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A PROCEDURE FOR OBTAINING STIFFNESSES AND MASSES OF A STRUCTURE FROM VIBRATION MODES AND SUBSTRUCTURE STATIC TEST DATA

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A PROCEDURE FOR OBTAINING STIFFNESSES AND MASSES OF A STRUCTURE FROM VIBRATION MODES AND SUBSTRUCTURE STATIC TEST DATA

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SUMMARY

The purpose of this investigation was to develop a component mode desynthesis procedure for determining the unknown vibration characteristics of a structural component (i. e., a launch vehicle) given the vibration characteristics of a structural system composed of that component combined with a known one (i. e., a payload). The desynthesis procedure developed does not require the vibration characteristics of the payload component but requires that at least one component static test be performed. This data is used in conjunction with the system measured frequencies and mode shapes to obtain the vibration characteristics of each component. The static test data is a catalyst in the desynthesis procedure and could be performed on any component or on the entire system. This method has direct application to determining the flight dynamics of an empty launch vehicle from measurements made on a vehicle/payload combination in conjunction with a static test on the payload. This procedure is applicable to a multicomponent system.

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I. INTRODUCTION

This model desynthesis procedure consists of the following steps. First, the coupled degrees of freedom for the structural system is established. Secondly, a component static test is performed. The unknown system stiffness and mass matrices are determined by parts, a row at a time, using the system measured eigenvalues and eigenvectors with the static test data. The stiffness and mass matrices of each component and the interface springs are determined from the system matrices. Finally, the component mass and stiffness matrices of the components are used in an eigenvalue analysis to determine the component dynamic responses. The method is demonstrated by the evaluation of a two beam, twelve degree-of-freedom system.

SYMBOLS

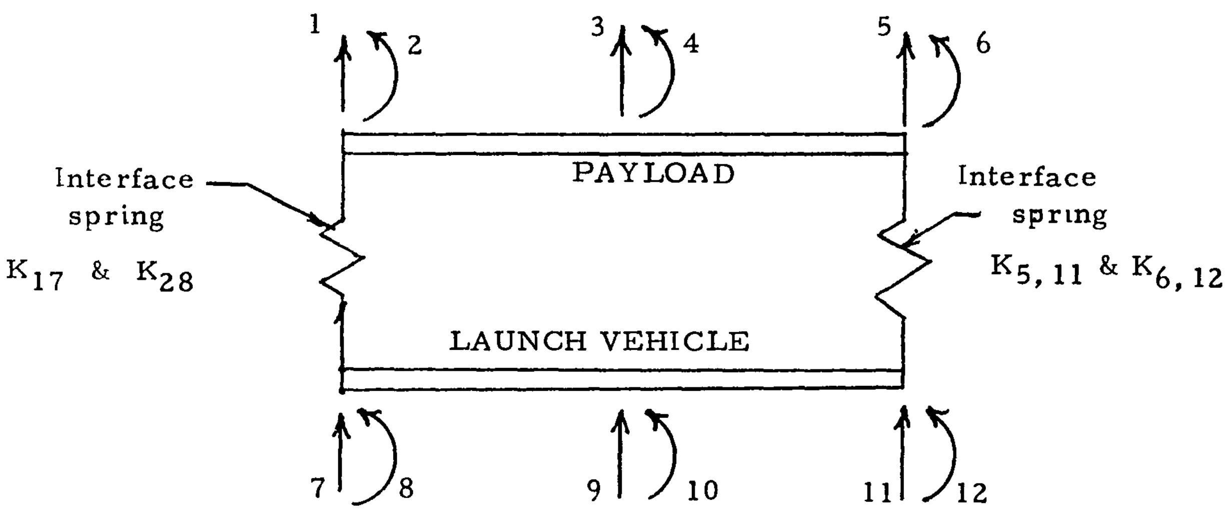
K_p or K_{ijp}	Payload stiffness
K or K_{ij}	System stiffness
M or M_{ij}	System mass
F_t or F_{it}	Static test force
u_t or u_{it}	Static test displacement
u or u_{ir}	Eigenvector, i = degree of freedom, r = mode number
ω_r^2	Eigenvalue
$\omega_{ij} = - \omega_r^2 \; u_{ij}$	
δ_t or δ_i	Measured static test displacement
α_t, θ_i or β_i	Measured static test rotations
F or P	Measured static test force
T	Measured static test torque
b_i	Column matrix
superscript or subscript	
r	Mode number

III. DESYNTHESIS PROCEDURE

3.1 Determine the Coupled Degrees of Freedom

The first step in this desynthesis procedure is to determine all of the structural coupling terms in the mass and stiffness matrices. If the structure has stiff bulkheads so that they respond as rigid bodies, the first and third bulkhead are uncoupled in the mass and stiffness matrix. Furthermore, if the motion of one bulkhead in a given direction results in a net force of zero on an adjacent fixed bulkhead these two degrees of freedom are uncoupled. The unknown stiffness matrix with an x representing the coupled degrees of freedom is shown in equation (1) for a 12 degree of freedom system.

Coupled terms in a twelve degree-of-freedom system. - The two beam model is shown below. The model properties are shown in Figure 1.



The coupled degrees of freedom in K matrix are:

$$
[K] = \begin{array}{cc}
 & \begin{array}{cccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{array} &
\left[\begin{array}{cccccc|cccccc}
X & X & X & X &   &   & X &   &   &   &   &   \\
X & X & X & X &   &   &   & X &   &   &   &   \\
X & X & X &   & X & X &   &   &   &   &   &   \\
X & X &   & X & X & X &   &   &   &   &   &   \\
  &   & X & X & X & X &   &   &   &   & X &   \\
  &   & X & X & X & X &   &   &   &   &   & X \\
\hline
X &   &   &   &   &   & X & X & X & X &   &   \\
  & X &   &   &   &   & X & X & X & X &   &   \\
  &   &   &   &   &   & X & X & X &   & X & X \\
  &   &   &   &   &   & X & X &   & X & X & X \\
  &   &   &   & X &   &   &   & X & X & X & X \\
  &   &   &   &   & X &   &   & X & X & X & X
\end{array}\right]
\end{array} \tag{1}
$$

For this structure there is no coupling between degree of freedom 1 and 5, 6, 8, 9, 10, 11 and 12 because when the degree of freedom 1 is given a unit displacement with all others restrained, there is no reaction force at these uncoupled degrees of freedom. By walking through each degree of freedom in a similar manner for degree of freedom 2, 3, etc., the complete system unknown coupled K term locations are determined. There is no interface spring coupling between 1-8, 2-7, 5-12 and 6-11 because of knowledge of the interface spring design. There is no coupling between 3-4 and 9-10 because of the mid point location with uniform stiffness along the length. The mass matrix will have the identical coupled locations except the 1-7, 2-8, 5-11 and 6-12 couplings which are omitted because there is no mass coupling across the interface.

3.2 Component Static Test

The second step is to perform a static test on one of the components. The payload component will normally have fewer degrees of freedom then the launch vehicle component and would probably be more economical to test. Alternatively, the entire system could be statically tested to satisfy this requirement.

The component static test data can be expressed in matrix form as:

$$\left[K_p\right] \left\{u_t\right\} = \left\{F_t\right\} \tag{2}$$

where:

K_p = unknown payload stiffness matrix

u_t = static test displacements

F_t = static test forces (applied and reaction)

For the twelve degree-of-freedom system equation (2) becomes:

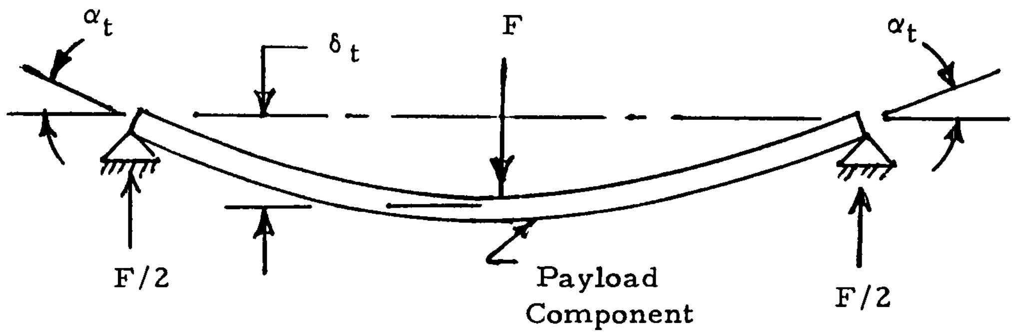
$$\begin{bmatrix} K_{11p} & K_{12} & K_{13} & K_{14} & 0 & 0 \\ K_{21} & K_{22p} & K_{23} & K_{24} & 0 & 0 \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} \\ 0 & 0 & K_{53} & K_{54} & K_{55p} & K_{56} \\ 0 & 0 & K_{63} & K_{64} & K_{65} & K_{66p} \end{bmatrix} \begin{Bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \\ u_{5t} \\ u_{6t} \end{Bmatrix} = \begin{Bmatrix} F_{1t} \\ F_{2t} \\ F_{3t} \\ F_{4t} \\ F_{5t} \\ F_{6t} \end{Bmatrix} \tag{3}$$

K_{34} and K_{43} are zero but were included in this analysis to demonstrate that uncoupled degree-of-freedom combinations can be overlooked to begin with and subsequently identified by the method.

The terms with the subscript p are the payload stiffness terms that are different than the system terms in equation (1) because of the coupling springs. The K_{ii} system terms in equation (1) are related to the payload terms of equation (3) and the interface stiffness terms by

$$
\begin{aligned}
K_{11} &= K_{11p} - K_{17} \\
K_{22} &= K_{22p} - K_{28} \\
K_{55} &= K_{55p} - K_{5,11} \\
K_{66} &= K_{66p} - K_{6,12}
\end{aligned}
\tag{4}
$$

<u>Static test for payload of twelve degree-of-freedom system</u>. - The payload component is simply supported at each end and loaded in bending as shown below. Other tests such as a cantilever beam test would be just as acceptable.



With a test force of F, the payload component has a measured δ_t deflection at the mid-point and a measured α_t rotation at each end. The test displacements and forces of equation (3) becomes:

$$\begin{Bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \\ u_{5t} \\ u_{6t} \end{Bmatrix} = \begin{Bmatrix} 0 \\ -\alpha_t \\ -\delta_t \\ 0 \\ 0 \\ \alpha_t \end{Bmatrix} \quad ; \quad \begin{Bmatrix} F_{1t} \\ F_{2t} \\ F_{3t} \\ F_{4t} \\ F_{5t} \\ F_{6t} \end{Bmatrix} = \begin{Bmatrix} F/2 \\ 0 \\ -F \\ 0 \\ F/2 \\ 0 \end{Bmatrix} \tag{5}$$

Equation (3) is revised by including the equalities of equation (4) to express all stiffness terms in system notation.

$$\begin{bmatrix} (K_{11}+K_{17}) & K_{12} & K_{13} & K_{14} & 0 & 0 \\ K_{21} & (K_{22}+K_{28}) & K_{23} & K_{24} & 0 & 0 \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} \\ 0 & 0 & K_{53} & K_{54} & (K_{55}+K_{5,11}) & K_{56} \\ 0 & 0 & K_{63} & K_{64} & K_{65} & (K_{66}+K_{6,12}) \end{bmatrix} \begin{Bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \\ u_{5t} \\ u_{6t} \end{Bmatrix} = \begin{Bmatrix} F_{1t} \\ F_{2t} \\ F_{3t} \\ F_{4t} \\ F_{5t} \\ F_{6t} \end{Bmatrix} \tag{6}$$

Imposing the condition that $K_{ij} = K_{ji}$ for $i \neq j$.- Equation (6) can be rearranged row by row, so that the unknown stiffnesses are contained in vectors (column matrices).

