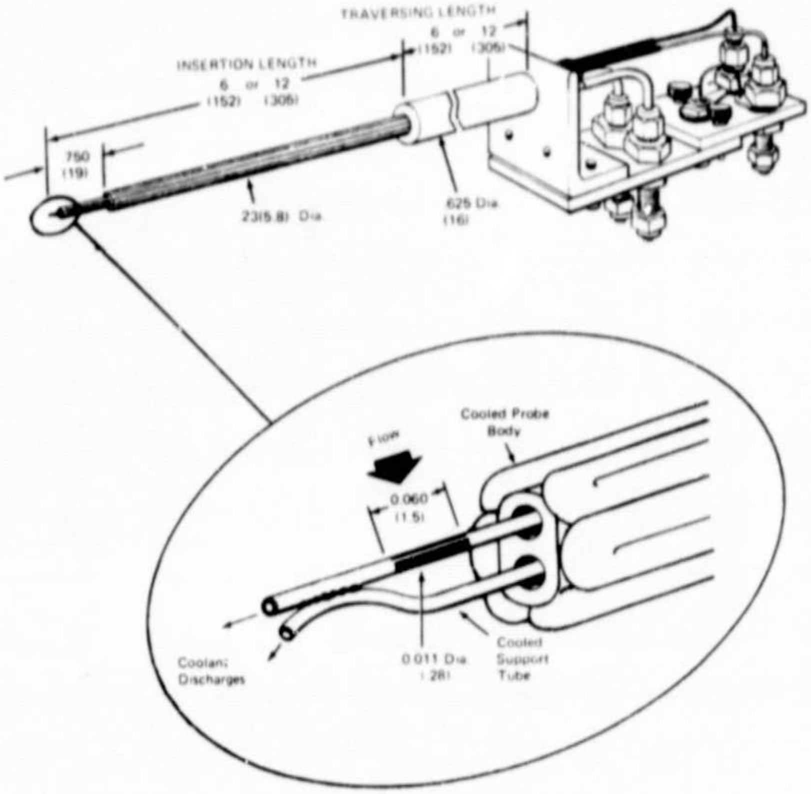
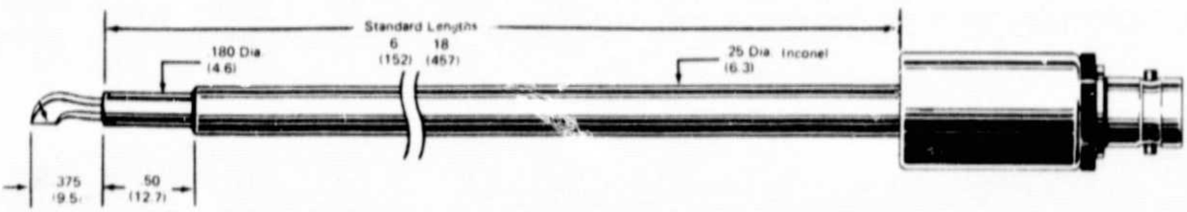


Figure 1. CF6-50 Frame Drawing Showing Instrumentation Port Location.



Water-Cooled Hot Film Probe



Hot Wire Probe

Figure 2. Turbulence Measurement Probes.

Table 2. Hot Wire and Cooled Film Probe Specifications.

Type	Thermo-Systems Designation	Sensor Diameter, cm	Sensor Length, cm	Upper Frequency Response, -3 db Point	Quantity
Platinum Iridium Hot Wire	1228-PI2.5-12	0.00064	0.127	50 kHz	2
Platinum Iridium Hot Wire	1228-PI5-12	0.00127	0.127	13 kHz	4
Platinum Iridium Hot Wire	1228-PI10-12	0.00254	0.127	0.5 kHz	2
Water-Cooled Film Probe	1725-6-6	0.028	0.152	14 kHz	2

mil diameter wire did survive through an engine idle test point but the test results show a strong probe support or probe tip resonance condition that invalidates the test data.

Cooling water for the cooled film probes was supplied from a water tank that was pressurized with a high-pressure nitrogen bottle. The cooling waterflow rate was set to reduce noise generation in the probe to levels below the background noise with no airflow.

A photograph of the electric motor-driven probe actuator mounted on a test rig is presented in Figure 3. A short-length adapter section was used to mount the actuator on the flange at the instrumentation port in the engine compressor rear frame. The radial immersion of the probe sensing element in the compressor exit flow passage was determined with a carefully calibrated remote reading digital counter to count actuator drive revolutions.

When not being used to obtain turbulence data, the probe sensing element was retracted out of the flow stream and up into the actuator adapter section. After setting an engine data point, turbulence measurements were made by quickly moving the probe sensing element into the airstream. Data were taken for 30 seconds at each of the three test locations and then the probe was returned to the retracted position. This procedure resulted in minimum probe exposure time in the particle-laden high-velocity airflow.

A schematic diagram of the data acquisition and reduction system for this program is presented in Figure 4. The turbulence probe was coupled to a constant temperature anemometer and signal conditioning system. An on-line readout system connected to the anemometer was used to check the data as it was acquired. A well-calibrated digital voltmeter displayed the signal DC level, a true RMS meter measured the AC level, and a small oscilloscope was used to visually observe the signal output. A magnetic tape recorder made a permanent record of the data which was used for the data reduction analysis.

Calibration procedures and equations for the turbulence probes are presented in Appendix A. This derivation is based on techniques developed by Thermo-Systems, Inc. (Reference 4) for the cooled film probes, and is used to find the rate of change of stream velocity with respect to output voltage, dV/dE, at each of the engine test points. The average stream velocity is the calculated passage velocity for the engine test conditions at the test data point. A schematic diagram of the constant temperature anemometer circuit is presented in Figure 5.

These calibration parameters were used to convert the magnetic tape data from AC and DC voltage level signals to instantaneous velocity values. A Time-Data Fast Fourier Transform (FFT) system was used for the data reduction and curve plotting procedures. This system includes a Digital Equipment Corporation PDP11/25 computer and a Tektronics CRT display and hard copy printout device. For the data reduction, 200 blocks of 512 samples of data were averaged in the data analysis procedures for each test point. The turbulence, turbulent intensity, microscale of turbulence, the probability

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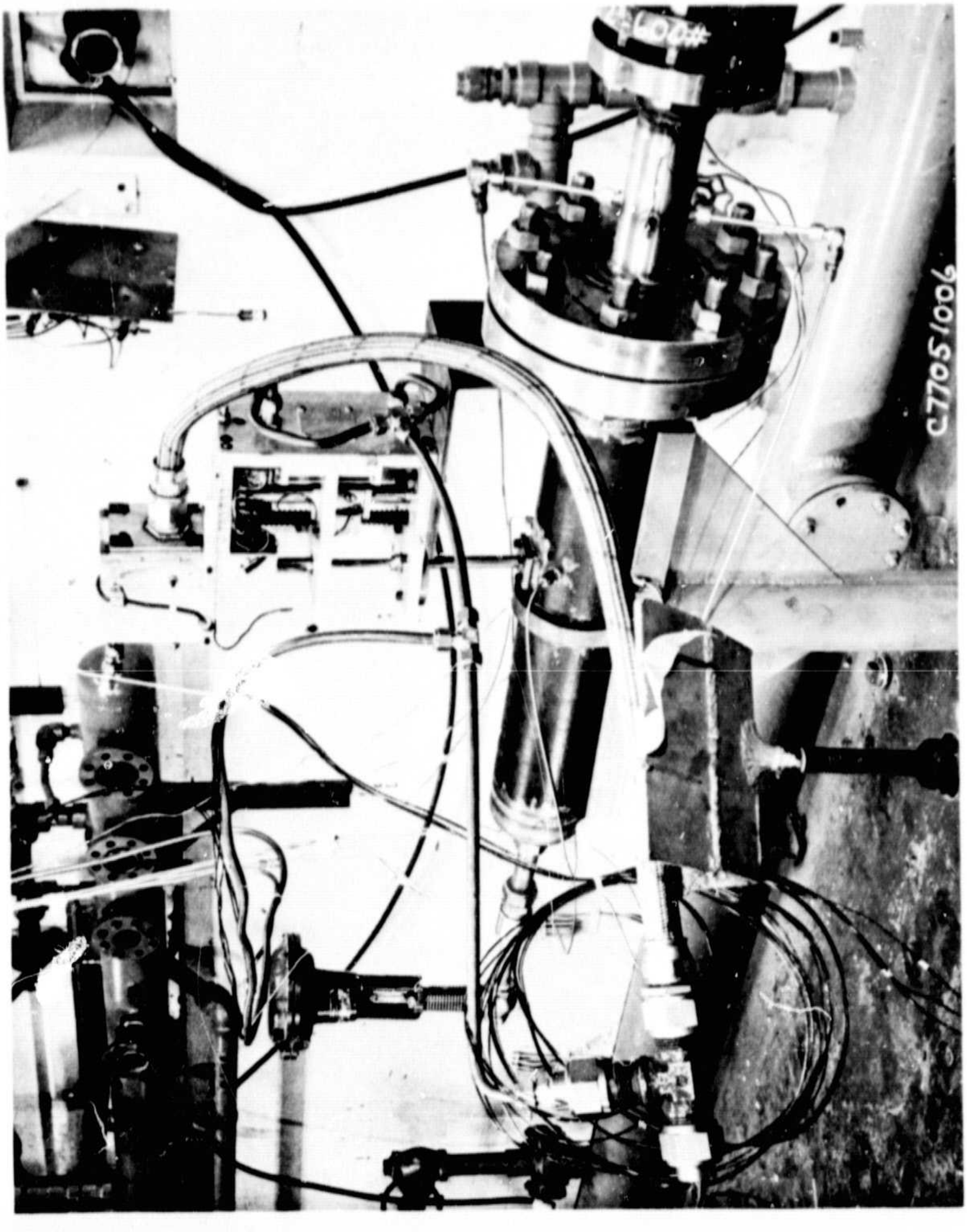

