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TECHNICAL MEMORANDUM (NASA) 63

LORAN-C TIME DIFFERENCE CALCULATIONS

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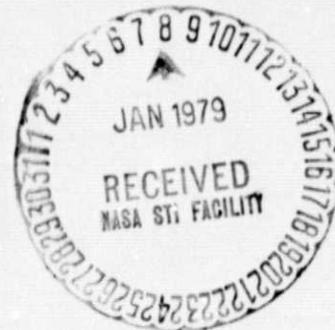
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A simplified approach to calculate  
Loran-C time differences from a  
given geographic location is presented.

by

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## I INTRODUCTION

Research at Ohio University is currently being done to develop a low-cost navigation package utilizing the Loran-C hyperbolic lattice system. One of the components of this package is a receiver to pick up the Loran-C signals and present them in the form of time differences. The time differences do not provide much information by themselves; it is necessary to convert them to a more useful form, such as geographic coordinates or range-bearing information. Although this has been done with charts and maps, today's high speed and inexpensive computing systems make the use of Loran-C much more attractive for aviation installations. This report will provide some of the simpler mathematical equations which may be used in Loran-C navigation calculations. Specifically, a technique will be presented to allow Loran-C time differences to be predicted at a location. This is useful for receiver performance work, and as a tool for more complex calculations, such as position fixing.

## II REVIEW OF LORAN-C SYSTEM

The standard Loran-C configuration consists of a master station and two or more secondary "slave" stations, synchronized to the master (see Figure 1). At the start of the group repetition interval (GRI), the master station transmits a set of pulses which are received at the secondary stations after an appropriate time delay. Each secondary then inserts a unique coding delay before re-transmitting the pulses. Reception of a master-secondary pair then defines a line-of-position (LOP) which, if plotted for a constant time difference (TD) reading, results in a hyperbola with the master and secondary as foci.<sup>[1]</sup> Reception of two master-secondary pairs result in two TD readings corresponding to the intersection of two LOP's. Conversions may now be done to obtain a more meaningful position fix. Using only two LOP's also results in a second ambiguous position, although this ordinarily does not present a serious problem in position fixing. The reception of Loran-C stations and generation of time difference information is a classic problem<sup>[2,3]</sup> and will not be considered in this text.

## III CALCULATION OF TIME DIFFERENCES

The receiver measures the time difference between the arrival of the master signal and the arrival of the secondary signal. This is sometimes known as the difference mode. Since the receiver cannot directly measure the distance to each of the stations, it effectively measures the time difference, or length, from the master to the secondary, plus the secondary coding delay, plus the distance from secondary to receiver, minus the distance from receiver to master. In equation form:

$$TD = \beta + \Delta + T_s - T_m$$

where  $\beta$  represents the baseline length from master to secondary,  $\Delta$  is the coding delay at the secondary, and  $T_s$  and  $T_m$  are the one-way baseline lengths from the receiver to the secondary and master respectively. The fixed quantities  $\beta$  and  $\Delta$  depend on the system geometry and the two baseline times are the unknowns which must be calculated.

