Monitoring Regional Crustal Deformation with Horizontal Geodetic Data

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Abstract. The National Ocean Survey is developing an automated system to derive parameters of horizontal crustal motion from existing geodetic data by the process of least-squares estimation. The estimated parameters will describe crustal motion as a function of geographic position. The system will first be tested in the Imperial Valley region of southern California, using data from 8 individual field projects spanning four decades of time.

Introduction

Global models for tectonic activity hypothesize the existence of rigid plates rotating with constant velocity. In contrast, local crustal motion as observed by geodetic and geophysical instrumentation varies from nearly continuous creep to the stop and go process associated with large earthquakes. To better understand the transition from global to local phenomena, the National Geodetic Survey of the National Ocean Survey is performing several studies of the geodetic data of the past 100 years on a regional level. These studies are designed to establish the pattern of horizontal crustal motion in areas from 100 to 300 km in diameter. This paper describes one of the techniques being used and some preliminary results obtained from a pilot study of the Imperial Valley area in southern California.

The participation of the National Geodetic Survey in crustal motion study is required by the forthcoming redefinition of the North American Horizontal Datum. A new adjustment of the entire U.S. control network will accompany this redefinition, and new positions will be published in 1983 for all stations of the control network. The following arguments are presented to support the geodetic community's need for a better understanding of crustal motion.

1. In any adjustment which incorporates data from different epochs, especially in an area of crustal movement, the observations need to be reduced to a model of the earth which allows geodetic positions to vary with time.
2. A model for crustal motion is needed for predicting the changes in position of published stations. To the extent feasible, parameters of motion could be published in 1983 along with station positions in a fashion similar to star catalogs.
3. A better understanding of crustal motion will help to better define requirements for reobserving disrupted sections of the control network.

The Technique

In most studies of crustal motion the usual technique is to compare different surveys of the same geodetic network, two epochs at a time. The technique to be discussed here will differ out of necessity. The geographic areas of study will generally be larger than the area covered by any one field project, and in most cases the various field projects will only partially overlap one another since most of them were observed to establish geodetic control where it previously did not exist, not for crustal motion study. The basic technique is to estimate parameters describing crustal motion by a simultaneous least-squares adjustment of all the pertinent geodetic data. This is accomplished by introducing into the adjustment a mathematical model which describes station positions as a function of time. The following paragraph describes the model which was used in the Imperial Valley pilot test.

In the model the region of study can consist of one or more subregions. Existing fault lines will usually provide the boundaries between subregions. The latitude ϕ_t and the longitude λ_t of a geodetic station in the i^{th} subregion at time t are given by the formulas.

$$\phi_t = \phi_{t_o} + f_{i,1}\,(t-t_o) + f_{i,3}\,(t-t_o)^2$$

$$\lambda_t = \lambda_{t_o} + f_{i,2}\,(t-t_o) + f_{i,4}\,(t-t_o)^2$$

Here t_o is a fixed time of reference, and $(\phi_{t_o},\lambda_{t_o})$ are the geodetic coordinates of the station at time t_o. Each $f_{i,j}$ for $1 \leq j \leq 4$ is a function over the variables ϕ_{t_o} and λ_{t_o}. In the first applications of the technique these functions will be of the form

$$f_{i,j}(\phi_{t_o},\lambda_{t_o}) = b_{i,j,1} + b_{i,j,2}\,\phi_{t_o} + b_{i,j,3}\lambda_{t_o} + b_{i,j,4}\phi^2_{t_o} + \dots$$

Note that this models the motion as a continuous function of time and a discontinuous function of position with the discontinuities occurring along the boundaries between subregions. Existing horizontal survey data in the form of directions, distances, and azimuths can be input into the adjustment process to obtain the least-squares estimates for the unknown coordinates $(\phi_{t_o},\lambda_{t_o})$ and the unknown coefficients $b_{i,j,k}$.