Figure 3. Probe Actuator Mounted on a Test Rig.

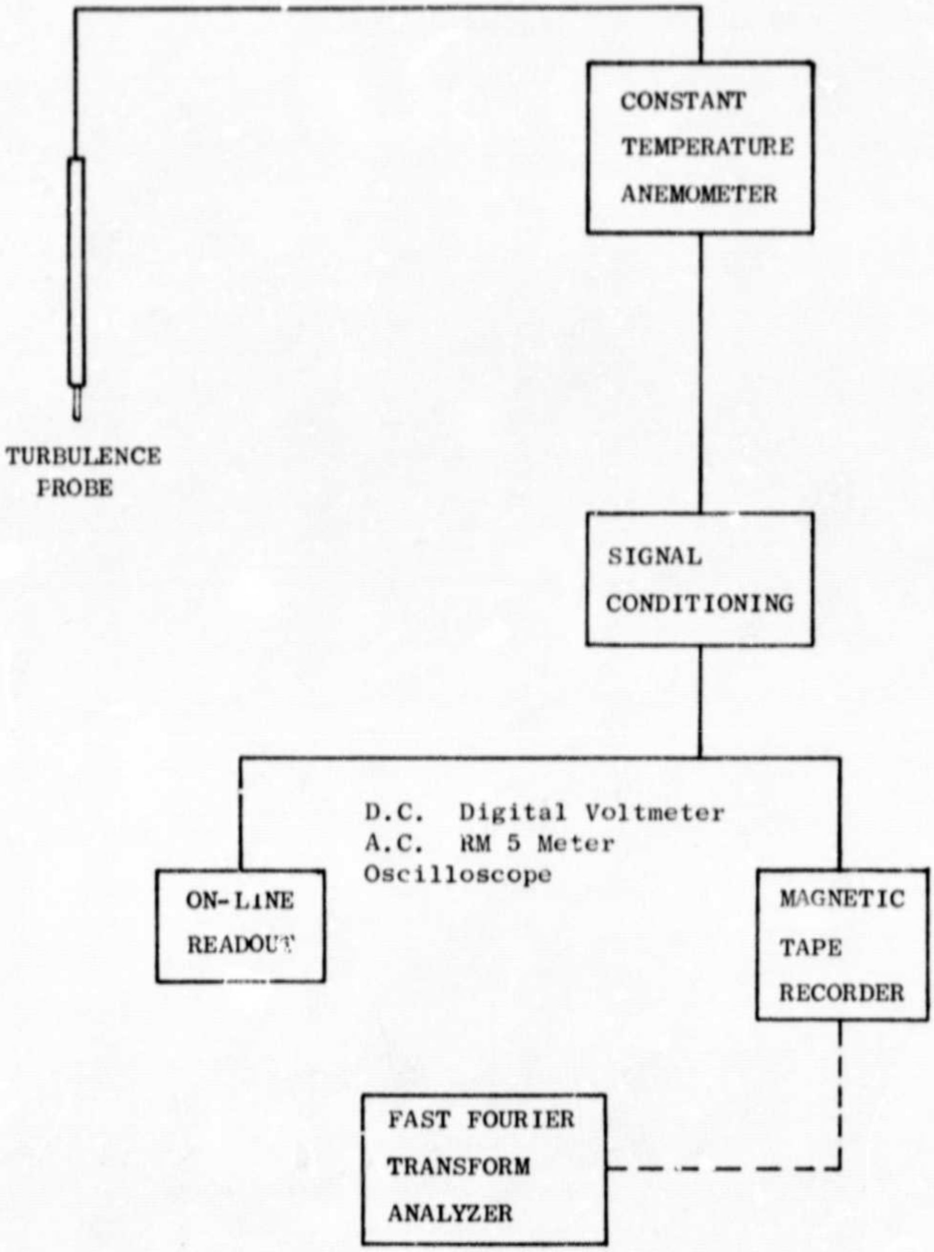

Figure 4. Data Acquisition and Reduction System.

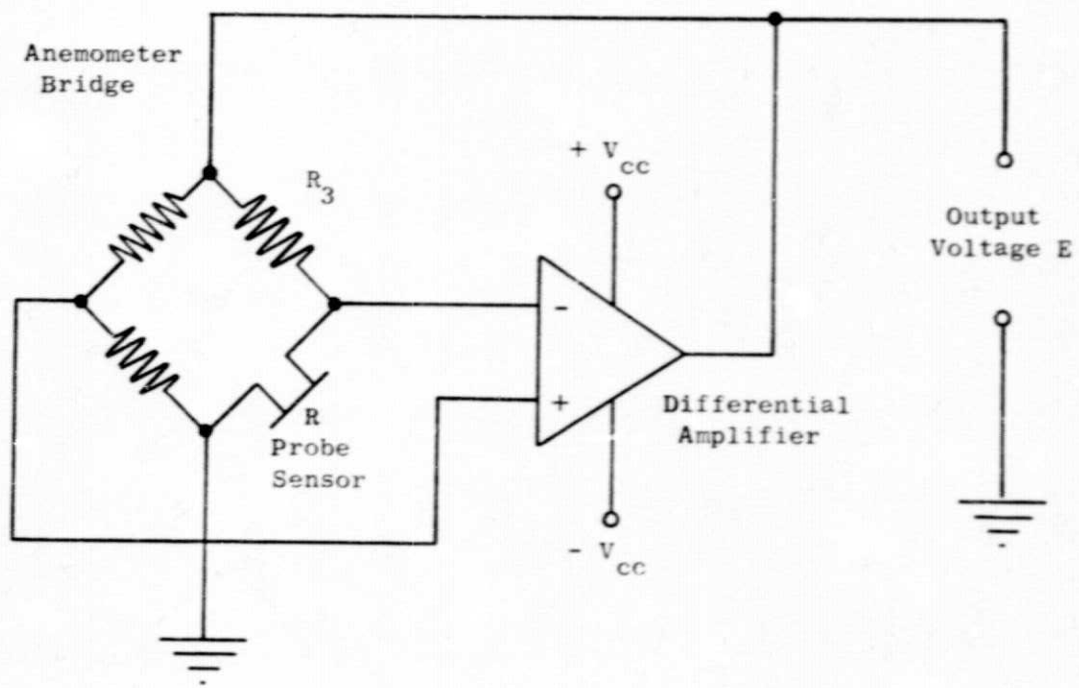

Figure 5. Constant Temperature Anemometer Circuit.

distribution of the turbulence were calculated for each test point. These parameters are defined in Appendix B (see Reference 5, for example). An FFT analysis of the sample records was used to find the power spectral density functions. The power spectral density distributions were plotted with a bandwidth of 50 Hz. An FFT conversion of this data was then used to calulate the autocorrelation curves. The turbulence scale for each test point was found by integrating the area under the normalized autocorrelation function to find an average time delay. The product of this average time delay and the stream velocity at the measurement point is the definition of the turbulence length scale. Mathematical expressions for the PSD and autocorrelation functions are also presented in Appendix B.

