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PILOT-OPTIMAL AUGMENTATION SYNTHESIS

by

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Abstract

Given adequate open-loop specifications, for example, aircraft handling qualities criteria, design techniques, particularly modern control approaches, are available to the system designer for synthesizing even the most complex flight control systems. Unfortunately, however, weaknesses exist in the handling qualities areas, particularly for "non-conventional" aircraft such as V/STOL and control configured vehicles (CCV's). In this paper, an augmentation synthesis method usable in the absence of quantitative handling qualities specifications, and yet explicitly including design objectives based on pilot-rating concepts, will be presented. The algorithm involves the unique approach of simultaneously solving for the stability augmentation system (SAS) gains, pilot equalization and pilot rating prediction via optimal control techniques. Simultaneous solution is required in this case since the pilot model (gains, etc.) depends upon the augmented plant dynamics, and the augmentation is obviously not a priori known. Another special feature is the use of the pilot's objective function (from which the pilot model evolves) to design the SAS.

Introduction

Given adequate design specifications, or aircraft handling qualities criteria, and a valid system model, design techniques, particularly modern control approaches, are available to the system designer for synthesizing even the most complex flight control systems. Unfortunately, however, weaknesses exist in the design specification area for non-conventional aircraft such as V/STOL and control configured vehicles (CCV's). The assertion here is that due to the "non-conventional," multi-variable nature of the vehicle (and the piloting task in the case of V/STOL), and due to the anticipated complexity of the systems involved, a "non-conventional" approach to the control design problem is worthy of investigation.

Since pilot acceptance is the ultimate criteria, in the absence of prior pilot opinion we must predict pilot rating. This is in contrast to design methods which attempt to a priori define "good" dynamics, and then use a model-following design technique[1], that is, design the augmentation so the augmented system will behave like the "good" model. One major drawback to this approach is that one is never sure that the pilot will agree with the

designers choice of "good" dynamics.

To predict pilot rating, some form of pilot model is required and two types of pilot models exist. Each have been used extensively; they include describing-function models[2] and optimal control models[3]. It is felt that for the problem at hand the optimal-control pilot model is ideal. It is more compatible with the multi-variable aspects of the problem and the advanced control design techniques already existing. Also, the form of the pilots equalization network is automatically determined, a very important property in this case.

Both pilot modeling approaches have been used primarily to study closed-loop system performance. Recent application areas for the optimal control model include low-visability landing of CTOL[4] and STOL aircraft[5] and the stability of the pilot aircraft system in maneuvering flight[6]. However, any stability augmentation systems in these studies were designed initially, from handling criteria for example, then the system performance evaluated as a separate step. That is, the SAS was designed first using the conventional approaches (e.g., pole placement), then the pilot model was added around the augmented system to evaluate "piloted" system performance.

Alternate approaches include the pilot as part of the plant[7], then the SAS design proceeds for the "pilot-augmented" plant. Buy the form of the pilot model must be assumed before beginning this design process, an undesirable situation for systems with non-conventional plant dynamics. The pilot is known to adapt his gain and form of equalization to the plant and task, but selecting the pilot model a priori would tend to imply knowledge of and invariance of the form of pilot model. Hence, the form of the pilot model should be determined as an integrated part of the system design. As stated previously, this is naturally accomplished with the optimal-control pilot model.

An analytical pilot model has also been used, although not as frequently, to predict pilot opinion. The most notable of these techniques, applied to the VTOL hover task, was the "paper pilot" developed by Anderson[8,9]. In this approach, parameters in the pilot describing function of assumed form are chosen such that a pilot-rating metric is minimized. This metric consists of a measure of performance (e.g., rms tracking error), and a measure of pilot workload (e.g., the amount of lead the pilot must introduce). In an assessment of this technique[10] it was found that a pilot rating functional based on easily measured motion quantities was adequate for pilot opinion prediction. However, the proposed pilot model was found to require some additions for better system performance predictions. Notably, these improvements included modification in form (describing function) and the addition of the pilot's remnant (or the "random" portion of the pilot's control input.) Hence, again we see the problems created by imposing an assumed form of the pilot's describing function.