This technique has several advantages over the standard technique of directly comparing two sets of measurements of the same quantities. It allows for the linking together of neighboring field projects into a single data set even though several years might exist between the times when the individual field projects were observed. It

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allows for the rigorous inclusion of astronomic azimuths which may have been observed separately and the inclusion of data at stations which have been destroyed. Additionally, the model provides a built-in mechanism for interpolating the values of velocity and acceleration over the entire region of study. Finally, the model generalizes the information contained in the data. This last point can be considered a disadvantage as well as an advantage. It is an advantage in so much as the concern is toward regional trends as opposed to local details. For example, local movement phenomena like hillside creep will be smoothed-out. On the other hand, it is a disadvantage in that unmodeled variations in regional motion will also be smoothed-out. For example, motion across an unmodeled, yet active, fault will be interpreted as a continuous function of position. For this reason the standard technique of directly comparing two sets of measurements over the same quantities will never be fully abandoned. Instead it will provide the standards by which to evaluate the accuracy of the mathematical model.

Although information is lost in the process of modeling the motion, a model is desired to provide a clear overall picture. When the inadequacies of a model are identified, the model can be refined. The study of the Imperial Valley data was conducted as a pilot test to evaluate the above model, identify its inadequacies, and suggest how the model might be changed to more accurately reflect the physical situation.

The Data

Of the seismic areas in the United States, the Imperial Valley and its immediate surroundings represent the most frequently observed part of the national network over any geographic area of comparable size. The basic network was observed in 1934. After the El Centro earthquake of May, 1940 (Richter scale magnitude = 7.1) the network was reobserved in 1941 to determine a new set of positions for several geodetic stations. Observations of the basic network were performed in 1954-55 and again in 1967 for the specific purpose of studying post-seismic activity in the area. In the period 1974-76 two field projects along the southern extent of the network were observed as part of the Transcontinental Traverse. This study also includes two minor field projects in the area, a 1942 triangulation survey around the southern half of the Salton Sea and a 1959 highway traverse survey extending parallel and about 10 km north of the Mexico-California border. Figure 1 shows the essential part of the network which was studied and its relationship to the fault system in the area.

Different parts of this data were previously investigated. The results of these investigations can be used to evaluate the performance of the model. Displacement vectors for the region were reported by Meade [1948] for the 1934-1941 period, Whitten [1956] for the 1941-55 period, and Gergen [1978] for the 1941-1976 period. Miller et al. [1970] published dis-

