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An Empirical Model for the Complex Dielectric Permittivity of Soils as a Function of Water Content

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ABSTRACT

The recent measurements on the dielectric properties of soils have shown that the variation of dielectric constant with moisture content depends on soil types. The observed dielectric constant increases only slowly with moisture content up to a transition point. Beyond the transition it increases rapidly with moisture content. The moisture value at transition region was found to be higher for high clay content soils than for sandy soils. Many mixing formulas reported in the literature were compared with, and were found incompatible with, the measured dielectric variations of soil-water mixtures. A simple empirical model was proposed to describe the dielectric behavior of the soil-water mixtures. This model employs the mixing of either the dielectric constants or the refrac- indices of ice, water, rock and air, and treats the transition moisture value as an adjustable parameter. The calculated mixture dielectric constants from the model were found to be in reasonable agreement with the measured results over the entire moisture range of 0-0.5 cm^3/cm^3. The transition moistures derived from the model range from 0.09 to 0.30 and are strongly correlated with the wilting points of the soils estimated from their textures. This relationship between transition moisture and wilting point provides a means of estimating soil dielectric properties on the basis of texture information.

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1. Introduction

The measurements of dielectric constants as a function of moisture content in soils have been carried out over a wide microwave frequency range in the past several years (1, 2, 3, 4, 5, 6, 7, 8, and 9). Some of these measurements were made for many soil samples with widely different texture structures and thus provided systematic studies on the variation of the dielectric constant with soil texture (1, 7, and 9). As a result of these studies, two distinct features associated with the relation between the soil dielectric constant and moisture content have emerged. First, for all soil samples the dielectric constant increases slowly with moisture content initially. After reaching a transition moisture value (7), the dielectric constant increases steeply with moisture content. Secondly, the transition moisture is found to vary with soil type or texture, being smaller for sandy soils than for clayey soils. Because of this variation of the transition moisture with soil types, the plots of the measured dielectric contents versus moisture content show differences for soils with different textures. These differences persist whether the moisture content is determined by percent by dry weight or by volume basis (9).

The main objective of this paper is to present a model to represent these measured data. Toward this objective, some mixing formulas reported in the literature were compared with the measured data and found to be inadequate in describing the dielectric behavior of various soil-water mixtures. Two simple empirical approaches are therefore proposed to describe the dependence of the

measured soil dielectric constants on the moisture content. In the first approach, the resultant dielectric constant of a soil-water mixture is expressed in terms of the direct mixing of the dielectric constants of the constituents. In the second approach, the complex indices of refraction of the constituents are mixed to give the resultant refractive index of the soil-water mixture. In contrast to the other previously reported mixing models, a bi-phase dielectric property for water in soils is assumed in both models here. For moisture contents below the transition moisture, the water in soils is thought to behave like ice and, consequently, the dielectric constant (or refractive index) for ice is used in the mixing, while above the transition moisture the dielectric properties of the liquid water are used. By varying the values of the transition moisture, both models are found to give a reasonably good fit to the measured data for soils with a wide range of textures. The transition moisture is observed to be strongly correlated with the wilting point of soils. Since the wilting point is related to soil texture (10), the observed correlation suggests the possibility of quantifying the measured dielectric constants according to soil types.

The experimental data used in this study were taken from Lundien (1) at 1.41 GHz, Newton (1977) at 1.4 GHz, and Wang et al (9) at 5 GHz, for the moisture contents in these reports were given both in percent by dry weight and by volume basis.

2. The Experimental Data

The measurements of dielectric constants at $\sim$5 GHz were made for four soil samples (9). The measured dielectric constants plotted as a function of the volumetric water content (W_c) are presented in Figure 1. The results of the dielectric measurements carried out at $\sim$1.4 GHz for a number of soil samples by Newton (7) and by Lundien (1) are displayed in Figure 2

and Figure 3 respectively. The textures, types and wilting points of all soils measured at these two frequencies were summarized in Table 1. The wilting point (WP) of soils in percent of dry weight was previously obtained by Schmugge et. al. (10) from a multiple regression of over 100 data sets of soil moisture characteristics. The expression for WP in terms of volumetric water content (cm^3/cm^3) was redetermined by the same procedure and the result is

$$WP = 0.06774 - 0.00064 \times SAND + 0.00478 \times CLAY \quad (1)$$

where CLAY and SAND are the clay and sand contents in percent of dry weight of a soil. The WP for each soil in Table 1 was calculated by the above equation, knowing the soil texture.

Each of the curves in Figures 1, 2, and 3 was obtained from the smoothing of the actually measured data points for a soil sample over the entire moisture range of 0-0.5 cm^3/cm^3. In each of the three figures the soil types could be distinguished by the numbers assigned to the curves in accordance with Table 1. It is noted from Figure 1 that the measured data at 5 GHz for all four soil samples showed a leveling-off of the dielectric constants at high $W_c \gtrsim 0.4$ cm^3/cm^3. The reason for this leveling-off was discussed in detail by Wang et. al. (9) and was most likely due to the leakage of soil water out of the apparatus when W_c was $\gtrsim$ the porosity (P) of the soil sample. No leveling-off was observed in the measured results at 1.4 GHz shown in Figures 2 and 3. The measured data of Lundien were limited to $W_c < P$, while in Newton's measurements the soil-water mixture was placed in a container such that water leakage out of the measurement system was not possible.

The measured data displayed in Figures 1, 2, and 3 clearly indicated two distinct regions in the variations of the soil dielectric constant with W_c. The

first region occurs at $W_c \lesssim W_t$ (transition moisture) where ϵ', the real part of the dielectric constant ϵ, increases slowly with Wc. In the second region at $W_c > W_t$, ϵ' increases steeply with W_c. The transition moisture W_t varies with soil texture, being larger for the clayey soils than for the sandy soils (1, 7, and 9). In the region $W_c \lesssim W_t$, most water molecules are tightly bound to the soil particles. Results of many studies (11, and references therein) on soil-water interactions have strongly suggested that the first few layers of water molecules around the surface of a soil particle form a definite configuration and behave like ice crystals. In such a configuration, it is difficult to polarize these water molecules and the bulk of water shows a smaller dielectric constant than that for the free water. The results of the dielectric measurements on soil-water mixtures by Hoekstra and Delaney (5) also suggested such a phenomenon. In comparing to the dielectric behavior of water in bulk, these authors reported a shift of the dielectric relaxation toward lower frequencies for soil-water mixtures with water contents of 5% and 10% by dry weight. Since the real part of the dielectric constant for water, ϵ'_ω, decreases with frequency near the dielectric relaxation area in the microwave region, ϵ'_ω is expected to be smaller for water in soil than for water in bulk.

By comparing Figures 1, 2, and 3 with Eq. (1), it is evident that W_t and WP are closely related. The WP characterizes a stage of W_c in the soil-water system, as briefly sketched in Figure 4, where the soil tension is about 15 atmospheres. Between WP and field capacity (FC) at soil tension of $\sim$1/3 bar water is held in pores by capillary attraction. For $W_c > FC$, water flows with gravity. At $W_c \lesssim WP$, it is difficult for plants extract water from soil. Water held at temperatures up to 105°C is referred to as hygroscopic water (soil tension of 31 bars and is virtually part of the mineral structure of the soil.

To make a crude estimate on the number of layers of water molecules around the soil particles when $W_c \approx W_t$ ($\approx$WP), the measured results of the H7 sample shown in Figure 1 are taken as an example. The W_t for this sample is estimated from Figure 1 to be $\sim 0.25\ cm^3/cm^3$, which gives 8.4×10^{21} water molecules per cm^3. Assuming the area of each water molecule to be 10^{-19} m^2(12), the total extended area corresponding to W_t is $\sim 8.4 \times 10^2$ m^2 per cm^3. The total surface area of 1 cm of dry H7 soil may be roughly estimated from the report of Rhoades, Raats, and Prather (13) in which the surface areas of four soils with different texture structures were determined. When the WP's for the four soils were calculated from Eq. (1), it was found that soils with larger surface areas were generally associated with higher WP's. Their Domino Clay has a WP of 0.19 which is close to that of H7 soil sample listed in Table 1. The surface area of H7 soil is not likely to be very different from the value of $\sim 78.3\ m^2/g$ determined for the Domino Clay. Since the dry density of H7 soil was measured to be $\sim 1.23\ g/cm^3$, the total surface area per cm^3 is in the order of $\sim 96\ m^2$. Thus, combining with the total extended area of water of $8.4 \times 10^2\ m^2/cm^3$ estimated above, the H7 soil particles are surrounded by ~8-9 layers of water molecules when $W_c = W_t \simeq 0.25\ cm^3/cm^3$. This estimate is close to or somewhat smaller than the thickness of the adsorbed ice-like water determined by many authors (2 and references therein).

A comparison between Figure 2 and Figure 3 reveals that the W_t's of the soil samples measured by Newton (7) appear to be generally larger than those measured by Lundien (1), although both measurements were carried out at about the same frequency of 1.4 GHz. The measurements of Lundien were made with a L band interferometer. On the other hand, the technique employed by Newton (14) was similar to the infinite transition line method used in the 5 GHz measurements (9). The soil samples used in both of these measurements covered a wide

texture range. However, the soil samples used in the Newton's measurements generally possess a slightly larger WP range than those used in Lundien's measurements as indicated in Table 1. These slight differences in the WP's are not likely to be the main cause for the observed differences in W_t's between the two measurements. The two different measurement techniques perhaps are responsible for the observed differences in the experimental results. Fortunately, this difficulty in the experimental data does not present a serious problem to the main objective of this paper.