1st row

$$\lfloor u_{1t} \quad u_{2t} \quad u_{3t} \quad u_{4t} \quad u_{1t} \rfloor \begin{Bmatrix} K_{11} \\ K_{12} \\ K_{13} \\ K_{14} \\ K_{17} \end{Bmatrix} = F_{1t} \tag{7a}$$

2nd row

$$\lfloor u_{1t} \quad u_{2t} \quad u_{3t} \quad u_{4t} \quad u_{2t} \rfloor \begin{Bmatrix} K_{12} \\ K_{22} \\ K_{23} \\ K_{24} \\ K_{28} \end{Bmatrix} = F_{2t} \tag{7b}$$

3rd row

$$\lfloor u_{1t} \quad u_{2t} \quad u_{3t} \quad u_{4t} \quad u_{5t} \quad u_{6t} \rfloor \begin{Bmatrix} K_{13} \\ K_{23} \\ K_{33} \\ K_{34} \\ K_{35} \\ K_{36} \end{Bmatrix} = F_{3t} \tag{7c}$$

4th row

$$\lfloor u_{1t} \; u_{2t} \; u_{3t} \; u_{4t} \; u_{5t} \; u_{6t} \rfloor \begin{Bmatrix} K_{14} \\ K_{24} \\ K_{34} \\ K_{44} \\ K_{45} \\ K_{46} \end{Bmatrix} = F_{4t} \tag{7d}$$

5th row

$$\lfloor u_{3t} \; u_{4t} \; u_{5t} \; u_{6t} \; u_{5t} \rfloor \begin{Bmatrix} K_{35} \\ K_{45} \\ K_{55} \\ K_{56} \\ K_{5,11} \end{Bmatrix} = F_{5t} \tag{7e}$$

6th row

$$\lfloor u_{3t} \; u_{4t} \; u_{5t} \; u_{6t} \; u_{6t} \rfloor \begin{Bmatrix} K_{36} \\ K_{46} \\ K_{56} \\ K_{66} \\ K_{6,12} \end{Bmatrix} = F_{6t} \tag{7f}$$

Decide which row of the K and M matrices will be solved first. - At this point it is necessary to decide which row of the system K matrix is to be solved first. Because of the zero static test forces in equation (5), the 2nd, 4th and 6th rows equations (7b), (7d), and (7f) are eliminated, since the purpose of the static test is to provide finite values on the right hand side of a set of simultaneous equations so as to obtain stiffnesses (and also masses). For this example, the first row (equation 7a) was selected for the first set of simultaneous equations.

3.3 System Measured Eigenvalues and Eigenvectors

The system stiffness and mass matrices are related to each measured eigenvalue and its associated eigenvector of the vehicle/payload system as

$$[K]\{u\}^{(r)} - \omega^2_{(r)} [M]\{u\}^{(r)} = 0 \qquad (8)$$

where

K = unknown vehicle/payload stiffness matrix

M = unknown vehicle/payload mass matrix

$\omega^2_{(r)}$ = the rth known system eigenvalue

$u^{(r)}$ = the rth set of eigenvectors

<u>Revise equation (8) for the twelve degree-of-freedom system.</u> - For the example two-beam system, equation 8 for the rth eigenvalue becomes equation (9) on the next page.

$$[K]\begin{Bmatrix} u_{1r}\\ u_{2r}\\ u_{3r}\\ u_{4r}\\ u_{5r}\\ u_{6r}\\ u_{7r}\\ u_{8r}\\ u_{9r}\\ u_{10r}\\ u_{11r}\\ u_{12r} \end{Bmatrix} - \omega^2_{(r)}\ [M]\begin{Bmatrix} u_{1r}\\ u_{2r}\\ u_{3r}\\ u_{4r}\\ u_{5r}\\ u_{6r}\\ u_{7r}\\ u_{8r}\\ u_{9r}\\ u_{10r}\\ u_{11r}\\ u_{12r} \end{Bmatrix} = \begin{Bmatrix} 0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0 \end{Bmatrix} \tag{9}$$

where the system K matrix of unknown coupled terms are identical to equation (1) with the 3-4 and 9-10 terms added.

$$[K] = \begin{bmatrix}
K_{11} & K_{12} & K_{13} & K_{14} & 0 & 0 & K_{17} & 0 & 0 & 0 & 0 & 0\\
K_{21} & K_{22} & K_{23} & K_{24} & 0 & 0 & 0 & K_{28} & 0 & 0 & 0 & 0\\
K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} & 0 & 0 & 0 & 0 & 0 & 0\\
K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & K_{53} & K_{54} & K_{55} & K_{56} & 0 & 0 & 0 & 0 & K_{5,11} & 0\\
0 & 0 & K_{63} & K_{64} & K_{65} & K_{66} & 0 & 0 & 0 & 0 & 0 & K_{6,12}\\
K_{71} & 0 & 0 & 0 & 0 & 0 & K_{77} & K_{78} & K_{79} & K_{7,10} & 0 & 0\\
0 & K_{82} & 0 & 0 & 0 & 0 & K_{87} & K_{88} & K_{89} & K_{8,10} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & K_{97} & K_{98} & K_{99} & K_{9,10} & K_{9,11} & K_{9,12}\\
0 & 0 & 0 & 0 & 0 & 0 & K_{10,7} & K_{10,8} & K_{10,9} & K_{10,10} & K_{10,11} & K_{10,12}\\
0 & 0 & 0 & 0 & K_{11,5} & 0 & 0 & 0 & K_{11,9} & K_{11,10} & K_{11,11} & K_{11,12}\\
0 & 0 & 0 & 0 & 0 & K_{12,6} & 0 & 0 & K_{12,9} & K_{12,10} & K_{12,11} & K_{12,12}
\end{bmatrix}$$

The system mass matrix is identical except the 1-7, 2-8, 5-11 and 6-12 terms are zero.

Imposing the symmetry condition that $K_{ij} = K_{ji}$ and $M_{ij} = M_{ji}$ for $i \neq j$. - Each row of equation (9) is rearranged to be

1st row

$$\lfloor u_{1r} \; u_{2r} \; u_{3r} \; u_{4r} \; u_{7r} \; W_{1r} \; W_{2r} \; W_{3r} \; W_{4r} \rfloor \begin{Bmatrix} K_{11} \\ K_{12} \\ K_{13} \\ K_{14} \\ K_{17} \\ M_{11} \\ M_{12} \\ M_{13} \\ M_{14} \end{Bmatrix} = 0 \qquad (10a)$$

where

$W_{ir} = -\omega_r^2 \, u_{ir}$

2nd row

$$\lfloor u_{1r} \; u_{2r} \; u_{3r} \; u_{4r} \; u_{8r} \; W_{1r} \; W_{2r} \; W_{3r} \; W_{4r} \rfloor \begin{Bmatrix} K_{12} \\ K_{22} \\ K_{23} \\ K_{24} \\ K_{28} \\ M_{12} \\ M_{22} \\ M_{23} \\ M_{24} \end{Bmatrix} = 0 \qquad (10b)$$

where

$W_{ir} = -\omega_r^2 \, u_{ir}$

3rd row

$$\lfloor u_{1r}\ u_{2r}\ u_{3r}\ u_{4r}\ u_{5r}\ u_{6r}\ W_{1r}\ W_{2r}\ W_{3r}\ W_{4r}\ W_{5r}\ W_{6r} \rfloor \begin{Bmatrix} K_{13} \\ K_{23} \\ K_{33} \\ K_{34} \\ K_{35} \\ K_{36} \\ M_{13} \\ M_{23} \\ M_{33} \\ M_{34} \\ M_{35} \\ M_{36} \end{Bmatrix} = 0 \qquad (10c)$$

where

$$W_{ir} = -\omega_r^2\ u_{ir}$$

4th row

$$\lfloor u_{1r}\ u_{2r}\ u_{3r}\ u_{4r}\ u_{5r}\ u_{6r}\ W_{1r}\ W_{2r}\ W_{3r}\ W_{4r}\ W_{5r}\ W_{6r} \rfloor \begin{Bmatrix} K_{14} \\ K_{24} \\ K_{34} \\ K_{44} \\ K_{45} \\ K_{46} \\ M_{14} \\ M_{24} \\ M_{34} \\ M_{44} \\ M_{45} \\ M_{46} \end{Bmatrix} = 0 \qquad (10d)$$

where

$$W_{ir} = -\omega_r^2\ u_{ir}$$

5th row

$$\lfloor u_{3r}\ u_{4r}\ u_{5r}\ u_{6r}\ u_{11r}\ W_{3r}\ W_{4r}\ W_{5r}\ W_{6r} \rfloor \begin{Bmatrix} K_{35} \\ K_{45} \\ K_{55} \\ K_{56} \\ K_{5,11} \\ M_{35} \\ M_{45} \\ M_{55} \\ M_{56} \end{Bmatrix} = 0 \qquad (10e)$$

where

$$W_{ir} = -\omega_r^2\, u_{ir}$$

6th row

$$\lfloor u_{3r}\ u_{4r}\ u_{5r}\ u_{6r}\ u_{12r}\ W_{3r}\ W_{4r}\ W_{5r}\ W_{6r} \rfloor \begin{Bmatrix} K_{36} \\ K_{46} \\ K_{56} \\ K_{66} \\ K_{6,12} \\ M_{36} \\ M_{46} \\ M_{56} \\ M_{66} \end{Bmatrix} = 0 \qquad (10f)$$

where

$$W_{ir} = -\omega_r^2\, u_{ir}$$

7th row

$$\lfloor u_{1r}\ u_{7r}\ u_{8r}\ u_{9r}\ u_{10r}\ W_{7r}\ W_{8r}\ W_{9r}\ W_{10r} \rfloor \begin{Bmatrix} K_{17} \\ K_{77} \\ K_{78} \\ K_{79} \\ K_{7,10} \\ M_{77} \\ M_{78} \\ M_{79} \\ M_{7,10} \end{Bmatrix} = 0 \qquad (10g)$$

where

$W_{ir} = -\omega_r^2 u_{ir}$

8th row

$$\lfloor u_{2r}\ u_{7r}\ u_{8r}\ u_{9r}\ u_{10r}\ W_{7r}\ W_{8r}\ W_{9r}\ W_{10r} \rfloor \begin{Bmatrix} K_{28} \\ K_{78} \\ K_{88} \\ K_{89} \\ K_{8,10} \\ M_{78} \\ M_{88} \\ M_{89} \\ M_{8,10} \end{Bmatrix} = 0 \qquad (10h)$$

where

$W_{ir} = -\omega_r^2 u_{ir}$

9th row

$$\lfloor u_{7r}\ u_{8r}\ u_{9r}\ u_{10r}\ u_{11r}\ u_{12r}\ W_{7r}\ W_{8r}\ W_{9r}\ W_{10r}\ W_{11r}\ W_{12r} \rfloor \begin{Bmatrix} K_{79} \\ K_{89} \\ K_{99} \\ K_{9,10} \\ K_{9,11} \\ K_{9,12} \\ M_{79} \\ M_{89} \\ M_{99} \\ M_{9,10} \\ M_{9,11} \\ M_{9,12} \end{Bmatrix} = 0 \quad (10i)$$

