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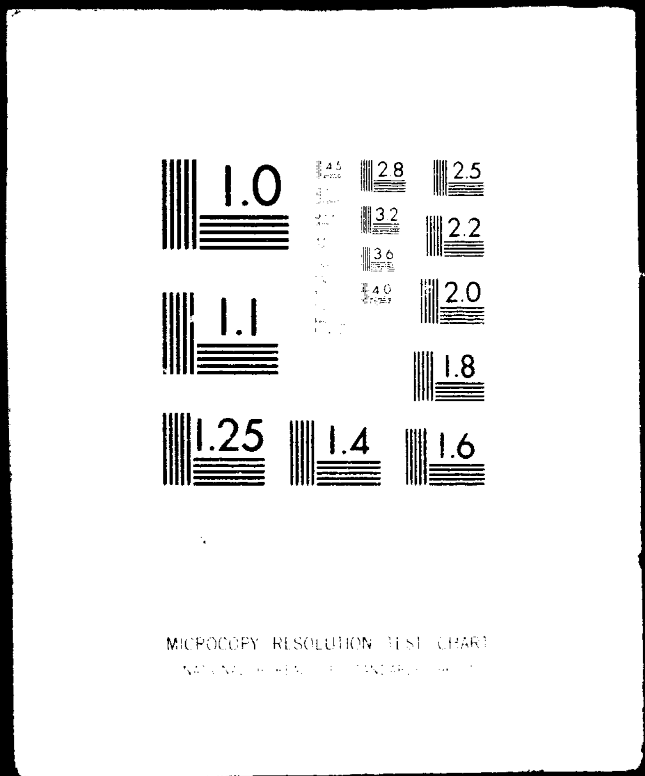

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Inertial Dynamics of a General Purpose Rotor Model

Ronald W. Du Val

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Table of Contents

List of Figures

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SYMBOLS

A	Acceleration
H	Angular momentum of rotor blade
w	Rotational rate
I	Moment of inertia
T	Coordinate transformation matrix
M	External moment
Ω	Rotor rate
ψ	Rotor azimuth
β	Flapping angle
δ	Lead-lag angle
p	x-axis component of rotation rate
q	y-axis component of rotation rate
r	z-axis component of rotation rate
i	Unit vector along x-axis
j	Unit vector along y-axis
k	Unit vector along z-axis
e	Hinge offset from rotor shaft
m	Mass of rotor blade
R	Distance to blade center of mass from hinge point

Subscripts and Superscripts

s	Fixed shaft-frame
s'	Rotating shaft-frame
b	Blade-frame
p	Principal axes frame
h	Blade hinge point
c	Blade center of mass
x	x-axis component
y	y-axis component
z	z-axis component

NOTATION

The subscripts, superscripts and symbols used in this paper are defined below. The notation conventions are as follows:

1. All vectors are denoted by arrows over a symbol. A symbol without an arrow represents a scalar magnitude. For example, $\vec{w}$ represents an angular rotation rate vector while w represents the magnitude of $\vec{w}$.

2. The reference point of the vector is given by the superscripts. For example, the symbol $\vec{w}^{s'-s}$ represents the rotational rate vector of the s'-frame with respect to the s-frame. The second superscript is dropped if the reference point is inertial. Instead of writing the angular rate of the s'-frame relative to the inertial frame as $\vec{w}^{s'-I}$ it is then written as $\vec{w}^{s'}$.

3. The second subscript denotes the reference frame into which a vector is resolved and the first subscript gives the axis of that frame. The symbol $w^{s'}_{xs}$ then gives the magnitude of the x component of the inertial rotation rate of the s'-frame resolved in s-frame coordinates. For conciseness, when a vector is resolved into the same coordinates frame that it references the superscript is dropped. For example, the symbol $w^{s'}_{xs'}$, refers to the magnitude of the x component of the inertial rotation rate of the s'-frame resolved in s'-frame coordinates, so it is written instead as $w_{xs'}$.

4. The symbols p, q and r are used in place of w_x, w_y and w_z respectively to represent magnitudes of the components of rotational rates. The symbol $w^{s'}_{xs}$, representing the magnitude of the x component of the inertial rotation rate of the s'-frame resolved in s-frame coordinates, is then written as $p^{s'}_{s}$.

SUMMARY

The inertial dynamics of a fully articulated stiff rotor blade are derived here with an emphasis on obtaining equations that facilitate an organized programming approach for simulation applications. The model for the derivation includes hinge offset and six degrees of freedom for the rotor shaft. Results are compared with the flapping and lead-lag equations currently used in the Rotor Systems Research Aircraft (RSRA) simulation model and differences are analyzed.

INTRODUCTION

A general purpose rotor model is a necessary starting point in the analysis and simulation of rotorcraft. In addition to allowing for flexibility in simulation such a model also serves as a consistent baseline from which a variety of more specialized models can be derived to satisfy specific analytical and simulation requirements.

This paper derives the inertial dynamics of a fully articulated, stiff rotor blade with offset hinges and six degrees of freedom of the shaft. Hingeless rotors can be represented by using appropriate values of spring stiffness and hinge offset in this fully articulated model.

A configuration of current interest is the Sikorsky S-61 rotor. This rotor is being utilized on the Rotor Systems Research Aircraft (RSRA) and mathematical models are given in Ref. 1 and Ref. 2. The current simulation utilizes the model of Ref. 1, however the complexity of the model has thus far prohibited real-time simulation of acceptable quality on existing computational facilities. Implementation is further complicated by the fact that these equations are not presented in a modular, building block form that permits efficient programming. Attempts to modify these equations to provide a real-time simulation on available computers have been hampered by the fact that their derivation is not documented. A lack of symmetry in some of the terms and an uncertainty as to the assumptions employed prompted the author of Ref. 2 to perform an independent derivation from basic principles as a basis for the analysis of computation requirements. His approach, however, was significantly different from that taken in Ref. 1, so no comparison with those equations was possible.

The derivation performed in this paper is intended to provide a comparison with the rotor inertial dynamics of Ref. 1 as well as to present these equations in a format more suitable for structured programming. It is hoped that the results will provide a basis for more specialized formulations as well as permit a more efficient programming approach for the general formulation.

In Chapter 1 the derivation is outlined and the most significant steps are presented along with the final results. The equations are first derived in the blade-frame since the moments of inertia are assumed constant in this frame (see Appendix C). The blade-frame variables are then expressed in terms of rotating shaft-frame components to obtain differential equations for flapping and lead-lag motions. Finally, the rotating shaft frame variables are given in terms of the fixed shaft-frame rates and accelerations. Chapter 2 presents a comparison of the results with the equations of Ref. 1 and Ref. 2 and Chapter 3 gives the conclusions and recommendations. A description of the coordinate systems and supporting derivations are given in the Appendices.