3. The Mixing Formulas

Many mixing formulas have been reported in the literature (15, 16, 17, 18, 19, 20, 21, 22, and 23) since the early work of Rayleigh (24). Some of these formulas were examined with the measured data on the mixtures of conducting particles in a pure dielectric host (25). Poe, Stogryn and Edgerton (26) also tested a few of these formulas with some measured data of soil permittivity-moisture variations. However, the data used by Poe et. al. were measured only at a few moisture values. As a result, it was difficult to assess whether any of the mixing formulas they examined had provided an adequate description on the dielectric behavior of soil-water mixtures. In recent years, extensive measurement on the dielectric properties of soils have been carried out at many frequencies and over the wide moisture range of 0-46% by dry weight (7, 9, and 27). It is appropriate now to re-examine those mixing formulas and find out if any of them will properly describe the variations of the dielectric constants with moisture contents for various soils.

Table 2 gives a list of the mixing formulas considered to be adequate for a comparison with the experimental data acquired in recent years. The majority

of these formulas dealt only with a mixture of two constituents. They basically implied a direct dependence of the mixture dielectric constant ϵ on the dielectric constants (ϵ_1 and ϵ_2) and the volume fractions (f_1 and f_2) of the constituents. The formulas of both Wiener and Pearce et. al. contain a free parameter which could be adjusted to fit the experimental data. Figure 5 showed the families of curves generated by these two formulas with several different values for the free parameters. The variations of ϵ with moisture content as described by the remaining eight expressions in Table 2 were shown in Figure 6. In both figures the real and imaginary parts of ϵ_1 for a dry soil were taken to be 3.0 and 0.01 respectively, which were close to the typical values measured at 1.4 GHz. The real and imaginary parts of ϵ_2 for pure water at 1.4 FHz were calculated to be 79.5 and 6.63 from a set of formulas (T. T. Wilheit, personal communication) resulting from the curve fitting to the data of Lane and Saxton (28). The measured dielectric constants at 1.4 GHz as a function of the volumetric water content for Yuma Sand and Vernon Clay (1) and Miller Clay (7) were also included in both figures for comparison. Only the real parts ϵ' of both the measured and the computed dielectric constants appear in the figures. The reasons for not comparing the measured and the computed values for the imaginary parts ϵ'' of the dielectric constants are two fold. First, the measured variations of ϵ'' with moisture content are small over the entire moisture range of 0-0.6 cm^3/cm^3. The sensitivity of the measured data sets is not good enough to distinguish one mixing formula from the other. Secondly, the ionic conductivity might contribute significantly to the values of ϵ'' for some soil-water mixtures (9). This tends to complicate the interpretation of the measured data even further as will be discussed in the next section. The omission of the comparison between the measured and computed ϵ'' does not change the outcome of the discussion below.

It is clear from Figure 5 that both formulas of Wiener and Pearce et al. do not give a good fit to the measured data. Note for example that the value for the crosses starts out below the calculated curves at low soil moisture and goes above the curves at high moisture content. Both formulas produces larger slope at moisture content $W_c \lesssim$ transition moisture Wt and smaller slope at $W_c \geq W_t$ than the measured data showed. Thus, it is not possible to adjust the free parameters A and B to obtain a good fit over the entire moisture range 0-0.5 cm^3/cm^3. This general remark applied to any of the measured data curves presented in Figures 2 and 3. The same difficulty arises when the comparison is made between the computed and the measured ϵ' at 5 GHz.

Similarly, the calculated variations of ϵ' with W_c from any one of the eight remaining mixing formulas do not give a good description to the measured results as shown in Figure 6. The calculated ϵ' from Botcher's formula matches that measured ϵ' of Miller Clay (Newton, 1977) quite well, but there is no free parameter to adjust for the other soil types. The Brown's formula gives a ϵ' versus moisture curve with a steep slope over the entire moisture range of interest. The steepness of the slope, with some offset constant, fits nicely with the observed rapid increase of ϵ' with W_c over the region $W_c > W_t$. However, the difficulty arises regarding the portion $W_c \lesssim W_t$. The formula Birchak et al. gives a ϵ'-moisture curve generally close to the measured data points but does not produce good fit over the entire moisture range for a given soil. The remaining formulas of Rayleigh, Bruggeman, Kharadly and Jackson, Nerpin and Chudnovskii, and Wagner are clearly not applicable in the soil-water mixture where $W_c \gtrsim 0.5$ cm^3/cm^3.

To generate a mixing formula which gives a reasonably good fit to the data shown in Figures 1, 2, and 3 for various soil types and over the entire moisture range of 0-0.5 cm^3/cm^3, it is necessary to take into account the reduced dielectric behavior of the initially adsorbed water. Since the evidence given in the

previous section suggests a similarity between the first few layers of the adsorbed water molecules around the soil particles and the ice crystals, the dielectric constant of ice may be used to describe the dielectric behavior of the soil-water mixture at $W_c \lesssim W_t$. Two empirical approaches were proposed in the following. The first approach deals with the direct mixing of the dielectric constants of the constituents. The expressions for the complex dielectric constants ϵ of a soil-water mixture are given by

$$\epsilon = W_c \epsilon_x + (P - W_c)\epsilon_a + (1 - P)\epsilon_r, W_c \lesssim W_t \tag{2}$$

with

$$\epsilon_x = \epsilon_i + (\epsilon_w - \epsilon_i)\frac{W_c}{W_t} \cdot \gamma \tag{3}$$

and

$$\epsilon = W_t \epsilon_x + (W_c - W_t)\epsilon_w + (P - W_c)\epsilon_a + (1 - P)\epsilon_r, W_c > W_t \tag{4}$$

with

$$\epsilon_x = \epsilon_i + (\epsilon_w - \epsilon_i)\gamma \tag{5}$$

Here P is the pososity of the dry soil. ϵ_a, ϵ_w, ϵ_r and ϵ_i, in sequential order, are the dielectric constants of air, water, rock and ice. ϵ_x stands for the dielectric constant of the initially adsorbed water. γ is a parameter which can be chosen to best fit equations (2) and (4) to the experimental data.

In the second approach, the mixing is done using the indices of refraction of water, ice, air and rock. The expressions for the mixing are in the same forms as Equations (2), (3), (4) and (5), with ϵ, ϵ_a, ϵ_w, ϵ_i, and ϵ_r replaced by the corresponding refractive indices. The numerical values for γ were found to be different between the two approaches, but those for W_t were generally very close for a given soil. Both approaches give a reasonably good fit to the experimental data and are essentially equivalent for practical purposes. Thus, only the first approach will be discussed in detail in the following sections.

4. Results

Before equations (2) through (5) can be used for calculations of ϵ and compared with the measured data, the soil porosity and the dielectric constants of all the constituents in the soil-water mixture have to be determined. The Porosity P of a dry soil is defined by (Brady, 1974)

$$P = 1 - \frac{\rho_s}{\rho_r} \tag{6}$$

where ρ_S is the density of the dry soil and ρ_r is the density of the associated solid rock. For the soil samples used in the dielectric measurements at 5 GHz and 1.4 GHz ρ_S lies in the range of 1.1-1.7 g/cm^3, while ρ_r varies between 2.6 and 2.75 g/cm^3(29), and for simplicity is assumed to be $\sim$2.65 g/cm^3. Entering the average value of ρ_S for the soil samples in Eq. (6) gives $P \simeq 0.5$, which will be used exclusively in this paper. It can be shown that with either P = 0.4 or P = 0.6, the calculated dielectric constants of a soil-water mixture differ only slightly from the ones with P = 0.5. The dielectric constants of ice, ϵ_i' and ϵ_i'', are assumed to be $\sim$3.2 and 0.1 respectively, and are frequency independent at frequencies $\gtrsim$1 GHz (30). ϵ_a' and ϵ_a'' of air are taken to be 1 and 0. ϵ_r' and ϵ_r'' of a solid rock vary (31), but the respective values of 5.5 and 0.2 fits well with the experimental value of the dry soils. For water ϵ_w' and ϵ_w'' change with frequency in the microwave region (5). They are calculated separately at 1.4 GHz and 5 GHz from the method mentioned in the previous section. Both ϵ_w' and ϵ_w'' at each frequency are calculated for a number of salinity values (S) in order to best fit the calculated ϵ'' to the measured data for various soils.

With the values of the dielectric constants of each constituent given, the real and imaginary parts of ϵ of a soil-water mixture were computed as a

function of moisture content Wc from Eqs. (2) through (4). The parameters γ and W_t were varied to obtain the best fit between the computed and the measured dielectric constants of the mixture. The values of W_t vary according to soil texture. The parameter γ is expected to be independent of soil type. It was found that, for all the experimental data shown in Figures 1, 2, and 3, $\gamma = 0.2$ ($\gamma = 0.9$ in the refractive index approach). Comparisons of some of the observed and computed mixture dielectric constants are shown in Figure 7 for the frequency of 1.4 GHz and in Figure 8 for the frequency of 5 GHz.

The experimental data in Figure 7 are the same as those shown in Figures 4 and 5, with the addition of the data for ϵ'', the imaginary part of the observed dielectric constant. Both ϵ' and ϵ'' for Yuma Sand in Figure 7 were found to compare reasonably well with the computed ϵ' and ϵ'' assuming $W_t = 0.09$ and S = 0. Similarly, the data for Vernon Clay could be fitted by the computed values using $W_t = 0.18$ cm^3/cm^3 with S = 7 PPT. The observed data for the heavier Miller Clay required a $W_t = 0.30$ cm^3/cm^3 and S = 10 PPT. Clearly, at the frequency of 1.4 GHz, the empirical formulas of Eqs. (2), (3), (4), and (5) give a much better description of the observed variations of dielectric constants with water content than the existing mixing formulas listed in Table 2.