This problem would appear to be alleviated by the use of the optimal pilot model. In fact, Hess[11] has found that the optimal control model can be used equally well for predicting pilot opinion, and has used this approach

in analytical display design for helicopters[12,13]. Use of the optimal pilot model for pilot rating allows for a natural pilot-rating metric via the pilot model objective function. Proper selection, based on the task, of the state and control weights in the objective function provides for the determination of the pilot gains, equalization, and pilot rating prediction simultaneously. As we have mentioned previously, this is a very important point in dealing with systems with non-conventional dynamics for which the pilot's describing function may not be known. If this approach is now integrated with the SAS design problem, a proposed design procedure results.

The Pilot Model

As presented in Reference 3, the optimal pilot model evolves from the assumption that the well-trained, well-motivated pilot selects his control input(s), u_p, subject to human limitations, such that the following objective is minimized,

$$J_p = E\left\{\lim_{T\to\infty} \frac{1}{T} \int_0^T (\bar{y}'Q\bar{y} + u_p'Ru_p + \dot{u}_p'G\dot{u}_p)dt\right\}$$

The dynamic system being controlled by the pilot is described by the familiar linear relation

$$\dot{\bar{x}} = A_p\bar{x} + B_p\bar{u}_p + \bar{w}$$

$$\bar{y} = C\bar{x} \qquad (1)$$

where $\bar{x}$ is the system state vector, $\bar{u}_p$ the pilot control vector, $\bar{y}$ the output vector, and $\bar{w}$ is the vector of zero-mean external disturbances with covariance

$$E[\bar{w}(t)\bar{w}'(t+\sigma)] = W\delta(\sigma)$$

Included as human limitations are observation delay, τ, and observation noise, $\bar{v}_y$. So the pilot actually perceives the noise-contaminated, delayed states, or

$$\bar{y}_p = C_p\bar{x}(t-\tau) + \bar{v}_y(t-\tau)$$

The covariance of the zero-mean observation noise may include the effects of perception thresholds and attention allocation, and is denoted

$$E[\bar{v}_y(t)\bar{v}_y'(t+\sigma)] = V_y\delta(\sigma)$$

Defining the augmented state vector, $\tilde{x} = \mathrm{col}[\bar{x},\bar{u}_p]$, the solution to the problem, or the pilot's control is given as

$$\dot{\bar{u}}_p^* = -G^{-1}[0 \vdots I]K_p\bar{x}$$

where K_p is the positive definite solution to the Riccati equation

$$-\left[\begin{array}{c:c} A_p & B_p \\ \hdashline 0 & 0 \end{array}\right]' K_p - K_p \left[\begin{array}{c:c} A_p & B_p \\ \hdashline 0 & 0 \end{array}\right] - \left[\begin{array}{c:c} C_p' QC_p & 0 \\ \hdashline 0 & R \end{array}\right]$$

$$+ K_p B_o G^{-1} B_o' K_p = \dot{K}_p \qquad (2)$$

It will be convenient to partition K_p such that

$$K_p = \left[\begin{array}{c:c} K_{p_1} & K_{p_2} \\ \hdashline K_{p_3} & K_{p_4} \end{array}\right]$$

and note that now the equations for the optimal control $\bar{u}_p^*$ is

$$\dot{\bar{u}}_p^* = -G^{-1}K_{p_3}\hat{\bar{x}} - G^{-1}K_{p_4}\bar{u}_p^{*\dagger}$$

or a linear feedback of the best estimate of the state, $\hat{\bar{x}}$, and some control dynamics. (These control dynamics have been shown to be equivalent to the pilot's neuro-muscular lag.)

Now, the state estimator consists of a Kalman filter and a least-mean square predictor, or

$$\dot{\hat{\bar{x}}}(t-\tau) = A_p\hat{\bar{x}}(t-\tau) + \Sigma C_p' V_y^{-1}[\bar{Y}_p(t) - C_p\hat{\bar{x}}(t-\tau)] + B_p\bar{u}_p^*(t-\tau)$$

$$\hat{\bar{x}}(t) = \bar{\beta}(t) + \ell^{A_p\tau}[\hat{\bar{x}}(t-\tau) - \bar{\beta}(t-\tau)]$$

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†To model the pilot's remnant, motor noise is usually added to the control equation. The final pilot's control is represented by

$$\dot{\bar{u}}_p^* = -G^{-1}K_{p_3}\hat{\bar{x}} - G^{-1}K_{p_4}\bar{u}_p^* + G^{-1}K_{p_4}\bar{v}_m$$

where

$$E[\bar{v}_m(t)\bar{v}_m'(t+\sigma)] = V_M\delta(\sigma)$$

with $\dot{\bar{\beta}} = A_p\bar{\beta} + B_p\bar{u}_p^*$

and the estimation error covariance matrix Σ is the solution of the Riccati equation

$$A_p\Sigma + \Sigma A_p' + W - \Sigma C_p' V_y^{-1} C_p \Sigma = [0]$$

This system of equations, when solved, determines the optimal-control pilot model.