Chapter I
Derivation of Rotor Dynamics

The inertial forces present in the flapping and lagging equations are derived based on the following assumptions:

1. The rotor shaft has six degrees of freedom.

2. The rotor blade is hinged in both the flapping and lead-lag axes.

3. The flapping and lead-lag hinges are co-located at an offset, e, from the shaft axis.

4. The rotor blade is stiff.

The equation for the rotational dynamics of a fully articulated rotor blade with co-located offset hinges, as shown in Fig. A.1, is (Ref. 3):

1.1) $$\vec{M} = m\vec{R} \times \vec{A}^h + \dot{\vec{H}}^h$$

The vector $\vec{M}^h$ represents the total external moments acting about the hinges and may be written in the blade coordinate systems as:

1.2) $$\vec{M}^h = M^h_{xb}\vec{i}_b + M^h_{yb}\vec{j}_b + M^h_{zb}\vec{k}_b$$

The vector $\vec{R}$ represents the position of the blade center of mass relative to the hinges and is written as:

1.3) $$\vec{R} = R\,\vec{j}_h$$

The inertial acceleration of the hinges, $\vec{A}^h$, is given in the blade coordinate system as:

1.4) $$\vec{A}^h = A^h_{xb}\vec{i}_b + A^h_{yb}\vec{j}_b + A^h_{zb}\vec{k}_b$$

The angular momentum vector of the blade about the hinges is given in the blade frame as:

1.5) $$\vec{H}^h = H^h_{xb}\vec{i}_b + H^h_{yb}\vec{j}_b + H^h_{zb}\vec{k}_b$$

We now substitute 1.2), 1.3), 1.4) and 1.5) into 1.1) and perform the indicated cross-products to obtain:

1.6) $$M^h_{xb} = mR\,A^h_{zb} + \dot{H}^h_{xb}$$

1.7) $$M^h_{yb} = \dot{H}^h_{yb}$$

1.8) $$M^h_{zb} = -mR\,A^h_{xb} + \dot{H}^h_{zb}$$

Equations 1.6) and 1.8) are the fundamental relationships from which blade flapping and lead-lag dynamics may be derived.

The components of hinge acceleration in the blade-frame are derived in Appendix D and given in terms of rotating shaft frame rates and accelerations as:

1.9) $$A^h_{xb} = \cos\delta\left[A^s_{xs'} - e(\dot{r}_{s'} - q_{s'}p_{s'})\right] - \sin\delta\left[A^s_{ys'} - e(p^2_{s'} + q^2_{s'})\right]$$

1.10) $$A^h_{yb} = -\sin\beta\left[A^s_{zs'} + e(\dot{p}_{s'} + q_{s'}r_{s'})\right] + \cos\beta\left\{\sin\delta\left[A^s_{xs'} - e(\dot{r}_{s'} - q_{s'}p_{s'})\right] + \cos\delta\left[A^s_{ys'} - e(p^2_{s'} + r^2_{s'})\right]\right\}$$

1.11) $$A^h_{zb} = \cos\beta\left[A^s_{zs'} + e(\dot{p}_{s'} + q_{s'}r_{s'})\right] + \sin\beta\left\{\sin\delta\,A^s_{xs'} - e(\dot{r}_{s'} - q_{s'}p_{s'}) + \cos\delta\left[A^s_{ys'} - e(p^2_{s'} + r^2_{s'})\right]\right\}$$

The blade-frame components of the time rate of change of the angular momentum of the blade about the hinge are derived in Appendix C and are given in terms of rotating shaft-frame rates as:

1.12) $$\dot{H}^h_{xb} = I_b\Big[\dot{p}_{s'}\cos\delta - \dot{q}_{s'}\sin\delta - (p_{s'}\sin\delta + q_{s'}\cos\delta)\dot{\delta} - \ddot{\beta} + (\cos^2\beta - \sin^2\beta)\left\{(p_{s'}\sin\delta + q_{s'}\cos\delta)(r_{s'} - \dot{\delta})\right\} - \sin\beta\cos\beta\left\{(r_{s'} - \dot{\delta})^2 - (p_{s'}\sin\delta + q_{s'}\cos\delta)^2\right\}\Big]$$

1.13) $$\dot{H}^h_{yb} = I_{yy}\left\{\left[\dot{p}_{s'}\sin\delta + \dot{q}_{s'}\cos\delta + (p_{s'}\cos\delta - q_{s'}\sin\delta)\dot{\delta} - (r_{s'} - \dot{\delta})\dot{\beta}\cos\beta\right] - \left[\dot{r}_{s'} - \ddot{\delta} + (p_{s'}\sin\delta + q_{s'}\cos\delta)\dot{\beta}\sin\beta\right]\right\}$$

1.14) $$\dot{H}^h_{zb} = I_b\Big[\sin\beta\left\{\dot{p}_{s'}\sin\delta + \dot{q}_{s'}\cos\delta - 2(r_{s'} - \dot{\delta})\dot{\beta} + r_{s'}(p_{s'}\cos\delta - q_{s'}\sin\delta)\right\} + \cos\beta\left\{2\dot{\beta}(p_{s'}\sin\delta + q_{s'}\cos\delta) + \dot{r}_{s'} - \ddot{\delta} - (p_{s'}\sin\delta + q_{s'}\cos\delta)(p_{s'}\cos\delta - q_{s'}\sin\delta)\right\}\Big]$$

Substituting 1.11) and 1.12) into 1.6) and solving for $\ddot{\beta}$ gives:

$$1.15) \quad \ddot{\beta} = \frac{mR}{I_b}\Big[\cos\beta\left\{A^s_{zs'} + e(\dot{p}_{s'} + q_{s'}r_{s'})\right\}$$
$$+ \sin\beta\left\{\sin\delta\left[A^s_{xs'} - e(\dot{r}_{s'} - q_{s'}p_{s'})\right] + \cos\delta\left[A^s_{ys'} - e(p_{s'}^2 + r_{s'}^2)\right]\right\}$$
$$+ \dot{p}_{s'}\cos\delta - \dot{q}_{s'}\sin\delta - (p_{s'}\sin\delta + q_{s'}\cos\delta)\dot{\delta}$$
$$+ (\cos^2\beta - \sin^2\beta)\left\{(p_{s'}\sin\delta + q_{s'}\cos\delta)(r_{s'} - \dot{\delta})\right\}$$
$$- \sin\beta\cos\beta\left\{(r_{s'} - \dot{\delta})^2 - (p_{s'}\sin\delta + q_{s'}\cos\delta)^2\right\}\Big] - \frac{M^h_{xb}}{I_b}$$

Substituting 1.9) and 1.14) into 1.8) and solving for $\ddot{\delta}$ gives

$$1.16) \quad \ddot{\delta} = \frac{-mR}{I_b\cos\beta}\left[\cos\delta\left[A^s_{xs'} - e(\dot{r}_{s'} - q_{s'}p_{s'})\right] - \sin\delta\left[A^s_{ys'} - e(p_{s'}^2 + r_{s'}^2)\right]\right]$$
$$+ \tan\beta\left\{\dot{p}_{s'}\sin\delta + \dot{q}_{s'}\cos\delta - 2(r_{s'} - \dot{\delta})\dot{\beta} + r_{s'}(p_{s'}\cos\delta - q_{s'}\sin\delta)\right\}$$
$$+ 2\dot{\beta}(p_{s'}\sin\delta + q_{s'}\cos\delta) + \dot{r}_{s'} - (p_{s'}\sin\delta + q_{s'}\cos\delta)(p_{s'}\cos\delta - q_{s'}\sin\delta)$$
$$- \frac{M^h_{zb}}{I_b\cos\beta}$$

