NASA Technical Memorandum 79011

(NASA-TM-79011) DECAY OF HOMOGENEOUS TURBULENCE FROM A GIVEN STATE AT HIGHER REYNOLDS NUMBER (NASA) 15 p HC A02/MF A01 CSCL 20D
N79-12361
Unclas
G3/34 38001

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TECHNICAL PAPER to be presented at the
Thirty-first Annual Meeting of the American Physical Society
Division of Fluid Dynamics
Los Angeles, California, November 19-21, 1978

DECAY OF HOMOGENEOUS TURBULENCE FROM A GIVEN STATE AT HIGHER REYNOLDS NUMBER

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E-9796

ABSTRACT

The turbulence equations are closed by specification of initial conditions (using either a Taylor or an exponential series) and by a modified Kovasznay-type closure. Good results for large times are obtained only for the initial-conditions closure used with four or more terms of an exponential series. The evolution of all of the initially-specified spectra can be calculated rather well from the theory. From a fundamental standpoint the method thus seems to be satisfactory.

I. INTRODUCTION

From a practical standpoint it would be advantageous to be able to calculate the evolution of the turbulent energy spectrum by specifying only that quantity initially. Unfortunately, because of the coupling between the members of the infinite hierarchy of multipoint correlation or spectral equations, it appears that we are not able to do that, so that a satisfactory theory would seem to require the initial specification of a number of interacting quantities. The prediction of the evolution of the energy spectrum in fact requires the specification of an infinite number of initial multipoint correlations or spectra (or functionals of these quantities).[1] Those quantities, together with the correlation or spectral equations, can be used to calculate the initial time derivatives of the correlations or spectra. The evolution of the turbulence can then be obtained by using an exponential time series which is an iterative solution of the Navier-Stokes equations. When the problem is formulated in this way, the correlation or spectral equations are closed by the initial specification of the turbulence and no assumption is necessary for closing those equations.

Of course in practice we can specify only a finite number of the lower-order quantities. This has been called the "gap problem."[2] It is the problem of bridging the gap between the infinite number of correlations which would theoretically be necessary for calculating the evolution of the turbulence, and a finite specifiable number of correlations. Most workers have attempted to bridge the gap by assuming that the initial distribution of turbulent fluctuations is exactly Gaussian (zero odd-order correlations). However, that is an artificial initial condition, probably never realized for real turbulence. The importance of accurate initial conditions is shown, for instance, by the data of Ling and Saad,[3] where measurements were made downstream from a turbulence-producing waterfall. The turbulence decay law for the initial conditions produced by the waterfall is considerably different from that for initial conditions produced by a grid. As will be seen later, the skewness factors for the velocity gradient are also different.

Here, as in Ref. 1, we bridge the gap simply by specifying a sufficient number of initial correlations or their spectral equivalents to calculate the evolution accurately. Fortunately, we do not have to specify the multipoint correlations or their spectral equivalents themselves (those would be extremely difficult to measure) but only two-point functionals of the multipoint spectral quantities.[1] It will be seen that the evolution of all the quantities which are specified initially can be calculated. If, on the other hand, a large number of initial conditions were specified in order to predict the evolution of say one quantity, it might be objected that the initial conditions were chosen to make the theory agree with experiment for that one quantity. However, that objection cannot be made if the evolution of all of the quantities which are specified initially can be calculated, as in the present theory. From a fundamental standpoint the calculation of the evolution of those quantities is all that might be expected from a theory of evolving turbulence.

II. ANALYSIS

In Refs. 1 and 4 we gave the basic theory for closure by specification of initial conditions and calculated results for low and moderate turbulence Reynolds numbers (R_λ between 3 and 70, where R_λ is the Reynolds number based on the Taylor microscale and the root-mean-square turbulent velocity fluctuation). Here we compare calculated results with the higher Reynolds number data of Ling and Saad.[3] The Reynolds numbers for those data were high enough to obtain a -5/3-power region in the energy spectrum (R_λ between 300 and 800).