where

$W_{ir} = -\omega_r^2\, u_{ir}$

10th row

$$\lfloor u_{7r}\ u_{8r}\ u_{9r}\ u_{10r}\ u_{11r}\ u_{12r}\ W_{7r}\ W_{8r}\ W_{9r}\ W_{10r}\ W_{11r}\ W_{12r} \rfloor \begin{Bmatrix} K_{7,10} \\ K_{8,10} \\ K_{9,10} \\ K_{10,10} \\ K_{10,11} \\ K_{10,12} \\ M_{7,10} \\ M_{8,10} \\ M_{9,10} \\ M_{10,10} \\ M_{10,11} \\ M_{10,12} \end{Bmatrix} = 0 \quad (10j)$$

where

$W_{ir} = -\omega_r^2\, u_{ir}$

11th row

$$\lfloor u_{5r}\ u_{9r}\ u_{10r}\ u_{11r}\ u_{12r}\ W_{9r}\ W_{10r}\ W_{11r}\ W_{12r} \rfloor \begin{Bmatrix} K_{5,11} \\ K_{9,11} \\ K_{10,11} \\ K_{11,11} \\ K_{11,12} \\ M_{9,11} \\ M_{10,11} \\ M_{11,11} \\ M_{11,12} \end{Bmatrix} = 0 \qquad (10k)$$

where

$$W_{ir} = -\omega_r^2\, u_{ir}$$

12th row

$$\lfloor u_{6r}\ u_{9r}\ u_{10r}\ u_{11r}\ u_{12r}\ W_{9r}\ W_{10r}\ W_{11r}\ W_{12r} \rfloor \begin{Bmatrix} K_{6,12} \\ K_{9,12} \\ K_{10,12} \\ K_{11,12} \\ K_{12,12} \\ M_{9,12} \\ M_{10,12} \\ M_{11,12} \\ M_{12,12} \end{Bmatrix} = 0 \qquad (10m)$$

where

$$W_{ir} = -\omega_r^2\, u_{ir}$$

3.4 Formulate Simultaneous Equations for Each Row of K and M Matrix

Each of the equations (10a) thru (10j) can be expanded by letting $r = 1, 2, \ldots, m$, to obtain a set of simultaneous equations that could be solved from the measured system eigenvalues and eigenvectors except that all the equations have a zero on the right hand side. To circumvent this problem one static test data equation (equation 7) is combined with the number of dynamic response equations (equation 10) required to equal the number of unknown K and M terms. Obviously, the right hand side of the static test equation (7) used, must not equal zero. For this example 12 degree-of-freedom system, equation (7a) for the first row was combined with (10a). Equation (10a) has 9 unknown system K and M terms. By combining the first 8 eigenvalues ($r = 1, 2, \ldots, 8$ in equation 10a) with the first row test equation (equation 7a), the 10 values are solved simultaneously.

Combine Equation (7a) and (10a) for the 1st row simultaneous equations. - This is assembled in matrix form to solve for the first row of the mass and stiffness matrix as shown in equation (11).

1st Row Simultaneous Equations

$$\begin{bmatrix} u_{11} & u_{21} & u_{31} & u_{41} & u_{71} & 0 & 0 & 0 & 0 \\ u_{12} & u_{22} & u_{32} & u_{42} & u_{72} & 0 & 0 & 0 & 0 \\ u_{13} & u_{23} & u_{33} & u_{43} & u_{73} & W_{13} & W_{23} & W_{33} & W_{43} \\ u_{14} & u_{24} & u_{34} & u_{44} & u_{74} & W_{14} & W_{24} & W_{34} & W_{44} \\ u_{15} & u_{25} & u_{35} & u_{45} & u_{75} & W_{15} & W_{25} & W_{35} & W_{45} \\ u_{16} & u_{26} & u_{36} & u_{46} & u_{76} & W_{16} & W_{26} & W_{36} & W_{46} \\ u_{17} & u_{27} & u_{37} & u_{47} & u_{77} & W_{17} & W_{27} & W_{37} & W_{47} \\ u_{18} & u_{28} & u_{38} & u_{48} & u_{78} & W_{18} & W_{28} & W_{38} & W_{48} \\ u_{1t} & u_{2t} & u_{3t} & u_{4t} & u_{1t} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} K_{11} \\ K_{12} \\ K_{13} \\ K_{14} \\ K_{17} \\ M_{11} \\ M_{12} \\ M_{13} \\ M_{14} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ F_{1t} \end{Bmatrix} \quad (11)$$

where

$$W_{ir} = -\omega_r^2 \; u_{ir}$$

The first two rows are rigid body modes and the last row is from the static test data, so 6 flexible system test eigenvalues and associated eigenvectors are required. An alternate approach would be to use 8 test modes with a static test equation and eliminate the rigid body modes.

The rigid body modes are constructed. - The first rigid body mode is pure translation and the 2nd is rotation about the mid points of both components. These rigid body eigenvalues are zero and the eigenvectors are obtained directly from the geometry of the structure. Other rigid body displacements could be used.

Combine equation (7b) and (10b) for the second row simultaneous equations. - In a manner similar to the derivation of equation (11), the K_{2i} and M_{2i} terms are solved by transposing the known K_{12} and M_{12} terms that were solved in equation (11). There are 7 unknowns remaining.

2nd Row Simultaneous Equations

$$\begin{bmatrix} u_{23} & u_{33} & u_{43} & u_{83} & W_{23} & W_{33} & W_{43} \\ u_{24} & u_{34} & u_{44} & u_{84} & W_{24} & W_{34} & W_{44} \\ u_{25} & u_{35} & u_{45} & u_{85} & W_{25} & W_{35} & W_{45} \\ u_{26} & u_{36} & u_{46} & u_{86} & W_{26} & W_{36} & W_{46} \\ u_{27} & u_{37} & u_{47} & u_{87} & W_{27} & W_{37} & W_{47} \\ u_{28} & u_{38} & u_{48} & u_{88} & W_{28} & W_{38} & W_{48} \\ u_{2t} & u_{3t} & u_{4t} & u_{2t} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} K_{22} \\ K_{23} \\ K_{24} \\ K_{28} \\ M_{22} \\ M_{23} \\ M_{24} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ F_{2t} \end{Bmatrix} - \begin{bmatrix} u_{13} & W_{13} \\ u_{14} & W_{14} \\ u_{15} & W_{15} \\ u_{16} & W_{16} \\ u_{17} & W_{17} \\ u_{18} & W_{18} \\ u_{1t} & 0 \end{bmatrix} \begin{Bmatrix} K_{12} \\ M_{12} \end{Bmatrix} \quad (12)$$

where

$$W_{ir} = -\omega_r^2 \; u_r$$

The two rigid body modes were not used. Other options are to use one rigid body mode and eliminate the static test equation or use 7 flexible modes.

Combine equation (7c) and (10c) for the third row simultaneous equations. - The K_{13} and M_{13} terms were solved in equation (11) and the K_{23} and M_{23} terms were solved in equation (12) and are transposed to the right hand side. There are 8 unknowns remaining.

3rd Row Simultaneous Equations

$$\begin{bmatrix} u_{32} & u_{42} & u_{52} & u_{62} & 0 & 0 & 0 & 0 \\ u_{33} & u_{43} & u_{53} & u_{63} & W_{33} & W_{43} & W_{53} & W_{63} \\ u_{34} & u_{44} & u_{54} & u_{64} & W_{34} & W_{44} & W_{54} & W_{64} \\ u_{35} & u_{45} & u_{55} & u_{65} & W_{35} & W_{45} & W_{55} & W_{65} \\ u_{36} & u_{46} & u_{56} & u_{66} & W_{36} & W_{46} & W_{56} & W_{66} \\ u_{37} & u_{47} & u_{57} & u_{67} & W_{37} & W_{47} & W_{57} & W_{67} \\ u_{38} & u_{48} & u_{58} & u_{68} & W_{38} & W_{48} & W_{58} & W_{68} \\ u_{3t} & u_{4t} & u_{5t} & u_{6t} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} K_{33} \\ K_{34} \\ K_{35} \\ K_{36} \\ M_{33} \\ M_{34} \\ M_{35} \\ M_{36} \end{Bmatrix} = \begin{Bmatrix} b_3 \end{Bmatrix} \qquad (13)$$

where

$$\begin{Bmatrix} b_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ F_{3t} \end{Bmatrix} - \begin{bmatrix} u_{12} & u_{22} & 0 & 0 \\ u_{13} & u_{23} & W_{13} & W_{23} \\ u_{14} & u_{24} & W_{14} & W_{24} \\ u_{15} & u_{25} & W_{15} & W_{25} \\ u_{16} & u_{26} & W_{16} & W_{26} \\ u_{17} & u_{27} & W_{17} & W_{27} \\ u_{18} & u_{28} & W_{18} & W_{28} \\ u_{1t} & u_{2t} & 0 & 0 \end{bmatrix} \begin{Bmatrix} K_{13} \\ K_{23} \\ M_{13} \\ M_{23} \end{Bmatrix}$$

$$W_{ir} = -\omega_r^2 \, u_{ir}$$

One rigid body mode and the static test was used. Other options would be to use the two rigid body modes and eliminate the static test equation or use 8 flexible modes.

Combine equation (7d) and (10d) for the fourth row simultaneous equations. - K_{14} and M_{14} from equation (13) are transposed to the right hand side. There are 6 unknowns remaining.

4th Row Simultaneous Equations

$$\begin{bmatrix} u_{43} & u_{53} & u_{63} & W_{43} & W_{53} & W_{63} \\ u_{44} & u_{54} & u_{64} & W_{44} & W_{54} & W_{64} \\ u_{45} & u_{55} & u_{65} & W_{45} & W_{55} & W_{65} \\ u_{46} & u_{56} & u_{66} & W_{46} & W_{56} & W_{66} \\ u_{47} & u_{57} & u_{67} & W_{47} & W_{57} & W_{67} \\ u_{48} & u_{58} & u_{68} & W_{48} & W_{58} & W_{68} \end{bmatrix} \begin{Bmatrix} K_{44} \\ K_{45} \\ K_{46} \\ M_{44} \\ M_{45} \\ M_{46} \end{Bmatrix} = \begin{Bmatrix} b_4 \end{Bmatrix} \tag{14}$$

where

$$\begin{Bmatrix} b_4 \end{Bmatrix} = - \begin{bmatrix} u_{13} & u_{23} & u_{33} & W_{13} & W_{23} & W_{33} \\ u_{14} & u_{24} & u_{34} & W_{14} & W_{24} & W_{34} \\ u_{15} & u_{25} & u_{35} & W_{15} & W_{25} & W_{35} \\ u_{16} & u_{26} & u_{36} & W_{16} & W_{26} & W_{36} \\ u_{17} & u_{27} & u_{37} & W_{17} & W_{27} & W_{37} \\ u_{18} & u_{28} & u_{38} & W_{18} & W_{28} & W_{38} \end{bmatrix} \begin{Bmatrix} K_{14} \\ K_{24} \\ K_{34} \\ M_{14} \\ M_{24} \\ M_{34} \end{Bmatrix}$$

$$W_{ir} = -\omega_r^2 \; u_{ir}$$

The first 6 flexible modes were used. Other options would be to include the static test equation and/or the rigid body modes.