The rotating shaft-frame components of rates and accelerations are derived in terms of fixed shaft-frame components in Appendix B. The resulting equations are:

$$1.17) \quad \begin{aligned} p_{s'} &= p_s\sin\psi + q_s\cos\psi \\ q_{s'} &= -p_s\cos\psi + q_s\sin\psi \\ r_{s'} &= r_s - \Omega \end{aligned}$$

$$1.18) \quad \begin{aligned} \dot{p}_{s'} &= \dot{p}_s\sin\psi + \dot{q}_s\cos\psi - q_{s'}\Omega \\ \dot{q}_{s'} &= -\dot{p}_s\cos\psi + \dot{q}_s\sin\psi + p_{s'}\Omega \\ \dot{r}_{s'} &= \dot{r}_s - \dot{\Omega} \end{aligned}$$

$$A^{s}_{xs'} = A_{xs}\sin\Psi + A_{ys}\cos\Psi$$

1.19) $$A^{s}_{ys'} = -A_{xs}\cos\Psi + A_{ys}\sin\Psi$$

$$A^{s}_{zs'} = A_{zs}$$

Equations 1.15) through 1.19) then allow determination of flapping acceleration and lead-lag acceleration from fixed shaft-frame components of rates and accelerations.

Chapter II

Comparison with RSRA Rotor Equations

In this chapter the results of Chapter I are compared with the equations of Ref. 1 and Ref. 2 to evaluate the relative merits of the equations for simulation and analytical studies. The blade flapping and lead-lag equations, as given in Ref. 1, are:

2.1
$$\ddot{\beta} = \frac{mR}{I_b}\Big\{\cos\beta A_{zs} + e\big[2\Omega(p_s\cos\psi - q_s\sin\psi) + \dot{p}_s\sin\psi + \dot{q}_s\cos\psi\big]$$
$$+ \sin\beta\cos\delta\big[A_{ys}\sin\psi - A_{xs}\cos\psi - e(r_s - \Omega)^2\big]\Big\}$$
$$+ \cos^2\beta\big[\cos\delta\,(\dot{p}_s\sin\psi + \dot{q}_s\cos\psi) - 2(\dot{\delta} + \Omega)(q_s\sin\psi - p_s\cos\psi)$$
$$- 2\Omega\sin\delta\,(p_s\sin\psi + q_s\cos\psi)\big]$$
$$+ \cos\beta\sin\beta\big[2\dot{\delta}(r_s - \Omega) - (r_s - \Omega)^2\big] - \frac{M^h_{xb}}{I_b}$$

2.2
$$\ddot{\delta} = \frac{mR}{I_b\cos\beta}\Big\{\sin\delta\big[A_{ys}\sin\psi - A_{xs}\cos\psi - e(r_s - \Omega)^2\big]$$
$$- \cos\delta\big[A_{xs}\sin\psi + A_{ys}\cos\psi + e(\dot{\Omega} - \dot{r}_s)\big]\Big\}$$
$$+ \tan\beta\big[2\dot{\beta}(\Omega + \dot{\delta} - r_s) + \dot{q}_s\sin(\psi + \delta) - \dot{p}_s\cos(\psi + \delta)\big]$$
$$+ \dot{r}_s - \dot{\Omega} + 2\dot{\beta}\big[\cos\delta(q_s\sin\Omega - p_s\cos\Omega) + \sin\delta(p_s\sin\psi + q_s\cos\psi)\big]$$
$$- \frac{M^h_{zb}}{I_b\cos\beta}$$

where the notation has been changed as required for consistency with this paper.

In order to compare equations we must write $\ddot{\beta}$ and $\ddot{\delta}$ in terms of s-frame variables. This is accomplished by substituting 1.17) through 1.19) into 1.15) and 1.16) to get:

2.3)
$$\ddot{\beta} = \frac{mR}{I_b}\Big\{\cos\beta\,A_{zs} + e\big[2\Omega(p_s\cos\psi - q_s\sin\psi)$$
$$+ \dot{p}_s\sin\psi + \dot{q}_s\cos\psi + r_s(q_s\sin\psi - p_s\cos\psi)\big]$$
$$+ \sin\beta\cos\delta\Big[A_{ys}\sin\psi - A_{xs}\cos\psi - e\big[(r_s - \Omega)^2 + (p_s\sin\psi + q_s\cos\psi)^2\big]\Big]$$
$$+ \sin\beta\sin\delta\Big[A_{xs}\sin\psi + A_{ys}\cos\psi - e\big[(\dot{r}_s - \dot{\Omega})$$
$$- (p_s\sin\psi + q_s\cos\psi)(q_s\sin\psi - p_s\cos\psi)\big]\Big]\Big\}$$
$$+ (\cos^2\beta - \sin^2\beta)\big[\cos\delta(r_s - \Omega - \dot{\delta})(q_s\sin\psi - p_s\cos\psi)$$
$$+ \sin\delta(r_s - \Omega - \dot{\delta})(p_s\sin\psi + q_s\cos\psi)\big]$$
$$+ \sin\beta\cos\beta\big\{\big[(p_s\sin\psi + q_s\cos\psi)\sin\delta + (q_s\sin\psi - p_s\cos\psi)\cos\delta\big]^2 - (r_s - \Omega - \dot{\delta})^2\big\}$$
$$+ \cos\delta\big[\dot{p}_s\sin\psi + \dot{q}_s\cos\psi - (\Omega + \dot{\delta})(q_s\sin\psi - p_s\cos\psi)\big]$$
$$- \sin\delta\big[\dot{q}_s\sin\psi - \dot{p}_s\cos\psi + (\Omega + \dot{\delta})(p_s\sin\psi + q_s\cos\psi)\big] - \frac{M^h_{xb}}{I_b}$$

$$2.4)\quad \ddot{\delta} = \frac{mR}{I_b\cos\beta}\Big[\sin\delta\Big\{A_{ys}\sin\psi - A_{xs}\cos\psi$$

$$- e\left[(r_s-\Omega)^2 + (p_s\sin\psi + q_s\cos\psi)^2\right]\Big\}$$

$$- \cos\delta\Big\{A_{xs}\sin\psi + A_{ys}\cos\psi$$

$$+ e\left[\dot{\Omega} - \dot{r}_s + (p_s\sin\psi + q_s\cos\psi)(q_s\sin\psi - p_s\cos\psi)\right]\Big\}\Big]$$

$$+ \tan\beta\Big\{2\dot{\beta}(\Omega + \dot{\delta} - r_s) + \dot{q}_s(\cos\psi\sin\delta + \sin\psi\cos\delta)$$

$$- \dot{p}_s(\cos\psi\cos\delta - \sin\psi\sin\delta)$$

$$+ r_s\left[p_s(\sin\psi\cos\delta + \cos\psi\sin\delta) + q_s(\cos\psi\cos\delta - \sin\psi\sin\delta)\right]\Big\}$$