The exponential-series expression for the energy spectrum function E was obtained in Ref. 1 as

$$E(\kappa, t) = B^2(\kappa)\exp\left[-2\nu\kappa^2(t - t_1)\right] + \sum_s B_s^2(\kappa)\exp\left[-2b_s(\kappa)(t - t_1)\right], \tag{1}$$

where κ is the wave number, t is the time, t_1 is the intial time, and ν is the kinematic viscosity. Equation (1) gives the evolution in time of the energy spectrum from an initial state which is specified by the B's and b's in the equation. The first term is the well-known expression for the decay of E in the final period (weak turbulence approximation). The rest of the terms in Eq. (1) therefore give the contribution of inertia to E. In the present note we will retain a maximum of four exponential terms in Eq. (1). This is one more term than it was necessary to retain for the low and moderate Reynolds number data considered in Refs. 1 and 4. With four terms retained, we will have to specify seven spectra at t_1 in order to evaluate the functions B, B_1, B_2, B_3, and b_1, b_2, and b_3. Evidently we need more spectra to describe the initial turbulence at higher Reynolds numbers because a wider range of eddy sizes is excited, and the turbulence structure is more complicated than at lower Reynolds numbers. The spectrum functions, in addition to $E(\kappa, t)$, which we will specify initially are $T(\kappa, t)$, $V(\kappa, t)$, $R(\kappa, t)$, $S(\kappa, t)$, $L(\kappa, t)$, and $M(\kappa, t)$, which are given by

$$T = \partial E/\partial t + 2\nu\kappa^2 E, \tag{2}$$

$$V = \partial T/\partial t + 2\nu\kappa^2 T, \tag{3}$$

$$R = \partial V/\partial t + 2\nu\kappa^2 V, \tag{4}$$

$$S = \partial R/\partial t + 2\nu\kappa^2 R, \tag{5}$$

$$L = \partial S/\partial t + 2\nu\kappa^2 S, \tag{6}$$

and

$$M = \partial L/\partial t + 2\nu\kappa^2 L. \tag{7}$$

The quantity T is the well-known energy-transfer function, and V, R, S, L, and M are, respectively, two-point functionals of three-, four-, five-, six-, and seven-point spectral quantities.[1] The latter five quantities are somewhat similar to T, inasmuch as they contain the effects of transfer between wave numbers of T, V, R, S, or L. However they differ, in that they also contain other effects, so that the areas under those spectra are not necessarily zero, as in the case of T. Equations (2) through (7) are obtained, respectively, from two-, three-, and four-, five-, six-, and seven-point spectral equations.[1] They can be thought of as two-point alternatives to the multipoint spectral equations and are much easier to work with.

For comparing the theory with the experiment of Ling and Saad,[3] the experimental input can be conveniently obtained from an empirical equation for E, Eq. (8) in their paper. The higher-order spectra were not measured directly in their experiment, but could be calculated from their equation for E and Eqs. (2) through (7). Except for experimental error those values will be the same as those that might have been measured directly. The B's and b's in Eq. (1) can be related to the initial spectra by successive differentiations of that equation with respect to time, setting $t = t_1$, and using Eqs. (2) through (7).

The use of Ling and Saad's Eq. (8) for obtaining the initial conditions might be taken as an indication that it is necessary to know the decay law to predict it. However that is not the case. Their equation was used because the initial spectra could not be measured directly in their experiment. Moreover we do not need to know E for the whole decay period, but only at enough early times to calculate the required initial spectra, the latter being a much smaller amount of information. At any rate the nature of the problem seems to be such that we do not have a reasonable alternative to the procedure followed here.

III. RESULTS AND DISCUSSION

Before giving results obtained from Eq. (1), we will consider a Taylor series with a maximum of seven initial spectra (the same maximum number that will be used with Eq. (1)), and a modified Kovasznay-type closure (modified to include an effect of initial T).[5] Results for those calculations are given in Fig. 1, where the initial state is specified at $t_1^* = 0.0048$. The quantity u_i is a velocity component, the overbar indicates an averaged value, and the stars indicate that quantities have been nondimensionalized by the kinematic viscosity and an experimental constant A (the proportionality constant in the power decay law for $\overline{u_1^2}$) having the dimensions $\left[(\text{length})^2 (\text{time})^{1.\bar{3}}\right]$.[3] The turbulent energy $(1/2)\overline{u_i u_i}$ is obtained by integrating E over all wave numbers. Figure 1 indicates that the Taylor-series results agree with experiment only for small times. Evidently, many more terms (and initial spectra) would be required in order to obtain accurate results for $\overline{u_i u_i}$ at large times. Note that even if enough terms were retained in the Taylor series to give accurate results for $\overline{u_i u_i}$, the decay of the higher-order spectra which would then have to be specified initially could not be accurately calculated. Thus the use of a Taylor series in the present problem does not give a satisfactory solution, regardless of the number of terms retained.

