AAA GUNNER MODEL BASED ON OBSERVER THEORY

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SUMMARY

The Luenberger observer theory is used to develop a predictive model
of a gunner's tracking response in antiaircraft artillery (AAA) systems.
This model is composed of an observer, a feedback controller and a remnant
element. An important feature of the model is that the structure is
simple, hence a computer simulation requires only a short execution time.
A parameter identification program based on the least squares curve
fitting method and the Gauss Newton gradient algorithm is developed to
determine the parameter values of the gunner model. Thus, a systematic
procedure exists for identifying model parameters for a given antiaircraft
tracking task. Model predictions of tracking errors are compared with
human tracking data obtained from manned AAA simulation experiments
conducted at the Aerospace Medical Research Laboratory, Wright-Patterson
AFB, Ohio. Model predictions are in excellent agreement with the
empirical data for several flyby and maneuvering target trajectories.

INTRODUCTION

A systematic study of threat effectiveness for antiaircraft artillery
(AAA) systems requires the development of a mathematical model for the
gunner's tracking response. The gunner model is then incorporated into
computer simulation programs as shown in reference 1 for predicting
aircraft attrition with respect to specific antiaircraft weapon systems.
Two of the fundamental design requirements of a gunner model are simplicity
in model structure and accuracy in the tracking error predictions. A
simple gunner model structure will shorten computer simulation execution
time. Obviously, accurate predictions of tracking error implies model
fidelity with respect to describing the gunner's tracking performance.
Then, the manned threat quantification in the threat analysis will be
reliable.

An antiaircraft gunner model based on the Luenberger observer theory in references 2, 3 and 4, is developed in this paper. It satisfies both the design requirements mentioned above. The structure of the model is simple and its predictions of tracking errors are accurate. It is composed of three main parts - an observer, a feedback controller, and a remnant element. An observer is itself a dynamic system whose output can be used as an estimate of the state of a given system. The simplicity of the observer design makes the observer an attractive design method. The estimated state is then used to implement a linear state variable feedback controller which represents the gunner's control function in the compensatory tracking task. The effects of all the randomness sources due to human psychophysical limitations and of modelling errors are lumped into one random remnant element in this model design. Another important feature of this model is that its parameters can be determined systematically instead of by trial-and-error. A parameter identification program based on the least squares curve-fitting method in reference 5 and the Gauss-Newton gradient algorithm in reference 6 is developed for this purpose. This program iteratively adjusts the parameter values to minimize the least squares error between the model prediction of tracking error and actual human tracking data obtained from manned AAA simulation experiments conducted at the Aerospace Medical Research Laboratory, WPAFB, Ohio. Thus, it provides a convenient procedure for model validation. In addition, a computer simulation program is developed with the designed model describing the gunner's response for a given AAA tracking task. The program provides time functions of the ensemble mean and standard deviation for the model's tracking error predictions (azimuth and elevation). Computer simulation results are in excellent agreement with the empirical data for several aircraft flyby and maneuvering trajectories. This verifies that the model can predict tracking errors accurately and thus is a reliable description of the gunner's compensatory tracking characteristics.

A comparison between this model and the optimal control model in references 7, 8 and 9 (by Kleinman, Baron, Levison) is also given. It can be shown that the model based on observer theory is as accurate as the optimal control model in predicting tracking errors. In addition, the computer execution time of the AAA closed loop system simulation utilizing this model is less than 15% of that using the optimal control model. This is a primary advantage of a model with simple structure.