A reasonable agreement at the frequency of 5 GHz was also obtained between the observed and computed dielectric constants of soil-water mixtures. The data points shown in Figure 8 were the measured results for Harlingen Clay, soil samples F2 and H7. The three pairs of curves were calculated from Eqs. (2), (3), (4), and (5) with different values for W_t and S; $W_t = 0.15$ cm^3/cm^3 and S = 0 PPT

for the solid curves, $W_t = 0.23$ cm^3/cm^3 and S = 3 ppT for the dashed curves, and $W_t = 0.27$ cm^3/cm^3 and S = 0 ppT for the dotted curves. The measured and the calculated values of ϵ' for each of three soils generally agreed well over the moisture range from 0 to 0.3-0.4 cm^3/cm^3 where the leveling-off of the measured ϵ' took place. The calculated and the measured values of ϵ'' for Harlingen Clay and soil sample H7 were found to be in agreement over the same moisture range. However, for soil sample F2, the calculated ϵ'' was generally higher than the measured ϵ'' at moisture content $\gtrsim 0.2$ cm^3/cm^3.

The measured dielectric constants of all the remaining soil samples were compared with the calculated values from Eqs. (2) through (5) in the same way. The values of W_t and S for each soil, which gave the best fit to the measured data, were entered in Table 1 for comparison. Two distinct features are found from an examination of the table. First, W_t is observed to vary with soil types, being larger for clayey soils than sandy soils. The data definitely show a positive correlation between W_t and WP of the soils. Next, the salinity required to give the best fit between the observed and the computed dielectric constants varies from one soil to another. Generally, clayey soils require higher salinity than the sandy soils. This result is further substantiated by the analysis of various soils showing that clay soils contain more phosphorus, potassium, and calcium than sandy soils (29). The experimental results of Rhoades, Raats, and Prather (13) also suggest more exchangeable ions for the clayey soils than the sandy soils.

To show the correlation between the transition moisture and wilting point more clearly, the values of W_t in Table 1 were plotted as a function of WP in Figure 9. A 1:1 line was drawn in the plot for comparison purpose. It is noted

that the majority of data points at WP < 0.3 cm^3/cm^3 fall above the 1:1 line in the figure. This suggests that the transition moisture as determined from the measured dielectric property is slightly larger than the wilting point moisture of a soil defined at the 15-bar moisture tension. Two additional features are also observed from this figure.

First, W_t determined from Newton's (7) data are found to be slightly higher than those determined from the other two sources (1,9). This again suggests a possible small discrepancy between the two sets of measurements at 1.4 GHz. Secondly, the W_t determined at 5 GHz are comparable to those determined at 1.4 GHz. Within the precision of the measurements, the data displayed in the figure are consistent with frequency independence of W_t. Measurements at frequencies other than the two reported here are clearly needed in order to study a possible frequency dependence of transition moisture.

Applying a linear regression analysis to the data in Figure 9 gives

$$W_t = 0.09 + 0.59 \times WP \qquad (7)$$

with a correlation coefficient of 0.94. The refractive index approach also gives the similar dependence of W_t on WP displayed in Figure 9. In both approaches, the high correlation coefficient suggests a close relationship between the transition moisture and the wilting point of soils.

5. Discussion

It was shown in the previous section that a simple model based on the straight forward combination of the dielectric constants of ice, water, air and rock was capable of describing the observed dielectric behavior of a soil-water mixture at both 1.4 GHz and 5 GHz frequencies. The fit to the experimental data by the model presented in this paper was far better than that obtained using the mixing

formulas listed in Table 2. It is emphasized that most of the mixing formulas in Table 2 were not derived specifically for describing the dielectric property of a soil-water mixture over a wide moisture range of 0-0.5 cm^3/cm^3. The ice-like behavior of the first adsorbed water molecules plays an important role in the observed variations of the soil dielectric constants with water content. The extent of the ice-like configuration of the initially adsorbed water could be approximately estimated by W_t knowing the soil texture structure.

The empirical model discussed in this paper provides a means to estimate a soil's dielectric properties from a knowledge of its texture. This relationship is important for estimating the brightness temperature T_B of soils in a general situation, since dielectric constants have been measured for only a limited number of cases. An example of the dependence of T_B on soil texture is given below.

Consider a uniform layer of soil with known moisture characteristics from which the wilting point of the soil can be derived. The soil layer is further assumed to be characterized by a uniform temperature of 300°K from the surface down to 1 m below. For simplicity, only the constant moisture profiles with values W_c ranging from 0.05 cm^3/cm^3 to 0.40 cm^3/cm^3 were considered. At the frequency of 1.4 GHz and the above moisture values, the contribution of the upwelling emission from the medium below 1 m is negligibly small. Under these conditions, the brightness temperatures were calculated by a plane stratified dielectric model (32) with ϵ' and ϵ'' given by eqs. (2) through (5). The results are shown in Figure 10 where the brightness temperature is plotted as a function of W_t for each moisture content indicated in the figure. The same values of the dielectric constants for air, rock, and ice given in the previous

section were used. The dielectric constant for fresh water at 1.4 GHz with zero salinity value was used in the computation. It can be shown that, by using moderate values for salinity, the resultant brightness temperature is not very different from that calculated using fresh water values. For example, with S = 10 PPT, the calculations resulted in a brightness temperature of 220.5°K at W_c = 0.40 cm^3/cm^3 and W_t = 0.05 cm^3/cm^3, which is only ~1°K smaller than the corresponding fresh water brightness temperatures shown in Figure 10. This small difference was also obtained at other values of W_t, but at the same W_c. For smaller values of W_c, the difference between the two temperatures was found to be smaller. This reduction in the temperature difference due to the changes in salinity at low W_c is expected from the observed variations of ϵ'' with W_c in Figures 2 and 3.

Figure 10 demonstrates the improvements in the estimation of the brightness temperature when the soil texture is known. Without the soil texture information, W_t can assume any value between ~0.08 cm^3/cm^3 for pure sand and ~0.34 cm^3/cm^3 for pure clay (see Figure 9). The corresponding uncertainty in the estimate of the brightness temperature at W_c = 0.40 cm^3/cm^3 could be as large as 30°K. With known soil texture information, the uncertainty in the brightness temperature estimate could be reduced. For example, for a soil with a wilting point moisture ~0.10 cm^3/cm^3 W_t is found from Figure 9 to be 0.14 cm^3/cm^3 with a probable error in the estimate of ~0.04 cm^3/cm^3. The corresponding brightness temperature from Figure 10 is ~223°K with an uncertainty of ~3°K for the soil at W_c = 0.40 cm^3/cm^3. An uncertainty of 3°K translates into a probable error of ~0.02 cm^3/cm^3 in the moisture estimate. Thus a better estimate in the moisture conetnt is possible when the empirical model is used.

The discussion above applies equally well when the dielectric constants of the soil-water mixture are computed by the refractive index approach. The uncertainties in the soil moisture estimate from both models can be shown to be comparable.

6. Conclusions

The available mixing formulas were examined with respect to the measured variations of soil dielectric constants with moisture content. It was shown that these formulas failed to account for the observed dielectric properties of various soil-water mixtures. The main reason for this failure was believed to be due to the ice-like behavior of the initially adsorbed water molecules. A simple empirical model was proposed to describe the observed dielectric constants of soil-water mixtures. In this model, the dielectric constant of a soil water mixture was computed from the known dielectric constants of air, ice, soil, and water, and the volume fraction of each constituent in the mixture. Two approaches were attempted for the model. The first approach deals with the direct mixing of the dielectric constants, while the second approach, with the mixing of the refractive indices of all constituents. Both approaches use the transition moisture as a parameter to fit the computed dielectric constants to the observed values for various soils. The major conclusions resulted from this study are:

- The empirical model gives a better description of the dielectric behavior of various soil-water mixtures than the previously existing mixing formulas. The main reason for the success of the model is that it takes into account the ice-like property of the initially adsorbed water molecules.

- The transition moistures are found to correlate positively with the wilting points of the soils. The correlation coefficient is found to be ~ 0.9. The dependence of the transition moisture on the wilting point is about the same at 1.4 GHz and 5 GHz.
- For the soils studied in this paper, the transition moistures are found to be generally higher than the wilting point moistures defined at the tension of 15 bars.
- For some soils the imaginary parts of the measured dielectric constants require the ionic conductivity term. The salinity values needed to fit the measured data are generally higher for the high clay content soils than for the sandy soils.
- The uncertainty in the brightness temperature estimate of a soil-water mixture is reduced appreciably when the soil texture effect is included. A typical numerical example given in this paper showed that the precision in the brightness temperature estimate could be improved by nearly 7 times, taking into account the soil texture effect.

References

[1] J. R. Lundien, "Terrain Analysis by Electromagnetic Means," Technical Report 3-727, U. S. Army Engineer Waterways Experiment Station, Vicksburg, Miss., February, 1971.

[2] M. L. Wiebe, "Laboratory Measurements of the Complex Dielectric Constant of Soils," Technical Report RSC-23, Texas A&M University, College Station, October 1971.

[3] Y. I. Leschanskiy, G. N. Lebedeva, and V. D. Schumilm, "Electrical Parameters of Sandy and Loamy Soils in the Range of Centimeter, Decimeter and Meter Wavelength," Izvestiya Vysschich Uchebrich Zavedenii, Radiofizika, Vol. 14 (4), pp 562-569, 1971.

[4] F. E. Geiger and D. Williams, "Dielectric Constants of Soils at Microwave Frequencies," NASA TMS-65987, April, 1972.

[5] P. Hoekstra and A. Delaney, "Dielectric Properties of Soils at UHF and Microwave Frequencies," J. Geophys. Res., Vol. 79, pp 1699-1708, 1974.

[6] J. E. Hipp, "Soil Electromagnetic Parameters as a Function of Frequency, Soil Density and Soil Moisture," Proc. IEEE, Vol. 62 (1), pp. 98-103, 1974.

[7] R. W. Newton, "Microwave Remote Sensing and its Application to Soil Moisture Detection," Technical Report RSC-81, Texas A&M University, College Station, Texas, January, 1977.

[8] E. G. Njoku, and J. A. Kong, "Theory for Passive Microwave Remote Sensing of Near-Surface Soil Moisture," J. Geophys. Res., Vol. 82 (20), pp. 3108-3118, 1977.