Finally, as noted previously, Hess has found that when the weightings on the state and control (i.e., Q anf R) in the pilot's objective function are appropriately selected, the resulting magnitude of the pilots objective function, after solving for the pilot model, is strongly correlated with the pilot's rating of the vehicle and task. If the pilot rating is given in the Cooper-Harper system, the relation is

$$\text{Pilot Rating (PR)} = 2.53 \text{ in } (10 J_p) + 0.28$$

Now, through this relation and the solution of the pilot model above, we now have not only a pilot-control model but a prediction of the pilot's rating of the dynamic system.

Augmentation Synthesis Method

In the determination of the pilot model parameters above, we have expressed the system dynamics in terms of the matrices A_p and B_p. However, since the augmentation has not been defined, the augmented plant, A_p and B_p is as yet unknown.

Consider the un-augmented plant dynamics to be described by

$$\dot{\bar{x}} = A\bar{x} + B\bar{u} + \bar{w}$$

where, as before, $\bar{x}$ is the system state vector and $\bar{w}$ is the same disturbance vector. However, A and B are now the un-augmented system matrices, and $\bar{u}$ is the control input vector. Now, the total control input to the plant will include pilot input, $\bar{u}_p$, plus augmentation input, $\bar{u}_{SAS}$, or

$$\bar{u} = \bar{u}_p + \bar{u}_{SAS}$$

Further, from the pilot model, we know that although the feedback gains (e.g., $G^{-1}K_{p_3}$, $G^{-1}K_{p_4}$) have not been determined, the pilot's control input in expressible as

$$\dot{\bar{u}}_p = -G^{-1}K_{p_3}\dot{\hat{\bar{x}}} - G^{-1}K_{p_4}\bar{u}_p \qquad (3)$$

Now, the estimate of the state, $\hat{\bar{x}}$, can be expressed in terms of the true state plus some estimation error, $\bar{\varepsilon}$, or

$$\hat{\bar{x}} = \bar{x} + \bar{\varepsilon}$$

By treating this error as another disturbance, $\bar{w}_p$, we can write the pilot's control equation as

$$\dot{\bar{u}}_p = -G^{-1}K_{p_3}\bar{x} - G^{-1}K_{p_4}\bar{u}_p + \bar{w}_p$$

(Note, the disturbance term, $\bar{w}_p$, can also include the pilot's remnant as well.) Combining this relation with the plant dynamic and pilot equations we have

$$\dot{\bar{\chi}} = \left[\begin{array}{c|c} A & B \\ \hline -G^{-1}K_{p_3} & -G^{-1}K_{p_4} \end{array}\right]\bar{\chi} + \left[\begin{array}{c} B \\ \hline 0 \end{array}\right]\bar{u}_{SAS} + \left[\begin{array}{c} \bar{w} \\ \hline \bar{w}_p \end{array}\right] \qquad (4)$$

where $\bar{\chi} = \mathrm{col}[\bar{x}, \bar{u}_p]$.

We now may proceed to determine an objective function for determining $\bar{u}_{SAS}$.

From the correlation between pilot rating and the pilot's objective function we clearly see that the best (i.e., lowest) pilot rating implies the lowest pilot objective function. Therefore, for optimum pilot rating, the control $\bar{u}_{SAS}$ should be chosen to minimize J_p as defined in the pilot rating method. (This method defines the state and control weights, Q and R, as the inverse of the maximum allowable deviations in the variables as perceived by the pilot.) Finally, to preclude infinite augmentation gains, we must also penalize augmentation control energy. Therefore, the augmentation is chosen to minimize

$$J_{SAS} = J_p + E\left\{\lim_{T\to\infty}\frac{1}{T}\int_0^T \bar{u}'_{SAS}\, F\, \bar{u}_{SAS}\, dt\right\}$$

or

$$J_{SAS} = E\left\{\lim_{T\to\infty}\frac{1}{T}\int_0^T (\bar{y}'Q\bar{y} + \bar{u}_p'R\bar{u}_p + \dot{\bar{u}}_p'G\dot{\bar{u}}_p + \bar{u}'_{SAS}F\bar{u}_{SAS})dt\right\}$$

and Q, R, and G are as chosen in the pilot's objective function, J_p.