$$+ \dot{r}_s - \dot{\Omega} + 2\dot{\beta}\left[\cos\delta(q_s\sin\psi - p_s\cos\psi) + \sin\delta(p_s\sin\psi + q_s\cos\psi)\right]$$

$$+\left[p_s(\cos\psi\cos\delta - \sin\psi\sin\delta) - q_s(\cos\psi\sin\delta + \sin\psi\cos\delta)\right]\left[p_s(\sin\psi\cos\delta\right.$$

$$\left. + \cos\psi\sin\delta) + q_s(\cos\psi\cos\delta - \sin\psi\sin\delta)\right]$$

$$-\frac{M^h_{zb}}{I_b\cos\beta}$$

Neglecting second order terms in shaft rotational rates (p_s, q_s, r_s) and using the trigonometric identities:

$$2.5)\quad \sin(\psi + \delta) = \cos\psi\sin\delta + \sin\psi\cos\delta$$

$$\cos(\psi + \delta) = \cos\psi\cos\delta - \sin\psi\sin\delta$$

we may rewrite 2.3) and 2.4) as:

$$2.6)\quad \ddot{\beta} = \frac{mR}{I_b}\Big[\cos\beta\Big\{A_{zs} + e\left[2\Omega(p_s\cos\psi - q_s\sin\psi) + \dot{p}_s\sin\psi + \dot{q}_s\cos\psi\right]\Big\}$$

$$+ \sin\beta\cos\delta\Big\{A_{ys}\sin\psi - A_{xs}\cos\psi - e(r_s - \Omega)^2\Big\}$$

$$+ \sin\beta\sin\delta\Big\{A_{xs}\sin\psi + A_{ys}\cos\psi - e(\dot{r}_s - \dot{\Omega})\Big\}\Big]$$

$$+ (\cos^2\beta - \sin^2\beta)\Big\{\cos\delta\left[-(\Omega + \dot{\delta})(q_s\sin\psi - p_s\cos\psi)\right]$$

$$+ \sin\delta\left[-(\Omega + \dot{\delta})(p_s\sin\psi + q_s\cos\psi)\right]\Big\}$$

$$+ \cos\beta\sin\beta\Big\{2\dot{\delta}(r_s - \Omega) - (r_s - \Omega)^2 - \dot{\delta}^2\Big\}$$

$$+ \cos\delta\Big\{\dot{p}_s\sin\psi + \dot{q}_s\cos\psi - (\Omega + \dot{\delta})\left[q_s\sin\psi - p_s\cos\psi\right]\Big\}$$

$$- \sin\delta\Big\{\dot{q}_s\sin\psi - \dot{p}_s\cos\psi + (\Omega + \dot{\delta})\left[p_s\sin\psi + q_s\cos\psi\right]\Big\} - \frac{M^h_{xb}}{I_b}$$

$$2.7)\quad \ddot{\delta} = \frac{mR}{I_b\cos\beta}\Big[\sin\delta\left\{A_{ys}\sin\psi - A_{xs}\cos\psi - e(r_s - \Omega)^2\right\}$$

$$- \cos\delta\left\{A_{xs}\sin\psi + A_{ys}\cos\psi + e(\dot{\Omega} - \dot{r}_s)\right\}\Big]$$

$$+ \tan\beta\left\{2\dot{\beta}(\Omega+\dot{\delta} - r_s) + \dot{q}_s\sin(\psi + \delta) - \dot{p}_s\cos(\psi + \delta)\right\}$$

$$+ \dot{r}_s - \dot{\Omega} + 2\dot{\beta}\left[\cos\delta\ (q_s\sin\psi - p_s\cos\psi) + \sin\delta(p_s\sin\psi + q_s\cos\psi)\right]$$

$$- \frac{M^h_{zb}}{I_b\cos\beta}$$

Comparing 2.7) and 2.2) we see that the equations are identical so the only additional assumption made in the lead-lag equation of Ref. 1 is that second order terms in shaft rates may be neglected.

Comparing 2.6) and 2.1) we see that further rearranging is required for comparison. Using the identity:

$$2.8)\quad \cos^2\beta + \sin^2\beta = 1$$

we multiply the last two lines of 2.6) by 2.8) and regroup to get:

$$2.9)\quad \ddot{\beta} = \frac{mR}{I_b}\Big[\cos\beta\left\{A_{zs} + e\left[2\Omega(p_s\cos\psi - q_s\sin\psi) + \dot{p}_s\sin\psi + \dot{q}_s\cos\psi\right]\right\}$$

$$+ \sin\beta\cos\delta\left\{A_{ys}\sin\psi - A_{xs}\cos\psi - e(r_s - \Omega)^2\right\}$$

$$+ \sin\beta\sin\delta\left\{A_{xs}\sin\psi + A_{ys}\cos\psi - e(\dot{r}_s - \dot{\Omega})\right\}\Big]$$

$$+ \cos^2\beta\left\{\cos\delta\left[\dot{p}_s\sin\psi + \dot{q}_s\cos\psi - 2(\Omega + \dot{\delta})(q_s\sin\psi - p_s\cos\psi)\right]\right.$$

$$\left. - \sin\delta\left[\dot{q}_s\sin\psi - \dot{p}_s\cos\psi + 2(\Omega + \dot{\delta})(p_s\sin\psi + q_s\cos\psi)\right]\right\}$$

$$+ \cos\beta\sin\beta\left\{2\dot{\delta}\ (r_s - \Omega) - (r_s - \Omega)^2 - \dot{\delta}^2\right\}$$

$$+ \sin^2\beta\left\{\cos\delta\left[\dot{p}_s\sin\psi + \dot{q}_s\cos\psi\right] - \sin\delta\left[\dot{q}_s\sin\psi - \dot{p}_s\cos\psi\right]\right\}$$

$$- \frac{M^h_{xb}}{I_b}$$

A comparison of 2.1) and 2.9) now reveals the following discrepancies in 2.1)

1. Line 3 in 2.9) has been neglected. This may be justified by assuming $\sin\delta$ is small.

2. All terms not including the factor Ω (rotor rate) have been dropped from line 5 in 2.9). This may be justified since the other factors are relatively small compared to Ω but the same is true for line 4 and similar terms have not been neglected there.

3. The square of the lead-lag rate, $\dot{\delta}$, has been neglected in line 6. It is not clear however, that this is negligible compared to squares of r_s and products of r_s and $\dot{\delta}$.

4. Line 7 in 2.9) has been neglected. This may be justified by assuming that $\sin^2\beta$ is small.

The approach of Ref. 2 is to substitute Euler's equations in the blade-frame into the moment equations and solve for the rotational accelerations in the blade-frame. In this paper, Euler's equations in the blade-frame are given in equation (C.3) and the moment equations are equations 1.6) through 1.8). Performing the substitutions we obtain:

2.10) $$\dot{p}_b = \frac{1}{I_b}\left(M^h_{xb} - mRA^h_{zb}\right) - q_b r_b$$

2.11) $$\dot{r}_b = \frac{1}{I_b}\left(M^h_{zb} + mRA^h_{xb}\right) + p_b q_b$$

Equation 2.10) agrees with its counterpart in Ref. 2, however the equation for $\dot{r}_b$ in Ref. 2 has a minus sign on the hinge acceleration term and appears to be in error.

