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EVALUATION OF THERMAL SOAK TIMES

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ABSTRACT

A mathematical model for the heat transfer within the electronics package of a Chaparral missile has been performed. The Grashof number for this configuration was less than 2000 which indicated that the primary mode of heat transfer was conduction. The Vodicka theory for heat conduction in laminated composite media was utilized to obtain the solution for the model.

INTRODUCTION

Modern missiles and various items of equipment required to support them are composed of many complex electrical and electronic systems. Many of these systems, if improperly insulated, can malfunction if exposed to excessively high or low temperatures. The thermal performance of these systems must many times be predicted by a mathematical model.

The purpose of the present investigation is to construct a mathematical model for the heat transfer within the electronics package of a Chaparral missile. For this particular configuration, the Grashof number was less than 2000 which indicated that the heat transfer is essentially all conduction. To model this system, the Vodicka theory for the heat conduction in laminated composite media was utilized. This method enables one to predict both the temperature of and the heat flux through the electronics package of the missile given ei her the surface temperature of the missile or the environmental temperature.

To check the validity of the theory an experimental test was conducted in which the electronics section of a Chaparral missile was placed in the thermal chamber at White Sands Missile Range. The electronics package was instrumented with thermocouples and temperatures were recorded as the chamber temperature was varied. The experimental results show good agreement with the mathematical model indicating that the original assumptions are valid for this situation. This agreement between the experimental results and the theory enables one not only to predict potentially harmful situations to the electronics package due to fluctuations in environmental temperature but to also design thermal tests of reasonable duration.

SYMBOLS

A_{in}, B_{in}, A_i, B_i	constant
C_{pi}	specific heat at constant pressure in the i^{th} section
D_i^2	thermal diffusivity in the i^{th} section
J_o	zero order Bessel function of the first kind
K_i	thermal conductivity in the i^{th} section
$L(r)$	function of r defined by Eq. (8)
ℓ_{ml}, N_m	constants defined by Eqs (5) and (6)
r	spatial coordinate
$T_i(r,t)$	temperature in the i^{th} section
$T_A(t)$	surface temperature
t	time
V	initial temperature distribution
$X_{in}(r)$	eigenfunction
Y_o	zero order Bessel function of the second kind
γ_n	eigenvalue
ρ_i	density in the i^{th} section

ANALYSIS

Experimental evidence [1,2] indicates that the heat transfer in enclosed air spaces is primarily due to conduction if the Grashof number is less than 2000 for vertical plates and less than 1700 for horizontal plates. The particular problem modeled in this paper is this type of situation where free convection is retarded and the heat transfer through the region can be simulated by the heat transfer through a laminated composite.

The heat conduction equation without heat generation for the i^{th} section of k solidly joined cylinders is

$$\frac{1}{D_i^2}\frac{\partial T_i}{\partial t}(r,t) = \frac{1}{r}\frac{\partial}{\partial r}\quad r\frac{\partial T}{\partial r}(r,t) \tag{1}$$

The boundary, internal and initial conditions for the composite are taken to be

$$\text{a)}\ \frac{\partial T_1}{\partial r}(r_1,t) = 0$$

$$\text{b)}\ T_k(V_{k+1},\ t) = T_A(t)$$

$$\text{c)}\ T_i(r,o) = V,\ r_i \le r \le r_{i+1},\ i = 1,\ 2,\ldots,k \tag{2}$$

$$\text{d)}\ T_i(r_{i+1},t) - T_{i+1}(r_{i+1},t)$$

$$\text{e)}\ K_i\frac{\partial T_i}{\partial r}(r_{i+1},t) = K_{i+1}\frac{\partial T_{i+1}}{\partial r}(r_{i+1},t)$$

Boundary condition (2b) assumes the wall temperature is known while (2a) assumes layer 1 is solid. For the actual situation, the first layer is hollow; but for practical applications there is no convenient method for monitoring the temperature at this point so (2a) was deemed a reasonable alternative.

The solution to Eq's (1) and (2) has been obtained [3] previously and is given by

$$T_i(r,t) = \sum_{n=1}^{\infty}\left\{g_n \exp(-\gamma_n^2 t) - \ell_{n1}\int_0^t \frac{dT_A}{d\tau}(\tau)\exp[-\gamma_n^2(t-\tau)]d\tau\right\}X_{in}(r) + L_i(r)\,T_A(t);\ r_i \le r \le r_{i+1},$$

$$i = 1,\ 2,\ \ldots,\ k \tag{3}$$

where

$$g_n = \frac{1}{N_n}\sum_{i=1}^{k}\rho_i C_{pi}\int_{r_i}^{r_{i+1}} rVX_{in}(r)dr - \ell_{n1}T_A(0),\ n = 1,\ 2,\ 3,\ \ldots \tag{4}$$

$$\ell_{n1} = \frac{1}{N_n}\sum_{i=1}^{k}\rho_i C_{pi}\int_{r_i}^{r_{i+1}} r\,L_i(r)X_{in}(r)dr,\ n = 1,\ 2,\ 3,\ \ldots \tag{5}$$