DESCRIPTION OF AN AAA GUN SYSTEM

The tracking task of an antiaircraft artillery (AAA) gun system can be described by a closed loop (single axis tracking loop) block diagram as shown in figure 1. Two gunners, one each for azimuth and

elevation axes, play the role of controller in this man-machine feedback control system. From his visual display, each gunner observes the tracking error, e_T (one for azimuth error and the other for elevation error), which is the difference between the target position angle θ_T and the gunsight line angle θ_g. Independently, the gunners operated the hand crank to control the gunsight system in order to align the gunsight line angle (output) with the target position angle (input). Therefore, the azimuth tracking task is decoupled from the elevation tracking task in this AAA system.
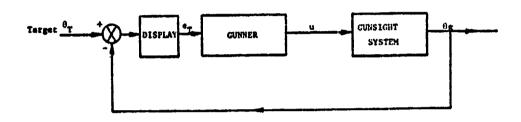

FIGURE 1: BLOCK DIAGRAM OF THE AAA CLOSED LOOP SYSTEM

The purpose of this paper is to develop a mathematical model of the response characteristics of a gunner in a compensatory tracking task. Therefore, in the following, we first describe the mathematical representation of the gunsight and rate-aided control dynamics (the gunsight system) and the target trajectories. In this paper the transfer function of the gunsight system considered is:

$$\frac{\Theta_g(s)}{U(s)} = \frac{1}{s} \qquad (1)$$

for the azimuth angle tracking as well as the elevation angle tracking. ($\Theta_g(s)$ and $U(s)$ are the Laplace transforms of $\theta_g(t)$ and $u(t)$ respectively.) It can be shown that this transfer function is a valid representation of many practical gunsight systems. Several flyby and maneuvering trajectories in reference 10 of the target aircraft of 45 seconds duration were selected as input to the AAA system of figure 1. These trajectories are deterministic functions of time. (But their dynamic properties $\dot{\theta}_T$, $\ddot{\theta}_T$, etc., are not known precisely to trackers. The state space equation of the gunsight system and the target motion can be derived as follows.

$$\dot{\underline{x}} = A\underline{x} + Bu + F\ddot{\theta}_T \qquad (2)$$

where $\underline{x}$ denotes the state vector having two components,

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \theta_T - \theta_g \\ \dot{\theta}_T \end{bmatrix}$$

and A, B, and F matrices are

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \quad F = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

and the scalars u and $\ddot{\theta}_T$ denote the control from the AAA gunner and the target acceleration. The tracking error e_T on the visual display is observed by the gunner and is expressed in the measurement equation:

$$y = C\underline{x} \tag{3}$$

where y is the observed tracking error and C is a row vector [1 0]. Equations (2) and (3) will be used in the next section to develop an AAA gunner model.

AAA GUNNER MODEL

This section presents a mathematical model of an antiaircraft gunner in the compensatory tracking task. The main design requirements for developing this model are:

- accurate model prediction of tracking errors

- simple model structure

- systematic determination of model parameters

In this paper, the Luenberger reduced order observer theory has been applied to design the gunner model which satisfies the above design requirements. Figure 2 shows the block diagram of this model consisting
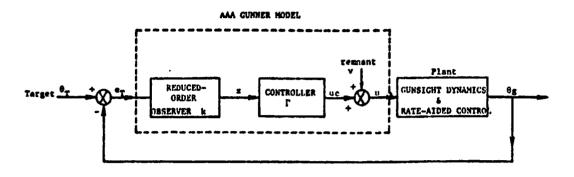
AAA GUNNER MODEL

FIGURE 2: BLOCK DIAGRAM OF THE STRUCTURE OF THE AAA GUNNER MODEL

of three main elements: observer, controller, and remnant. The first
element is a reduced-order observer which processes the gunner's
observation from the visual display to provide an estimate of the states
of the AAA system. It will be shown that the system equation (2) is a
2nd order system, but the reduced-order observer is only of the first
order. Since some components of the state as given by the system outputs
are already available by direct measurement. The estimation of these
components of the state is not necessary and will cause a certain degree
of redundancy. The use of a reduced-order observer eliminates this
redundancy and still provides sufficient information to reconstruct (or
estimate) the state of the observed system. The controller represents
the gunner's tracking function by an estimated-state linear feedback
control law. The observer and the controller consists of the deterministic
part of the gunner model. The effects of the various randomness sources
in the AAA man-machine closed loop system and of the modelling errors
are lumped into one element called remnant which is the stochastic part
of the gunner model. These randomness sources include the modelling error,
the observation error, the neuromotor noise, etc. Mathematical equations
of this model are given below.