As a part of this turbulence measurement program, a calibration test rig was designed and constructed to precalibrate the turbulence probes, check the operation of the probe actuator, and obtain operating experience with the probes and data reduction procedures prior to the engine test runs. This test rig used a high-pressure blowdown air supply and a small preburner to simulate the pressures, temperatures, and stream velocities of the engine for the full range of engine operating conditions. This test rig is shown in Figure 3 with the probe actuator installed on the rig.

Several hot wire probes were tested in this calibration test rig and a considerable amount of test experience was obtained, but nearly all of the hot wire probes failed before completing the full range of simulated engine operating conditions. One of the P15 probes (see Table 2) survived through the calibration test range, but this probe failed on the engine before test data were acquired. Both of the water-cooled probes failed in the calibration test rig and were returned to the vendor for repairs. The "ruggedized" repair procedure described earlier was developed during the engine testing phase of this program. Evidently, a considerable amount of probe development work will be required to develop turbulence measurement probes that can withstand the very severe operating conditions encountered in the compressor exit flow of a gas turbine engine.

4.0 RESULTS AND DISCUSSION

Engine test parameters for three different test conditions at engine idle power settings are presented in Table 3. These parameters were calculated from engine test data, using data reduction techniques described in Reference 1. These parameters include the calculated values for the average airflow velocities at the turbulence measurement station.

Turbulent intensity levels, microscale values, and length scale values, as defined in Appendix B, are presented in Table 4 for six data points. Data for the inner, center, and outer probe positions are shown for engine test point No. 106A. For engine test point No. 106, data for the inner and outer probe locations are presented and for engine test point No. 95, data for the inner probe location is presented. The three data points not presented were judged to be invalid data.

At engine idle conditions, the turbulence intensity ranged from 4.8 percent to 5.6 percent and the length scale ranged from 5.64 cm to 6.95 cm. These turbulence intensity values are considerably higher than those for fully developed pipe flow (about 3 percent) in the center of the pipe at the same Reynolds number (Reference 6). The length scale values are somewhat larger than the passage height at the measurement plane (5.54 cm), which indicates that the shape of the turbulent eddies are elongated in the axial direction. The microscale values range from about 0.73 cm to about 0.98 cm.

Power spectral density distributions (PSD) for each of the six test data points are presented in Figures 6 through 11. These curves show turbulent energy distribution as a function of frequency. The largest proportion of the turbulent energy in the compressor exit flow appears to occur at frequencies below about one kilohertz. The sharp peaks at the higher frequencies are probably generated by the compressor rotor vanes as the flow between the rotor vanes is intercepted by the stator vanes at the blade passing frequency. The blade passing frequency of the last compressor stage at idle conditions is about 8000 Hz which corresponds fairly closely to the highest frequency goals in the PSD curves.

A typical velocity probability density curve for the turbulence test data is presented in Figure 12, and a typical probability distribution curve is presented in Figure 13. These curves, which are plotted as a function of standard deviations from the mean velocity, show that the data follows the expected Gaussian distribution. The probability density function for random data describes the probability that the velocity will have a value within some defined range of values at any assumed instant of time. For perfectly random data, the probability density function curve would have the classical bell-shaped Gaussian form. Small deviations from the perfect Gaussian form, as illustrated by the probability density curve of Figure 12, indicate that a small amount of nonrandom, or sinusoidal, content is present in the test data. The probability distribution function is equal to the integral of the probability density function from minus infinity to any assumed value of

Table 3. Engine Test Parameters.

Data Point Number	95	106	106A
Engine Power Setting	Idle	Idle	Idle
Engine Thrust - N	8069	7015	7564
Fuel Flow Rate - kg/hr	706	630	638
Core Airflow Rate - kg/sec	17.46	17.05	17.41
Bleed Airflow Rate - kg/sec	0.943	0.921	0.943
Core Compressor Speed - rpm	6505	6360	6424
Engine Pressure Ratio	2.97	2.80	2.90
Percent Rated Takeoff Thrust	3.63	3.16	3.36
Percent Bleed	5.40	5.41	5.41
Ambient Pressure - atm	0.984	0.987	0.987
Ambient Temperature - K	300	295	295
Ambient Humidity - g/kg	14.3	13.3	13.3
Compressor Exit Total Pressure - atm	2.93	2.76	2.86
Compressor Exit Total Temperature - K	452	439	442
Compressor Exit Average Air Velocity - m/sec	110.9	111.5	110.6
Measurement Station Average Air Velocity - m/sec	68.3	69.2	68.6

Table 4. Turbulence Test Data.

Data Point Number	95	106	106	106A	106A	106A
Probe Position	Inner	Inner	Outer	Inner	Center	Outer
Calculated Velocity - m/sec	68.3	69.2	69.2	68.6	76.2	68.6
Turbulence - m/sec	3.26	3.66	3.44	3.60	4.27	3.84
Turbulent Intensity - %	4.8	5.3	5.0	5.2	5.6	5.6
Turbulent Microscale - cm	0.73	0.94	0.85	0.79	0.98	0.91
Turbulent Length Scale - cm	6.58	6.04	5.73	5.64	6.95	5.97

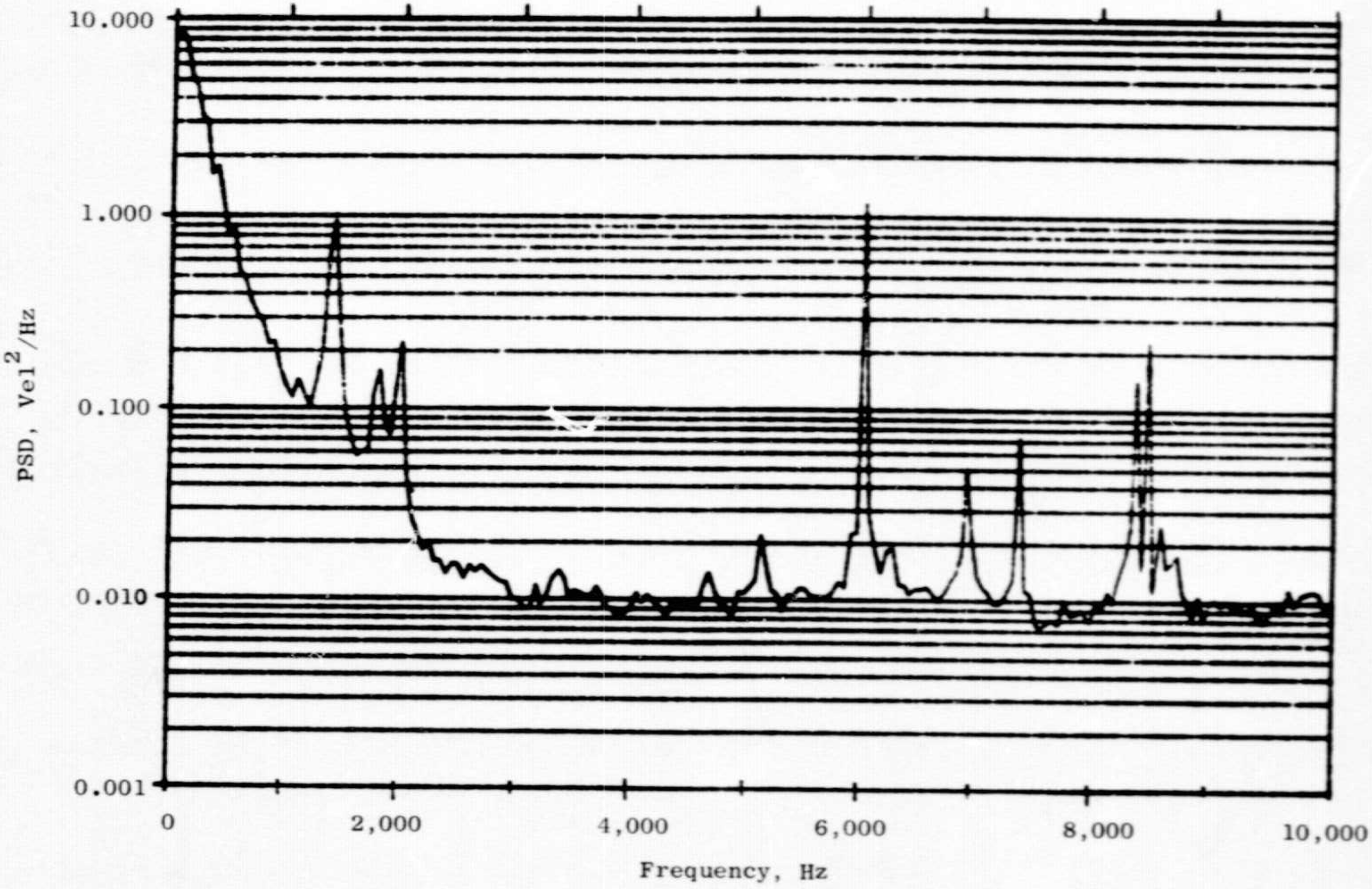

Figure 6. Power Spectral Density for Data Point 95, Inner Probe Position.