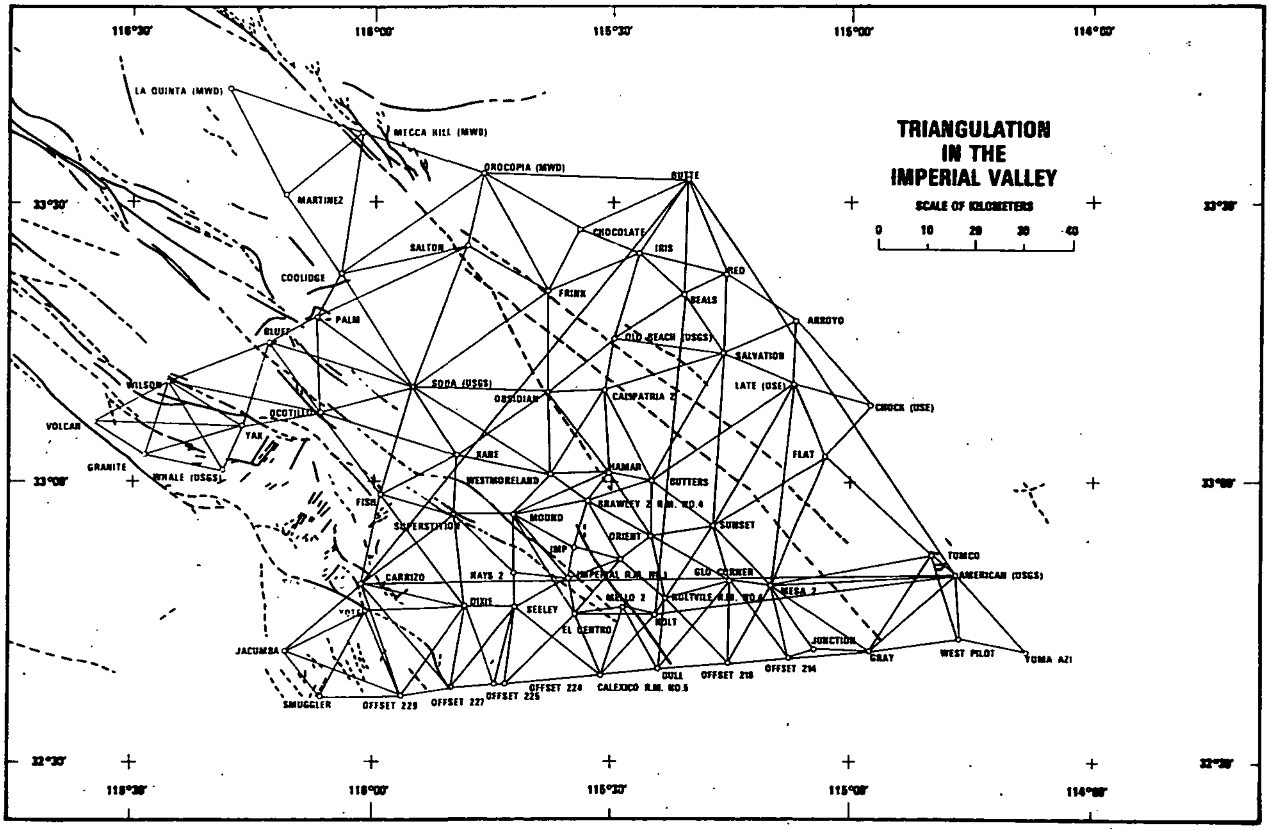

Fig. 1. Triangulation in the Imperial Valley and its relationship to the fault system.

placement vectors and strain components for various periods using appropriate subsets of the 1934-41-55-67 data. A geophysical interpretation of the data was given by Scholz and Fitch [1969] by comparing the 1941-55 data to the dislocation model of Chinnery [1961]. Interpretations of the data were also performed by Savage and Burford [1970], Barker [1976], and Thatcher [1978]. Each of these last three papers analyzed the strain components derived from the 1941-55-67 data by the method of Frank [1966].

Experiments with the Model

The model as programmed for this pilot test allows the user to partition the region of study into three subregions and to solve for 15 coefficients (all terms in ϕ and λ up to degree four) for each of the four polynomials associated with a subregion. This gives the user a total of 180 parameters with which to describe the motion. The experiments are to determine the appropriateness of these parameters. However, it was first necessary to establish the location of active faults. This was accomplished with geological maps and an adjustment of the data to a model which does not include any time parameters. The geological maps located the known faults. The residuals obtained from the adjustment identified which of these faults were the most active. In some instances the maps were ambiguous as to the location of a station relative to the fault. Station B in figure 2 illustrates the problem. These ambiguities were resolved by assuming one sense for the relative motion between opposite sides of the fault and checking whether the angle α at B measured clockwise from A to C is increasing or decreasing with time. In figure 2 right-lateral motion is assumed. Thus, if B were to the left of the fault, then the angle α would decrease in size with time. If B were to the right of the fault, the angle would increase. Note that the situation is reversed if left-lateral motion is assumed.

Once the location of the active faults were incorporated into the model, the data was readjusted several times. The first readjustment revealed an inadequacy of the model in that it could not accommodate large discontinuities in motion as a function of time such as those which occurred along the Imperial fault as a result of the 1940 El Centro earthquake. To continue with the test all pre-1940 data was removed from the data set except for a 1935 astronomic azimuth near station YOTE (see figure 1). This azimuth was retained for better orientation control over time. In retaining the azimuth it is assumed that the distance between the location of this observation and that of the earthquake is sufficiently large that the orientation of the observed line did not change discontinuously at the time of the earthquake. Future mathematical models need a feature to accommodate the relatively instantaneous shifts in position associated with major earthquakes.

With the pre-1940 data removed, the remaining data revealed a velocity pattern corresponding to a general shrinking of the network, indicating a problem with scale. The 1959 highway traverse, the 1967 observations, and the two Transcontinental Traverse projects 1974-76 all have sufficient electronic distance measurements to render good scale control for these epochs. However, closer examination of the 27 observed distances of 1959 indicated that they were too long by an estimated 13 parts per million. These observations were accordingly rescaled for the purpose of this test. Further investigation of these distances is being undertaken.