Combine equation (7e) and (10e) for the fifth row simultaneous equations. - K_{15}, M_{15}, K_{25}, M_{25}, K_{35}, M_{35}, K_{45}, and M_{45} have previously been solved in equation (11) thru (14) and are transposed to the right hand side. There are 5 unknowns remaining.

5th Row Simultaneous Equations

$$\begin{bmatrix} u_{53} & u_{63} & u_{11,3} & W_{53} & W_{63} \\ u_{54} & u_{64} & u_{11,4} & W_{54} & W_{64} \\ u_{55} & u_{65} & u_{11,5} & W_{55} & W_{65} \\ u_{56} & u_{66} & u_{11,6} & W_{56} & W_{66} \\ u_{57} & u_{67} & u_{11,7} & W_{57} & W_{67} \end{bmatrix} \begin{Bmatrix} K_{55} \\ K_{56} \\ K_{5,11} \\ M_{55} \\ M_{56} \end{Bmatrix} = - \begin{bmatrix} u_{33} & u_{43} & W_{33} & W_{43} \\ u_{34} & u_{44} & W_{34} & W_{44} \\ u_{35} & u_{45} & W_{35} & W_{45} \\ u_{36} & u_{46} & W_{36} & W_{46} \\ u_{37} & u_{47} & W_{37} & W_{47} \end{bmatrix} \begin{Bmatrix} K_{35} \\ K_{45} \\ M_{35} \\ M_{45} \end{Bmatrix} \qquad (15)$$

where

$$W_{ir} = -\omega_r^2 \; u_{ir}$$

The first 5 flexible modes were used. Singularity problems were encountered when both rigid body modes and the test equation was used with 2 flexible modes.

Combined equation (7f) and (10f) for the sixth row simultaneous equations. - K_{36}, K_{46}, K_{56}, M_{36}, M_{46} and M_{56} have previously been solved in equations (13) thru (15) and are transposed to the right hand side. There are 3 unknowns remaining.

6th Row Simultaneous Equations

$$\begin{bmatrix} u_{63} & u_{12,3} & W_{63} \\ u_{64} & u_{12,4} & W_{64} \\ u_{65} & u_{12,5} & W_{65} \end{bmatrix} \begin{Bmatrix} K_{66} \\ K_{6,12} \\ M_{66} \end{Bmatrix} = - \begin{bmatrix} u_{33} & u_{43} & u_{53} & W_{33} & W_{43} & W_{53} \\ u_{34} & u_{44} & u_{54} & W_{34} & W_{44} & W_{54} \\ u_{35} & u_{45} & u_{55} & W_{35} & W_{45} & W_{55} \end{bmatrix} \begin{Bmatrix} K_{36} \\ K_{46} \\ K_{56} \\ M_{36} \\ M_{46} \\ M_{56} \end{Bmatrix} \quad (16)$$

where

$$W_{ir} = -\omega_r^2 \; u_{ir}$$

The first 3 flexible modes were used. One rigid body mode or the static test equation could be used with two of the flexible modes. The number of flexible modes used should be greater than the sum of the rigid body modes to eliminate singularity problems.

Expand equation (10g) for the seventh row simultaneous equations. - K_{17} has previously been solved in equation (11) and is transposed to the right hand side. There are 8 unknowns remaining.

7th Row Simultaneous Equations

$$\begin{bmatrix} u_{71} & u_{81} & u_{91} & u_{10,1} & 0 & 0 & 0 & 0 \\ u_{72} & u_{82} & u_{92} & u_{10,2} & 0 & 0 & 0 & 0 \\ u_{73} & u_{83} & u_{93} & u_{10,3} & W_{73} & W_{83} & W_{93} & W_{10,3} \\ u_{74} & u_{84} & u_{94} & u_{10,4} & W_{74} & W_{84} & W_{94} & W_{10,4} \\ u_{75} & u_{85} & u_{95} & u_{10,5} & W_{75} & W_{85} & W_{95} & W_{10,5} \\ u_{76} & u_{86} & u_{96} & u_{10,6} & W_{76} & W_{86} & W_{96} & W_{10,6} \\ u_{77} & u_{87} & u_{97} & u_{10,7} & W_{77} & W_{87} & W_{97} & W_{10,7} \\ u_{78} & u_{88} & u_{98} & u_{10,8} & W_{78} & W_{88} & W_{98} & W_{10,8} \end{bmatrix} \begin{Bmatrix} K_{77} \\ K_{78} \\ K_{79} \\ K_{7,10} \\ M_{77} \\ M_{78} \\ M_{79} \\ M_{7,10} \end{Bmatrix} = - \begin{Bmatrix} u_{11} \\ u_{12} \\ u_{13} \\ u_{14} \\ u_{15} \\ u_{16} \\ u_{17} \\ u_{18} \end{Bmatrix} K_{17} \qquad (17)$$

where

$$W_{ir} = -\omega_r^2 \, \upsilon_{ir}$$

This is the first row of K and M terms in the launch vehicle component that was not tested statically. Six measured system eigenvalues and eigenvectors were used with the 2 rigid body modes. Eight flexible modes could be used with no rigid body modes.

Expand equation (10h) for the eighth row simultaneous equations. - K_{28}, K_{78} and M_{78} have previously been solved in equations (12) and (17) and are transposed to the right hand side. There are 6 unknowns remaining.

8th Row Simultaneous Equations

$$\begin{bmatrix} u_{83} & u_{93} & u_{10,3} & W_{83} & W_{93} & W_{10,3} \\ u_{84} & u_{94} & u_{10,4} & W_{84} & W_{94} & W_{10,4} \\ u_{85} & u_{95} & u_{10,5} & W_{85} & W_{95} & W_{10,5} \\ u_{86} & u_{96} & u_{10,6} & W_{86} & W_{96} & W_{10,6} \\ u_{87} & u_{97} & u_{10,7} & W_{87} & W_{97} & W_{10,7} \\ u_{88} & u_{98} & u_{10,7} & W_{88} & W_{98} & W_{10,8} \end{bmatrix} \begin{Bmatrix} K_{88} \\ K_{89} \\ K_{8,10} \\ M_{88} \\ M_{89} \\ M_{8,10} \end{Bmatrix} = - \begin{bmatrix} u_{23} & u_{73} & W_{73} \\ u_{24} & u_{74} & W_{74} \\ u_{25} & u_{75} & W_{75} \\ u_{26} & u_{76} & W_{76} \\ u_{27} & u_{77} & W_{77} \\ u_{28} & u_{78} & W_{78} \end{bmatrix} \begin{Bmatrix} K_{28} \\ K_{78} \\ M_{78} \end{Bmatrix} \quad (18)$$

where

$$W_{ir} = - \omega_r^2 \; u_{ir}$$

The first 6 flexible modes were used. The 2 rigid body modes could have been used.

Expand equation (10i) for the ninth row simultaneous equations. -

K_{79}, K_{89}, M_{79} and M_{89} have previously been solved in equations (17) and (18) and are transposed to the right hand side. There are 8 unknowns remaining.

9th Row Simultaneous Equations

$$\begin{bmatrix} u_{91} & u_{10,1} & u_{11,1} & u_{12,1} & 0 & 0 & 0 & 0 \\ u_{92} & u_{10,2} & u_{11,2} & u_{12,2} & 0 & 0 & 0 & 0 \\ u_{93} & u_{10,3} & u_{11,3} & u_{12,3} & W_{9,3} & W_{10,3} & W_{11,3} & W_{12,3} \\ u_{94} & u_{10,4} & u_{11,4} & u_{12,4} & W_{9,4} & W_{10,4} & W_{11,4} & W_{12,4} \\ u_{95} & u_{10,5} & u_{11,5} & u_{12,5} & W_{9,5} & W_{10,5} & W_{11,5} & W_{12,5} \\ u_{96} & u_{10,6} & u_{11,6} & u_{12,6} & W_{9,6} & W_{10,6} & W_{11,6} & W_{12,6} \\ u_{97} & u_{10,7} & u_{11,7} & u_{12,7} & W_{9,7} & W_{10,7} & W_{11,7} & W_{12,7} \\ u_{98} & u_{10,8} & u_{11,8} & u_{12,8} & W_{9,8} & W_{10,8} & W_{11,8} & W_{12,8} \end{bmatrix} \begin{Bmatrix} K_{99} \\ K_{9,10} \\ K_{9,11} \\ K_{9,12} \\ M_{99} \\ M_{9,10} \\ M_{9,11} \\ M_{9,12} \end{Bmatrix} = \begin{Bmatrix} b9 \end{Bmatrix} \quad (19)$$

where

$$\begin{Bmatrix} b_9 \end{Bmatrix} = - \begin{bmatrix} u_{71} & u_{81} & 0 & 0 \\ u_{72} & u_{82} & 0 & 0 \\ u_{73} & u_{83} & W_{73} & W_{83} \\ u_{74} & u_{84} & W_{74} & W_{84} \\ u_{75} & u_{85} & W_{75} & W_{85} \\ u_{76} & u_{86} & W_{76} & W_{86} \\ u_{77} & u_{87} & W_{77} & W_{87} \\ u_{78} & u_{88} & W_{78} & W_{88} \end{bmatrix} \begin{Bmatrix} K_{79} \\ K_{89} \\ M_{79} \\ M_{89} \end{Bmatrix}$$

$W_{ir} = - \omega_r^2 \; u_{ir}$

The 2 rigid body modes and the first 6 flexible modes were used. Another option would be to use 8 flexible modes.

Expand equation (10j) for the tenth row simultaneous equations. -

$K_{7,10}$, $K_{8,10}$, $K_{9,10}$, $M_{7,10}$, $M_{8,10}$ and $M_{9,10}$ have been solved in equations (17), (18) and (19) and are transposed to the right hand side. There are 6 unknowns remaining.

10th Row Simultaneous Equations

$$\begin{bmatrix} u_{10,3} & u_{11,3} & u_{12,3} & W_{10,3} & W_{11,3} & W_{12,3} \\ u_{10,4} & u_{11,4} & u_{12,4} & W_{10,4} & W_{11,4} & W_{12,4} \\ u_{10,5} & u_{11,5} & u_{12,5} & W_{10,5} & W_{11,5} & W_{12,5} \\ u_{10,6} & u_{11,6} & u_{12,6} & W_{10,6} & W_{11,6} & W_{12,6} \\ u_{10,7} & u_{11,7} & u_{12,7} & W_{10,7} & W_{11,7} & W_{12,7} \\ u_{10,8} & u_{11,8} & u_{12,8} & W_{10,8} & W_{11,8} & W_{12,8} \end{bmatrix} \begin{Bmatrix} K_{10,10} \\ K_{10,11} \\ K_{10,12} \\ M_{10,10} \\ M_{10,11} \\ M_{10,12} \end{Bmatrix} = \begin{Bmatrix} b_{10} \end{Bmatrix} \quad (20)$$

where

$$\begin{Bmatrix} b_{10} \end{Bmatrix} = - \begin{bmatrix} u_{73} & u_{83} & u_{93} & W_{73} & W_{83} & W_{93} \\ u_{74} & u_{84} & u_{94} & W_{74} & W_{84} & W_{94} \\ u_{75} & u_{85} & u_{95} & W_{75} & W_{85} & W_{95} \\ u_{76} & u_{86} & u_{96} & W_{76} & W_{86} & W_{96} \\ u_{77} & u_{87} & u_{97} & W_{77} & W_{87} & W_{97} \\ u_{78} & u_{88} & u_{98} & W_{78} & W_{88} & W_{98} \end{bmatrix} \begin{Bmatrix} K_{7,10} \\ K_{8,10} \\ K_{9,10} \\ M_{7,10} \\ M_{8,10} \\ M_{9,10} \end{Bmatrix}$$

$W_{ir} = -\omega_r^2 \; u_{ir}$

The first 6 flexible modes were used. Another option would be to use 2 rigid body modes with 4 flexible modes.