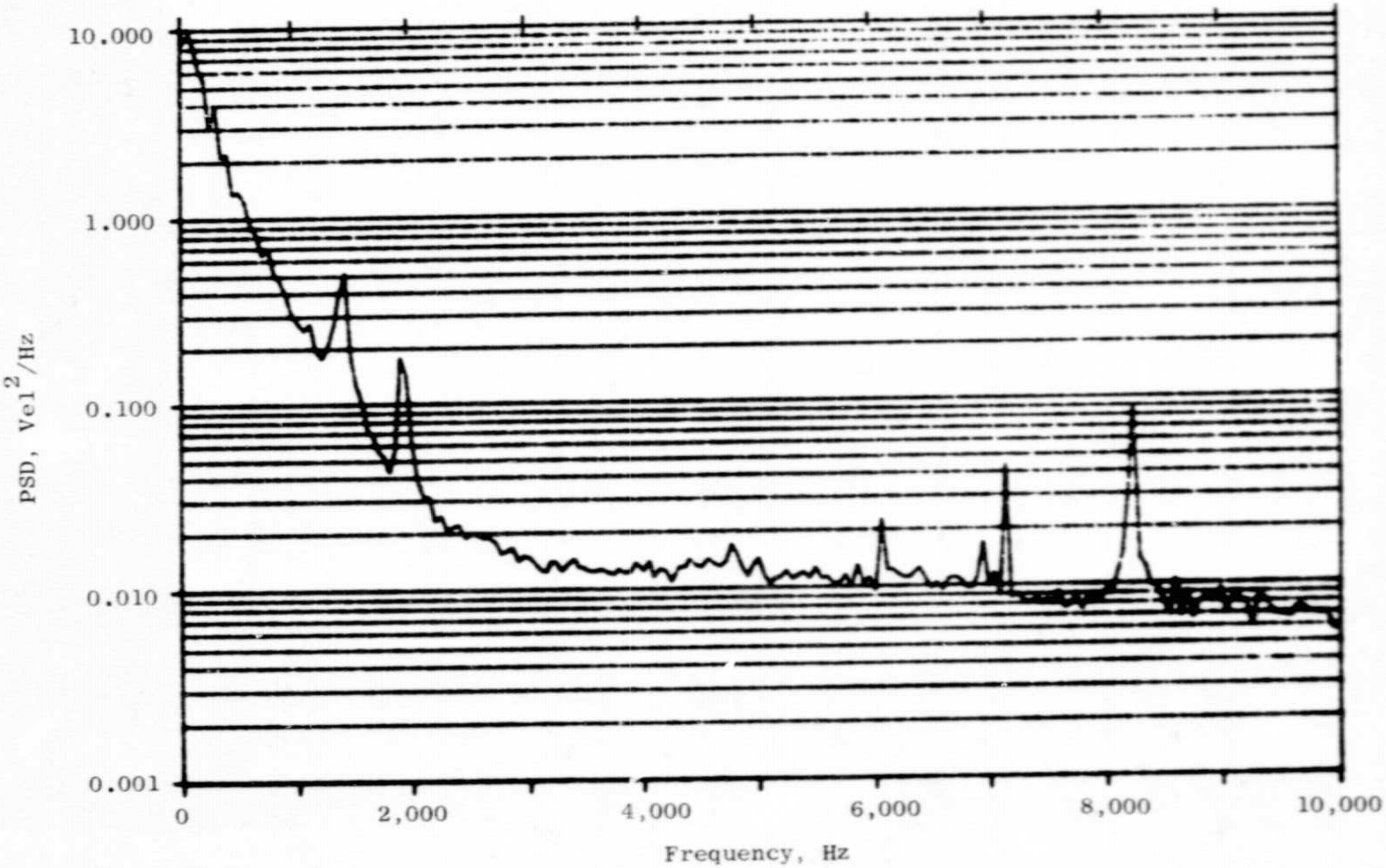

Figure 7. Power Spectral Density for Data Point 106, Inner Probe Position.

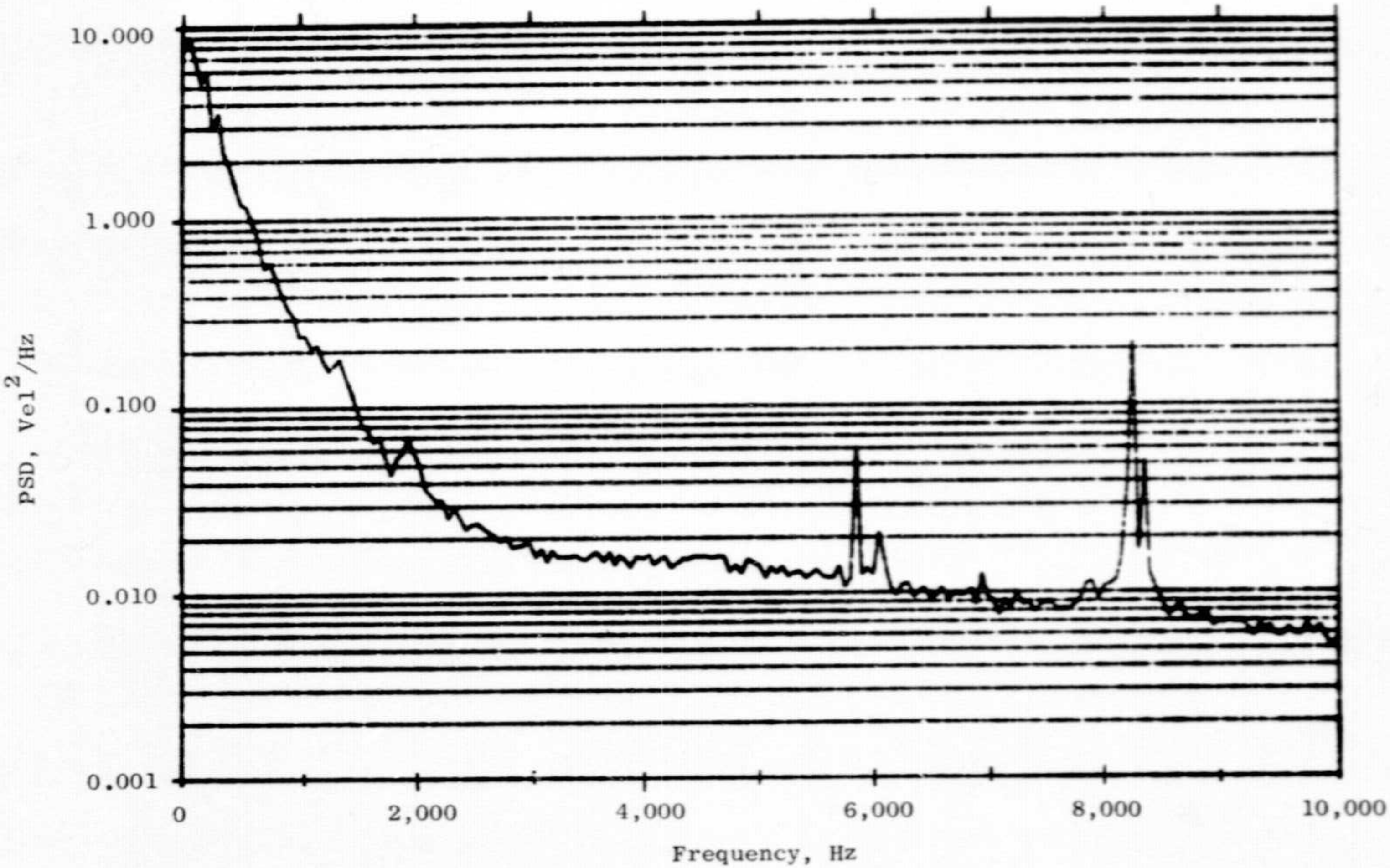

Figure 8. Power Spectral Density for Data Point 106, Outer Probe Position.