Model Equations

System equations (2) and (3) are used in the design of the gunner
model. However, the gunner doesn't have the precise information about the
target dynamics, so the term representing target acceleration, $\ddot{\theta}_T$, in
Eq. (2) will not be included in the design of the observer equation.
The effect on the tracking error due to the modelling error of the gunner's
uncertainty about target dynamics will be included in the remnant element.
Now from Eq. (3),

$$y = C\underline{x} = x_1$$

the tracking error is available from direct observation. Thus, it is only
necessary to estimate the second component x_2 of the state vector $\underline{x}$ in
order to implement a state variable feedback control law. By the

67

reduced-order Luenberger observer theory in reference 4, an estimate $\hat{x}_2$ of the state variable x_2 can be obtained by

$$\dot{\hat{x}}_2 = (a_{22} - ka_{12})\,\hat{x}_2 + k\dot{y} + (a_{21} - ka_{11})\,y + (b_2 - kb_1)\,u_c \tag{4}$$

where a_{ij} and b_k are the elements of matrices A and B in Eq. (2), the scalar k is the observer gain, y and $\dot{y}$ are the observed tracking error and error rate respectively, and u_c is the linear feedback control law (the controller) with the form:

$$u_c = -[\gamma_1\ \gamma_2] \begin{bmatrix} y \\ \hat{x}_2 \end{bmatrix}$$

where the feedback control gains γ_1 and γ_2 are two constants determined in reference 10. Note that the state feedback is composed of y (the observed variable which is x_1) and $\hat{x}_2$ (the estimated state of x_2). It can be shown that the system (2) and (3) is completely observable. (The definition of observability and the conditions of a system to be observable can be found in reference 11). Then, by the observer theory, there always exists an observer gain k to make the eigenvalue of the observer (Eq. (4)) negative. Thus, the output of the observer will be a good estimation to the state of the observed system. This shows the existence of proper observer gain k in Eq. (4). Actually, the value of observer gain k is determined by a curve-fitting identification program. The required differentiation of y in Equation (4) can be avoided by introducing the following variable:

$$z(t) = \hat{x}_2 - ky(t) \tag{5}$$

Hence the observer dynamics can be represented by

$$\dot{z} = (a_{22} - ka_{12})z + (a_{22} - ka_{12})ky + (a_{21} - ka_{11})y + (b_2 - kb_1)u_c \tag{6}$$

Next, the actual output of this model is expressed as the sum of the output u_c of the controller and the remnant element v.

$$\begin{aligned} u &= u_c + v \\ &= -[\gamma_1\gamma_2] \begin{bmatrix} y \\ \hat{x}_2 \end{bmatrix} + v \end{aligned} \tag{7}$$

where the remnant term v(t) is modeled as a white noise and its statistical properties are selected to be

$$E\,[v(t)] = 0 \qquad \text{for all } t \tag{8}$$

$$E\,[v(t)\,v(\tau)] = q(t)\,\delta(t - \tau) \qquad \text{for all } t \text{ and } \tau \text{ where}$$

E is the expectation operator, $\delta(t)$ is the Dirac delta function and the

covariance function $q(t)$ is assumed as a function of estimated target dynamics,

$$q(t) = \alpha_1 + \alpha_2 \hat{\dot{\theta}}_T^2(t) + \alpha_3 \hat{\ddot{\theta}}_T^2(t) \tag{9}$$

where α_1, α_2, and α_3 are three nonnegative constants to be determined, and $\hat{\dot{\theta}}_T$ and $\hat{\ddot{\theta}}_T$ are estimated target angle rate and acceleration respectively.