In Ref. 2, $\dot{p}_b$ and $\dot{r}_b$ are integrated and the flapping and lead-lag rates obtained as a function of the difference in blade-frame and rotating shaft-frame rates. The appropriate equations in this paper are given in (B.16) and the resulting expressions for flapping and lead-lag rates are:

2.12) $\dot{\beta} = p_{s'}\cos\delta - q_{s'}\sin\delta - p_b$

2.13) $\dot{\delta} = \left[p_{s'}\sin\delta + q_{s'}\cos\delta\right]\sin\beta + r_{s'}\cos\beta - r_b$

No differential equation for q_b has been obtained since the moment of inertia about the y-axis has been assumed negligible so there is a problem in obtaining values of q_b for use in 2.10) and 2.11). In Ref. 2 it is suggested that, with the exception of blade pitch control, q_b may be negligible. This seems unlikely, however, since the rotational degrees of freedom of the rotor shaft as well as the lead-lag rate ($\dot{\delta}$) and the rotational rate (Ω) can couple into the y-axis of the blade-frame. A solution to this problem can be obtained by noting that 2.12) and 2.13) have been obtained from the p_b and r_b equations of (B.16) so an independent equation for q_b remains and is given as:

2.14) $q_b = \left[p_{s'}\sin\delta + q_{s'}\cos\delta\right]\cos\beta - (r_{s'} - \dot{\delta})\sin\beta$

where $\dot{\delta}$ is obtained from 2.13). The hinge accelerations in the blade-frame and the rotating shaft frame rates are required in 2.10) through 2.14) and may be obtained from the fixed shaft-frame rates and accelerations by 1.9) through 1.11) and 1.17) through 1.19).

Chapter III

Conclusions

The equations for the inertial dynamics of a rotor blade as derived in Chapter I are intended to serve as both a general purpose simulation model and as a baseline for more specialized analytical studies. In this paper, no assumptions as to the relative magnitude of the rates have been made. For most applications it should be possible to further simplify these equations by appropriate assumptions. This derivation has been intended to serve as a baseline for more specialized applications; hence, only the most generally applicable assumptions have been utilized.

By writing the flapping and lead-lag equations in terms of rotating shaft-frame rates and accelerations the equations have been significantly simplified over the formualtion in Ref. 1 where these equations were given in terms of fixed shaft-frame variables. In this paper the rotating shaft frame variables are obtained by a separate transformation from the fixed shaft frame variables. This allows for a more structured approach to programming and thereby improves the efficiency of the simulation. Programming errors are also more readily located and corrected in this format.

The validity of the equations of Ref. 1, which are used in the current RSRA simulation, was checked by combining the modules derived in Chapter I to obtain differential equations for blade flapping and lead-lag motion as a function of fixed shaft frame variables. By neglecting higher order terms in some rates the lead-lag equation was found to be identical to the lead-lag equation of Ref. 1, but discrepancies were found in the flapping equation. The discrepancies appear to be the result of inconsistancies in applying small angle and low rate approximations in the equations of Ref. 1. No attempt has been made in this paper to determine the validity of these assumptions or the numerical significance of the discrepancies.

The approach used in Ref. 2 is to obtain equations for the rotational accelerations of the blade-frame and integrate these accelerations. The flapping and lead-lag rates are then found as a function of the difference between blade-frame and rotating shaft-frame rates. This approach was repeated in Chapter II for comparison purposes and a discrepancy was found in the $\dot{r}_b$ equation. The sign of the hinge acceleration term in this equation appears to be in error in Ref. 2.

A problem with the approach taken in Ref. 2 is that no differential equation for q_b can be obtained since the y-axis moment of inertia of the blade is assumed negligible and the suggestion made in Ref. 2 that q_b may be small and can be neglected does not appear to be justified. A solution to this problem is suggested in Chapter II of this paper and involves solving for q_b as a function of rotating shaft axis rates and lead-lag rates. With this addition, the equations of Ref. 2 may be well suited to simulation since they are presented in an extremely modular format. For analytical work, however, explicit differential equations for blade flapping and lead-lag motions are required rather than expressions for these variables in terms of the output of other differential equations. The equations derived in this paper are intended to satisfy both simulation and analytical requirements.

Appendix A

Coordinate Systems

The four coordinate frames used in this analysis are shown in Fig. A. 1. They are:

1) The fixed shaft-frame (s).

2) The rotating shaft-frame (s').

3) The blade-frame (b).

4) The principal axes frame (p).

The fixed shaft-frame (s) is centered in the rotor hub with its x-axis in the x-z plane of the aircraft and its z-axis normal to the plane of the rotor hub. The s-frame is the starting point of this analysis. Rotational rates and accelerations and translational accelerations of this frame are assumed available in terms of rates and accelerations at the aircraft center of gravity.

The rotating shaft-frame (s') is shown relative to the fixed shaft-frame in Fig. A.2. This frame is centered in the hub but rotates with the blade. Its y-axis is directed through the co-located hinges and its z-axis is normal to the rotor hub. The x-axis of the s'-frame is aligned with the y-axis of the s-frame at $\psi = 0$.

The relative orientations of the blade-frame and the rotating shaft-frame are shown in Fig. A.3. The rotating shaft-frame is centered at the co-located hinges and fixed to the blade, so its origin is displaced from the s-frame origin by the amount of hinge offset. The y-axis of this frame is aligned with the blade and its x-axis is normal to the plane of the flapping hinge.

The origin of the principal axes frame (p) is co-located with the origin of the blade-frame but the principal axes are fixed in the blade and oriented to eliminate all products of inertia.

The relative orientation of the principal axes frame and the blade-frame is shown in Fig. A.4. It has been assumed that the y_b axis (axis of feathering) passes through the center of gravity of each blade cross section. The y_b axis is then a principal axis since the x-y and z-y products of inertia vanish. The x and z principal axes are consequently in the x-z plane of the blade-frame at an angle Θ_p from the x_b and z_b axes, where Θ_p is chosen to make the x-z product of inertia vanish. Note that Θ_p is a function of the blade twist and the impressed blade pitch, so it will vary with cyclic and collective control inputs.