An adjustment with the rescaled distances revealed a velocity pattern corresponding to a rotation about the fixed station, indicating an orientation problem. The data includes 13 astronomic azimuth observations, one observed in 1935, one in 1967, and the remaining 11 as part of the two Transcontinental Traverse projects 1974-76. These azimuths are not suspected to contain any serious non-random error. Instead the results indicate a case of modeling observational errors as movement. The model is a second degree polynomial in time and the above azimuths essentially represent three epochs, i.e., three points on the graph of network orientation versus time, the minimum required to determine a second degree polynomial. Hence all error in these three epochs of orientation is absorbed into the model. For the first two epochs, 1935 and 1967, the orientation is determined by a single observation. It is not unreasonable for an astronomic azimuth to be in error by one arc second which corresponds to 0.485 meters at distance of 100 km from the fixed station. Further evidence of this effect was revealed by the high correlation coefficients between the estimated parameters which are linear in time and the corresponding parameters which involve the second power of time. Consequently, the data does not allow for the solution of an acceleration term. One way to overcome this weakness in the data would be to enlarge the network so as to include additional orientation control from nearby projects. There is a limit, however, to the effectiveness

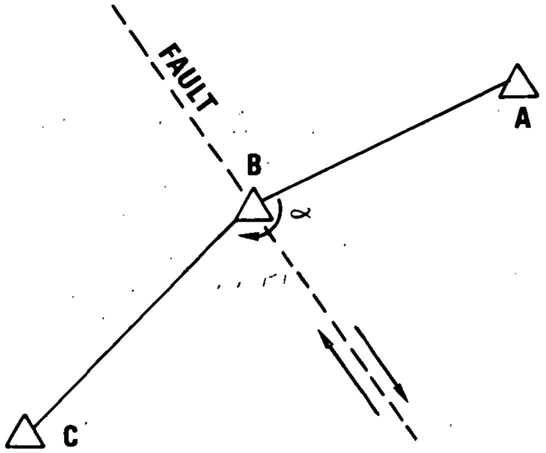

Fig. 2. The position of station B relative to the fault can be established by the direction of change in the angle α with time.

of this technique. The more distant the observations are from the location of interest, the smaller is their relative information content.

With the elimination of all terms involving the second power of time, a solution involving 16 time parameters was obtained. In this solution the polynomial in the ith area is of the form

$$f_{i,j}\ (\phi t_o,\lambda t_o) = b_{i,j,1} + b_{i,j,2}\ (\phi t_o - \bar{\phi}) + b_{i,j,3}\ (\lambda t_o - \bar{\lambda})$$

where $1 \leq i \leq 3$, $1 \leq j \leq 2$ and $(\bar{\phi},\bar{\lambda})$ are the coordinates assigned to the station ORIENT. The constraints $b_{1,1,1} = b_{1,2,1} = 0$ were imposed. This corresponds to the assumption that station ORIENT in subregion 1 did not move with time. The estimated values for several parameters in this solution were below the estimated values of their standard errors. Hence, additional constraints need to be imposed to compensate for the inadequacies of the data. Some experimental adjustments were performed constraining different combinations of weakly determined parameters to specific values. Figure 3 illustrates the velocity vectors relative to station ORIENT obtained in one of these experiments. Here the five constraints $b_{1,2,2} = b_{2,1,1} = b_{3,1,3} = b_{3,2,1} = b_{3,2,3} = 0$ were imposed in addition to fixing station ORIENT. The heavy wavy lines in figure 3 correspond the subregion boundaries input to the solution and dividing the region into three subregions. The error ellipses in figure 3 indicate the 95% confidence limits for the velocity vectors. Note that error ellipses are relative to the origin and depend on the choice of constraints.

Statistical analysis in the form of an F-test indicate that the 11 parameter solution of figure 3 is overconstrained relative to the 16 parameter solution at the 0.01 significance level. A few more adjustments were attempted to find the optimum set of constraints utilizing the statistical concept of fixing a parameter whenever there is insufficient information to significantly estimate its value. Sometimes more realistic constraints can be derived from the physical theory itself. Both the 16 and 11 parameter solutions result in nonsymmetric strain matrices for each subregion. Physically this corresponds to a rotation of the network with time. The average rotation of the network obtained from these solutions is of the order of 0.1 (10^{-6}) radians/yr. Since the estimated standard error in astronomic azimuths is 5.3 (10^{-6}) radians [Strange and Pettey, 1977], this rotation is probably only noise in the 13 observed azimuths. If it is physically plausible that the overall network does not rotate with