The modified Kovasznay-type closure is in somewhat better agreement with experiment than the Taylor series, but at large times the agreement is still not good. This is in contrast to its good agreement at moderate and low Reynolds numbers.[5] It was introduced in Ref. 5 in an effort to reduce the required number of initial spectra. Evidently that effort is successful only for moderate and low Reynolds numbers. It is possible, of course, that a more sophisticated method might be more successful (e.g. see Ref. 6).

A comparison between theory and experiment using the exponential series (Eq. (1)) is given in Figs. 2 to 8, where the initial state is again specified at $t_1^* = 0.0048$. As in Refs. 1 and 4, unphysical singularities occasionally occurred in the theoretical spectra. Inasmuch as the unphysical values were localized, particularly in the higher-order approximations, they were simply omitted in plotting the curves.

Figure 2 gives a comparison between theory and experiment for the decay of turbulent energy. Theoretical curves are shown for one, two, three, and four exponential terms retained in Eq. (1). The curve for four terms is in good agreement with experiment for the whole decay period. Comparison of the curve in Fig. 1 for one term retained (weak turbulence approximation) with the experimental curve shows the effect of inertia on the decay process. In contrast to the results for moderate Reynolds number,[1, 4] where inertia and viscous effects were of the same order of magnitude, the inertia effects for the present higher Reynolds number results are at least an order of magnitude greater than the viscous effects. Thus most of the decay at high Reynolds numbers is due to inertial self-interaction of the turbulence, rather than to viscous effects. Figures 3 and 4 show how the energy and the transfer spectra decay with time.

Figures 5 and 6 compare theory and experiment at a late time for all of the spectra which are initially specified to describe the initial turbulence. The prediction of the decay of all seven of the spectra which are specified initially is rather good. Thus the present theory appears to solve an initial-value problem for higher Reynolds number turbulence in which the decay of seven initially-specified spectra is predicted.

Although the initial dissipation spectrum $\kappa^2 E$ is not specified independently, because of its importance in turbulence theory it is compared with experiment and with the energy spectrum at a late time in Fig. 7. Again, good agreement is indicated. The separation of the energy and dissipation spectra is good evidence that we are dealing with high Reynolds number turbulence.

Another important quantity is the skewness factor of the velocity gradient. That quantity can be calculated from the spectra of E and T.[7] The plot in Fig. 8 indicates excellent agreement between theory and experiment for the skewness factor. The difference between the trend in Fig. 8 and that in grid-generated turbulence[8] is probably due to the difference between initial conditions for grid-generated turbulence and the waterfall-generated turbulence considered here.[3]

ACKNOWLEDGMENT

I should like to acknowledge the work of Frank Molls in working out the methods used for obtaining the numerical results given here.

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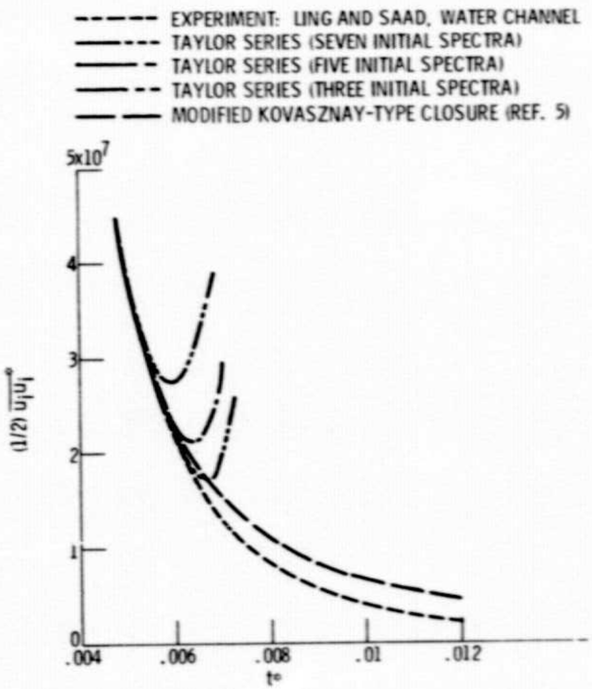

Figure 1. - Comparison of theory with the experiment of reference 3 for decay of turbulent energy. Turbulence equations closed by specification of initial conditions using a Taylor series and by a modified Ko[illegible]znay-type closure.