The transformation from the blade-frame to the principal axis-frame consists of a single rotation, Θ_p, about the y_b axis and is given by

$$(A.1) \qquad T_{p/b} = \begin{bmatrix} \cos\Theta & 0 & -\sin\Theta \\ 0 & 1 & 0 \\ \sin\Theta & 0 & \cos\Theta \end{bmatrix}$$

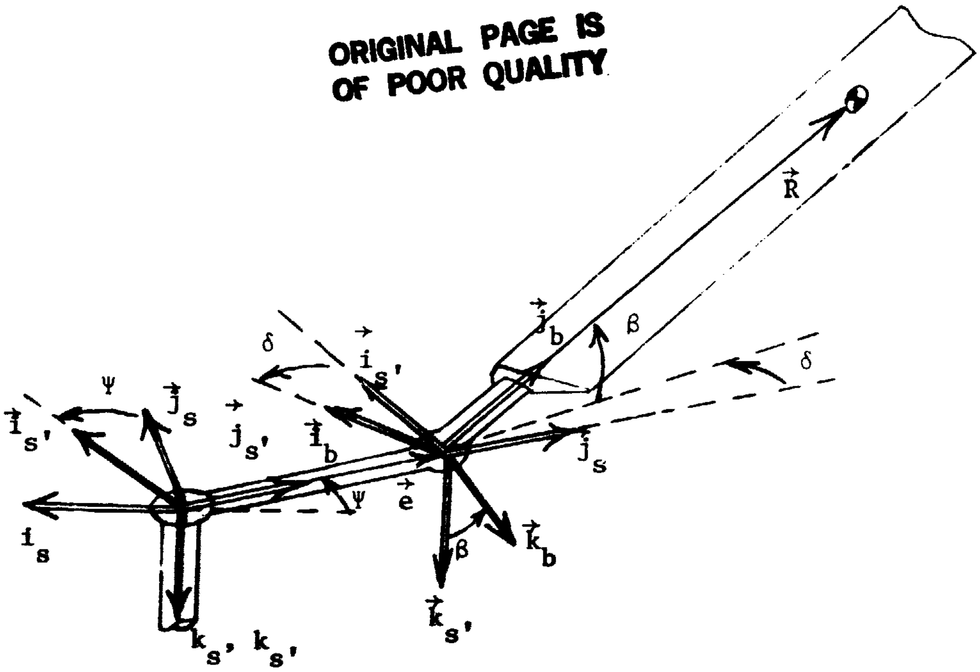

Figure A.1. Coordinate Systems

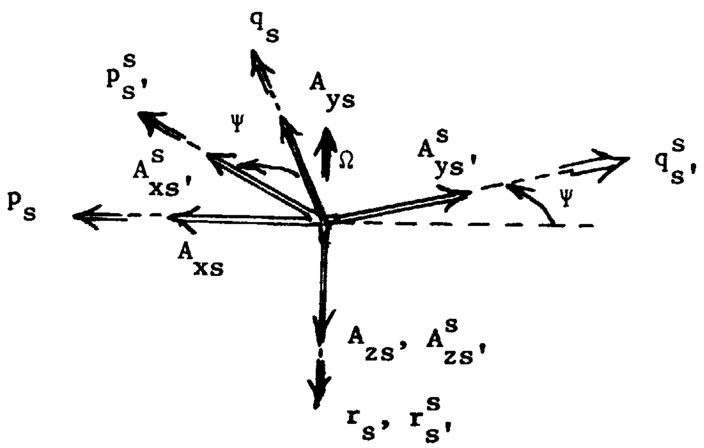

Figure A.2. Fixed and Rotating Shaft Frames

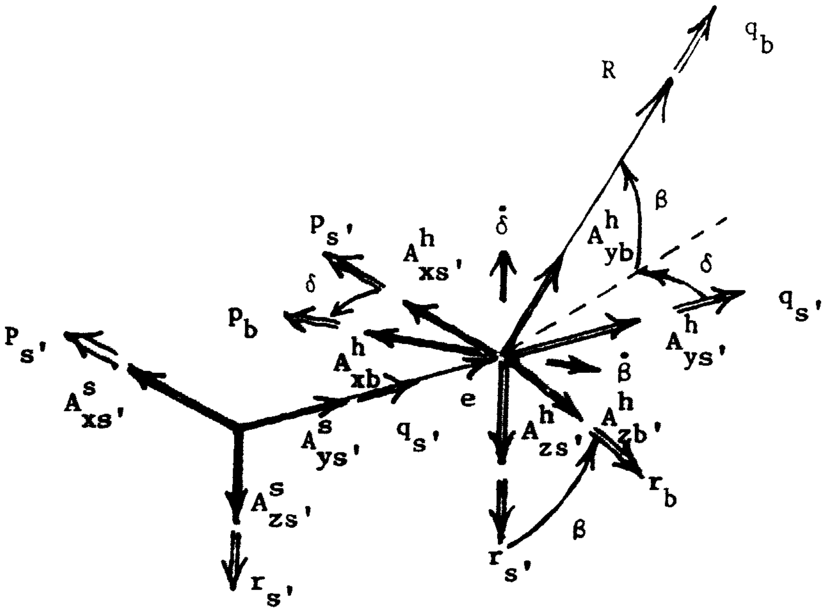

Figure A. 3. Rotating Shaft and Blade Frames

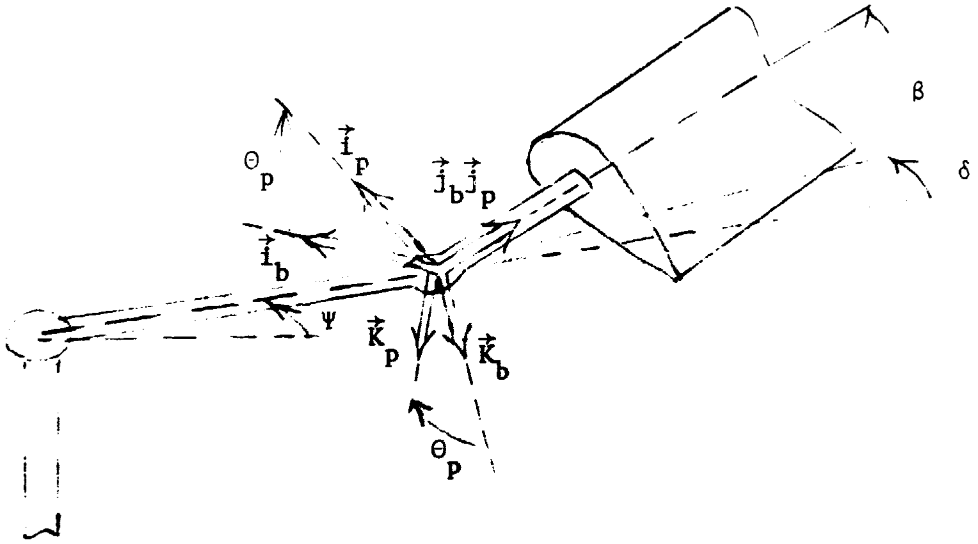

Figure A.4. Principal Axes and Blade

Appendix B

Rotational Rate of Blade Frame

Starting from the fixed shaft-frame, s, the rotational rate of the rotating shaft frame, s', is given by:

$$\text{(B.1)} \qquad \vec{w}^{s'} = \vec{w}^{s} + \vec{w}^{s'-s}$$

where $\vec{w}^{s}$ is the rate of the s frame and is given in s-frame components as:

$$\text{(B.2)} \qquad \vec{w}^{s} = p_s\vec{i}_s + q_s\vec{j}_s + r_s\vec{k}_s$$

The vector $\vec{w}^{s'-s}$ is the rate of the s'-frame relative to the s-frame and from Fig. A.2 is seen to be:

$$\text{(B.3)} \qquad \vec{w}^{s'-s} = -\Omega\,\vec{k}_s$$

where Ω is the rotor rate.