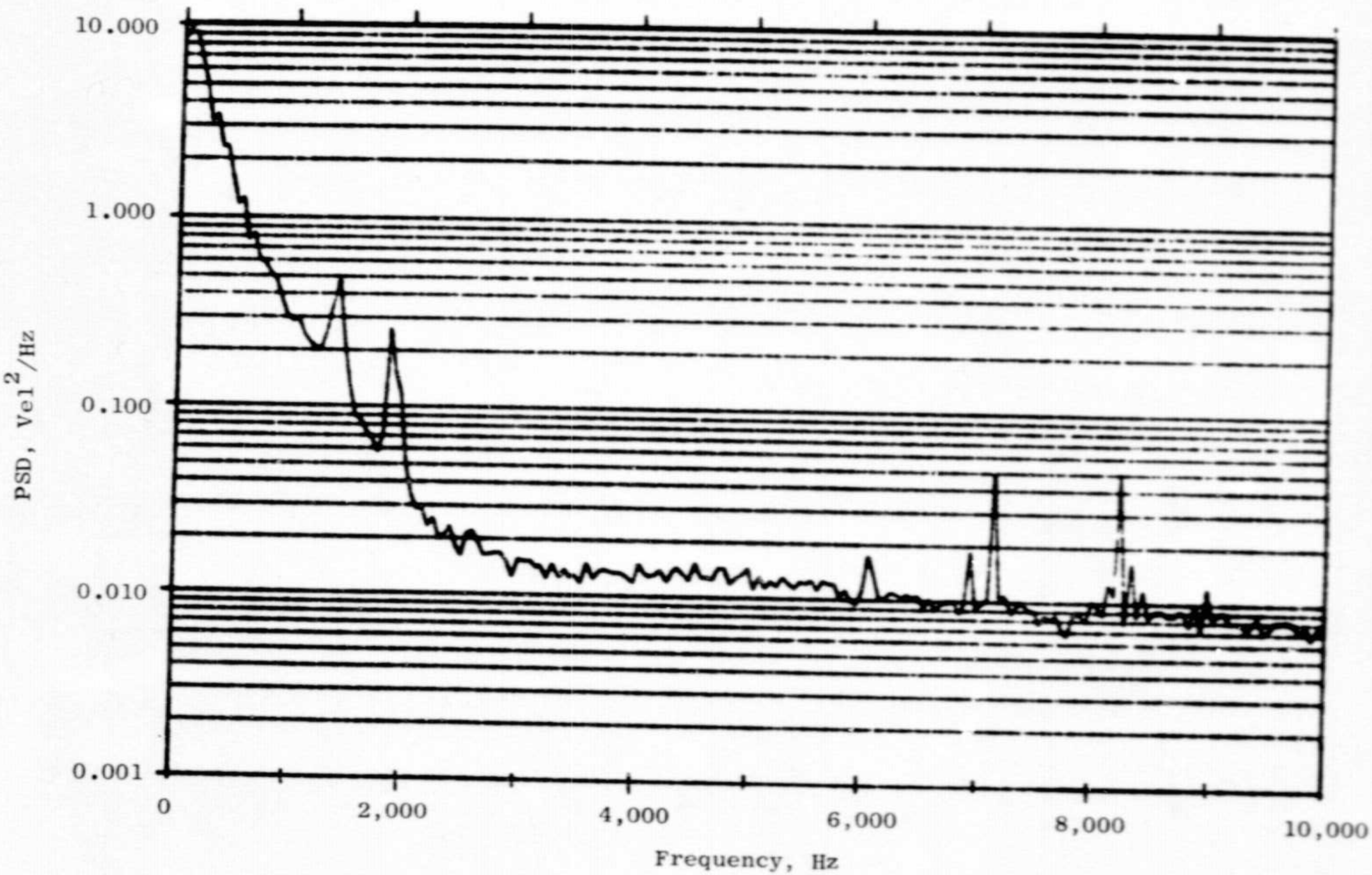

Figure 9. Power Spectral Density for Data Point 106A, Inner Probe Position.

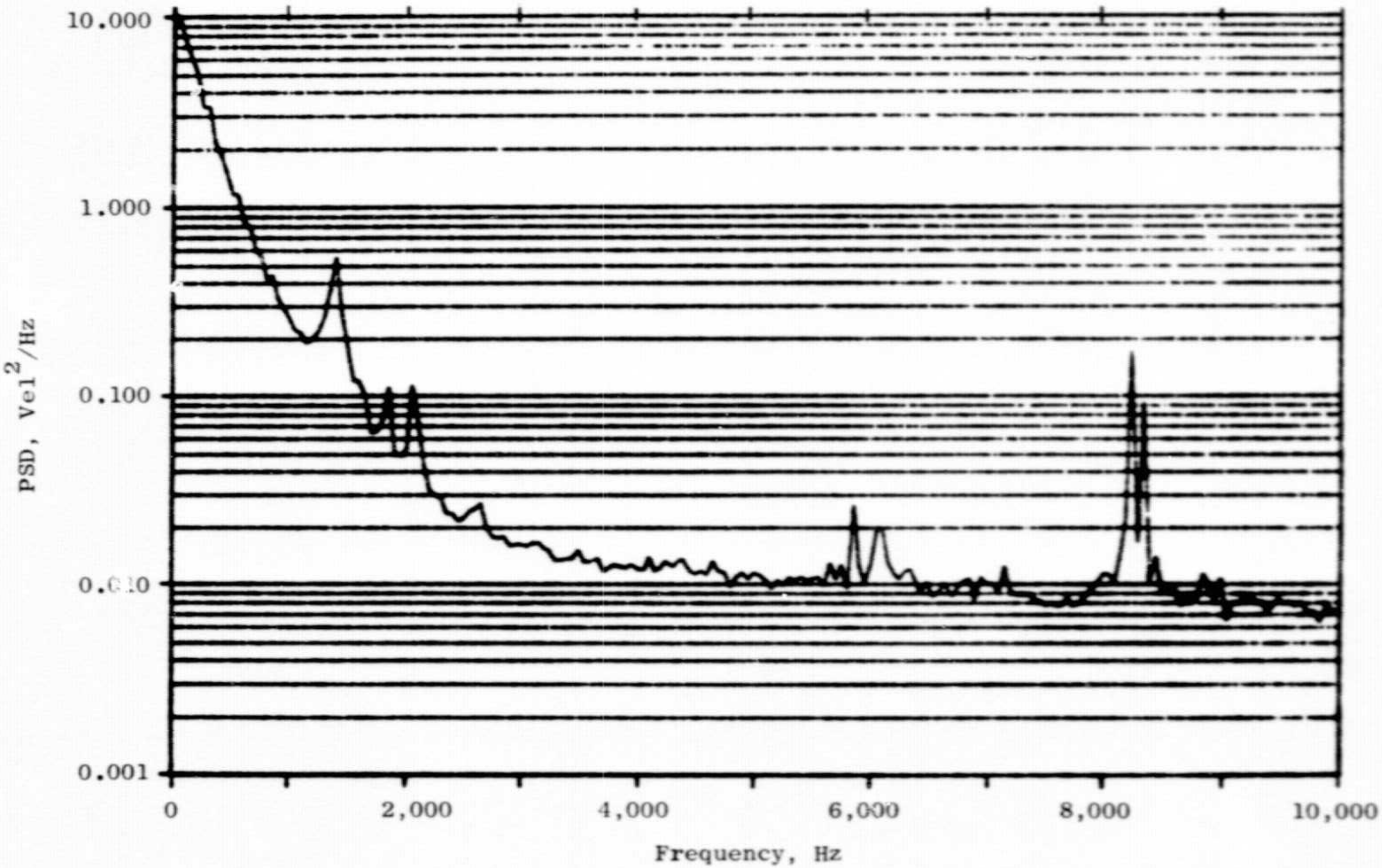

Figure 10. Power Spectral Density for Data Point 106A, Center Probe Position.