[9] J. Wang, T. Schmugge, and D. Williams, "Dielectric Constants of Soils at Microwave Frequencies – II," NASA Technical Paper 1238, May 1978.

[10] T. Schmugge, T. Wilheit, W. Webster, Jr., and P. Gloersen, "Remote Sensing of Soil Moisture with Microwave Radiometers – II," NASA TN D-8321, September, 1976.

[11] R. E. Grim, Clay Mineralogy, New York: McGraw-Hill Book Company, 1968

[12] S. Iwata, "Thermodynamics of Soil Water: IV. Chemical Potential of Soil Water," Soil Science, Vol. 117 (3), pp. 135-139, 1974.

[13] J. D. Rhoades, P. A. C. Raats, and R. J. Prather, "Effects of Liquid-Phase Electrical Conductivity, Water Content, and Surface Conductivity on Bulk Soil Electrical Conductivity," Soil Sci. Soc. Am. J., Vol. 40, pp. 651-655, 1976.

[14] R. E. Newton, and W. R. McClellan, "Permittivity Measurements of Soils at L-Band," Technical Report RSC-58, Texas A&M University, College Station, Texas, June 1975.

[15] C. J. F. Böttcher, Theory of Electric Polarization, Elsevier, Amsterdam, 1952.

[16] O. Wiener, "Leipziger Berichte," Vol. 62, p. 256, 1910.

[17] C. A. R. Pearce, "The Permittivity of Two-Phase Mixtures," Brit. J. Appl. Phys. Vol. 6, pp. 358-361, October 1955.

[18] W. F. Brown, "Dielectrics," in Encyclopedia of Physics, 17, Berlin: Springer, 1956.

[19] J. R. Birchak, C. G. Gardner, J. E. Hipp, and J. M. Victor, "High Dielectric Constant Microwave Probes for Sensing Soil Moisture," Proc. IEEE, Vol. 62 (1), January 1974, pp. 93-98.

[20] K. W. Wagner, Arch. Elektrochem., Vol. 3, 1914, p. 100.

[21] D. A. G. Bruggeman, "Berechnung Verschiedener Physikallischer Konstanten von Heterogenen Substanzen," Ann. Phys. (Leipzig), Vol. 24, 1935, pp. 636-664.

[22] M. M. Z. Khardly and W. Jackson, "Properties of Artificial Dielectrics Comprising Arrays of Conducting Elements," Proc. Inst. Elec. Eng., Vol. 100, 1953, pp. 199-212.

[23] S. V. Nerpin and A. F. Chudnovskii, Physics of the Soil, Israel Program for Scientific Translations, Jerusalem, 1970, p. 317.

[24] Lord Rayleigh, "On the Influences of Obstacles Arranged in Rectangular Order on the Properties of a Medium," Phil. Mag., Vol. 34, 1892, pp. 481-502.

[25] W. S. Rothwell, "Complex Permittivity of Conductor-Dielectric Mixtures," IEEE Trans. Microwave Theory Tech., Vol. MTT-19, April 1971, pp. 413-414.

[26] G. Poe, A. Stogryn, and A. T. Edgerton, "Determination of Soil Moisture Content.

[27] J. Cihlar, and F. T. Ulaby, "Dielectric Properties of Soils as a Function of Moisture Content," RSL Technical Report 177-47, Univ. Kansas, Lawrence, November 1974.

[28] J. A. Lane, and J. A. Saxton, "Dielectric Dispersion in Pure Polar Liquids at Very High Radio-Frequencies, Measurements on Water, Methyl and Ethyl Alcohols," Proc. Roy. Soc., Vol. A213, 1952, pp. 400-408.

[29] N. C. Brady, The Nature and Properties of Soils, MacMillan Publishing Co., Inc., New York, 1974.

[30] S. Evans, "Dielectric Properties of Ice and Snow – A Review," J. Glaciology, Vol. 5 (42), 1965, pp. 773-792.

[31] M. J. Campbell and J. Ulrichs, "Electrical Properties of Rocks and Their Significance for Lunar Radar Observations," J. Geophys. Res., Vol. 74 (25), November 1969, pp. 5867-5881.

[32] T. T. Wilheit, "Radiative Transfer in a Plane Stratified Dielectric, "IEEE Trans. Geosci. Electronics, Vol. GE-16 (2), April 1978, pp. 138-143.

Table 1
Different Types of Soils Used for the Dielectric Measurements as a Function of Water Content at 1.4 GHz and 5 GHz

No.	Soil Type	Texture (%)			Wilting* Point, WP cm^3/cm^3	Transition' Moisture, Wt cm^3/cm^3	Salinity,‡ S PPT	Remarks
		Sand	Silt	Clay				
1	M5	88.0	7.3	4.7	0.034	0.110	0	Measurement Frequency = 5 GHz (Wang et. al., 1978)
2	F2	56.0	26.7	17.3	0.115	0.150	0	
3	H7	19.3	46.0	34.7	0.220	0.230	3	
4	Harlingen Clay	2.0	37.0	61.0	0.358	0.270	0	
5	Yuma Sand	100.0	0	0	0.004	0.090	0	Measurement Frequency = 1.412 GHz (Lundien, 1971)
2	Eufaula Fine Sand	90.0	7.0	3.0	0.024	0.100	0	
3	Dougherty Fine Sand	82.0	14.0	4.0	0.034	0.100	0	
4	Minco Very Fine Sand	70.0	22.0	8.0	0.051	0.100	0	
5	Openwood Street Silt	22.0	70.0	8.0	0.092	0.150	2	
6	Chickasha Loam	58.0	28.0	14.0	0.098	0.150	2	
7	Zaneis Loam	48.0	36.0	16.0	0.114	0.160	2	
8	Collinville Loam	45.0	39.0	16.0	0.115	0.170	2	
9	Kirkland Silt Loam	26.0	56.0	18.0	0.137	0.150	2	
10	Tabler Silt Loam	22.0	56.0	22.0	0.159	0.140	2	
11	Vernon Clay Loam	16.0	56.0	28.0	0.192	0.180	7	
12	Long Lake Clay	6.0	54.0	40.0	0.255	0.220	9	
1	Sand	86.0	7.0	7.0	0.046	0.14	0	Measurement Frequency = 1.4 GHz (Newton, 1977)
2	Samples 4 and 5	40.0	26.0	34.0	0.205	0.26	9	
3	Samples 7 and 18	36.0	29.0	35.0	0.212	0.23	6	
4	Samples 14 and 15	52.0	9.0	39.0	0.221	0.26	8	
5	Sample 13	44.0	12.0	44.0	0.250	0.27	9	
6	Miller Clay	3.0	35.0	62.0	0.361	0.30	10	

*Computed from Eq. (1)
'Determined from Eqs. (2) and (3)
‡Determined from Eqs. (2) and (3)

Table 2
The Mixing Formulas Used for Comparison with the Measured Dielectric Constants of Soil-Water Mixtures

Source	Formula	Remarks
1. Rayleigh [24]	$\frac{\epsilon - 1}{\epsilon + 2} = f_0 \frac{\epsilon_0 - 1}{\epsilon_0 + 2} + f_1 \frac{\epsilon_1 - 1}{\epsilon_1 + 2}$	$f_0 + f_1 = 1$
2. Böttcher [15]	$\frac{\epsilon - \epsilon_1}{3\epsilon} = f_0 \frac{\epsilon_0 - \epsilon_1}{\epsilon_0 + 2\epsilon}$	
3. Brown [18]	$\epsilon = f_0\epsilon_0 + f_1\epsilon_1$	$f_0 + f_1 = 1$
4. Birchak et. al. [19]	$\sqrt{\epsilon} = f_0 \sqrt{\epsilon_0} + f_1 \sqrt{\epsilon_1}$	$f_0 + f_1 = 1$
5. Bruggeman [21]	$\frac{\epsilon}{\epsilon_1}(1 - f_0)^3 = \left(\frac{\epsilon_0 - \epsilon}{\epsilon_0 - \epsilon_1}\right)^3$	
6. Wagner [20]	$\frac{\epsilon - \epsilon_1}{3\epsilon_1} = f_0 \frac{\epsilon_0 - \epsilon_1}{\epsilon_0 + 2\epsilon_1}$	
7. Kharadly and Jackson [22]	$\frac{\epsilon - \epsilon_1}{\epsilon + 2\epsilon_1} = f_0 \frac{\epsilon_0 - \epsilon_1}{\epsilon_0 + 2\epsilon_1}$	
8. Nerpin and Chudnovskii [23]	$\epsilon = 1 \frac{3\sum_i \frac{\epsilon_i - 1}{\epsilon_i + 2} f_i}{1\sum_i \frac{\epsilon_i - 1}{\epsilon_i + 2} f_i}$	f_i = fractional volume of each constituent
9. Wiener [16]	$\epsilon = \frac{f_0\epsilon_0 U + \epsilon_1 f_1}{f_0 U + f_1}; \quad U = \frac{\epsilon_1 + F}{\epsilon_0 + F}$	$f_0 + f_1 = 1$. F is an adjustable parameter
10. Pearce [17]	$\epsilon = \epsilon_0 + \frac{f_1(1 - F)}{1 - f_1 F}(\epsilon_1 - \epsilon_0)$	$-1 \leq F \leq 1$