Expand equation (10k) for the eleventh row simultaneous equations. - $K_{5,11}$; $K_{9,11}$; $K_{10,11}$; $M_{9,11}$ and $M_{10,11}$ have been solved in equations (15), (19) and (20) and are transposed to the right hand side. There are 4 unknowns remaining.

11th Row Simultaneous Equations

$$\begin{bmatrix} u_{11,3} & u_{12,3} & W_{11,3} & W_{12,3} \\ u_{11,4} & u_{12,4} & W_{11,4} & W_{12,4} \\ u_{11,5} & u_{12,5} & W_{11,5} & W_{12,5} \\ u_{11,6} & u_{12,6} & W_{11,6} & W_{12,6} \end{bmatrix} \begin{Bmatrix} K_{11,11} \\ K_{11,12} \\ M_{11,11} \\ M_{11,12} \end{Bmatrix} = - \begin{bmatrix} u_{53} & u_{93} & u_{10,3} & W_{93} & W_{10,3} \\ u_{54} & u_{94} & u_{10,4} & W_{94} & W_{10,4} \\ u_{55} & u_{95} & u_{10,5} & W_{95} & W_{10,5} \\ u_{56} & u_{96} & u_{10,6} & W_{96} & W_{10,6} \end{bmatrix} \begin{Bmatrix} K_{5,11} \\ K_{9,11} \\ K_{10,11} \\ M_{9,11} \\ M_{10,11} \end{Bmatrix} \quad (21)$$

where

$$W_{ir} = -\omega_r^2 \; u_{ir}$$

The first 4 flexible modes were used.

Expand equation (10m) for the twelfth row simultaneous equations. -

$K_{6,12}$, $K_{9,12}$, $K_{10,12}$, $K_{11,12}$, $M_{9,12}$, $M_{10,12}$ and $M_{11,12}$ have been solved in equations (16), (19), (20) and (21) and are transposed to the right hand side. There are two unknowns remaining.

12th Row Simultaneous Equations

$$\begin{bmatrix} u_{12,2} & W_{12,2} \\ u_{12,6} & W_{12,6} \end{bmatrix} \begin{Bmatrix} K_{12,12} \\ M_{12,12} \end{Bmatrix} = - \begin{bmatrix} u_{62} & u_{92} & u_{10,2} & u_{11,2} & W_{92} & W_{10,2} & W_{11,2} \\ u_{66} & u_{96} & u_{10,6} & u_{11,6} & W_{96} & W_{10,6} & W_{11,6} \end{bmatrix} \begin{Bmatrix} K_{6,12} \\ K_{9,12} \\ K_{10,12} \\ K_{11,12} \\ M_{9,12} \\ M_{10,12} \\ M_{11,12} \end{Bmatrix} \tag{22}$$

where

$$W_{ir} = -\omega_r^2 \, u_{ir}$$

The second rigid body mode and the fourth flexible mode were used. The fourth flexible mode was used because of a high rotational degree of freedom 12 dynamic response.

IV. ANALYSIS AND DISCUSSION

The twelve degree-of-freedom two beam model shown in Section 3.1 and Figure 1 was evaluated using the simultaneous equations (11) thru (22). The simple beam static test shown in Section 3.2 was used in this evaluation.

The desynthesis computer code SIMUL3 was written. - The program uses the NASA Langley library program ITIMP to solve the simultaneous equations. The program input is the measured eigenvalues and eigenvectors, the rigid body eigenvectors, and the static test displacements and forces. The program stashes the data in matrix form into equation (11) and solves for the 9 values in the first row of the K and M matrices. The program then uses this output to evaluate the second row using equation (12) and continues through equation (22) when all of the system stiffness and mass properties are determined.

System dynamic data was simulated by a NASTRAN analysis. - To represent the first 6 measured system flexible eigenvalues and eigenvectors, the NASTRAN program was used with the eigenvalues normalized to unity (MAX and the NASTRAN EIGR card). The NASTRAN data for the 6 flexible modes and 2 rigid body modes are tabulated in Table 1.

The payload static test data was simulated by a NASTRAN anaylsis. - A simple beam unit load (F= 1.0) was applied at the mid point and the simulated test values of the end slope (α_t) and the center deflection (δt) of equation (5) were calculated using the properties in Figure 1.

Four different evaluations using the computer code SIMUL3 were made. -

Analysis No. 1. - The use of static test data with no measurement errors was evaluated with the system dynamic properties from Table 1. The stiffness and mass terms were compared with the correct values (from NASTRAN). The comparison is shown in Table 2 as a percentage change in mass or stiffness. No problems were encountered until the evaluation of M_{66}

(equation 16) and $M_{12,12}$ (equation 22). The M_{66} value was only 84.1% of the NASTRAN value but the original $M_{12,12}$ value was 1366% higher than the NASTRAN value so different options were used to evaluate $K_{12,12}$ and $M_{12,12}$ using equation (22).

Inputs	% Change $\left[\frac{\text{Desynthesis}}{\text{NASTRAN}} \times 100\right]$ $K_{12,12}$	$M_{12,12}$
1. 1st flex. mode 2nd flex. mode	99.6	1366.2
2. 2nd flex. mode 3rd flex. mode	99.0	-117.4 [a]
3. 2nd rigid body 1st flex. mode	100.4	-HIGH [a]
4. 2nd rigid body 2nd flex. mode	100.4	184.1
5. 2nd rigid body 4th flex. mode	100.4	103.6

The final values used were from the fifth input combination of the 2nd rigid body mode and the 4th flexible mode. The variation depends on what two modes are used and could be due to many variables. The use of the 4th flexible mode could have improved the results because this mode has a higher rotational dynamic response then the 1st, 2nd and 3rd flexible modes. Another possibility is that numerical problems because of small difference of large numbers in equation (22). The most likely cause could be the order in which the simultaneous equations were solved (equation 11 thru 22). The M_{77}, M_{99} and $M_{11,11}$ translation terms have very large values relative to the $M_{12,12}$ rotational

a. Negative values indicate the Desynthesis analysis value had the wrong sign.

term indicating that the $M_{12,12}$ should not be solved last.[b] If the order of solving the rows in the K and M matrix are changed so that the M_{77}, M_{99} or $M_{11,11}$ is the last equation solved, this $K_{12,12}$ and $M_{12,12}$ discrepancy will probably be eliminated. M_{77} is 39 times larger than $M_{12,12}$ and M_{99} is 78 times larger than $M_{12,12}$. Examination of Table 2 indicates that the same problem exists for the M_{66} terms. The rearrangement of the order of analysis to solve for the M_{66} before M_{55} will probably eliminate the M_{66} discrepancy.

Even though this problem had not been eliminated, the K and M values of the payload component were evaluated for their dynamic response using SCAMP and compared with the dynamic response using the K and M values obtained from NASTRAN. The expected poor comparison is shown in Table 6.

Analysis No. 2. - The static test slope (α_t) measurement at each end was increased 1% and the displacement at the center was maintained at the correct value. The resulting K and M terms are compared with the results of Analysis No. 1 in Table 3. All of the K and M terms in the first row from equation (11) increased to 103.1% of the Analysis No. 1 results. The values in the second row varied from 84.2 to 106.7%. The values in the third row varied from 32.2% to 123.3%. The error increased as the analysis continued through the payload component. However, when row 7, the first row in the vehicle component, was evaluated using equation (17), the error decreased back to 103.1%, the value identical to row 1. In general, all of the stiffness terms become larger and the mass terms become smaller.

b. See Appendix for verification by subsequent analysis.

The erratic changes in K_{66}, M_{66}, $K_{12,12}$ and $M_{12,12}$ are probably caused by the same problems that was encountered and discussed in Analysis No. 1. The K_{66} and M_{66} actually changed sign. The K or M matrix of the vehicle component was entered into SCAMP to determine the dynamic response and it was determined that the mass matrix was not positive definite and no eigenvalue solution was obtained.

Analysis No. 3. - The static test displacement (δ_t) measurement at the mid point was increased 1% and the slopes at the end points were maintained at the correct value. The resulting K and M terms are compared with the results of Analysis No. 1 in Table 4. All of the K and M terms in row 1 decreased to 96.2% of the Analysis No. 1 values. After the first row the K terms tended to decrease and the M terms tended to increase in value. In the vehicle component the 7th row values were again the same as the row 1 values at 96.2%. No eigenvalue solution was attempted.

Analysis No. 4. - The combined static test measurement error of Analysis No. 2 and Analysis No. 3 was used. This included an increase in the slope at each end of 1% and an increase in displacement at the center 1%. The resulting K and M terms are compared with the results of Analysis No. 1 in Table 5. As can be seen every value was 99.0% of that in Analysis No. 1. Putting these vehicle component M and K terms into SCAMP resulted in identical eigenvalues to that of Analysis No. 1 as shown in Table 6. If the static test measurements are changed so that all readings are high or low the same percentage, the desynthesis analysis will give the correct vehicle component dynamic response.

V. EVALUATION OF LANGLEY DYNAMIC RESEARCH MODEL

A preliminary evaluation was made to investigate the use of the Langley Dynamic Research Model in this desynthesis procedure. There was some system free-free dynamic response test data and some static test data available on the payload. This model has a vehicle carrier and a payload component. The payload component shown in Figure 2 has 48 degrees of freedom. The payload and carrier together have 288 degrees of freedom. There are 8 triangular bulkheads that are fairly rigid, connected by three longerons at the three corners. There are additional diagonal elements not shown to give the structure the shear and torsional capability. Each bulkhead can be considered to move as a rigid body with six degrees of freedom each or a total of 48. The degrees of freedom of the interconnecting springs between the payload and the carrier are shown in Figure 2. To gain some insight into the type of static test required, the uncoupled stiffness (K) terms are shown for the payload as an X at each coupled location in Figure 3. The first 6 degrees of freedom represent the first bulkhead in the six system directions (3 translations and 3 rotations). The second set of 6 degrees of freedom (7 thru 12) represent bulkhead 2, etc. By studying Figure 3, row by row, it can be seen that the 1st bulkhead is coupled to the 2nd bulkhead (rows 1 thru 6). The second bulkhead is coupled to the 1st and 3rd bulkhead (rows 7 thru 12). Each bulkhead is coupled to the bulkheads adjacent to them. Also, some of the 6 degrees of freedom within each bulkhead are uncoupled. The diagonals between bulkheads (not shown) in figure 2 are arranged so that if the first bulkhead has a rigid bydy motion in the 3 direction with the second bulkhead fixed, the net load in the 1 direction on bulkhead 1 is zero. Therefore, there are no 1-3 coupling terms in Figure 3. When the first bulkhead is moved in the 2 direction there is a net force in the 1 direction. Therefore, the 1-2 coupling terms are present. Rows 3, 25, 26, 27, 28, 29, and 30 have interface spring coupling terms not shown in Figure 3.

