Analysis of the absolute value of Titan's albedo and its variation with increasing phase angle has yielded constraints on the optical properties and average particle size of the aerosols responsible for the scattering of visible light. The real index of refraction of the scattering material lies within the range $1.5 < n_r < 2.0$ and the average particle size is somewhere between $0.2 \mu m$ and $0.4 \mu m$. The amount of limb darkening produced by these models leads to an occultation radius of $\sim 2700$ km.

The reflection of visible light by Titan is believed to be due chiefly to scattering by an optically thick layer of particles analogous to the blue absorbing "Axel dust" present in the upper atmospheres of the outer planets. By comparing observed properties of Titan with model calculations, we have obtained preliminary estimates of some of the properties of the aerosols, including particle size and spectral absorption characteristics. In addition, these characteristics allow us to predict Titan's limb darkening, thereby removing an ambiguity in the determination of Titan's radius found from lunar occultation observations.

We evaluated the scattering characteristics of our aerosol models with a computer program based on the doubling method that provides an accurate solution to the multiple scattering problem. The single scattering properties of the aerosols were
computed using a scheme developed for nonspherical particles (Pollack et al., 1977). The free parameters of the model include the real part of the index of refraction \(n_r\); the imaginary index \(n_i\); optical depth of the aerosol layer at a reference wavelength of 0.55 \(\mu\)m, \(\tau\); particle size distribution function \(f(r)\), where \(r\) is the radius of an equal volume sphere; and three parameters, \(\alpha_o\), FTB, and SAR, which are related to the nonspherical nature of the particles. We used the two parameter size distribution proposed by Hansen and Hovenier (1974), since the scattering properties of the aerosol layer depend almost entirely on one of these parameters; \(\bar{r}\), the cross section (geometric) weighted average particle radius. The second parameter \(b\) is a measure of the width of the distribution function. The nonspherical parameters \(\alpha_o\), FTB, and SAR are, respectively, the ratio of particle circumference to wavelength below which the particles act like spherical scatterers and above which they depart from Mie scattering; the ratio of light singly scattered into the forward hemisphere to that scattered into the backward hemisphere for the non-Mie domain; and the ratio of actual surface area to that of an equal volume sphere.

In the calculations discussed below, we selected the following values for the above parameters: \(n_r = 1.3, 1.4, 1.5, 1.7, 1.8, 2.0, 2.5, 3.0\); \(n_i(\lambda)\) to be found from the observations; \(\tau = 10\) (essentially infinite optical depth); \(\bar{r}\) to be determined from the observations; \(b = 0.05\) (a narrow distribution); \(\alpha_o = 8\) (typical of particles lacking sharp edges); FTB = 2; and SAR = 1.3.

Even though the phase angle dependence of Titan can only be observed over a 6.5' range from Earth, it is still possible to obtain some information on the mean particle size \(\bar{r}\) from this data. The magnitude of the phase angle variation depends on the shape of the single scattering phase function near a scattering angle \(\theta\) of 180°. This in turn depends on \(\bar{r}\). When \(\bar{\sigma} = 2\pi\bar{r}/\lambda << 1\), where \(\lambda\) is the wavelength, the aerosol exhibits a Rayleigh scattering phase function, which decreases only slightly with decreasing \(\theta\) when \(\theta\) is near 180°. When \(\bar{\sigma} \gg \alpha_o\), the phase function increases with decreasing \(\theta\), but when \(1 < \bar{\sigma} < \alpha_o\), it decreases significantly as \(\theta\) decreases slightly from 180°. Thus, the second \(\bar{\sigma}\) domain exhibits the smallest decrease in brightness with increasing phase angle (decreasing \(\theta\)), the third \(\bar{\sigma}\) domain exhibits the largest decrease, and the first \(\bar{\sigma}\) domain exhibits an intermediate behavior. Hence, some bounds on \(\bar{r}\) can be obtained from the observed phase effect.
We have computed the phase effect of our model aerosol layers for a wide range of values of \( r \). For each choice of \( r \), we have evaluated \( n^i(\lambda) \) by demanding that the computed geometric albedo match the observed value at wavelength \( \lambda \) (Nelson and Hapke, 1978). We compared the predicted phase effects with those observed by Noland, et al. (1974) at six wavelengths ranging from 0.35 \( \mu \)m to 0.75 \( \mu \)m. The large decrease in brightness with increasing phase angle found at the shorter wavelengths implies that \( 1 < \tilde{a} < \alpha_0 \), or 0.05 \( \mu \)m < \( \tilde{r} \) < 0.45 \( \mu \)m. Within this size domain it is possible to obtain an actual value for \( \tilde{r} \) from the data. Figure 1 shows a plot of the ratio of Titan's disk integrated intensity at 6.4° and 0° phase angle as a function of wavelength. Filled squares and vertical lines indicate the observed values and their associated error bars, as found from a least squares fit to the observations, while the curves indicate the predicted behavior of several models that come closest to fitting the data. The middle curve corresponds to a model with \( n_r = 1.5, \tilde{r} = 0.35 \mu \)m, while the top and bottom curves refer to \( \tilde{r} = 0.32 \mu \)m and \( \tilde{r} = 0.40 \mu \)m, respectively. We see that the middle curve is not only capable of matching the observed phase effect at two wavelengths, as might be expected since there are two free parameters \( n_r \) and \( \tilde{r} \), but is also able to reproduce the observed spectral dependence over all six wavelengths. We also see that \( \tilde{r} \) can be altered by at most a few hundredths of a micron from its optimum value before the fit to the observations becomes unacceptable. However, the value of \( \tilde{r} \) can be changed by a somewhat larger amount if \( n_r \) is also varied.