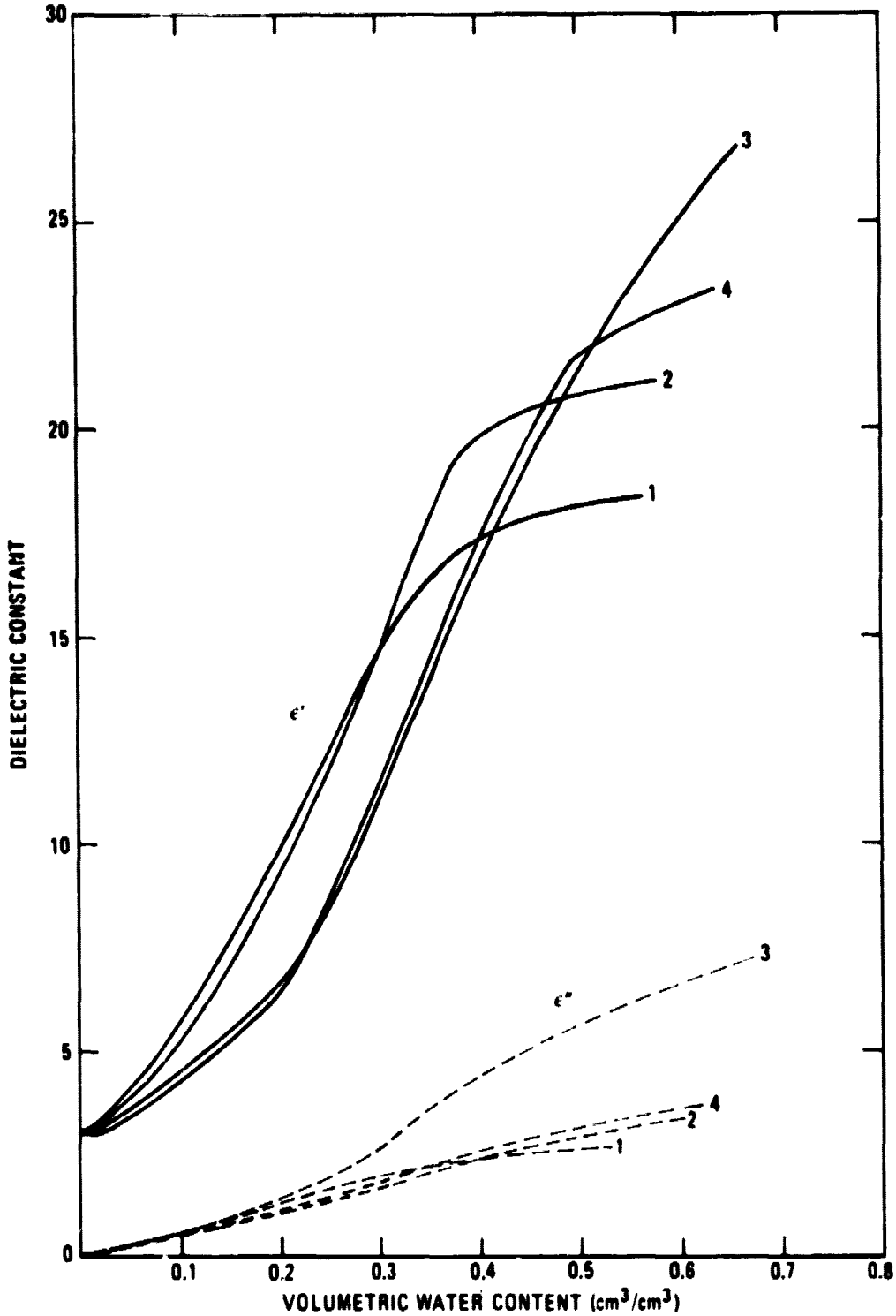

Figure 1. The dielectric constants versus volumetric water content for 4 soils measured at 5 GHz. Soil types were identified by the numbers assigned to the curves in accordance with Table 1

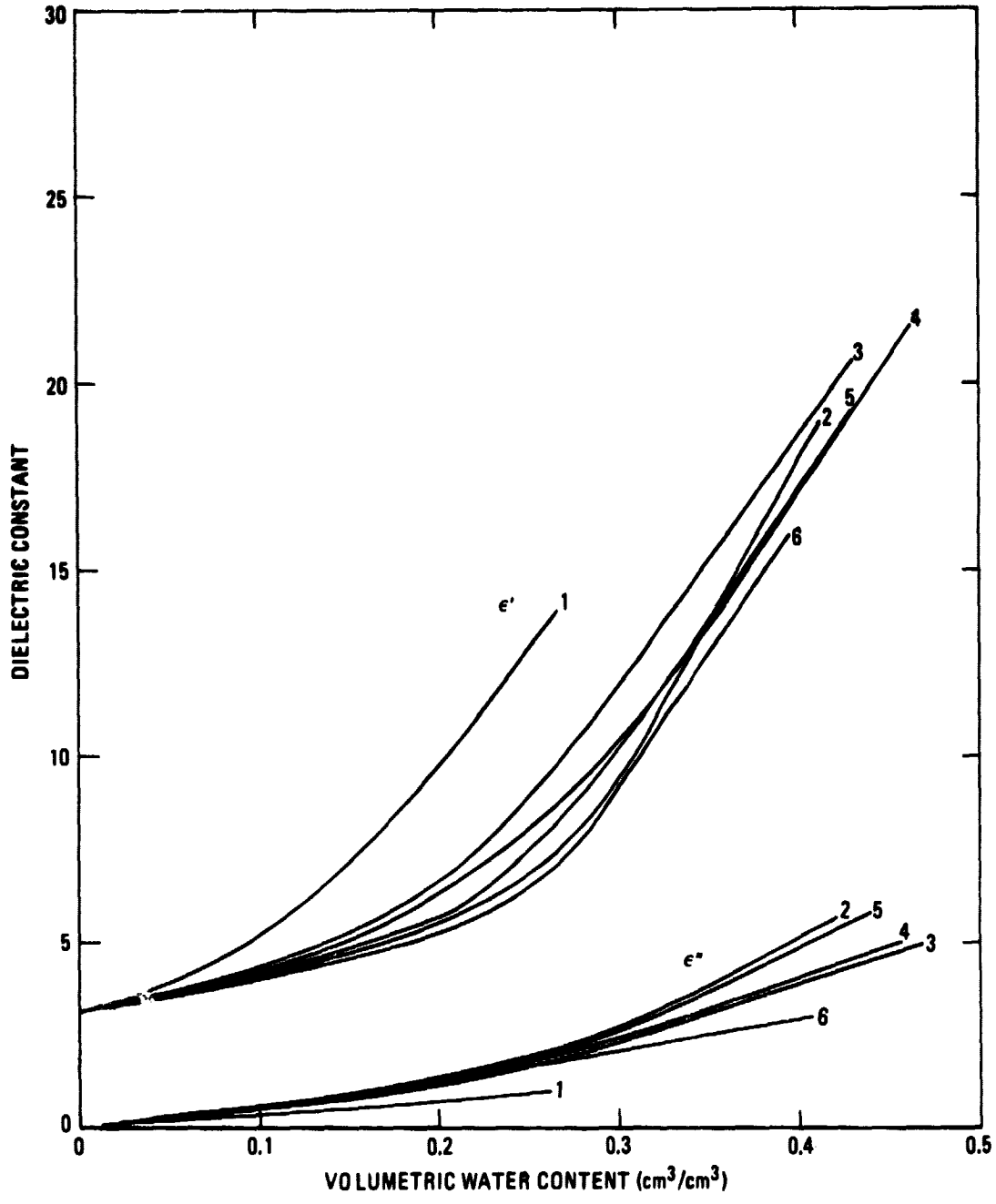

Figure 2. The dielectric constants versus volumetric water content for 6 soils measured at 1.4 GHz. Soil types were identified by the numbers assigned to the curves in accordance with Table 1

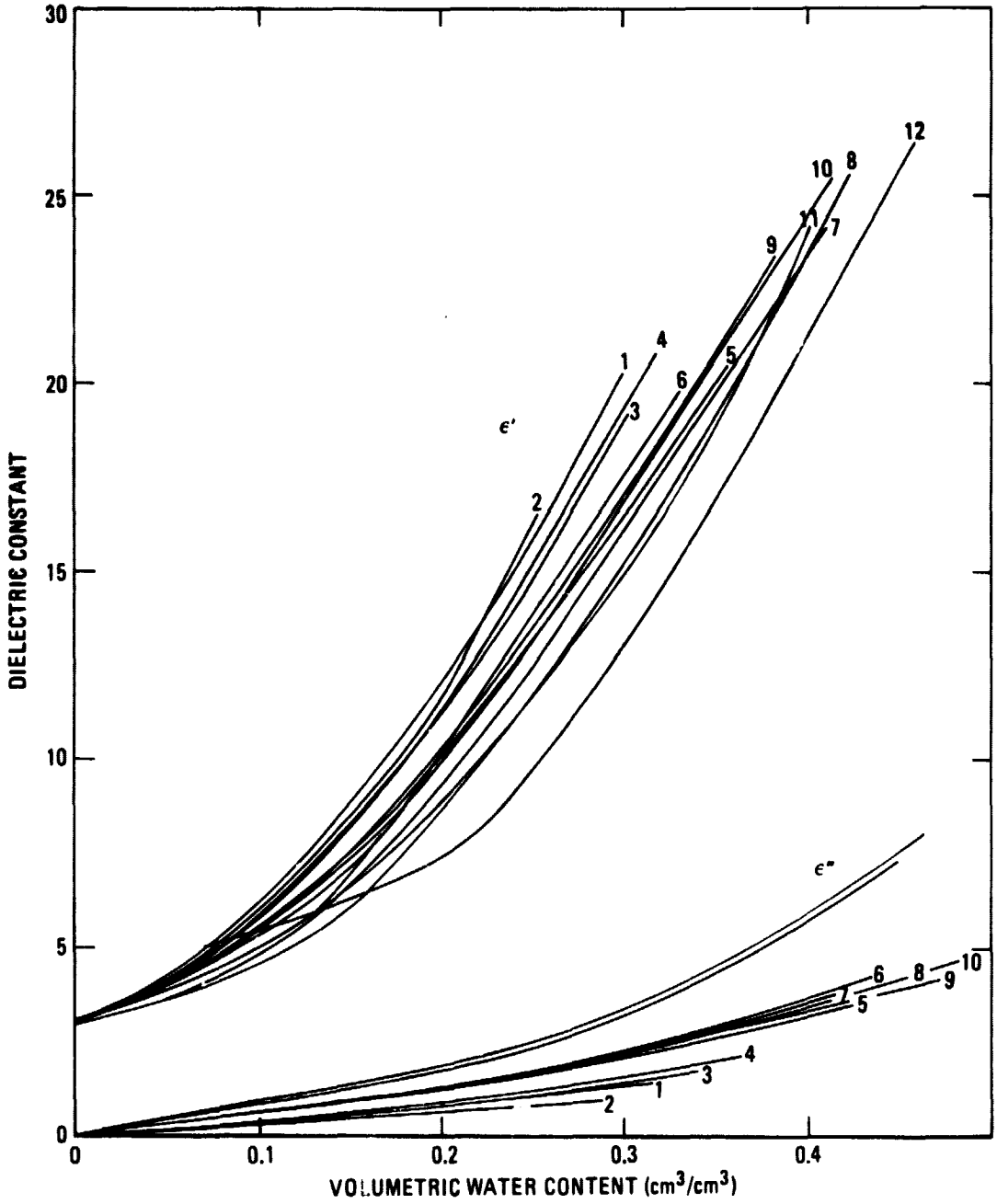

Figure 3. The dielectric constants versus volumetric water content for 12 soils measured at 1.412 GHz. Soil types were identified by the numbers assigned to the curves in accordance with Table 1