<u>The static test equation</u>. - From equation (2) and equation (3), the first six rows of a static test equation (3) for this system is:

Test degree of freedom

$$
\begin{array}{cc}
 & \begin{array}{cccccccccccc} u_{1t} & u_{2t} & u_{3t} & u_{4t} & u_{5t} & u_{6t} & u_{7t} & u_{8t} & u_{9t} & u_{10t} & u_{11t} & u_{12t} \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} &
\left[\begin{array}{cccccccccccc}
K_{11} & K_{12} & 0 & K_{14} & K_{15} & K_{16} & K_{17} & K_{18} & 0 & K_{1,10} & K_{1,11} & K_{1,12} \\
K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} & K_{27} & K_{28} & 0 & K_{2,10} & K_{2,11} & K_{2,12} \\
0 & K_{32} & K_{33p} & K_{34p} & K_{35} & K_{36} & 0 & 0 & K_{39} & K_{3,10} & K_{3,11} & K_{3,12} \\
K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} & K_{47} & K_{48} & K_{49} & K_{4,10} & K_{4,11} & K_{4,12} \\
K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} & K_{57} & K_{58} & K_{59} & K_{5,10} & K_{5,11} & K_{5,12} \\
K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} & K_{67} & K_{68} & K_{69} & K_{6,10} & K_{6,11} & K_{6,12}
\end{array}\right]
\end{array}
\left\{ u_t \right\} = \left\{ F_t \right\}
$$

First Bulkhead System Degree of Freedom						Second Bulkhead System Degree of Freedom					
1	2	3	4	5	6	1	2	3	4	5	6

where

$$K_{33} = K_{33p} - K_{3,93}$$

$$K_{34} = K_{34p} - K_{3,94}$$

It can be seen that static test data is required on the first two bulkheads only if one of the first six rows are used to solve the first set of simultaneous equations.

Test data available. - The test data available on the payload consisted of a cantilever test, a bending test and a torque test. Any one of these tests is adequate provided sufficient data is measured in each test. The data needed and the data measured is a follows:

$$\begin{Bmatrix} u_{it} \\ u_{2t} \\ u_{3t} \\ u_{4t} \\ u_{5t} \\ u_{6t} \\ u_{7t} \\ u_{8t} \\ u_{9t} \\ u_{10t} \\ u_{11t} \\ u_{12t} \end{Bmatrix} = \begin{Bmatrix} F_{it} \end{Bmatrix}$$

Cantilever Test

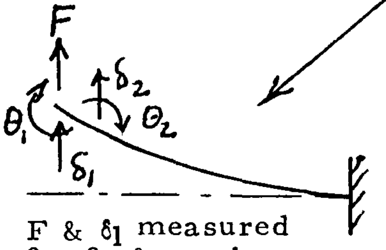
$$\begin{Bmatrix} 0 \\ 0 \\ \delta_1 \\ 0 \\ \theta_1 \\ 0 \\ 0 \\ 0 \\ \delta_2 \\ 0 \\ +\theta_2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ F \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Bending Test

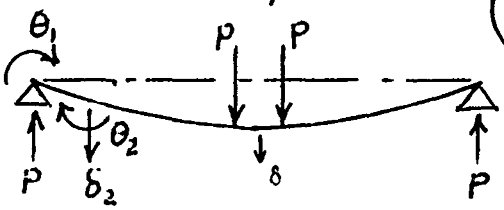
$$\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \theta_1 \\ 0 \\ 0 \\ 0 \\ -\delta_2 \\ 0 \\ \theta_2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Torque Test

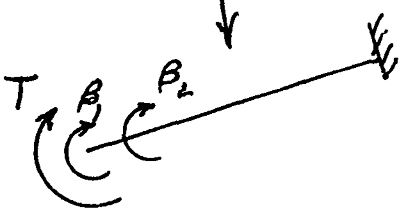
$$\begin{Bmatrix} 0 \\ 0 \\ 0 \\ \beta_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \beta_2 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$



F & δ_1 measured
θ_1, θ_2 & δ_2 also needed.



P & δ measured
θ_1, θ_2 & δ_2 also needed.



T & β_1 measured
β_2 also needed.

The major test displacements are shown on the previous page. However, all the degrees of freedom of the 1st and 2nd bulkhead having coupling terms in Figure 3 should be measured. The static test data available was not adequate because it only measured the vertical displacement of the first bulkhead (δ_1) in the cantilever test, the central deflection (δ) in the beam bending test and the torsional rotation of the first bulkhead (θ_1) in the torque test. More precise test data is required, but only on the first two bulkheads in this example. For the cantilever test and the bending test, row 3 would have to be the first solution to have the finite values (F or P) on the right hand side. If the torque static test is used, row 4 would have to be used for the first solution. The degrees of freedom required for the measurements in each static test are:

	Bulkhead No. 1	Bulkhead No. 2
Cantilever test (Row 3)	2, (3)[c], 4, 5, &6	3, 4, 5, & 6
Bending test (Row 3)	2, 3, 4, 5, & 6	3, 4, 5, & 6
Torque test (Row 4)	1, 2, 3, (4)[c], 5, & 6	1, 2, 3, 4, 5, & 6

c. Values that were measured in static test.

The number of system measured eigenvalues and eigenvectors required depends on the row selected for the first solution. As previously stated the third row would be selected for the cantilever test or the bending test so as to have a force on the right hand side in the simultaneous equations solution. The fourth row would be used for the torque test. There are eleven unknown K terms in row 3 plus 9 unknown M terms or a total of 20 unknowns. In row 4 there are 12 unknown M terms for a total of 24. If we used row 3 for the first set of equations to solve, the number of flexible modes required would be 13. There are 6 rigid body modes and one test equation. In examining the number of measured flexible modes available, it was found to be just short of the 13 that had all 6 degrees of freedom measured. The important point here is that for this 288 system degrees of freedom only 13 measured eignevalues and eigenvectors are required to solve the first set of simultaneous equations. Examining the coupled terms in the vehicle structure indicates that more measured eigenvalues may be required to solve the vehicle mass and stiffness matrix.

VI. CONCLUSIONS AND RECOMMENDATIONS

1. A desynthesis procedure has been developed to combine static test data from the payload component with a few payload/vehicle system eigenvalues and mode shapes to derive the stiffness matrix and mass matrix of both components. The mass and stiffness matrix of each component is then used to extract the component eigenvalues and mode shapes.

2. The desynthesis procedure was demonstrated on a two beam, 12 degree of freedom system with nearly complete success. The last row of the payload component, K_{66} and M_{66} terms in equation (16) and the last row of the vehicle component $K_{12,12}$ and $M_{12,12}$ terms in equation (22) resulted in erroneous results as shown in Table 2. It is recommended that the order of row solution be revised so that the 5th row and 6th row solutions are interchanged and the 11th row and 12th row solutions are interchanged. The row solution order would then be 1, 2, 3, 4, 6, 5, 7, 8, 9, 10, 12 and 11. Equations (15), (16), (21) and (22) would have to be revised and the computer code would have to be changed to reflect these alterations.

3. It is recommended that the computer program be updated so that the variables can be interchanged on input to automate the following.

- Change the order of row evaluation by an input variable.
- By input variables, change the number of rigid body modes used, the number of flexible modes used, and the number of static test equations used in each set of row simultaneous equations.
- Change the number of degrees of freedom of the system by input variables.
- Change the location of coupled terms in each row by input variables

With these changes made the computer program would be automated to allow iterations on the solution of a given system and would have the capability to evaluate any system size and/or coupled degree of freedom location within the stiffness and mass matrix.

4. It has been demonstrated that the accuracy of the solution is very sensitive to the accuracy of the static test data. It was demonstrated that in a simple beam bending test of the payload component, that a 1% error in the slope measurement or deflection in the center independently would cause considerable errors in M and K values. However, if they both had a 1% high measurement error, the correct M and K terms were derived. This desynthesis method has a large potential provided the accuracy that can be measured in the laboratory is sufficient. It is recommended that static test data be measured on a laboratory payload structure component and that this payload with a laboratory vehicle structure have the eigenvalues and mode shape measured. This data would then be used in this desynthesis analysis to determine if the measurement accuracy of the test data is adequate. This laboratory structure could be a simple two beam model or the existing Langley Dynamic Research Model.

5. The preliminary evaluation of the 288 degree-of-freedom Langley Dynamic Research Model consisting of a payload and a vehicle structure indicated that only 27 measured system eigenvalues and mode shapes and one static test of the payload should be sufficient to perform the desynthesis analysis. The payload static test could be a cantelever test, a simple

beam bending test or a cantelever torque test. Exact measurements of 6 degree of freedom displacements are required on two bulkheads at one end of the structure in each test. No measurements are required on the remaining 6 bulkheads.

6. This desynthesis procedure should be applicable to a multicomponent system with a static test on one component.

TABLE 1

EIGENVALUES AND EIGENVECTORS USED IN THE DESYNTHESIS ANALYSIS (VALUES FROM NASTRAN NORMALIZED TO ONE)

$u_{ij} =$

	1	2	3	4	5	6	7	8
1	1.0	1.0	-.9793407	-.8183043	-.4514553	.1480432	-.2259740	.0368018
2	0	-1.0	-.0038746	.6158865	1.0	-.2254144	1.0	-.1144421
3	1.0	0	-.9271466	.0688562	.2874545	-.1117015	-.0051818	.0058164
4	0	-1.0	.1287581	1.0	-.0258628	.0497821	-.8684909	.2216226
5	1.0	-1.0	-.7688752	.7703924	-.4519291	.1266950	.2222345	-.0226015
6	0	-1.0	.1225218	.2846892	-.9194526	.0297036	.9141464	.0413581
7	1.0	1.0	1.0	.8224029	-.1748622	-.4540260	-.0435387	-.2237451
8	0	-1.0	-.0556343	-.7372945	.4297894	1.0	.2246833	1.0
9	1.0	0	.9164300	-.0512733	.1002866	.2895597	.0039955	-.0033242
10	0	-1.0	-.1216034	-.9325306	.0165073	-.0248525	-.1292969	-.8428792
11	1.0	-1.0	.7802890	-.8388912	.1002866	-.4542416	.0396152	.2217029
12	0	-1.0	-.1181919	-.5815038	-.4588584	-.9219962	.2503231	.9470547

EIGENVALUES

ω_1^2 0.0

ω_2^2 0.0

ω_3^2 $2.155839\,(10)^5$

ω_4^2 $1.674222\,(10)^6$

ω_5^2 $1.526401\,(10)^6$

ω_6^2 $3.078905\,(10)^7$

ω_7^2 $1.367141\,(10)^8$

ω_8^2 $2.625948\,(10)^8$

TABLE 2

ACCURACY OF STIFFNESS AND MASS TERMS GENERATED BY DESYNTHESIS PROCEDURE

PAYLOAD COMPONENT

	K_{i1}	K_{i2}	K_{i3}	K_{i4}	K_{i5}	K_{i6}
Row 1	100.0	100.0	100.0	100.0	-	-
Row 2		100.0	100.0	100.0	-	-
Row 3			100.0	-	100.0	100.0
Row 4				100.0	100.0	100.0
Row 5					100.0	100.1
Row 6						99.1

	M_{i1}	M_{i2}	M_{i3}	M_{i4}	M_{i5}	M_{i6}
Row 1	100.0	100.0	100.0	100.0	-	-
Row 2		100.0	100.0	100.0	-	-
Row 3			100.0	-	99.9	99.9
Row 4				100.0	99.8	99.9
Row 5					100.0	100.1
Row 6						84.1