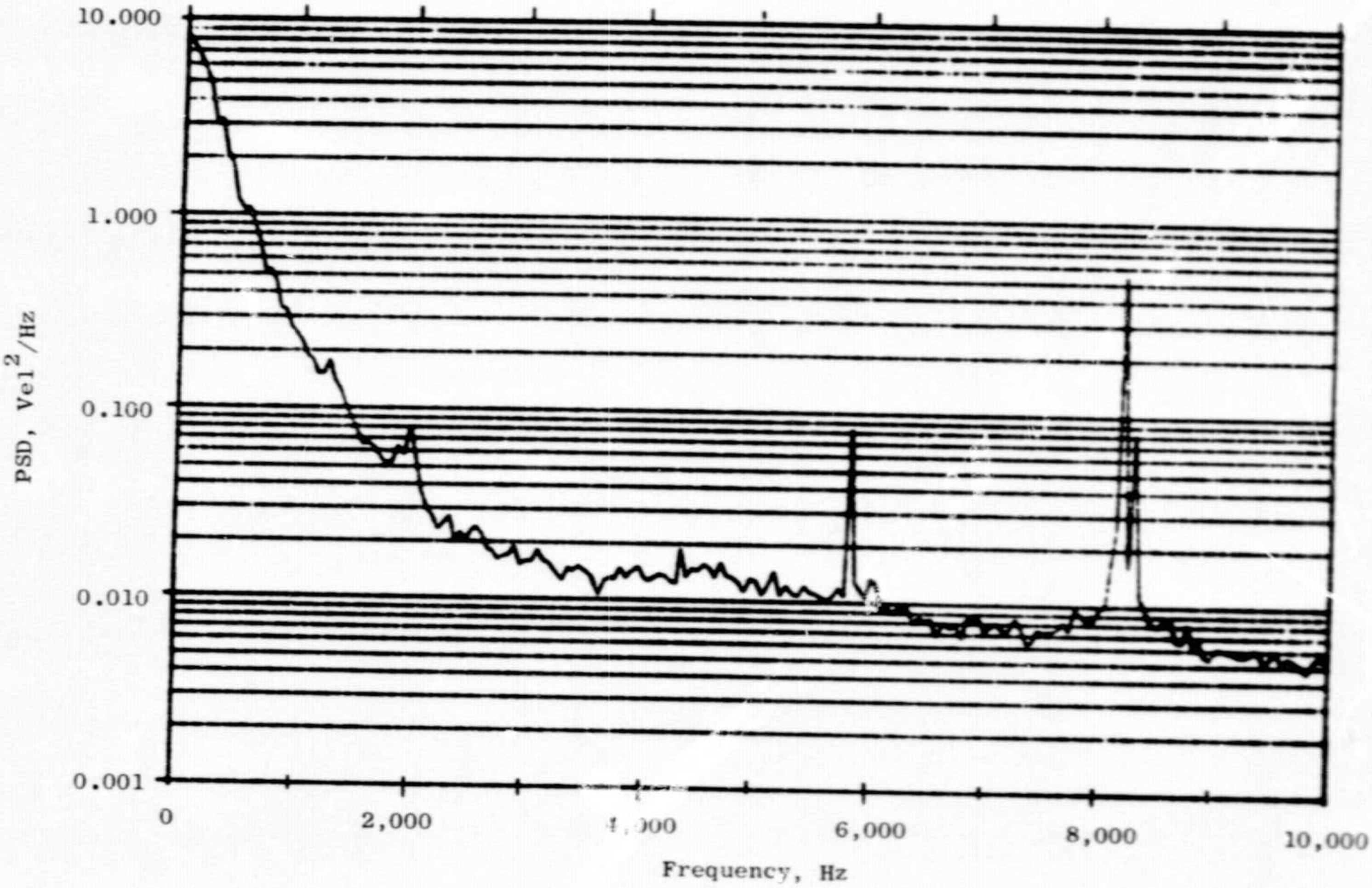

Figure 11. Power Spectral Density for Data Point 106A, Outer Probe Position.

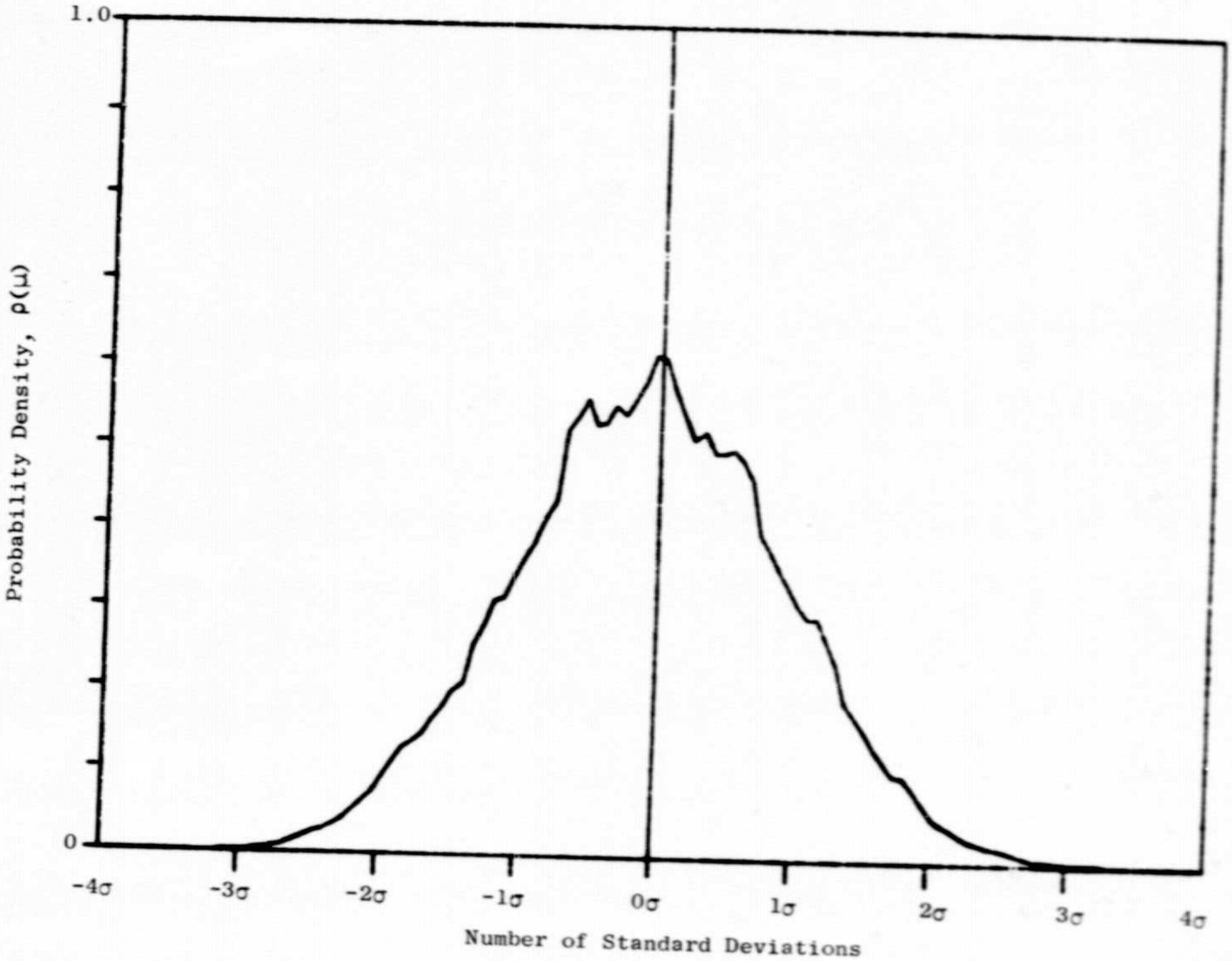

Figure 12. Probability Density for Data Point 106A, Center Probe Position.