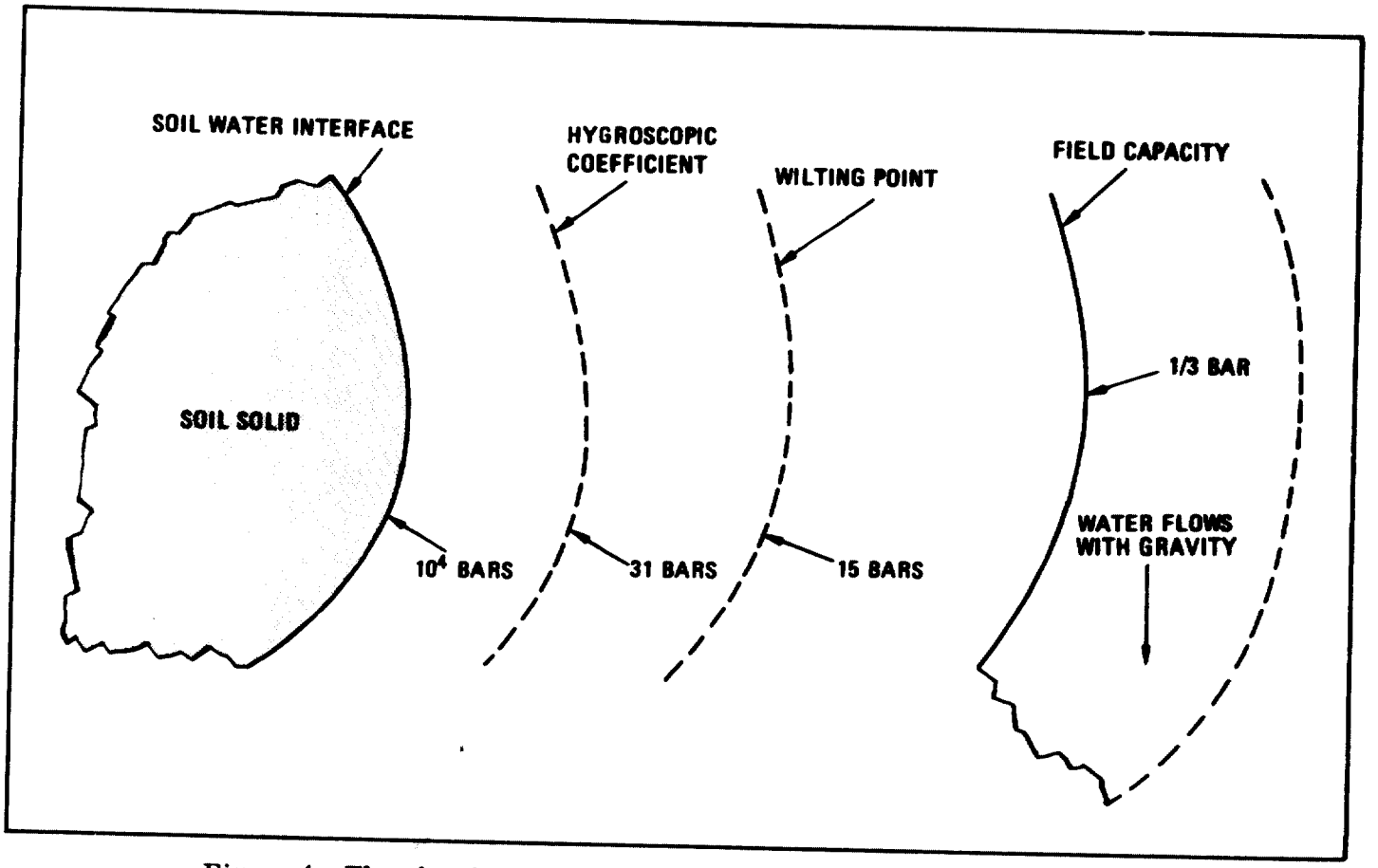

Figure 4. The sketch of a typical soil-water system. The wilting point and field capacity are defined at the tension of 15 bars and 1/3 bar respectively.

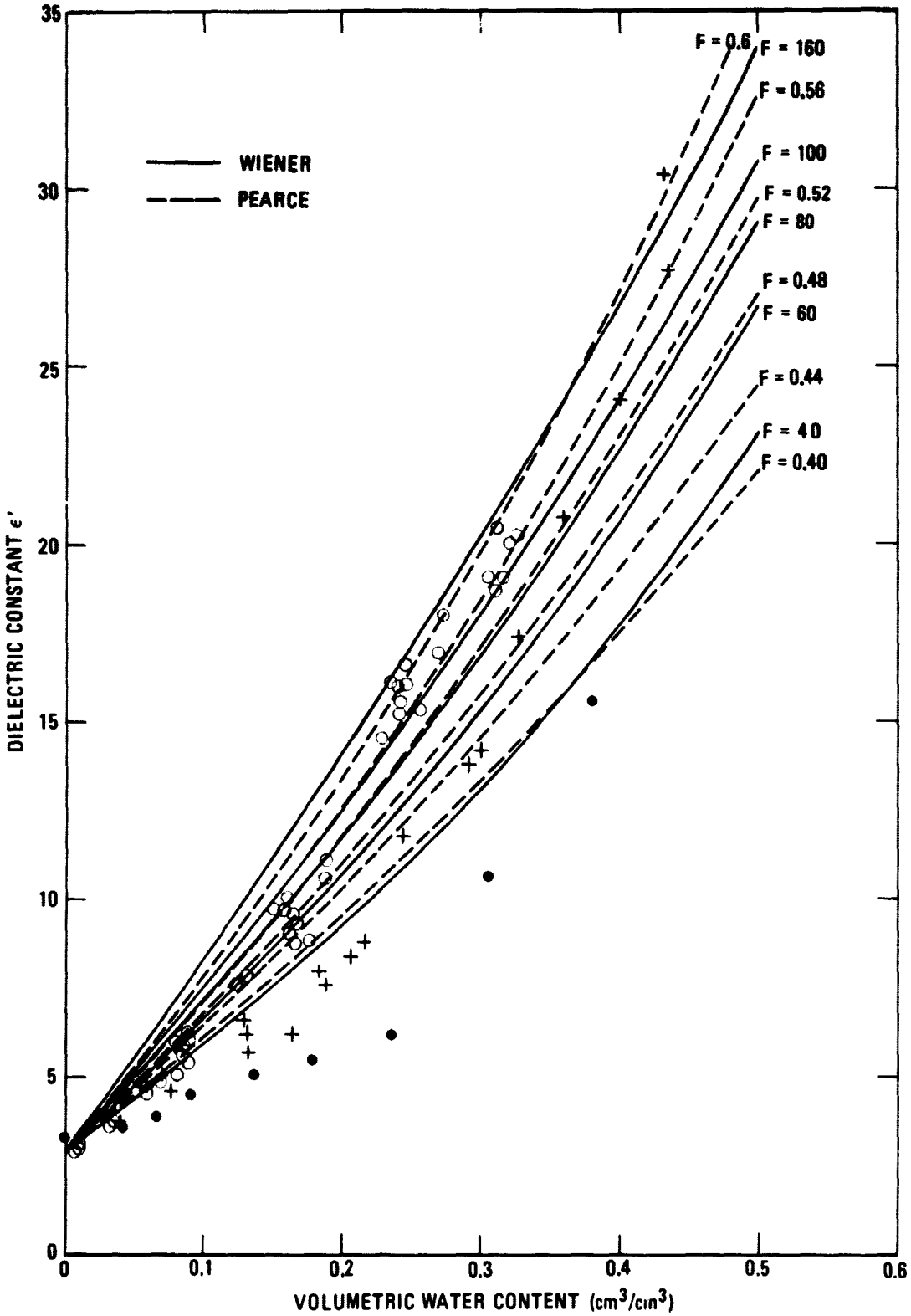

Figure 5. A comparison of the mixing formulas by Wiener and Pearce with the dielectric constants of 3 soils as a function of water content