$$N_n = \sum_{i=1}^{k} \rho_i C_{pi} \int_{r_i}^{r_{i+1}} r[X_{in}(r)]^2 dr, \quad n = 1, 2, 3, \ldots \tag{6}$$

$$X_{in}(r) = A_{in} J_o\left(\frac{\gamma_n}{D_i} r\right) + B_{in} Y_o\left(\frac{\gamma_n}{D_i} r\right), \quad \begin{matrix} n = 1, 2, 3, \ldots \\ i = 1, 2, \ldots, k \end{matrix} \tag{7}$$

$$L_i(r) = A_i \ell_n r + B_i \tag{8}$$

The coefficients A_{in}, B_{in} for i = 1, 2, ..., k, n = 1, 2, ... the coefficients A_i and B_i for i - 1, 2, ..., k and the eigenvalues γ_n, n = 1, 2, 3, ... are obtained in a straightforward manner by applying the boundary and internal conditions to the assumed solution. Details are given in [3].

MISSILE SYSTEM

The electronics system of a Chaparral missile can be simulated as a five layer composite. Layer one and layer five are strainless steel, layers two and four are air and layer three is the electronics package. The physical dimensions and thermal properties were taken as

<u>Layer</u>

K(w/m-°K)	16.264	0.0271	9.08×10^{-3}	0.0271	16.264
ρ(gm/cm^3)	7.824	0.00114	0.0818	0.00114	7.824
C_p(cal/gm-°K)	0.11	0.24	0.45	0.24	0.11

$r_1 = 0$, $r_2 = 1.111$ cm, $r_3 = 2.85$ cm, $r_4 - 3.85$ cm, $r_5 = 5.97$ cm,

$r_6 = 6.35$ cm

Proceeding in the same manner as outlined previously, we obtain the solution as

$$T_i(r,t) \sum_{m=1}^{\infty} \left\{ V\ell_m \exp(-\gamma_m^2 t) - \ell_m T_A(t) + \gamma_m^2 \ell_m T_A(t) * \exp(-\gamma_m^2 t) \right\} X_{im}(r) + T_A(t) \tag{9}$$

where

$$\ell_m = -\frac{K_5 r_6}{\gamma_m^2 N_n} \frac{dX_{5m}(r_6)}{dr} \tag{10}$$

REFERENCES

1. M. Jakob, "Free Convection Through Enclosed Plane Gas Layers," Trans. ASME, Vol. 68, p. 189 (1946).

2. M. Jakob, Heat Transfer, Vol. 1, John Wiley & Sons, Inc., New York (1949).

3. G. P. Mulholland and M. H. Cobble, "Diffusion Through Composite Media," Int. J. Heat Mass Transfer, Vol. 15, pp. 147-159 (1972).

APPENDIX A

The eigenvalues for the electronics section of the missile configuration described previously are found by solving the matrix equation

$$\begin{bmatrix} M_{01}(r_2) & -M_{02}(r_2) & -N_{02}(r_2) & & & & & \\ A_1 M_{11}(r_2) & -M_{12}(r_2) & -N_{12}(r_2) & & & & & \\ 0 & M_{02}(r_3) & N_{02}(r_3) & -M_{03}(r_3) & -N_{03}(r_3) & & & \\ 0 & 0 & 0 & -M_{03}(r_4) & -N_{13}(r_3) & & & \\ 0 & 0 & 0 & M_{03}(r_4) & N_{03}(r_4) & & & \\ 0 & 0 & 0 & A_3 M_{13}(r_4) & A_3 N_{13}(r_4) & -M_{14}(r_4) & -N_{04}(r_4) & 0 \\ 0 & 0 & 0 & 0 & 0 & & M_{04}(r_5) & N_{04}(r_5) P_o \\ 0 & 0 & 0 & 0 & 0 & & A_4 M_{14}(r_5) & A_4 N_{14}(r_5) P_1 \end{bmatrix} = 0$$

where

$$M_{ij}(r) = J_i\left(\frac{Y_m}{D_j} r\right)$$

$$N_{ij}(r) = Y_i\left(\frac{Y_m}{D_j} r\right)$$

$$A_i = \frac{K_i}{K_{i+1}} \quad \frac{D_{i+1}}{D_i}$$

$$P_i = \frac{-M_{i5}(r_5) N_{05}(r_6) + M_{05}(r_6) N_{i5}(r_5)}{N_{05}(r_6)}$$

$$N_m = \sum_{\ell=1}^{4} \frac{r_{\ell+1}^2}{2} \Bigg\{ (\rho_\ell C_{p\ell} - \rho_{\ell+1} C_{p\ell+1}) X_{\ell m}^2 (r_{\ell+1}) + \frac{1}{\gamma_m^2} \left(\frac{K_\ell K_{\ell+1} - K_\ell^2}{K_{\ell+1}} \right) \left(\frac{dX_{\ell m}}{dr} (r_{\ell+1}) \right)^2 \Bigg\} + \frac{r_6^2}{2} \frac{K_5}{\gamma_m^2} \left(\frac{dX_{5m}}{dr} (r_6) \right)^2$$

and where the ℓ_m are the roots of the eigenvalue equation given in Appendix A.