Combining (B.1), (B.2) and (B.3) the rotational rate of the s' frame may be written as:

$$\text{(B.4)} \qquad \vec{w}^{s'} = p_s^{s'}\,\vec{i}_s + q_s^{s'}\,\vec{j}_s + r_s^{s'}\,\vec{k}_s$$

where:

$$\text{(B.5)} \qquad \begin{aligned} p_s^{s'} &= p_s \\ q_s^{s'} &= q_s \\ r_s^{s'} &= r_s - \Omega \end{aligned}$$

From Fig. A.2 the transformation from the shaft to the rotating shaft axes is seen to be:

$$\text{(B.6)} \qquad T_{s'/s} = \begin{bmatrix} \sin\psi & \cos\psi & 0 \\ -\cos\psi & \sin\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where ψ is the azimuth of the blade.

The rotational rate of the s'-frame may then be resolved into s' components using (B.6). We then have:

$$\vec{w}^{s'} = p_{s'}\,\vec{i}_{s'} + q_{s'}\,\vec{j}_{s'} + r_{s'}\,\vec{k}_{s'} \qquad \text{(B.7)}$$

where:

$$\begin{aligned} p_{s'} &= p_s \sin\psi + q_s \cos\psi \\ q_{s'} &= -p_s \cos\psi + q_s \sin\psi \\ r_{s'} &= r_s - \Omega \end{aligned} \qquad \text{(B.8)}$$

Differentiating (B.8) the rotational accelerations in the s'-frame are found to be:

$$\begin{aligned} \dot{p}_{s'} &= \dot{p}_s \sin\psi + \dot{q}_s \cos\psi - q_{s'}\,\Omega \\ \dot{q}_{s'} &= -\dot{p}_s \cos\psi + \dot{q}_s \sin\psi + p_{s'}\,\Omega \\ \dot{r}_{s'} &= \dot{r}_s - \dot{\Omega} \end{aligned} \qquad \text{(B.9)}$$

Using (B.6) the acceleration of the origin of the fixed shaft-frame may be written in rotating shaft-frame coordinates as:

$$\begin{aligned} A^s_{xs'} &= A_{xs} \sin\psi + A_{ys} \cos\psi \\ A^s_{ys'} &= -A_{xs} \cos\psi + A_{ys} \sin\psi \\ A^s_{zs'} &= A^s_{zs} \end{aligned} \qquad \text{(B.10)}$$

The rotational rate of the blade-frame, b, is given by:

$$\vec{w}^{b} = \vec{w}^{s'} + \vec{w}^{b-s'} \qquad \text{(B.11)}$$

where $\vec{w}^{b-s'}$ is the rate of the blade-frame relative to the rotating shaft-frame. From Fig. A.3 we may write $\vec{w}^{b-s'}$ in blade coordinates as:

$$\vec{w}^{b-s'} = -\dot{\beta}\vec{i}_b + \dot{\delta}\sin\beta\,\vec{j}_b - \dot{\delta}\cos\beta\,\vec{k}_b \qquad \text{(B.12)}$$

where β is the flapping angle and δ is the lead-lag angle.

The transformation from rotating shaft-frame to blade-frame is found from Fig. A.3 to be:

$$(B.13) \qquad T_{b/s'} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} \cos\delta & -\sin\delta & 0 \\ \sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

or

$$(B.14) \qquad T_{b/s'} = \begin{bmatrix} \cos\delta & -\sin\delta & 0 \\ \cos\beta\sin\delta & \cos\beta\cos\delta & -\sin\beta \\ \sin\beta\sin\delta & \sin\beta\cos\delta & \cos\beta \end{bmatrix}$$

Using (B.14) to resolve (B.7) into the blade-frame and combining the result with (B.11) and (B.12) we get:

$$(B.15) \qquad \vec{w}^{b} = p_b \vec{i}_b + q_b \vec{j}_b + r_b \vec{k}_b$$

where:

$$p_b = p_{s'}\cos\delta - q_{s'}\sin\delta - \dot{\beta}$$

$$(B.16) \qquad \begin{aligned} q_b &= \left[p_{s'}\sin\delta + q_{s'}\cos\delta\right]\cos\beta - (r_{s'} - \dot{\delta})\sin\beta \\ r_b &= \left[p_{s'}\sin\delta + q_{s'}\cos\delta\right]\sin\beta + (r_{s'} - \dot{\delta})\cos\beta \end{aligned}$$

$$(B.17) \qquad \begin{aligned} \dot{p}_b &= \dot{p}_{s'}\cos\delta - \dot{q}_{s'}\sin\delta - \ddot{\beta} - \left[p_{s'}\sin\delta + q_{s'}\cos\delta\right]\dot{\delta} \\ \dot{q}_b &= \Big\{\dot{p}_{s'}\sin\delta + \dot{q}_{s'}\cos\delta + \left[p_{s'}\cos\delta - q_{s'}\sin\delta\right]\dot{\delta} \\ &\quad - (r_{s'} - \dot{\delta})\dot{\beta}\Big\}\cos\beta \\ &\quad - \Big\{\dot{r}_{s'} - \ddot{\delta} + \left[p_{s'}\sin\delta + q_{s'}\cos\delta\right]\dot{\beta}\Big\}\sin\beta \\ \dot{r}_b &= \Big\{\dot{p}_{s'}\sin\delta + \dot{q}_{s'}\cos\delta + \left[p_{s'}\cos\delta - q_{s'}\sin\delta\right]\dot{\delta} \\ &\quad - (r_{s'} - \dot{\delta})\dot{\beta}\Big\}\sin\beta \\ &\quad + \Big\{\dot{r}_{s'} - \ddot{\delta} + \left[p_{s'}\sin\delta + q_{s'}\cos\delta\right]\dot{\beta}\Big\}\cos\beta \end{aligned}$$

Appendix C

Euler's Equations in the Blade Frame

The angular momentum vector in the blade-frame may be written in terms of the principal moments of inertia as (Ref. 3):

$$\vec{H}^h_b = T^T_{p/b} \; I_p T_{p/b} \; \vec{w}_b \tag{C.1}$$

where $T_{p/b}$ is the transformation from the blade-frame to the principal axis frame, as given by (A.1), I_p is the principal moment of inertia tensor, and $\vec{w}_b$ is the angular velocity of the blade-frame, as given by (B.15).

Substituting (A.1) and (B.15) into (C.1) gives:

$$\begin{aligned}
\vec{H}^h_{xb} &= (I_{xx} \cos^2\Theta_p + I_{zz} \sin^2\Theta_p)\, p_b + (I_{zz} - I_{xx}) \sin\Theta_p \cos\Theta_p r_b \\
\vec{H}^h_{yb} &= I_{yy} q_b \\
\vec{H}^h_{zb} &= (I_{zz} - I_{xx}) \sin\Theta_p \cos\Theta_p p_b + (I_{xx} \sin^2\Theta_p + I_{zz} \cos^2\Theta_p) r_b
\end{aligned} \tag{C.2}$$

The time rate of change of the angular momentum vector is then:

$$\dot{\vec{H}}^h = \dot{\vec{H}}^{h-b} + \vec{w}_b \times \vec{H} \tag{C.3}$$

A significant simplification can be obtained by the following assumption:

$$I_{xx} \simeq I_{zz} \triangleq I_b \tag{C.4}$$

The angular momentum components are then:

$$\begin{aligned}
\vec{H}^h_{xb} &= I_b p_b \\
\vec{H}^h_{yb} &= I_{yy} q_b \\
\vec{H}^h_{zb} &= I_b r_b
\end{aligned} \tag{C.5}$$

Note that this assumption has eliminated all Θ_p terms, making the moments of inertia constant in the blade-frame.