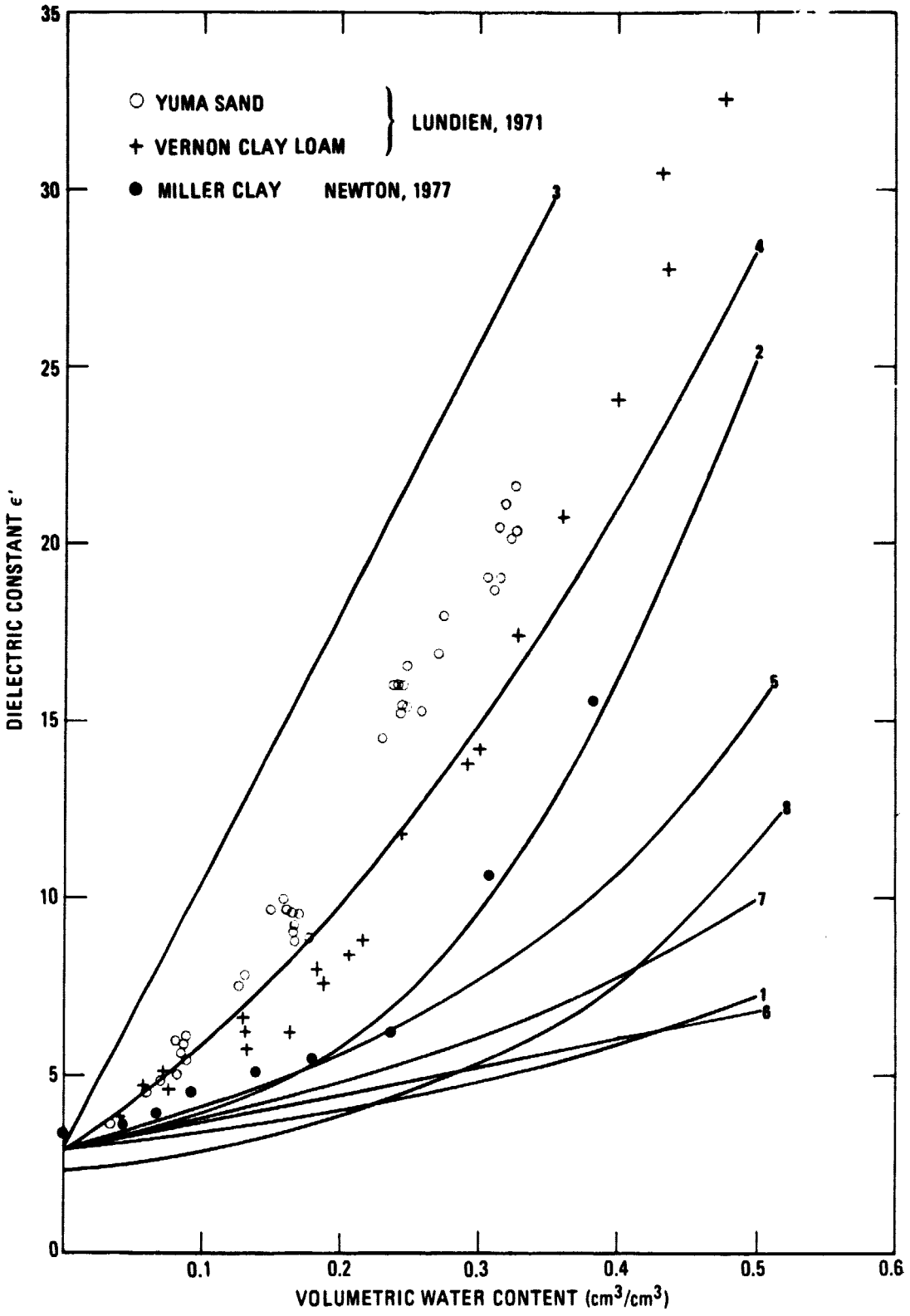

Figure 6. A comparison between some mixing formulas reported in the literature and the measured dielectric constants of 3 soils as a function of water content. Curves are labeled according to Table 2.

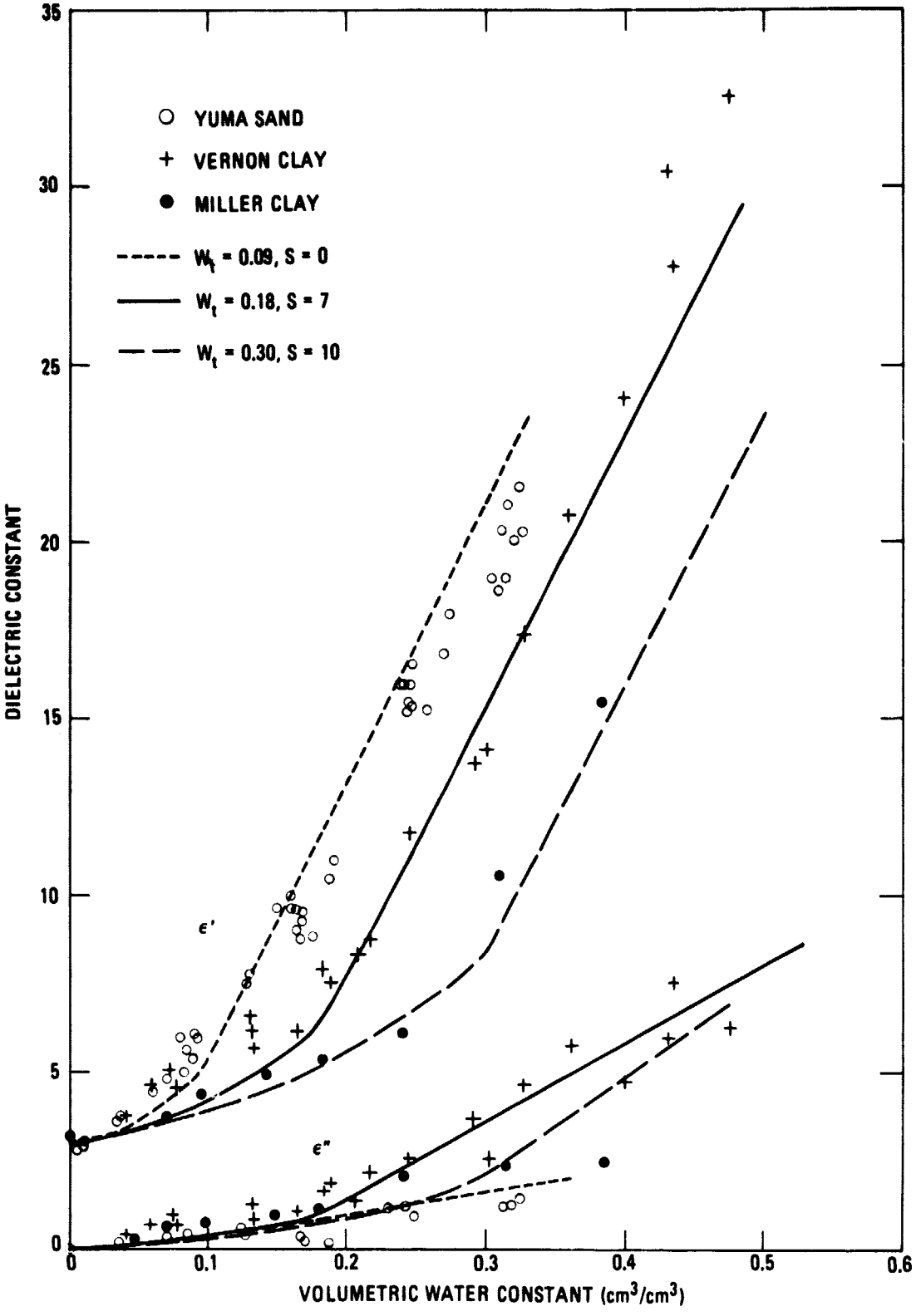

Figure 7. A comparison between the calculated dielectric constants from the empirical model and the measured values at 1.4 GHz

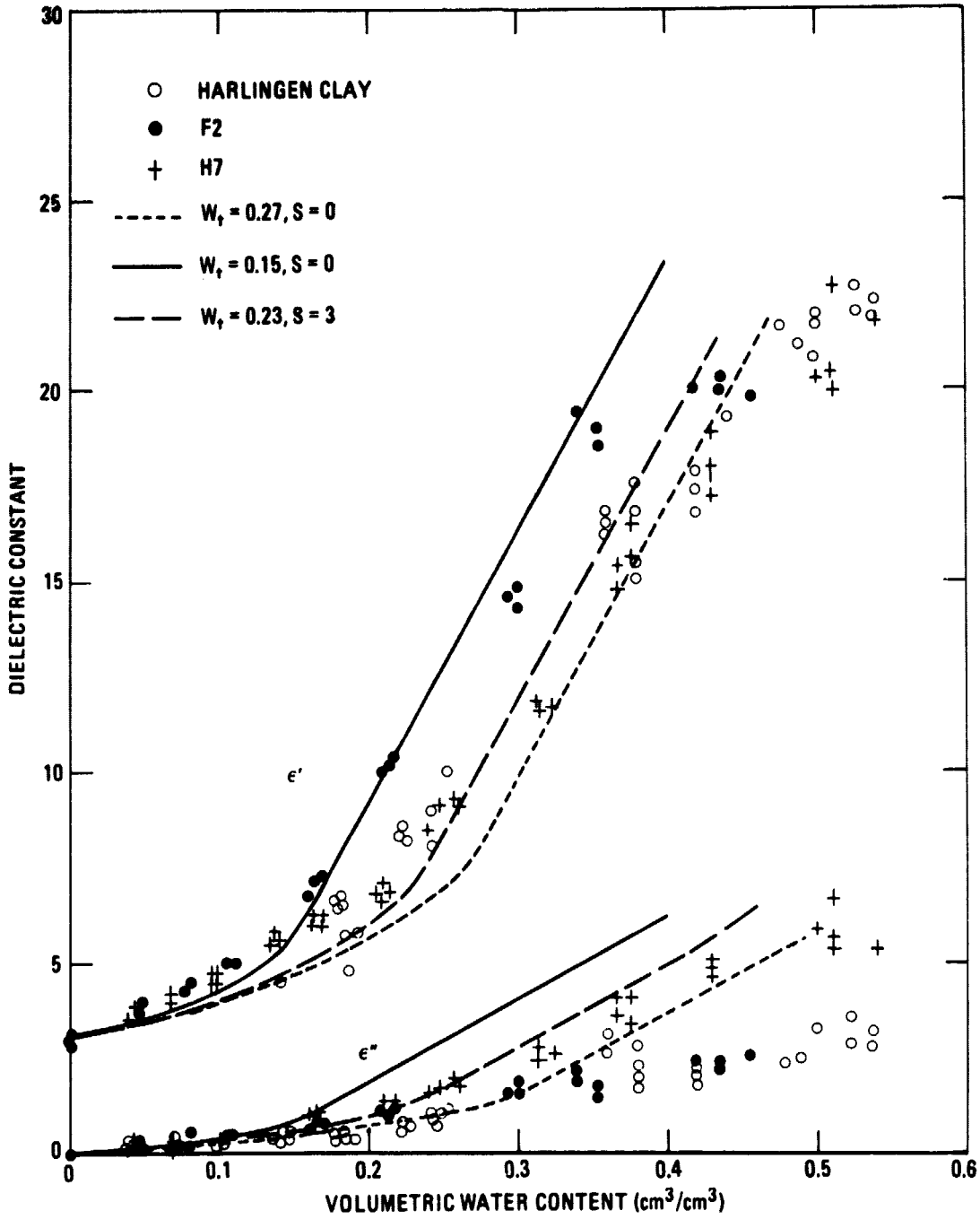

Figure 8. A comparison between the calculated dielectric constants from the empirical model and the measured values at 5 GHz