In Figure 2, the model curves coming closest to fitting the observations are shown for several different values of \( n_r \). In addition to the cases shown, fits to the observations were attempted using \( n_r = 1.3, 1.4, 2.5, \) and 3.0. A refractive index of 1.4 or less is unable to produce a phase variation much less than 0.98, which makes it clearly unacceptable at the shorter wavelengths. Refractive indices greater than 2 yield insufficient variation of the phase function with wavelength, a trend which can already be detected in the model curve shown for \( n_r = 2.0 \). So judging from the results shown in Figure 2, \( 1.5 \leq n_r \leq 2.0 \) and 0.2 \( \mu \)m \( \leq \tilde{r} \leq 0.4 \mu \)m.

The inferred value of \( \tilde{r} \) is consistent with the requirements of the inversion model of Titan (Danielson et al., 1973). This model requires that the mean size of the aerosols absorbing some of the incident sunlight be small enough so that they are poor radiators in the thermal region of the spectrum, i.e. \( \tilde{a} \ll 1 \) for \( \lambda \sim 10 \mu \)m or longer. This condition allows the upper atmosphere to assume higher temperatures than the effective temperature.
Figure 1. Illustration of the phase effect calculated for Titan as a function of wavelength, for $n_\tau = 1.5$ and three different mean particle sizes. The values derived from the observations of Noland et al. (1974) are indicated by filled squares with error bars.
MODEL FITS TO PHASE VARIATION

Figure 2. The best fit to the observed spectral dependence of the phase variation obtainable by varying \( \tilde{r} \), for four different values of \( n_r \).
Figure 3 displays the imaginary index of refraction as a function of wavelength, as found by matching Titan's geometric albedo for an assumed visible "surface" radius of 2700 km. The four curves of this figure correspond to the four models shown in Figure 2. We see that the absorption coefficient of the aerosols decreases by about one order of magnitude between 0.35 \( \mu \)m and about 0.6 \( \mu \)m, but that it flattens toward longer wavelengths. Both the deduced spectral shape and approximate absolute value of \( n_i \) should provide useful constraints on the composition of the aerosols.

Finally, we consider the limb darkening behavior of our most successful models. In Figure 4, \( I(\mu)/I(\mu = 1) \) is plotted as a function of \( \mu \) at wavelengths of 0.49 \( \mu \)m and 0.62 \( \mu \)m, where \( \mu \) is the cosine of the angle between the local vertical and the line of sight. These predictions refer to zero degrees phase angle. Also given in the figure for comparison is the behavior of a Lambert surface.

These results have relevance for the inference of Titan’s radius from lunar occultation observations (Elliot et al., 1975). These data do not permit the simultaneous solution of both the limb darkening law and the satellite's radius at the occultation level. For a uniformly bright satellite \( I(\mu)/I(\mu = 1) \), the occultation radius is about 2500 km, while for a Lambert law, it has a value of 2900 km. The predicted limb darkening illustrated in Figure 4 implies that the occultation radius is about 2600 to 2700 km. We plan to make a more precise determination of this important quantity.