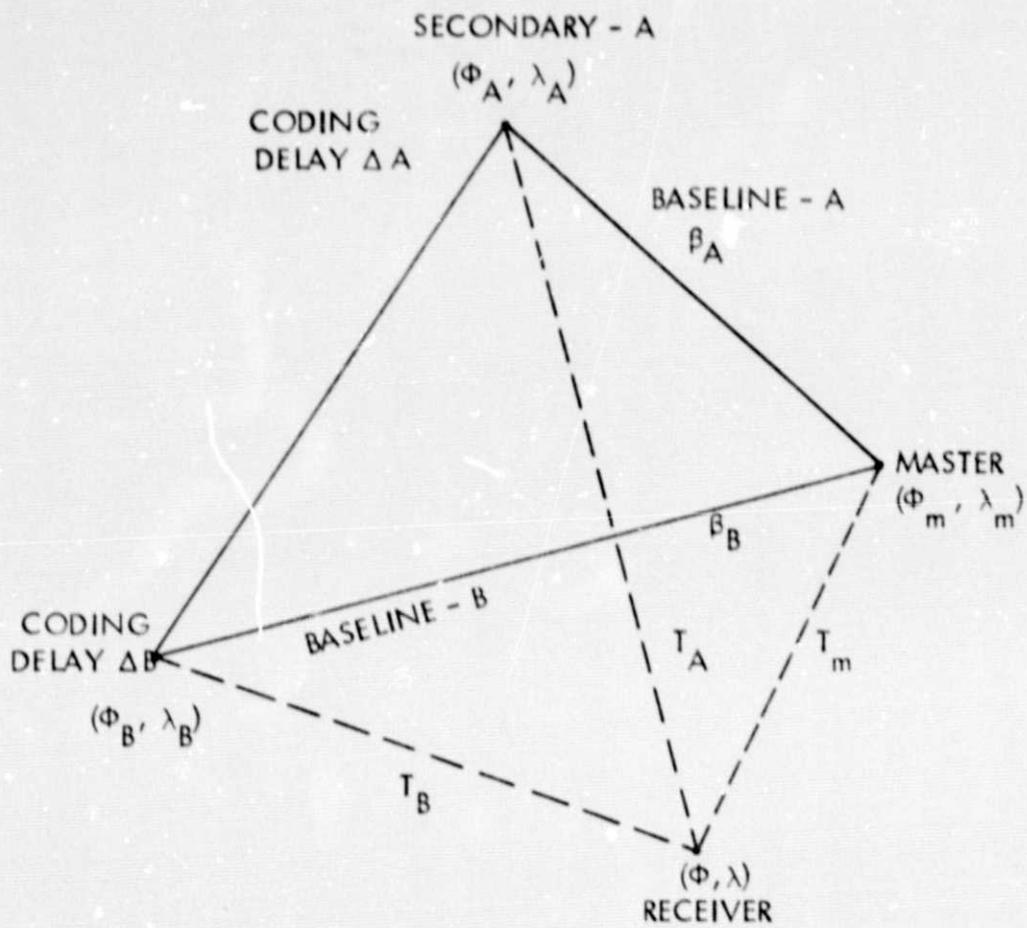


Figure 1. General Loran-C Configuration.

A geodesic model of the earth will now be developed to calculate distances between two points. The following discussion will be done in conjunction with Figure 2. A sphere with radius  $a$  is circumscribed around the best-fitting ellipsoid of the earth. The angle  $\Phi$  (latitude) is found from a segment perpendicular to the tangent at a point  $P$  on the ellipsoid. Angle  $\beta$  is found from the segment joining the vertical projection of  $P$  on the spheroid. From Figure 2:

$$x = a \cos \beta$$

$$y = b \sin \beta$$

and

$$\tan \Phi = -dx/dy$$

substituting

$$\tan \beta = (b/a) \tan \Phi.$$

If two points having coordinates  $(\Phi_1, \lambda_1)$  and  $(\Phi_2, \lambda_2)$  are considered, the circular arc distance is: [4]

$$\cos x = \sin \beta_1 \sin \beta_2 + \cos \beta_1 \cos \beta_2 \cos (\lambda_1 - \lambda_2)$$

By multiplying the arc distance just obtained by the semi-major radius of the earth, the distance may be obtained in the same units used for the radius (miles, kilometers, microseconds, etc.). Solving the distance on the geoid proceeds as follows [4]

$$P = (a-b) (x - \sin x) / 4(1 + \cos x)$$

$$Q = (a-b) (x + \sin x) / 4(1 - \cos x)$$

$$A_0 = (\sin \beta_1 + \sin \beta_2)^2$$

$$B_0 = (\sin \beta_1 - \sin \beta_2)^2$$

Finally

$$d = D - A_0 P - B_0 Q$$

where  $d$  is the geodesic arc length and  $D$  is the spherical arc length (in the same units).

By following the above procedure, the arc length may be found for receiver-to-master and receiver-to-secondary and used to solve the difference equation given earlier.

Figure 3 shows the above procedure as implemented in a computer program using the standard FORTRAN-IV programming language. The subprogram "DIS" calculates the geodesic arc length from the coordinates of two points. Figure 4 gives an explanation of the variables