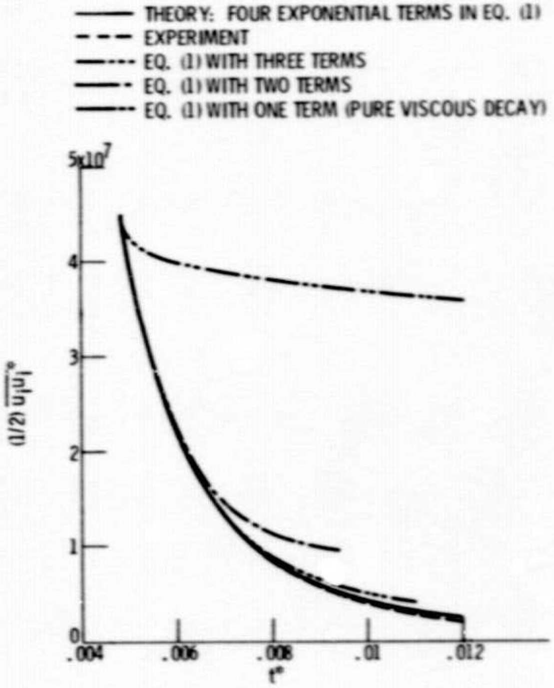

Figure 2. - Comparison of theory with the experiment of reference 3 for decay of turbulent energy. Turbulence equations closed by specification of initial conditions using an exponential series (eq. (1)).

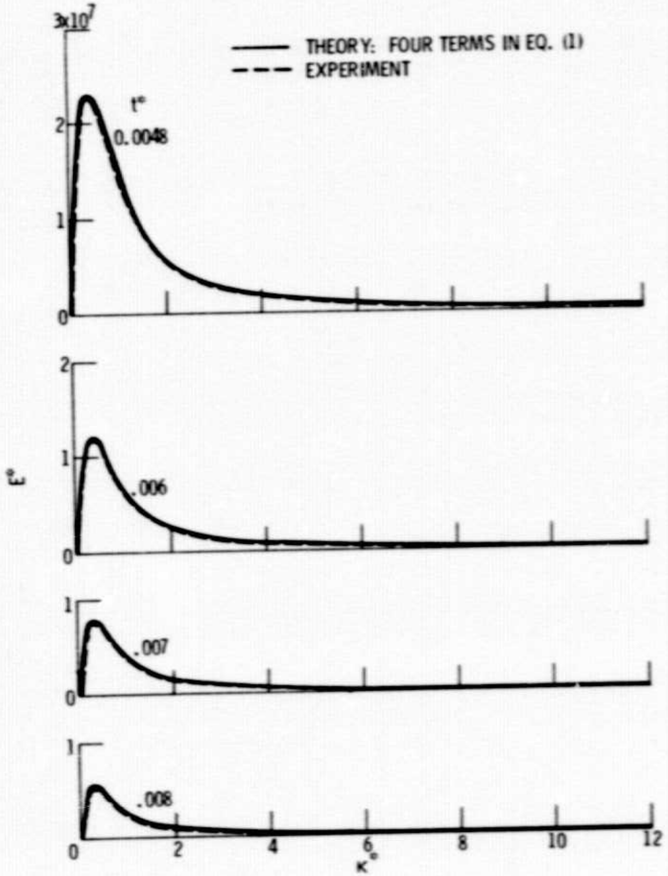

Figure 3. - Comparison of theory with the experiment of reference 3 for decay of three-dimensional energy spectra.