REFERENCES

Figure 3. Imaginary index of refraction as a function of wavelength for the cases shown in Figure 2. Numbering of the curves is the same as in Figure 2.
TITAN LIMB DARKENING

![Graph showing limb darkening at wavelengths](image)

DISCUSSION

D. MORRISON: It seems clear that the surface properties as they could be derived from photometric or polarimetric measurements are important for distinguishing between the models. You could have a surface, John, where Don has the top of a cloud level. I would like to read to you from Veverka’s paper given at the Titan Workshop (p. 54)—"the single most important conclusion to be drawn from the photometry and polarimetry of Titan is that a Saturn-like cloud model may be required to explain the sum of the observations." Can you comment on this?

D. HUNTING: That is a perfectly reasonable interpretation, but it’s not unique. On p. 57 of that book I point out that Titan could also be paved with a glassy layer of asphalt, which would have polarization properties indistinguishable from those of a cloud. And that’s not an entirely improbable kind of appearance for a Titan surface.

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J. CALDWELL: I don't think it's quite right to conclude that the cloud has to be like Saturn. The optical depth of the atmosphere and dust layer, even in these low surface pressure models, is very large in the visible and ultraviolet. It seems to me perfectly reasonable that the polarization might be due to the properties of the dust.

B. SMITH: What optical depth do you use?

J. CALDWELL: Podolak and Danielson calculate an optical depth due to dust of \( \approx 5 \) at 5000 Å.

B. SMITH: Either way, there is no hope of seeing the surface.

D. HUN TEN: And no hope of seeing the cloud top of my model either.

J. POLLACK: The successful model by Rages and myself shows, starting about 0.6 \( \mu m \), a tremendously sharp increase in the imaginary index to shorter wavelengths; at somewhat longer wavelengths, it tends to flatten out. So, in effect, whatever is making up this layer is something that has a very strong and very sharp ultraviolet absorption band. I think that both the shape and the absolute absorption coefficient here may allow us to choose between different compositional possibilities.

With a model we can, of course, define what the limb darkening of Titan would be like, and the relevance of that is that the lunar occultation measurement of the size of Titan is very dependent upon what limb darkening law you assume. If you assume a uniformly bright surface, which would be a flat line in Figure 4, then you get a radius of 2500 km. If you assume a Lambert law, you get about 2900 km.

What our models would say is that you're somewhere in between, but closer to the Lambert surface, so therefore, the occultation radius would be something on the order of 2600 to 2700 km.

Finally, I would like to note that I've become increasingly impressed that the apparent secular brightening of Titan is, in fact, a real phenomenon, and there's now something like six years of observations that indicate that Titan's brightness has increased by maybe five or so percent. If we assume that that's right, and we also assume that what we see is a photochemical layer, then it's quite conceivable that plausible solar variability could indeed have a very interesting climatic feedback.

For example, if you have ultraviolet solar variations, which we know from satellite observation do occur, variations in intensity will cause a variation in the production rate of the smog, which in turn could affect the size of the particles somewhat, and
therefore, affect the overall brightness of Titan. If this sort of linkage is true, it's very interesting in the sense that a very small energy change in the Sun is able to enormously amplify the amount of solar energy that is deposited in Titan.

D. HUNTEN: But we should also remember that ordinary seasonal variations may be the answer.

J. CALDWELL: Podolak and Danielson have done a lot of work to derive the photometric properties of the dust, and they have a size of about 0.1 μm.

J. POLLACK: This size (0.1 μm) is incompatible with the phase angle variations (see Figure 1 of this article). Also, they make some arbitrary assumptions about the analytic dependence of the imaginary index; they assumed a power law. I think you can see from Figure 3 of this article that this is not true over the whole spectral range of the observations.

B. SMITH: What do you mean by the particle size?

J. POLLACK: Any time you derive particle size information from brightness measurements, what you're really doing is just deriving one gross property of the size distribution, namely, the cross-sectional averaged particle size. For Podolak and Danielson, the number given is the maximum radius of a flat distribution, and the effective size is a bit smaller.

J. POLLACK: On the difference between the equivalent widths on Titan and Saturn for the methane bands, it could equally well be the result of the differences in the properties of the scattering medium that's present in the two atmospheres.

L. TRAFTON: Possibly. But look at the differences between the spectrum of Jupiter and Saturn where you have appreciably different haze layers. The haze layer of ammonia is really thin on Jupiter, it's really thick on Saturn, and yet these big differences in the aerosol structure between the two planets lead to only small differences in the shapes of the spectra when you compare them against the shapes of methane in Titan's atmosphere.