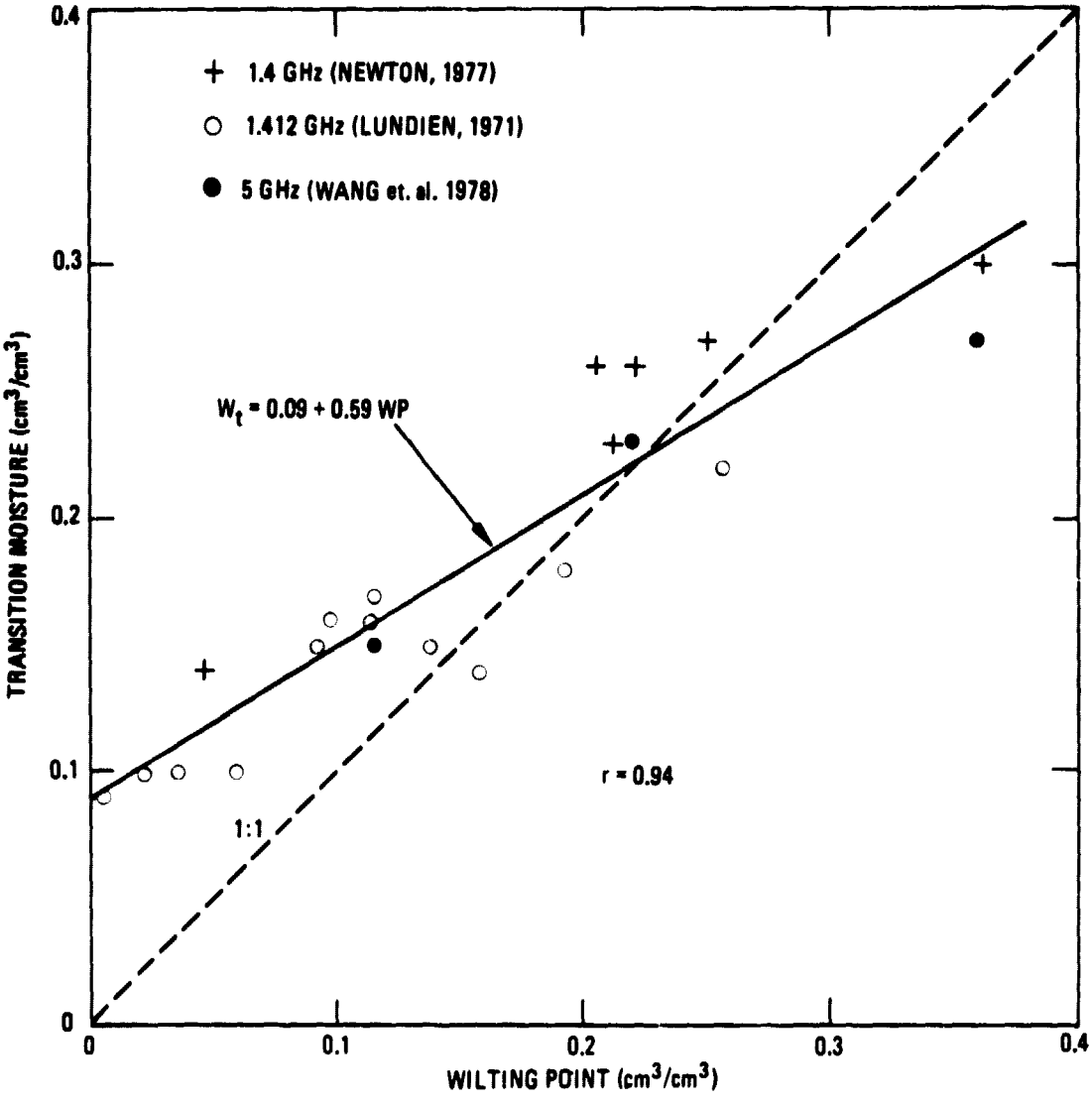

Figure 9. The variations of the transistion moistures derived from the model with the wilting points of soils. The solid line was derived from linear regression, while the dashed 1:1 line was drawn for comparison.

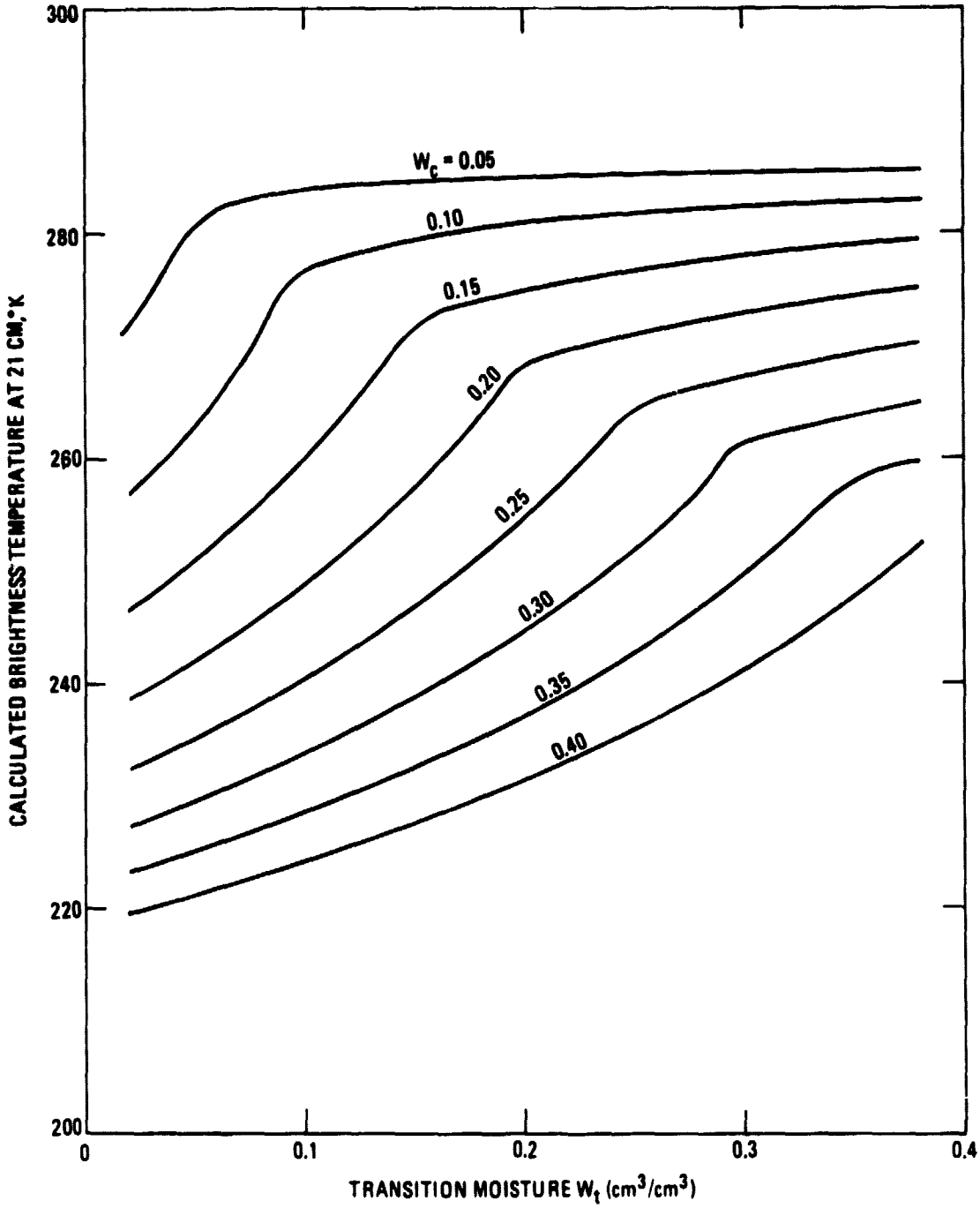

Figure 10. The calculated soil brightness temperature as a function of transition moisture. The uniform moisture profile was assumed and the moisture values were indicated on the curves.

Figure Captions

Figure 1. The dielectric constants versus volumetric water content for 4 soils measured at 5 GHz. Soil types were identified by the numbers assigned to the curves in accordance with Table 1

Figure 2. The dielectric constants versus volumetric water content for 6 soils measured at 1.4 GHz. Soil types were identified by the numbers assigned to the curves in accordance with Table 1.

Figure 3. The dielectric constants versus volumetric water content for 12 soils measured at 1.412 GHz. Soil types were identified by the numbers assigned to the curves in accordance with Table 1.

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