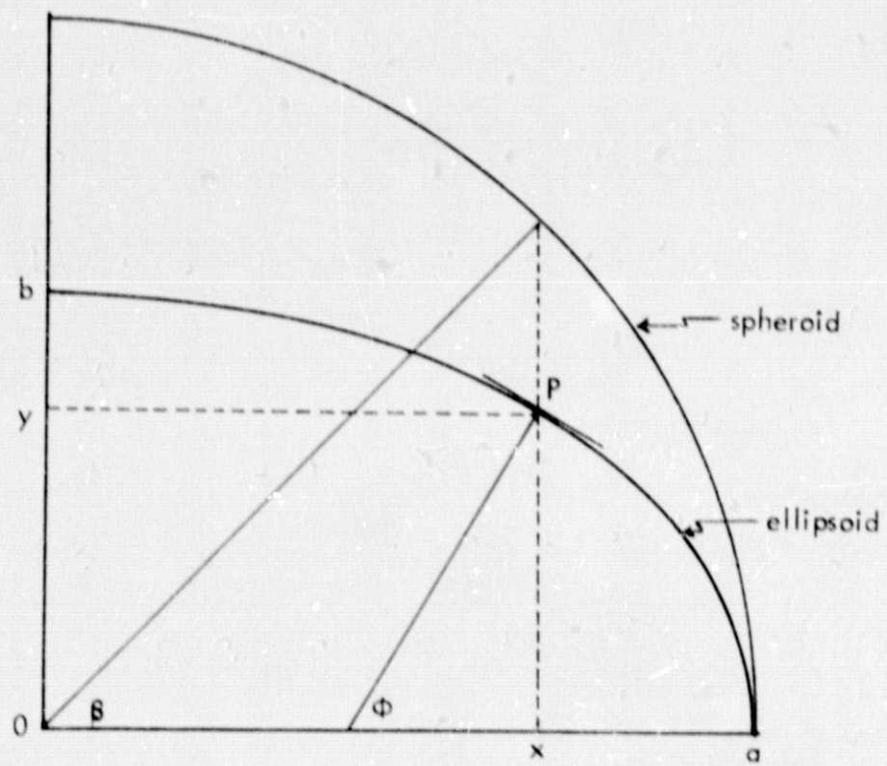


Figure 2. Relationship of Ellipsoid and Spheroid.

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IMPLICIT REAL(L)
DIMENSION CORD(6),STAT(5,2),BETA(4),DELTA(4),TD(4),T(5)
COMMON PHIR,LAMR
DATA STAT/0.59450743,0.47181301,0.81638789,0.72000571,0.6955499,1.
:35983819,1.39826820,0.92806991,1.221333821,1.52693106/
DATA BETA/2695.51,8389.65,3541.31,3560.71/
DATA DELTA/1.1E4,2.8E4,4.9E4,6.5E4/
DATA RDS/1.74532925E-2/
16 PRINT 6
6 FORMAT('CENTER LATITUDE AND LONGITUDE OF POSITION; **CR** TO TERMINATE')
:NATE,'/' SDDD MM SS.S')
READ(5,1,END=15)CORD
1 FORMAT(F4.0,1X,F2.0,1X,F4.0)
PHIR=RDS*(CORD(1)+(CORD(2)+CORD(3)/60.)/60.)
LAMR=RDS*(CORD(4)+(CORD(5)+CORD(6)/60.)/60.)
DO 10 NST=1,5
10 T(NST)=DIS(STAT(NST,1),STAT(NST,2))
DO 11 K=1,4
11 TD(K)=BETA(K)+DELTA(K)+T(K+1)-T(1)
PRINT 7,TD
7 FORMAT('DUS EAST RR = 9930'/' W = ',F9.3,5X,' X = ',F9.3,5X,' Y = ',
:F9.3,5X,' Z = ',F9.3)
GOTO 16
15 STOP
END
REAL FUNCTION DIS(PHI1,LAM1)
IMPLICIT REAL(L)
COMMON PHI,LAM
DATA A/2.128233935E4/,B/2.121099393E4/,C/9.9664767E-1/,F/3.3523298
:69E-3/
BETA=ATAN(C*TAN(PHI))
BETA1=ATAN(C*TAN(PHI1))
X=ARCOS(SIN(BETA)*SIN(BETA1)+COS(BETA)*COS(BETA1)*COS(LAM-LAM1))
D=A*X
P=(A-B)*(X-SIN(X))/(4.*(1.+COS(X)))
G=(A-B)*(X+SIN(X))/(4.*(1.-COS(X)))
AO=(SIN(BETA)+SIN(BETA1))*2
BO=(SIN(BETA)-SIN(BETA1))*2
DIS=ABS(D-AO*P-BO*G)
RETURN
END

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Figure 3. Time Difference Program.