VEHICLE COMPONENT

	K_{i7}	K_{i8}	K_{i9}	K_{i10}	K_{i11}	K_{i12}
Row 7	100.0	100.0	100.0	100.0	-	-
Row 8		100.0	100.0	100.0	-	-
Row 9			100.0	-	100.0	100.0
Row 10				100.0	100.0	100.0
Row 11					100.0	100.1
Row 12						100.4

	M_{i7}	M_{i8}	M_{i9}	M_{i10}	M_{i11}	M_{i12}
Row 7	100.0	100.0	100.0	100.0	-	-
Row 8		100.1	99.8	99.9	-	-
Row 9			100.1	-	99.9	99.9
Row 10				100.1	100.0	100.0
Row 11					100.0	100.0
Row 12						103.6

INTERFACE SPRINGS

K_{17} = 100.0%; K_{28} = 100.0%; $K_{5,11}$ = 99.4%; $K_{12,6}$ = 102.5%

Numbers in matrix represent the percent accuracy compared to the values generated by the NASTRAN program

$$\% = \frac{\text{K or M (Desynthesis)}}{\text{K or M (NASTRAN)}} \times 100$$

TABLE 3

EFFECT OF STATIC TEST MEASUREMENT ERROR
ON K AND M, SLOPE MEASURED 1% HIGH ON BOTH ENDS

PAYLOAD COMPONENT

	K_{i1}	K_{i2}	K_{i3}	K_{i4}	K_{i5}	K_{i6}
Row 1	103.1	103.1	103.1	103.1	-	-
Row 2		101.4	103.0	106.7	-	-
Row 3			112.6	-	123.3	128.2
Row 4				107.9	119.4	130.1
Row 5					126.9	152.8
Row 6						-265.3*

	M_{i1}	M_{i2}	M_{i3}	M_{i4}	M_{i5}	M_{i6}
Row 1	103.1	103.1	103.1	103.1	-	-
Row 2		104.3	84.2	94.8	-	-
Row 3			97.8	-	32.2	48.7
Row 4				90.3	5.3	42.0
Row 5					90.6	144.2
Row 6						-7676.*

VEHICLE COMPONENT

	K_{i7}	K_{i8}	K_{i9}	K_{i10}	K_{i11}	K_{i12}
Row 7	103.1	103.1	103.1	103.1	-	-
Row 8		101.6	103.1	106.3	-	-
Row 9			103.6	-	104.2	106.0
Row 10				101.8	102.3	104.2
Row 11					106.6	110.1
Row 12						226.8

	M_{i7}	M_{i8}	M_{i9}	M_{i10}	M_{i11}	M_{i12}
Row 7	103.1	103.1	103.1	103.1	-	-
Row 8		111.4	75.7	86.6	-	-
Row 9			106.1	-	91.3	79.6
Row 10				107.8	93.2	94.9
Row 11					76.6	30.1
Row 12						13252.

Numbers are the percentage related to the static test with no error

$$\% = \frac{\text{K or M (Slope 1\% High)}}{\text{K or M (No Error)}} \times 100$$

* Negative - indicates the values had opposite signs.

TABLE 4

EFFECT OF STATIC TEST MEASUREMENT ERROR ON K AND M, DEFLECTION MEASURED 1% HIGH AT CENTER

PAYLOAD COMPONENT

	K_{i1}	K_{i2}	K_{i3}	K_{i4}	K_{i5}	K_{i6}
Row 1	96.2	96.2	96.2	96.2	-	-
Row 2		97.8	96.2	92.7	-	-
Row 3			88.1	-	78.9	74.3
Row 4				92.3	82.6	73.4
Row 5					75.1	52.0
Row 6						432.2

	M_{i1}	M_{i2}	M_{i3}	M_{i4}	M_{i5}	M_{i6}
Row 1	96.2	96.2	96.2	91.2	-	-
Row 2		95.1	114.0	103.9	-	-
Row 3			101.2	-	159.8	145.5
Row 4				107.2	180.7	149.0
Row 5					108.8	63.1
Row 6						7191.

VEHICLE COMPONENT

	K_{i7}	K_{i8}	K_{i9}	K_{i10}	K_{i11}	K_{i12}
Row 7	96.2	96.2	96.2	96.2	-	-
Row 8		97.6	96.2	93.1	-	-
Row 9			95.6	-	95.1	93.4
Row 10				97.4	96.9	95.1
Row 11					92.8	89.4
Row 12						-17.1*

	M_{i7}	M_{i8}	M_{i9}	M_{i10}	M_{i11}	M_{i12}
Row 7	96.2	96.1	96.2	96.2	-	-
Row 8		88.3	122.0	114.5	-	-
Row 9			93.3	-	107.3	118.3
Row 10				91.7	105.5	103.9
Row 11					122.5	169.5
Row 12						-10035*

Numbers are the percentage related to the static test with no error

$$\% = \frac{\text{K or M (Deflection 1\% High)}}{\text{K or M (No Error)}} \times 100$$

* Negative - indicates the values had opposite signs.

TABLE 5

EFFECT OF STATIC TEST MEASUREMENT ERROR
ON K AND M, END SLOPES AND CENTER DEFLECTION MEASURED
1% HIGH

PAYLOAD COMPONENT

	K_{i1}	K_{i2}	K_{i3}	K_{i4}	K_{i5}	K_{i6}
Row 1	99.0	99.0	99.0	99.0	-	-
Row 2		99.0	99.0	99.0	-	-
Row 3			99.0	-	99.0	99.0
Row 4				99.0	99.0	99.0
Row 5					99.0	99.0
Row 6						99.0

	M_{i1}	M_{i2}	M_{i3}	M_{i4}	M_{i5}	M_{i6}
Row 1	99.0	99.0	99.0	99.0	-	-
Row 2		99.0	99.0	99.0	-	-
Row 3			99.0	-	99.0	99.0
Row 4				99.0	99.0	99.0
Row 5					99.0	99.0
Row 6						99.0

VEHICLE COMPONENT

	K_{i7}	K_{i8}	K_{i9}	K_{i10}	K_{i11}	K_{i12}
Row 7	99.0	99.0	99.0	99.0	-	-
Row 8		99.0	99.0	99.0	-	-
Row 9			99.0	-	99.0	99.0
Row 10				99.0	99.0	99.0
Row 11					99.0	99.0
Row 12						99.0

	M_{i7}	M_{i8}	M_{i9}	M_{i10}	M_{i11}	M_{i12}
Row 7	99.0	99.0	99.0	99.0	-	-
Row 8		99.0	99.0	99.0	-	-
Row 9			99.0	-	99.0	99.0
Row 10				99.0	99.0	99.0
Row 11					99.0	99.0
Row 12						99.0

Numbers are the percentage related to the static test with no error

$$\% = \frac{\text{K or M (Slope \& Deflection 1\% High)}}{\text{K or M (No Error)}} \times 100$$

TABLE 6

VEHICLE COMPONENT EIGENVALUES
USING SCAMP

Mode No.	Using NASTRAN M and K (Hz) **	Using Desynthesis M and K		
		Analysis No. 1 NO ERROR (Hz) **	Analysis No. 2 Slope 1% High (Hz) *	Analysis No. 4 Slope & Deflection 1% High (Hz)
3	782.5	910.4	-	910.4
4	2449.0	2656.0	-	2656.0
5	6124.0	6074.0	-	6074.0
6	9783.0	9254.0	-	9254.0

* Mass matrix not positive definite so SCAMP could not extract eigenvalues

** The discrepancy between NASTRAN and the desynthesis values is due to the fact that numerical problems were encountered in solving for $K_{12,12}$ and $M_{12,12}$ (see Table 2). The $M_{12,12}$ term is 3.6 percent high, and the $K_{12,12}$ term is 0.4 percent high. As discussed in Section IV, this discrepancy is probably due to the order of solution of K and M terms.

TABLE 7

ACCURACY OF STIFFNESS AND MASS TERMS
GENERATED BY DESYNTHESIS PROCEDURE
(WITH ROW SOLUTION ORDER 1, 2, 3, 4, 6, 5, 7, 8, 9, 12, AND 11)

PAYLOAD COMPONENT

	K_{i1}	K_{i2}	K_{i3}	K_{i4}	K_{i5}	K_{i6}
Row 1	100.0	100.0	100.0	100.0	-	-
Row 2		100.0	100.0	100.0	-	-
Row 3			100.0	-	100.0	100.0
Row 4				100.0	100.0	100.0
Row 5					100.1	100.0
Row 6						100.1

	M_{i1}	M_{i2}	M_{i3}	M_{i4}	M_{i5}	M_{i6}
Row 1	100.0	100.0	100.0	100.0	-	-
Row 2		100.0	100.0	100.0	-	-
Row 3			100.0	-	99.9	99.9
Row 4				100.0	99.8	99.9
Row 5					100.1	100.0
Row 6						100.0

VEHICLE COMPONENT

	K_{i7}	K_{i8}	K_{i9}	K_{i10}	K_{i11}	K_{i12}
Row 7	100.0	100.0	100.0	100.0	-	-
Row 8		100.0	100.0	100.0	-	-
Row 9			100.0	-	100.0	100.0
Row 10				100.0	100.0	100.0
Row 11					100.1	100.1
Row 12						100.1

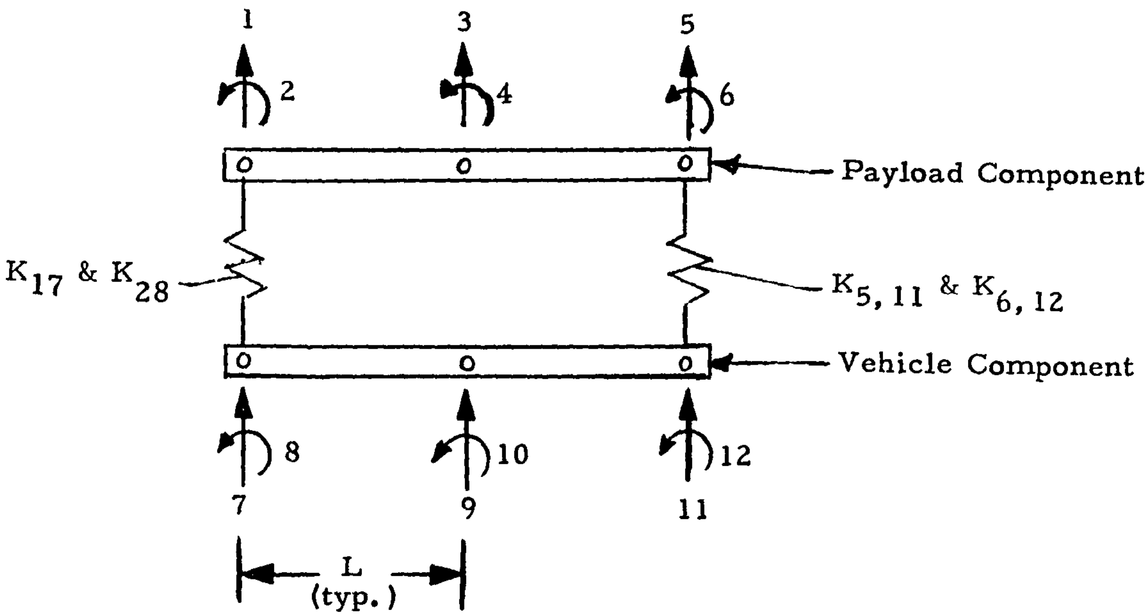
	M_{i7}	M_{i8}	M_{i9}	M_{i10}	M_{i11}	M_{i12}
Row 7	100.0	100.0	100.0	100.0	-	-
Row 8		100.1	99.8	99.9	-	-
Row 9			100.1	-	99.9	99.9
Row 10				100.1	100.0	100.0
Row 11					100.2	100.2
Row 12						100.7

INTERFACE SPRINGS

K_{17} = 100.0%; K_{28} = 100.0%; $K_{5,\ 11}$ = 97.6%; $K_{6,\ 12}$ = 100.1%.