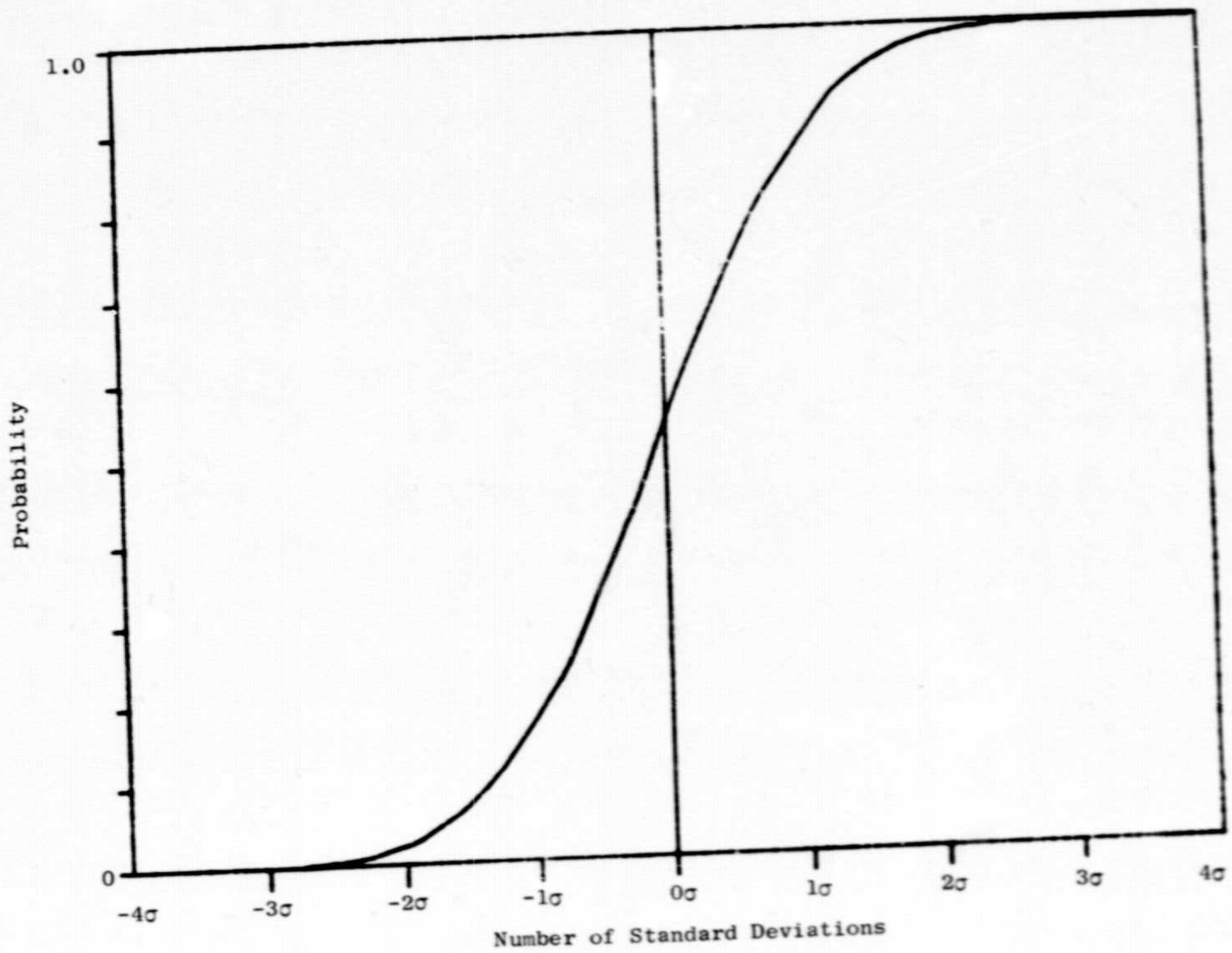

Figure 13. Probability Distribution for Data Point 106A, Center Probe Position.

velocity, or, in this case, to a value of the number of standard deviations from the mean velocity. This function is the probability that the instantaneous value of velocity is less than or equal to the assumed value of velocity. Mathematical definitions of these random functions are presented in Appendix B; more detailed descriptions are presented in Reference 5.

A typical normalized autocorrelation curve for the test data is presented in Figure 14. The integral length scale values presented in Table 4 are computed from the product of the average stream velocity and the integral of the normalized autocorrelation parameter. This integral is performed numerically over a finite time interval with the upper limit determined by the time at which the correlation reaches zero. The small periodic variations in this curve are caused by the pure tone frequencies that appear in the corresponding PSD curves. Precise definition of the autocorrelation parameter and integral length scale are presented in Appendix B.

Turbulence data acquired at the 30 percent and 85 percent engine power conditions were judged to be invalid because a high degree of random noise content was evident in the PSD curves for these conditions. However, these data indicate that the turbulent intensity values increase as the engine power level increases.

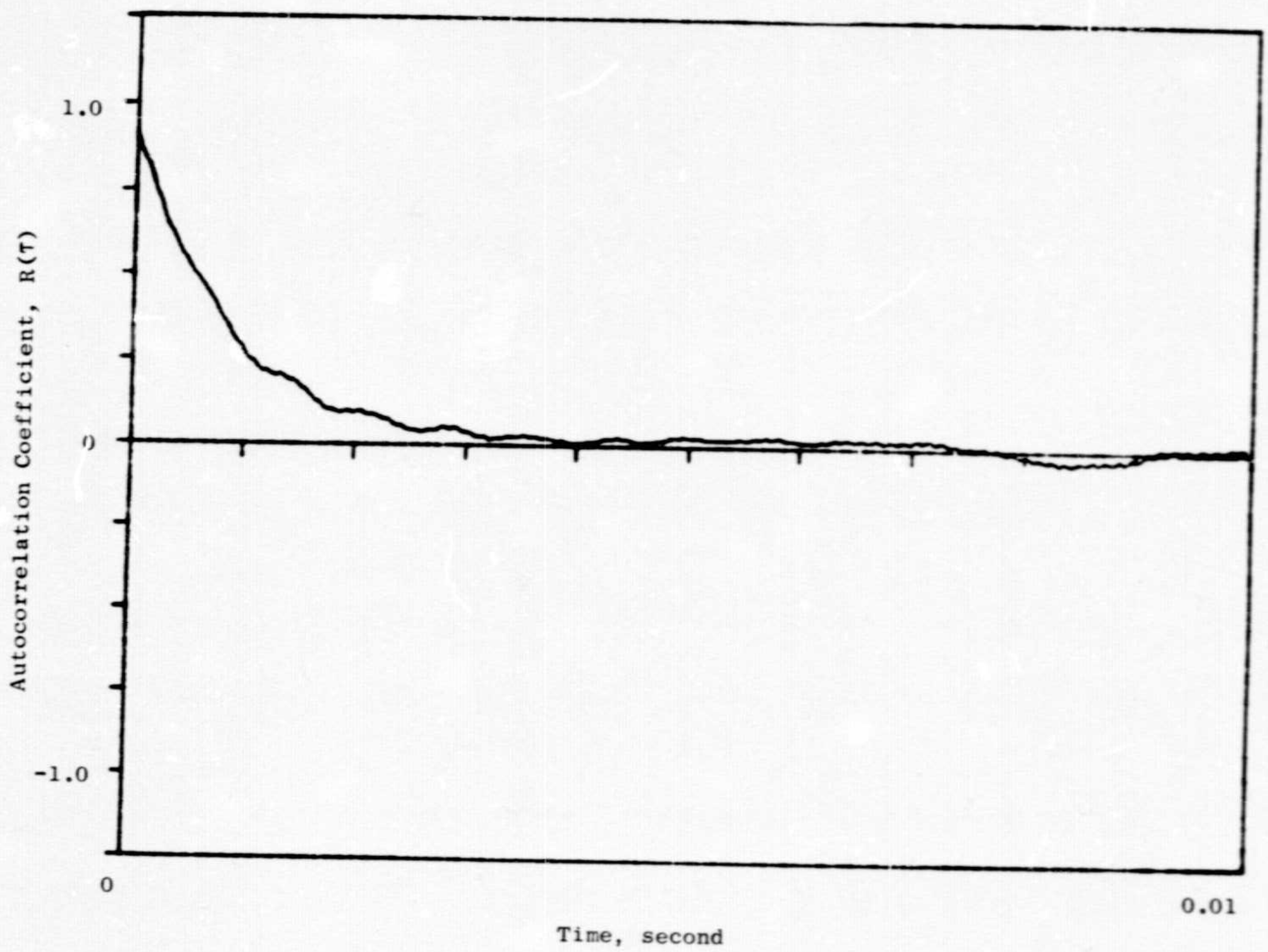

Figure 14. Autocorrelation Coefficient for Data Point 106A, Center Probe Position.

5.0 CONCLUSIONS

From the results of this turbulence measurement program, several conclusions can be summarized as follows:

1. Turbulence intensity levels at idle conditions in the compressor exit flow stream of the CF6-50 gas turbine engine at a point 15.5-cm downstream of the compressor outlet guide vanes are about 5 percent of the average stream velocity, and turbulence length scale values at this location are rather large (about 6 cm).

2. The turbulent eddies are evidently elongated in the axial direction since the passage height at the measurement plane is 5.5 cm which is less than the average length scale value. Elongated eddies will be difficult to simulate in a combustor test rig.