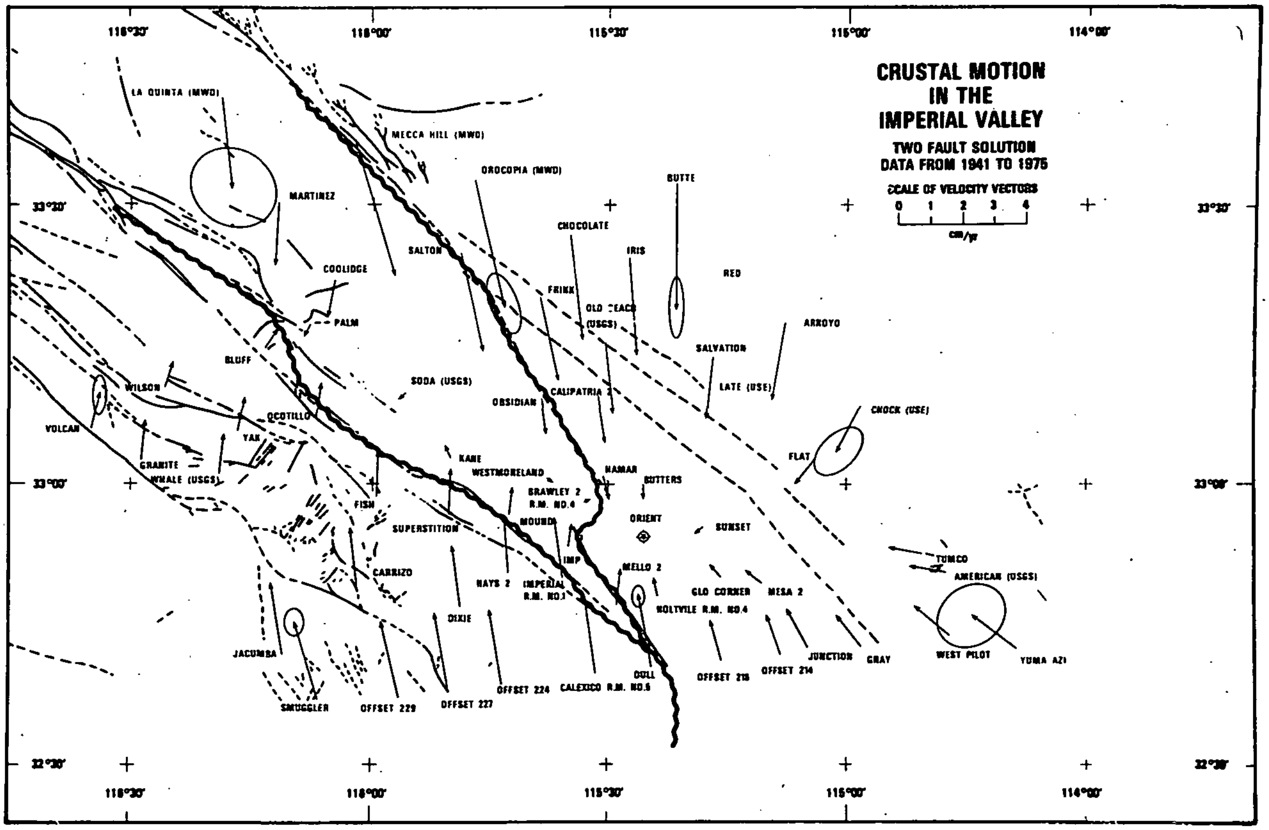

Fig. 3. The velocity vectors relative to station ORIENT as obtained from an 11 parameter solution of the 1941-76 data. The ellipses are the 95% confidence limits, and the heavy wavy lines represent the subregional boundaries supplied in the solution.

time, then a more realistic type of constraint is to restrict some or all of the strain matrices to a symmetric form. Experiments with this type of constraint could not be performed in time to include them in this report.