This may be written as

$$J_{SAS} = E\left\{\lim_{T\to\infty} \frac{1}{T} \int_0^T (\bar{x}'P\bar{x} + \bar{u}'_{SAS}F\bar{u}_{SAS})dt\right\}$$

where

$$P = \left[\begin{array}{c|c} C_p'QC_p + K_{P_3}'G^{-1}K_{P_3} & K_{P_3}'G^{-1}K_{P_4} \\ \hline K_{P_4}'G^{-1}K_{P_3} & R+K_{P_4}'G^{-1}K_{P_4} \end{array}\right]$$

and instead of Equation 3 being substituted for $\dot{\bar{u}}_p$ in the above J_{SAS}, we have invoked a sort of separation principle and substituted the relation

$$\dot{\bar{u}}_p = -G^{-1}K_{P_3}\bar{x} - G^{-1}K_{P_4}\bar{u}_p$$

The justification for this relation being used lies in the fact that we wish to synthesize the augmentation based on how the pilot is <u>trying</u> to perform the control function rather than on how the pilot is capable of doing so.

With this objective function and the system dynamics given in Equation 4, the problem is now stated in conventional form, except K_{P_3} and K_{P_4} are as yet undetermined of course. If we assume, for example, full state feedback, the solution of this problem is known to be

$$\bar{u}^*_{SAS} = -F^{-1}[B' \mid 0]K_{SAS}\bar{x}$$

or

$$\bar{u}^*_{SAS} = -F^{-1}B'K_{SAS_1}\bar{x} - F^{-1}B'K_{SAS_2}\bar{u}_p$$

where

$$K_{SAS} = \left[\begin{array}{c|c} K_{SAS_1} & K_{SAS_2} \\ \hline K_{SAS_3} & K_{SAS_4} \end{array}\right]$$

is the solution to the Riccati equation

$$-\left[\begin{array}{c|c} A & B \\ \hline -G^{-1}K_{P_3} & -G^{-1}K_{P_4} \end{array}\right]' K_{SAS} - K_{SAS}\left[\begin{array}{c|c} A & B \\ \hline -G^{-1}K_{P_3} & -G^{-1}K_{P_4} \end{array}\right]$$

$$- P + K_{SAS}\left[\begin{array}{c} B \\ \hline 0 \end{array}\right]F^{-1}[B' \mid 0]K_{SAS} = \dot{K}_{SAS} \qquad (5)$$

We see in this expression that the solution for K_{SAS} obviously depends on K_p (or K_{p_3} and K_{p_4}). Returning to the Riccati equation for the pilot gain (Equation 2), we also see that that equation depends in turn on the SAS gains (or K_{SAS}) since the pilot Riccati equation involves the augmented plant matrices A_p and B_p. As a result of the SAS design procedure just presented, we now know, however, the SAS structure. Returning to the pilot model, we may now include this SAS structure specifically, so that A_p and B_p (as in Equation 1) may in fact be expressed as

$$A_p = A - BF^{-1}B'K_{SAS_1}$$

and $$B_p = B(I - F^{-1}B'K_{SAS_2})$$

Substituting these expressions in the pilot Riccati equation yields two coupled Riccati equations, one for the pilot gains, Equation 2, and one for the SAS gains, Equation 5. These may be solved simultaneously for K_{SAS} and K_p by integrating both equations backward. Note that this solution does not involve a two-point-boundary-value problem. The system is represented in Figure 1.

A Simple Numerical Example

Consider a simple tracking task with the controlled element (plant) dynamics considered in Ref. 11,

$$\theta(s)/\delta(s) = K/s^2, \quad K = 11.7$$

The command signal, θ_c, is white noise, w, passed through the filter

$$\theta_c(s)/w(s) = 3.67/(s^2 + 3s + 2.25)$$

and $$E(w) = 0, \quad \sigma_w^2 = 1.0$$

If we define the state vector as $\bar{x} = \text{col}(\theta_c, \dot{\theta}_c, \theta, \dot{\theta})$, we have the plant

$$\dot{\bar{x}} = A\bar{x} + B\delta + 3.67\bar{w}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2.25 & -3. & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B' = [0,0,0,11.7]$$