Numbers in matrix represent the program accuracy compared to the values generated by the NASTRAN program

$$\% = \frac{\text{K or M (Desynthesis)}}{\text{K or M (NASTPAN)}} \times 100$$



$I = 3.330\ (10)^{-4}\ cm^4\ (8.0\ (10)^{-6}\ in^4)$

$E = 86185\ N/cm^2\ (125000\ psi)$

Density = $27.68\ g/cm^3\ (0.1\ lb/in^3)$

$A = .06452\ cm^2\ (.010\ in^2)$

$L = 2.54\ cm\ (1.0\ in)$

$EI = 28.7\ N - cm^2\ (1.0\ lb - in^2)$

FIGURE 1. TWO BEAM MODEL
12 D.O.F. SYSTEM

Figure 1. (Continued)

STIFFNESS MATRIX - 2 BEAM MODEL

$$
\begin{array}{cc}
 & \begin{array}{cccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{array} &
\left[
\begin{array}{cccccc|cccccc}
\frac{12EI}{L^3}+K_A & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & 0 & -K_A & 0 & 0 & 0 & 0 & 0 \\
\frac{6EI}{L^2} & \frac{4EI}{L}+\theta_A & -\frac{6EI}{L^2} & \frac{2EI}{L} & 0 & 0 & 0 & -\theta_A & 0 & 0 & 0 & 0 \\
-\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{24EI}{L^3} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{8EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3}+K_B & -\frac{6EI}{L^2} & 0 & 0 & 0 & 0 & -K_B & 0 \\
0 & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L}+\theta_B & 0 & 0 & 0 & 0 & 0 & -\theta_B \\
\hline
-K_A & 0 & 0 & 0 & 0 & 0 & \frac{24EI}{L^3}+K_A & \frac{12EI}{L^2} & -\frac{24EI}{L^3} & \frac{12EI}{L^2} & 0 & 0 \\
0 & -\theta_A & 0 & 0 & 0 & 0 & \frac{12EI}{L^2} & \frac{8EI}{L}+\theta_A & -\frac{12EI}{L^2} & \frac{4EI}{L} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{24EI}{L^3} & -\frac{12EI}{L^2} & \frac{48EI}{L^3}+K_B & 0 & -\frac{24EI}{L^3} & \frac{12EI}{L^2} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{12EI}{L^2} & \frac{4EI}{L} & 0 & \frac{16EI}{L}+\theta_B & -\frac{12EI}{L^2} & \frac{4EI}{L} \\
0 & 0 & 0 & 0 & -K_B & 0 & 0 & 0 & -\frac{24EI}{L^3} & -\frac{12EI}{L^2} & \frac{24EI}{L^3} & -\frac{12EI}{L^2} \\
0 & 0 & 0 & 0 & 0 & -\theta_B & 0 & 0 & \frac{12EI}{L^2} & \frac{4EI}{L} & -\frac{12EI}{L^2} & \frac{8EI}{L}
\end{array}
\right]
\end{array}
$$

Figure 1. (Concluded)

MASS MATRIX - 2 BEAM MODEL

$$
\frac{MAL}{420} \times
\begin{array}{rl}
 & \begin{array}{cccccccccccc} \underline{1} & \underline{2} & \underline{3} & \underline{4} & \underline{5} & \underline{6} & \underline{7} & \underline{8} & \underline{9} & \underline{10} & \underline{11} & \underline{12} \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{array} &
\left[\begin{array}{cccccc|cccccc}
156 & -22L & 54 & 13L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-22L & 4L^2 & -13L & -3L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
54 & -13L & 312 & 0 & 54 & 13L & 0 & 0 & 0 & 0 & 0 & 0 \\
13L & -3L & 0 & 8L^2 & -13L & -3L & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 54 & -13L & 156 & 22L & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 13L & -3L^2 & 22L & 4L^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 156 & -22L & 54 & 13L & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -22L & 4L^2 & -13L & -3L & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 54 & -13L & 312 & 0 & 54 & 13L \\
0 & 0 & 0 & 0 & 0 & 0 & 13L & -3L & 0 & 8L^2 & -13L & -3L \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 54 & -13L & 156 & 22L \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13L & -3L^2 & 22L & 4L^2
\end{array}\right]
\end{array}
$$

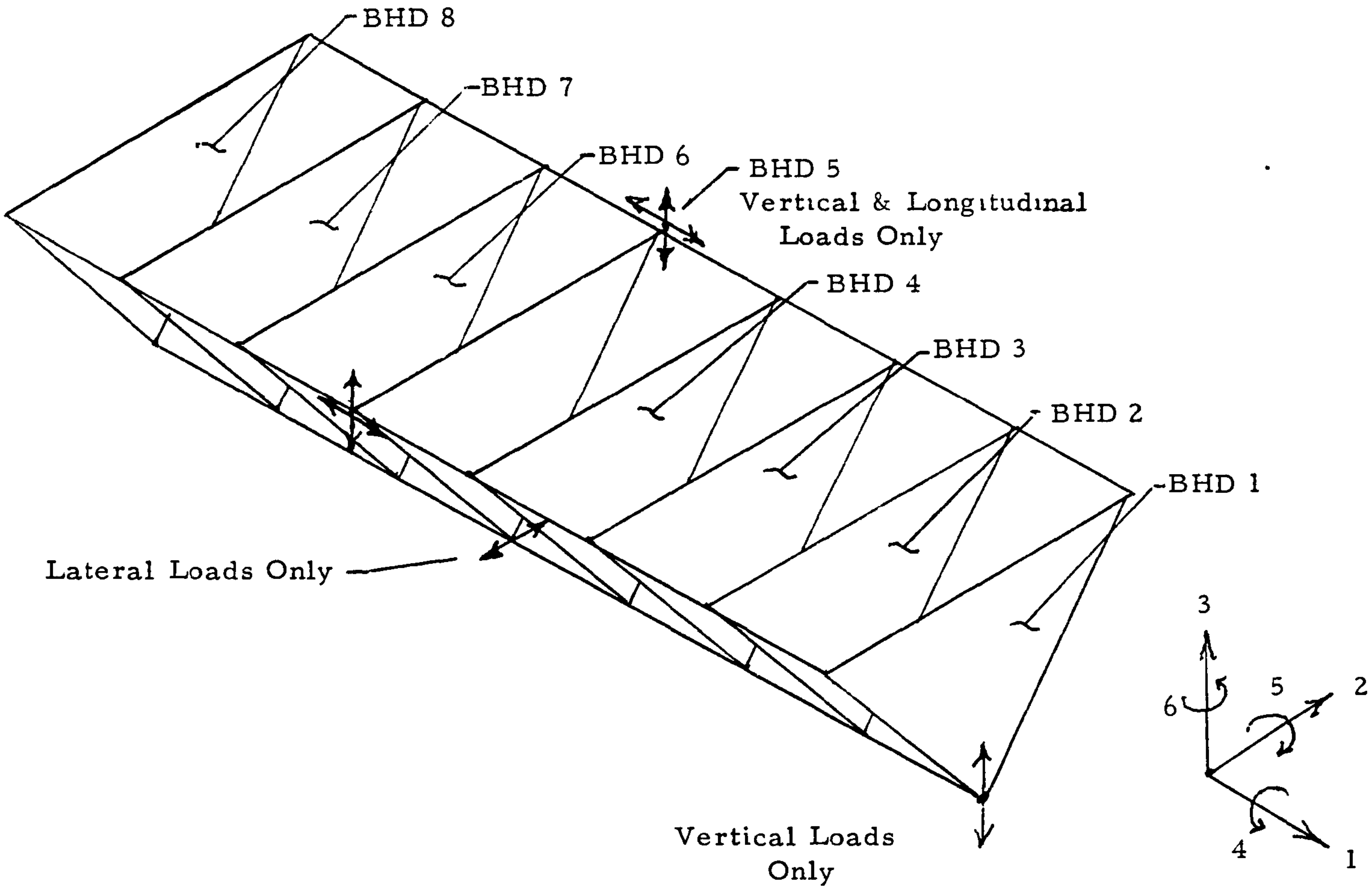


FIGURE 2. - PAYLOAD - LANGLEY DYNAMIC RESEARCH MODEL

FIGURE 3.

COUPLED DEGREES OF FREEDOM FOR
PAYLOAD OF LANGLEY DYNAMIC RESEARCH MODEL

```
 COLUMN..         10        20        30        40        50        60
  ROW              .         .         .         .         .         .
  ...    ............................................................
   .
     .  D* ***** ***
     .  *D****** ***
     .   *D***  ****
     .  ***D********
  5 ..  ****D*******
     .  *****D******
     .  ** ***D* ***** ***
     .  ** ****D ***** ***
     .    ****  D***  ****
 10 ..  *********D********
     .  **********D*******
     .  ***********D******
     .        ** ***D* ***** ***
     .        ** ****D****** ***
 15 ..          **** *D***  ****
     .        *********D********
     .        **********D*******
     .        ***********D******
     .              ** ***D* ***** ***
 20 ..              ** ****D ***** ***
     .                ****  D***  * **
     .              *********D********
     .              **********D*******
     .              ***********D******
 25 ..                    ** ***D* ***** ***
     .                    ** ****D****** ***
     .                      **** *D***  ****
     .                    *********D********
     .                    **********D*******
 30 ..                    ***********D******
     .                          ** ***D* ***** ***
     .                          ** ****D ***** ***
     .                            ****  D***  ****
     .                          *********D********
 35 ..                          **********D*******
     .                          ***********D******
     .                                ** ***D   **** **$
     .                                ** *** D**  ** **$
     .                                  **** *D*    ***$
 40 ..                                ****** **D  *****$
     .                                *******   D******$
     .                                *******   *D*****$
     .                                      ** ***D* **$
     .                                      ** ****D **$
 45 ..                                        ****  D**$
     .                                      *********D*$
     .                                      **********D$
     .                                      $$$$$$$$$$$D
     .
 50 ..
```

APPENDIX

EFFECT OF SEQUENCE OF SIMULTANEOUS EQUATIONS ON ACCURACY OF STIFFNESSES AND MASSES

As recommended in the second paragraph of the conclusions, the sequence of simultaneous equations used in the solution of equation (8) was revised so that stiffnesses and masses in the sixth matrix row were solved before those in the fifth row and, similarly, stiffnesses and masses in the twelfth row were solved before those in the eleventh row. Thus, the new row solution order used was 1, 2, 3, 4, 6, 5, 7, 8, 9, 10, 12, and 11. The original row order was 1 through 12 in numerical sequence. The results of the revised analysis are tabulated in Table 7 and should be compared with the original results in Table 2. By making this change, the inaccurate results for K_{66}, M_{66}, $K_{12,\ 12}$ and $M_{12,\ 12}$ in Table 2 were eliminated. All of the mass and stiffness terms of the payload component and the launch vehicle component were accurate to within one percent of the values from the NASTRAN solution. The $K_{5,\ 11}$ interface spring stiffness term had the largest error, namely 97.6 percent of the NASTRAN value.

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