Equations of the Closed-loop AAA System

In the previous section, the gunner model equations of the observer, the controller, and the remnant have been derived. These equations are combined with system equations (2) and (3) to obtain the mathematical model of the closed loop AAA system. Since $x_1 = y$, Eqs. (2) and (6) can be rewritten as follows:

$$\dot{y} = a_{11}y + a_{12}x_2 + b_1 u + f_1 \ddot{\theta}_T$$

$$\dot{x}_2 = a_{21}y + a_{22}x_2 + b_2 u + f_2 \ddot{\theta}_T$$

$$\dot{z} = (a_{22} - ka_{12})z + (a_{22} - ka_{12})ky + (a_{21} - ka_{11})y + (b_2 - kb_1)u_c \tag{10}$$

$$u = u_c + v$$

$$u_c = -[\gamma_1 \ \gamma_2] \begin{bmatrix} y \\ \hat{x}_2 \end{bmatrix}$$

By introducing new variables:

$$x_3 = x_2 - ky$$

and

$$e = x_3 - z \tag{11}$$

Eq. (10) can be rewritten as

$$\dot{X} = A_1 X + F_1 \ddot{\theta}_T + D_1 v \tag{12}$$

where X is the state vector of the overall system with components:

$$X = \begin{bmatrix} y \\ x_3 \\ e \end{bmatrix} = \begin{bmatrix} y \\ x_2 - ky \\ x_3 - z \end{bmatrix}$$

69

and A_1, F_1, and D_1 are matrices defined as follows:

$$A_1 = \begin{bmatrix} a_{11} + a_{12}k - b_1(\gamma_1 + k\,\gamma_2) & a_{12} - b_1\,\gamma_2 & b_1\,\gamma_2 \\ \begin{matrix}(a_{22}-ka_{12})k+a_{21}-ka_{11} \\ -(b_2-kb_1)\,(\gamma_1+k\gamma_2)\end{matrix} & a_{22}-ka_{12}-(b_2-kb_1)\gamma_2 & (b_2-kb_1)\gamma_2 \\ 0 & 0 & a_{22} - ka_{12} \end{bmatrix}$$

$$F_1 = \begin{bmatrix} f_1 \\ f_2 - kf_1 \\ f_2 - kf_1 \end{bmatrix} \quad , \quad D_1 = \begin{bmatrix} b_1 \\ b_2 - kb_1 \\ b_2 - kb_1 \end{bmatrix}$$

Once the structure of the model is designed, the next step is to determine the parameters associated with this model (i.e. k, γ_1, γ_2, α_1, α_2, α_3 in Eqs. (6), (7) and (9)). It is important to have a systematic method to determine these parameters for a given AAA system. A parameter identification program based on the least squares curve-fitting method and the Gauss Newton gradient algorithm has been developed by the authors. This program can easily determine the parameters of the gunner model by minimizing the difference between the model prediction of the tracking error and the corresponding empirical data. The following equations (mean equation and covariance equation) are used in the curve-fitting program. Letting the expectation value of X be $\bar{X}$, then we have,

$$\dot{\bar{X}} = A_1\bar{X} + F_1\ddot{\theta}_T$$

(13)

and the covariance matrix of X (t) is $P(t) = E[\ (X(t) - \bar{X}(t))(X(t)-\bar{X}(t))^T]$; then it can be shown in reference 11 that the covariance matrix is governed by

$$\dot{P} = A_1P + PA_1^T + D_1\ q(t)\ D_1^T$$

(14)

Equations (13) and (14) are used in the paremeter identification program to fit the empirical data obtained from the manned AAA simulation experiments conducted at the Aerospace Medical Research Laboratory, Wright-Patterson AFB, Ohio. The detail of the identification program and the curve-fitting procedure can be found in reference 10. The results

of curve-fitting parameter identification program are shown in the following table:

	k	γ_1	γ_2	α_1	α_2	α_3
Azimuth Tracking	2.94	-2.87	-1.00	.0496	.0024	.103
Elevation Tracking	3.02	-3.01	-1.00	.0032	.00047	.259