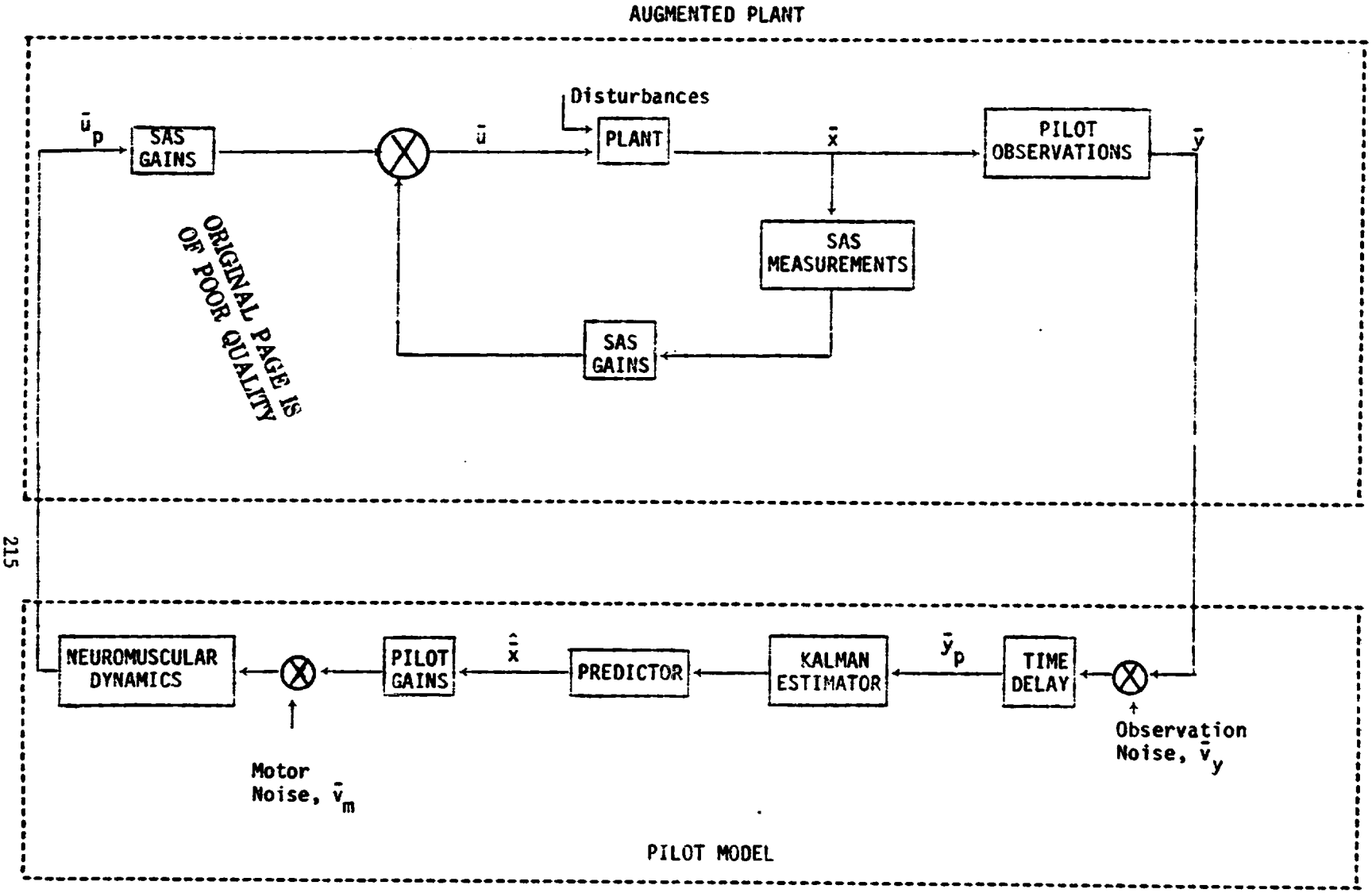

Figure 1 Piloted Vehicle Schematic

For this system, error and error rate are perceived by the pilot or

$$\bar{y}_p = \begin{bmatrix} \theta_c - \theta \\ \dot{\theta}_c - \dot{\theta} \end{bmatrix} = \begin{bmatrix} 1. & 0 & -1. & 0 \\ 0 & 1. & 0. & -1. \end{bmatrix} \bar{x}$$