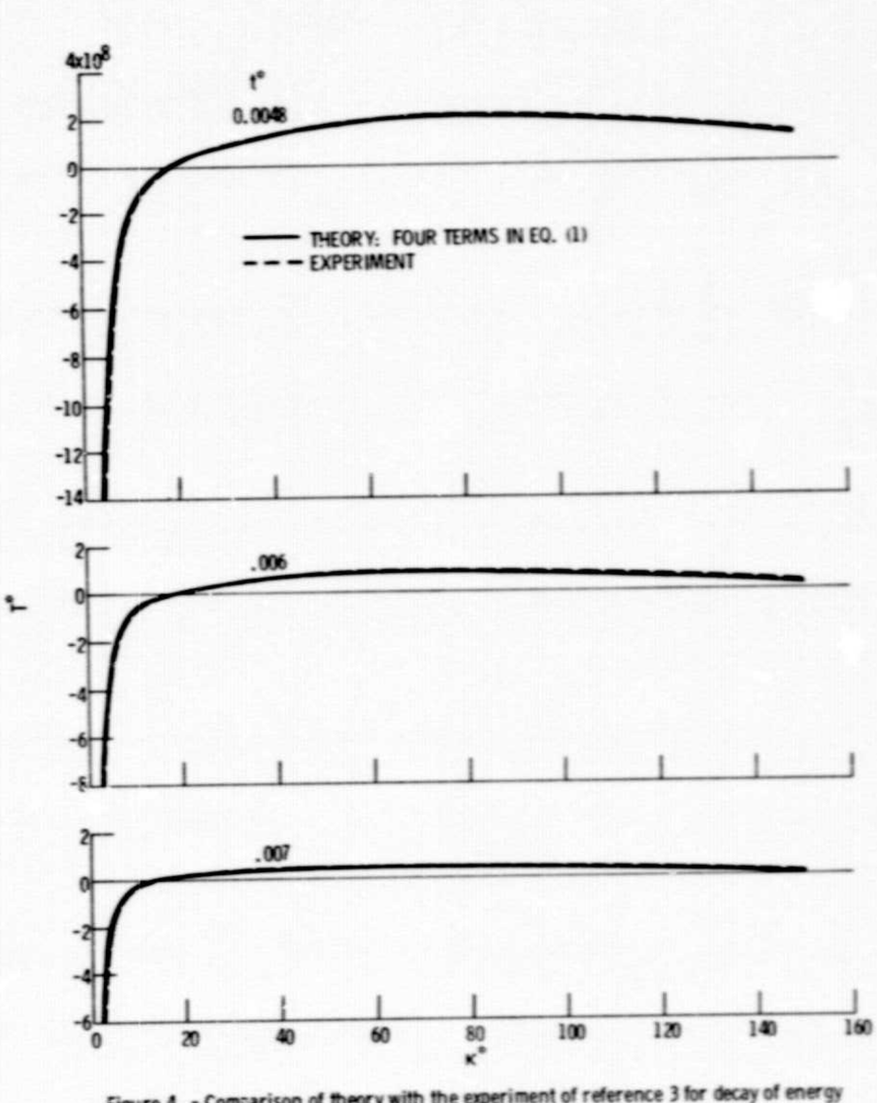

Figure 4. - Comparison of theory with the experiment of reference 3 for decay of energy transfer spectra.

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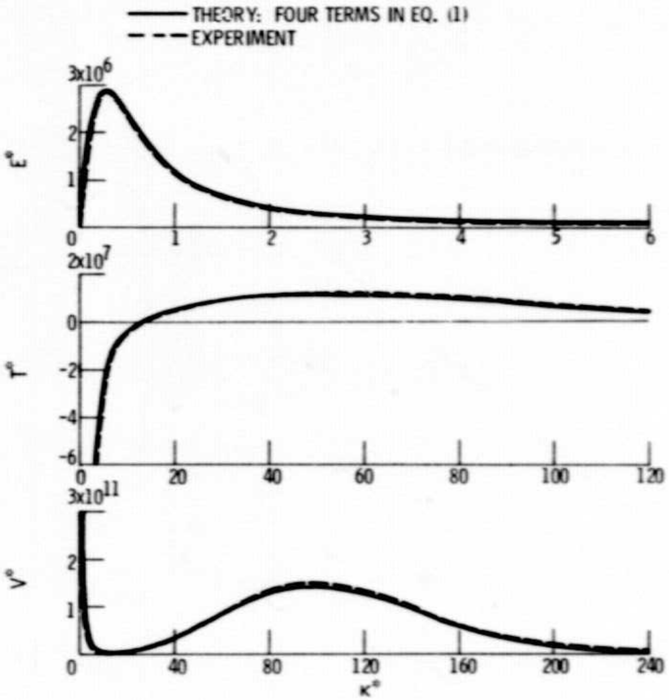