### Main Program

BETA(1)	Baseline - Carolina Beach to Jupiter	(W)
BETA(2)	Baseline - Carolina Beach to Cape Race	(X)
BETA(3)	Baseline - Carolina Beach to Nantucket	(Y)
BETA(4)	Baseline - Carolina Beach to Dana	(Z)
CORD(1,2,3)	Latitude of receiver; degree-minute-second	
CORD(4,5,6)	Longitude of receiver; degree-minute-second	
DELTA(1)	Secondary Coding Delay at Jupiter	(W)
DELTA(2)	Secondary Coding Delay at Cape Race	(X)
DELTA(3)	Secondary Coding Delay at Nantucket	(Y)
DELTA(4)	Secondary Coding Delay at Dana	(Z)
LAMR	Decimal value of longitude	
PHIR	Decimal value of latitude	
RDS	$\pi/180$	
STAT(1,1)	Latitude of Master (Cape Fear)	
STAT(2,1)	Latitude of Secondary W (Jupiter)	
STAT(3,1)	Latitude of Secondary X (Cape Race)	
STAT(4,1)	Latitude of Secondary Y (Nantucket)	
STAT(5,1)	Latitude of Secondary Z (Dana)	
STAT(1,2)	Longitude of Master (Cape Fear)	
STAT(2,2)	Longitude of Secondary W (Cape Race)	
STAT(3,2)	Longitude of Secondary X (Jupiter)	
STAT(4,2)	Longitude of Secondary Y (Nantucket)	
STAT(5,2)	Longitude of Secondary Z (Dana)	
T	Distance from receiver to each of the stations	
TD	Time differences for all four pairs	

Figure 4. List of Variables.

Subprogram DIS

A	Semi-major radius of earth (microseconds)
AO	Used in curvature correction
B	Semi-minor radius of earth (microseconds)
BETA	Angle of projected point on sphere
BETA1	Angle of projected point on sphere
BO	Used in curvature correction
C	$B / A$
D	Spherical arc length
DIS	Geodesic arc length
F	Flattening of the earth, 1/298.2
LAM	Longitude of receiver
LAM1	Longitude of Loran station
P	Used in curvature correction
PHI	Latitude of receiver
PHI1	Latitude of Loran station
Q	Used in curvature correction
X	Angular arc length

Figure 4 (Cont.). List of Variables.

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used in this program. The program operates as follows: The known values for all the station coordinates of the U.S. East Coast chain are initialized at the beginning of the program along with the known values of the baseline lengths and coding delays. All distances in this program are in microseconds and angles are in radians. When the program runs, it requests the entry of the geographics coordinates in the standard degree-minute-second format. These coordinates are then converted to decimal numbers in radians. Subprogram "DIS" is called five times to calculate the arc length from the receiver to the master and the four secondary stations. These values are then used to solve the time difference equation and the time differences just computed are printed out for the four pairs. Although this program is set up for the U.S. East Coast chain (9930), it may be changed to other repetition rates by changing the coordinates of the stations, the baseline lengths, and the coding delays. Information on the earth used in this program is derived from the Fischer Spheroid System.<sup>[5]</sup>

#### IV ADDITIONAL CONSIDERATIONS

To get a more accurate prediction of Loran-C time differences at a location, there are two factors which should be taken into account. These are: a more accurate model of the earth's curvature, and the effect of signal propagation delays. For absolute work, there are methods<sup>[6,7]</sup> of making estimates of the earth's shape using techniques such as oblate spheroids with unequal axes or power series expansions of the surface. These methods are usually of academic interest; they are too complicated for a small computer system in an aircraft. Using the techniques discussed earlier allows time difference calculations to be made to within three to four microseconds.

Propagation delays are difficult to model in a general sense. The terrain over which the signals must travel have a significant effect on delays.<sup>[8]</sup> Many time difference calculations are done assuming a vacuum and then corrected for salt water retardation. For propagation over land, a profile of the surface features may be made, and a typical retardation value for the profile may be calculated and applied to subsequent time difference calculations. By doing this, accuracy may be improved by two to three microseconds over a vacuum depending on the distances from the transmitters and the surface profile used. Other factors which can either retard or accelerate signals are atmospheric and ionospheric anomalies and local weather conditions. These would have to be monitored daily if absolute accuracy is to be obtained.

Many studies have been made of the effects of the earth's curvature and propagation delays on the accuracy of predicting Loran time differences. Various levels of corrections may be applied depending on the individual situation.

#### V CONCLUSIONS

The foregoing has presented a technique which is simple enough to be used on small computer systems, yet provides sufficient accuracy for most Loran-C work. A variation of

the computer program presented has been implemented successfully on a small hand-held calculator.<sup>9</sup> Although using elementary models for curvature and propagation delays, there is enough flexibility to allow more accurate corrections to be made. The program discussed has been a great aid to receiver performance work and the testing of position-fixing calculations.

## VI ACKNOWLEDGEMENTS

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