The performance index, chosen consistent with Hess's rating hypothesis is

$$J_p = E\left\{\lim_{T\to\infty} \frac{1}{T} \int_0^T [(\theta_c - \theta)^2 + .01\delta_p^2 + g\dot{\delta}_p^2]dt\right\}$$

and g is chosen to yield a neuromuscular lag, $1/\tau_N = G^{-1}K_{P_4} = 10.$, or $\tau_N = 0.1$ seconds. Unagumented, the pilot Riccati equations are solved with the following noise statistics (human limitations)

1) Equal attention allocation between error and error rate.
2) Observation thresholds on error and error rate = 0.5(units of display displacement.)
3) Sensor noise, $(V_y)_{ii}/E(y_i^2) = -20dB \quad i=1,2$
4) Motor noise, $(Vu)/E(u_c^2) = -20db$, where $u_c = -G^{-1}K_{p_3}\hat{\bar{x}}$
5) Observation delay, $\tau = 0.1$ seconds

The "piloted" system performance is given in the following table.

Table 1, Un-augmented System Performance

$(\theta_c - \theta)$ rms	$\dot{\delta}_p$ rms	J_p	P.R.
1.17	1.00	1.39	6.9*

* This pilot rating has been verified by experiment

Assuming full-state feedback, the augmentation control law is

$$\delta_{SAS} = -K_1\theta_c - K_2\dot{\theta}_c - K_3\theta - K_4\dot{\theta} - K_\delta\delta_p$$

The SAS objective function is

$$J_{SAS} = J_p + E\left\{\lim_{T\to\infty} \frac{1}{T} \int_0^T f\delta_{SAS}^2 \, dt\right\}$$

so the piloted plant, including augmentation will be

$$\dot{\bar{x}} = A_p\bar{x} + B_p\delta_p + 3.67\bar{w}$$

where

$$A_p = \begin{bmatrix} 0 & 1. & 0 & 0 \\ -2.25 & -3. & 0 & 0 \\ 0 & 0 & 0 & 1. \\ -11.7K_1 & -11.7K_2 & -11.7K_3 & -11.7K_4 \end{bmatrix}$$

$$B_p' = [0,0,0,11.7(1-K_\delta)]$$

Solving the pilot and SAS Riccati equations simultaneously, and then determining piloted system performance as before yields the results given in the following table.

Table 2 Augmented System Performance

f	$(\theta_c - \theta)$ rms	δ_p rms	J_p *	P.R.**
100.0	1.10	0.89	1.21	6.6
10.0	0.79	0.61	0.62	4.9
1.0	0.38	0.35	0.15	1.3

* Note this is the numerical value of the pilot's objective function, J_p, not J_{SAS}.
** Predicted pilot rating based on J_p.

The augmentation gains, K_1 - K_4 and K_δ, for the three cases above are given in Table 3, along with the augmented plant eigenvalues.

Table 3 Augmentation Gains

f	K_1	K_2	K_3	K_4	K_5	Plant Eigenvalues*
100.0	-.009	-.002	.009	.003	.004	$-.017 \pm .331j$
10.	-.078	-.016	.084	.024	.036	$-.142 \pm .982j$
1.	-.513	-.090	.542	.130	.155	$-.758 \pm 2.40j$

* Not including noise filter eigenvalues of course.

Summary

In summary, we have cited the flight-dynamic and control problems of non-conventional flight vehicles (V/STOL and CCV) due to the complexity of augmentation required and the lack of handling qualities objectives. We have presented a methodology intended to be suitable for this type of problem. The method uses an optimal control pilot model, not only to predict

piloted performance but pilot rating as well. With the optimal-control model structure, we were able to formulate the augmentation synthesis problem as an optimal control problem with the parameters in plant matrices depending on the pilot model, and vice versa. This necessitates simultaneous solution of the two (pilot and augmentation) problems. We have included the form of the solution under the assumption of full-state variable feedback and no measurement noise, and a simple numerical example.

The first extension to be addressed will be the solution for the case of limited state feedback. This case is actually closer to pure plant augmentation than the case addressed here. In our solution, and in the example, we have closed the tracking loop, and pure plant augmentation would only feed back plant states. However, the primary purpose of our discussion here was to provide the problem structure which would be unchanged regardless of augmentation approach.

Further extensions will also include the cases with state estimation, with and without measurement noise. Also, the necessity of pre-tuning the pilot model will be investigated.

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