Figure 5. - Comparison of theory with experiment (ref. 3) at a late time (t° = 0.01) for the lower-order spectral quantities used to specify the initial turbulence.

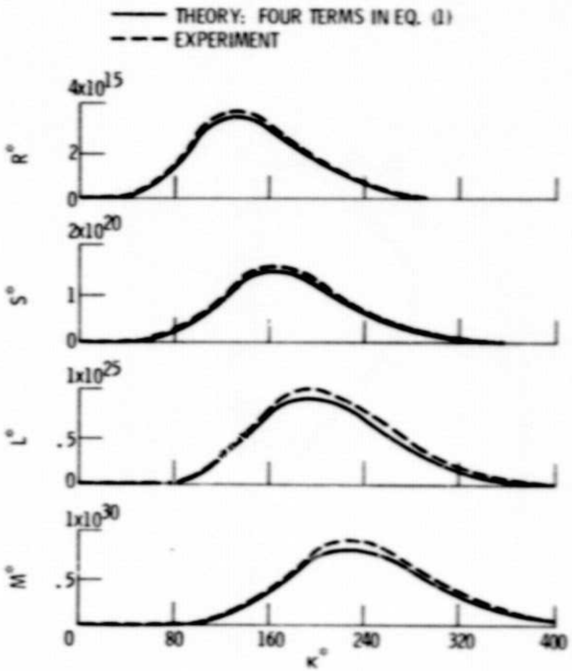

Figure 6. - Comparison of theory with experiment (ref. 3) at a late time (t° = 0.01) for the higher-order spectral quantities used to specify the initial turbulence.

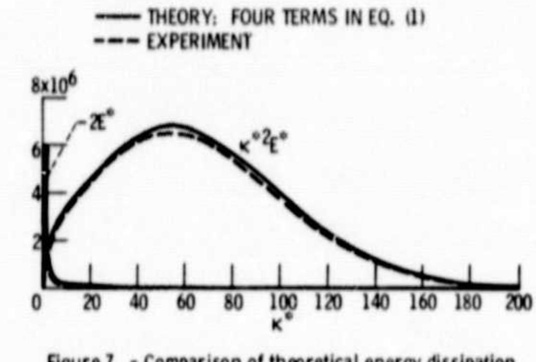

Figure 7. - Comparison of theoretical energy dissipation spectrum at a late time (t° = 0.01) with experiment of reference 3 and with energy spectra.

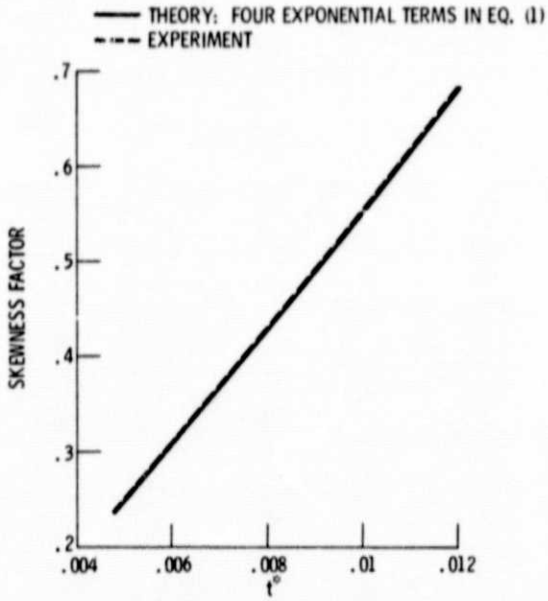

Figure 8. - Comparison of evolution of theoretical skewness factor of velocity gradient with experiment of reference 3.