Evaluation of the Model

The results of the various solutions translate into north-south contraction in the Imperial Valley in addition to right-lateral strike slip motion along the faults. The interpretation of north-south contraction was at first questioned as a possible result of poor scale control in the data. However, the same interpretation was obtained [Savage et al., 1978] from trilateration networks observed by the U.S. Geological Survey between 1972 and 1978. The data used by Savage et al. and the data of this study have no observations in common. Possible corroboration of the north-south contraction hypothesis is also provided by the observed subsidence in the area to the southeast of the Salton Sea (vicinity of station CALIPATRIA in figure 1) relative to the area just north of the Mexico-California boundary [Reese, 1977]. This subsidence is based on three epochs of leveling data spanning the period 1972-76.

To further check the accuracy of the model, the 16 parameter solution is compared in table 1 to the results obtained by Miller et al. [1970] and Savage et al. [1978]. In general the velocity vectors of this study have a more north-south trend than those of the other two studies. However, the difference is not statistically significant. Since a symmetric strain matrix was

TABLE 1. Comparison of velocities relative to OLD BEACH deduced from 1941-67 triangulation [Miller et al., 1970], 1972-78 trilateration [Savage et al., 1978], and the 1941-76 triangulation and trilateration of this study.

Station	Epoch	μ_1 (East) (mm/yr)	μ_2 (North) (mm/yr)
BUTTE	41-67	16	2
	72-78	3 ± 9	-2 ± 12
	41-76	2 ± 8	-15 ± 5
OROCOPIA	41-67	1	1
	72-78	6 ± 4	-4 ± 4
	41-76	4 ± 9	-8 ± 8
ALAMO	41-67	0	3
	72-78	1 ± 2	-1 ± 3
	41-76	0 ± 3	5 ± 3
SODA	41-67	-16	9
	72-78	-3 ± 3	13 ± 4
	41-76	-10 ± 5	18 ± 10
KANE	41-67	-20	19
	72-78	-7 ± 5	16 ± 4
	41-76	-10 ± 7	23 ± 8
FISH	41-67	-15	26
	72-78	-12 ± 5	24 ± 4
	41-76	-11 ± 10	33 ± 12
DIXIE	41-67	-11	33
	72-78	-22 ± 9	27 ± 4
	41-76	-13 ± 14	35 ± 9
OFFSET 225	41-67	-5	43
	72-78	-31 ± 14	22 ± 10
	41-76	-15 ± 18	38 ± 9
CARRIZO	41-67	-15	33
	72-78	-19 ± 8	29 ± 5
	41-76	-13 ± 14	36 ± 13
OFFSET 229	41-67	-16	35
	72-78	-24 ± 14	33 ± 5
	41-76	-16 ± 19	40 ± 12

assumed by Savage et al., the standard errors associated with their values are in general smaller than those of this study.

In table 1 the numbers representing the solution of this study would change by varying the number of parameters which are estimated. Some variations of the model that deserve investigation are the inclusion of the Sand Hill fault which runs from station GRAY northwesterly to station FRINK (figure 1), and the inclusion of a fault line in the vicinity of station OFFSET 227. In addition to varying the allowable coefficients of this model, it is desirable to refine the entire mathematical model so that the estimated parameters correspond closer to physically observable quantities like the elasticity of the crust or the depth of faulting. On the other hand, even if the model were perfect the results of this study could differ from the results of Miller and Savage. The results of Miller are based only on the 1941 and 1967 networks with the assumption that three stations were fixed in time. The results of Savage represent different data over a significantly shorter time span and were obtained by assuming one station and one azimuth fixed in time. Finally, recall that the model is unable to extract acceleration information from this particular data set. Thus it is impossible to check Thatcher's [1978] result that the average velocity across the extent of the fault zone decreased from 82 ± 11 mm/yr for the 1941-54 period to 23 ± 15 mm/yr for the 1954-67 period.

Conclusion

The pilot test in the Imperial Valley demonstrated the advantages of fitting a model to the data. In particular, data from several sources can be assimilated. Observations of scale, orientation, and triangulation which could not be used directly by the technique of comparing two sets of observations over the same quantities have been included in a single data set. In the same manner the model will allow for the merger of classical geodetic observations with data derived from radio interferometry, creepmeters, Doppler, and satellite lasar-ranging observations. However, before embarking on such an ambitious project, a model is sought which corresponds more closely to physical reality. This pilot test was a preliminary step in constructing such a model. It provides a departing point for future models and it reveals to some extent the information content of classical geodetic data.

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