3. The compressor exit turbulence at the measurement plane used for this program is heavily dominated by frequencies below one kilohertz, as shown in the power spectral density distributions.

4. The feasibility of using cooled film probes to measure compressor exit turbulence in an operating gas turbine engine has been demonstrated.

5. Additional turbulence probe development is needed to develop probes that can withstand the severe flow stream environment behind a compressor at high-power engine operating conditions and deliver reliable turbulence data at these conditions.

APPENDIX A

DERIVATION OF PROBE CALIBRATION EQUATIONS

The calibration equation for the cooled film probe is derived from the basic heat transfer and electrical energy relationships for the probe sensing element. This derivation parallels the derivation presented in Reference 4, with several important differences.

According to McAdams, Reference 7, convective heat transfer from a circular cylinder with a cross-flow Reynolds number of about 2×10^3 may be correlated with the following expression:

$$\frac{hD}{k_f} = BR_e^{\ m} \tag{1}$$

where the correlation coefficient B and exponent m are functions of Reynolds number.

The time rate of heat transfer is:

$$Q = A_s h \Delta T \tag{2}$$

Or, from equation (1), with a circular cylinder of length L:

$$Q = \pi k_f L B R_e^{\ m} \Delta T_s \tag{3}$$

From Reference 4, the electrical power input to the sensor is:

$$P = \frac{E^2 R}{(R + R_3)^2} \tag{4}$$

The anemometer circuit for the probe sensor is illustrated in Figure 5.

A heat balance for the cooled sensor can be expressed as the sum of the heat transferred to the airstream and to the cooling water, as follows:

$$P = Q + Q_w \tag{5}$$

Using equations (3) and (4), we can rewrite equation (5) as:

$$\frac{E^2 R}{(R + R_3)^2} = \pi k_f L B R_e^{\ m} \Delta T_s + Q_w \tag{6}$$

For this test program, a calibration reading of the cooled film probe DC voltage level was taken at a zero velocity condition in the compressor exit diffuser before the engine was started at the beginning of the test run. At this condition, the probe sensor film temperature and cooling waterflows were set at the proper test values. For quiescent flow conditions, the probe sensor heat transfer to the external air is by natural convection. The Nusselt number for this case is then, from Reference 7:

$$\frac{hD}{k_f} = CX^n \tag{7}$$

where X is the product of the Grashof and Prandtl numbers.

Then, from equation (2), the time rate of heat transfer in this case is:

$$Q_o = \pi k_f LCX^n \Delta T_o \tag{8}$$

And the heat balance, from equations (4), (5) and (8), becomes:

$$\frac{E_o^{\;2} R}{(R + R_3)^2} = \pi k_f LCX^n \Delta T_o + Q_w \tag{9}$$

Subtracting equation (9) from equation (6) eliminates Q_w, as follows:

$$\frac{(E^2 - E_o^{\;2})R}{(R + R_3)^2} = \pi k_f L(BR_e^{\;m} \Delta T_s - CX^n \Delta T_o) \tag{10}$$

Since the Reynolds number, R_e, is a function of stream velocity, V:

$$R_e = \frac{\rho VD}{\mu} \tag{11}$$

expression (10) can be differentiated to find dV/dE as follows:

$$2EdE \; \frac{R}{(R + R_3)^2} = \pi kLB \left(\frac{\rho D}{\mu}\right)^m \Delta T_s mV^{m-1} dV \tag{12}$$

then, dividing equation (12) by equation (10) and rearranging terms, we have:

$$\frac{dV}{dE} = \frac{2EV}{(E^2 - E_o^{\;2})} \; \frac{BR_e^{\;m} - CX^n \left(\frac{\Delta T_o}{\Delta T_e}\right)}{mBR_e^{\;m}} \tag{13}$$

which is the calibration equation used for this test program.

Reynolds number values for this test program, at engine idle conditions, were about 2000, and at this Reynolds number, the natural convection term of equation (13) is a small fraction (less than 2 percent) of the forced convection term. Therefore, for practical considerations, the natural convection term could be neglected (or, an approximate value could be used), which results in considerable simplification of equation (13), as follows:

$$\frac{dV}{dE} = \frac{2EV}{m\ (E^2 - E_o^2)} \tag{14}$$

This expression is identical to the calibration equation derived in Reference 4.

SYMBOLS FOR APPENDIX A

A_s - Area of heat transfer surface - m^2

B - Coefficient for forced convection heat transfer

C - Coefficient for natural convection heat transfer (~0.53)

$C\rho$ - Specific heat of fluid - cal/k_s - K

D - Diameter of probe sensor element - μ

E - Electrical bridge output potential - Volts

E_o - Output potential for zero velocity - Volts

G_v - Grashof number - $D^3 \rho_f g \Delta T_o / T_f \mu_f^2$

g - Acceleration of gravity - m/sec^2

h - Convective heat transfer coefficient - cal/sec-m^2- K

k - Thermal conductivity - cal/sec-m^2- K/m

L - Length of probe sensor element - m

m - Reynolds number exponent for forced convection

n - Natural convection correlation exponent (~0.25)

P - Power - Watts

P_r - Prandtl number - $(C\rho\mu / k)_f$

Q - Time rate of heat transfer - cal/sec

Q_w - Heat transfer to cooling water - cal/sec

R - Electrical resistance of probe sensor - ohms

R_3 - Bridge series resistance - ohms

R_e - Reynolds number - $\rho VD/\mu$

T - Temperature - K

ΔT_s - Difference between sensor surface temperature and airstream temperature - K

ΔT_o - Temperature difference at zero velocity - K

V - Stream velocity - m/sec

X - Product of Grashot and Prandtl numbers

ρ - Airstream density - kg/m^3

μ - Airstream viscosity - kg/sec-m

Subscript:

f - Evaluate at average film temperature

APPENDIX B

DEFINITION OF TURBULENCE PARAMETERS

The turbulence parameters calculated for this program are defined by the following expressions:

Turbulence -

$$\sigma_u^{\,2} = \frac{1}{N-1} \sum_{k=1}^{n} (u_k - \overline{V})^2$$

Turbulent Intensity -

$$I_t = \frac{\sigma_u}{\overline{V}}$$

Microscale of Turbulence -

$$\lambda = \overline{V} \left[\frac{\sigma_u^2}{\frac{1}{N} \sum_{k=1}^{n} \left(\frac{U_k - U_{k-1}}{(\Delta t)_k} \right)^2} \right]$$

Probability Density Function -

$$p\,(u) = \frac{N_u}{N(\Delta\sigma)}$$

Probability Distribution -

$$p\,(u) = \int_{-\sigma_e}^{\sigma} p\,(u)\, d\sigma$$

The expression for the turbulence length scale is as follows:

$$L_s = \overline{V} \int_{o}^{\tau_o} R\,(\tau)\, d\tau$$

where R (τ) is the autocorrelation coefficient, which may be defined as:

$$R(\tau) = \frac{1}{\sigma_u^2} \frac{1}{N} \sum_{k=1}^{N-\tau} u_k (u_{k+\tau})$$

However, for this program, the autocorrelation coefficient was found by executing a Fourier transform of the power spectral density distribution as described in Reference 5, page 312. The power spectral density is defined as follows:

$$G_x(f_k) = \frac{2\Delta t}{N} \left| Z_k \right|^2$$

where the Fourier components, Z_k, are found from the data samples, using the following formula:

$$Z_k = \sum_{n=o}^{N-1} u_n \exp\left[-j \frac{2\pi kn}{N}\right]$$

The autocorrelation coefficient is then:

$$R(\tau) = \int_o^{f_m} G_x(f) \cos 2\pi f \tau df$$

which is the inverse Fourier transform of the power spectral density function, $G_x(f)$.

SYMBOLS FOR APPENDIX B

f - Frequency - cycles/sec

G_x - Power spectral density function - cm^2/sec

I_t - Turbulent intensity

L_s - Turbulence length scale - cm

N - Total number of samples

N_u - Number of samples within defined $\Delta\sigma$ range

$p(u)$ - Probability density function

$P(u)$ - Probability distribution function

$R(\tau)$ - Autocorrelation coefficient

Δt - Time interval - sec

u_k - Instantaneous velocity - m/sec

$\overline{V}$ - Average velocity - m/sec

Z_k - Fourier component of velocity - m/sec

λ - Microscale of turbulence - cm

σ_u - Standard deviation of velocity - m/sec

$\Delta\sigma$ - Finite increment of standard deviation

τ - Autocorrelation time delay - sec