SIMULATION RESULTS AND DISCUSSION

The numerical values of the parameters of this gunner model were determined in the previous section with respect to the gunsight dynamic system (Eq. (2)) and a specific target trajectory. The gunner model is now ready to be used for computer simulation. A computer simulation program of the AAA system with this model representing the gunner response was developed. The input to this program is the target motion trajectory. The outputs are the model predictions of the ensemble mean and standard deviation of the tracking error. A typical result is plotted in fig. 3 for a specific target trajectory. The solid line in fig. 3 denotes the
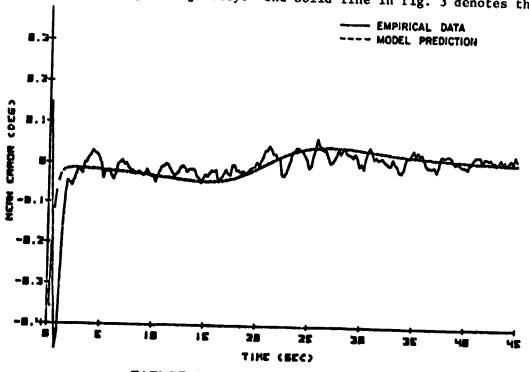

FIGURE 3.   MEAN TRACKING ERROR

empirical data of the sample ensemble mean for azimuth tracking errors.
The corresponding model prediction is denoted by the dotted line in fig 3.
There is an excellent match between these two curves. Similar comparison
between model predictions and empirical data for several other flyby and
maneuvering target trajectories can be found in reference 10. All the
simulation results show that this gunner model with the same parameter
values can predict accurately the tracking errors for various target
trajectories with similar frequency band widths. Therefore, it is a
predictive model. The values of the model paremeters depends on the gun-
sight dynamic system. Furthermore, this model is adaptive with respect to
the target motion and this adaptive property is considered in the
structure of the covariance function (Eq. (9)) of the remnant element.

A comparison of the model prediction accuracy between this model
and the optimal control model in reference 7 has been done for several
target trajectories. All the results show that both models give accurate
predictions of tracking errors. A typical result is shown in fig. 4
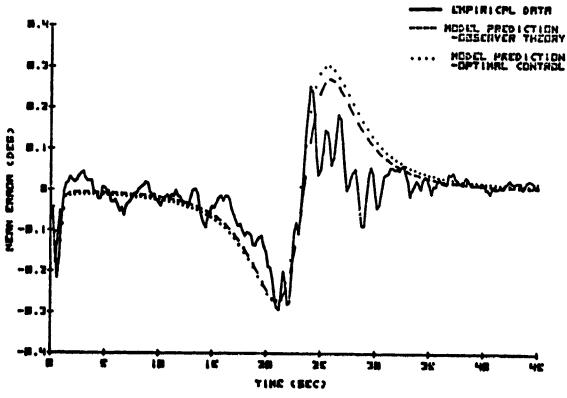for a flyby trajectory. It is obvious that the gunner model developed in

FIGURE 4.   MEAN TRACKING ERROR
AZIMUTH

this paper can predict the tracking errors as accurately as those obtained by the optimal control model. However the computer execution time of simulating the AAA gun system using the gunner model is less than 15% of that by the optimal control model. It is a primary advantage of a model with simple structure. It can be concluded that the gunner model based on observer theory is very useful in the analysis of the performance of the AAA gun system.

CONCLUSION

The Luenberger observer theory has been applied to design an antiaircraft gunner model which is composed of a reduced-order observer, a state variable feedback controller and a remnant element. The high-lights of this model are simple in the structure and accurate in the model prediction of tracking errors. The key design requirement is to make the model structure simple so that it can shorten computer simulation time. It has also been shown in figures 3 and 4 that this model can predict the tracking errors accurately. In addition, a parameter identification program based on the least squares curve-fitting method and the Gauss Newton algorithm has been used to systematically determine the numerical values of the model parameters. This gunner model has been used to study the AAA effectiveness of several air defense weapon systems at the Aerospace Medical Research Laboratory, Wright-Patterson AFB. All the results show that it is an accurate and efficient antiaircraft gunner model.

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