Figure 1 is a comparison between the experimentally and anlytically obtained temperatures of the outside surface of the electronics package during a test conducted in the thermal chamber at White Sands Missile Range. The differences between the two curves for $t > 50$ minutes is most likely due to the inexact values of density, specific hear and thermal conductivity used in the model. The package was assumed to be cork at 68°F.

Since the package consists of numerous electronic items of different materials, the true evaluation of these properties would have to be made in a separate test.

The encouraging portion of this figure is for $t > 50$ minutes. In this region where the effects of density and specific heat are less important, the two curves converge. Since the purpose of the study was to evaluate the time necessary for the electronics package to reach a steady-state operating condition, it is felt that this model will accomplish that function. Better temperature profiles can be obtained if more accurate values for the density and specific heat of the electronics package are utilized.

CONCLUSIONS

A mathematical model for the temperature distribution and heat flux across the electronics package of a Chaparral missile has been obtained. This model can be utilized in situations where natural convection currents are suppressed and the heat transfer can be simulated by conduction through laminate composites.

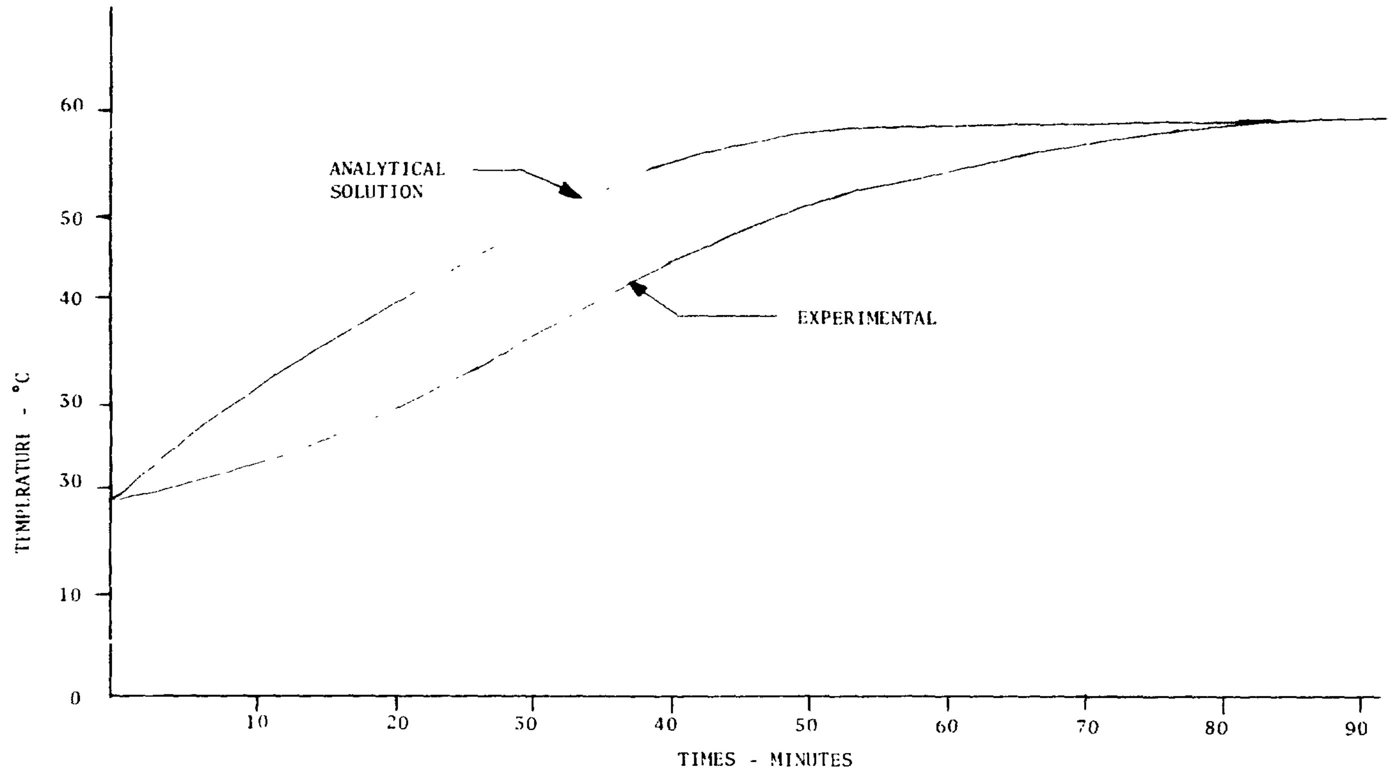

FIGURE 1. TEMPERATURE-TIME HISTORY OF ELECTRONICS PACKAGE