J. POLLACK: Let me be a little bit more specific. In the case of Jupiter and Saturn, there is an optically thin haze layer and then a fairly thick, dense cloud layer beneath that. In the case of Titan, there is an optically thick haze layer which means that a lot more multiple scattering happens in Titan's atmosphere, while on Jupiter and Saturn it's closer to simple reflecting.
1. TRAFTON: Yes, but there are really three different regimes. I really
don't see Jupiter and Saturn being in one regime and Titan being entirely different.
But in terms of differences in the spectrum, I do see very little differences between
the shapes of the spectrum of Jupiter and Saturn.

J. POLLACK: That is what I would expect for the photochemical haze; it
is much thinner in the case of Jupiter and Saturn than it is on Titan. I believe most
of the line formation takes place within the actual dust in the case of Titan.

D. MORRISON: Why should Titan have so much more of this dust or smog
than Jupiter or Saturn?

J. POLLACK: I think it goes back to abundances; for one thing, the fractional
abundance of methane is a lot more on Titan.

D. HUNten: I don't agree that it's a lot more; what is different is the absence
of hydrogen in the photochemical processes, so that the reducing power of the atmos-
phere is negligible.

J. POLLACK: Yes, and nitrogen in Titan's atmosphere could also be a
critical element there.

1. TRAFTON: What is needed is laboratory data for methane at a very low
pressure like a hundredth atmosphere to maybe a tenth to know whether or not there
is indeed any pressure dependence at the methane level.

J. CALDWELL: Such a measurement requires a very long path length.
Lutes applied tremendous lengths already to do this. You may be asking the
impossible.

SUMMARY OF DISCUSSION AMONG TRAFTON, HUNten, AND POLLACK: Fink,
Benner, and Dick (1977) find below 8000 Å that the bands obey Beer's law and are pressure-
independent. Their explanation is that there are many lines per half-width even at
zero pressure. Pressure effects begin to be evident at longer wavelengths and might
be expected in the wings of all bands. Podolak and Giver have explored possible
saturation effects for the lowest pressures. Temperature effects must also be kept
in mind.

M. KLEIN: (addressed to Huntten) If you really have this cloud at the 77 K level
of 600 g cm⁻², the opacity to microwaves at 3 mm is no longer unappreciable. This
effect may not be insignificant.
D. HUN TEN: The particles would have to be very large to have a significant scattering opacity. Pure methane, either liquid or solid, should have very low millimeter-wave absorption, because the molecule is nonpolar. I think it would need polar impurities to be much of an absorber.

In addition, I don't really believe the cloud is that dense. I believe that most of that mass must precipitate out and leave a much thinner cloud. All I did in the paper was follow the Lewis-type prescription in which you condense out all the mass that is available at each height and call it a cloud.

D. STROBEL: Do your methane cloud properties satisfy the observations that Low and Rieke (Astrophys. J. 190, 1143, 1974) made at 5 µm? It could be thermal emission at 165 K or solar reflection with an albedo of 0.10.

D. HUN TEN: My methane cloud top is about 70 or 80 K, but I don't see why it couldn't explain the observations as reflected sunlight.

J. CALDWELL: Is your cloud opaque at all in the 10 µm region? There are many transparent gaps between the fundamental bands of C₂H₆ and C₂H₄ in this region.

D. HUN TEN: I would think so. In one of my models I did simply postulate an opaque cloud. I rationalize this by suggesting that Axel dust dissolves in methane and gives it some additional absorption that pure methane wouldn't have.

J. CALDWELL: In my models, the optical depth of the Axel dust is the order of 0.05. If the optical depth gets much larger than that, the emission of the dust will go up at 10 µm in contradiction to the observations.

D. HUN TEN: But I'm not talking of dust itself, rather the same material after it's dissolved in the cold methane clouds.

J. CALDWELL: My reason for emphasizing this point is that any model must be consistent with Gillett's observations at 10 µm, which exclude a high brightness temperature. If there's a 200 K surface below the haze, something has to hide that from an outside observer.

D. HUN TEN: I think pressure induced hydrogen will do that. Even at 10 µm, with twenty atmospheres of nitrogen and several km-A of hydrogen at the surface, that's a very opaque medium. It's very much like Venus.

G. SISCOE: What is the hydrogen escape rate from Titan?

D. HUN TEN: My estimate in Planetary Satellites is 9 x 10⁹ molecules cm⁻² s⁻¹. That is based simply on photolysis of the methane we see to be present, and is firm unless someone can find a stronger source.