Using (C.5) in (C.3) gives:

$$\begin{aligned} \dot{H}^h_{xb} &= I_b\dot{p}_b + (I_b - I_{yy})q_b r_b \\ \dot{H}^h_{yb} &= I_{yy}\dot{q}_b \qquad \text{(C.6)} \\ \dot{H}^h_{zb} &= I_b\dot{r}_b + (I_{yy} - I_b)\, p_b q_b \end{aligned}$$

If we further assume:

$$I_{yy} << I_{bb} \qquad \text{(C.7)}$$

then (C.6) becomes:

$$\begin{aligned} \dot{H}^h_{xb} &= I_b(\dot{p}_b + q_b r_b) \\ \dot{H}^h_{yb} &= I_{yy}\dot{q}_b \qquad \text{(C.8)} \\ \dot{H}^h_{zb} &= I_b(\dot{r}_b - p_b q_b) \end{aligned}$$

Using (B.16) and (B.17) in (C.8) gives:

$$\begin{aligned} \dot{H}^h_{xb} = I_b \Big\{ & \dot{p}_{s'}\cos\delta - \dot{q}_{s'}\sin\delta - \left[p_{s'}\sin\delta + q_{s'}\cos\delta\right]\dot{\delta} - \ddot{\beta} \\ & + \left[\cos^2\beta - \sin^2\beta\right]\left[p_{s'}\sin\delta + q_{s'}\cos\delta\right](r_{s'} - \dot{\delta}) \\ & - \sin\beta\cos\beta\left[(r_{s'} - \dot{\delta})^2 - (p_{s'}\sin\delta - q_{s'}\cos\delta)^2\right]\Big\} \end{aligned} \qquad \text{(C.9)}$$

$$\begin{aligned} \dot{H}^h_{yb} = I_{yy}\Big\{ & \left[\dot{p}_{s'}\sin\delta + \dot{q}_{s'}\cos\delta + (p_{s'}\cos\delta - q_{s'}\sin\delta)\dot{\delta} - (r_{s'} - \dot{\delta})\dot{\beta}\right]\cos\beta \\ & - \left[\dot{r}_{s'} - \ddot{\delta} + (p_{s'}\sin\delta + q_{s'}\cos\delta)\dot{\beta}\right]\sin\beta\Big\} \end{aligned} \qquad \text{(C.10)}$$

$$\begin{aligned} \dot{H}^h_{zb} = I_b\Big\{ & \sin\beta\left[\dot{p}_{s'}\sin\delta + \dot{q}_{s'}\cos\delta - 2(r_{s'} - \dot{\delta})\dot{\beta} + r_{s'}\left[p_{s'}\cos\delta - q_{s'}\sin\delta\right]\right] \\ & + \cos\beta\Big[2\dot{\beta}\left[p_{s'}\sin\delta + q_{s'}\cos\delta\right] + \dot{r}_{s'} - \ddot{\delta} \\ & - \left[p_{s'}\sin\delta + q_{s'}\cos\delta\right]\left[p_{s'}\cos\delta - q_{s'}\sin\delta\right]\Big]\Big\} \end{aligned} \qquad \text{(C.11)}$$

Appendix D

Acceleration of Hinge Point

The acceleration of the hinge may be written as:

$$\text{(D.1)} \qquad \vec{A}^{h} = \vec{A}^{s} + \vec{A}^{h-s}$$

where $\vec{A}^{s}$ is the inertial acceleration of the origin of the s-frame and $\vec{A}^{h-s}$ is the acceleration of the hinge relative to the s-frame.

Using the transformation of(B.6) we may write $\vec{A}^{s}$ interms of its components in the s'-frame as:

$$\text{(D.2)} \qquad \vec{A}^{s} = A^{s}_{xs'}\,\vec{i}_{s'} + A^{s}_{ys'}\,\vec{j}_{s} + A^{s}_{zs'}\,\vec{k}_{s'}$$

From Fig. A.1 the acceleration of the hinge relative to the s-frame is seen to be:

$$\text{(D.3)} \qquad \vec{A}^{h-s} = \ddot{\vec{e}}$$

where:

$$\text{(D.4)} \qquad \vec{e} = e\vec{j}_{s'}$$

We may write the derivative of (D.4) in terms of the rotational rate of the s'-frame as:

$$\text{(D.5)} \qquad \dot{\vec{e}} = \dot{e}\,\vec{j}_{s'} + \vec{w}^{s'} \times \vec{e} = \vec{w}^{s'} \times \vec{e}$$

where we have noted that $\dot{e}$ is zero in the s'-frame. The second derivative is then:

$$\text{(D.6)} \qquad \ddot{\vec{e}} = \dot{\vec{w}}^{s'} \times \vec{e} + \vec{w}^{s'} \times \dot{\vec{e}}$$

combining (D.1), (D.3) and(D.6) then gives:

$$\text{(D.7)} \qquad \vec{A}^{h} = \vec{A}^{s} + \dot{\vec{w}}^{s'} \times \vec{e} + \vec{w}^{s'} \times (\vec{w}^{s'} \times \vec{e})$$

Using the definition of $\vec{w}^{s'}$ in (B.7) and its derivative along with (D2) and (D.4) we obtain:

$$\text{(D.8)} \qquad \vec{A}^{h} = A^{h}_{xs'}\,\vec{i}_{s'} + A^{h}_{ys'}\,\vec{j}_{s'} + A^{h}_{zs'}\,\vec{k}_{s'}$$

where:

$$
\text{(D.9)}\qquad
\begin{aligned}
A^h_{xs'} &= A^s_{xs'} - e(\dot{r}_{s'} - q_{s'}\, p_{s'})\\
A^h_{ys'} &= A^s_{ys'} - e(p_{s'}{}^2 + r_{s'}{}^2)\\
A^h_{zs'} &= A^s_{zs'} + e(\dot{p}_{s'} + q_{s'}\, r_{s'})
\end{aligned}
$$

Using the transformation of (B.14) we may then write $\vec{A}^h$ in terms of its components in the blade-frame as:

$$
\text{(D.10)}\qquad \vec{A}^h = A^h_{xb}\,\vec{i}_b + A^h_{yb}\,\vec{j}_b + A^h_{zb}\,\vec{k}_b
$$

where:

$$
\text{D.11)}\qquad
\begin{aligned}
A^h_{xb} &= A^h_{xs'}\cos\delta - A^h_{ys'}\sin\delta\\
A^h_{yb} &= (A^h_{xs'}\sin\delta + A^h_{ys'}\cos\delta)\cos\beta - A^h_{zs'}\sin\beta\\
A^h_{zb} &= (A^h_{xs'}\sin\delta + A^s_{ys'}\cos\delta)\sin\beta + A^h_{zs'}\cos\beta
\end{aligned}
$$

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